

Beyond Ricardo: Assignment Models in International Trade*

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Abstract

International trade has experienced a Ricardian revival. In this article, we offer a user guide to assignment models, which we will refer to as Ricardo-Roy (R-R) models, that have contributed to this revival.

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1 Introduction

International trade has experienced a Ricardian revival. For almost two hundred years, David Ricardo’s theory of comparative advantage has been perceived as a useful pedagogical tool with little empirical content. The seminal work of [Eaton & Kortum \(2002\)](#) has shattered this perception and led to a boom in quantitative work in the field, nicely surveyed in [Eaton & Kortum \(2012\)](#). As part of this Ricardian revival, trade economists have also developed assignment models that incorporate multiple factors of production into Ricardo’s original model. In recent years, these models have been used to study a broad set of issues ranging from the impact of trade on the distribution of earnings to its mitigating effect on the consequences of climate change in agricultural markets. The goal of this article is to offer a user guide to these multi-factor generalizations of the Ricardian model, which we will refer to as Ricardo-Roy (R-R) models.

By an R-R model, we formally mean a trade model in which production functions are linear, as in the original Ricardian model, but one in which countries may be endowed with more than one factor, as in the Roy model. Total output in any given sector and country, say wine in Portugal, can thus be expressed as

$$Q(\text{Wine, Portugal}) = \sum_f A(f, \text{Wine, Portugal})L(f, \text{Wine, Portugal}),$$

where $A(f, \text{Wine, Portugal})$ denotes the productivity of factor f , if employed in the wine sector in Portugal, and $L(f, \text{Wine, Portugal})$ denotes the employment of that factor. When the number of factors in each country is equal to one, the R-R model collapses to the Ricardian model. Depending on the particular application, different factors may correspond to different types of labor, capital, or land, whereas different sectors may correspond to different industries, occupations, or tasks. But regardless of what the particular application may be, the key feature of R-R models is that factors’ marginal products, and hence marginal rates of technical substitution, are constant. As a result, comparative advantage—i.e., relative differences in productivity—drives the assignment of factors to sectors around the world.

The first part of our survey uses R-R models to revisit a number of classical questions in the field. Among other things, we discuss how cross-country differences in technologies and factor endowments affect the pattern of international trade, as in [Costinot \(2009\)](#), as well as how changes in the economic environment—including opening up to trade—affect factor allocation and factor prices, as in [Costinot & Vogel \(2010\)](#). Answering these questions in the context of an R-R model requires new tools and techniques. Because

of the linearity of the production function, corner solutions in R-R models are the norm rather than the exception. Hence, the main issue when solving for competitive equilibria is to characterize the extensive margin, that is the set of sectors to which a given factor should be assigned. Fortunately, standard mathematical notions and results from the monotone comparative statics literature, such as log-supermodularity and [Milgrom & Shannon \(1994\)](#)'s Monotonicity Theorem, are well suited to deal with this and other related issues. We briefly review these mathematical tools in [Section 2](#).

Compared to previous neoclassical trade models, R-R models offer a useful compromise. They are more general than Ricardian models, which makes them amenable to study how factor endowments shape international specialization as well as the distributional consequences of trade, yet since marginal rates of technical substitution are constant, they remain significantly more tractable than general neoclassical trade models with arbitrary numbers of goods and factors. Predictions derived in such general models tend to be either weak or unintuitive. For example, the “Friends and Enemies” result of [Jones & Scheinkman \(1977\)](#) states that a rise in the price of some good causes a disproportionately larger increase in the price of some factor; but depending on the number of goods and factors, it may or may not lead to a disproportionately larger decrease in the price of some other factor. A common theme in that older literature, reviewed by [Ethier \(1984\)](#), is that predictions in high-dimensional environments hinge on the answer to one fairly abstract question: Are there more goods than factors in the world?

In [Section 3](#), we demonstrate that R-R models deliver sharp predictions in economies with large numbers of goods and factors. First, they offer variations of classical theorems—e.g., Factor Price Equalization, Rybczynski, and Stolper-Samuelson theorems—whose empirical content is no weaker than their famous counterparts in the two-good-two-factor Heckscher-Ohlin model. Second, R-R models offer new predictions regarding the impact of changes in the distribution of prices, factor endowments, or factor demands with no counterparts in the two-good-two-factor Heckscher-Ohlin model. These theoretical results are useful because they open the door for general equilibrium analyses of recent phenomena that have been documented in the labor and public finance literatures, but would otherwise fall outside the scope of standard trade theory. These recent phenomena include changes in inequality at the top of the income distribution as well as wage and job polarization; see e.g. [Piketty & Saez \(2003\)](#), [Autor, Katz & Kearney \(2008\)](#), and [Goos & Manning \(2007\)](#), respectively.

[Section 4](#) presents various extensions of R-R models. We first introduce imperfect competition, as in [Sampson \(2014\)](#). When good markets are monopolistically competitive à la [Melitz \(2003\)](#), we show how the same tools and techniques can also shed light on the

relationship between firm heterogeneity, worker heterogeneity, and international trade. We also incorporate sequential production, as in [Costinot, Vogel & Wang \(2013\)](#), to study how vertical specialization shapes inequality and the interdependence of nations. We conclude by discussing a number of generalizations and variations of the basic linear production functions at the core of R-R models.

The last two sections focus on quantitative and empirical work. In Section 5, we emphasize parametric applications of R-R models using Generalized Extreme Value (GEV) distributions of productivity shocks. We draw a distinction between models that feature unobserved heterogeneity across goods, as in the influential Ricardian model of [Eaton & Kortum \(2002\)](#), and models that feature unobserved heterogeneity across factors, as in the more recent work of [Lagakos & Waugh \(2013\)](#), [Hsieh et al. \(2013\)](#), or [Burstein, Morales & Vogel \(2014\)](#). In both cases, we discuss how to conduct counterfactual and welfare analysis and highlight the key differences associated with these two distinct approaches. In Section 6, we turn to non-parametric applications of R-R models to agricultural markets based on detailed micro-level data from the Food and Agriculture Organization's (FAO) Global Agro-Ecological Zones (GAEZ) project. These non-parametric applications include empirical tests of Ricardo's theory of comparative advantage ([Costinot & Donaldson, 2012](#)), the measurement of the gains from economic integration ([Costinot & Donaldson, 2014](#)), and a quantitative analysis of the consequences of climate change ([Costinot, Donaldson & Smith, 2014](#)).

R-R models are related to an older literature on linear programming in economics, see [Dorfman, Samuelson & Solow \(1958\)](#). Since production functions are linear in R-R models, solving for efficient allocations in such models amount to solving linear programming problems, an observation made by [Whitin \(1953\)](#) in the context of the Ricardian model. [Ruffin \(1988\)](#) was the first to point out that multiple-factor generalizations of the Ricardian model may provide a useful alternative to Heckscher-Ohlin models with arbitrary neoclassical production functions. He offers a number of examples with two countries and two or three factors in which simpler theorems about trade, welfare, and factor payments can be derived. A similar idea can be found in [Ohnsorge & Trefler \(2007\)](#) who use the log-normal specification of the Roy model to derive variations of the Rybczynski and Heckscher-Ohlin theorems in economies with heterogeneous workers.

Though labor markets are not the only possible application of R-R models, it is an important one. Assignment models, in general, and the Roy model, in particular, have been fruitfully applied by labor economists to study the effect of self-selection on the distribution of earnings as well as the assignment of workers to tasks; see e.g. [Roy \(1951\)](#), [Heckman & Sedlacek \(1985\)](#), [Borjas \(1987\)](#), [Heckman & Honore \(1990\)](#), [Teulings \(1995\)](#),

Teulings (2005), and Acemoglu & Autor (2011). Sattinger (1993) provides an early survey of that literature that clarifies the relationship between the Roy model and other assignment models. Some of these alternative assignment models, such as Becker (1973), Lucas (1978), and Garicano (2000), have also been fruitfully applied in an open economy context to study the effects of international trade and offshoring on heterogeneous workers or entrepreneurs; see e.g. Grossman & Maggi (2000), Kremer & Maskin (2006), Antras, Garicano & Rossi-Hansberg (2006), Nocke & Yeaple (2008), Monte (2011), and Grossman, Helpman & Kircher (2013). Like R-R models, these alternative assignment models can be thought of as very simple neoclassical models—in the sense that very strong assumptions on the complementarity between factors of production are imposed—which makes them well-suited to study economies with a large number of factors of production. The surveys of Antras & Rossi-Hansberg (2009) and Garicano & Rossi-Hansberg (2014) in this journal as well as Grossman (2013) offer nice overviews of recent work in this area.

2 The Mathematics of Comparative Advantage

The premise of David Ricardo’s theory of comparative advantage is that some individuals or countries are *relatively more* productive in some activities than others. In his famous example, England is relatively better than Portugal at producing cloth than wine. Assuming that labor is the only factor of production in each country and that technology is subject to constant returns to scale, the previous statement can be expressed as

$$A(\text{Cloth, England}) / A(\text{Wine, England}) \geq A(\text{Cloth, Portugal}) / A(\text{Wine, Portugal}), \quad (1)$$

where $A(\cdot, \cdot)$ denotes labor productivity in a given sector and country. According to inequality (1), England has a comparative advantage in cloth and, if inequality (1) did not hold, it would have a comparative advantage in wine.

Now let us move beyond David Ricardo’s example and consider a world economy with any number of goods and countries. How would one generalize inequality (1) to formalize the notion that some countries may have a comparative comparative advantage in some sectors? A fruitful way to proceed is to assume that each country and sector can be described by a scalar, call them γ and σ , respectively. For instance, γ and σ may reflect the quality of a country’s financial institutions and the dependence of a sector on external financing, as in Matsuyama (2005); the level of rigidities in a country’s labor market and the volatility of sectoral productivity or demand shocks, as in Melitz & Cunat (2012); or more generally, the level of development of a country and the technological intensity of

a sector, as in [Krugman \(1986\)](#). In such environments, statements about the comparative advantage of high- γ countries in high- σ sectors can still be expressed as

$$A(\sigma', \gamma') / A(\sigma, \gamma') \geq A(\sigma', \gamma) / A(\sigma, \gamma), \text{ for all } \sigma' \geq \sigma \text{ and } \gamma' \geq \gamma. \quad (2)$$

Mathematically, inequality (2) implies that A is log-supermodular in (σ, γ) . This particular form of complementarity captures the idea that increasing one variable is *relatively more* important when the other variables are high and is intimately related to the notion of comparative advantage introduced by David Ricardo.

The previous idea easily extends to situations in which σ and γ are multi-dimensional. For any $x, x' \in \mathbb{R}^n$, let $\max(x, x')$ be the vector of \mathbb{R}^n whose i th component is $\max(x_i, x'_i)$, and $\min(x, x')$ be the vector whose i th component is $\min(x_i, x'_i)$. Given the previous notation, a function $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$ is log-supermodular if for all $x, x' \in \mathbb{R}^n$,

$$g(\max(x, x')) \cdot g(\min(x, x')) \geq g(x) \cdot g(x').$$

Inequality (2) above corresponds to the special case in which $x \equiv (\sigma', \gamma)$ and $x' \equiv (\sigma, \gamma')$. If g is strictly positive, then g is log-supermodular if and only if $\ln g$ is supermodular. Accordingly, g is log-supermodular if and only if for all i and j , g is log-supermodular in (x_i, x_j) when regarded as a function of the arguments (x_i, x_j) only. If g also is twice differentiable, then the latter condition is equivalent to $\frac{\partial^2 \ln g}{\partial x_i \partial x_j} \geq 0$. If the above inequality holds with a strict inequality, we say that g is strictly log-supermodular and if the above inequality is reversed, we say that g is log-submodular.

Most of our theoretical results build on three properties of log-supermodular functions:

Property 1. *If $g, h: \mathbb{R}^n \rightarrow \mathbb{R}^+$ are log-supermodular, then gh is log-supermodular.*

Property 2. *If $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$ is log-supermodular, then $G(x_{-i}) \equiv \int g(x_i, x_{-i}) dx_i$ is log-supermodular, where x_{-i} denotes the vector x with the i th component removed.*

Property 3. *If $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$ is log-supermodular, then $x_i^*(x_{-i}) \equiv \arg \max_{x_i \in \mathbb{R}} g(x_i, x_{-i})$ is increasing in x_{-i} with respect to the component-wise order.*

Properties 1 and 2 state that log-supermodularity is preserved by multiplication and integration. Property 1 derives from the definition of log-supermodularity. A general proof of Property 2 can be found in [Karlin & Rinott \(1980\)](#). Since log-supermodularity is a strong form of complementarity—stronger than quasi-supermodularity and the single crossing property—Property 3 derives from [Milgrom & Shannon's \(1994\)](#) Monotonicity Theorem. Note that in Property 3, $x_i^*(x_{-i})$ may not be a singleton. If so, the monotonicity

of $x_i^*(x_{-i})$ is expressed in terms of the strong set order. In this case, both the supremum and the infimum of the set $x_i^*(x_{-i})$ are increasing in x_{-i} .

As we demonstrate next, log-supermodularity offers a powerful way to parametrize cross-country differences in technology, preferences, and endowments in order to study their implications for the global allocation of factors and the distribution of earnings.

3 The R-R Model

In this section we introduce our baseline version of the R-R model and derive cross-sectional and comparative static predictions in this environment.

3.1 Assumptions

Consider a world economy with many countries indexed by $\gamma \in \Gamma \subset \mathbb{R}^3$. The vector of country characteristics, γ , comprises a technology shifter, γ_A , a taste shifter, γ_D , and a factor endowment shifter, γ_L . These three variables capture all potential sources of international specialization. Each country is populated by a representative agent endowed with multiple factors indexed by $\omega \in \Omega \subset \mathbb{R}$. The representative agent has homothetic preferences over multiple goods or sectors indexed by $\sigma \in \Sigma \subset \mathbb{R}$. All markets are perfectly competitive and all goods are freely traded across countries.¹ $p(\sigma)$ denotes the world price of good σ . Factors are immobile across countries and perfectly mobile across sectors. $L(\omega, \gamma_L) \geq 0$ denotes the inelastic supply of factor ω in country γ and $w(\omega, \gamma)$ denotes the price of factor ω in country γ .

The defining feature of R-R models is that production functions are linear. Output of good σ in country γ is given by

$$Q(\sigma, \gamma) = \int_{\Omega} A(\omega, \sigma, \gamma_A) L(\omega, \sigma, \gamma) d\omega, \quad (3)$$

where $A(\omega, \sigma, \gamma_A) \geq 0$ denotes the exogenous productivity of factor ω in country γ if employed in sector σ and $L(\omega, \sigma, \gamma)$ denotes the endogenous quantity of factor ω used to produce good σ in country γ .²

¹Free trade in goods will play the same role in an R-R model as in a standard neoclassical trade model. It will be crucial for R-R versions of the Factor Price Equalization and the Heckscher-Ohlin Theorems, which rely on good prices being equalized around the world. It will play no role for predictions like the Stolper-Samuelson Theorem, which hold for arbitrary vectors of good prices. Like in the Ricardian model, trade costs can be easily incorporated into R-R models when productivity shocks are drawn from a GEV distribution, as in [Eaton & Kortum \(2002\)](#). We discuss such parametric applications in Section 5.

²There may be a continuum or a discrete number of factors in Ω . Whenever the integral sign “ \int_{Ω} ”

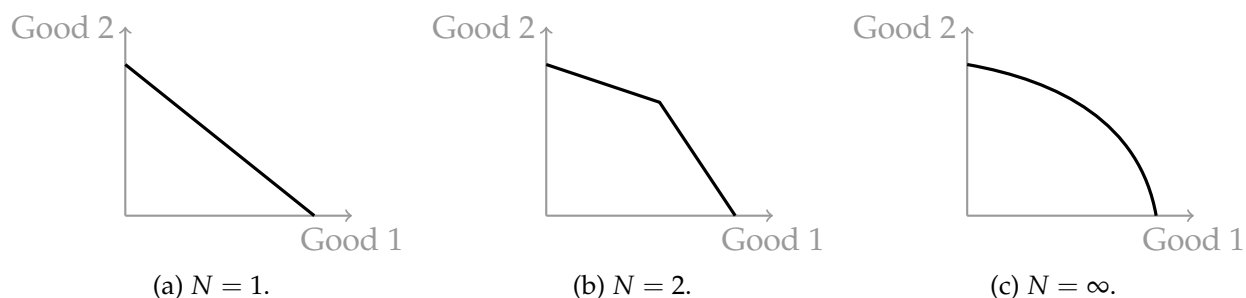


Figure 1: PPF in R-R model with 2 goods and $N = 1, 2, \infty$ factors.

The Ricardian model corresponds to the special case in which there is only one factor of production in each country. In this situation, the production possibility frontier in any country γ reduces to a straight line; see Figure 1a. In an R-R model more generally, countries may be endowed with multiple factors of production, leading to kinks in the production possibility frontier; see Figure 1b. As the number of factors goes to infinity, the production possibility frontier becomes smooth, as in a standard Heckscher-Ohlin or specific factor model; see Figure 1c.³

This is an important observation. Holding the number of factors fixed, an R-R model with a linear production function is necessarily more restrictive than a standard neoclassical trade model. But the number of factors needs not be fixed. In particular, an R-R model with a continuum of factors does not impose more *a priori* restrictions on the data than a Heckscher-Ohlin model with two factors. To take an analogy from the literature on discrete choice models in industrial organization, assuming that a continuum of heterogeneous consumers have constant marginal rates of substitution may not lead to different implications for aggregate demand than assuming a representative agent with a general utility function; see e.g. Anderson, de Palma & Thisse (1992). We come back to related issues in Section 5 when discussing parametric applications of R-R models.

appears, one should therefore think of a Lebesgue integral. If there is a finite number of factors, " \int_{Ω} " is simply equivalent to " \sum_{Ω} ." Integrals over country and sector characteristics should be interpreted in a similar manner.

³In an Arrow-Debreu economy, which R-R models are special cases of, one can always think of factors located in different countries as different factors. In the absence of trade costs and cross-country differences in preferences, the closed economy of an R-R model with N factors is therefore equivalent to the world economy of a Ricardian model with N countries.

3.2 Competitive Equilibrium

In a competitive equilibrium, consumers maximize utility, firms maximize profits, and markets clear.

Consumers. Let $D(p, I(\gamma)|\sigma, \gamma_D)$ denote the Marshallian demand for good σ in country γ as a function of the schedule of world prices, $p \equiv \{p(\sigma)\}$, and the income of country γ 's representative agent, $I(\gamma) \equiv \int_{\Omega} w(\omega, \gamma) L(\omega, \gamma_L) d\omega$. By definition of the Marshallian demand, utility maximization requires the consumption of good σ in country γ to satisfy

$$D(\sigma, \gamma) = D(p, I(\gamma)|\sigma, \gamma_D). \quad (4)$$

Firms. For future reference, it is useful to start by studying the cost minimization problem of a representative firm, which is a necessary condition for profit maximization. By equation (3), the unit-cost function of a firm producing good σ in country γ is given by

$$c(\sigma, \gamma) \equiv \min_{l(\omega, \sigma, \gamma) \geq 0} \left\{ \int_{\Omega} w(\omega, \gamma) l(\omega, \sigma, \gamma) d\omega \mid \int_{\Omega} A(\omega, \sigma, \gamma_A) l(\omega, \sigma, \gamma) d\omega \geq 1 \right\}.$$

The linearity of the production function immediately implies

$$c(\sigma, \gamma) = \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\}. \quad (5)$$

In turn, the set of factors, $\Omega(\sigma, \gamma) \equiv \{\omega \in \Omega : L(\omega, \sigma, \gamma) > 0\}$, demanded by firms producing good σ in country γ satisfies

$$\Omega(\sigma, \gamma) \subset \arg \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\}. \quad (6)$$

Having characterized the unit-cost function of a representative firm, its profit function can be expressed as $\pi(\sigma, \gamma) \equiv \max_{q \geq 0} \{p(\sigma)q - c(\sigma, \gamma)q\}$. Profit maximization then requires

$$p(\sigma) \leq c(\sigma, \gamma), \text{ with equality if } \Omega(\sigma, \gamma) \neq \emptyset. \quad (7)$$

Market clearing. Factor and good market clearing finally require

$$\int_{\Sigma} L(\omega, \sigma, \gamma) d\sigma = L(\omega, \gamma_L), \text{ for all } \omega, \gamma, \quad (8)$$

$$\int_{\Gamma} D(\sigma, \gamma) d\gamma = \int_{\Gamma} Q(\sigma, \gamma) d\gamma, \text{ for all } \sigma. \quad (9)$$

To summarize, a competitive equilibrium corresponds to consumption, $D : \Sigma \times \Gamma \rightarrow \mathbb{R}^+$, output, $Q : \Sigma \times \Gamma \rightarrow \mathbb{R}^+$, factor allocation, $L : \Omega \times \Sigma \times \Gamma \rightarrow \mathbb{R}^+$, good prices, $p : \Sigma \rightarrow \mathbb{R}^+$,

and factor prices, $w : \Omega \times \Gamma \rightarrow \mathbb{R}^+$, such that equations (3)-(9) hold.

3.3 Cross-Sectional Predictions

In this section, we follow Costinot (2009) and focus on the cross-sectional predictions of an R-R model. Formally, we take good prices, $p(\sigma)$, as given and explore how factor allocation, factor prices, and aggregate output vary across countries and industries in a competitive equilibrium. Accordingly, demand considerations and the good market clearing conditions—equations (4) and (8)—play no role here.

3.3.1 Factor allocation

A central question in assignment models is: Who works where? In the context of an R-R model, one may be interested in characterizing the set of workers employed in particular sectors or, conversely, the set of goods produced by particular countries. To make progress on these issues, a common practice in the literature is to impose restrictions on technology that generate Positive Assortative Matching (PAM).

Assumption 1. $A(\omega, \sigma, \gamma_A)$ is strictly log-supermodular in (ω, σ) and (σ, γ_A) .

According to Assumption 1, high- ω factors are relatively more productive in high- σ sectors and high- γ countries are relatively more productive in high- σ sectors. A simple example of a log-supermodular function is $A(\omega, \sigma, \gamma_A) \equiv \exp(\omega\sigma)$ or $\exp(\sigma\gamma_A)$, as in Krugman (1986), Teulings (1995), and Ohnsorge & Trefler (2007).⁴

By Property 3, Assumption 1 implies that $\arg \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\}$ is increasing in σ in any country γ . Since the log-supermodularity of A in (ω, σ) is strict, one can further show that for any pair of sectors, $\sigma \neq \sigma'$, there can be at most one factor ω_0 such that $\omega_0 \in \arg \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\} \cap \arg \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma', \gamma_A)\}$. Combining the two previous observations, we obtain our first result.

PAM (I). Suppose that Assumption 1 holds. Then for any country γ , $\Omega(\sigma, \gamma)$ is increasing in σ .

This is intuitive. In a competitive equilibrium, high- ω factors should be employed in the high- σ sectors in which they have a comparative advantage.

⁴The strict log-supermodularity of $A(\omega, \sigma, \gamma_A)$ in (ω, σ) formally rules out the possibility that two distinct factors are perfect substitutes across all sectors. At a theoretical level, this restriction is purely semantic. If two workers only differ in terms of their absolute advantage, one can always refer to them as one factor and let the efficiency units that they are endowed with vary. This is the convention we adopt in this article. Since we assume the existence of a representative agent, the distribution of these efficiency units is irrelevant for any of our theoretical results and is, therefore, omitted. At an empirical level, though, one should keep in mind that the distribution of earnings depends both on the schedule of prices per efficiency units, which we refer to as the factor price, $w(\omega, \gamma)$, and the distribution of these efficiency units.

We can follow a similar strategy to analyze patterns of international specialization. Let $\Sigma(\omega, \gamma) \equiv \{\sigma : L(\omega, \sigma, \gamma) > 0\}$ denote the set of sectors in which factor ω is employed in country γ . Conditions (5) and (7) imply that the value of the marginal product of a factor ω in any sector σ should be weakly less than its price,

$$p(\sigma)A(\omega, \sigma, \gamma_A) \leq w(\omega, \gamma), \text{ for all } \sigma, \quad (10)$$

with equality if factor ω is employed in that sector, $\omega \in \Omega(\sigma, \gamma)$. Since $\sigma \in \Sigma(\omega, \gamma)$ if and only if $\omega \in \Omega(\sigma, \gamma)$, condition (10) further implies that

$$\Sigma(\omega, \gamma) \subset \arg \max_{\sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}. \quad (11)$$

This condition states that factors from any country should be employed in the sector that maximizes the value of their marginal product, an expression of the efficiency of perfectly competitive markets.

Starting from condition (11) and using the exact same logic as above, we obtain the following prediction.

PAM (II). *Suppose that Assumption 1 holds. Then for any factor ω , $\Sigma(\omega, \gamma)$ is increasing in γ_A .*

In a competitive equilibrium, there must also be a “ladder” of countries with high- γ countries in high- σ sectors. This is the prediction at the heart of many Ricardian models such as the technology gap model developed by [Krugman \(1986\)](#) as well as many models in the institutions and trade literature reviewed by [Nunn & Trefler \(2014\)](#). As discussed in [Costinot \(2009\)](#), Assumption 1 is critical for such patterns in the sense that it cannot be dispensed with for PAM to arise in all economic environments satisfying the assumptions of Section 3.1.⁵

An important special case in the literature is the case in which $\Sigma(\omega, \gamma)$ is a singleton. This corresponds to a situation in which each factor is only assigned to one sector. A sufficient condition for $\Sigma(\omega, \gamma)$ to be a singleton is that $\arg \max_{\sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}$ is itself a singleton. Graphically, this situation arises when production occurs at a vertex of the production possibility frontier in [Figure 1](#). As the numbers of factors and hence vertices increase, this restriction becomes milder and milder. If there is a continuum of factors, then $\Sigma(\omega, \gamma)$ must be a singleton for almost all ω .⁶

⁵The symmetry between PAM (I) and PAM (II) should not be surprising. As already discussed earlier, factors in different countries can always be defined as different factors in an Arrow-Debreu economy. Under this alternative interpretation, PAM within and between countries are two sides of the same coin.

⁶This is true regardless of whether there is a finite number of goods or a continuum of goods with finite measure. To see this, note for any $\omega \neq \omega'$, the overlap between $\Sigma(\omega, \gamma)$ and $\Sigma(\omega', \gamma)$ must be measure

3.3.2 Factor prices

Conditions (5) and (7) also have strong implications for the distribution of factor prices within and between countries. Starting from condition (10) and noting that there must exist a σ such that $\omega \in \Omega(\sigma, \gamma)$ by factor market clearing, we obtain

$$w(\omega, \gamma) = \max_{\sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}. \quad (12)$$

Now consider two countries with the same technology, $\gamma_A = \gamma'_A$, but potentially different endowments and preferences. Equation (12) immediately implies $w(\omega, \gamma) = w(\omega, \gamma') \equiv w(\omega)$. In other words, we always have factor price equalization (FPE), as originally noted by Ruffin (1988) and summarized below.

R-R FPE Theorem. *If there are no technological differences between countries, then factor prices are equalized under free trade, $w(\omega, \gamma) = w(\omega)$ for all γ .*

For all subsequent results, we restrict ourselves to an economy with a continuum of factors. As discussed above, this implies that $\Sigma(\omega, \gamma)$ is a singleton. Thus, the allocation of factors to sectors can be summarized by a matching function, M , such that $\Sigma(\omega, \gamma) = \{M(\omega, \gamma)\}$. In Section 3.4, we will add the assumption of a continuum of goods.

Under the assumption of a continuum of factors, we can analyze the distribution of factor prices within each country by differentiating equation (12) with respect ω . By the Envelope Theorem, we must have

$$\frac{d \ln w(\omega, \gamma)}{d\omega} = \frac{\partial \ln A(\omega, M(\omega, \gamma), \gamma_A)}{\partial \omega}. \quad (13)$$

Equation (13) is one of the key equilibrium conditions used in our comparative static analysis. Intuitively, if two distinct factors, ω_1 and ω_2 , were to be employed in the same sector σ , then their relative prices should exactly equal their relative productivities, $w(\omega_1, \gamma)/w(\omega_2, \gamma) = A(\omega_1, \sigma, \gamma_A)/A(\omega_2, \sigma, \gamma_A)$, or in logs,

$$\Delta \ln w(\omega_1, \gamma) - \Delta \ln w(\omega_2, \gamma) = \Delta \ln A(\omega_1, \sigma, \gamma_A) - \Delta \ln A(\omega_2, \sigma, \gamma_A).$$

Equation (13) expands on this observation by using the fact that reallocations of factors across sectors must have second-order effects on the value of a factor's marginal product.⁷

zero under Assumption 1. So if the set of factors for which $\Sigma(\omega, \gamma)$ is not a singleton had strictly positive measure, the set of goods to which they are assigned would have to have infinite measure.

⁷Here, we implicitly assume that $w(\cdot, \gamma)$ is differentiable. In economies with a continuum of goods, this property follows from assuming that $A(\omega, \sigma, \gamma)$ is differentiable. In economies with a discrete number of goods, $w(\cdot, \gamma)$ would necessarily feature a discrete number of kinks. In such environments, the Envelope

Finally, note that whereas equation (12) relies on perfect competition in good markets—through the first-order condition (7)—equation (13) does not. Condition (5) and factor market clearing alone imply that $w(\omega, \gamma) = \max_{\sigma} \{c(\sigma, \gamma)A(\omega, \sigma, \gamma_A)\}$. Starting from this expression and invoking the Envelope Theorem, we again obtain equation (13). This observation will play a central role in extending R-R models to environments with imperfectly competitive good markets.

3.3.3 Aggregate output

We have already established that Assumption 1 imposes PAM. PAM, however, only imposes a restriction on the extensive margin of employment, that is whether a factor should be employed in a sector in a particular country; it does not impose any restriction on the intensive margin of employment, and in turn, aggregate output.

To derive cross-sectional predictions about aggregate output, we now impose the following restriction on the distribution of factor endowments.

Assumption 2. $L(\omega, \gamma_L)$ is log-supermodular.

For any pair of countries, $\gamma'_L \geq \gamma_L$, and factors, $\omega' \geq \omega$, such that $L(\omega, \gamma_L), L(\omega, \gamma'_L) \neq 0$, Assumption 2 implies

$$L(\omega', \gamma'_L)/L(\omega, \gamma'_L) \geq L(\omega', \gamma_L)/L(\omega, \gamma_L).$$

According to Assumption 2, high- γ_L countries are relatively abundant in high- ω factors. Formally, it is equivalent to the assumption that the densities of countries' factor endowments can be ranked in terms of monotone likelihood ratio dominance. [Milgrom \(1981\)](#) offers many examples of density function that satisfy this assumption including the normal (with mean γ_L) and the uniform (on $[0, \gamma_L]$). This is the natural generalization of the notion of skill abundance in a two-factor model. Note that Assumption 2 also allows us to consider situations in which different sets of factor are available in countries γ and γ' . In such situations, the highest- ω factor must be in country γ and the lowest- ω factor in country γ' .

Since $\Sigma(\omega, \gamma)$ is a singleton, employment of a factor ω in a particular sector σ must now be equal to the total endowment of that factor, $L(\omega, \gamma_L)$, whenever $\omega \in \Omega(\sigma, \gamma)$. Thus, output of good σ can be expressed as

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma, \gamma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega.$$

Theorem of [Milgrom & Segal \(2002\)](#) provides a strict generalization of equation (13).

If there are no technological differences between countries, FPE further implies that the allocation of factors to sectors must be the same in all countries, $\Omega(\sigma, \gamma) \equiv \Omega(\sigma)$, so that the previous expression simplifies into

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega. \quad (14)$$

Using equation (14) together with PAM and Properties 1 and 2, which imply that log-supermodularity is preserved by multiplication and integration, [Costinot \(2009\)](#) establishes the following Rybczynski-type result.

R-R Rybczynski Theorem. *Suppose that Assumptions 1 and 2 hold. Then $Q(\sigma, \gamma)$ is log-supermodular in (σ, γ_L) .*

For any pair of goods, $\sigma \leq \sigma'$, and any pair of countries with identical technology, $\gamma_A = \gamma'_A$, but different endowments, $\gamma_L \leq \gamma'_L$, the previous property implies that

$$\frac{Q(\sigma', \gamma')}{Q(\sigma, \gamma')} \geq \frac{Q(\sigma', \gamma)}{Q(\sigma, \gamma)}.$$

In other words, the country that is relatively more abundant in the high- ω factors, i.e. country γ' , produces relatively more in the sector that is “intensive” in those factors under PAM, i.e. sector σ' . This is akin to the predictions of the Rybczynski Theorem in a two-by-two Heckscher-Ohlin model. Here, however, the previous prediction holds for an arbitrarily large number of goods and factors. If one further assumes that countries have identical preferences, $\gamma_D = \gamma'_D$, the Rybczynski Theorem above implies that high- γ_L countries are net exporters of high- σ goods, whereas low- γ_L countries are net exporters of low- σ_L goods, in line with the predictions of the two-by-two Heckscher-Ohlin Theorem, a point emphasized by [Ohnsorge & Trefler \(2007\)](#).

As shown in [Costinot \(2009\)](#), one can use a similar logic to establish that aggregate employment and aggregate revenue in a country and sector must also be log-supermodular functions of (σ, γ_L) . Using U.S. data on cities’ skill distributions, sectors’ skill intensities, and cities’ sectoral employment, [Davis & Dingel \(2013\)](#) provide empirical support for such predictions.

3.4 Comparative Static Predictions

The goal of this subsection is to go from cross-sectional predictions to comparative static predictions about the effects of various shocks on factor allocation and factor prices. We start by revisiting the Stolper-Samuelson Theorem, which emphasizes shocks to good

prices. We then turn to the consequences of factor endowment and taste shocks.⁸ Following Costinot & Vogel (2010), we do so in the case of a continuum of both goods and factors: $\Sigma = [\underline{\sigma}, \bar{\sigma}]$ and $\Omega = [\underline{\omega}, \bar{\omega}]$. Under mild regularity conditions on productivity, endowments, and demand functions, this guarantees that the schedule of factor prices and the matching function are differentiable, which we assume throughout. Comparative static results in the discrete case can be found in Costinot & Vogel (2009).

3.4.1 Price shocks

Consider a small open economy whose characteristics γ are held fixed, whereas country characteristics in the rest of the world, which we summarize by ϕ , are subject to a shock. Using this parametrization, a foreign shock to technology, tastes, or factor endowments simply corresponds to a change from ϕ to ϕ' . In a neoclassical environment, foreign shocks only affect the small open economy γ through their effects on world prices. To make that relationship explicit here, we now let $p(\sigma, \phi)$ denote the world price of good σ as a function of foreign characteristics ϕ .

In line with the analysis with the analysis of Section 3.3, we restrict ourselves to foreign shocks that satisfy the following restriction.

Assumption 3. $p(\sigma, \phi)$ is log-supermodular in (σ, ϕ) .

For any pair of goods, $\sigma' \geq \sigma$, a shock from ϕ to $\phi' \geq \phi$ corresponds to an increase in the relative price of good σ' , which is the good intensive in high- ω factors under PAM. In the context of the two-by-two Heckscher-Ohlin model, the Stolper-Samuelson Theorem predicts that the relative price of the skill-intensive good should lead to an increase in the relative price of skilled workers. We now demonstrate that in an R-R model, a similar prediction extends to economies with an arbitrary large number of goods and factors.

For the purposes of this subsection, and this subsection only, we let $w(\cdot, \gamma, \phi)$ and $M(\cdot, \gamma, \phi)$ denote the schedule of factor prices and the matching function in country γ as a function of the foreign shock, ϕ . Using this notation, we can rewrite equation (12) as

$$w(\omega, \gamma, \phi) = \max_{\sigma} \{A(\omega, \sigma, \gamma_A)p(\sigma, \phi)\}.$$

Starting from the previous equation and invoking the Envelope Theorem, now with re-

⁸If one reinterprets goods as tasks used to produce a unique final good, as in Costinot & Vogel (2010), then taste shocks can also be interpreted as technological shocks to that final good production function.

spect to a change in ϕ , we obtain

$$\frac{d \ln w(\omega, \gamma, \phi)}{d\phi} = \frac{\partial \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \phi}. \quad (15)$$

Since PAM implies that M is increasing in ω , Assumption 3 further implies that

$$\frac{d}{d\omega} \left(\frac{\partial \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \phi} \right) = \frac{dM(\omega, \gamma, \phi)}{d\omega} \frac{\partial^2 \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \sigma \partial \phi} \geq 0.$$

Combining the previous inequality with equation (15), we obtain the following Stolper-Samuelson-type result.

R-R Stolper-Samuelson Theorem. *Suppose that Assumptions 1 and 3 hold. Then $w(\omega, \gamma, \phi)$ is log-supermodular in (ω, ϕ) .*

Economically speaking, the previous result states that increase in the relative price of high- σ goods (caused by a shock from ϕ to ϕ') must be accompanied by an increase in the relative price of high- ω factors (that tend to be employed in the production of these goods). The intuition is again simple. Take two factors, $\omega' \geq \omega$, employed in two sectors, $\sigma' \geq \sigma$, before the shock. If both factors were to remain employed in the same sector after the shock, then the change in their relative prices would just be equal to the change in the relative prices of the goods they produce,

$$\ln \left[\frac{w(\omega', \gamma, \phi')}{w(\omega, \gamma, \phi')} \right] - \ln \left[\frac{w(\omega', \gamma, \phi)}{w(\omega, \gamma, \phi)} \right] = \ln \left[\frac{p(\sigma', \phi')}{p(\sigma, \phi')} \right] - \ln \left[\frac{p(\sigma', \phi)}{p(\sigma, \phi)} \right].$$

Hence, an increase in the relative price of good σ' would mechanically increase the relative price of factor ω' . Like in Section 3.3.2, the previous Stolper-Samuelson-type result expands on this observation by using the fact that factor reallocations across sectors must have second-order effects on the value of a factor's marginal product.

Under the assumption that the small open economy is fully diversified, both before and after the shock, the previous result further implies the existence of a factor $\omega^* \in (\underline{\omega}, \bar{\omega})$ such that real factor returns decrease for all factors below ω^* and increase for all factors above ω^* . In other words, a foreign shock must create winners and losers. Intuitively, factor $\underline{\omega}$ must lose because it keeps producing good $\underline{\sigma}$, whose price decreases relative to all other prices. Conversely, factor $\bar{\omega}$ must win because it keeps producing good $\bar{\sigma}$, whose price increases relative to all other prices.

3.4.2 Endowment and taste shocks

We proceed in two steps. We first study the consequences of endowment and taste shocks in a closed economy. Using the fact that the free trade equilibrium reproduces the integrated equilibrium, we then discuss how these comparative static results under autarky can be used to study the effects of opening up to trade.

Consider a closed economy with characteristic γ . A competitive equilibrium under autarky corresponds to $(D^a, Q^a, L^a, p^a, w^a)$ such that equations (3)-(8) hold and the good market clearing condition (9) is given by

$$D^a(\sigma, \gamma) = Q^a(\sigma, \gamma), \text{ for all } \sigma \text{ and } \gamma. \quad (16)$$

We start by expressing the competitive equilibrium of a closed economy in a compact form as a system of two differential equations in the schedule of factor prices, w^a , and the matching function, M^a .

Given PAM, the factor market clearing condition (8) can be rearranged as

$$\int_{\sigma}^{M^a(\omega, \gamma)} Q^a(\sigma, \gamma) / A\left((M^a)^{-1}(\sigma, \gamma), \sigma, \gamma_A\right) d\sigma = \int_{\omega}^{\omega} L(v, \gamma_L) dv, \text{ for all } \omega, \quad (17)$$

From utility maximization and the good market clearing condition—equations (4) and (16)—we also know that

$$Q^a(\sigma, \gamma) = D^a(p^a, I^a(\gamma) | \sigma, \gamma_D)$$

Substituting into equation (17) and differentiating with respect to ω , we obtain after rearrangements,

$$\frac{dM^a(\omega, \gamma)}{d\omega} = \frac{A(\omega, M^a(\omega, \gamma), \gamma_A) L(\omega, \gamma_L)}{D(p^a, I^a(\gamma) | M^a(\omega, \gamma), \gamma_D)}. \quad (18)$$

In a competitive equilibrium, the slope of the matching function is set such that factor supply equals factor demand. The higher the supply of a given factor, $L(\omega, \gamma_L)$, relative to its demand, $D(p^a, I^a(\gamma) | M^a(\omega, \gamma), \gamma_D) / A(\omega, M^a(\omega, \gamma), \gamma_A)$, the “faster” it should get assigned to sectors for markets to clear.

Costinot & Vogel (2010) derive a number of comparative static predictions in the case in which demand functions are CES:

$$D(p, I(\gamma) | \sigma, \gamma_D) = \frac{B(\sigma, \gamma_D) p^{-\varepsilon}(\sigma) I(\gamma)}{p^{1-\varepsilon}(\gamma_D)}, \quad (19)$$

where $B(\sigma, \gamma_D)$ is a demand-shifter of good σ and $P(\gamma_D) = (\int_{\Sigma} B(\sigma, \gamma_D) p^{1-\varepsilon}(\sigma) d\sigma)^{1/(1-\varepsilon)}$ denotes the CES price index. In the rest of this article, we refer to an economy in which equation (19) holds as a CES economy. In such an economy, normalizing the CES price index to one, equation (18) can be rearranged as

$$\frac{dM^a(\omega, \gamma)}{d\omega} = \frac{A^{1-\varepsilon}(\omega, M^a(\omega, \gamma), \gamma_A) (w^a(\omega, \gamma))^\varepsilon L(\omega, \gamma_L)}{B(M^a(\omega, \gamma), \gamma_D) \int_{\Omega} w^a(\omega', \gamma) L(\omega', \gamma_L) d\omega'}, \quad (20)$$

where we have used $p^a(M^a(\omega, \gamma)) = w^a(\omega, \gamma) / A(\omega, M^a(\omega, \gamma), \gamma_A)$, by conditions (5) and (7), and $I^a(\gamma) = \int_{\Omega} w^a(\omega', \gamma) L(\omega', \gamma_L) d\omega'$.

Equations (13) and (20) offer a system of two differential equations in (M^a, w^a) . The characterization of a competitive equilibrium is completed by the two boundary conditions, $M^a(\underline{\omega}, \gamma) = \underline{\sigma}$ and $M^a(\bar{\omega}, \gamma) = \bar{\sigma}$, which state that the lowest and highest factors should be employed in the lowest and highest sectors, an implication of PAM.

Given equations (13) and (20), one can study how shocks to factor supply and factor demand, parametrized as changes in γ_L and γ_D , respectively, affect factor allocation, $M^a(\omega, \gamma)$, and factor prices, $w^a(\omega, \gamma)$. As we did for technology and factor endowments, we impose the following restriction on how demand shocks, γ_D , affect the relative consumption of various goods.

Assumption 4. $B(\sigma, \gamma_D)$ is log-submodular in (σ, γ_D) .

Given equation (19), Assumption 4 implies that an increase in γ_D lowers the relative demand for high- σ goods.⁹ For any pair of goods, $\sigma \geq \sigma'$, and countries, $\gamma'_D \geq \gamma_D$, such that $B(\sigma, \gamma_D), B(\sigma, \gamma'_D) \neq 0$, we must have

$$D(p, I(\gamma) | \sigma', \gamma'_D) / D(p, I(\gamma) | \sigma, \gamma'_D) \leq D(p, I(\gamma) | \sigma', \gamma_D) / D(p, I(\gamma) | \sigma, \gamma_D).$$

In this environment, [Costinot & Vogel \(2010\)](#) show the two following comparative static results about factor allocation and factor prices.

Comparative Statics (I): Factor Allocation. *Suppose that Assumptions 1, 2, and 4 hold in a CES economy under autarky. Then $M^a(\omega, \gamma)$ is decreasing in γ_D and γ_L .*

Comparative Statics (II): Factor Prices. *Suppose that Assumptions 1, 2, and 4 hold in a CES economy under autarky. Then $w^a(\omega, \gamma)$ is log-submodular in (ω, γ_D) and (ω, γ_L) .*

Consider first an endowment shock from γ_L to $\gamma'_L \geq \gamma_L$. By Assumption 2, this corresponds to an increase in the relative supply of high- ω factors. In the new equilibrium,

⁹Assuming log-submodularity rather than log-supermodularity is purely expositional. This convention guarantees that changes in γ_L and γ_D have symmetric effects on factor allocation and factor prices.

this must be accompanied by an increase in the set of sectors employing higher- ω factors, which is achieved by a downward shift in the matching function. Having established that the matching function must shift down, one can then use equation (13) to sign the effect of a change in relative factor supply on relative factor prices:

$$\frac{d^2 \ln w^a(\omega, \gamma)}{d\gamma_L d\omega} = \frac{dM^a(\omega, \gamma)}{d\gamma_L} \frac{\partial^2 \ln A(\omega, M^a(\omega, \gamma), \gamma_A)}{\partial \sigma \partial \omega} \leq 0,$$

where the previous inequality uses $\frac{dM^a(\omega, \gamma)}{d\gamma_L} \leq 0$ and $\frac{\partial^2 \ln A(\omega, M^a(\omega, \gamma), \gamma_A)}{\partial \sigma \partial \omega} \geq 0$ by Assumption 1. As intuition would suggest, if the relative supply of high- ω factors go up, their relative price must go down.

The intuition regarding the effect of a taste shock is similar. By Assumption 4, an increase in γ_D corresponds to a decrease in the relative demand for high- σ goods. This change in factor demand must be accompanied by factors moving into lower- σ sectors, which explains why $M^a(\omega, \gamma)$ is decreasing in γ_D . Conditional on the change in the matching function, the effects on relative factors prices are the same as in the case of a shock to factor endowments. If factors move into lower- σ sectors in which low- ω factors have a comparative advantage, low- ω factors will be relatively better off.

As shown in [Costinot & Vogel \(2010\)](#), the same approach can be used to study richer endowment and taste shocks, e.g. shocks that disproportionately affect “middle” factors or sectors. While the economic forces at play are similar to those presented here, such extensions are important since they allow for the analysis of recent labor market phenomena such as job and wage polarization, as emphasized by [Acemoglu & Autor \(2011\)](#).

To go from the previous closed-economy results to the effect of opening up to trade, we can use the fact that under factor price equalization, the free trade equilibrium replicates the integrated equilibrium. Hence in the absence of technological differences across countries, factor allocation and prices in any country γ , $M(\omega, \gamma)$ and $w(\omega, \gamma)$, must be equal to those of a fictitious world economy under autarky, $M^a(\omega, \gamma^w)$ and $w^a(\omega, \gamma^w)$, with

$$\frac{dM^a(\omega, \gamma^w)}{d\omega} = \frac{A^{1-\varepsilon}(\omega, M^a(\omega, \gamma^w), \gamma_A^w) (w^a(\omega, \gamma^w))^\varepsilon L(\omega, \gamma_L^w)}{B(M^a(\omega, \gamma^w), \gamma_D^w) \int_{\Omega} w^a(\omega', \gamma^w) L(\omega', \gamma_L^w) d\omega' } \quad (21)$$

$$\frac{d \ln w^a(\omega, \gamma^w)}{d\omega} = \frac{\partial \ln A(\omega, M^a(\omega, \gamma^w), \gamma_A^w)}{\partial \omega}. \quad (22)$$

In the previous system of equations, γ_A^w corresponds to the technological parameter com-

mon across countries, whereas γ_L^w and γ_D^w are implicitly defined such that

$$\begin{aligned} L(\omega, \gamma_L^w) &= \int_{\Gamma} L(\omega, \gamma_L) d\gamma_L, \\ B(\sigma, \gamma_D^w) &= \int_{\Gamma} \frac{\int_{\Omega} w^a(\omega, \gamma^w) L(\omega, \gamma_L) d\omega}{\int_{\Omega} w^a(\omega, \gamma^w) L(\omega, \gamma_L^w) d\omega} P^{\varepsilon-1}(\gamma) B(\sigma, \gamma_D) d\gamma_D. \end{aligned}$$

In the two-country case, one can check that if $\gamma \leq \gamma'$, then $\gamma^w \in [\gamma, \gamma']$. This simple observation implies that the consequences of opening up to trade in country γ are isomorphic to an increase in γ_D and γ_L under autarky, with effects on factor allocation and factor prices as described above. Trade will lead to sector downgrading for all factors, i.e. a downward shift in the matching function, and to a pervasive decrease in the relative price of high- ω factors. The opposite is true in country γ' . Like in the case of a closed economy, the previous logic can also be used to study the effects of trade integration between countries that differ in terms of “diversity,” as emphasized in [Grossman & Maggi \(2000\)](#).

We conclude by pointing out that although we have presented the above comparative static results as closed economy results in an R-R model with a continuum of factors, they can always be interpreted as open economy results in a Ricardian model with a continuum of countries, as in [Matsuyama \(1996\)](#) and [Yanagawa \(1996\)](#). To do so, one simply needs to define factors in different countries as different factors. Under this interpretation, the previous results can be used, for instance, to shed light on the impact of growth in a subset of countries on patterns of specialization—as captured by the matching function—and the world income distribution—as captured by the schedule of factor prices.

4 Theoretical Extensions

The baseline R-R model presented above is special along two dimensions: good markets are perfectly competitive and production functions are linear. In this section, we relax these assumptions about market structure and technology and show how to apply the tools and techniques introduced in Section 3 to these alternative environments.

4.1 Monopolistic Competition

We first follow [Sampson \(2014\)](#) and introduce monopolistic competition with firm-level heterogeneity à la [Melitz \(2003\)](#) into an otherwise standard R-R model.¹⁰ We focus on

¹⁰Other recent papers introducing monopolistic competition into an R-R model include [Edwards & Peroni \(2014\)](#), [Gaubert \(2014\)](#), and [Grossman & Helpman \(2014\)](#).

a world economy comprising $n + 1$ symmetric countries and omit for now the vector of country characteristics γ . Goods markets are monopolistically competitive and preferences are CES over a continuum of symmetric varieties. There is an unbounded pool of potential entrants that are ex-ante identical. To enter, a firm incurs a sunk cost, $f_e > 0$. Entry costs and all other fixed costs are proportional to the CES price index, which we normalize to one. Upon entry, a firm randomly draws a blueprint with characteristic σ from a distribution G . If the firm incurs an additional fixed cost $f > 0$, it can produce a differentiated variety for the domestic market using the same linear production function as in Section 3.1,

$$q(\sigma) = \int_{\Omega} A(\omega, \sigma) l(\omega, \sigma) d\omega,$$

where $A(\omega, \sigma)$ denotes the productivity of the firm if it were to hire $l(\omega, \sigma)$ units of factor $\omega \in [\underline{\omega}, \bar{\omega}]$. We further assume that $A(\omega, \sigma)$ is strictly increasing in σ so that σ is an index of firm-level productivity. The production function in Melitz (2003) corresponds to the special case in which there is only one factor of production and $A(\omega, \sigma) \equiv \sigma$. Finally, in order to export, a firm must incur a fixed cost $f_x \geq 0$ per market and a per-unit iceberg trade cost $\tau \geq 1$.

Like in Section 3.2, consumers maximize their utility, firms maximize their profits, and markets clear. The key difference is that firms have market power. Thus profit maximization now requires marginal cost to be equal to marginal revenue rather than price,

$$\begin{aligned} \frac{dr(q, \sigma)}{dq} &= \frac{w(\omega)}{A(\omega, \sigma)}, \\ \frac{dr_x(q_x, \sigma)}{dq_x} &= \frac{\tau w(\omega)}{A(\omega, \sigma)}, \end{aligned}$$

where $r(q, \sigma)$ and $r_x(q_x, \sigma)$ denote a firm's revenue if it sells $q > 0$ and $q_x > 0$ units in the domestic and foreign markets, respectively. In contrast, the cost minimization problem of the firm is unchanged. Given the linearity of the production function, conditions (5) and (6) must still hold. Under Assumption 1, this immediately implies that we must have PAM in this alternative environment: high- ω factors will be employed in high- σ firms. Since high- σ firms will also be larger in terms of sales and more likely to be exporters, as in Melitz (2003), R-R models with monopolistic competition therefore provide simple micro-foundations for the well-documented firm-size and exporter wage premia.¹¹

¹¹Yeaple (2005) provides an early example of a monopolistically competitive model with firm and worker heterogeneity in which PAM arises under Assumption 1. Alternative micro-foundations for the firm-size and exporter wage premia based on extensions of Melitz (2003) with imperfectly competitive labor markets can be found in Davidson, Matusz & Shevchenko (2008), Helpman, Itskhoki & Redding (2010), and Egger & Kreickemeier (2012), among others.

As discussed in Section 3.3, since equation (5) still holds, we must also have $w(\omega) = \max_{\sigma} \{A(\omega, \sigma)c(\sigma)\}$. By the Envelope Theorem, this implies

$$\frac{d \ln w(\omega)}{d\omega} = \frac{\partial \ln A(\omega, M(\omega))}{\partial \omega},$$

exactly as in the baseline R-R model. Combining the goods and factor market clearing conditions, which are unchanged, one can then use the same strategy as in Section 3.4 to show that

$$\frac{dM(\omega)}{d\omega} = \frac{A(\omega, M(\omega))L^w(\omega)}{D^w(p, E^w|M(\omega))},$$

where $L^w(\omega)$ denotes world endowment of factor ω ; E^w denotes world expenditure, which includes both spending by consumers and firms; and $D^w(p, E^w|\sigma)$ denotes world absorption for σ varieties. In short, the two key differential equations characterizing factor prices and the matching function remain unchanged under monopolistic competition.

Of course, one should not infer from the previous observation that monopolistically competitive models do not have new implications. In the present environment, world absorption, $D^w(p, E^w|\sigma)$, depends both on the level of variable trade costs, τ , as well as the fixed costs, f_e, f , and f_x , which determine the entry and exit decisions of firms across markets. This opens up new and interesting channels through which trade integration—modeled as a change in τ, f_x , or n —may affect the distribution of earnings.

Let $\underline{\sigma}$ denote the productivity cut-off above which firms choose to produce and $\underline{\sigma}_x$ denote the productivity cut-off above which they choose to export. Under the assumption that preferences are CES, one can then express world demand for σ varieties as

$$D^w(p, E^w|\sigma) = \frac{B^w(\sigma, \gamma_D^w)p^{-\varepsilon}(\sigma)g(\sigma)E^w}{\int_{\Sigma} B^w(\sigma', \gamma_D^w)p^{1-\varepsilon}(\sigma')g(\sigma')d\sigma'}$$

where world demand characteristics, γ_D^w , and the demand shifter for σ varieties, $B^w(\sigma, \gamma_D^w)$, are such that

$$\begin{aligned} \gamma_D^w &\equiv (\tau, n, \underline{\sigma}, \underline{\sigma}_x), \\ B^w(\sigma, \gamma_D^w) &\equiv \left(1 + n\tau^{1-\varepsilon}\mathbb{I}_{[\underline{\sigma}_x, \infty)}(\sigma)\right)\mathbb{I}_{[\underline{\sigma}, \infty)}(\sigma). \end{aligned}$$

In the previous expression, $\mathbb{I}_{[\underline{\sigma}, \infty)}(\cdot)$ and $\mathbb{I}_{[\underline{\sigma}_x, \infty)}(\cdot)$ are indicator functions that capture the selection of different firms into domestic production and export, respectively. Although the cut-offs $\underline{\sigma}$ and $\underline{\sigma}_x$ are themselves endogenous objects that depend on fixed and variable trade costs through standard zero-profit conditions, it will be convenient to study how

trade integration shapes inequality in two steps: (i) treat the demand shifters, B^w , as functions of σ and $\gamma_D^w \equiv (\tau, n, \underline{\sigma}, \underline{\sigma}_x)$; and (ii) analyze how $\underline{\sigma}$ and $\underline{\sigma}_x$ vary with τ , f , and f_x .

To apply the results of Section 3.4.2 in this environment, one only needs to check that B^w is log-submodular in (σ, τ) and log-supermodular in (σ, n) , $(\sigma, \underline{\sigma})$, and $(\sigma, \underline{\sigma}_x)$, which is a matter of simple algebra. From our previous analysis, we then obtain that *ceteris paribus*, a decrease in trade costs, τ , or an increase in the number of countries, n , and the productivity cut-offs, $\underline{\sigma}$ and $\underline{\sigma}_x$, should lead to an upward shift in the matching function and a pervasive increase in the relative price of high- ω factors around the world.

This is an important difference between the models in Sections 3 and 4.1. Whereas trade integration in Section 3.4 leads to lead to opposite effects at home and abroad—because endowments and demand in the integrated economy lie in-between the endowments and demand in the two countries—selection effects à la Melitz (2003) imply that trade integration—modeled as a reduction in trade costs or an increase in the number of countries—have the same effects on the distribution of earnings around the world. Intuitively, if trade integration increases the relative demand for high- σ firms everywhere, it must also increase the relative price of the high- ω factors that are employed in these firms.

As alluded to above, the total effect of a change in variable trade costs or the number of countries is more subtle. In addition to their direct effects, they also indirectly affect entry and exit decisions, which is reflected in changes in $\underline{\sigma}$ and $\underline{\sigma}_x$. This last effect tends to work in the opposite direction: when variable trade costs fall, this lowers the export cut-off, $\underline{\sigma}_x$, which then increases the relative demand of firms below that cut-off. Sampson (2014) analyzes these countervailing forces and provides further extensions, including endogenous technology adoption as in Yeaple (2005).

4.2 Vertical Specialization

Up to this point, we have focused on the implications of R-R models for trade in goods. Although these goods may be intermediate goods or tasks, the previous analysis abstracts from global supply chains in which countries specialize in different stages of a good's production sequence, a phenomenon which Hummels, Ishii & Yi (2001) refer to as vertical specialization. Building on earlier work by Dixit & Grossman (1982) and Sanyal (1983), Costinot, Vogel & Wang (2012) and Costinot, Vogel & Wang (2013) develop variants of R-R models with sequential production to study how vertical specialization shapes inequality and the interdependence of nations. We now briefly describe their framework

and summarize their results.¹²

There is a unique final good whose production requires a continuum of stages $\sigma \in [\underline{\sigma}, \bar{\sigma}]$. At the end of each stage σ , firms can use factors of production and the input from that stage in order to perform the next stage, $\sigma + \delta\sigma$. If firms from country γ combine $Q(\omega, \sigma, \gamma)$ units of intermediate good σ with $L(\omega, \sigma, \gamma)$ units of factor ω for all $\omega \in \Omega$, then total output of intermediate good $\sigma + \delta\sigma$ in country γ is equal to

$$Q(\sigma + \delta\sigma, \gamma) = \int_{\Omega} A(\omega, \gamma_A) \min\{Q(\omega, \sigma, \gamma), L(\omega, \sigma, \gamma)/\delta\sigma\} d\omega, \quad (23)$$

where total factor productivity, $A(\omega, \gamma_A)$, is such that

$$A(\omega, \gamma_A) \equiv \exp(-\lambda(\omega, \gamma_A)/\delta\sigma). \quad (24)$$

$\lambda(\omega, \gamma_A)$ can be interpreted as the constant Poisson rate at which “mistakes” occur along a given supply chain, as in [Sobel \(1992\)](#) and [Kremer \(1993\)](#). At any given stage, the likelihood of such mistakes may depend on the quality of workers and machines, indexed by ω , as well as the quality of infrastructure and institutions in a country, indexed by γ_A , but is assumed to be constant across stages.

When $\lambda(\omega, \gamma_A)$ is strictly decreasing in ω and γ_A , so that high- ω factors and high- γ_A countries have an absolute advantage in all stages, there must be vertical specialization in any free trade equilibrium with more productive factors or countries specializing in later stages of production. Mathematically, PAM arises for the same reason as in earlier sections. By equations (23) and (24), the cumulative amount of factor ω necessary to produce all stages from $\underline{\sigma}$ to σ in country γ is equal to $\exp((\sigma - \underline{\sigma})\lambda(\omega, \gamma))$, which is log-submodular in both (ω, σ) and (σ, γ) . Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage.¹³

Once PAM has been established, competitive equilibria can still be described as a system of differential equations that jointly characterize the schedule of factor prices and the matching function. In a Ricardian version of this model—with only one factor of production per country—[Costinot, Vogel & Wang \(2013\)](#) use this system to contrast the effects of technological change in countries located at the bottom and the top of a supply chain. In a two-country version of this model with a continuum of factors, [Costinot, Vogel &](#)

¹²[Yi \(2003\)](#), [Yi \(2010\)](#), and [Johnson & Moxnes \(2013\)](#) offer examples of quantitative work using Ricardian models with sequential production. The implications of contractual imperfections in such environments are explored in [Antras & Chor \(2013\)](#).

¹³[Costinot, Vogel & Wang \(2013\)](#) also study cases in which the rate of mistakes, and hence factor productivity, vary across stages. If the stage-varying Poisson rate, $\lambda(\omega, \sigma, \gamma_A)$, is submodular in (ω, σ) and (σ, γ_A) , then factor productivity is log-supermodular in these variables and PAM still applies.

Wang (2012) use a similar approach to analyze the consequences of trade integration between countries with different factor endowments. While the effects of trade integration on the matching function are the same as in Section 3.4, sequential production leads to new and richer predictions about the effects of trade on inequality. Namely, standard Stolper-Samuelson forces operate at the bottom of the chain, but the opposite is true at the top.

4.3 Other Extensions

In the baseline R-R model as well as the previous extensions, factors of production are characterized by their exogenous productivity in various economic activities. In practice, productivity may be neither exogenous nor the only source of heterogeneity among factors. The marginal product of labor may vary with the stock of capital; workers may have different preferences over working conditions; and workers may vary in terms of how costly it is for them to acquire skills. Fortunately, such considerations can all be incorporated into an R-R model.

As shown in our online Appendix, the tools and techniques of Section 3 can be used to derive similar cross-sectional and comparative static predictions in economies with:

- i. **Factor complementarity**, if output of good σ in country γ is given by

$$Q(\sigma, \gamma) = F[K^{agg}(\sigma, \gamma), L^{agg}(\sigma, \gamma) | \sigma, \gamma],$$

where $F(\cdot, \cdot | \sigma, \gamma)$ is a constant returns to scale production function; $K^{agg}(\sigma, \gamma)$ and $L^{agg}(\sigma, \gamma)$ denote the aggregate amounts of capital and labor, respectively, with

$$L^{agg}(\sigma, \gamma) = \int_{\Omega} A(\omega, \sigma, \gamma_A) L(\omega, \sigma, \gamma) d\omega.$$

- ii. **Heterogeneous preferences**, if the utility of a worker with characteristic ω receiving a wage $w^c(\sigma, \gamma)$ in sector σ and country γ is given by

$$V(\omega, \sigma, \gamma) \equiv w^c(\sigma, \gamma) U(\omega, \sigma, \gamma_U),$$

where γ_U is a new exogenous preference shifter and $U(\omega, \sigma, \gamma_U)$ is strictly log-supermodular in (ω, σ) and (σ, γ_U) .¹⁴

¹⁴When thinking about heterogeneity in preferences, a natural interpretation of σ is location rather than industry. In practice, different individuals may choose to live in different cities because they value their various amenities differently. With this interpretation in mind, R-R models also provide a useful framework

- iii. **Endogenous skills**, if firms from country γ need to pay $S(\omega, \sigma, \gamma_S) > 0$ in order to train a worker of type ω in sector σ —with learning costs proportional to the consumer price index—and $S(\omega, \sigma, \gamma_S)$ is strictly submodular in (ω, σ) and (σ, γ_S) .¹⁵

5 Parametric Applications

The two previous sections have derived a number of sharp cross-sectional and comparative static predictions, especially in economies with a continuum of goods and factors. In the data, however, one always observes a discrete number of factors and sectors. Furthermore, PAM never perfectly holds for these observed groups of factors and sectors; all factors are likely to be employed in all sectors, albeit with different intensity.

One way to bridge the gap between theory and data is to maintain the assumption that there is a continuum of factors or goods that are perfectly observed by consumers and firms, but add the assumption that the econometrician only observes coarser measures of these characteristics. Under these assumptions, there may therefore be unobserved heterogeneity, from the point of view of the econometrician, within a given group of factors or goods.

A number of papers in the trade literature have followed the previous approach. The empirical content of such papers then crucially depends on the distributional assumptions imposed on unobserved heterogeneity across goods or factors. By far the most common assumption in the existing literature is to assume Generalized Extreme Value (GEV) distributions of productivity, as in the influential work of [Eaton & Kortum \(2002\)](#).¹⁶ Sections 5.1 and 5.2 discuss the implications of GEV distributions of productivity shocks across goods and factors, respectively.

5.1 Unobserved Productivity Shocks across Goods

Consider first a R-R model with a discrete number of factors and countries and a continuum of goods. For notational convenience, true characteristics, which are perfectly ob-

to analyze the relationship between migration and trade, both within and between countries.

¹⁵This is the approach followed by [Blanchard & Willmann \(2013\)](#).

¹⁶This distributional assumption is standard in the analysis of discrete choice models in industrial organization, see e.g. [McFadden \(1974\)](#) and [Berry \(1994\)](#), as well as in the matching literature, see e.g. [Choo & Siow \(2006\)](#). Following the seminal work of [Roy \(1951\)](#), numerous papers in the labor literature have focused instead on environments in which the distribution of worker skills is log-normally distributed, see e.g. [Heckman & Sedlacek \(1985\)](#). In the international trade literature, [Ohnsorge & Trefler \(2007\)](#) also impose log-normality. [Liu & Trefler \(2011\)](#) propose an alternative empirical approach based on linearized versions of the Roy model's estimating equations.

served by all market participants, are now indexed by $(\omega^*, \sigma^*, \gamma^*)$, whereas characteristics observed by the econometrician are indexed by (ω, σ, γ) . We maintain this convention throughout this section. Here, factor and country characteristics are perfectly observed by the econometrician, $\omega^* = \omega$ and $\gamma^* = \gamma$, but good characteristics are not, $\sigma^* \neq \sigma$. Specifically, for each observed value of σ , we assume that there exists a continuum of goods $\sigma^* \in [0, 1]$ with the same observable characteristic and refer to this measure one of goods as an industry. In practice, the pair of observables (ω, γ) may refer to “worker with a college degree from the United States,” whereas the observable σ may measure “the share of the total wage bill associated with workers with a college degree” in a given industry. In this case, unobserved heterogeneity across goods may reflect the fact that goods with different unobservable characteristic, σ^* , but identical observable characteristic, σ , may employ different types of workers ω in a competitive equilibrium.

Factor productivity, $A(\omega^*, \sigma^*, \gamma_A^*)$, is independently drawn across all factors, goods, and countries from a Fréchet distribution. Given observables (ω, σ, γ) , we assume that

$$\Pr\{A(\omega, \sigma^*, \gamma_A) \leq a|\sigma\} = \exp\left(-[a/T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)}\right), \quad (25)$$

where $T(\omega, \sigma, \gamma_A) \geq 0$ and $\theta(\sigma) > \varepsilon(\sigma) - 1$, with $\varepsilon(\sigma)$ the elasticity of demand to be introduced below. The first parameter, $T(\omega, \sigma, \gamma_A)$, is a locational shifter that can be thought of as the fundamental productivity of a given factor and country in an industry. The second parameter, $\theta(\sigma) > 1$, is a shape parameter that captures the extent of intra-industry heterogeneity and is assumed to be constant across factors and countries. The Ricardian model developed by [Eaton & Kortum \(2002\)](#) corresponds to the special case in which there is only one factor, i.e. a unique ω , and one industry, i.e. a unique σ . Multi-industry extensions considered in [Levchenko & Zhang \(2011\)](#), [Costinot, Donaldson & Komunjer \(2012\)](#), and [Caliendo & Parro \(2014\)](#), among others, correspond to cases in which σ can take multiple values. Here we further allow for multiple factors within each country.

In line with the existing literature, we assume a two-tier utility function in which the upper-level is Cobb-Douglas—across industries with observables σ —and the lower-level is CES—across goods σ^* within the same industry. Specifically, the demand function for a good σ^* with observable characteristic σ is given by

$$D(p, I(\gamma)|\sigma^*, \gamma_D) = \frac{B(\sigma, \gamma_D)p^{-\varepsilon(\sigma)}(\sigma^*)I(\gamma)}{P^{1-\varepsilon(\sigma)}(\sigma)}, \quad (26)$$

where $P(\sigma) \equiv (\int_0^1 p^{1-\varepsilon(\sigma)}(\sigma^*)d\sigma^*)^{1/(1-\varepsilon(\sigma))}$ denotes the industry-level price index and $B(\sigma, \gamma_D) \in [0, 1]$ now refers to the exogenous share of expenditure on all goods with

observable characteristic σ . The rest of the assumptions are the same as in Section 3.

Since none of the equilibrium conditions derived in Section 3.2 depend on either the number of goods, factors, and countries or the distribution of productivity across goods, factors, and countries, a competitive equilibrium remains characterized by the same system of equations. Given the specific distributional assumptions imposed by equation (25), however, one can further show that for an industry with observable characteristic σ , the probability that factor ω in country γ offers the minimum cost of producing a good σ^* is given by

$$\pi(\omega, \gamma | \sigma) = \frac{[w(\omega, \gamma) / T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)}}{\sum_{\gamma'} \sum_{\omega'} [w(\omega', \gamma') / T(\omega', \sigma, \gamma'_A)]^{-\theta(\sigma)}}. \quad (27)$$

As is well-known, equation (25) also implies that conditional on offering the minimum cost of producing a good, the distribution of unit costs across goods σ^* is the same for all factors and countries. Thus $\pi(\omega, \gamma | \sigma)$ is also equal to the share of expenditure on goods with observable characteristics σ that are produced using factor ω in country γ .¹⁷

Using the previous observation and equation (26), which implies that a share $B(\sigma, \gamma_D)$ of total expenditure in country γ is allocated to industry σ , one can rearrange the factor market clearing condition (8) as

$$w(\omega, \gamma) L(\omega, \gamma_L) = \sum_{\gamma'} \sum_{\sigma} \pi(\omega, \gamma | \sigma) B(\sigma, \gamma'_D) I(\gamma'), \text{ for all } \omega, \gamma. \quad (28)$$

Equations (27) and (28) uniquely pin down the schedule of factor prices $w(\omega, \gamma)$ up to a choice of numeraire. To go from good prices to factor prices, one can then use the fact that the lower-level utility is CES. Under this assumption, equation (25) implies that the CES price index associated with goods with observable characteristic σ is given by

$$P(\sigma) = \chi(\sigma) \times \left(\sum_{\gamma} \sum_{\omega} [w(\omega, \gamma) / T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)} \right)^{-1/\theta(\sigma)}, \quad (29)$$

with $\chi(\sigma) \equiv \left(\Gamma \left(\frac{\theta(\sigma)+1-\varepsilon(\sigma)}{\theta(\sigma)} \right) \right)^{1/(1-\varepsilon(\sigma))}$.¹⁸ Knowledge of factor prices, $w(\omega, \gamma)$, and price indices, $P(\sigma)$, is then sufficient to conduct welfare analysis.

Compared to Section 3.3, equation (27) implies a weaker form of PAM in the cross-

¹⁷In line with the analysis of Section 3, we assume that all goods are freely traded. Therefore shares of expenditures are constant across all importing countries, as can be seen from equation (27). As shown in Eaton & Kortum (2002) and as already mentioned in Section 3.1, it is easy to introduce iceberg trade costs in such an environment. It is equally easy to introduce them in the R-R models considered in Section 5.2.

¹⁸ $\Gamma(\cdot)$ denotes the gamma function, i.e., $\Gamma(t) = \int_0^{\infty} u^{t-1} \exp(-u) du$.

section. Consider the following “stochastic” version of Assumption 1.

Assumption 1 [Fréchet]. $T(\omega, \sigma, \gamma_A)$ is strictly log-supermodular in (ω, σ) and (σ, γ_A) .

This new version of Assumption 1 also captures the idea that high- ω factors and high- γ_A countries are relatively more productive in high- σ industries, but it does not require this to be true for all goods σ^* within an industry. Starting from equation (27), one can check that if this version of Assumption 1 holds and $\theta(\sigma)$ does not vary across industries, $\theta(\sigma) \equiv \theta$, then $\pi(\omega, \gamma|\sigma)$ is also strictly log-supermodular in (ω, σ) and (σ, γ_A) . This implies that high- ω factors and high- γ_A countries should tend to sell relatively more in high- σ industries. Because of equation (25) and the assumption of a continuum of goods with the same observable characteristic σ , all factors in all countries will now be used in all industries. But for a given factor, ω , if one compares the distribution of sales across industries of two countries such that $\gamma'_A \geq \gamma_A$, then $\pi(\omega, \gamma'_A|\cdot)$ must dominate $\pi(\omega, \gamma_A|\cdot)$ in terms of the Monotone Likelihood Ratio Property. This is the idea behind the revealed measure of comparative advantage developed in Costinot, Donaldson & Komunjer (2012).

Equation (27) also creates a tight connection between good prices and factor prices. Combining equations (27) and (29) yields

$$w(\omega, \gamma) = P(\sigma) T(\omega, \sigma, \gamma_A) \left[\gamma(\sigma)^{\theta(\sigma)} \pi(\omega, \gamma|\sigma) \right]^{-1/\theta(\sigma)}, \text{ for all } \sigma. \quad (30)$$

This is akin to equation (12) in Section 3.3 with two important differences. First, $P(\sigma)$ corresponds to the CES price index of all goods with characteristics σ . Second, the term $\left[\gamma(\sigma)^{\theta(\sigma)} \pi(\omega, \gamma|\sigma) \right]^{-1/\theta(\sigma)}$ adjusts for the effect of self-selection of factors from different countries across industries, which creates a wedge between fundamental productivity, $T(\omega, \sigma, \gamma_A)$, and average productivity.

Last but not least, starting from equations (27) and (28), one can use the “exact hat algebra,” as in Dekle, Eaton & Kortum (2008) and other quantitative trade models discussed in Costinot & Rodríguez-Clare (2013), to conduct comparative static and welfare analysis. Although few analytical results are available in this environment, counterfactual simulations can be performed using only estimates of $\theta(\sigma)$ that can be obtained from the value of trade elasticities in a gravity equation. In the next subsection, we discuss how richer analytical results can be obtained when one goes from unobserved heterogeneity across goods to unobserved heterogeneity across factors.

5.2 Unobserved Productivity Shocks across Factors

Consider now the polar case of an R-R model with a discrete number of sectors and countries and a continuum of factors. Compared to Section 5.1, sector and country characteristics are perfectly observed by the econometrician, $\sigma^* = \sigma$ and $\gamma^* = \gamma$, but factor characteristics are not, $\omega^* \neq \omega$. In line with the analysis of the previous subsection, for each value of ω , we assume that there exists a continuum of factors $\omega^* \in [0, 1]$ with the same observable characteristic. In this environment, factors with different unobservable characteristic, ω^* , but identical observable characteristic, ω , may therefore be allocated to sectors with different characteristics σ in a competitive equilibrium.

The distributional assumption imposed on factor productivity, $A(\omega^*, \sigma^*, \gamma_A^*)$, is similar to the one imposed in Section 5.1. Factor productivity remains independently drawn across all factors, goods and countries from a Fréchet distribution, but given observables (ω, σ, γ) , we now assume that

$$\Pr \{A(\omega^*, \sigma, \gamma_A) \leq a | \omega\} = \exp \left(- [a/T(\omega, \sigma, \gamma_A)]^{-\theta(\omega, \gamma_A)} \right), \quad (31)$$

with $T(\omega, \sigma, \gamma_A) \geq 0$ and $\theta(\omega, \gamma_A) > 1$. Besides the fact that unobserved heterogeneity now derives from ω^* rather than σ^* , the only difference between equations (25) and (31) is that the shape parameter θ is now specific to a group of factors within a country. Fréchet distributions of productivity shocks across factors have been imposed in the recent closed-economy models of [Lagakos & Waugh \(2013\)](#) and [Hsieh et al. \(2013\)](#) as well as the open economy models of [Burstein, Morales & Vogel \(2014\)](#), [Costinot, Donaldson & Smith \(2014\)](#), and [Fajgelbaum & Redding \(2014\)](#).

In line with earlier sections, we restrict ourselves to CES demand functions. Here again, a competitive equilibrium is characterized by the same system of equations as in Section 3.2. Similarly, for a factor with observable characteristic ω in country γ , the probability that the value of the marginal product is maximized in sector σ is given by

$$\pi(\sigma | \omega, \gamma) = \frac{[p(\sigma) T(\omega, \sigma, \gamma_A)]^{\theta(\omega, \gamma_A)}}{\sum_{\sigma'} [p(\sigma') T(\omega, \sigma', \gamma_A)]^{\theta(\omega, \gamma_A)}}. \quad (32)$$

Because of equation (31), the distribution of prices for factors with observable characteristic ω in country γ is now the same across all sectors σ . Hence, $\pi(\sigma | \omega, \gamma)$ is also the share of income of factors from country γ with observable characteristic ω that is earned in sector σ . Together with the assumption of CES demand functions, this implies that the

goods market clearing condition (9) can be rearranged as

$$\sum_{\gamma} B(\sigma, \gamma_D) (p(\sigma))^{1-\varepsilon} I(\gamma) = \sum_{\gamma'} \sum_{\omega} \bar{w}(\omega, \gamma') \pi(\sigma|\omega, \gamma') L(\omega, \gamma'_L), \text{ for all } \sigma, \quad (33)$$

where $\bar{w}(\omega, \gamma)$ denotes the average return across all factors ω^* from country γ with observable characteristic ω , which is constant across all sectors, and the CES price index has been normalized to one. Finally, equation (31) implies that $\bar{w}(\omega, \gamma)$ satisfies

$$\bar{w}(\omega, \gamma) = \chi(\omega, \gamma_A) \times \left(\sum_{\sigma} [p(\sigma) T(\omega, \sigma, \gamma_A)]^{\theta(\omega, \gamma_A)} \right)^{1/\theta(\omega, \gamma_A)}, \quad (34)$$

where $\chi(\omega, \gamma_A) \equiv \Gamma\left(\frac{\theta(\omega, \gamma_A)-1}{\theta(\omega, \gamma_A)}\right)$. Equations (32)-(34) fully pin down good prices, $p(\sigma)$, and average factor prices, $\bar{w}(\omega, \gamma)$, up to a normalization.

Before studying the differences between the models of Sections 5.1 and 5.2, it is useful to understand their similarities. Like equation (27) in Section 5.1, equation (32) provides a simple relationship between fundamental productivities, T s, shape parameters, θ s, and factor allocations, π s.¹⁹ Under the Fréchet version of Assumption 1 and the restriction that $\theta(\omega, \gamma_A) \equiv \theta$, equation (32) also generates a weaker form of PAM: $\pi(\sigma|\omega, \gamma)$ is strictly log-supermodular in (ω, σ) and (σ, γ_A) . In terms of price predictions, equations (32) and (34) imply that

$$\bar{w}(\omega, \gamma) = p(\sigma) T(\omega, \sigma, \gamma_A) \left[\gamma(\omega, \gamma_A)^{\theta(\omega, \gamma_A)} \pi(\sigma|\omega, \gamma) \right]^{-1/\theta(\omega, \gamma_A)}, \text{ for all } \sigma, \quad (35)$$

which again highlights the importance of self-selection. Furthermore, given the “CES-like” functional forms in equations (32)-(34), one can still use the exact algebra of [Dekle, Eaton & Kortum \(2008\)](#) to conduct comparative static and welfare analysis in this environment, an approach implemented by [Burstein, Morales & Vogel \(2014\)](#).²⁰

Given these similarities, it would be tempting to view this subsection as a simple re-labeling of factors and goods in Section 5.1. It is not. The equilibrium system of equations here—equations (32)-(34)—cannot be obtained by permutations of σ , ω , and γ in equations (27)-(29). To take one example, $\bar{w}(\omega, \gamma)$ here is an arithmetic average, whereas $P(\sigma)$ in Section 5.1 is not. Such distinctions are consequential. Whereas we have very few an-

¹⁹[Hsieh et al. \(2013\)](#) and [Burstein, Morales & Vogel \(2014\)](#) use this relationship, estimates of θ s, and data on factor allocation to recover the productivity of different worker groups across occupations as well as the implicit taxes or wedges that they may face.

²⁰[Galle, Rodríguez-Clare & Yi \(2014\)](#) follow a similar strategy in a quantitative trade model that combines both unobserved good and factor heterogeneity drawn from Fréchet distributions.

alytical results about the effects of trade integration or other shocks in the environment considered in Section 5.1, our online Appendix establishes that the FPE Theorem, the Rybczynski Theorem, as well as the the Stolper-Samuelson Theorem all generalize to the present environment.

Similar results can be derived in an environment with unobserved preference shocks, as in Artuc, Chaudhuri & McLaren (2010), Dix-Carneiro (2014), and Redding (2014). Such extensions are important since they provide a natural framework to pursue quantitative work on the consequences of international trade in economies with imperfect factor mobility across sectors and locations. We also discuss them in details in our online Appendix.

6 Non-Parametric Applications

Ideally, when applying R-R models to the analysis of counterfactual questions, one would like to have information about the productivity of all factors in all sectors and countries. Unfortunately, such detailed information is rarely available. Typically, researchers may be able to observe “factual” productivity, that is the productivity of factors in the sectors in which they are currently employed, but not their “counterfactual” productivity, that is their productivity in the sectors to which they would be reallocated in response to a counterfactual shock.

In Section 5, we have discussed parametric applications of R-R models that deal with the previous issue by assuming that unobserved factor productivity is drawn from Fréchet distributions. We now turn to alternative applications of R-R models that aim to dispense as much as possible with functional-form assumptions. The basic idea behind these applications is to focus on agriculture, a sector of the economy in which the scientific knowledge of how essential inputs such as water, soil and climatic conditions map into outputs is uniquely well understood. As a result, agronomists are able to predict with great accuracy how productive various parcels of land would be were they to be used to grow any one of a set of crops. In short, both factual and counterfactual productivities are known in this particular context.

All applications in this section are based on detailed micro-level data from the Food and Agriculture Organization’s (FAO) Global Agro-Ecological Zones (GAEZ) project. This dataset records potential crop yields for millions of 5 arc-minute grid-cells covering the surface of the earth; see Nunn & Qian (2011) and Costinot & Donaldson (2014) for additional information. In Section 6.1, we first illustrate how the GAEZ data can be used to test the cross-sectional implications of David Ricardo’s theory of comparative advantage. In Sections 6.2 and 6.3, we then discuss how the same agronomic data can be used to

measure the historical gains from economic integration as well as to quantify the consequences of climate change in agricultural markets, respectively.

6.1 Testing Ricardo’s Theory of Comparative Advantage

David Ricardo’s theory of comparative advantage, upon which R-R models build, is one of the oldest and most distinguished theories in economics. However, it is a difficult theory to confront to the data. As already noted by [Deardorff \(1984\)](#): “Problems arise, however, most having to do with the observability of [productivity by industry and country]. The...problem is implicit in the Ricardian model itself...[because] the model implies complete specialization in equilibrium... This in turn means that the differences in labor requirements cannot be observed, since imported goods will almost never be produced in the importing country.” Accordingly, empirical work on the cross-sectional implications of the Ricardian model has proceeded by imposing ad-hoc specifications, as in [MacDougall \(1951\)](#), [Stern \(1962\)](#), [Balassa \(1963\)](#), and [Golub & Hsieh \(2000\)](#), or strong distributional assumptions on the extent of unobserved labor productivity, as in [Costinot, Donaldson & Komunjer \(2012\)](#).

[Costinot & Donaldson \(2012\)](#) propose an alternative empirical strategy that relies instead on productivity and price data from the FAO for 17 major agricultural crops and 55 major agricultural countries. As discussed above, the GAEZ project reports “total production capacity” in tons per hectare for each 5 arc-minute grid cell within these 55 countries. [Costinot & Donaldson \(2012\)](#) interpret each grid-cell as a distinct factor ω and “total production capacity” and the area of each grid-cell as the empirical counterparts of $A(\omega, \sigma, \gamma_A)$ and $L(\omega, \gamma_L)$, respectively. Under the assumption that $\Sigma(\omega, \gamma)$ is a singleton,²¹ the aggregate output of crop σ in country γ is given

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma, \gamma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega, \quad (36)$$

where $\Omega(\sigma, \gamma)$ is the set of factors allocated to crop σ in country γ ,

$$\Omega(\sigma, \gamma) = \{ \omega \in \Omega \mid A(\omega, \sigma, \gamma_A) / A(\omega, \sigma', \gamma_A) > p(\sigma', \gamma) / p(\sigma, \gamma) \text{ for all } \sigma' \neq \sigma \}. \quad (37)$$

In the previous expression, $p(\sigma, \gamma)$ denotes the local price of crop σ in country γ , which is allowed to vary across countries because of transportation costs or taxes and subsidies.

Combining equations (36) and (37) with productivity and price data, [Costinot & Donaldson \(2012\)](#) then compute predicted output levels for all crops and countries and com-

²¹This assumption is satisfied for all but two grid cells in Brazil.

pare them to those observed in the data. Their empirical results show that the output levels predicted by Ricardo’s theory of comparative advantage agree reasonably well with actual data on worldwide agricultural production.

6.2 Measuring the Gains from Economic Integration

In Section 6.1, we have shown how to use data on agricultural output, productivity, and prices to “test” the cross-sectional predictions of R-R models. Alternatively, in the absence of price data, one could use the cross-sectional predictions of R-R models, together with agricultural output and productivity data, in order to “back out” model-consistent prices. This is the approach followed by [Costinot & Donaldson \(2014\)](#) in the context of U.S. agricultural markets from 1880 to 1997.

Combining agronomic data from the GAEZ project with historical data from the U.S. Census, the authors first demonstrate how to identify the spatial distribution of crop prices across U.S. counties over time, while still allowing for crop-county-year specific productivity shocks. By comparing local crop prices, estimated from the model, and prices in major wholesale markets, observed in historical data, the authors then estimate trade costs for each U.S. county from 1880 to 1997. Finally, armed with the previous estimates, they compute the aggregate productivity gains due to changes in trade costs over that time period, which they refer to as the gains from economic integration.

In terms of the empirical validity of R-R models, [Costinot & Donaldson \(2014\)](#) find a robust, positive correlation between county-level prices inferred from the model and historical state-level prices, which are the most disaggregated price data available over the long time period. Since the estimation of local prices in [Costinot & Donaldson \(2014\)](#) does not use any direct information on prices—it relies instead on information about historical output levels and modern productivity—these additional empirical results also suggest that R-R models have significant explanatory power in the data.

6.3 Quantifying the Consequences of Climate Change

Another attractive feature of the GAEZ dataset is that potential crop yields are available under various climate change scenarios. Hence, using both pre- and post-climate change crop yields as inputs into an R-R model, one can explore the macro-level consequences of climate change in agricultural markets. This is the starting point of the quantitative analysis in [Costinot, Donaldson & Smith \(2014\)](#).

The authors develop an R-R model that combines detailed micro-level data with otherwise parsimonious parametric assumptions. Compared to the applications reviewed

in Sections 6.1 and 6.2, it features observed productivity heterogeneity across grid-cells, as reported in the GAEZ dataset, but also unobserved productivity heterogeneity within grid-cells, which is assumed to be drawn from a Fréchet distribution across a continuum of parcels. Closed-form solutions for factor allocation and aggregate output levels are therefore akin to those presented in Section 5.2, which dramatically simplifies the computation of competitive equilibria in an environment with 10 crops, 50 countries, and 1.7 million grid-cells.

Using this trade model, Costinot, Donaldson & Smith (2014) quantify the impact of climate change on agricultural markets. This new type of Computational General Equilibrium (CGE) models—based on rich micro-level data and a tight connection between theory and empirics—could be applied to many other outstanding questions in the field. Sotelo (2013) provides an interesting example. By combining the GAEZ data with a theoretical framework similar to the one in Costinot, Donaldson & Smith (2014), he quantifies the contribution of trade frictions to the low agricultural productivity of Peru.

7 Concluding Remarks

In this article, we have provided a user guide to multi-factor generalizations of the Ricardian model of international trade, which we have referred to as Ricardo-Roy (R-R) models. We have presented tools and techniques for theoretical, quantitative, and empirical work in this class of assignment models. We have also reviewed a number of applications of these models. We conclude by discussing a number of open questions and promising avenues for future research.

On the theoretical front, we have described how the baseline R-R model could be extended along a number of dimensions, including the introduction of monopolistic competition and sequential production. R-R models could also accommodate many of the other features that have previously been studied in a Ricardian environment, such as external economies of scale as in Ethier (1982), Grossman & Rossi-Hansberg (2010), and Matsuyama (2013), or learning by doing, as in Krugman (1987) and Young (1991). R-R models could also be used to study the relationship between trade and labor market frictions, perhaps building on the closed economy models of Teulings (2003) and Teulings & Gautier (2004), or endogenous technological change, as in the work of Acemoglu (2002). In an R-R model, technological change may be biased towards particular tasks rather than factors, as in Atkinson & Stiglitz (1969), with factors endogenously reallocating between these tasks. It would also be interesting to explore the implications of R-R models for trade policy, both from a positive and a normative perspective. Such models may provide

a useful alternative to the traditional specific-factor model. The methodology developed by [Costinot et al. \(2014\)](#) to characterize optimal trade taxes in a Ricardian model should prove helpful in this more general environment.

On the empirical front, R-R models are particularly well-suited to study the distributional consequences of international trade. Building on the important work of [Artuc, Chaudhuri & McLaren \(2010\)](#), one could imagine using variations of the parametric models and methods presented in Section 5 to quantify the importance of previous trade liberalization episodes across sectors and regions. These simple static models may offer a useful compromise between reduced-form work, as in [Goldberg & Pavcnik \(2007\)](#), and richer dynamic structural models, as in [Dix-Carneiro \(2014\)](#). As shown in [Sampson \(2014\)](#), R-R models can also accommodate firm-level heterogeneity in imperfectly competitive markets. One could also imagine using parametric versions of such R-R models to help analyze the increasing number of matched employer-employee datasets that have become available.

Of course, future empirical and quantitative work does not have to be limited to parametric applications. It is well-known that GEV distributions impose strong restrictions on own-price and cross-price elasticities. [Galichon & Salanié \(2012\)](#) offer an example of how to relax these distributional assumptions in one-to-one matching models. It may be interesting to follow a similar approach when studying the effects of international trade in local labor markets. Finally, agricultural trade policy is one of the most salient issues in today's world economy. R-R models and the GAEZ data described in Section 6 offer a unique opportunity to shed light on this key policy issue. What are the effects on the Common Agricultural Policy on the misallocation of resources in European agricultural markets? How do agricultural trade policies set in developed economies affect households in developing countries? These are central questions about which we know fairly little, but that could be addressed using R-R models.

In sum, much remains to be done.

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