Capital-Skill Complementarity in Firms and in the Aggregate Economy

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October 4, 2022
Capital-skill complementarity?

Hypothesis:

capital-skill complementarity + ↓ equipment prices ⇒ ↑ inequality
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- Conceptually intuitive idea
- Under calibrated aggregate elasticities, accounts for evolution of U.S. skill premium

Krusell, Ohanian, Rios-Rull, & Violante (2000)
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Critiques: hypothesis faces two central critiques

- Identification: aggregate elasticities not identified in aggregate time series
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- Generality: global ↓ equipment price, yet weak cross-country evidence (e.g., France)
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This paper: tackle both issues using theory + empirics using French manufacturing data
This Paper

Micro-to-Macro identification of capital-skill complementarity:

- Credibly estimate substitution elasticities within firms using micro data
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- Aggregate using theory featuring heterogeneous firm-level CRS technologies

equipment price ↓ ⇒ skill premium ↑ ≡ within-firm complementarity

+ cross-firm cov. of capital and skill intensity
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- + cross-firm cov. of capital and skill intensity
- + cross-firm cov. of capital intensity and price shocks

Study French manufacturing (1960-2005) where equipment price and skill premium stable

Moderate capital-skill complementarity: fall in equipment prices predicts 6% rise in skill premium

Without heterogeneity in equipment price shocks, only half as much

Additional questions:
- Impact of equipment prices on the labor share
- Beyond equipment shock (in progress): impact of changes in wage “wedges” across firms
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A Sample of Prior Work

**SBTC:** vast literature measures SBTC as a residual
Katz & Murphy (1992); and numerous followup contributions

**Capital-skill complementarity:** provides empirical proxy
Grilliches (1969); Greenwood & Yorukoglu (1997); Krusell, Ohanian, Rios-Rull, & Violante (2000); and many more

**Aggregate elasticities from micro elasticities:** alternative to time-series identification
Oberfield & Raval (2021); Baqee & Farhi (2019); Lashkari, Bauer, & Boussard (2022)

**Alternative approaches to observing SBTC:** focus on IT, automation, etc
Caroli & Van Reenen (2001); Bresnahan, Brynjolfsson, & Hitt (2002); Akerman, Gaarder, & Mogstad (2015), Acemoglu & Restrepo (2018, 2020, ...); Caunedo, Jaume, & Keller (2021); Adao, Beraja, & Pandalai-Nayar (2022), and many more
Roadmap

Theory

Data and Estimation

Results

Conclusion
Environment: Firms, Production, and Demand

Firms and production:

- Continuum of monopolistically competitive single-prod firms $i \in \mathcal{I}$
- Factor inputs $X_{fi}$ with $f \in \{\ell, h, e\}$: low-skill labor ($X_{\ell i}$), high-skill labor ($X_{hi}$), and equipment ($X_{ei}$)
- Arbitrary CRS technologies given by unit factor demand $(A_{\ell i}, A_{hi}, A_{ei})$ such that $X_{fi} = A_{fi}Y_i$
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Demand:

- Nested consumer demand over outputs of firms (sector $s$ firms $\mathcal{I}_s \subseteq \mathcal{I}$):

$$Y_i = \Phi_i \left( \frac{P_i}{P_s} \right)^{-\epsilon} \left( \frac{P_s}{P} \right)^{-\eta} Y$$

- $\Phi_i$ & $P_i$: firm demand shifter & price
- $P_s$: sector-specific price index
- $P$: aggregate price index
Simplified Environment: Factor Markets

Factor supplies: inelastic aggregate supplies $X_\ell$, $X_h$, and $X_e$

Factor prices equate aggregate supply and demand (assume common factor prices) ($W_\ell \equiv 1$)

$$X_\ell = \sum_i X_{\ell i} \quad X_h = \sum_i X_{hi} \quad X_e = \sum_i X_{ei}$$
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Goal: study shock to $X_e$ that lowers equipment price $W_e$
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Skill premium: $W_h / W_\ell = W_h$
Skill Premium, Equipment Price, and Macro Elasticities of Substitution

Response of skill premium to equipment price shock with common factor prices:

\[
\frac{d \ln (W_h/W_\ell)}{d \ln W_e} = -\frac{\sigma_{le} - \sigma_{he}}{\sigma_{lh}}
\]

Macro (aggregate-level) factor demand elasticity of substitution:

\[
\sigma_{ff'} \equiv \frac{\partial \ln (X_f/X_{f'})}{\partial \ln W_{f'}}
\]
Aggregate elasticity between factors $f$ and $f'$:

$$\sigma_{ff'} = \sum_i \left( \Lambda_{fi} \sigma_{ff',i} - \Lambda_{f'i} \sigma_{ff',i} \right) \theta_{f'i} + \varepsilon \sum_i \left( \Lambda_{f'i} - \Lambda_{fi} \right) \theta_{f'i} - (\varepsilon - \eta) \sum_s \left( \Lambda_{f's} - \Lambda_{fs} \right) \theta_{f's}$$

- **within-firm substitution**
- **cross-firm substitution**
- **cross-sector substitution**

**Factor intensity** (share of all firm’s factor payments that goes to factor $f$)

$$\theta_{fi} \equiv \frac{W_f X_{fi}}{\sum_{f'} W_{f'} X_{f'i}}$$

**Firm size** (share of aggregate factor employment)

$$\Lambda_{fi} \equiv \frac{X_{fi}}{\sum_{f'} X_{f'i}}$$

**Micro (firm-level) elasticity of substitution between factors**

$$\sigma_{ff',i} \equiv \frac{1}{\theta_{f'i}} \frac{\partial \ln \Lambda_{fi}}{\partial \ln W_{f'}}$$
Baseline Environment: Heterogeneous Equipment Prices

Equipment prices: firm-specific price $W_{ei} = W_e T_{ei}$

“Shadow” equipment price: $W_e$

Firm-specific equipment price “wedge”: $T_{ei}$
Baseline Environment: Heterogeneous Equipment Prices

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“Shadow” equipment price: \( W_e \)
Firm-specific equipment price “wedge”: \( T_{ei} \)

Microfoundation: \( X_{ei} \) a CRS aggregate across \( J \) varieties of equipment, \( G_i (X_{ei1}, \cdots, X_{eiJ}) \)

- Firm\times variety-specific price distortions (\( \sim \) iceberg costs) \( \Rightarrow W_{eij} = T_{eij} W_e \)
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Empirical Setting: use customs data to identify firm-level composition of equipment investment

- variety $j \equiv$ equipment product × origin country of product
Skill Premium, Equipment Prices, and Macro Elasticities

Response of skill premium to heterogeneous equipment price shocks:

\[
\frac{d \ln \left( \frac{W_h}{W_\ell} \right)}{d \ln W_e} = - \frac{\sigma_{le}^\omega - \sigma_{he}^\omega}{\sigma_{lh}}
\]
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\]

Heterogeneous equipment price shocks: firm \( i \) experiences price change \( d \ln W_{ei} \)

\[
d \ln W_e \equiv \sum_i \Lambda_{ei}^* d \ln W_{ei} \quad \Lambda_{fi}^* \equiv \frac{W_{fi} X_{fi}}{\sum_{i'} W_{fi'} X_{fi'}}
\]

Generalized aggregate elasticity of substitution w.r.t. equipment price shocks:

\[
\sigma_{fe}^\omega \equiv \frac{\partial \ln \left( \frac{X_f}{X_e} \right)}{\partial \ln W_e}
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Micro to Generalized Macro

Generalized aggregate elasticity of substitution w.r.t. equipment price shocks:

$$\sigma_{fe}^\omega \equiv \frac{\partial \ln (X_f/X_e)}{\partial \ln W_e}$$

From micro to macro:

$$\sigma_{fe}^\omega = \sum_i (\Lambda_{fi}\sigma_{fe,i} - \Lambda_{ei}\sigma_{ee,i}) \theta_i \omega_i + \varepsilon \sum_i (\Lambda_{ei} - \Lambda_{fi}) \theta_i \omega_i + (\varepsilon - \eta) \sum_s (\Lambda_{es} - \Lambda_{fs}) \theta_{es} \omega_{es}$$

where

$$\omega_i \equiv \frac{d \ln W_{ei}}{d \ln W_e} \quad \Rightarrow \quad d \ln W_{ei} = \omega_i \ d \ln W_e$$

$$\omega_{es} \equiv \sum_{i \in I_s} \Lambda_{ei|s}^* \omega_i$$
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Requirements

Elasticities:

- Nested CES demand: $\varepsilon$, $\eta$
- Production function elasticities by firm and factor pair: $\sigma_{f'f,i}$

Equilibrium “shares”:

- Firm-level factor intensities: $\theta_{fi}$
- Firm-level factor employment shares: $\Lambda_{fi}$

Additional requirements for heterogeneous shocks:

- Firm-level factor payment share: $\Lambda^*_{ei}$
- Elasticity of firm-level equipment price change to average: $\omega_{ei}$
Micro Elasticities: CRESH Firm-Level Technology

CRESH firm-level production function:

Hanoch (1971)

\[ \sum_f (Z_{fi}A_{fi})^{\frac{\sigma_f-1}{\sigma_f}} = 1 \]
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Micro-level elasticity of substitution for CRESH firm-level technology (\(\bar{\sigma}_i \equiv \sum_f \theta_{fi} \sigma_f\)):

\[
\sigma_{ff',i} = \frac{\sigma_f \sigma_{f'}}{\bar{\sigma}_i} - \frac{\sigma_f}{\theta_{f,i}} \Pi_{ff'} \Rightarrow \frac{\sigma_{le,i}}{\sigma_{he,i}} = \frac{\sigma_l}{\sigma_h}
\]

Generalizes CES, where \(\sigma_f \equiv \sigma\) for all \(f\)

Relative to nested CES: no \textit{a priori} nesting choice required
Requirements

Elasticities:

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- Production function elasticities: $\sigma_\ell, \sigma_h, \sigma_e$

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Data

Administrative records from the universe of French firms (1994-2007)

**FICUS Data:** firm balance sheet information (based on tax records)
- Value added (Revenue) $R_{it} = P_{it} Y_{it}$
- Total equipment investment of firms
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**DADS Data:** employee-level data matched to employer firms
- Composition-adjusted wage and employment of skill groups $W_{fit}$ and $X_{fit}$ for $f \in \{\ell, h\}$
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**Customs Data:** firm export/import quantities and unit values
- Use import composition to build firm-level equipment stocks and prices \( W_{eit} \) and \( X_{eit} \)
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Data further used to construct instruments needed for identifying micro elasticities
Demand elasticity $\varepsilon$: sectoral CES demand gives firm revenue

$$\Delta r_{it} = -(\varepsilon - 1) \Delta p_{it} + \alpha_{st} + \Delta \phi_{it}$$

- Use firm-level export quantities and prices (five-year differences everywhere)
- **Standard endogeneity concern:** firm-specific demand shifter $\Delta \phi_{it}$ correlated with price $\Delta p_{it}$
Demand elasticity $\varepsilon$: sectoral CES demand gives firm revenue

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- IV for price change $\Delta p_{it}$: import exposure to exchange rate shocks
Estimating Equations #2

Production function elasticities $\sigma_f$: CRESH factor demand implies

$$\Delta x_{eit} - \Delta x_{\ell it} = -\sigma_\ell \left( \Delta w_{eit} - \Delta w_{\ell it} \right) + \left( \frac{\sigma_\ell}{\sigma_c} - 1 \right) \left( \frac{\epsilon}{\epsilon-1} \Delta r_{it} - \Delta x_{eit} \right) + \beta_{\ell it} + \nu_{\ell it}$$

$$\Delta x_{eit} - \Delta x_{hit} = -\sigma_h \left( \Delta w_{eit} - \Delta w_{hit} \right) + \left( \frac{\sigma_h}{\sigma_c} - 1 \right) \left( \frac{\epsilon}{\epsilon-1} \Delta r_{it} - \Delta x_{eit} \right) + \beta_{ht} + \nu_{hit}$$

- **Endogeneity concern**: Residual a function of
  - firm-specific factor-augmenting productivity shocks + demand shifter
  - sectoral price index
Production function elasticities $\sigma_f$: CRESH factor demand implies

$$\Delta x_{eit} - \Delta x_{l_{it}} = -\sigma_l \left( \Delta w_{eit} - \Delta w_{l_{it}} \right) + \left( \frac{\sigma_l}{\sigma_e} - 1 \right) \left( \frac{\epsilon}{\epsilon - 1} \Delta r_{it} - \Delta x_{eit} \right) + \beta_{lt} + \nu_{l_{it}}$$

$$\Delta x_{eit} - \Delta x_{h_{it}} = -\sigma_h \left( \Delta w_{eit} - \Delta w_{h_{it}} \right) + \left( \frac{\sigma_h}{\sigma_e} - 1 \right) \left( \frac{\epsilon}{\epsilon - 1} \Delta r_{it} - \Delta x_{eit} \right) + \beta_{ht} + \nu_{h_{it}}$$

- **Endogeneity concern**: Residual a function of
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  - sectoral price index
- **IV for equipment**: equipment import exposure to bilateral transport cost shocks
- **IV for revenues**: import exposure to origin supply shocks
- **IVs for wages**: local exposure to national sector-level labor demand shocks
## Estimation Results: Demand Elasticity $\varepsilon$

<table>
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<tr>
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<td>0.98 (0.01)</td>
<td>2.92 (0.80)</td>
<td>3.63 (1.350)</td>
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Estimation Results: Production Function Elasticities $\sigma_f$

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<td>$\Delta w_e - \Delta w_h$</td>
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<td>$Pr[\sigma_l = \sigma_h]$</td>
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Joint Estimation
Roadmap

Theory

Data and Estimation

Results

Conclusion
Estimation Results: Firm Size and the Elasticities of Substitution

Elasticities of substitution with equipment $\sigma_{fe,i} \equiv \sigma_f \sigma_e / \bar{\sigma}_i$ slightly fall in firm size
Larger firms have lower intensities in low-skilled labor and higher intensities of equipment.
Aggregation Results: Predicted Skill Premium/Uniform Shock

![Graph showing data and model predictions for predicted skill premium/uniform shock over time from 1997 to 2007. The graph compares data points (black line) with model predictions (red dots) with a shaded area indicating the range of alternative specifications. The x-axis represents years (1997 to 2007), and the y-axis represents the skill premium/uniform shock. The legend includes categories such as Alternative Specifications, Aggregate Elasticities, and Labor Share.](image-url)
Aggregation Results: Predicted Skill Premium/Heterogeneous Shock
### Aggregation Results: Predicted Responses to Equipment Price Shock

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Roadmap

Theory

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Results

Conclusion
Conclusion

Theory: multi-factor/sector model with arbitrary CRS technologies + het. equipment prices

- Skill premium response to factor price shock in terms of generalized agg. substitution elasticities
- Characterized aggregate elasticities in terms of firm-level elasticities + demand elasticity


- Credibly estimate micro elasticities using trade + labor market instruments
- Aggregate to predict response of skill premium to observed fall in equipment price(s)

Moderate agg. capital-skill complementarity: observed shock leads to 6% rise in skill premium
Roadmap

Appendix
Microfoundations for Equipment Wedges

Equipment inputs for firm $i$ is a CRS aggregate across $J$ varieties of equipment capital

$$X_{ei} = G_i (X_{e1i}, \ldots, X_{eij}, \ldots, X_{eij})$$

- Firm $\times$ variety-specific price distortions ($\sim$ iceberg costs) $\Rightarrow$ $W_{eij} = T_{eij} W_{ej}$
- Marginal products bounded above $\Rightarrow$ variety demand has a choke price

$$\frac{\partial G_i}{\partial X_{eij}} \leq B < \infty, \quad \forall i, j$$

Empirical Setting: use customs data to identify firm-level composition of equipment investment

- variety $j \equiv$ equipment product $\times$ origin country of product

back
Composite Factors (Common Labor Wages)

Consider a composite factor defined as a combination of other factors:

\[ X_c = \sum_{f \in \mathcal{F}_c} X_f \]

○ Example: labor \( n \) as a composite of both high and low skilled labor \( \mathcal{F}_n \equiv \{ \ell, h \} \)

Elasticities of Substitution (defined in terms of payments):

\[ \sigma_{cf'} \equiv 1 + \frac{\partial \ln (\theta_c / \theta_{f'})}{\partial W_{f'}} = \sum_{f \in \mathcal{F}_c} \frac{\theta_f}{\theta_c} \sigma_{ff'} \quad \text{where} \quad \theta_c \equiv \sum_{f \in \mathcal{F}_c} \theta_f \]
Macro Elasticities: CES Firm-Level Technology

Micro-level elasticity of substitution for CES firm-level technology:

\[
\sigma_{ff',i} = \begin{cases} 
\sigma, & f \neq f' \\
\sigma \frac{\theta_{fi} - 1}{\theta_{fi}}, & f = f'
\end{cases}
\]

Macro-level elasticities of substitution assuming single sector (\(\varepsilon = \eta\)):

\[
\sigma_{f'f} = \sigma + (\varepsilon - \sigma) \sum_i (\Lambda_{f'i} - \Lambda_{fi}) \theta_{f'i}
\]

\[
\sigma^*_{f'f} = \sigma + (\varepsilon - \sigma) \sum_i (\Lambda^*_{f'i} - \Lambda^*_{fi}) \theta_{f'i}
\]
Macro Elasticities: CRESH Firm-Level Technology

Macro-level elasticities of substitution assuming single sector ($\epsilon = \eta$):

$$\sigma_{ff'} = \sigma_{f'} \left[ 1 - \sum_i \left( \Lambda_{f'i} \frac{\sigma_{f'}}{\sigma_i} - \Lambda_{fi} \frac{\sigma_f}{\sigma_i} \right) \theta_{f'i} \right] + \epsilon \sum_i \left( \Lambda_{f'i} - \Lambda_{fi} \right) \theta_{f'i}$$

$$\sigma_{f^*f'} = \sigma_{f'} \left[ 1 - \sum_i \left( \Lambda_{{f^*}i} \frac{\sigma_{f'}}{\sigma_i} - \Lambda_{{f^*}i} \frac{\sigma_f}{\sigma_i} \right) \theta_{f'i} \right] + \epsilon \sum_i \left( \Lambda_{{f^*}i} - \Lambda_{{f^*}i} \right) \theta_{f'i}$$
Macro Elasticities: Many Factors

Shock to equipment price, holding factor wedges and other factor supplies constant

Let $\mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{\ell, e\}}$ be vector of shadow prices, and define matrix and vectors ($\sigma_{ff} \equiv 0$):

$$
\Sigma \equiv (\sigma_{ff'})_{f, f' \in \mathcal{F}/\{\ell, e\}} \quad \sigma_f. \equiv (\sigma_{f f'})_{f \in \mathcal{F}/\{\ell, e\}} \quad \sigma.f \equiv (\sigma_{f' f})_{f \in \mathcal{F}/\{\ell, e\}}
$$
Macro Elasticities: Many Factors

Shock to equipment price, holding factor wedges and other factor supplies constant

Let \( \mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{\ell,e\}} \) be vector of shadow prices, and define matrix and vectors \((\sigma_{ff} \equiv 0)\):

\[
\Sigma \equiv (\sigma_{ff'})_{f,f' \in \mathcal{F}/\{\ell,e\}} \quad \sigma_f. \equiv (\sigma_{ff'})_{f \in \mathcal{F}/\{\ell,e\}} \quad \sigma_{.f} \equiv (\sigma_{f.f})_{f \in \mathcal{F}/\{\ell,e\}}
\]

Response of shadow skill premium:

\[
\frac{d \ln \mathbf{W}}{d \ln W_e} = (1 \sigma'_\ell. - \Sigma)^{-1}(\sigma_{.e} - \sigma_{\ell e})
\]
Macro Elasticities: Many Factors

Shock to equipment price, holding factor wedges and other factor supplies constant

Let \( \mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{l,e\}} \) be vector of shadow prices, and define matrix and vectors (\( \sigma_{ff} \equiv 0 \)):

\[
\Sigma^* \equiv (\sigma^*_{ff'})_{f,f' \in \mathcal{F}/\{l,e\}} \quad \sigma_f^* \equiv (\sigma^*_{ff'})_{f \in \mathcal{F}/\{l,e\}} \quad \sigma^* \equiv (\sigma^*_{ff'})_{f \in \mathcal{F}/\{l,e\}}
\]

Response of shadow skill premium:

\[
\frac{d \ln \mathbf{W}}{d \ln W_e} = (\mathbf{1} \sigma'_e - \Sigma)^{-1} (\sigma_e - \sigma_{le})
\]

Response of observed skill premium:

\[
\frac{d \ln \Psi}{d \ln W_e} = \frac{d \ln \mathbf{W}}{d \ln W_e} + \sigma^*_{he} - \sigma^*_{le} + (\sigma^*_{h} - \sigma^*_{l})' \frac{d \ln \mathbf{W}}{d \ln W_e}
\]
Within and Between-Firm Substitution: Examples

Response of the skill premium to an equipment price shock with uniform factor prices

\[
\frac{d \ln (W_h/W_e)}{d \ln W_e} = - \frac{\sigma_{le} - \sigma_{he}}{\sigma_{lh}}
\]
Within and Between-Firm Substitution: Examples

Response of the skill premium to an equipment price shock with uniform factor prices

\[ \frac{d \ln \left( \frac{W_h}{W_e} \right)}{d \ln W_e} = -\frac{\sigma_{le} - \sigma_{he}}{\sigma_{\ell h}} \]

Only within-firm: homogenous firms with a CRESH production function:

\[ \frac{d \ln \left( \frac{W_h}{W_e} \right)}{d \ln W_e} = -\frac{\sigma_{le,i} - \sigma_{he,i}}{\sigma_{\ell h,i}} = -\frac{\sigma_{l} - \sigma_{h}}{\sigma_{\ell} \sigma_{h}} \sigma_{e} \]
Within and Between-Firm Substitution: Examples

Response of the skill premium to an equipment price shock with uniform factor prices

\[
\frac{d \ln (W_h/W_e)}{d \ln W_e} = -\frac{\sigma_{le} - \sigma_{he}}{\sigma_{lh}}
\]

Only within-firm: homogenous firms with a CRESH production function:

\[
\frac{d \ln (W_h/W_e)}{d \ln W_e} = -\frac{\sigma_{le,i} - \sigma_{he,i}}{\sigma_{lh,i}} = \frac{-\sigma_{l} - \sigma_{h}}{\sigma_{l} \sigma_{h}} \sigma_{e}
\]

Only cross-firm: heterogenous firms with CES production functions \((\sigma_f \equiv \sigma)\) and single sector:

\[
\frac{d \ln (W_h/W_e)}{d \ln W_e} = -\frac{(\varepsilon - \sigma) \sum_i (\Lambda_{hi} - \Lambda_{li}) \theta_{ei}}{\sigma + (\varepsilon - \sigma) \sum_i (\Lambda_{hi} - \Lambda_{li}) \theta_{hi}}
\]
Macro Elasticities: Many Factors

Shock to equipment prices, holding other factor wedges and supplies constant.

Let $\mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{\ell,e\}}$ be vector of shadow prices, and define matrix and vectors ($\sigma_{ff} \equiv 0$):

$$
\Sigma^\omega \equiv (\sigma^\omega_{ff'})_{f,f' \in \mathcal{F}/\{\ell,e\}}, \quad \sigma^\omega_f \equiv (\sigma^\omega_{ff'})_{f \in \mathcal{F}/\{\ell,e\}}, \quad \sigma^\omega \cdot \equiv (\sigma^\omega_{ff'})_{f \in \mathcal{F}/\{\ell,e\}}
$$
Macro Elasticities: Many Factors

Shock to equipment prices, holding other factor wedges and supplies constant

Let $\mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{\ell,e\}}$ be vector of shadow prices, and define matrix and vectors ($\sigma_{ff} \equiv 0$):

$$\Sigma^\omega \equiv (\sigma^\omega_{f'f})_{f,f' \in \mathcal{F}/\{\ell,e\}} \quad \sigma^\omega_{f} \equiv (\sigma^\omega_{f'f})_{f \in \mathcal{F}/\{\ell,e\}} \quad \sigma^\omega_{f'} \equiv (\sigma^\omega_{f'f})_{f \in \mathcal{F}/\{\ell,e\}}$$

Response of shadow skill premium:

$$d \ln \mathbf{W} = \left[ \mathbf{1} (\sigma^\omega_{\ell'})' - \Sigma^\omega \right]^{-1} \left[ (\sigma^\omega_{\ell e} - \sigma^\omega_{\ell e}) \, d \ln \mathbf{W}_e + \sigma^\omega_{\ell} \, d \ln \mathbf{T}_{\ell} - \sigma^\omega_{\ell h} \mathbf{1}_h \, d \ln \mathbf{T}_{h} \right]$$
Macro Elasticities: Many Factors

Shock to equipment prices, holding other factor wedges and supplies constant

Let \( \mathbf{W} \equiv (W_f)_{f \in \mathcal{F}/\{\ell,e\}} \) be vector of shadow prices, and define matrix and vectors (\( \sigma_{ff} \equiv 0 \)):

\[
\Sigma^{*,\omega} \equiv (\sigma^{*,\omega}_{ff'})_{f,f' \in \mathcal{F}/\{\ell,e\}}, \quad \sigma^{*,\omega}_{f.} \equiv (\sigma^{*,\omega}_{ff'})_{f \in \mathcal{F}/\{\ell,e\}}, \quad \sigma^{*,\omega}_{.f} \equiv (\sigma^{*,\omega}_{f'f})_{f \in \mathcal{F}/\{\ell,e\}}
\]

Response of shadow skill premium:

\[
d \ln \mathbf{W} = \left[ \mathbf{1} (\sigma^{*,\omega}_{.\ell}') - \Sigma^{\omega} \right]^{-1} \left[ (\sigma^{\omega}_{.e} - \sigma^{\omega}_{.\ell e}) d \ln \mathbf{W}_e + \sigma^{\omega}_{.\ell} d \ln \mathbf{T}_\ell - \sigma^{\omega}_{.\ell h} \mathbf{1}_h d \ln \mathbf{T}_h \right]
\]

Response of observed skill premium:

\[
d \ln \Psi = d \ln \mathbf{W}_h + (\sigma^{*,\omega}_{.he} - \sigma^{*,\omega}_{.\ell e}) d \ln \mathbf{W}_e + (\sigma^{*,\omega}_{.h.} - \sigma^{*,\omega}_{.\ell .})' d \ln \mathbf{W} + \sigma^{*,\omega}_{.\ell h} d \ln \mathbf{T}_h - \sigma^{*,\omega}_{.h \ell} d \ln \mathbf{T}_\ell
\]
### Summary Statistics

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Data: Skill Groups, Wages, and Employment

Skill groups: assign workers to skill groups based on occupation
Caliendo et al. (2015), Caluccio et al. (2015)
- High-skilled: Managers, Middle managers and professionals, Qualified workers
- Low-skilled: Clerks, Blue-collar

Change in firm-skill-specific wages:
- Estimate firm-level wage changes by running worker-level wage regression on stayers:

\[ w_{ji,t} - w_{ji,t-1} = \beta'_t \mathbf{X}_{jit} + F_{E_{fit}} + e_{jit} \]

\[ \Delta w_{fit} \equiv \sum_{\tau=0}^{4} F_{E_{fit-t}} \]

Worker-\( j \) characteristics \( \mathbf{X}_{jit} \): age, sex, 2-digit occupation category belonging to skill type \( f \)
- Find firm-level change in demand for each skill type by deflating wage bill change with wage change
Data: Equipment Data

Equipment investment: infer using change in the book value of two asset types in FICUS-BRN

- Machinery, equipment and tools (AR - Installations techniques, matriel et outillage industriels)
- Other tangible fixed assets (AT - Autres immobilisations corporelles)

Sources of equipment investment: use imports information in customs data:

- BEC codes: Imported Capital goods–except transport equipment (41) & Parts and accessories (42)
- Define domestic equipment investment as the BRN total minus the customs total

Prices:

- Imported: unit values at firm-year-origin-product level
- Domestic: INSEE series at sector-year-asset level (transport, hardware, & other machinery)
Data: Equipment Stocks and Prices

Firm-level Price Index for Equipment Investment:

- Sato-Vartia index $P_{it}^{e}$ aggregates price change across origin/products (domestic $\equiv$ one product)
- Quantity of equipment investment $Q_{it}^{e} = I_{it}^{e}/P_{it}^{r}$ with $P_{it}^{e} \equiv \prod_{\tau=1}^{t} P_{i\tau}^{e}$

Perpetual Inventory Method:

$$X_{it}^{e} = \begin{cases} \frac{1}{\delta_{st}^{e}} \sum_{\tau=0}^{T_{i}-1} Q_{i\tau}^{e} / T_{i}, & \text{if } t = 0, \\ (1 - \delta_{st}^{e})Q_{it-1}^{e} + Q_{it}^{e}, & \text{if } t > 0, \end{cases}$$

with $\delta_{it}^{e}$ sector-specific deprecation rate (KLEMS), $T_{i} - 1$ number of periods firm in data

User Cost of Equipment:

$$W_{eit} = P_{it}^{e} \left( R_{t}^{e} + \delta_{st}^{e} - \frac{p_{s, t+1}^{e} - p_{s, t-2}^{e}}{3} \right)$$
Instruments: RER Shocks As Cost Shifters

Instrument for Firm-level Prices: shifts marginal costs & uncorrelated with demand shocks $\Delta \phi_{it}$

$$\Delta RER_{it} = \sum_c \frac{M_{ci,t-5}}{M_{i,t-5}} \Delta \ln \left( \frac{NER_{ct}}{NER_{FR,t}} \cdot \frac{CPI_{ct}}{CPI_{FR,t}} \right)$$

- $M_{ci,t}$: imports of firm $i$ from country $c$ in period $t$
- $NER_{ct}$: nominal exchange rate (euros per country $c$ currency)
- $CPI_{ct}$: consumer price index in country $c$ in period $t$

Real depreciation of country $c$ currency relative to euro raises $\Delta RER_{it}$ and lowers $\Delta p_{it}$
Instruments: Transport Cost Shocks as Equipment Price Shifters

Instrument for Equipment Prices: shifts costs of importing equipment goods

Hummels et al. (2014)

\[ \Delta ET C_{it} = \sum_c \sum_{k \in K_e} \frac{M_{cki,t-5}^e}{M_{i,t-5}^e} \Delta \ln TC_{ckt} \]

- \( M_{cki,t-5}^e \): equipment imports of firm \( i \) from country \( c \) in period \( t \)
- \( \Delta \ln TC_{ckt} \): change in transport costs predicted due to oil/jet fuel price shocks

Transport costs \( TC_{ckt} \) predicted based on:

- Distance to country of origin \( c \)
- Product \( k \) frequency of the modes of transport
- Elasticity of mode of transport charges to oil/fuel prices
Instruments: World Supply Shocks as Value Added Shifters

Instrument for Firm-Level Value Added: shifts from revenues

Hummels et al. (2014)

\[ \Delta WES_{it} = \sum_c \sum_{k \in K_i} \frac{M_{cki,t-5}}{M_{i,t-5}} \Delta \ln Exp_{ckt} \]

- \( \Delta \ln Exp_{ckt} \): growth of exports of country \( c \) in product \( k \) in period \( t \) to all countries except France
- \( M_{cki,t-5} \): intermediate good imports of firm \( i \) from country \( c \) in period \( t \)

Productivity growth in origin uncorrelated with demand/factor augmenting productivity shocks
Instruments: Shift-Share Instruments for Local Wages

Generalized Bartik IV: shifts firm-level skill premium

$$\Delta SSW_{it} = \sum_r \sum_s \left( \frac{X_{ni,t-5}^r}{X_{ni,t-5}} \right) \cdot \left( \frac{X_{ns,t-5}^r}{X_{n,t-5}} \right) \cdot \left( \frac{X_{hs,t-5}^r}{X_{f,s,t-5}} \right) \cdot \Delta \ln GO_{st}$$

- $X_{ni,t}^r$: firm $i$ employment in region $r$ at time $t$ (firm exposure to local wage shocks)
- $X_{ns,t}^r$: region $r$ employment in sector $s$ at time $t$
- $X_{f,s,t}^r$: region $r$ employment of skill-type $f$ in sector $s$
- $\Delta \ln GO_{st}$: growth in French gross output of sector $s$

National expansion in a sector that is skill intensive in a given region raises local skill premium
Estimation Results: Demand Elasticity $\varepsilon$ with Alternative IV

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Only use intermediate inputs in the construction $\Delta RER_{it}$
### Estimation Results: Joint Estimation of $\varepsilon$ and $\sigma_f$

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</table>
Aggregation Results: Predicted Skill Premium/Uniform Shock

Alternative Specifications

Baseline, $\epsilon = 2.3$, $\epsilon = 3.6$, Year-Sector FEs
Measuring Wedges

Firm-specific wages $W_{fi} \equiv W_f T_{fi}$ ($f \in \{\ell, h\}$) give us wedges $T_{fi}$ subject to multiplicative factor

Shadow wage $\widehat{W}_{ft} \equiv \widehat{W}_{ft} W_{ft}$ in euros $\Rightarrow$ observed wages $\widehat{W}_{fi,t} \equiv \widehat{W}_{ft, t} T_{fi,t}$ for $f \in \{\ell, h\}$

Assumption: two residuals (uniform wedge shocks) $\overline{T}_{f,t} \equiv \frac{1}{I} \sum_i T_{fi,t}$

- Observed unweighted mean wage rate
  \[
  \overline{W}_{f,t} \equiv \frac{1}{I} \sum_i \widehat{W}_{fi,t} = \widehat{W}_{f,t} \times \overline{T}_{f,t} \quad f \in \{\ell, h\}
  \]

- Observed wedge shocks
  \[
  \widehat{T}_{fi,t} \equiv \frac{\widehat{W}_{fi,t}}{\widehat{W}_{f,t}} = \frac{T_{fi,t}}{\overline{T}_{f,t}} \quad f \in \{\ell, h\}
  \]

- Observed numeraire $\overline{W}_{\ell,t} \equiv W_{\ell,t} \equiv 1 \Rightarrow \overline{W}_{h,t} \equiv W_{h,t}$
Aggregation Results: Heterogenous Price Shocks

Equipment Wedges ($\omega_{ei}$)
Aggregation Results: Heterogenous Price Shocks

Difference in the Change in Labor Wedges ($\omega_{hi} - \omega_{li}$) by Low-Skilled Aggregate Payment Share ($\Lambda_{li}^*$)
Aggregation Results: Predicted Labor Share/Uniform Shock

![Graph showing labor share predictions over time]

- Data
- Model Prediction
Aggregation Results: Predicted Labor Share/Uniform Shock

Alternative Specifications

- Baseline
- $\epsilon = 2.3$
- $\epsilon = 3.6$
- Year-Sector FE
Aggregation Results: Predicted Labor Share/Heterogeneous Shock

![Graph showing the predicted labor share/heterogeneous shock with data, uniform shock, and equipment shocks over the years 1997 to 2007.](image-url)
Aggregation Results: Predicted Responses to Equipment Price Shock

<table>
<thead>
<tr>
<th></th>
<th>Homogeneous Shock</th>
<th>Heterogeneous Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) ( \Delta \ln \theta_n/(1 - \theta_n) )</td>
<td>(2) ( \Delta \ln \theta_n/(1 - \theta_n) )</td>
</tr>
<tr>
<td>Equipment</td>
<td>-0.211</td>
<td>-0.169</td>
</tr>
<tr>
<td></td>
<td>[-0.449, -0.055]</td>
<td>[-0.367, -0.051]</td>
</tr>
<tr>
<td>Observed</td>
<td>-0.171</td>
<td>-0.171</td>
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</table>
### Predicted Change in the Skill Premium: Decomposition

<table>
<thead>
<tr>
<th>Category</th>
<th>Equipment</th>
<th>High-skilled</th>
<th>Low-skilled</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within-firm</td>
<td>0.001</td>
<td>-0.062</td>
<td>0.041</td>
<td>-0.019</td>
</tr>
<tr>
<td>Cross-firm reallocation</td>
<td>0.059</td>
<td>-0.050</td>
<td>0.009</td>
<td>0.018</td>
</tr>
<tr>
<td>Cross-sectoral reallocation</td>
<td>0.003</td>
<td>0.008</td>
<td>-0.001</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.064</strong></td>
<td><strong>-0.104</strong></td>
<td><strong>0.049</strong></td>
<td><strong>0.009</strong></td>
</tr>
<tr>
<td>Cumulated shock</td>
<td>-0.416</td>
<td>0.094</td>
<td>0.051</td>
<td></td>
</tr>
</tbody>
</table>
Micro Elasticities of Substitution

Factor intensity: share of all firm i’s factor payments that goes to factor f

\[ \theta_{fi} \equiv \frac{W_{fi}X_{fi}}{\sum_{f'} W_{f'i}X_{f'i}} \]
Micro Elasticities of Substitution

Factor intensity: share of all firm $i$’s factor payments that goes to factor $f$

$$\theta_{fi} \equiv \frac{W_{fi}X_{fi}}{\sum_{f'} W_{f'i}X_{f'i}}$$

Micro (firm-level) elasticity of substitution:

- Firm-specific CRS technology given by unit factor demand $(A_{ki}, A_{hi}, A_{ei})$ such that $X_{fi} = A_{fi}Y_i$
- Factor demand elasticity of substitution:

$$\sigma_{f'j,i} \equiv \frac{1}{\theta_{fi}'} \frac{\partial \ln A_{fi}}{\partial \ln W_{f'}}$$
From Micro to Macro Elasticities

Aggregate elasticities $\sigma_{ff'}$ and $\sigma^*_{ff'}$ between factors $f$ and $f'$:

$$
\sigma_{ff'} = \sum_i \left( \Lambda_{f_i} \sigma_{f'f',i} - \Lambda_{f'f_i} \sigma_{f'f',i} \right) \theta_{f'i} + \epsilon \sum_i \left( \Lambda_{f'i} - \Lambda_{f_i} \right) \theta_{f'i} - (\epsilon - \eta) \sum_s \left( \Lambda_{f's} - \Lambda_{f_s} \right) \theta_{f's}
$$

$$
\sigma^*_{ff'} = \sum_i \left( \Lambda^*_{f_i} \sigma_{f'f',i} - \Lambda^*_{f'f_i} \sigma_{f'f',i} \right) \theta_{f'i} + \epsilon \sum_i \left( \Lambda^*_{f'i} - \Lambda^*_{f_i} \right) \theta_{f'i} - (\epsilon - \eta) \sum_s \left( \Lambda^*_{f's} - \Lambda^*_{f_s} \right) \theta_{f's}
$$

- within-firm substitution
- cross-firm substitution
- cross-sector substitution
From Micro to Macro Elasticities

Aggregate elasticities $\sigma_{ff'}$ and $\sigma^*_{ff'}$ between factors $f$ and $f'$:

\[
\sigma_{ff'} = \sum_i (\Lambda_{fi} \sigma_{f'i'i},i - \Lambda_{f'i'i},i - \Lambda_{f'i'i},i) \theta_{f'i'i} + \varepsilon \sum_i (\Lambda_{f'i'i} - \Lambda_{f'i}) \theta_{f'i'i} - (\varepsilon - \eta) \sum_s (\Lambda_{f's'i} - \Lambda_{f'i}) \theta_{f's'i'}
\]

\[
\sigma^*_{ff'} = \sum_i (\Lambda^*_{fi} \sigma^*_{f'i'i},i - \Lambda^*_{f'i'i},i - \Lambda^*_{f'i'i},i) \theta_{f'i'i} + \varepsilon \sum_i (\Lambda^*_{f'i'i} - \Lambda^*_{f'i}) \theta_{f'i'i} - (\varepsilon - \eta) \sum_s (\Lambda^*_{f's'i} - \Lambda^*_{f'i}) \theta_{f's'i'}
\]

within-firm substitution  \hspace{2cm} cross-firm substitution  \hspace{2cm} cross-sector substitution

Aggregate generalized elasticities $\sigma^\omega_{ff'}$ and $\sigma^*_{ff'}^\omega$ between factors $f$ and $f'$:

\[
\sigma^\omega_{ff'} = \sum_i (\Lambda_{fi} \sigma_{f'i'i},i - \Lambda_{f'i'i},i - \Lambda_{f'i'i},i) \theta_{f'i'i} \omega_{f'i'i} + \varepsilon \sum_i (\Lambda_{f'i'i} - \Lambda_{f'i}) \theta_{f'i'i} \omega_{f'i'i} - (\varepsilon - \eta) \sum_s (\Lambda_{f's'i} - \Lambda_{f'i}) \theta_{f's'i} \omega_{f's'i'}
\]

\[
\sigma^*_{ff'}^\omega = \sum_i (\Lambda^*_{fi} \sigma^*_{f'i'i},i - \Lambda^*_{f'i'i},i - \Lambda^*_{f'i'i},i) \theta_{f'i'i} \omega_{f'i'i} + \varepsilon \sum_i (\Lambda^*_{f'i'i} - \Lambda^*_{f'i}) \theta_{f'i'i} \omega_{f'i'i} - (\varepsilon - \eta) \sum_s (\Lambda^*_{f's'i} - \Lambda^*_{f'i}) \theta_{f's'i} \omega_{f's'i'}
\]
Heterogeneous Equipment Price and Labor Wedge Shocks

Firm-level shares of aggregate factor demand and payment:

\[
\Lambda_{fi} \equiv \frac{X_{fi}}{\sum_{i'} X_{fi'}} \quad \Lambda^*_f \equiv \frac{W_{fi}X_{fi}}{\sum_{i'} W_{fi'}X_{fi'}}
\]

Heterogeneous equipment price shocks: firm \(i\) experiences price change \(d \ln W_{ei}\)

\[
d \ln \overline{W}_e \equiv \sum_i \Lambda^*_{ei} d \ln W_{ei} \quad \omega_{ei} \equiv \frac{dW_{ei}}{dW_e} \quad \Rightarrow \quad d \ln W_{ei} = \omega_{ei} d \ln \overline{W}_e
\]
Heterogeneous Equipment Price and Labor Wedge Shocks

Firm-level shares of aggregate factor demand and payment:

\[
\Lambda_{fi} \equiv \frac{X_{fi}}{\sum_{i'} X_{f'i'}} \\
\Lambda^*_{fi} \equiv \frac{W_{fi}X_{fi}}{\sum_{i'} W_{f'i'}X_{f'i'}}
\]

Heterogeneous equipment price shocks: firm \(i\) experiences price change \(d \ln W_{ei}\)

\[
d \ln \overline{W}_e \equiv \sum_i \Lambda^*_{ei} d \ln W_{ei} \\
\omega_{ei} \equiv \frac{dW_{ei}}{dW_e} \quad \Rightarrow \quad d \ln W_{ei} = \omega_{ei} d \ln \overline{W}_e
\]

Shocks to labor wedges: firm \(i\) experiences shock to its labor wedges \(dT_{fi}\) for \(f \in \{\ell, h\}\):

\[
d \ln T_f \equiv \sum_i \Lambda^*_{f'i} d \ln T_{fi} \\
\omega_{fi} \equiv \frac{dT_{fi}}{dT_f} \quad \Rightarrow \quad d \ln T_{fi} = \omega_{fi} d \ln T_f
\]
## Aggregation Results: Aggregate Elasticities

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial \omega_h}{\partial \omega_c}$</td>
<td>-0.054</td>
<td>-0.045</td>
<td>-0.060</td>
<td>-0.084</td>
</tr>
<tr>
<td></td>
<td>[-0.093, -0.013]</td>
<td>[-0.074, -0.012]</td>
<td>[-0.101, -0.011]</td>
<td>[-0.168, -0.013]</td>
</tr>
<tr>
<td>$\sigma_{he}^*$</td>
<td>1.323</td>
<td>1.217</td>
<td>1.450</td>
<td>1.274</td>
</tr>
<tr>
<td>$\sigma_{le}^*$</td>
<td>1.379</td>
<td>1.263</td>
<td>1.515</td>
<td>1.359</td>
</tr>
<tr>
<td>$\sigma_{lh}^*$</td>
<td>1.046</td>
<td>1.013</td>
<td>1.091</td>
<td>1.020</td>
</tr>
<tr>
<td>$\frac{1}{1-\eta_c} \frac{d \ln \theta_c}{d \omega_c}$</td>
<td>0.359</td>
<td>0.244</td>
<td>0.494</td>
<td>0.325</td>
</tr>
<tr>
<td></td>
<td>[0.091, 0.762]</td>
<td>[.]</td>
<td>[.]</td>
<td>[.]</td>
</tr>
<tr>
<td>$\sigma_{ne}^*$</td>
<td>1.350</td>
<td>1.239</td>
<td>1.482</td>
<td>1.315</td>
</tr>
<tr>
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<td>[1.090, 1.737]</td>
<td>[1.002, 1.549]</td>
<td>[1.150, 2.774]</td>
<td>[0.806, 2.357]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>2.92</td>
<td>2.321</td>
<td>3.627</td>
<td>2.92</td>
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<td>$\eta$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$\sigma_{l}$</td>
<td>0.968</td>
<td>0.969</td>
<td>0.967</td>
<td>1.001</td>
</tr>
<tr>
<td>$\sigma_{h}$</td>
<td>0.841</td>
<td>0.864</td>
<td>0.824</td>
<td>0.798</td>
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<tr>
<td>$\sigma_{e}$</td>
<td>1.111</td>
<td>1.092</td>
<td>1.126</td>
<td>1.014</td>
</tr>
</tbody>
</table>