Capital-Skill Complementarity in Firms and in the Aggregate Economy

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Abstract

We study capital-skill complementarity in a multi-sector theory of aggregate technology featuring firms with arbitrary and heterogeneous multi-factor production functions. Importantly, the theory allows for heterogeneity in factor prices across firms. We characterize the elasticity of the aggregate skill premium to the price of equipment capital in terms of firm-level elasticitites of substitution across factors, the elasticities of substitution across firms and sectors, and the distribution of factor intensities across firms. Using micro data from France, we estimate the two sets of elasticities assuming a firm-level production function featuring heterogeneous cross-factor elasticities. The model predicts that the observed fall in equipment prices contributes to a sizable rise in the skill premium (around 6%), implying a modest degree of capital-skill complementarity. Using our framework we show how this rise is mostly counteracted by the relative decline of the skill premium among firms with higher skill intensities in France.

1 Introduction

A vast literature concludes that capital-skill complementarity plays a substantial role in generating skill-biased technical change and increasing inequality. The intellectual foundation of this literature—first formalized by Griliches (1969) and implemented by Krusell et al. (2000)—is an aggregate production function featuring capital equipment and two skill groups of labor. If equipment is relatively more complementary with skilled than unskilled labor, then declining equipment prices will increase the *skill premium* (the relative wage of more to less skilled workers). The framework is conceptually attractive and has proven successful in accounting for the evolution of the skill premium in the United States given particular elasticities between equipment and skilled and unskilled labor Krusell et al. (2000).

Despite its virtues, the capital-skill complementarity hypothesis faces two central critiques. First, the key aggregate elasticities of substitution that determine the extent of capital-skill complementarity are not identified in the aggregate time series in the presence of any other mechanism that generates a trend in the skill-bias of technical change (Acemoglu, 2002). Hence, whether capital is complementary with skilled labor at the aggregate level remains an open question. Second, while the decline in equipment prices is global, cross-country evidence supportive of capital-skill complementarity, is not particularly strong (see, e.g., Duffy et al., 2004; Henderson, 2009). For instance, as we will discuss below, the skill premium in a major developed economy such as France remains stable despite a substantial fall in equipment prices.

In this paper, we tackle both of these issues using theory and French manufacturing and international trade data. First, we leverage micro-level data to measure the strength of capital-skill complementarity at the aggregate level. We credibly estimate the extent of capital-skill complementarity within firms, embed this within a macroeconomic model, and perform an aggregation to study the macro implications of those micro elasticities (in the spirit of Oberfield and Raval, 2021 and Baqaee and Farhi, 2019). Second, we depart from the assumption of homogeneous factor prices. We allow for exogenous firm-and-factor specific input price wedges. These wedges affect the measurement of aggregate capital-skill complementarity. We find that within and between firm adjustments generate aggregate capital-skill complementarity: a reduction in the price of equipment both increases firm-level skill intensities and reallocates labor toward skill-intensive firms, each of which increases the aggregate skill premium. Moreover, changes in wedges also have a direct effect on the skill premium. In spite of not targeting the evolution of the aggregate skill premium in French manufacturing, we find that the combination of equipment price declines and changes in wedges can together explain the stability of the French skill premium between 1998 and 2007. We additionally study changes in the labor share of French manufacturing and similarly find that changes in equipment prices and wedges match its observed decline in spite of not targeting this moment.

In Section 2, we construct a theory of multi-factor and multi-sector aggregate technology that allows for arbitrary constant returns-to-scale firm-level production technologies. We aggregate across firms assuming monopolistic competition and nested constant elasticity of substitution product demand across sectors and across firms within sectors. We incorporate distortions to factor markets characterized by exogenous firm-specific deviations in factor prices from an unobserved, common, latent *shadow* factor price that clears the factor market.[1] In the presence of firm-specific factor prices, the *observed* skill premium depends on both the shadow skill premium, i.e., the relative shadow wage of high skilled to low-skilled workers, and the distribution of skill premia and labor across firms.

We first focus on changes in a common equipment price, assuming common factor prices across firms. In this case, the observed skill premium equals the shadow skill premium. The response of the shadow skill premium depends on aggregate elasticities of substitution in factor *demand*, which account for the effect of changes in the price of a given factor on the relative demand for another factor. We fully characterize the aggregate elasticities of substitution for factor demand in terms of micro-level elasticities of substitution and the distribution of factor employment across firms. Aggregate complementarity may emerge either if capital and skill are complementarity in the firm-level production function or if capital and skill intensities positively covary across firms.

We then allow for fixed, firm-specific factor price wedges. In this case, the response of the observed skill premium to a common equipment price shock depends both on the response of the shadow skill premium and on the reallocation of skilled and unskilled labor across firms that differ in their wages. This new channel depends on aggregate elasticities of substitution in factor demand as well as on aggregate elasticities of substitution in factor *payments*. The latter accounts for the effect of changes in the price of a given factor on the relative share of payments going to another factor. While these two types of aggregate elasticities coincide with common factor prices, they deviate in the presence of distortions. We fully characterize aggregate elasticities of substitution in factor payments in terms of micro-elasticities of substitution and the distribution of factor payments across firms. Now, even if the shadow skill premium is not affected by a decline in the equipment price, aggregate complementarity may emerge if equipment intensive firms pay relatively higher skill premia.

[1]We provide micro-foundations for these wedges in Appendix C.

Our final theoretical result generalizes the analysis to allow for joint shocks to the price of equipment and to firm-specific wedges. We show that we can provide yet another generalization of the conept of the aggregate elasticity of substitution that captures the effect of shocks to firm-specific wedges to a given factor on the demand and payments to another factor. We again characterize these aggregate elasticities and show how they jointly determine the response of the shadow and observed skill premia to an equipment price shock and firm-specific factor-market wedge shocks.

To use our framework to assess the impact of falling equipment prices on the skill premium, in Section 3 we use matched employer-employee micro data from France spanning 1997-2007. In the data, we can measure variables that are key to our theoretical aggregation results, including equipment, low-skilled labor, and high-skilled labor intensities across firms, using worker occupation as a proxy for skill. We provide a set of stylized facts that play a central role in shaping aggregate capital-skill complementarity. Low-skilled labor intensities fall monotonically and substantially in firm size in the cross section, with the smallest and largest firms having a low-skill labor intensity of approximately 50% and 20%, respectively. Equipment intensity rises in firm size throughout the size distribution from below 20% to above 40%. These facts imply that falling equipment prices reallocate inputs toward larger and more equipment intensive firms, which also demand relatively more high-skilled labor. Thus, through this channel between-firm reallocation generates capital-skill complementarity in the aggregate. We also document that the skill premium is decreasing in firm size among firms with more than 50 employees. In contrast to the first channel, as falling equipment prices reallocate inputs toward larger and equipment intensive firms, this second channel lowers the observed skill premium. The second channel also highlights the importance of incorporating factor-price wedges in countries like France with strong labor-market regulations.

We estimate the key micro-level elasticities needed for our aggregation. For this exercise, we impose a functional form assumption on firm-level technology and consider the CRESH family of production functions (Hanoch, 1971). This family generalizes the CES production function by allowing varying degrees of substitutability among factor inputs without requiring ex ante assumptions about the nesting of factors (unlike the nested CES specifications). We estimate three factor-specific elasticity parameters of the CRESH production function using five-year changes in firm-specific factor employments, factor prices, and revenue. To identify these elasticities, we use two cost-shift instruments related to firm-specific international trade connections, and an additional cost-shift instrument that moves the skilled and unskilled wages firms face. Across several different specifications, we identify a modest, but statistically significant, degree of capital-skill complementarity at the firm level. Finally, we also estimate the within-sector elasticity of substitution across firms using five-year changes in sales and prices, instrumenting for firm-specific price changes using the interaction between real exchange rate movements and the initial allocation of firm imports across origin countries as a cost shifter.

The estimated elasticities, when combined with the observed distribution of factor intensities and wedges, allow us to perform our main aggregation exercise. As a first step, we measure the aggregate elasticity of the observed skill premium using our micro estimates and the average firm-level shares across all years in our sample (1997-2007). Without factor-price wedges, we find that this elasticity is -0.054, so that a 1% decline in the price of equipment generates a 0.054% increase in the observed skill premium in France. We then feed the observed annual changes in equipment prices based on the macro series to the model to predict the response of the skill premium each year in the data, and evaluate the cumulative impact on the skill premium over the entire period. Since our theory takes the shadow wage of low-skilled labor as the numeraire, the relevant shock is the price of equipment relative to the low-skill shadow wage. We measure each in a model-consistent way. The cumulative change between 1997 and 2007 in this relative price is a fall of approximately 58 log points. The model predicts a rise of around 3.1% in the skill premium.

Finally, we turn our main quantitative exercise, in which we incorporate labor wedges and heterogeneity in changes in the price of equipment. Using observed data on volumes and unit values of imported equipment goods, we measure changes in firm-level equipment prices and feed them into the model. We additionally back out firm-level labor wedges and their changes in the data and feed them into the model. The model predicts both make quantitatively sizable contributions to the observed skill premium. Within the period under consideration, incorporating factor-price variation across firms increases the predicted rise in the skill premium caused by declines in equipment prices from 3.1% to 6.4%. In contrast, changes in labor wedges lower the skill premium, to the point that the combined predicted change is only 1%. This overall prediction is close to the change in the observed skill premium in the data, which is roughly zero.

Literature. Our paper contributes to a large literature in labor, trade, and macro economics that studies the implications of technological change on inequality across skill groups. Following Katz and Murphy (1992), many earlier studies considered a broad notion of skill-biased technical change, identified empirically as a residual to match the observed evolution of the skill premium given the observed evolution of the aggregate relative supply of skill (see Acemoglu, 2002; Card and DiNardo, 2002, for reviews).[2]

Our approach builds more directly on the studies of capital-embodied technical change, which focus on changes in the quality-adjusted price of capital equipment as a well-defined empirical proxy for the technological shock affecting the labor market (Greenwood and Yorukoglu, 1997; Hornstein et al., 2005). This strand of the literature in turn builds on earlier work on aggregate technology by Griliches (1969, 1970) who first hypothesized capital-skill complementarity. In a seminal paper, Krusell et al. (2000) use this idea to provide an account of the rise of the skill premium in the U.S. by calibrating a four-factor aggregate production function representation of the U.S. economy.

By providing a framework for explicit aggregation of micro-level firm responses, we also build on recent work by Oberfield and Raval (2021), Lashkari et al. (2022), and Baqaee and Farhi (2019). The first two consider two-factor models with firm heterogeneity under constant returns-to-scale and variable returns-to-scale technologies, respectively. The latter considers a multi-factor model without firm heterogeneity. Unlike these papers, we allow for deviations from perfect factor markets. We note that the relationship between micro-level and macro-level substitution is also widely studied in the context of the effects of trade liberalization on the skill premium (e.g., Parro, 2013; Burstein et al., 2013; Burstein and Vogel, 2017).

Alternative approaches to studying the effect of technical change on skill demand and the skill premium focus on the roles of information technology (e.g., Caroli and Van Reenen, 2001; Bresnahan et al., 2002; Akerman et al., 2015; Harrigan et al., 2021; Adao et al., 2021) or automation (e.g., Acemoglu and Restrepo, 2018, 2020); our contribution relative to this literature is aggregating from micro estimates to quantify macro implications. Our paper is also closely related to a number of recent empirical contributions. Caunedo et al. (2021) study the inequality consequences of capital-embodied technical change by studying the occupation-level elasticity of substitution between labor and capital. Lindner et al. (2021) use micro data from Hungary and Norway to study the effect of innovation and R&D on firm-level demand for skill assuming imperfect labor markets.[3]

The paper is organized as follows. Section 2 presents our framework and theoretical results. Section 3 discusses our data. Section 4 presents our strategy for estimating the firm-level production function along with the estimation results. Section 5 discusses the

[2]A more recent approach in the literature emphasizes the importance of studying the endogenous assignment of production tasks to different skill groups for understanding the effect of technology on inequality (Costinot and Vogel, 2010; Acemoglu and Autor, 2011).

[3]Our results documenting the empirical association between the firm-level skill premia and capital intensity are also related to the results of Aghion et al. (2017) who document a negative association between firm-level skill premia and R&D intensity.

aggregation results and Section 6 concludes the paper.

2 Theory

2.1 Environment

Firms and Production. There is a large set of firms indexed by $i \in \mathcal{I}$ with arbitrary constant returns to scale production functions. We denote by $\mathcal{I}_s \subseteq \mathcal{I}$ the set of firms in sector s. The goods market is monopolistically competitive within each sector, and we assume nested CES preferences across sectors so that demand for firm $i \in \mathcal{I}_s$ is given by

$$Y_i = \Phi_i \left(\frac{P_i}{P_s}\right)^{-\varepsilon} \left(\frac{P_s}{P}\right)^{-\eta} Y \tag{1}$$

where ε is the elasticity of substitution across firms within each sector, η is the elasticity of substitution across sectors, Φ_i is a firm-specific demand shifter, P_s is the price index within sector s, and P is the aggregate price index. With slight abuse of notation, we also use P_i to denote the price that firm i charges for its output.

There are multiple factors of production factors X_f where $f \in \mathcal{F}$. We will focus in particular on three such factors $\{\ell, h, e\}$: low-skilled labor (X_ℓ), high-skilled labor (X_h), and capital equipment (X_e). Firms produce with heterogeneous CRS production functions, which we characterize in terms of a corresponding collection of unit factor requirements for all factors $\left(A_{fi}\right)_{f\in\mathcal{F}}$ as a function of the factor prices faced by the firm, such that the demand of firm i for factor f is given by $X_{fi} = A_{fi}Y_i$. For some of the results below, we will consider a specific parametric assumption on the structure of these unit factor requirements, but more generally we allow for arbitrary functions of factor prices.

Factor Markets. In the aggregate, we assume that each factor f has an exogenous and inelastic supply X_f. Firms face wedges in factor markets and pay firm-and-factor specific prices for each factor (which we sometimes refer to as wages). We assume a fixed and exogenous wedge T_{fi} for each firm i and factor f. The firm-level factor price is $W_{fi} = W_f T_{fi}$, where we refer to W_f as the *shadow price (or wage)* of factor f, since it is unobservable in the data. The shadow factor price is common across firms and equates supply and demand, $X_f = \sum_i X_{fi}$. We take the shadow wage of low-skilled workers as the numeraire and normalize $W_\ell = 1$. In Appendix C, we provide a micro-foundation for firm-level heterogeneity in the price of equipment, motivating our equipment capital wedges. We also present a micro-foundation for the labor market wedges based on firm-and-labor-type specific compensating differentials in competitive factor markets.

We define the *observed skill premium* Ψ as the average wage of high-skilled workers relative to the average wage of low-skilled workers,

$$\Psi \equiv \frac{\overline{W}_h}{\overline{W}_\ell} = \frac{\sum_i W_{hi} X_{hi} / X_h}{\sum_{i'} W_{\ell i'} X_{\ell i'} / X_\ell}, \tag{2}$$

where $X_\ell \equiv \sum_i X_{\ell i}$ and $X_h \equiv \sum_i X_{hi}$ are the aggregate demands for high- and low-skilled labor, respectively. We aim to characterize the response of the skill premium to a shock to the prices of different factors, particularly the price of equipment prices. In our model, such changes in equipment prices can stem from changes in the aggregate supply of equipment goods X_e or from changes in firm-specific wedges T_{ei}.

We characterize the response of the observed skill premium to such shocks in terms of a number of aggregate elasticities of substitution. To this end, we begin in Section 2.2 by defining these aggregate elasticities and in turn characterizing them in turns of firm-level production elasticities and other model fundamentals.

2.2 Micro and Macro Elasticities of Substitution

Each firm takes its own input prices as given and chooses its factor inputs to maximize profit. We define $\theta_{fi} \equiv W_{fi} X_{fi} / \left(\sum_{f'} W_{f'i} X_{f'i} \right)$ as firm i's factor-f intensity (i.e., i's payments to factor f relative to i's total factor payments). Let lower-case variables indicate the logarithm of upper-case variables, $v \equiv \ln V$ for any variable V, so that, e.g., $a_{fi} \equiv \ln A_{fi}$. We define the elasticity of (factor) substitution between factors f and f' at the firm-i-level as

$$\sigma_{ff',i} \equiv \frac{1}{\theta_{f'i}} \frac{\partial a_{fi}}{\partial w_{f'}} \tag{3}$$

and at the aggregate level as[4]

$$\sigma_{ff'} \equiv \frac{\partial \ln \left(X_f / X_{f'} \right)}{\partial w_{f'}}. \tag{4}$$

[4]The definition in Equation (4) is commonly referred to as the Morishima elasticity of substitution (MES), as contrasted from the Allen-Uzawa elasticity of substitution defined in Equation (3) (Blackorby and Russell, 1989). In our setting, it will prove more convenient to use the former to characterize the aggregate-level and the latter to characterize the firm-level elasticity of substitution. With more than two factors, the two definitions are not identical. For example, unlike the Allen-Uzawa elasticity, the Morishima elasticity is not symmetric between the two factors.

Note that the variation in the price of factor f' in both definitions (3) and (4) correspond to shifts in the shadow factor price. Such changes in shadow wages proportionally shift factor prices across all firms.

In the presence of factors wedges, we may define an distinct measure of substitutability across factors in terms of factor payments. Let $\sigma^*_{ff'}$ denote the aggregate elasticity of factor payment substitution as

$$\sigma^*_{ff'} \equiv 1 + \frac{\partial \ln \left(\theta_f / \theta_{f'}\right)}{\partial w_{f'}}, \tag{5}$$

where $\theta_f \equiv \left(\sum_i W_{fi} X_{fi}\right) / \left(\sum_{f',i'} W_{f'i'} X_{f'i'}\right)$ denotes the aggregate factor-f intensity of all factor payments in the economy. When there are no wedges ($T_{fi} \equiv T_f$ for all i), the two aggregate elasticities of substitution defined in Equations (4) and (5) coincide, $\sigma_{ff'} \equiv \sigma^*_{ff'}$. More generally, however, the two elasticities may diverge due to the fact that reallocations across firms have distinct effects on aggregate factor demand and factor payments.

Let $\Lambda_{fi} \equiv X_{fi} / \left(\sum_{i'} X_{fi'}\right)$ and $\Lambda^*_{fi} \equiv W_{fi} X_{fi} / \left(\sum_{i'} W_{fi'} X_{fi'}\right)$ denote firm i's share of aggregate demand and of aggregate payments for factor f, respectively. With a slight abuse of notation, let $\Lambda_{fs} \equiv \sum_{i \in \mathcal{I}_s} \Lambda_{fi}$ and $\Lambda^*_{fs} \equiv \sum_{i \in \mathcal{I}_s} \Lambda^*_{fi}$ denote the corresponding shares for all firms in sector s. The following lemma characterizes the two aggregate elasticities of substitution using these definitions.

Lemma 1. *The aggregate elasticities of substitution $\sigma_{ff'}$ and $\sigma^*_{ff'}$ between factors f and f' defined by Equations (4) and (5) are given by*

$$\sigma_{ff'} = \sum_i \left(\Lambda_{fi} \sigma_{ff',i} - \Lambda_{f'i} \sigma_{f'f',i}\right) \theta_{f'i} + \varepsilon \sum_i \left(\Lambda_{f'i} - \Lambda_{fi}\right) \theta_{f'i} - (\varepsilon - \eta) \sum_s \left(\Lambda_{f's} - \Lambda_{fs}\right) \theta_{f's}, \tag{6}$$

$$\sigma^*_{ff'} = \sum_i \left(\Lambda^*_{fi} \sigma_{ff',i} - \Lambda^*_{f'i} \sigma_{f'f',i}\right) \theta_{f'i} + \varepsilon \sum_i \left(\Lambda^*_{f'i} - \Lambda^*_{fi}\right) \theta_{f'i} - (\varepsilon - \eta) \sum_s \left(\Lambda^*_{f's} - \Lambda^*_{fs}\right) \theta_{f's}, \tag{7}$$

where $\theta_{fs} \equiv \left(\sum_{i \in \mathcal{I}_s} W_{fi} X_{fi}\right) / \left(\sum_{f',i' \in \mathcal{I}_s} W_{f'i'} X_{f'i'}\right)$ denotes the intensity of payments for factor f in sector s.

Proof. See Appendix B. □

In the presence of exogenous distortions, Equations (6) and (7) decompose the two aggregate elasticities of substitution into three components: the first term accounts for the appropriate weighted average of the within-firm elasticities of substitution across firms, the second term accounts for the effect of within-sector reallocations across firms, and the last term accounts for the effect of cross-sectoral reallocations. In the case of the aggregate elasticity of substitution for demand (payments), the averages across firms are weighted by their shares in aggregate demand for (payment to) factors.

Parametric Forms for Firm-Level Production Functions To further unpack the within-firm substitution component, consider two alternative examples for the specification of firm-level production functions.

Example 1 (CES Production Function)**.** Consider a CES production function with elasticity of substitution σ. Cross-factor elasticities of substitution in firm-level technology are given by $\sigma_{ff',i} \equiv \sigma$ for all $f \neq f'$ and $\sigma_{ff,i} = \sigma\left(\theta_{fi} - 1\right)/\theta_{fi}$.

Assuming a single sector model (equivalently, $\eta = \varepsilon$), the CES production function implies that the aggregate elasticity of substitution for factor demand and factor payments are given by

$$\sigma_{ff'} = \sigma + (\varepsilon - \sigma)\sum_i \left(\Lambda_{f'i} - \Lambda_{fi}\right)\theta_{f'i}, \qquad \sigma^*_{ff'} = \sigma + (\varepsilon - \sigma)\sum_i \left(\Lambda^*_{f'i} - \Lambda^*_{fi}\right)\theta_{f'i}.$$

If we assume that the elasticity of demand exceeds the elasticity of substitution across factors ($\varepsilon > \sigma$), the contribution of cross-firm reallocations to the aggregate elasticity of substitution between two factors f and f' is positive if firms that are more intensive in factor f' on average account for a larger share of aggregate payments to factor f' compared to factor f (Oberfield and Raval, 2021).

Example 2 (CRESH Production Function (Hanoch, 1971))**.** Consider a production function implicitly defined through the following constraint on unit factor requirements:[5]

$$\sum_{f\in\{\ell,h,e\}} \left(Z_{fi}A_{fi}\right)^{\frac{\sigma_f - 1}{\sigma_f}} = 1, \tag{8}$$

where Z_{fi} are firm-specific factor-augmenting productivity and $\sigma_f > 0$ are factor-specific parameters. The production function defined in Equation (8) is a homothetic generalization of the standard CES production function allowing for different degrees of substitutability among different factor inputs. The CES production function is nested in this specification for the case of constant parameters $\sigma_f \equiv \sigma$ across all factors f.

[5] Hanoch (1971) introduced this class of production functions and referred to them as CRESH (constant ratios of elasticities of substitution with homotheticity). The advantage of the specification in Equation (8), compared to a nested CES specifications like that of Krusell et al. (2000), is that it does not a require an a priori nesting structure between different factors in the firm-level production function.

According to the production function in Equation (8), the elasticity of substitution between factors f and f' for firm i satisfies[6]

$$\sigma_{ff',i} = \frac{\sigma_f \sigma_{f'}}{\overline{\sigma}_i} - \frac{\sigma_f}{\theta_{f,i}} \mathbb{I}_{ff'}, \tag{9}$$

where we have defined $\overline{\sigma}_i \equiv \sum_f \theta_{fi}\sigma_f$ as a producer-specific weighted mean of σ_f, weighing by factor cost shares, and where $\mathbb{I}_{ff'}$ is an indicator function that takes the value 1 when $f = f'$ and 0 otherwise. Let us now compare the firm-level elasticity of substitution between equipment and high-skilled labor to that between equipment and low-skilled labor. From Equation (9) it follows that the ratio of these two elasticities satisfies

$$\frac{\sigma_{he,i}}{\sigma_{\ell e,i}} = \frac{\sigma_h}{\sigma_\ell}. \tag{10}$$

Thus, parameters σ_h and σ_ℓ characterize the relative substitutability of high- and low-skilled workers with equipment. If $\sigma_\ell > \sigma_h$, then low-skilled workers are more substitutable with equipment.

Assuming again a single-sector ($\eta = \varepsilon$), Equations (6) and (9) yield

$$\sigma_{ff'} = \sigma_{f'} \left[1 - \sum_i \left(\Lambda_{f'i} \frac{\sigma_{f'}}{\overline{\sigma}_i} - \Lambda_{fi} \frac{\sigma_f}{\overline{\sigma}_i} \right) \theta_{f'i} \right] + \varepsilon \sum_i \left(\Lambda_{f'i} - \Lambda_{fi} \right) \theta_{f'i}.$$

We can also find the expression for the aggregate elasticity of substitution for factor payments $\sigma^*_{ff'}$ in a similar fashion, replacing the firms' shares in aggregate factor demand Λ_{fi} and $\Lambda_{f'i}$ with those in aggregate factor payments Λ^*_{fi} and $\Lambda^*_{f'i}$.

Composite Factors We can use our results to derive the elasticities of substitution for composite factors. Let factor c be defined as a composite of a set $\mathcal{F}_c$ of other factors. For instance, we typically define labor (which we denote with label n) as the sum of all workers irrespective of their skill type such that $X_n \equiv X_\ell + X_h$ where we have $\mathcal{F}_n \equiv \{\ell, h\}$. We can show that the aggregate elasticity of factor payments substitution between a composite factor c and any other factor f' is simply the convex combination of all the factors constituting the composite, that is,

$$\sigma^*_{cf'} \equiv 1 + \frac{\partial \ln \left(\theta_c / \theta_{f'} \right)}{\partial w_{f'}} = \sum_{f \in \mathcal{F}_c} \frac{\theta_f}{\theta_c} \sigma^*_{ff'}, \tag{11}$$

[6]See Appendix B for the solution of the cost minimization problem and the derivation of Equation (9).

where the intensity of the composite factor is sum of the intensities of all the factors included in it, $\theta_c \equiv \sum_{f \in \mathcal{F}_c} \theta_f$, and where $\sigma^*_{ff'}$ is the aggregate elasticity of factor payments between f and f', given by Equation (7). In the example of labor as the composite factor, the aggregate elasticity of substitution between labor and equipment is a convex combination of the aggregate elasticities of the factor payments for the two skill groups, with each group's weight given by the share of that skill group in labor payments, that is, $\sigma^*_{ne} = \frac{\theta_\ell}{\theta_n}\sigma^*_{\ell e} + \frac{\theta_h}{\theta_n}\sigma^*_{he}$.

The following result characterizes the elasticity of the aggregate share of a composite factor c with respect to the price of factor f'.

Lemma 2. *The elasticity of the aggregate intensity of a composite factor c relative to the price of a given factor f' is given by*

$$\frac{\partial \ln \theta_c}{\partial w_f} = \begin{cases} \sigma^*_{cf} - \theta_{\overline{f}}\, \sigma^*_{\overline{f}f} - \theta_f, & f \notin \mathcal{F}_c, \\ \frac{\theta_{\overline{c}_f}}{\theta_c}\, \sigma^*_{\overline{c}_f f} - \theta_{\overline{f}}\, \sigma^*_{\overline{f}f} + \theta_f \frac{1-\theta_c}{\theta_c}, & f \in \mathcal{F}_c, \end{cases} \tag{12}$$

where we have defined two composite factors: a composite factor $\overline{f}$ as the set of all factors other than f (with $\mathcal{F}_{\overline{f}} \equiv \mathcal{F} / \{f\}$) and composite factor $\overline{c}_f$ as the set of all factors in the composite factor c other than f (with $\mathcal{F}_{\overline{c}_f} \equiv \mathcal{F}_c / \{f\}$).

Proof. See Appendix B. □

In the example of the composite labor factor n, and in a three-sector model with $\mathcal{F} = \{\ell, h, e\}$, Equation (12) implies that the elasticity of the labor share with respect to the price of equipment and the wage of high skilled workers are given by $\frac{\partial \ln \theta_n}{\partial w_e} = \theta_e \left(\sigma^*_{ne} - 1\right)$, and $\frac{\partial \ln \theta_n}{\partial w_h} = \theta_e \left(\frac{\theta_\ell}{\theta_n}\sigma^*_{\ell h} - \sigma^*_{eh} + \frac{\theta_\ell}{\theta_n}\right)$ since in this case $\mathcal{F}_{\overline{n}_h} = \{\ell\}$ and $\mathcal{F}_{\overline{e}} = \mathcal{F}_n$.

2.3 The Skill Premium and the Equipment Price

In the model, a shock to the equipment price may stem from an underlying shock to the aggregate stock of equipment. We focus our attention to characterizing the responses of the shadow factor prices and the observed skill premium as a function of the size of the shock to the price of equipment, since the latter object is the one we can more easily measure in the data.

To state our main result, we need to express the aggregate elasticities of substitution in vector and matrix form. Since low-skilled labor is the numeraire and the equipment is the factor experiencing the factor price shock, we do not include them in the vector and matrix form, defining the matrix $\mathbf{\Sigma}$ and vectors $\boldsymbol{\sigma}_{f\cdot}$ and $\boldsymbol{\sigma}_{\cdot f}$ as

$$\mathbf{\Sigma} \equiv \left(\sigma_{ff'}\right)_{f,f'\in\mathcal{F}/\{\ell,e\}}, \qquad \boldsymbol{\sigma}_{f\cdot} \equiv \left(\sigma_{ff'}\right)_{f\in\mathcal{F}/\{\ell,e\}}, \qquad \boldsymbol{\sigma}_{\cdot f} \equiv \left(\sigma_{f'f}\right)_{f\in\mathcal{F}/\{\ell,e\}}, \tag{13}$$

and where the aggregate substitution elasticity of factor demand $\sigma_{ff'}$ is given by Equation (6) with $\sigma_{ff} \equiv 0$ for all f. Using these definitions, the following proposition characterizes the response.

Proposition 1. *Consider a shock to the supply of equipment that leads to a change dw_e in the price of equipment, holding all other factor supplies and wedges constant. Let $\boldsymbol{w} \equiv \left(w_f\right)_{f\in\mathcal{F}/\{\ell,e\}}$ be the vector of the logarithm of the price of all factors other than equipment and unskilled labor. The elasticity of this vector with respect to the price of capital equipment is given by*

$$\frac{d\boldsymbol{w}}{dw_e} = \left(\mathbf{1}\,\boldsymbol{\sigma}'_{\ell\cdot} - \mathbf{\Sigma}\right)^{-1}\,\left(\boldsymbol{\sigma}_{\cdot e} - \sigma_{\ell e}\right), \tag{14}$$

where $\mathbf{1}$ is a vector of ones, and where the matrix $\mathbf{\Sigma}$ and vectors $\boldsymbol{\sigma}_{f\cdot}$ and $\boldsymbol{\sigma}_{\cdot f}$ are defined by Equation (13). Moreover, the response of the observed skill premium deviates from that of the shadow skill premium and is given by

$$\frac{d\psi}{dw_e} = \frac{dw_h}{dw_e} + \sigma^*_{he} - \sigma^*_{\ell e} + \left(\boldsymbol{\sigma}^*_{h\cdot} - \boldsymbol{\sigma}^*_{\ell\cdot}\right)'\,\frac{d\boldsymbol{w}}{dw_e}, \tag{15}$$

where we have defined a vector $\boldsymbol{\sigma}^*_{f\cdot}$ following Equation (13), replacing elasticities of factor substitution $\sigma_{ff'}$ with elasticities of factor payments substitution $\sigma^*_{ff'}$ with $\sigma^*_{ff} \equiv 0$.

Proof. See Appendix B. □

To better understand the implications of Proposition 1, let us first consider the case with perfect factor markets, where all firms pay common factor prices, $W_{fi} = W_f$. In this case the share of firm i in both aggregate factor demand and factor payments are equal $\Lambda^*_{fi} = \Lambda_{fi}$, and thus Lemma 1 implies that $\sigma^*_{ff'} = \sigma_{ff'}$ for all f and f'. As expected, it is easy to check that equations (14) and (15) together imply that the responses of the observed and shadow skill premium coincide with uniform factor prices, that is, $d\psi/dw_e = dw_h/dw_e$.

In the presence of firm-specific factor wedges, the employment Λ_{fi} and wage bill distributions differ Λ^*_{fi} from one another. Equation (14) (along with Lemma 1) shows that the employment distribution still determines the elasticity of the shadow skill premium since the shadow skill premium equates labor supply with labor demand. Proposition 1 additionally shows that when employment Λ_{fi} and wage bill distributions differ Λ^*_{fi}, the response of the observed skill premium deviates from that of the shadow skill premium, as seen on the right hand side of Equation (15). The gap between the responses of the observed and the shadow premia accounts for the contribution of labor reallocation across firms to the change in the observed skill premium at a fixed value of the shadow skill premium.

To understand the intuition for this gap in Equation (15), note that taking a full derivative of Equation (2) we find

$$d\psi = dw_h + \sum_i \left(\left(\Lambda^*_{hi} - \Lambda_{hi} \right) d \ln \Lambda_{hi} - \left(\Lambda^*_{\ell i} - \Lambda_{\ell i} \right) d \ln \Lambda_{\ell i} \right), \tag{16}$$

where we have used the fact that $\sum_i \Lambda_{fi} \, d\Lambda_{fi} = 0$. If firms pay common wages, then $\Lambda_{fi} = \Lambda^*_{fi}$ and labor reallocation has no direct effect on the observed skill premium (i.e., holding the general equilibrium shadow wages fixed); this explains why the second term is zero under the case of perfect labor markets. If firms do not pay common wages, so that there exist firms for which $\Lambda_{fi} \neq \Lambda^*_{fi}$, then labor reallocation has a direct effect on the observed skill premium. If skilled labor reallocates towards firms paying higher skilled wages (firms for which $\Lambda_{hi} > \Lambda^*_{hi}$) or unskilled labor reallocates towards firms paying lower unskilled wages (firms for which $\Lambda_{\ell i} < \Lambda^*_{\ell i}$), then the observed skill premium rises for a given shadow skill premium.

Let us now specialize the results of Proposition 1 to case of a three factor model where $\mathcal{F} = \{\ell, h, e\}$. In this case, we only need to find the response of the high skilled wages (shadow skill premium) to the equipment price shock, which leads to the following expressions for the responses of the shadow and obseverd skill premia

$$\frac{dw_h}{dw_e} = -\frac{\sigma_{\ell e} - \sigma_{he}}{\sigma_{\ell h}}, \tag{17}$$

$$\frac{d\psi}{dw_e} = \frac{dw_h}{dw_e} - \sigma^*_{\ell h} \left(\frac{dw_h}{dw_e} + \frac{\sigma^*_{\ell e} - \sigma^*_{he}}{\sigma^*_{\ell h}} \right). \tag{18}$$

First, comparing Equations (17) and (18), we see that the gap between the observed and the shadow skill premia exists to the extent that the aggregate elasticities in terms of factor demand and factor payments deviate from one another, that is, $\frac{\sigma_{\ell e} - \sigma_{he}}{\sigma_{\ell h}} \neq \frac{\sigma^*_{\ell e} - \sigma^*_{he}}{\sigma^*_{\ell h}}$. Substituting from the results of Lemma 1, we can expand Equation (17) as

$$\frac{dw_h}{dw_e} = -\frac{\sum_i \left(\Lambda_{\ell i} \sigma_{\ell e,i} - \Lambda_{hi} \sigma_{he,i} \right) \theta_{ei} - \varepsilon \sum_i \left(\Lambda_{\ell i} - \Lambda_{hi} \right) \theta_{ei} + (\varepsilon - \eta) \sum_s \left(\Lambda_{\ell s} - \Lambda_{hs} \right) \theta_{es}}{\sum_i \left(\Lambda_{\ell i} \sigma_{\ell h,i} - \Lambda_{hi} \sigma_{hh,i} \right) \theta_{hi} + \varepsilon \sum_i \left(\Lambda_{hi} - \Lambda_{\ell i} \right) \theta_{hi} - (\varepsilon - \eta) \sum_s \left(\Lambda_{hs} - \Lambda_{\ell s} \right) \theta_{hs}}. \tag{19}$$

To further unpack Equation (19), let us consider the case without heterogeneity, where all firms have identical production functions. In this case, under the CES production function (Example 1), we have $dw_h/dw_e = 0$, and under the more general CRESH production function (Example 2), Equation (17) simplifies to

$$\frac{dw_h}{dw_e} = -\frac{\sigma_{\ell e,i} - \sigma_{he,i}}{\sigma_{\ell h,i}} = -\frac{\sigma_\ell - \sigma_h}{\sigma_\ell \sigma_h}\sigma_e. \tag{20}$$

Under these assumptions, Equation (20) shows that falling equipment prices raise the skill premium when the firm-level elasticity of substitution between equipment and low-skilled labor exceeds that between equipment and high-skilled labor. As we saw in Equation (10), the restriction $\sigma_\ell > \sigma_h$ on the parameters of the production function is a necessary and sufficient condition for this result.

In the presence of heterogeneity, Equation (17) shows that the same intuition as in the firm-level case we saw in Equation (20) still caries over to the aggrgate level. The sign of the response of the skill premium to changes in equipment prices again depends on the difference between the degrees of aggregate substitutability between equipment versus the two labor types. Under the CES production function, and assuming a single sector model ($\eta = \varepsilon$), we find

$$\frac{dw_h}{dw_e} = -\frac{(\varepsilon - \sigma)\sum_i (\Lambda_{hi} - \Lambda_{\ell i})\,\theta_{ei}}{\sigma + (\varepsilon - \sigma)\sum_i (\Lambda_{hi} - \Lambda_{\ell i})\,\theta_{hi}}. \tag{21}$$

Unlike the model with homogenous CES production functions, in the presence of heterogeneity the elasticity of the skill premium with respect to equipment may differ from zero. In particular, assuming $\varepsilon > \sigma$, if firms with high equipment-intensities on average contribute a larger share of the wage bill of high-skilled workers, then a fall in equipment prices leads to a rise in the skill premium. In this case, there is complementarity between equipment and skill *across* firms. Under the CRESH production function, and assuming again a single sector model ($\eta = \varepsilon$), we find

$$\frac{dw_h}{dw_e} = -\frac{\sigma_e \sum_i \left(\Lambda_{\ell i}\frac{\sigma_\ell - \overline{\sigma}_i}{\overline{\sigma}_i} + \Lambda_{hi}\frac{\overline{\sigma}_i - \sigma_h}{\overline{\sigma}_i}\right)\theta_{ei} + (\varepsilon - \sigma_e)\sum_i (\Lambda_{hi} - \Lambda_{\ell i})\,\theta_{ei}}{\sigma_h\left[1 + \sum_i \left(\Lambda_{\ell i}\frac{\sigma_\ell}{\overline{\sigma}_i} - \Lambda_{hi}\frac{\sigma_h}{\overline{\sigma}_i}\right)\theta_{hi}\right] + \varepsilon \sum_i (\Lambda_{hi} - \Lambda_{\ell i})\,\theta_{hi}}. \tag{22}$$

In this case, when the firm-level production function features sufficient complementarity between equipment and skill within firms, in the sense that $\sigma_h \leq \overline{\sigma}_i \leq \sigma_\ell$, then the first term in the numerator of Equation (22) is positive. In such a setting, we have complementarity between equipment and skill both across and within firms.

As another corollary of Proposition 1, we can use to derive an expression for the general equilibrium elasticity of substitution between labor and equipment, which accounts both for the direct effect of equipment prices on substitution between equipment and labor as well as the endogenous responses of all other factor prices. In particular, using the results of Lemma 2, we find that the general equilibrium response of the share of labor in factor income $\theta_n \equiv \theta_\ell + \theta_h$ to the shock defined in Proposition 1 in a three-sector model with $\mathcal{F} = \{\ell, h, e\}$ satisfies

$$\frac{1}{1-\theta_n}\frac{d\ln\theta_n}{dw_e} = (\sigma^*_{ne} - 1) - \left(\frac{\theta_\ell}{\theta_n}\left(\sigma^*_{\ell h} - 1\right) - \left(\sigma^*_{eh} - 1\right)\right)\frac{\sigma_{\ell e} - \sigma_{he}}{\sigma_{\ell h}}, \tag{23}$$

where σ^*_{ne} is given by Equation (11), and where we have used the fact that $\theta_e = 1 - \theta_n$ in the three-sector model. Assuming that the endogenous response of the shadow skill premium is positive (when $\sigma_{\ell e} > \sigma_{he}$), then whether or not the labor share rises or falls may deviate from a prediction based solely on the elasticity of substitution between labor and equipment σ^*_{ne}. In particular, the extent of this deviation depends on the aggregate elasticities of substitution between low- and high-skilled labor payments $\sigma^*_{\ell h}$ and that between high-skilled labor and equipment payments σ^*_{eh}.

The intuition for the above result is simple: falling equipment prices (relative to the wages of low-skilled workers) raise the skill premium and the average wage. The more substitutable the payments to high-skilled labor are with those to equipment (the higher the σ^*_{eh}), the stronger the shift away is from labor payments toward equipment. In contrast, the more substitutable the two skill types are (the higher the $\sigma^*_{\ell h}$), the weaker the shift away is from labor payments, since low-skilled labor receives some benefits from rising skill premia.

2.4 Heterogenous Shocks to Equipment Price and Labor Wedges

In this section, we study the case in which the changes in the price of equipment are heterogeneous across firms. Let dw_{ei} denote a small change in the log price of equipment faced by firm i, and define $d\overline{w}_e \equiv \sum_i \Lambda^*_{fi} dw_{ei}$ and shares ω_{ei} satisfying $dw_{ei} = \omega_{ei} d\overline{w}_e$. In addition, we may also consider exogenous shocks that heterogeneously shift labor wedges across firms. In particular, assume labor wedges T_{fi} for $f \in \{L, H\}$ shift infinitesimally by an amount $d\tau_{fi} \equiv d\ln T_{fi}$ characterized by

$$d\tau_{fi} = \omega_{fi}\, d\tau_f, \qquad f \in \{L, H\}, \tag{24}$$

where ω_{fi} is a firm-specific (potentially negative) weight satisfying $\sum_i \Lambda^*_{fi}\omega_{fi} = 1$.

Under the above assumptions, we can generalize the decomposition in Equation (16) as

$$d\psi = dw_h + \sum_i \Lambda^*_{hi}\, d\ln\Lambda_{hi} - \sum_i \Lambda^*_{\ell i}\, d\ln\Lambda_{\ell i} + \sum_i \Lambda^*_{hi}\, d\ln\tau_{hi} - \sum_i \Lambda^*_{\ell i}\, d\tau_{\ell i}, \tag{25}$$

where dw_h now stands for the relative response of the shadow skill premium to the *joint* shocks to the equipment prices as well as the labor wedges. The two additional terms in Equation (25) compared to Equation (16) account for the direct effects of changes in the labor wedges on the skill premium.

In parallel to our definition of the aggregate elasticities of substitution (4) and (5), we can define the aggregate wedge elasticities capturing the effect of the shock to factor-f' wedges on the relative demand and payments for factor f as

$$\sigma^{\omega}_{ff'} \equiv \frac{\partial \ln\left(X_f/X_{f'}\right)}{\partial \tau_{f'}}, \qquad \sigma^{*,\omega}_{ff'} \equiv 1 + \frac{\partial \ln\left(\theta_f/\theta_{f'}\right)}{\partial \tau_{f'}}, \tag{26}$$

where the superscript ω indicates the fact that the elasticity is defined with respect to a given set of firm-specific weights $(\omega_{f'i})$ following Equation (24). In the case of equipment capital $f = e$, we let $\tau_e \equiv \overline{w}_e$ in Equation (26). Note that the aggregate elasticities of substitution defined in Equations (4) and (5) are special cases of the aggregate wedge elasticity defined in Equation (26) corresponding to the case of $\omega_{fi} \equiv 1$. The next proposition characterizes the aggregate wedge elasticities in their general form.

Lemma 3. *The aggregate wedge elasticities of substitution $\sigma^{\omega}_{ff'}$ between factors f and f' defined by Equations (26) are given by*

$$\begin{aligned}\sigma^{\omega}_{ff'} = &\sum_i \left(\Lambda_{fi}\sigma_{ff',i} - \Lambda_{f'i}\sigma_{f'f',i}\right)\theta_{f'i}\omega_{f'i} \\ &+ \varepsilon\sum_i\left(\Lambda_{f'i} - \Lambda_{fi}\right)\theta_{f'i}\omega_{f'i} - (\varepsilon-\eta)\sum_s\left(\Lambda_{f's} - \Lambda_{fs}\right)\theta_{f's}\omega_{f's},\end{aligned} \tag{27}$$

$$\begin{aligned}\sigma^{*,\omega}_{ff'} = &\sum_i \left(\Lambda^*_{fi}\sigma_{ff',i} - \Lambda^*_{f'i}\sigma_{f'f',i}\right)\theta_{f'i}\omega_{f'i} \\ &+ \varepsilon\sum_i\left(\Lambda^*_{f'i} - \Lambda^*_{fi}\right)\theta_{f'i}\omega_{f'i} - (\varepsilon-\eta)\sum_s\left(\Lambda^*_{f's} - \Lambda^*_{fs}\right)\theta_{f's}\omega_{f's},\end{aligned} \tag{28}$$

where we have defined the mean sector-specific wedge shock ω_{fs} as

$$\omega_{fs} \equiv \sum_{i\in\mathcal{I}_s} \Lambda^*_{fi|s}\omega_{fi}, \tag{29}$$

with $\Lambda^*_{fi|s}$ accounting for the share of sector-s payments to factor f paid by firm i.

Proof. See Appendix B. □

Comparing the results of Lemmas 1 and 3, we find that the expressions for the aggregate wedge elasticities simply parallel those for the aggregate elasticities of substitution with one modification: they replace the firm-specific factor-f' intensity $\theta_{f'i}$ with $\theta_{f'i}\omega_{f'i}$. This result is intuitive. The structure of the wedge shocks assumed in Equation (24) implies different pass-throughs from the overall factor-level wedge shock $d\tau$ to firm-specific factor prices. In so far as we are interested in the aggregate effects of the shock, the heterogeneous pass-throughs translate into effective factor-intensities that are smaller or larger than the underlying firm-level factor intensities depending on the size of the pass-through.

The next proposition characterizes the responses of the shadow and observed skill premia to joint shocks to the shadow equipment prices and to labor wedges. Here, we again use the vector and matrix forms to represent the aggregate elasticities, defining matrices $\boldsymbol{\Sigma}^{\omega}$ ($\boldsymbol{\Sigma}^{*,\omega}$), and vectors $\boldsymbol{\sigma}^{\omega}_{f\cdot}$ and $\boldsymbol{\sigma}^{\omega}_{\cdot f}$ ($\boldsymbol{\sigma}^{*,\omega}_{f\cdot}$ and $\boldsymbol{\sigma}^{*,\omega}_{\cdot f}$) by replacing aggregate elasticities of factor substitution $\sigma_{ff'}$ with generalized aggregate elasticities $\sigma^{\omega}_{ff'}$ ($\sigma^{*,\omega}_{ff'}$) in Equation (13), where the generalized aggregate elasticities are defined by Equation (26) with $\sigma^{\omega}_{ff} \equiv 0$ ($\sigma^{*,\omega}_{ff} \equiv 0$) for all f.

Proposition 2. *Consider exogenous shocks to the aggregate stock of equipment and to the equipment and labor wedges $d\tau_{fi} \equiv \omega_{fi} d\tau_f$ for $f \in \{e, \ell, h\}$ with lead to a shock to firm-level equipment prices, holding fixed the aggregate supplies of all factors. Letting $\boldsymbol{w} \equiv (w_f)_{f\in\mathcal{F}/\{\ell,e\}}$ be the vector of the logarithm of the price of all factors other than equipment and unskilled labor, the first-order change in the elements of this vector in response to the shocks is given by*

$$d\boldsymbol{w} = \left[\mathbf{1}\ (\boldsymbol{\sigma}^{\omega}_{\ell\cdot})' - \boldsymbol{\Sigma}^{\omega}\right]^{-1} \left[\left(\boldsymbol{\sigma}^{\omega}_{\cdot e} - \sigma^{\omega}_{\ell e}\right) d\overline{w}_e + \boldsymbol{\sigma}^{\omega}_{\cdot \ell}\, d\tau_\ell - \sigma^{\omega}_{\ell h}\, \mathbf{1}_{\cdot h}\, d\tau_h\right], \tag{30}$$

where $\mathbf{1}$ is a vector of ones, $\mathbf{1}_{\cdot h}$ is a vector of zeros with a one in the element corresponding to factor h, and where matrices $\boldsymbol{\Sigma}^{\omega}$ and vectors $\boldsymbol{\sigma}^{\omega}_{f\cdot}$ and $\boldsymbol{\sigma}^{\omega}_{\cdot f}$ are defined as above. The first-order response of the observed skill premium is then given by

$$d\psi = dw_h + \left(\sigma^*_{he} - \sigma^*_{\ell e}\right) d\overline{w}_e + \left(\boldsymbol{\sigma}^{*,\omega}_{h\cdot} - \boldsymbol{\sigma}^{*,\omega}_{\ell\cdot}\right)' d\boldsymbol{w} + \sigma^{*,\omega}_{\ell h}\, d\tau_h - \sigma^{*,\omega}_{h\ell}\, d\tau_\ell, \tag{31}$$

where we have similarly defined vectors $\boldsymbol{\sigma}^{*,\omega}_{h\cdot}$ and $\boldsymbol{\sigma}^{*,\omega}_{\ell\cdot}$.

Proof. See Appendix B. □

Proposition 2 characterizes the responses of shadow and observed skill premia to the joint shocks to equipment prices and labor wedges across firms to the first order of approximation in the magnitude of the shocks. We can specialize this result to the case of a three-factor model to find

$$dw_h = -\frac{\sigma_{\ell e}^{\omega} - \sigma_{he}^{\omega}}{\sigma_{\ell h}} d\overline{w}_e - \frac{\sigma_{\ell h}^{\omega}}{\sigma_{\ell h}} d\tau_h + \frac{\sigma_{h\ell}^{\omega}}{\sigma_{\ell h}} d\tau_\ell, \tag{32}$$

$$d\psi = dw_h - \sigma_{\ell h}^{*} \left(dw_h + \frac{\sigma_{\ell e}^{*,\omega} - \sigma_{he}^{*,\omega}}{\sigma_{\ell h}^{*}} d\overline{w}_e + \frac{\sigma_{\ell h}^{*,\omega}}{\sigma_{\ell h}^{*}} d\tau_h - \frac{\sigma_{hl}^{*,\omega}}{\sigma_{\ell h}^{*}} d\tau_\ell \right). \tag{33}$$

In addition to the effect of heterogeneous equipment shocks, both the shadow and observed skill premia now account for the direct effect of shocks to labor wedges as well.

It is straightforward to show that we can generalize the results of Lemma 2 to the case of heterogeneous firm-level price shocks to find $\partial \ln \theta_f / \partial \tau_{f'}$. The proof of lemma shows that the resulting expressions are similar to those in Equation (12), only repalcing aggregate elasticities of substitution $\sigma_{ff'}^{*}$ with generalized elasticities $\sigma_{ff'}^{*,\omega}$. It then follows that we can characterize the response of the labor share to the combination of shocks considered in Proposition 2 in the three-factor case as

$$\begin{aligned} \frac{d \ln \theta_n}{1-\theta_n} = & \left(\frac{\theta_\ell}{\theta_n} \sigma_{\ell e}^{*,\omega} + \frac{\theta_h}{\theta_n} \sigma_{he}^{*,\omega} - 1 \right) d\overline{w}_e \\ & + \left(\frac{\theta_\ell}{\theta_n} (\sigma_{\ell h}^{*} - 1) - (\sigma_{eh}^{*} - 1) \right) dw_h \\ & + \left(\frac{\theta_\ell}{\theta_n} (\sigma_{\ell h}^{*,\omega} - 1) - (\sigma_{eh}^{*,\omega} - 1) \right) d\tau_h + \left(\frac{\theta_h}{\theta_n} (\sigma_{h\ell}^{*,\omega} - 1) - (\sigma_{e\ell}^{*,\omega} - 1) \right) d\tau_\ell, \end{aligned} \tag{34}$$

where we have again used the fact that $\theta_e = 1 - \theta_n$ in the three-factor model, where dw_h is given by Equation (32). As we will see, to use these results in practice, we can discipline the changes in the labor wedges by the observed evolution of the skill premia across firms.

3 Data

3.1 Data sources

We exploit a number of French administrative data sources on firms, workers, and firm-level export and import transactions.

Balance sheet and administrative information for the near universe of French enterprises are retrieved from FICUS, a dataset jointly administered by the French Institute National de la Statistique et des Études Économiques (INSEE) and the Direction Générale des Finances Publiques (DGFiP). The detailed breakdown of firm capital by asset type is retrieved from the BRN dataset, which covers all firms that are subject to the BRN (Bénéfice Réel Normal) tax regime over the period 1993-2009.[7] It includes over 60% of French firms, which account for 79% of employment and value added compared to the full set of firms contained in FICUS; moreover, they account for over 90% of the aggregate value of the trade flows on the customs records.

Firm-level international transaction data are provided by the French Directorate-General of Customs and Indirect Taxes (DGDDI), which provides information on the annul value of imports and exports by country of origin/destination and 8-digit CN product for all firms involved in international transactions (with simplified declarations below certain thresholds). We make use of the data over the period 1994-2007 and focus on manufacturing firms. Our final dataset describes the international trade flows of French firm concerning over 5,000 products from/to 161 countries.

As we will discuss in detail in Section 3.2 below, firm-level employment and wage bill data are computed aggregating worker-level data on hours worked and wages. These data, together with worker characteristics, are retrieved from the DADS (Déclarations Annuelles de Données Sociales) Poste, a matched employer-employee dataset provided by INSEE that covers the whole population of French workers. Summary statistics are displayed in Table 1 for the full sample used in the aggregation exercise of Section 5 and the reduced sample used in the estimation of the firm-level production elasticities of Section 4.

Sectoral data on employment, investment, and capital by asset type are retrieved from INSEE National Accounts. Industry-level data on depreciation rates by asset type come from the EU KLEMS Database.

[7] Firms with revenues above a certain threshold must be affiliated with the BRN regime. In 2007, the thresholds were 763,000 euros if a firm operates in trade or real estate sectors and 230,000 euros otherwise.

Table 1: Summary Statistics

		All firms						Reduced Sample					
	Source	Obs. (Nb)	Mean	Sd	p25	p50	p75	Obs. (Nb)	mean	sd	p25	p50	p75
Output	FICUS	475,438	15,681,216	299,923,380	741,636	1,687,545	4,967,409	96,841	49,643,200	336,116,805	4,667,909	10,301,318	28,229,455
Value Added	FICUS	475,438	4,329,647	71,316,215	319,909	672,364	1,698,909	96,841	14,590,585	128,657,845	1,585,636	3,307,591	8,502,182
Low-Skill Working Hours	DADS	475,438	72,655	594,771	8,877	20,350	50,306	96,841	213,750	588,840	40,789	82,043	198,556
Low-Skill Hourly Wage	DADS	475,438	12	3	10	12	13	96,841	13	3	11	12	14
Low-Skill Wage Bill ($W_\ell X_\ell$)	DADS	475,438	962,397	9,096,444	103,932	239,494	597,769	96,841	2,904,784	8,639,068	489,212	1,007,062	2,494,455
High-Skill Working Hours	DADS	475,438	40,354	444,336	3,454	6,824	17,212	96,841	129,998	558,184	15,113	32,657	82,727
High-Skill Hourly Wage	DADS	475,438	23	9	17	21	26	96,841	25	7	20	23	27
High-Skill Wage Bill ($W_h X_h$)	DADS	475,438	1,028,991	12,120,242	66,437	155,623	399,760	96,841	3,383,120	15,825,433	360,887	775,668	1,972,223
Equipment Stock ($W_e X_e$)	BRN	475,438	16,577,251	332,594,816	340,187	924,150	3,185,276	96,841	57,803,739	475,338,376	2,958,942	7,636,314	24,890,984
Exports - Total	Customs	475,438	3,648,894	52,623,201	0	0	170,442	96,841	14,217,365	80,351,406	112,017	1,034,075	5,637,990
Imports - Total	Customs	475,438	2,269,708	34,576,308	0	0	235,696	96,841	8,955,206	66,861,640	260,961	1,004,400	3,836,091
Imports - Intermediate products	Customs	475,438	1,678,365	24,086,984	0	0	128,939	96,841	6,586,778	43,398,599	151,553	676,140	2,756,554
Imports - Equipment products	Customs	475,438	531,716	11,940,700	0	0	932	96,841	2,160,954	21,786,515	3,627	69,098	488,002

Notes: Observations is the number of firm-year pairs in the manufacturing sector from BRN over the 1998-2007 period. The units for all variables are euros except for those involving working hours. The full sample include all firms used in the aggregation exercise of Section 5, which is conditional on firms with positive equipment capital stock and employing both low- and high-skill workers. The reduced sample is restricted to the set of firms used in the estimation of the firm-level production elasticities (Table 3), which further conditions on firms importing both intermediate and equipment products (and having both high-skill and low-skill stayers).

3.2 Variable definitions

In this section we provide a brief overview of how we compute the main variables of interest used in the analysis. For more details see Appendix D.

Firm revenue. Our baseline model doesn't include intermediate inputs. Revenue and value added are therefore identical in the model, and we use value added in the data as the counterpart to revenue in the model. Annual value added for each firm i is obtained from FICUS as gross output less intermediate expenditures, where gross output is defined as the sum of sales and changes in inventories while intermediates include all the purchases of the firm.

Skill groups. The DADS data does not provide information on the education of workers, but includes their detailed occupational codes. Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on their occupation, using occupational classifications in French social security data (CS - catégories socioprofessionnelles). High-skill workers are those employed as Managers, Middle managers and professionals, and Qualified workers. Low-skill workers are employed as Clerks and Blue-collar.

Change in firm-and-skill-specific wages and changes in employment. In the model, all workers in a skill group are identical but in the data workers within the same group vary in observables. To measure five-year firm-and-skill specific wage changes in the data (Δw_{hit} and $\Delta w_{\ell it}$), we correct changes in observed wages for both changes in returns to worker observable characteristics and changes in worker composition. To do so, we estimate a national Mincerian regression of log wage changes between each consecutive time periods $t-1$ and t. In this regression, we control for worker observables (age, sex, 2-digit CS category), including additionally a firm-skill-time effect, in the sample of workers in a given skill group who are employed by the same firm between $t-1$ and t.[8] We define the year-on-year change in the firm-and-skill-specific wage to be equal to the firm-skill-year fixed effect, that is, the average log change in wages controlling for observables on the estimation sample. We chain these year-on-year changes together to obtain five-year changes. We measure five-year changes in employment (Δx_{hit} and $\Delta x_{\ell it}$), by deflating changes in firm-and-skill-specific wage bills by the corresponding wage change (Δw_{hit} and $\Delta w_{\ell it}$).

The level and change in equipment stock. We measure the equipment stock on the basis of data on investments in equipment products, retrieved from BRN, and transaction-level

[8]We exclude from the data atypical workers and those who did not meet a minimum hours-worked threshold.

data on the imports of equipment products, obtained from the customs dataset.[9] We aggregate the transaction-level data to compute quantities and unit values for the imports in each equipment product classification and from each origin country for all firms. We compute changes in firm-level equipment prices as the average of equipment product-origin-firm-specific year-on-year log unit price changes weighed by Sato-Vartia weights, where we use the series of equipment prices from the French national accounts for all firms' domestic equipment purchases. We then chain these equipment price changes to construct a firm-specific equipment price level in each year (P^e_{it}). Equipment investment quantities are obtained by dividing firm-level equipment investment values by the firm-level equipment price. Equipment quantities are then used to obtain the equipment stock on the basis of the perpetual inventory method. Five-year log changes in equipment quantities are obtained directly from this series of firm-and-year-specific equipment stocks.

User cost of Equipment. We define the firm-level user cost of equipment W_{eit} (the effective rental price of equipment capital) as follows:

$$W_{eit} = P^e_{it}\left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3}\right)$$

where P^e_{it} is the firm-specific price of equipment products purchased by firm i in time t defined above, R^e_t is the required rate of return on equipment investments, and p^e_{st} and δ^e_{st} are the log price and depreciation rate of equipment products at the sector level from the national accounts, respectively.

4 The Estimation of Elasticities

In Section 2, we provided a theory that chacaterizes the elasticity of the observed skill premium and the labor share with respect to the equipment price for arbitrary constant returns to scale firm-level production functions. In this section, we assume that the firm-level production function belongs to the CRESH family presented in Example 2 (specified in Equation (8)) with three factors (low/high skilled labor and equipment capital), and estimate this production function using our micro data. We additionally use the data to estimate the key remaining elasticities that we need to perform the aggregation exercise presented in Section 2.

[9] In the customs data, we define equipment products as the ones belonging to category 4 of the BEC Rev. 4 classification (*Capital goods (except transport equipment), and parts and accessories thereof*). For more details, see Section D.2 in the Appendix.

4.1 Estimating Equations

To perform the aggregation, we require the following micro elasticities: the demand elasticity across sectors η, the demand elasticity across firms within each sector ε, and the factor-specific elasticities σ_ℓ, σ_h, and σ_e that together characterize the CRESH production function. We impose $\eta = 1$ and estimate the remaining parameters.

Demand Elasticity ε. Given CES demand across firms within each sector, we have

$$\Delta y_{it} = \alpha_{st} - \varepsilon \Delta p_{it} + \Delta \phi_{it} \tag{35}$$

where α_{st} is a sector-s and time-t fixed effect, Δp_{it} is the change in firm i's log price, and $\Delta \phi_{it}$ is the log change in firm i's demand shifter.

In our baseline, we estimate Equation (35) using five-year changes in export quantities and prices at the firm level, where $\Delta x_t \equiv x_t - x_{t-5}$ for any variable x. We use export data since we do not observe unit prices for domestic sales. For each exporting firm i, we measure the firm-specific year-on-year log export price change as the average of product- and export-market-specific log unit price changes weighed by Sato-Vartia weights. We then chain together these year-on-year price changes to obtain five-year price changes, Δp_{it}. We then measure firm i's five-year export quantity change Δq_{it} by deflating firm i's five-year change in log total export revenue by the constructed export price index Δp_{it}.

Production Function Elasticities σ_ℓ, σ_h, and σ_e. The production function specified in Equation (8) implies that firm i's factor demand satisfies

$$\Delta x_{eit} - \Delta x_{\ell it} = \beta_{\ell t} - \sigma_\ell \left(\Delta w_{eit} - \Delta w_{\ell it}\right) + \left(\frac{\sigma_\ell}{\sigma_e} - 1\right) \left(\tfrac{\varepsilon}{\varepsilon - 1} \Delta r_{it} - \Delta x_{eit}\right) + \nu_{\ell it} \tag{36}$$

$$\Delta x_{eit} - \Delta x_{hit} = \beta_{ht} - \sigma_h \left(\Delta w_{eit} - \Delta w_{hit}\right) + \left(\frac{\sigma_h}{\sigma_e} - 1\right) \left(\tfrac{\varepsilon}{\varepsilon - 1} \Delta r_{it} - \Delta x_{eit}\right) + \nu_{hit}, \tag{37}$$

where x_{fit} is the logarithm of employment of factor f in firm i at time t and where we have replaced the change in firm i's output with the change in revenue r_{it} using the structure imposed by the CES demand for firm products. The structural residuals $\nu_{\ell it}$ and ν_{hit}, therefore, depend not only on changes in firm-specific factor-augmenting productivities Δz_{fi}, but also the change in firm i's demand shifter and the change in the relevant sectoral price index. See the proof of Equations (36) and (37) in Appendix B.

In our baseline, we estimate equations (36) and (37) using five-year changes in factor employment, prices, and revenues. We measure firm i's employment and payment of each factor as described in Section 3.2 and we obtain the respective five-year changes by chaining the firm-specific year-on-year log changes. Our approach ensures that the measurement of firm-level wage change for each skill group is not affected by by changes in the composition of the firm's workforce.[10]

4.2 Estimation Approach and Instruments

We first describe our overall estimation strategy and then describe our instruments.

Estimation Strategy. We first estimate ε using Equation (35) via 2SLS. We then use the implied value ε to estimate σ_ℓ, σ_h, and σ_e using equations (36) and (37) via GMM, since the three structural parameters we aim to identify depend on parameter estimates from both equations.[11] We bootstrap the full estimation so that our confidence intervals for σ_ℓ, σ_h, and σ_e incorporate the dispersion of our estimated ε.

Instrument for Demand Elasticity Estimation. In general, changes in firm-level price Δp_{it} my covary with firm-specific changes in demand shifters $\Delta \phi_{it}$ due, e.g., to shocks to the quality of firm products. This creates an endogeneity problem since changes in demand shifters are correlated with the residuals on the right hand sides of Equations (36) and (37). To address this problem, we construct an instrument that shifts firm i's marginal cost independently of changes in its demand shifter.

We leverage the differences in firm-specific import exposure across source countries and changes in real exchange rates between France and its trading partners. If firm i obtains a large share of its imports from a given source country c and the real exchange rate of country c depreciates relative to France, then firm i's cost of imports falls, thereby reducing firm i's marginal cost. We refer to this as the real-exchange-rate instrument and define it formally as

$$\Delta RER_{it} = \sum_{c} \frac{M_{ci,t-5}}{M_{i,t-5}} \Delta \ln \left(NER_{ct} \cdot \frac{CPI_{ct}}{CPI_{FR,t}} \right) \tag{38}$$

Here, NER_{ct} is the nominal exchange rate (defined as euros per country c currency), CPI_{ct} is the consumer price index (CPI) in the origin country c and $CPI_{FR,t}$ is the CPI of France in year t. Exposure shares are constructed as firm i's total import value in year $t-5$ from country c, $M_{ci,t-5}$, relative to total imports by firm i in $t-5$, $M_{c,t-5}$. A real depreciation of country c's currency relative to the euro is reflected in an increase in ΔRER_{it}.

[10] There is an alternative approach to estimating σ_f that does not require the inclusion of output (or revenue) as an independent variable. There are two benefits of using equations (36) and (37). First, the inclusion of additional factors of production leave these estimating equations unchanged. Hence, our estimates of these parameters is robust to additional factors. Second, these equations can be estimated using 2SLS.

[11] In an analysis of the robustness of our results, we estimate equations (36) and (37) separately using 2SLS.

Instruments for Firm-Level Production Elasticities Estimation. The structural residuals $\nu_{\ell it}$ and ν_{hit} in Equations (36) and (37) depend on changes in firm-specific factor-augmenting productivities Δz_{fi}, and on changes in firm i's demand shifter and changes in the relevant sectoral price index.[12] To address these endogeneity problems, we construct instruments that shift firm i's (i) the cost of equipment relative to low-skilled labor, and (ii) the cost of equipment relative to high-skilled labor, and (iii) revenue relative to its equipment stock. These instruments should be independent of changes in the firm's factor-augmenting productivities and its demand shifter.

Our first instrument is intended to shift firm i's equipment price. Its logic is similar to our real-exchange-rate instrument. A decrease in the transport cost of shipping type-k equipment from origin country c to France will reduce the equipment cost of a firm i for which this country-equipment pair constitutes a large share of equipment imports. Following Hummels et al. (2014), we create an instrument that leverages differences in firm-specific equipment import exposure across source countries and predicted changes in shipping costs between France and its trading partners induced by changes in oil and jet fuel prices. Using French micro-data, we obtain mode-of-transport frequencies and weight-value ratios by HS6 product. Exploiting different sources, we retrieve data on oil and jet fuel prices, weighted distances between France and all other countries, and transportation charge elasticities. For each origin country c and equipment product $k \in \mathcal{K}_e$, we compute predicted five-year changes in transport costs and construct a firm-specific average weighting by initial equipment import shares. We refer to this as the equipment transit-cost instrument and define it formally as

$$\Delta ETC_{it} = \sum_{c} \sum_{k \in \mathcal{K}_e} \frac{M^e_{cki,t-5}}{M^e_{i,t-5}} \Delta \ln TC_{ckt} \tag{39}$$

Here, $\ln TC_{ckt}$ is the predicted transport cost of equipment product k from country c to France (given oil and fuel prices, the distance between France and country c, and the predicted shipping mode). Exposure shares are constructed as firm i's import value of equipment products in year $t-5$ from country c, denoted by $M^e_{cki,t-5}$, relative to the value of total imports of equipment products in year $t-5$ by firm i, denoted by $M^e_{c,t-5}$.

Our second instrument is intended to shift firm revenues. Its logic is also similar to our real-exchange-rate instrument. An increase in productivity for origin country c in producing intermediate input k will reduce the marginal cost of a firm i for which this country-input pair constitutes a large share of intermediate imports. Again following Hummels et

[12] See the derivation of Equations (36) and (37) in Appendix B.

al. (2014), we create an instrument that leverages differences in firm-specific intermediate good import exposure across source countries and intermediates and changes in world export supplies. For each exporting country c and HS6 product k in the set of intermediate products $\mathcal{K}_I$, we measure the value of total exports towards all countries excluding France in year t, denoted by Exp_{ckt}. We use this to construct a firm-i-specific average of changes in export supplies weighting by initial intermediate good and origin country import shares. We refer to this as the world-export-supply instrument and define it formally as

$$\Delta WES_{it} = \sum_{c} \sum_{k \in \mathcal{K}_I} \frac{M^I_{cki,t-5}}{M^I_{i,t-5}} \Delta \ln Exp_{ckt} \tag{40}$$

Here, $\Delta \ln Exp_{ckt}$ is the change in the exports of intermediate product $k \in \mathcal{K}_I$ from origin country c to the rest of the world (excluding France). Exposure shares are constructed as firm i's import value of intermediate good k in year $t-5$ from country c, $M^I_{cki,t-5}$, relative to the value of total imports of intermediate goods in year $t-5$ by firm i, $M^I_{c,t-5}$.

Our third instrument is intended to shift the wage of skilled and unskilled workers facing each firm. A growth in French output of a sector s that is particularly skilled (or unskilled) intensive will raise the skilled (unskilled) wage in a region with a large share of employment in that sector. This will raise the effective skilled (unskilled) wage facing firm i if it has a large share of its employment in such regions. Following this intuition, our third instrument leverages differences in the spatial compositions of firm production, the industrial mix of these French regions, and the skill intensities and growth of these industries. We refer to this as the skill-specific-wage instrument and define it formally as

$$\Delta SSW_{it} = \sum_{r} \sum_{s} \left(\frac{X^r_{ni,t-5}}{X_{ni,t-5}} \right) \cdot \left(\frac{X^r_{ns,t-5}}{X^r_{n,t-5}} \right) \cdot \left(\frac{X^r_{hs,t-5}}{X^r_{\ell s,t-5}} \right) \cdot \Delta \ln GO_{st} \tag{41}$$

Here, $\Delta \ln GO_{st}$ is the change in gross output for sector s at the national level in France. Exposure shares are constructed as the product of three components: (i) firm i's employment in region r, $X^r_{ni,t-5}$, relative to its total employment, $X_{ni,t-5}$ across all domestic regions; (ii) employment in sector s in region r, $X^r_{ns,t-5}$, relative to total employment in region r, $X^r_{n,t-5}$; and (iii) employment of skilled labor in sector s, $X^r_{hs,t-5}$, relative to employment of unskilled labor in sector s, $X^r_{\ell s,t-5}$, both defined at the national level.

4.3 Estimation Results

Demand Elasticity Estimates. Since we estimate Equation (35) using export quantities and prices, in our baseline we restrict the sample of firms to those exporting on average

Table 2: Between-Firm Demand Elasticity Estimates

	OLS	2SLS				
	(1)	**(2)**	(3)	(4)	(5)	(6)
ε	0.978	**2.920**	3.627	2.206	2.929	2.321
	(0.010)	**(0.794)**	(1.350)	(0.452)	(0.803)	(0.514)
Observations	72,202	**45,435**	55,101	45,435	45,435	44,992
Year FE	-	**-**	-	Yes	-	-
Year-Sector FE	Yes	**Yes**	Yes	No	Yes	Yes
Controls	No	**No**	No	No	Yes	No
Exp/GO$\geq$ 5%	Yes	**Yes**	No	Yes	Yes	Yes
IV	-	**RER**	RER	RER	RER	RER+SSW
KP F stat	-	**12.75**	9.81	16.39	12.60	9.446

Notes: This table reports estimation results of Equation (35). The dependent variable is the 5-year change in firm i's export quantity and the independent variable is $-1\times$ the corresponding change in firm i's export price. Columns 2-6 use the RER instrument defined in Equation (38) and column 6 additionally uses the SSW instrument defined in equation (41). Year fixed effects are included in column 4 and year-sector fixed effects are included in all other columns. Controls indicates the inclusion of the average import share in the pre-sample period (1996-7) interacted with a time-varying coefficient. Exp/GO$\geq$5% indicates that the sample is restricted to firms that export at least 5% of gross output. We report the Kleibergen-Paap rk Wald (KP F stat) of the first stage regression. Robust standard errors are clustered at the firm level and reported in parenthesis.

at least 5% of their gross output over the period. Column 1 of Table 2 displays the OLS estimate of ε in Equation (35) while columns 2–6 display the 2SLS estimates together with standard errors clustered by firm. Whereas the OLS estimate is close to one, the 2SLS estimates range from 2.2 to 3.6. Column 2 represents our baseline estimate, $\varepsilon = 2.92$, which is close to the median value of 2.7 reported by Broda and Weinstein (2006) for products at the five-digit level. In our baseline specification, the first-stage KP F-statistic is approximately 13 and it remains broadly stable across specifications.

In column 3, we omit the sample restriction that firms export on average at least 5% of their gross output and the estimate increases to 3.6. In column 4 we replace year-sector fixed effects with year fixed effects, and our estimate falls slightly to 2.2. In column 5, we additionally control for the average import share interacted with time dummies, following Adão et al. (n.d.). Our estimated elasticity of substitution is largely unchanged including this set of controls. Finally, in column 6, we use both our real-exchange-rate instrument and our skill-specific-wage instrument and our estimate falls slightly to 2.3.[13]

Firm-Level Production Elasticity Estimates. The top panel of Table 3 displays coefficient estimates in Equations (36) in the odd columns and (37) in the even columns. The bottom panel displays the corresponding estimates of the structural parameters of interest, σ_ℓ, σ_h,

[13] Table E.1 replicates Table 2 using an alternative instrument in which exposure shares are constructed as firm i's import value of intermediate goods only. Results are robust.

Table 3: Firm-Level Production Elasticities

	(1)	**(2)**	(3)	(4)	(5)	(6)	(7)	(8)
	$\Delta x_e - \Delta x_\ell$	$\Delta x_e - \Delta x_h$	$\Delta x_e - \Delta x_\ell$	$\Delta x_e - \Delta x_h$	$\Delta x_e - \Delta x_\ell$	$\Delta x_e - \Delta x_h$	$\Delta x_e - \Delta x_\ell$	$\Delta x_e - \Delta x_h$
$\Delta w_e - \Delta w_\ell$	**-0.968**		-0.969		-0.967		-1.001	
	(0.179)		(0.165)		(0.179)		(0.456)	
$\frac{\varepsilon}{\varepsilon-1}\Delta r - \Delta x_e$	**-0.129**		-0.113		-0.140		-0.0123	
	(0.061)		(0.055)		(0.070)		(0.163)	
$\Delta w_e - \Delta w_h$		**-0.841**		-0.864		-0.824		-0.798
		(0.147)		(0.145)		(0.143)		(0.339)
Observations	**35,142**		35,142		35,142		35,188	
Year FE	**Yes**		Yes		Yes		No	
Year-Sector FE	**No**		No		No		Yes	
σ_ℓ	**0.9679**		0.9689		0.9674		1.0012	
	(0.179)		(0.165)		(0.179)		(0.456)	
σ_h	**0.8412**		0.864		0.8236		0.7979	
	(0.147)		(0.145)		(0.143)		(0.339)	
σ_e	**1.1107**		1.0921		1.1255		1.0136	
	(0.247)		(0.215)		(0.252)		(0.672)	
$\Pr[\sigma_\ell \leq \sigma_h]$	**0.005**		0.005		0.005		0.015	
ε	**2.92**		2.321		3.627		2.92	

Notes: This table reports regression coefficients for the system of Equations (36) and (37) estimated using GMM including year effects (columns 1-6) or year-sector effects (columns 7-8) for a given value of ε. Corresponding point estimates of σ_ℓ, σ_h, and σ_e are displayed. Bootstrap standard errors are reported in parenthesis, obtained by bootstrapping the estimation of ε and of this system 200 times and $\Pr[\sigma_\ell \leq \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h; if $\sigma_\ell > \sigma_h$ in each of our 200 replications, we set this value to 1/200.

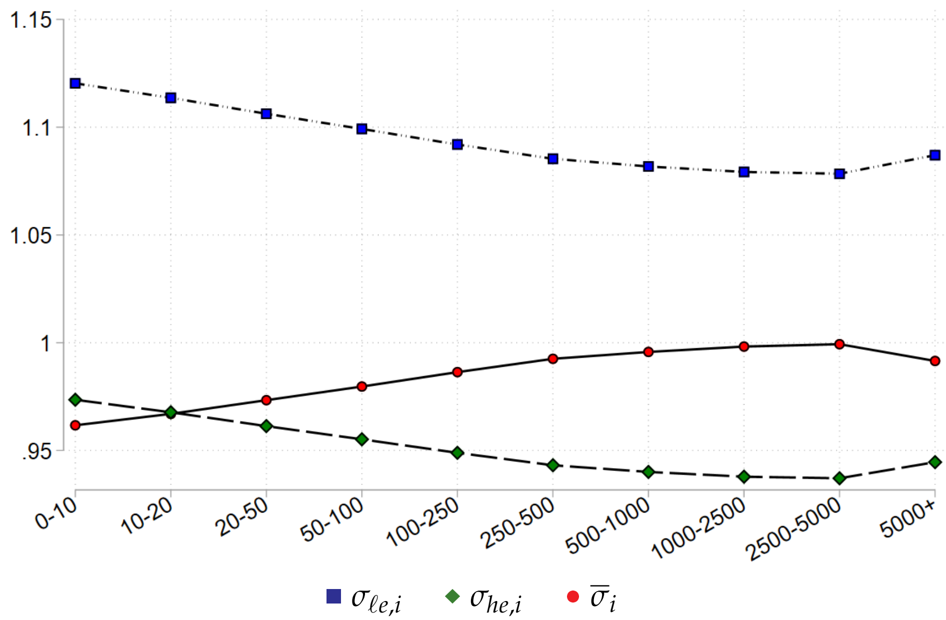
and σ_e, estimated using GMM for different values of ε and using different fixed effects. Columns 1 and 2 displays our baseline estimates of $\sigma_\ell = 0.97$, $\sigma_h = 0.84$, and $\sigma_e = 1.11$, using our baseline estimate of $\varepsilon = 2.92$ and including year effects. We find that $\sigma_\ell > \sigma_h$, which implies that equipment is more substitutable with low- than high-skilled labor: at the firm level, production exhibits equipment-skill complementarity. This difference in parameters is statistically significant, as shown in the bottom panel of Table 3.

In columns 3 and 4, we use the lowest estimate of ε reported in Table 2 (conditional on sector-year fixed effects) and in columns 5 and 6 we use its highest estimate. In columns 7 and 8 we include year-sector effects instead of year effects. Across all columns, we find that $\sigma_e > \sigma_\ell > \sigma_h$ and that the difference between σ_ℓ and σ_h is statistically significant.[14]

Figure 1 displays the implications of these estimates for firm-level elasticities of substitution between low-skilled labor and equipment $\sigma_{\ell e,i}$ and between high-skilled labor and equipment $\sigma_{he,i}$ across the firm size distribution, as given by Equation (9). The levels of $\sigma_{\ell e,i}$ and $\sigma_{he,i}$ decline across the firm-size distribution because larger firms have higher

[14] In Table E.2 in Appendix, we perform a joint GMM estimation of the demand elasticity and the production elasticities on the subsample of firms that are simultaneously importers and exporters. Our conclusions are mostly unchanged and we obtain very similar estimates for all parameters, featuring in particular the same ranking for production elasticities.

Figure 1: Firm Size and Elasticities of Substitution Between Factors



Notes: This figure displays firm-level elasticities of substitution between low-skilled labor and equipment $\sigma_{\ell e,i}$ and between high-skilled labor and equipment $\sigma_{he,i}$ across the firm size distribution. The elasticities are constructed using Equation (9), our baseline estimates of σ_ℓ, σ_h, σ_e and the values of factor intensities θ_{fi}, measured in 2003, for each f constructed within the relevant firm-size bin.

equipment intensities θ_{ei}, as shown in Panel (c) of Figure 2, and σ_e is greater than both σ_ℓ or σ_h, as shown in Table 3. This implies that $\overline{\sigma}_i$ increases in firm size, as shown in Figure 1.

5 Aggregation

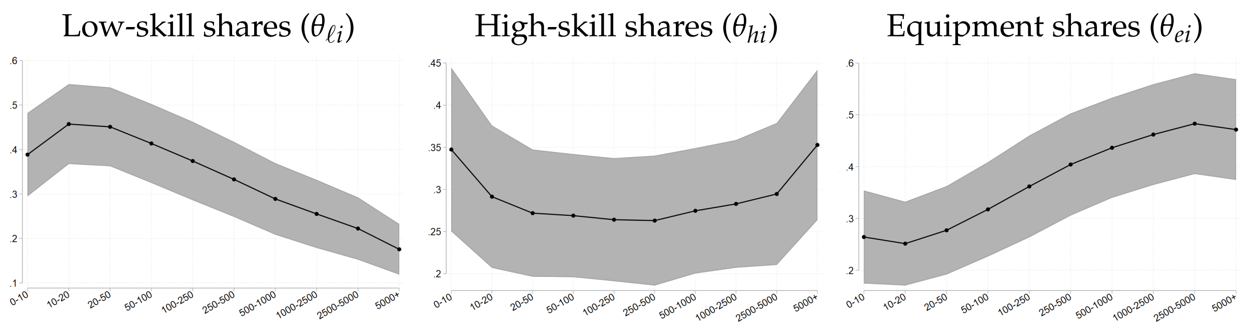
Before discussing the quantitative aggregation results in Section 5.2, we begin by presenting two cross-sectional facts that play a key role in shaping the aggregate patterns of capital-skill complementarity. As with Section 4, we consider the model with three factors (low/high skilled labor and equipment capital).

5.1 Stylized Facts

We document two sets of stylized facts ion how factor intensities and skill premia vary across French manufacturing firms. We measure factor intensities, θ_{fi} for skilled and unskilled labor using wage bills for skilled and unskilled workers. For equipment, we use the payments to equipment $W_{eit}X_{eit}$ constructed as described in Section 3.2 above.

Figure 2 shows, for each factor f, the average factor intensity in each bin of firm employment size, together with its corresponding standard deviation. Low-skill labor inten-

Figure 2: Firm Size and Factor Shares



Notes: The figure plots the average factor intensity across firms and the corresponding standard deviation (in grey) by size bins over the 1998-2007 period.

sity declines monotonically and substantially in firm size, from approximately 50% for the smallest firms to below 20% for the largest. Larger firms are, therefore, more intensive in the combination of high-skill labor and equipment. High-skill intensity is flat across most of the firm-size distribution except for the first two and the last two size bins. Equipment intensity, on the other hand, rises with firm size throughout the size distribution from below 20% to above 40%.

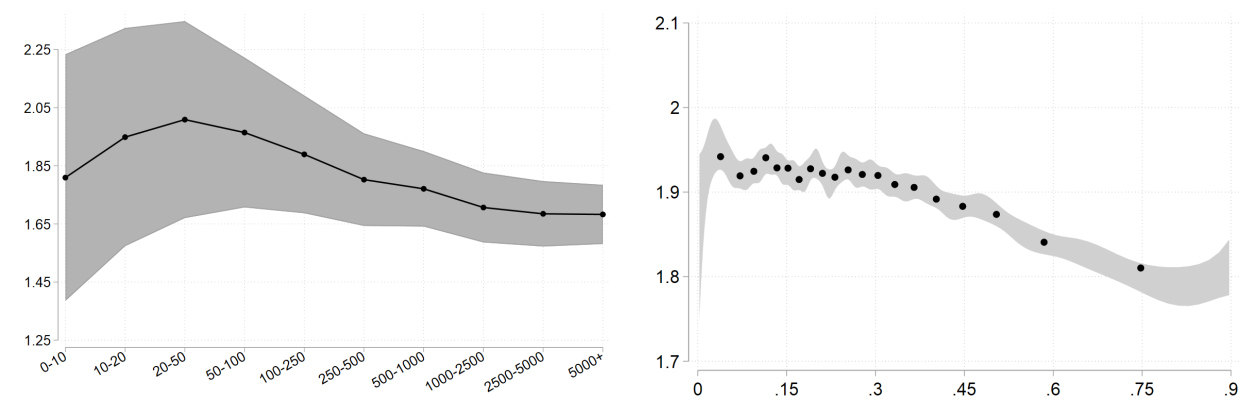
Figure 2 implies a correlation between equipment intensity and the skill intensity of labor across the firm size distribution. Such a correlation has important implications for the impact of a decline in equipment prices on the skill premium, even in the absence of capital-skill complementarity at the firm level. In this case, a decline in equipment prices induces factors to reallocate towards equipment-intensive firms and raises the relative demand for skilled labor, since equipment-intensive firms are relatively skill intensive. As we saw in our theoretical analysis in Section 2, in response to falling equipment prices this reallocation mechanism leads to an increase in the shadow and, therefore, observed skill premium.

We next document the pattern of observed skill premia across the firm-size distribution and by equipment intensity. We construct the observed skill premium averaging across all sample years. Figure 3a displays the results by firm-size bins, including the corresponding standard deviation. Among small firms, the skill premium is increasing in size up to firms with fifty employees. Thereafter, the skill premium is monotonically decreasing in firm size. Figure 3b displays a binscatter of the average skill premium by equipment intensity, together with a confidence band. Once again, the skill premium is monotonically decreasing in equipment intensity. Thus, a decline in equipment prices reallocates labor towards firms with high equipment intensity and lower skill premia. Our theoretical analysis in Section 2 predicts that, in response to falling equipment prices,this reallocation mechanism leads to a decrease in the observed skill premium when holding shadow wages fixed.

Figure 3: Skill Premium by Size and Equipment Intensity

(a) By Firm Size

(b) By Equipment Shares (θ_{ei})



Notes: Panel (a) plots the average skill premium across firms by size bins and the corresponding standard deviation (in grey). Panel (b) plots the average skill premium across firms by bins of equipment intensity and the corresponding confidence band (piecewise polynomial of degree 3 with 3 smoothness constraints). The firm-level skill premium is measured as the ratio of the observed average hourly wage of high skill vs low skill workers employed by firm, computed as the wage bill divided by the number of hours worked in each skill group. Averages are computed over the 1998-2007 period.

5.2 Aggregate Results

We aim to use our framework to study the implications of observed shocks to firm-level equipment prices on the skill premium. In line with our theory, we consider several different settings. We first rely on Proposition 1 to consider a standard setting in which the economy faces a uniform fall in the relative price of equipment, in line with the exercise in Krusell et al. (2000). We then use the framework developed in Section 2.4 to use our data to examine the effect of the observed, heterogenous firm-level price shocks, with or without accounting for the contemporaneous shocks to labor wedges.

5.2.1 Uniform Shock to the Equipment Price

We first quantify the effects of uniform changes in equipment prices on the skill premium and the labor share over time in France. This exercise is in line with the aggregate production function framework of Krusell et al. (2000) that does not feature wedges in the prices faced by different firms. In line with their exercise, we calibrate the magnitude of the shock using aggregate data, relying on the changes in the price of equipment relative to minimum wage based on the series produced by the INSEE. Throughout, we consider the three-factor model with CRESH technology.

We first compute the elasticity of the skill premium and the labor share with respect to the price of equipment following the results in Section 2.3. Table 4 displays the elasticity of the observed skill premium with respect to equipment prices $d\psi / dw_e$ constructed using Equation (18), the elasticity of the labor share with respect to equipment prices $d \ln \theta_n / dw_e$ constructed using Equation (23), and the aggregate elasticity of substitution between labor and equipment σ_{ne}^* constructed using Equation (11), under various parameterizations, all calculated using shares averaged across all years.[15]

Column 1 is our baseline specification, corresponding to the micro elasticities presented in Columns 1 and 2 of Table 3, and using average shares over the sample period. The results imply that a decrease in the equipment price of 1% increases the skill premium by approximately 0.05% and decreases the rescaled labor share by 0.36%.[16]. The fact that the labor share rises implies that the elasticity of substitution between equipment and labor is greater than one. The third row of Table 4 confirms this; σ_{ne} is approximately 1.4.

Columns 2 and 3 replace ε with our lower and upper bound of estimates from Table 2 and the corresponding estimates of σ_ℓ, σ_h, and σ_e from Columns 3-4 and 5-6 of Table 3, respectively. Finally, Column 4 uses values of σ_ℓ, σ_h, and σ_e estimated including year-sector fixed effects from Columns 7 and 8 of Table 3. Qualitative results—a decline in the price of equipment raises the skill premium and lowers the labor share—are robust across specifications. Quantitative elasticities are largely robust as well, with the elasticity of the skill premium ranging from -0.045 and -0.083 and the elasticity of the labor share ranging from 0.24 and 0.50.

Next, we use our framework to quantify the contribution of the *observed* fall in the relative price of equipment in France to the evolution of the skill premium and the labor share in France in our sample period. Doing so requires, in the addition to the elasticity of the skill premium with respect to the equipment price, presented in Table 4, measuring the change in the equipment price in the data. Because we normalize the shadow wage of low-skilled labor in our theory in Section 2, we must measure this relative price change.

As a starting point we consider uniform changes in the equipment price and the low-skill shadow wage aggregating industry-specific equipment investment prices taken from French national accounts, industry-specific depreciation rates from EU-KLEMS, and the nominal hourly minimum wage ('Smic') sourced from the French Ministry of Labor. The cumulative log change in the relative equipment price is approximately 58 log points over the period 1997-2007, with changes in equipment prices and changes in low-skill shadow

[15]Table E.3 reports the aggregate elasticities of substitution for factor demand and factor payments for our baseline specification.

[16]The labor share is rescaled by the equipment share $(1 - \theta_n)$, which is approximately 0.53 on average over the period.

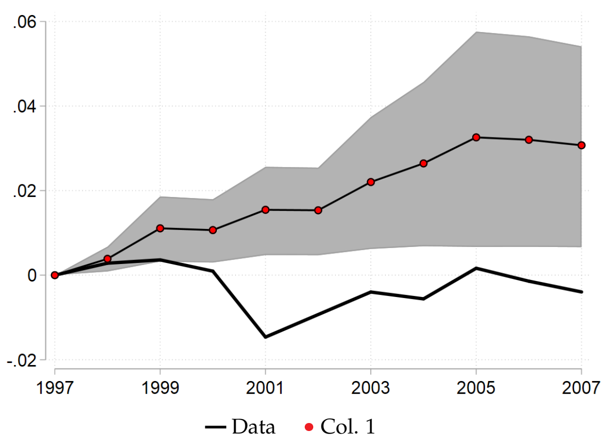
Table 4: Aggregate elasticities

	(1)	(2)	(3)	(4)
$\frac{d\psi}{dw_e}$	**-0.054**	-0.045	-0.058	-0.083
	[-0.093, -0.013]	[-0.074, -0.012]	[-0.101, -0.011]	[-0.168, -0.013]
$\frac{1}{1-\theta_n}\frac{d\ln\theta_n}{dw_e}$	**0.36**	0.244	0.497	0.326
	[0.091, 0.762]	[,]	[,]	[,]
σ^*_{ne}	**1.350**	1.239	1.482	1.315
	[1.090, 1.737]	[1.002, 1.549]	[1.150, 2.774]	[0.806, 2.357]
ε	**2.92**	2.321	3.627	2.92
η	**1**	1	1	1
σ_ℓ	**0.968**	0.969	0.967	1.001
σ_h	**0.841**	0.864	0.824	0.798
σ_e	**1.111**	1.092	1.126	1.014

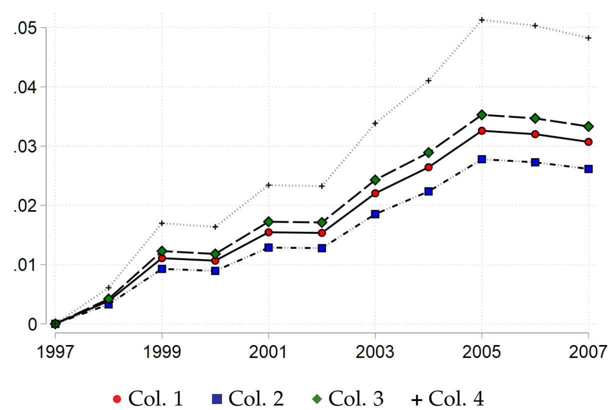
Notes: $d\psi/dw_e$ and $d\ln\theta_n/dw_e$ are the elasticities of the observed skill premium and the labor share with respect to the equipment price and σ^*_{ne} is the elasticity of substitution between equipment and labor. These elasticities are constructed using firm factor shares averaged across all sample years. Column 1 is our baseline, columns 2 and 3 use our lowest and highest estimates of ε, and column 4 uses our estimates of σ_ℓ, σ_h, and σ_e including year-sector FE. The table displays 95% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

Figure 4: Predicted Skill Premium, Uniform Shock

(a) Baseline (Col. 1 of Table 4)



(b) Alternative Estimates



Notes: Panel (a) plots the evolution of the predicted skill premium (ψ) in response to the observed fall in the equipment price relative to the low skill shadow wage. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage ('Smic') sourced from the French Ministry of Labour. The figure displays a 95% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 4.

wages each accounting for roughly half of this decline (see details in Table E.4 in the appendix).

Panel (a) of Figure 4 and the left panel of Table 5 show the model-predicted impact of the fall in the relative equipment price on the skill premium between 1997-2007 (along with the associated 95% confidence interval). To compute the predicted change in the skill premium between years $t-1$ and t, we compute the aggregate elasticities using the distribution of factor intensities in year $t-1$ and use the relative equipment price change between $t-1$ and t as the corresponding shock. We then compute the implied cumulative change in the skill premium starting from 1997. The confidence interval is constructed by re-estimating the micro-level elasticities of substitution across bootstrapped samples of the data (200 times). Between 1997 and 2007, the decline in the price of equipment relative to the shadow wage of low-skill labor generates a 3.1 log point increase in the skill premium, which is statistically different from zero. Panel (b) displays the robustness of our quantitative results across the estimation specifications displayed in Table 3. Consistent with the elasticities in Table 4, we obtain very similar results across specifications.

Figure E.1 in the appendix displays the corresponding results for the predicted change in the (relative) labor share in France over the period. Between 1997 and 2007, the decline in the price of equipment relative to the shadow wage of low-skill labor generates a 21.2 log point decrease in the relative labor share (rescaled by the equipment share), which is close to the 17 log point decrease that we observe in our data. The predicted change is approximately equivalent to a 10% fall in the actual labor share.[17]

Table 5: Predicted Change in the Skill Premium and Labor Share, 1998-2007

	Homogeneous Shocks		Heterogeneous Shocks	
	(1) $\Delta\psi$	(2) $\Delta \ln \theta_n/(1-\theta_n)$	(3) $\Delta\psi$	(4) $\Delta \ln \theta_n/(1-\theta_n)$
Equipment	0.031	-0.212	0.064	-0.170
	[0.007, 0.054]	[-0.449, -0.055]	[0.036, 0.094]	[-0.367, -0.051]
Equipment & Labor			0.01	-0.167
			[-0.024, 0.026]	[-0.38, -0.041]
Labor wedges			*-0.054*	*0.003*
Residual			*-0.014*	*-0.004*
Observed	-0.004	-0.171	-0.004	-0.171

Notes: This table reports the predicted log change in the skill premium (ψ) and relative labor share ($\theta_n/(1-\theta_n)$) in response to the observed fall in the equipment price relative to the low skill shadow wage. The first two columns present the results obtained using uniform shocks computed from national account data. Columns 3 and 4 report the estimates obtained using heterogeneous price shocks.

We obtain similar results when we perform a simpler exercise multiplying the total cumulative change in the relative equipment (rental) price by the average aggregate elas-

[17]Note that we express the observed change in the labor share relative to the equipment share and in log changes. The observed change is comparable to the change in percentage points found by Cette et al. (2019) for the business sector over the corresponding period.

ticity over the entire period from column 1 of Table 4. The results are displayed in Table E.4 and they deliver virtually identical conclusions over the period for both the skill premium and the labor share. At the same time, this approach allows us to extend the time frame of the analysis and obtain the predicted changes over almost three decades. Our model generates an approximately 6.9% rise in the skill premium and a 36% fall in the relative labor share in France between 1990 and 2015.

5.2.2 Heterogenous Shocks to Equipment Prices and Labor Wedges

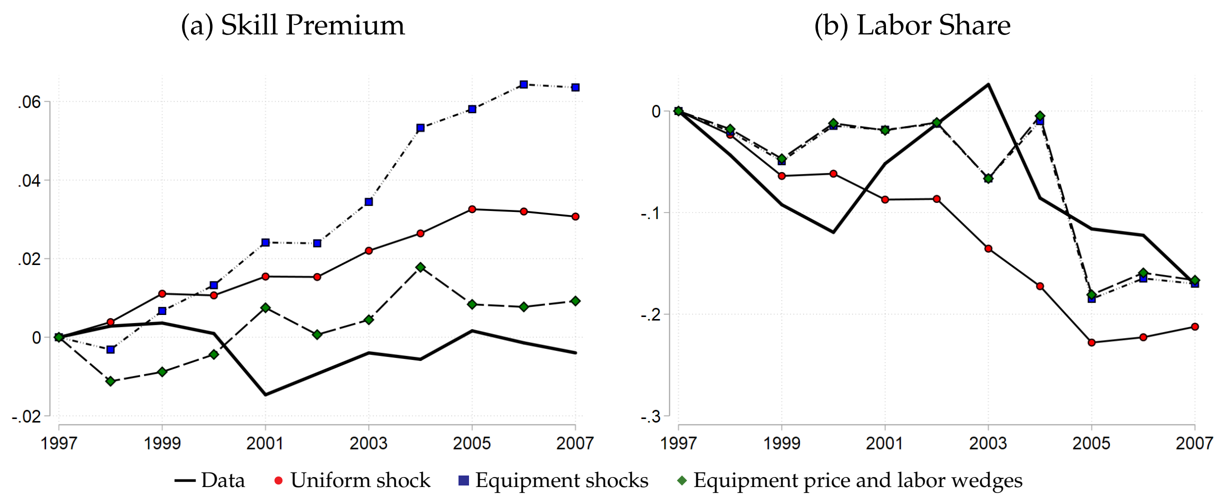
We next use the theory presented in section 2.4 to account for the heterogeneity in shocks experienced by different firms. In particular, we measure firm-specific equipment prices and labor wedges as outlined in Appendix A.1. We then construct model-consistent year-on-year changes in the skill premium and the (relative) labor share following the methodology in Appendix A.3.

The results of the exercise are shown in the right panel of Table 5 and Figure 5. Compared to the case of the uniform shock studied before, the decline in the firm-specific relative price of equipment now generates a larger 6.4 log point increase in the skill premium. This result is driven by the positive covariance between prices changes and equipment intensity, i.e., firms that employ a higher share of equipment are subject to larger falls in the relative price of equipment (Figure E.2a). This reinforced effect on the skill premium holds against the backdrop of the virtually flat pattern of the skill premium observed in the data over the period. In this respect, allowing for shocks that heterogeneously shift labor wedges across firms is key for replicating the observed pattern in the French data. Figure E.3 shows that the difference in the change between high-skilled and low-skilled wedges is negatively correlated with firm size measured as the share of aggregate low-skilled labor payments ($\Lambda^*_{\ell i}$), which, according to Equation (25), will result into a direct negative contribution to the skill premium. Indeed we find that the overall contribution of the labor wedges to the predicted skill premium is negative, which helps explain the stability of the skill premium in the French data over this period.

Table E.5 decomposes the predicted skill premium into all margins. It shows that the heterogeneous changes in equipment prices result into a within-sector reallocation across firms that contributes the most to the rise in the skill premium. The high-skilled wedges push down the skill premium, with the within-firm component delivering the largest negative contribution to the skill premium. Indeed not only are equipment intensive firms subject to larger falls in the relative price of equipment (Figure E.2a), but they also experience relatively lower increases in the high-skill labor wedges (Figure E.2b).[18]

When it comes to the labor share (Figure 5b), the model-consistent and firm-specific equipment price changes generate a fall in the labor share that is virtually identical to what we observed in the data, with labor wedges not contributing to the decrease.

Figure 5: Predicted Skill Premium and Labor Share, Heterogenous Shocks



Notes: The figure plots the evolution of the predicted skill premium (ψ, Panel a) and of the predicted labor share ($\theta_n/(1-\theta_n)$, Panel b) in response to firm-specific equipment prices and labor wedges shocks (see Section A.1 of the Appendix). As a comparison, the figure also plots the predicted changes in response to the uniform shock defined in the previous Section and the change in the skill premium and the labor share observed in the data.

6 Conclusion

Does the aggregate production technology feature capital-skill complementarity? If so, is this driven primarily by capital-skill complementarity at the firm level, or by a positive correlation between firm-level capital intensity and skill intensity? How do labor-market distortions affect capital-skill complementarity in the aggregate?

To make progress on these questions, we have characterized the response of the observed skill premium to changes in the price of equipment capital in a multi-factor, multi-sector framework featuring arbitrary constant returns to scale production technologies at the firm-level and arbitrary distortions to firm-level factor prices. We have shown that generalized definitions of the aggregate elasticities of substitution between factors—the aggregate elasticity of substitution in factor *demand* and the aggregate elasticity of substitution in factor *payments*—shape the answers to these questions.

[18] Additionally, the change in low-skill labor wedges tend to increase with equipment intensity.

To leverage our theoretical results, we used matched employer-employee data from France to document a set of stylized facts that play a central role in shaping aggregate capital-skill complementarity. We further relied on the data to measure firm-level factor intensities and wages for each skill group, and estimate the micro-level elasticities of substitution, under a particular functional form assumption on firm-level technology. We identified statistically significant capital-skill complementarity at the firm level.

The estimated elasticities, when combined with the observed distribution of factor intensities, imply that a 1% decline in the price of equipment generates a 0.054% increase in the observed skill premium in France. Together with an approximately 44 percent decline in the relative price of equipment, capital-skill complementarity generated an approximately 3.1% rise in the French skill premium between 1997 and 2007.

We additionally show that accouting for the heterogeneous changes in the price of equipment and the labor wedges can make quantitatively sizable contributions in this exercise. Within the period under consideration, while the former raises the predicted rise in the skill premium from 3.1% to 6.4%, the latter in fact lowers it substantially, to the point that the overall predicted change in the skill premium is only 1%. The change in the observed data is close to zero.

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A Aggregation in the Data

Notation Throughout, we use the notation $\Delta v_t \equiv v_t - v_{t-1}$ and $\widetilde{v}_t \equiv \frac{1}{2}(v_{t-1} + v_t)$ for any variable v.

A.1 Measuring Labor Wedges in the Data

Let $\widehat{W}_{f,t}$ denote the shadow price of each factor f at time t denoted in constant euros, as observed in the data, such that $\widehat{W}_{f,t} \equiv W_{f,t}\widehat{W}_{\ell,t}$ with $W_{\ell,t} \equiv 1$ for all t. Thus, we observe the price $\widehat{W}_{fi,t} = \widehat{W}_{f,t}T_{fi,t}$ of each factor f for firm i at time t, which we can readily use to measure the heterogeneity in the shocks to labor wedges. Since we do not observe the shadow price $W_{f,t}$ of labor $f \in \{\ell, h\}$, we can infer the corresponding labor wedges $T_{fi,t}$ subject to one multiplicative factor.

Let the unweighted mean of wedges $\overline{\overline{T}}_{f,t} \equiv \frac{1}{I}\sum_i T_{fi,t}$ be a uniform, unobserved wedge shock to the wage of type f workers. These two uniform shocks $\overline{\overline{T}}_{f,t}$ for $f \in \{\ell, h\}$ constitute the two main *residuals* in our framework that include forces such as changes in the factor augmenting productivities or other unobserved factor specific distortions and/or taxes. Defining the unweighted mean wage rate $\overline{\overline{W}}_{f,t} \equiv \frac{1}{I}\sum_i \widehat{W}_{fi,t}$ for $f \in \{\ell, h\}$ across firms, we have that it is the product of the shadow wage rate and the corresponding residual:

$$\overline{\overline{W}}_{f,t} = \widehat{W}_{f,t} \times \overline{\overline{T}}_{f,t}, \qquad f \in \{\ell, h\}.$$

We are going to decompose the wedge shocks into two components. First, we have the observed wedge shocks:

$$\widehat{T}_{fi,t} \equiv \frac{\widehat{W}_{fi,t}}{\overline{\overline{W}}_{f,t}} = \frac{T_{fi,t}}{\overline{\overline{T}}_{f,t}}, \qquad f \in \{\ell, h\}.$$

Note that since the shadow low-skilled wage is the numeraire we have:

$$W_{ei,t} \equiv \frac{\widehat{W}_{ei,t}}{\widehat{W}_{\ell,t}} = \widehat{T}_{ei,t} \times \overline{\overline{T}}_{\ell,t},$$

where we have defined:

$$\widehat{T}_{ei,t} \equiv \frac{\widehat{W}_{ei,t}}{\overline{\overline{W}}_{\ell,t}}.$$

Since we have let the shadow price of low-skilled labor be the numeraire, the assumption above also implies that we can state all prices in the model by letting $\overline{\overline{W}}_{\ell,t} \equiv W_{\ell,t} \equiv 1$

for all times t. Note that with these assumptions, we find that the shadow skill premium equals the unweighted mean of high-skilled workers $\overline{\overline{W}}_{h,t} \equiv W_{h,t}$.

Mapping the observed changed in the wedges to the framework developed in Section 2.4, we define

$$\Delta\widehat{\tau}_{f,t} \equiv \sum_i \widetilde{\Lambda}^*_{fi,t} \Delta\widehat{\tau}_{fi,t}, \tag{42}$$

$$\omega_{fi,t} \equiv \frac{\Delta\widehat{\tau}_{fi,t}}{\Delta\widehat{\tau}_{f,t}}, \tag{43}$$

where, as before, we have $\widehat{\tau}_{fi,t} \equiv \ln \widehat{T}_{fi,t}$ as defined above.

A.2 A Decomposition of Change in the Aggregate Skill Premium

Consider the change $\Delta\psi_t$ in log skill premium as time t. We can approximate the change in the skill premium to the second order of approximation in chnages in prices and wedges as:

$$\begin{aligned}
\Delta\psi_t &= \Delta w_{h,t} + \Delta \ln\left(\sum_i \Lambda_{hi,t} T_{hi,t}\right) - \Delta \ln\left(\sum_i \Lambda_{\ell i,t} T_{\ell i,t}\right), \\
&\approx \Delta w_{h,t} + \sum_i \widetilde{\Lambda}^*_{hi,t}\left(\Delta \ln \Lambda_{hi,t} + \Delta\tau_{hi,t}\right) - \sum_i \widetilde{\Lambda}^*_{\ell i,t}\left(\Delta \ln \Lambda_{\ell i,t} + \Delta\tau_{\ell i,t}\right), \\
&= \underbrace{\Delta w_{h,t} + \Delta\tau_{h,t} - \Delta\tau_{\ell,t}}_{\text{within-firm change}} + \underbrace{\sum_i \widetilde{\Lambda}^*_{hi,t}\Delta \ln \Lambda_{hi,t} - \sum_i \widetilde{\Lambda}^*_{\ell i,t}\Delta \ln \Lambda_{\ell i,t}}_{\text{across-firm change}},
\end{aligned}$$

where in the approximation in the second line, we have used Lemma 4 (presented in Appendix B), and in the last equality, we have used the definition (42) above. The first term accounts for within-firm changes in the skill premia, driven either by the general equilibrium change in the shadow skill premium, with a positive covariance between the shocks to the high-skilled wedges that positively covary with firms' shares of high-skilled factor payments, or a negative covariance between the shocks to the low-skilled wedges that positively covary with firms' shares of low-skilled factor payments. Note that we can write this term only in terms of observables:

$$\begin{aligned}
\Delta w_{h,t} + \Delta\tau_{h,t} - \Delta\tau_{\ell,t} &= \Delta\widehat{w}_{h,t} - \Delta\widehat{w}_{\ell,t} + \Delta\widehat{\tau}_{h,t} - \Delta\widehat{\tau}_{\ell,t} + \Delta\overline{\overline{\tau}}_{h,t} - \Delta\overline{\overline{\tau}}_{\ell,t}, \\
&= \Delta\overline{\overline{w}}_{h,t} - \Delta\overline{\overline{w}}_{\ell,t} + \Delta\widehat{\tau}_{h,t} - \Delta\widehat{\tau}_{\ell,t}.
\end{aligned}$$

The second terms (across-firms) only exists due to the presence of cross-sectional heterogeneity in wage premia across firms. We can furthermore decompose this across-firm change to within and across sectors:

$$\sum_i \widetilde{\Lambda}^*_{hi,t}\Delta \ln \Lambda_{hi,t} - \sum_i \widetilde{\Lambda}^*_{\ell i,t}\Delta \ln \Lambda_{\ell i,t} = \underbrace{\sum_s \widetilde{\Lambda}^*_{hs,t}\Delta \ln \Lambda_{hs,t} - \sum_s \widetilde{\Lambda}^*_{\ell s,t}\Delta \ln \Lambda_{\ell s,t}}_{\text{across-sector change}}$$
$$+ \underbrace{\sum_i \widetilde{\Lambda}^*_{hi,t}\Delta \ln\left(\frac{\Lambda_{hi,t}}{\Lambda_{hs,t}}\right) - \sum_i \widetilde{\Lambda}^*_{\ell i,t}\Delta \ln\left(\frac{\Lambda_{\ell i,t}}{\Lambda_{\ell s,t}}\right)}_{\text{within-sector across-firm change}}.$$

A.3 Applying Aggregation Formulas

In this section, we provide a second order generalization of Proposition 3. Define

$$\widetilde{\sigma}^{\omega}_{ff'i,t} \equiv \frac{1}{2}\left(\sigma^{\omega_t}_{ff'i,t-1} + \sigma^{\omega_t}_{ff'i,t}\right), \tag{44}$$
$$\widetilde{\sigma}^{*,\omega}_{ff'i,t} \equiv \frac{1}{2}\left(\sigma^{*,\omega_t}_{ff'i,t-1} + \sigma^{*,\omega_t}_{ff'i,t}\right), \tag{45}$$

where in both equations we have used the same set of wedge shock weights $\left(\omega_{f'i,t}\right)$ defined in Equation (43) (all for time t). Note that in the special case of uniform prices shocks, this leads us to the definitions:

$$\widetilde{\sigma}_{ff'i,t} \equiv \frac{1}{2}\left(\sigma_{ff'i,t-1} + \sigma_{ff'i,t}\right), \tag{46}$$
$$\widetilde{\sigma}^{*}_{ff'i,t} \equiv \frac{1}{2}\left(\sigma^{*}_{ff'i,t-1} + \sigma^{*}_{ff'i,t}\right). \tag{47}$$

We now have the following proposition re-states Proposition 3 as a second order approximation of the relative changes in the skill premia in terms of firm-level factor price shocks.

Proposition 3. *Consider exogenous shocks to relative the supply of skill $\Delta h_t - \Delta \ell_t$, and to the firm-level equipment prices $\Delta w_{ei,t} \equiv \omega_{ei,t}\Delta\overline{w}_{e,t}$ and labor wedges $\Delta\tau_{fi,t} \equiv \omega_{fi}\Delta\tau_{f,t}$ for $f \in \{\ell, h\}$. To the second order of approximation in the size of these shocks, the response of the shadow and the aggregate skill premia are given by*

$$\Delta w_{h,t} = -\frac{\Delta h_t - \Delta \ell_t}{\widetilde{\sigma}_{\ell h,t}} - \frac{\widetilde{\sigma}^{\omega}_{\ell e,t} - \widetilde{\sigma}^{\omega}_{he,t}}{\widetilde{\sigma}_{\ell h,t}}\Delta\overline{w}_{e,t} - \frac{\widetilde{\sigma}^{\omega}_{\ell h,t}}{\widetilde{\sigma}_{\ell h,t}}\Delta\tau_{h,t} + \frac{\widetilde{\sigma}^{\omega}_{h\ell}}{\widetilde{\sigma}_{\ell h,t}}\Delta\tau_{\ell,t} \tag{48}$$

$$\Delta\psi_t = \Delta w_{h,t} - \widetilde{\sigma}^*_{\ell h,t}\left(\Delta w_{h,t} + \frac{\widetilde{\sigma}^{*,\omega}_{\ell e,t} - \widetilde{\sigma}^{*,\omega}_{he}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\overline{w}_{e,t} + \frac{\widetilde{\sigma}^{*,\omega}_{\ell h,t}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\tau_{h,t} - \frac{\widetilde{\sigma}^{*,\omega}_{hl}}{\widetilde{\sigma}^*_{\ell h}}\Delta\tau_{\ell,t}\right) \tag{49}$$

where the aggregate elasticities are given by Equations (6), (7), (68), (69), (46), (47), (44), and (45).

Proof. See Appendix B. □

Finally, applying this expression to the measured shocks characterized in Section A.1 with $\Delta\tau_{f,t} = \Delta\widehat{\tau}_{f,t} + \Delta\overline{\overline{\tau}}_{f,t}$ for $f \in \{\ell, h\}$ and $\Delta\overline{w}_{e,t} = \Delta\widehat{\tau}_{e,t} + \Delta\overline{\overline{\tau}}_{\ell,t}$, we find:

$$\begin{aligned}\Delta w_{h,t} = &\overbrace{-\frac{\Delta h_t - \Delta\ell_t}{\widetilde{\sigma}_{\ell h,t}} - \frac{\widetilde{\sigma}^{\omega}_{\ell e,t} - \widetilde{\sigma}^{\omega}_{he,t}}{\widetilde{\sigma}_{\ell h}}\Delta\widehat{\tau}_{e,t} - \frac{\widetilde{\sigma}^{\omega}_{\ell h,t}}{\widetilde{\sigma}_{\ell h,t}}\Delta\widehat{\tau}_{h,t} + \frac{\widetilde{\sigma}^{\omega}_{h\ell}}{\widetilde{\sigma}_{\ell h,t}}\Delta\widehat{\tau}_{\ell,t}}^{\text{observed term}\equiv\Delta\widehat{w}^{obs}_{h,t}} \\ &\underbrace{-\frac{\widetilde{\sigma}_{\ell e,t} - \widetilde{\sigma}_{he,t}}{\widetilde{\sigma}_{\ell h}}\Delta\overline{\overline{\tau}}_{\ell,t} - \frac{\widetilde{\sigma}^{\omega}_{\ell h,t}}{\widetilde{\sigma}_{\ell h,t}}\Delta\overline{\overline{\tau}}_{h,t} + \frac{\widetilde{\sigma}^{\omega}_{h\ell}}{\widetilde{\sigma}_{\ell h,t}}\Delta\overline{\overline{\tau}}_{\ell,t}}_{\text{unobserved residuals}\equiv\Delta w^{res}_{h,t}},\end{aligned} \tag{50}$$

$$\begin{aligned}\Delta\psi_t = &\overbrace{\Delta\widehat{w}^{obs}_{h,t} - \widetilde{\sigma}^*_{\ell h,t}\left(\Delta\widehat{w}^{obs}_{h,t} + \frac{\Delta h_t - \Delta\ell_t}{\widetilde{\sigma}^*_{\ell h,t}} + \frac{\widetilde{\sigma}^{*,\omega}_{\ell e,t} - \widetilde{\sigma}^{*,\omega}_{he}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\widehat{\tau}_{e,t} + \frac{\widetilde{\sigma}^{*,\omega}_{\ell h,t}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\widehat{\tau}_{h,t} - \frac{\widetilde{\sigma}^{*,\omega}_{hl}}{\widetilde{\sigma}^*_{\ell h}}\Delta\widehat{\tau}_{\ell,t}\right)}^{\text{observed term}\equiv\Delta\widehat{\psi}^{obs}_t} \\ &\underbrace{+\Delta w^{res}_{h,t} - \widetilde{\sigma}^*_{\ell h,t}\left(\Delta\widehat{w}^{res}_{h,t} + \frac{\widetilde{\sigma}^*_{\ell e,t} - \widetilde{\sigma}^*_{he}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\overline{\overline{\tau}}_{\ell,t} + \frac{\widetilde{\sigma}^*_{\ell h,t}}{\widetilde{\sigma}^*_{\ell h,t}}\Delta\overline{\overline{\tau}}_{h,t} - \frac{\widetilde{\sigma}^*_{hl}}{\widetilde{\sigma}^*_{\ell h}}\Delta\overline{\overline{\tau}}_{\ell,t}\right)}_{\text{unobserved residual}\,\equiv\,\Delta\psi^{res}_t}.\end{aligned} \tag{51}$$

B Proofs and Derivations

Proof for Equation (9). Let us first find the output elasticity of factor f:

$$\frac{X_f \partial Y}{Y \partial X_f} = \frac{\left(\frac{\sigma_f - 1}{\sigma_f}\right)\left(\frac{Z_f A_{fi}}{Y_i}\right)^{\frac{\sigma_f - 1}{\sigma_f}}}{\sum_{f'}\left(\frac{\sigma_{f'} - 1}{\sigma_{f'}}\right)\left(\frac{Z_{f'} X_{f'i}}{Y_i}\right)^{\frac{\sigma_{f'} - 1}{\sigma'_f}}}.$$

We compute the factor cost shares through solving the problem of minimizing unit costs for the firm:

$$\min_{(A_f)} \sum_f W_{fi} A_{fi} + \Xi_i \left(1 - \sum_{f \in \{\ell, h, e\}} \left(Z_{fi} A_{fi}\right)^{\frac{\sigma_f - 1}{\sigma_f}}\right),$$

where Ξ_i is the Lagrange multiplier on the constraint defining the production function in Equation (8), where we have substituted for the unit factor-f requirement as $A_{fi} \equiv \frac{X_{fi}}{Y_i}$.. The first order condition with respect to factor f gives us:

$$W_{fi} = \Xi_i \frac{\sigma_f - 1}{\sigma_f} \frac{1}{A_{fi}} \left(Z_{fi} A_{fi}\right)^{\frac{\sigma_f - 1}{\sigma_f}}.$$

Using the constraint in Equation (8) then yields:

$$\Xi_i = \sum_f \frac{\sigma_f}{\sigma_f - 1} W_{fi} A_{fi}.$$

Moreover, the cost share of factor f is given by:

$$\theta_{fi} \equiv \frac{W_{fi} A_{fi}}{\sum_{f'} W_{f'i} A_{f'i}} = \frac{\left(\frac{\sigma_f - 1}{\sigma_f}\right) \left(Z_f A_{fi}\right)^{\frac{\sigma_f - 1}{\sigma_f}}}{\sum_{f'} \left(\frac{\sigma_{f'} - 1}{\sigma_{f'}}\right) \left(Z_{f'} X_{f'i}\right)^{\frac{\sigma_{f'} - 1}{\sigma_f'}}} = \frac{X_{fi}}{Y} \frac{\partial Y}{\partial X_{fi}}.$$

It follows that the ratio of payments to factors f and f' satisfy:

$$\frac{W_{fi}}{W_{f'i}} = \frac{\frac{\sigma_f - 1}{\sigma_f} Z_{fi}^{\frac{\sigma_f - 1}{\sigma_f}} A_{fi}^{-\frac{1}{\sigma_f}}}{\frac{\sigma_{f'} - 1}{\sigma_{f'}} Z_{fi}^{\frac{\sigma_{f'} - 1}{\sigma_{f'}}} A_{fi}^{-\frac{1}{\sigma_{f'}}}}. \tag{52}$$

Thus, assuming constant factor-augmenting productivities, we find:

$$dw_f - dw_{f'} = -\frac{1}{\sigma_f} da_{fi} - \frac{1}{\sigma_{f'}} da_{f'i}. \tag{53}$$

Using the constraint (8) again, we find:

$$0 = \sum_f \left(\frac{\sigma_f - 1}{\sigma_f}\right) \left(Z_f A_{fi}\right)^{\frac{\sigma_f - 1}{\sigma_f}} da_{fi} = \sum_f \left(\frac{\sigma_f - 1}{\sigma_f}\right) \theta_{fi} da_{fi}. \tag{54}$$

Combining Equations (53) and (54) yields:

$$da_{\ell i} = \sigma_\ell \sigma_h \frac{\theta_{hi}}{\overline{\sigma}_i} dw_h + \sigma_\ell \sigma_e \frac{\theta_{ei}}{\overline{\sigma}_i} dw_e, \tag{55}$$

$$da_{hi} = \sigma_h \left(1 - \sigma_h \frac{\theta_{hi}}{\overline{\sigma}_i}\right) dw_h + \frac{\sigma_h \sigma_e}{\overline{\sigma}_i} \theta_{ei} dw_e, \tag{56}$$

$$da_{ei} = \sigma_e \left(1 - \sigma_e \frac{\theta_{ei}}{\overline{\sigma}_i}\right) dw_e + \frac{\sigma_h \sigma_e}{\overline{\sigma}_i} \theta_{hi} dw_h, \tag{57}$$

which together lead to the desired result. □

Proof for Lemma 1. To compute the aggregate elasticities of substitution defined in Equations (4) and (5), first note that

$$d \ln x_f = \sum_i \Lambda_{fi} \left(dy_i + d \ln a_{fi}\right), \tag{58}$$

$$d \ln \theta_f = \sum_i \Lambda^*_{fi} \left(ds_i + d \ln \theta_{fi}\right), \tag{59}$$

where we have $\theta_f = \sum_i S_i \theta_{fi}$ with S_i standing for the share of firm i in all factor payments, and where we have let $s_i \equiv \ln S_i$ denote the log of the share of firm i in all factor payments. Given that we have CRS technology at the producer level, we can express θ_{fi} in terms of unit factor requirements A_i, as $\theta_{fi} = W_f A_{fi} / C_i$, where $C_i \equiv \sum_{f'} W_{f'} A_{f'i}$ is the minimum unit cost of producer i. This allows us to write $d \ln \theta_{fi} = dw_f + da_{fi} - dc_i$.

Let us first compute the aggregate elasticity of substiution for factor demand $\sigma_{ff'}$ in Equation (4). We have $da_{fi} = \sum_{f'} \sigma_{ff',i} \theta_{f'i} dw_{f'}$, and with monopolistic competition and CES demand in Equation (1) and cost minimization, we obtain

$$dy_i = dy - \varepsilon \left(dc_i - dc_s\right) - \eta \left(dc_s - dc\right),$$

where we have $dc_s \equiv \frac{1}{S_s} \sum_{i \in \mathcal{I}_s} S_i dc_i$ and $dc \equiv \sum_i S_i dc_i$. Combining theese expressions with Equation (58) gives us:

$$\begin{aligned} d \ln x_f &= \sum_i \Lambda_{fi} \left(da_{fi} - \varepsilon dc_i\right) + (\varepsilon - \eta) \sum_s \Lambda_{fs} dc_s + \eta \, dc, \\ &= \sum_i \Lambda_{fi} \theta_{f'i} \left(\sigma_{ff',i} - \varepsilon\right) dw_{f'} + (\varepsilon - \eta) \sum_s \Lambda_{fs} \theta_{f's} dw_{f'} + \eta \, dc, \end{aligned} \tag{60}$$

where we have used the relsut that $dc_s = \sum_f \theta_{fs} dw_f$, and with slight abuse of notation, defined $\Lambda_{fs} \equiv \sum_{i \in \mathcal{I}_s} \Lambda_{fi}$. Equation (6) immediately follows from the above result.

Next, Since we compute the aggregate elasticity of substiution for factor payments $\sigma^*_{ff'}$ in Equation (5). We have $d \ln \theta_{fi} = da_{fi} + dw_f - dc_i$ and, under the assumption of constant and equal firm-level markups, the share of input costs paid by producer i, S_i, can be expressed as $S_i = C_i^{1-\varepsilon} / \sum_{i'} C_{i'}^{1-\varepsilon} \times$, in which case we have

$$ds_i = (1-\varepsilon)\left(dc_i - dc_s\right) + (1-\eta)\left(dc_s - dc\right).$$

Using again the definition of (3), Equation (59) can be expanded as:

$$d\ln\theta_f = dw_f + \sum_i \Lambda^*_{fi}\theta_{f'i}\left(\sigma_{ff',i} - \varepsilon\right) dw_{f'} + (\varepsilon - \eta)\sum_s \Lambda^*_{fs}\theta_{f's} dw_{f'} + (\eta - 1)\, dc, \quad (61)$$

where we have again defined $\Lambda^*_{fs} \equiv \sum_{i\in\mathcal{I}_s}\Lambda^*_{fi}$. Equation (7) immediately follows from the above result. □

Proof of Lamma 2. First, we compute the elasticity of the intensity of factor f with respect to its own price. We rely on the equality $1 = \theta_f\left(1 + \sum_{f'\neq f}\frac{\theta_{f'}}{\theta_f}\right)$, and computing the elasticity with respect to the price of factor f', we find

$$0 = \frac{\partial\ln\theta_f}{\partial w_f} + \frac{\sum_{f'\neq f}\frac{\partial}{\partial w_f}\left(\frac{\theta_{f'}}{\theta_f}\right)}{1 + \sum_{f'\neq f}\frac{\theta_{f'}}{\theta_f}} = \frac{\partial\ln\theta_f}{\partial w_f} + \frac{\sum_{f'\neq f}\frac{\theta_{f'}}{\theta_f}\left(\sigma^*_{f'f} - 1\right)}{1 + \sum_{f'\neq f}\frac{\theta_{f'}}{\theta_f}}.$$

This allows to write the elasticity as

$$\frac{\partial\ln\theta_f}{\partial w_f} = 1 - \theta_f - \sum_{f'\neq f}\theta_{f'}\sigma^*_{f'f} = (1-\theta_f)\left(1 - \sum_{f'\neq f}\frac{\theta_{f'}}{1-\theta_f}\sigma^*_{f'f}\right) = \theta_{\overline{f}}\left(1 - \sigma^*_{\overline{f}f}\right),$$

where we have defined a composite factor $\overline{f}$ corresponding to the set of all factors other than f, such that $\mathcal{F}_{\overline{f}} \equiv \mathcal{F}/\{f\}$, and have used the definition of the corresponding elasticity of substitution from Equation (11). Using this result, we can now compute the elasticity of the intensity of any other factor $f' \neq f$ with respect to the price of factor f:

$$\begin{aligned}\frac{\partial\ln\theta_{f'}}{\partial w_f} &= \frac{\partial\ln\left(\theta_{f'}/\theta_f\right)}{\partial w_f} + \frac{\partial\ln\theta_f}{\partial w_f},\\ &= \sigma^*_{f'f} - 1 - \theta_{\overline{f}}\left(\sigma^*_{\overline{f}f} - 1\right),\\ &= \sigma^*_{f'f} - \theta_{\overline{f}}\sigma^*_{\overline{f}f} - \theta_f.\end{aligned}$$

Now, we can compute the elasticity of the intensity of a composite factor c with respect to some factor f' not in c:

$$\begin{aligned}
\frac{\partial \ln \theta_c}{\partial w_f} &= \frac{1}{\theta_c} \sum_{f' \in \mathcal{C}_c} \frac{\partial \theta_{f'}}{\partial w_f} = \sum_{f \in \mathcal{C}_c} \frac{\theta_f}{\theta_c} \frac{\partial \ln \theta_f}{\partial w_{f'}}, \\
&= \sum_{f' \in \mathcal{C}_c} \frac{\theta_{f'}}{\theta_c} \left(\sigma^*_{f'f} - \theta_{\overline{f}} \sigma^*_{\overline{f}f} - \theta_f \right), \\
&= \sigma^*_{cf} - \theta_{\overline{f}} \sigma^*_{\overline{f}f} - \theta_f.
\end{aligned}$$

Finally, if the composite factor c includes factor f, we have:

$$\begin{aligned}
\frac{\partial \ln \theta_c}{\partial w_f} &= \frac{1}{\theta_c} \sum_{f' \in \mathcal{C}_c} \frac{\partial \theta_{f'}}{\partial w_f} = \sum_{f' \in \mathcal{C}_c} \frac{\theta_{f'}}{\theta_c} \frac{\partial \ln \theta_{f'}}{\partial w_f}, \\
&= \sum_{f' \in \mathcal{C}_c, f' \neq f} \frac{\theta_{f'}}{\theta_c} \left(\sigma^*_{f'f} - \theta_{\overline{f}} \sigma^*_{\overline{f}f} - \theta_f \right) - \frac{\theta_f}{\theta_c} \theta_{\overline{f}} \left(\sigma^*_{\overline{f}f} - 1 \right), \\
&= \sum_{f' \in \mathcal{C}_c, f' \neq f} \frac{\theta_{f'}}{\theta_c} \sigma^*_{f'f} - \theta_{\overline{f}} \sigma^*_{\overline{f}f} - \frac{\theta_c - \theta_f}{\theta_c} \theta_f + \frac{\theta_f}{\theta_c} \theta_{\overline{f}}, \\
&= \frac{\theta_c - \theta_f}{\theta_c} \sigma^*_{\overline{c}_f f} - \theta_{\overline{f}} \sigma^*_{\overline{f}f} + \theta_f \frac{1 - \theta_c}{\theta_c},
\end{aligned}$$

where in the second line, we have separated the term $\frac{\partial \ln \theta_f}{\partial w_f}$ from all other terms, and where in the last line, we have defined a composite factor $\overline{c}_f$ by removing f from the composite factor c, that is, $\mathcal{F}_{\overline{c}_f} \equiv \mathcal{F}_c / \{f\}$. □

Proof of Proposition 1. We keep the aggregate supply of high, H, and low skilled labor, L, constant. Hence, we have

$$\begin{aligned}
0 = d \ln \left(\frac{X_\ell}{X_f} \right) &= \frac{\partial \ln \left(X_\ell / X_f \right)}{\partial w_f} dw_f + \sum_{f' \neq \ell, e, f} \left[\frac{\partial \ln \left(X_\ell / X_{f'} \right)}{\partial w_{f'}} - \frac{\partial \ln \left(X_f / X_{f'} \right)}{\partial w_{f'}} \right] dw_{f'} \\
&\quad + \left[\frac{\partial \ln \left(X_\ell / X_e \right)}{\partial w_e} - \frac{\partial \ln \left(X_f / X_e \right)}{\partial w_e} \right] dw_e, \\
&= \sigma_{\ell f}\, dw_f + \sum_{f' \neq \ell, e, f} \left(\sigma_{\ell f'} - \sigma_{f f'} \right) dw_{f'} + \left(\sigma_{\ell e} - \sigma_{fe} \right) dw_e,
\end{aligned}$$

which gives us Equation (14) using the definition (4).

To derive Equation (15), we take the full derivative of Equation (2) to find

$$-d\psi = d\ln\left(\frac{\overline{W}_l L}{\overline{W}_h H}\right) = d\ln\left(\frac{\theta_\ell}{\theta_h}\right) = \frac{\partial \ln\left(\theta_\ell/\theta_h\right)}{\partial w_h} dw_h + \left[\frac{\partial \ln\left(\theta_\ell/\theta_e\right)}{\partial w_e} - \frac{\partial \ln\left(\theta_h/\theta_e\right)}{\partial w_e}\right] dw_e$$
$$+ \sum_{f\notin\{\ell,h,e\}} \left[\frac{\partial \ln\left(\theta_\ell/\theta_f\right)}{\partial w_f} - \frac{\partial \ln\left(\theta_h/\theta_f\right)}{\partial w_f}\right] dw_f,$$

which gives us the desired result when substituting for the aggregate elasticities in terms of factor payment from Equation (5). □

Proof for Lemma 3. Using Equations (58) and (59), and taking steps similar to those for the proof of Lemma 1, we find:

$$d\ln x_f = \sum_i \Lambda_{fi}\left(da_{fi} - \varepsilon dc_i\right) + (\varepsilon-\eta)\sum_s \Lambda_{fs} dc_s + \eta\, dc. \tag{62}$$

We now have that $da_{fi} = \sum_{f'} \theta_{f'i}\sigma_{ff',i} d\tau_{f'i} = \sum_{f'} \theta_{f'i}\sigma_{ff',i}\omega_{f'i} d\tau_{f'}$ and $dc_i = \sum_{f'} \theta_{f'i} d\tau_{f'i} = \sum_{f'} \theta_{f'i}\omega_{f'i} d\tau_{f'}$, and

$$dc_s \equiv \sum_{i\in\mathcal{I}_s} \tfrac{S_i}{S_s} dc_i = \sum_{f'}\sum_{i\in\mathcal{I}_s} \tfrac{S_i}{S_s}\theta_{f'i}\omega_{f'i} d\tau_{f'} = \sum_{f'} \omega_{f's}\theta_{f's} d\tau_{f'},$$

where we have defined ω_{fs} as in Equation (29) using $\Lambda^*_{fi|s} \equiv \frac{S_i}{S_s}\left(\frac{\theta_{fi}}{\theta_{fs}}\right)$. Substituting these expressions in Equation (62), we find

$$d\ln x_f = \sum_i \Lambda_{fi}\theta_{f'i}\left(\sigma_{ff',i} - \varepsilon\right)\omega_{f'i} d\tau_{f'} + (\varepsilon-\eta)\sum_s \Lambda_{fs}\theta_{f's}\omega_{f's} dw_{f'} + \eta\, dc. \tag{63}$$

The above expression immediately leads to Equation (27).

From the definition of θ_f, Equations (59) follows. We can now write $d\ln\theta_{fi} = d\tau_{fi} + da_{fi} - dc_i$, leading to:

$$d\ln\theta_f = d\tau_f + \sum_i \Lambda^*_{fi}\left(da_{fi} - \varepsilon dc_i\right) + (\varepsilon-\eta)\sum_s \Lambda^*_{fs} dc_s + (\eta-1)\, dc, \tag{64}$$

where we have used the normalization $\sum_i \Lambda_{fi}\omega_{fi} = 1$ in the first term on the right hand side. Substituting again for the expressions for da_{fi}, dc_i, and dc_s , we find

$$d\ln\theta_f = d\tau_f + \sum_i \Lambda^*_{fi}\sum_{f'}\theta_{f'i}\left(\sigma_{ff',i} - \varepsilon\right)\omega_{f'i} d\tau_{f'} + (\varepsilon-\eta)\sum_{f'} \Lambda^*_{f's}\theta_{f's}\omega_{f's} d\tau_{f'} + (\eta-1)\, dc. \tag{65}$$

Equation (28) immediately follows. $\square$

Proof of Proposition 2. We keep the aggregate supply of high, H, and low skilled labor, L, constant. Hence, we have

$$
\begin{aligned}
0 = d\ln\left(\frac{X_\ell}{X_f}\right) &= \frac{\partial \ln\left(X_\ell/X_f\right)}{\partial w_f}\, dw_f + \sum_{f'\neq \ell,e,f}\left[\frac{\partial \ln\left(X_\ell/X_{f'}\right)}{\partial w_{f'}} - \frac{\partial \ln\left(X_f/X_{f'}\right)}{\partial w_{f'}}\right] dw_{f'} \\
&\quad + \frac{\partial \ln\left(X_\ell/X_h\right)}{\partial \tau_h}\, d\tau_h - \frac{\partial \ln\left(X_h/X_\ell\right)}{\partial \tau_\ell}\, d\tau_\ell + \left[\frac{\partial \ln\left(X_\ell/X_e\right)}{\partial \tau_e} - \frac{\partial \ln\left(X_f/X_e\right)}{\partial \tau_e}\right] d\tau_e, \\
&= \sigma_{\ell f}\, dw_f + \sum_{f'\neq \ell,e,f}\left(\sigma_{\ell f'} - \sigma_{f f'}\right) dw_{f'} + \sigma^{\omega}_{\ell h} d\tau_h \mathbb{I}_{fh} - \sigma^{\omega}_{f\ell}\, d\tau_\ell + \left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{fe}\right) d\overline{w}_e,
\end{aligned}
$$

which leads to Equation (30) when written in vector/matrix form.

We first derive Equation (49). The change in the skill premium satisfies (letting $\tau_e \equiv \overline{w}_e$):

$$
\begin{aligned}
-d\psi = d\ln\left(\frac{\theta_\ell/L}{\theta_h/H}\right) &= \frac{\partial \ln\left(\theta_\ell/\theta_h\right)}{\partial w_h}\, dw_h + \left[\frac{\partial \ln\left(\theta_\ell/\theta_e\right)}{\partial \tau_e} - \frac{\partial \ln\left(\theta_h/\theta_e\right)}{\partial \tau_e}\right] d\tau_e \\
&\quad + \sum_{f\notin\{\ell,h,e\}}\left[\frac{\partial \ln\left(\theta_\ell/\theta_f\right)}{\partial w_f} - \frac{\partial \ln\left(\theta_h/\theta_f\right)}{\partial w_f}\right] dw_f \\
&\quad + \frac{\partial \ln\left(\theta_\ell/\theta_h\right)}{\partial \tau_h}\, d\tau_h - \frac{\partial \ln\left(\theta_h/\theta_\ell\right)}{\partial \tau_l}\, d\tau_l, \qquad (66) \\
&= \left(\sigma^{*}_{\ell h} - 1\right) dw_h + \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) d\tau_e + \sum_{f\notin\{\ell,h,e\}}\left(\sigma^{*}_{\ell f} - \sigma^{*}_{hf}\right) dw_f \\
&\quad + \sigma^{*,\omega}_{\ell h} d\tau_h - \sigma^{*,\omega}_{h\ell} d\tau_\ell,
\end{aligned}
$$

leading to the desired result. $\square$

Proof of Equations (36) and (37). Writing Equation (52) in differences, we find for $f \in \{\ell, h\}$:

$$
\begin{aligned}
\Delta w_{eit} - \Delta w_{fit} &= \frac{1}{\sigma_f}\left(\Delta x_{fit} - \Delta y_{it}\right) - \frac{1}{\sigma_e}\left(\Delta x_{eit} - \Delta y_{it}\right) + \left(1 - \frac{1}{\sigma_e}\right)\Delta z_{eit} - \left(1 - \frac{1}{\sigma_\ell}\right)\Delta z_{fit}, \\
&= \left(\frac{1}{\sigma_e} - \frac{1}{\sigma_f}\right)\left(\Delta y_{it} - \Delta x_{eit}\right) - \frac{1}{\sigma_f}\left(\Delta x_{eit} - \Delta x_{fit}\right) + \widetilde{\nu}_{fit},
\end{aligned}
$$

where we have define the residual $\widetilde{\nu}_{fit} \equiv \left(1 - \frac{1}{\sigma_e}\right)\Delta z_{eit} - \left(1 - \frac{1}{\sigma_\ell}\right)\Delta z_{fit}$ in the second equality. We can write this result as

$$\Delta x_{eit} - \Delta x_{fit} = -\sigma_f \left(\Delta w_{eit} - \Delta w_{fit}\right) + \left(\frac{\sigma_f}{\sigma_e} - 1\right) \left(\Delta y_{it} - \Delta x_{eit}\right) + \sigma_f \, \widetilde{\nu}_{fit}. \tag{67}$$

Since we do not observe the growth in firm-level quantity Δy_{it}, we use the demand system in Equation (1) to write it in terms of revenue. The demand system implies:

$$\Delta p_{it} = -\frac{1}{\varepsilon} \left(\Delta y_{it} - \Delta y - (\varepsilon - \eta) \, \Delta p_{st} - \eta \, \Delta p - \Delta \phi_{it}\right),$$

which leads to the following relationship between growth in revenue and the growth in quantities:

$$\Delta r_{it} = \Delta p_{it} + \Delta y_{it} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \left(\Delta y_{it} + \Delta \widetilde{\phi}_{it}\right),$$

where we have define $(\varepsilon - 1) \, \Delta \widetilde{\phi}_{it} \equiv \Delta \phi_{it} + (\varepsilon - \eta) \, \Delta p_{st} + \eta \, \Delta p + \Delta y$. Substituting for Δy_{it} in Equation (67), we find the desired results with the choice of

$$\nu_{fit} \equiv \sigma_f \, \widetilde{\nu}_{fit} + \left(\frac{\sigma_f}{\sigma_e} - 1\right) \Delta \widetilde{\phi}_{it}.$$

□

Proof of Proposition 3. To prove this result, it is sufficient to follow the logic of the proof of Proposition 3, which is stated in terms of first-order changes in firm-level wedge/price shocks in Equations 66 and **??**, but instead provide second order approximations for the changes in relative factor demand $X_f / X_{f'}$ and relative factor payments $\theta_f / \theta_{f'}$.

Applying Lemma 4 below for the vector of all firm-level shocks, we need to characterize the partial derivatives of relative factor payments and relative factor demands with respect to each shock. For each firm-level shock to the firm j, e.g., $\Delta \tau_{fi,t}$, we can define a correpsonding vector $\omega^j_{fi,t}$ by letting $\omega^j_{fi,t} \equiv \Delta \tau_{fi,t} / \Delta \tau^j_{f,t}$ for $i = j$ and $\omega^j_{fi,t} = 0$ otherwise, where we have additionally defined $\Delta \tau^j_{f,t} = \widetilde{\Lambda}^*_{fi,t} \Delta \tau_{fi,t}$. Using these definitions, Lemma 3 provides us with the derivatives of aggregate factor demand and factor payment functions with respect to such a shock at time t:

$$\sigma^{\omega^j}_{ff',t} = \Bigg[\left(\Lambda_{fj,t} \sigma_{ff',j,t} - \Lambda_{f'j,t} \sigma_{f'f',j,t}\right) \theta_{f'j,t1} + \varepsilon \left(\Lambda_{f'j,t} - \Lambda_{fj,t}\right) \theta_{f'j,t}$$
$$- (\varepsilon - \eta) \left(\Lambda_{f's,t} - \Lambda_{fs,t}\right) \theta_{f's,t} \Lambda^*_{fi|s,t} \Bigg] \frac{1}{\widetilde{\Lambda}^*_{f'j,t}}, \qquad \text{for } s \text{ such that } j \in \mathcal{I}_s, \tag{68}$$

$$\sigma^{*,\omega j}_{ff',t} = \Bigg[\left(\Lambda^*_{fj,t}\sigma_{ff',j,t} - \Lambda^*_{f'j,t}\sigma_{f'f',j,t} \right) \theta_{f'j,t} + \varepsilon \left(\Lambda^*_{f'j,t} - \Lambda^*_{fj,t} \right) \theta_{f'j,t}$$

$$- (\varepsilon - \eta) \left(\Lambda^*_{f's,t} - \Lambda^*_{fs,t} \right) \theta_{f's,t} \Lambda^*_{fi|s,t} \Bigg] \frac{1}{\widetilde{\Lambda}^*_{f'j,t}}, \qquad \text{for } s \text{ such that } j \in \mathcal{I}_s. \quad (69)$$

Using Lemma 4, we can now express the changes in relative factor demand $X_f / X_{f'}$ and relative factor payments $\theta_f / \theta_{f'}$ with respect to the vector of firm-level factor price shocks $\Delta\tau_{f'i}$ as

$$\Delta \ln \left(\frac{X_{f,t}}{X_{f',t}} \right) \approx \frac{1}{2} \sum_j \left(\sigma^{\omega j}_{ff',t-1} + \sigma^{\omega j}_{ff',t} \right) \Delta\tau^j_{f',t} = \frac{1}{2} \left(\sigma^{\omega}_{ff',t-1} + \sigma^{\omega}_{ff',t} \right) \Delta\tau_{f,t},$$

$$\Delta \ln \left(\frac{\theta_{f,t}}{\theta_{f',t}} \right) \approx \frac{1}{2} \sum_j \left(\sigma^{*,\omega j}_{ff',t-1} + \sigma^{*,\omega j}_{ff',t} \right) \Delta\tau^j_{f',t} = \frac{1}{2} \left(\sigma^{\omega,*}_{ff',t-1} + \sigma^{\omega,*}_{ff',t} \right) \Delta\tau_{f,t},$$

where we have used the definition $\Delta\tau^j_{f',t} = \widetilde{\Lambda}^*_{fi,t} \Delta\tau_{fi,t}$ as well as the definitions of $\sigma^{\omega}_{ff'}$ and $\sigma^{\omega,*}_{ff'}$ in Equations (68) and (69) and the definition of wedges $\omega_{f'i,t} = \Delta\tau_{f'i,t} / \Delta\tau_{f',t}$ with $\Delta\tau_{f',t} \equiv \sum_i \widetilde{\Lambda}^*_{fi,t} \Delta\tau_{f'i,t}$. Applying this result to each term in Equations 66 and **??** leads to the desired result. $\square$

Lemma 4. *For a second-order continuously differentiable function $f(\boldsymbol{x})$, we have:*

$$f(\boldsymbol{y}) = f(\boldsymbol{x}) + \frac{1}{2} \sum_i \left(\frac{\partial f(\boldsymbol{x})}{\partial x_i} + \frac{\partial f(\boldsymbol{y})}{\partial y_i} \right) (y_i - x_i) + O\left(\max_i |y_i - x_i|^3 \right).$$

Proof. Using Taylor's expansion, up to the second order in $\boldsymbol{y} - \boldsymbol{x}$, we have:

$$f(\boldsymbol{y}) = f(\boldsymbol{x}) + \sum_{i=1}^{I} \frac{\partial f(\boldsymbol{x})}{\partial x_i} (y_i - x_i) + \frac{1}{2} \sum_{i,j=1}^{I} \frac{\partial^2 f(\boldsymbol{x})}{\partial x_i \partial x_j} (y_i - x_i) (y_j - x_j),$$

$$f(\boldsymbol{x}) = f(\boldsymbol{y}) + \sum_{i=1}^{I} \frac{\partial f(\boldsymbol{y})}{\partial x_i} (x_i - y_i) + \frac{1}{2} \sum_{i,j=1}^{I} \frac{\partial^2 f(\boldsymbol{y})}{\partial x_i \partial x_j} (y_i - x_i) (y_j - x_j).$$

Together, the two equations imply:

$$f(\boldsymbol{y}) = f(\boldsymbol{x}) + \frac{1}{2} \sum_{i=1}^{I} \left[\frac{\partial f(\boldsymbol{y})}{\partial y_i} + \frac{\partial f(\boldsymbol{x})}{\partial x_i} \right] (y_i - x_i) + \frac{1}{4} \sum_{i,j=1}^{I} \left[\frac{\partial^2 f(\boldsymbol{x})}{\partial x_i \partial x_j} - \frac{\partial^2 f(\boldsymbol{y})}{\partial x_i \partial x_j} \right] (y_i - x_i) (y_j - x_j).$$

This gives us the desired result since:

$$\frac{\partial^2 f(\boldsymbol{y})}{\partial x_i \partial x_j} - \frac{\partial^2 f(\boldsymbol{x})}{\partial x_i \partial x_j} = \sum_k \frac{\partial^3 f(\boldsymbol{x})}{\partial x_k \partial x_i \partial x_j} (y_k - x_k) .$$

$\square$

C Microfoundations for the Factor Price Wedges

C.1 Heterogeneous Equipment Equipment

Here, we provide a microfoundation for firm- and skill-type-specific wages stemming from variations in compensating differentials across firms. In this microfoundation, the labor market is perfectly competitive.

Suppose that firms are characterized by variations in amenities that are specific to worker skill. More specifically, if a high-skill (or low-skill) worker is employed by firm i, then her real income is divided by T_{hi} (or $T_{\ell i}$). The market equilibrium then leads to firm-specific compensating differentials that are equivalent to amenity differences.

For two firms i and i' to have positive employment of skilled (or unskilled) labor in a competitive labor market, their wages must satisfy $W_{hi}/T_{hi} = W_{hi'}/T_{hi'}$ (or $W_{\ell i}/T_{\ell i} = W_{\ell i'}/T_{\ell i'}$). Letting W_h and W_ℓ denote the shadow wages of skilled and unkilled labor, which we set equal to the wage of a (potentially counterfactual) firm i' with $T_{hi'} = 1$ or $T_{\ell i'} = 1$, we then have $W_{hi} = W_h T_{hi}$ and $W_{\ell i} = W_\ell T_{\ell i}$ for all i, exactly as in our model of Section 2.

C.2 Compensating Differentials

Here, we provide a microfoundation for firm- and skill-type-specific wages stemming from variations in compensating differentials across firms. In this microfoundation, the labor market is perfectly competitive.

Suppose that firms are characterized by variations in amenities that are specific to worker skill. More specifically, if a high-skill (or low-skill) worker is employed by firm i, then her real income is divided by T_{hi} (or $T_{\ell i}$). The market equilibrium then leads to firm-specific compensating differentials that are equivalent to amenity differences.

For two firms i and i' to have positive employment of skilled (or unskilled) labor in a competitive labor market, their wages must satisfy $W_{hi}/T_{hi} = W_{hi'}/T_{hi'}$ (or $W_{\ell i}/T_{\ell i} = W_{\ell i'}/T_{\ell i'}$). Letting W_h and W_ℓ denote the shadow wages of skilled and unkilled labor, which we set equal to the wage of a (potentially counterfactual) firm i' with $T_{hi'} = 1$ or $T_{\ell i'} = 1$, we then have $W_{hi} = W_h T_{hi}$ and $W_{\ell i} = W_\ell T_{\ell i}$ for all i, exactly as in our model of Section 2.

D Empirical Appendix

D.1 Measuring firm-and-skill-specific wages and employment

Our model-consistent measures of firm- and skill-specific wages are computed exploiting data retrieved from DADS Poste. This dataset provides, for each worker, the observed wage in t and $t-1$ as well as the number of hours worked, the duration of the employment contract and a set of individual characteristics.[19] In the model, all workers in a skill group are identical. As a result, we need to correct the observed changes in firm- and skill-specific wages for worker-level unobservables and changes in returns to worker observable characteristics. To do so, we estimate a national Mincerian regression on log wage changes between $t-1$ and t controlling for worker observables (age, sex, 2-digit occupation category) and a firm-skill-time effect on a sample of workers in a given skill group who are employed by the same firm between $t-1$ and t. We exclude from our sample atypical workers, cross-border workers, trainees and workers who were employed for a period shorter than a month or for less than 160 hours over the year. Worker-level hourly wages are computed by dividing individual wage bills by the reported number of hours worked. Finally, we drop observations reporting an hourly wage below 80% of the minimum wage.

We estimate the following linear regression separately for low- and high-skilled workers

$$wage_{ji,t} - wage_{ji,t-1} = \boldsymbol{b}_t' \boldsymbol{X}_{jit} + \gamma_{fit} + e_{jit},$$

where worker j is classified as either h- or ℓ skilled depending on its occupation (catégories socioprofessionnelles) at time t, as defined in the main text. The year-on-year change in the firm-and-skill specific wage is the estimated firm-skill-year fixed effect. We chain the year-on-year changes together to obtain five-year changes, so that $\Delta w_{fit} = \sum_{\tau=0}^{4} \widehat{\gamma}_{fi,t-\tau}$ with $f \in \{h, \ell\}$. We further measure five-year changes in employment, Δx_{hit} and $\Delta x_{\ell it}$, by deflating changes in firm-and-skill-specific wage bills by the corresponding five-year wage change, Δw_{hit} and $\Delta w_{\ell it}$.

[19]The dataset is at the job-spell level. Starting in 2002, a unique worker identifier is provided so that job spells can be correctly aggregated at the worker level. Before 2002, we cannot implement this aggregation and we consider each spell as a separate worker.

D.2 Measuring equipment stock

The data that we use to build our measure of the stock of equipment come from the BRN and the customs datasets. Firm-level data available in BRN include a breakdown of tangible capital by asset type. Our measure of domestic equipment includes the following two asset categories: *Machinery, equipment and tools (AR - Installations techniques, matriel et outillage industriels)* and *Other tangible fixed assets (AT - Autres immobilisations corporelles)*.

Investment Data. Since the version of BRN available to external researchers does not include information on investments, we compute a proxy for investments in each asset category by exploiting data on the book value of the stock (K_{it}), of disposed assets (U_{it}), and on the value of the depreciation account (D_{it}). Assuming an average useful life $T = 3$ years, we compute our proxy for equipment investment as:[20]

$$I^e_{it} = \frac{T}{T-1} \cdot \left[(K_{it} - K_{it-1}) + (D_{it-1} - D_{it}) + \sum_{s=t-T}^{t-1} \frac{I^e_{is}}{T} + \psi_{it} + U_{it} - \lambda_{it-1} \right],$$

where

$$\psi_{it} = \begin{cases} \min\left\{ \frac{K_{i0}}{T}, K_{i0} - D_{i0} - \frac{K_{i0}}{T}(t-1) \right\}, & \text{if } K_{i0} - D_{i0} - \frac{K_{i0}}{T}(t-1) > 0 \\ 0, & \text{otherwise ,} \end{cases}$$

and

$$\lambda_{it} = \begin{cases} 0, & \text{if } t = 1, \\ \min\left\{ U_{i1}, \psi + \frac{I^e_{i1}}{T} \right\}, & \text{if } t = 2, \\ \min\left\{ U_{it-1} + \max\left\{ U_{it-2} - \sum_{s=t-T-1}^{t-2} \frac{K_{i0}}{T}, 0 \right\}, \sum_{s=t-T}^{t-1} \frac{K_{i0}}{T} \right\}, & \text{if } t > 2. \end{cases}$$

For the imported part of equipment investment, we rely on customs data, which provide information on partner country-, CN8 product-, firm-, and year-specific transaction value and quantities (in kgs). We define equipment products as the ones that belong to category 4 of the BEC Rev. 4 classification: *Capital goods (except transport equipment), and parts and accessories thereof*. Subsequently, we obtain a proxy for domestic equipment investment by subtracting imported equipment from the total value of equipment investment obtained above.

Price Data. Prices for imported equipment products are defined as unit values at the firm-

[20] We validated our proxy against real investment data, only accessible to INSEE employees, obtaining a correlation above 0.9. We are grateful to Jocelyn Boussard for help with this analysis.

, year-, product-, and country of origin-level. For domestic investment, we do not have firm-level information on prices. Therefore we exploit sector-, asset-, and year-specific data provided by the French Statistical Institute (INSEE). Our proxy for equipment price is computed aggregating data on 3 broad ESA 2010 asset categories, namely *Transport equipment (N1131G)*, *Computer hardware (N11321G)* and *Other machinery and equipment and weapons systems (N11OG)*.

Firm-Level Price Index for Equipment Investment At this point, we have annual firm-level data on domestic equipment investments and imported equipment investments at the level of product-origins, together with their prices. We next compute a firm-level price index for equipment to deflate the investments values and infer the quantity of equipment investment.

Our index is a slight variation on standard the Sato-Vartia price index. We treat the domestic investment as a single equipment product originating from France, and define the share of equipment product k coming from the country of origin c as $S^e_{ckit} = I^e_{ckit}/\tilde{I}^e_{it}$, where $\tilde{I}^e_{it}$ is the total firm-level equipment investment at time t on product-country varieties common between $t-1$ and t. Similarly, we define the share of equipment products k coming from country of origin c at $t-1$ as $S^e_{ckit-1} = I^e_{ckit-1}/\tilde{I}^e_{it-1}$, where $\tilde{I}^e_{it-1}$ is the total firm-level equipment investment at time $t-1$ on common (product-country) varieties between $t-1$ and t. Using these shares, we define product-country weights ω^e_{ckit} as:

$$\omega^e_{ckit} = \begin{cases} \frac{S^e_{ckit} - S^e_{ckit-1}}{s^e_{ckit} - s^e_{ckit-1}} & \text{if } S^e_{ckit} \neq S^e_{ckit-1}, \\ S^e_{ckit} & \text{if } S^e_{ckit} = S^e_{ckit-1}, \end{cases}$$

and we compute the firm-level Sato-Vartia log price indices as:

$$\mathbb{P}^e_{it} = \exp\left(\sum_{ck} (p_{cki,t} - p_{cki,t-1}) \frac{\omega^e_{ckit}}{\sum_{ck} \omega^e_{ckit}}\right).$$

Subsequently, we compute equipment investment quantities Q^e_{it} using equipment investment values (I^e_{it},) and the price indices computed in the previous stage: $Q^e_{it} = I^e_{it}/P^e_{it}$ where $P^e_{it} \equiv \prod_{\tau=0}^{t} \mathbb{P}^e_{i\tau}$.

Construction of the Equipment Stocks. Finally, we compute the equipment stock X_{eit}, applying the perpetual inventory method:

$$X^e_{it} = \begin{cases} \frac{1}{\delta^e_{st}} \frac{\sum_{\tau=0}^{T_i - 1} Q^e_{i\tau}}{T_i}, & \text{if } t = 1, \\ (1 - \delta^e_{st}) Q^e_{it-1} + Q^e_{it}, & \text{if } t > 1, \end{cases}$$

where δ^e_{st} is a sector-specific depreciation rate obtained from the EU KLEMS dataset. Following Mueller (2008), we set the initial capital stock for each firm i as the average investment over all T_i years in which the firm is present in the sample divided by the depreciation rate.

E Additional Tables and Figures

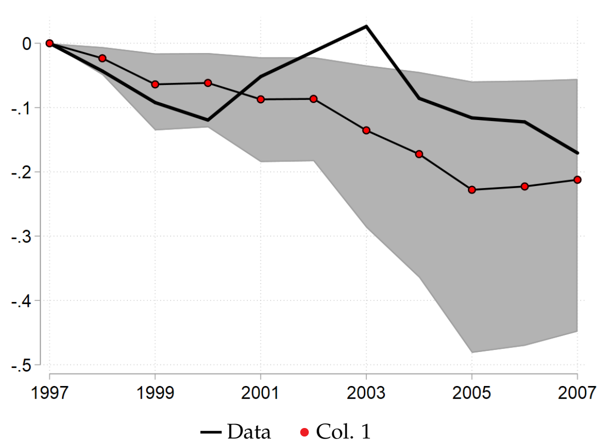
Table E.1: Between-Firm Demand Elasticity Estimates (Alternative Instrument)

	OLS	2SLS				
	(1)	(2)	(3)	(4)	(5)	(6)
ε	0.978	2.325	2.696	2.084	2.330	2.108
	(0.010)	(0.589)	(0.860)	(0.396)	(0.593)	(0.496)
Observations	72,202	44,020	53,171	44,020	44,020	43,579
Year FE	-	-	-	Yes	-	-
Year-Sector FE	Yes	Yes	Yes	No	Yes	Yes
Controls	No	No	No	No	Yes	No
Exp/GO$\geq$ 5%	Yes	Yes	No	Yes	Yes	Yes
IV	-	RER ($\mathcal{K}_I$)	RER ($\mathcal{K}_I$)	RER ($\mathcal{K}_I$)	RER ($\mathcal{K}_I$)	RER + SSW
KP F stat	-	13.06	8.34	16.76	12.92	8.67

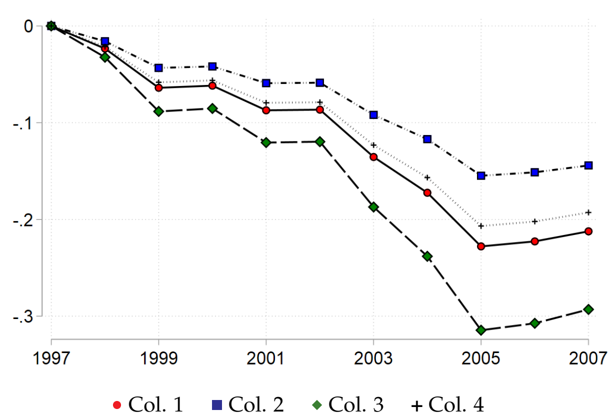
Notes: This table reports estimation results of equation (35). The dependent variable is the 5-year change in firm i's export quantity and the independent variable is $-1\times$ the corresponding change in firm i's export price. Columns 2-6 use the RER instrument defined in Equation (38), with the difference that exposure shares are based on the import of intermediate products only. Column 6 additionally uses the SSW instrument defined in equation (41). Year fixed effects are included in column 4 and year-sector fixed effects are included in all other columns. Controls indicates the inclusion of the average import share in the pre-sample period (1996-7) interacted with a time-varying coefficient. Exp/GO$\geq$5% indicates that the sample is restricted to firms that export at least 5% of gross output. We report the Kleibergen-Paap rk Wald (KP F stat) of the first stage regression. Robust standard errors are clustered at the firm level and reported in parenthesis.

Figure E.1: Predicted Labor Share, Uniform Shock

(a) Baseline (Col. 1 of Table 4)



(b) Alternative Estimates



Notes: Panel (a) plots the evolution of the predicted labor share ($\theta_n/(1-\theta_n)$) in response to the observed fall in the equipment price relative to the low skill shadow wage. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage ('Smic') sourced from the French Ministry of Labour. The figure displays a 95% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. Panel (b) plots the variation in the predicted labor share across the different estimated aggregate elasticities of Table 4.

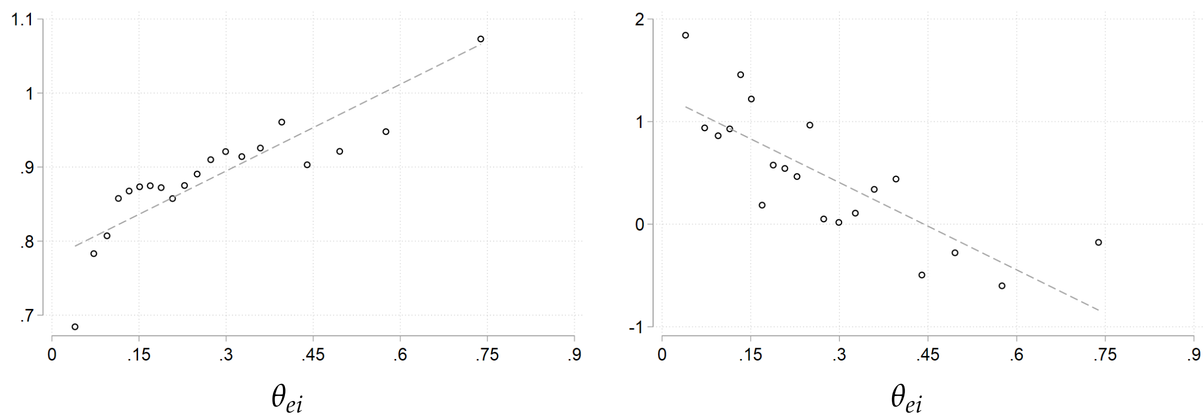
Table E.2: Joint Estimation of Between-Firm Demand Elasticity and Firm-Level Production Elasticities

	(1)	(2)	(3)
	$\Delta x_e - \Delta x_\ell$	$\Delta x_e - \Delta x_h$	Δq
$\Delta w_e - \Delta w_\ell$	-0.922		
	(0.147)		
$\frac{\varepsilon}{\varepsilon-1}\Delta r - \Delta x_e$	-0.0705		
	(0.0467)		
$\Delta w_e - \Delta w_h$		-0.817	
		(0.128)	
Δp			-1.927
			(0.418)
Observations		**26,845**	
Year FE		**Yes**	
σ_ℓ		**0.922**	
σ_h		**0.817**	
σ_e		**0.992**	
ε		**1.93**	
$\Pr[\sigma_\ell \leq \sigma_h]$		**0.015**	

Notes: This table reports regression coefficients for the system of equations (35), (36), and (37) jointly estimated using GMM. Corresponding point estimates of σ_ℓ, σ_h, σ_e, and ε are displayed. Robust standard errors are clustered at the firm level and reported in parenthesis. $\Pr[\sigma_\ell \leq \sigma_h]$ reports a one-sided Wald test.

Figure E.2: Change in Equipment and Labor Wedges (1998-2008) by Equipment Intensity

(a) Equipment wedges (ω_{ei}) (b) High-skill wedges (ω_{hi})



Notes: The figure plots the change in firm-specific equipment price wedges (ω_{ei}, Panel a) and high-skill labor wedges (ω_{hi}, Panel b) by bins of equipment shares (θ_{ei}).

Table E.3: Aggregate Elasticities of Substitution between Factors

σ_{he}	σ^*_{he}	σ_{le}	σ^*_{le}	σ_{lh}	σ^*_{lh}
1.321	1.323	1.386	1.379	1.043	1.046
[1.069, 1.694]	[1.071, 1.697]	[1.120, 1.791]	[1.112, 1.782]	[0.835, 1.277]	[0.838, 1.280]

Notes: The table reports the aggregate elasticities of substitution between factors constructed using Equation (6) and the aggregate elasticities of factor payment substitution using Equation (7).

Table E.4: Predicted Change in the Skill Premium and Labor Share, Uniform Shock, Back-of-the-envelope Calculation

	1990-2015	1997-2007
Δw_e	-0.491	-0.246
Δw_ℓ	0.745	0.339
Δw_e-Δw_ℓ	-1.236	-0.585
$\Delta\psi$	0.067	0.032
$\Delta \ln \theta_n/(1-\theta_n)$	-0.445	-0.211

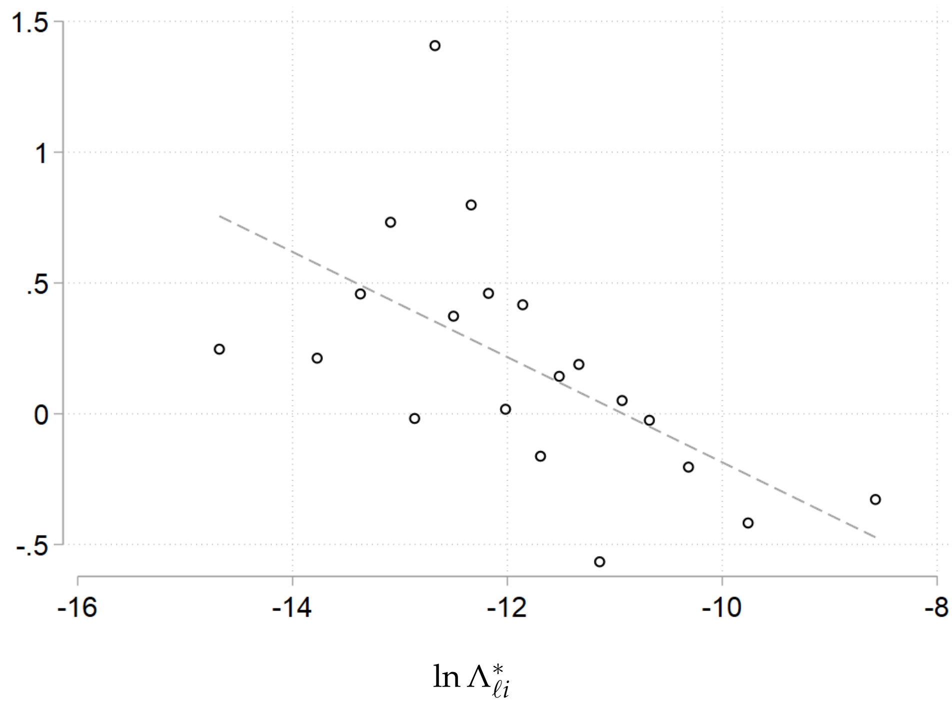
Notes: This table reports the predicted log change in the skill premium (ψ) and the labor share ($\theta_n/(1-\theta_n)$) in response to the observed fall in the equipment price relative to the low skill shadow wage. The predicted change is obtained by multiplying the total cumulated change in the relative equipment price by the average aggregate elasticities over the entire period from column 1 of Table 4. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage ('Smic') sourced from the French Ministry of Labour.

Table E.5: Predicted Change in the Skill Premium, Decomposition

	Equipment	High-skilled	Low-skilled	**Total**
Within-firm	0.001	-0.062	0.041	**-0.019**
Cross-firm reallocation	0.059	-0.050	0.009	**0.018**
Cross-sectoral reallocation	0.003	0.008	-0.001	**0.010**
Total	**0.064**	**-0.104**	**0.049**	**0.009**
Cumulated shock	-0.416	0.094	0.051	

Notes: This table reports the predicted log change in the skill premium (ψ) and relative labor share ($\theta_n/(1-\theta_n)$) in response to the observed fall in the firm-specific price of equipment and labor wedges. The within and reallocation components are obtained decomposing the aggregate wedge elasticities of substitution in the terms defined in Lemma 3.

Figure E.3: Difference in the Change in Labor Wedges ($\omega_{hi} - \omega_{\ell i}$) by Low-Skilled Aggregate Payment Share ($\Lambda^*_{\ell i}$)



Notes: The figure plots the difference between the change in high-skilled wedges and the change in low-skilled wedges ($\omega_{hi} - \omega_{\ell i}$) by the logged share of aggregate low-skilled labor payments ($\Lambda^*_{\ell i}$). The change in the wedges in computed over the period 1998-2007.