Trade shocks and labor-market adjustment*

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Abstract

We develop a framework to analyze the impact of trade shocks on a range of labor-market adjustment margins in economies with a large number of sectors and labor groups. We provide analytic results characterizing equilibria. We show that labor groups earning a greater share of wage income in sectors with relative price declines experience a relative increase in unemployment and non-participation and decrease in wages and welfare. Our framework provides a guide for quantitative and empirical investigations into the labor-market impacts of trade shocks.

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1 Introduction

We develop a framework to analyze the impact of trade shocks on a wide range of labor-market adjustment margins in economies with a large number of sectors and labor groups. We endogenize the extensive margin of labor supply (labor force participation), the sector in which each worker searches for a job conditional on labor force participation, the probability of successful employment conditional on job search in a given sector (frictional unemployment), and hours worked and average wages conditional on successful employment. We provide analytic results characterizing equilibria and showing how changes in sectoral prices affect each margin of adjustment as well as welfare for each labor group.

We consider a static assignment model of trade with many sectors and labor groups (where labor groups are defined by observable characteristics such as region, education, gender, etc.) featuring search frictions. Each agent is characterized by a vector of productivities: one in each sector and one in non-participation. Production requires that a worker and a firm match, matching is subject to frictions, and search is directed. An agent in the labor force chooses the sector in which to apply for a job and each firm targets its vacancy at the labor group and sector of its choice. Workers and firms are atomistic, treating the vector of probabilities of successfully matching—with one probability per sector and labor group pair—as exogenous and these probabilities are determined in equilibrium. A worker who successfully matches chooses how many hours to work and the worker's wage is determined by the outcome of a bargaining process between the firm and worker.

We aim to provide the simplest, most parsimonious and tractable extension possible of canonical models of international trade. Nevertheless, our framework remains rich. Specifically, our model generates a group g-specific extensive margin elasticity of labor supply, another group g-specific intensive margin elasticity of labor supply, and another group g-specific labor supply elasticity across sectors conditional on labor force participation.

Combining worker and firm optimization, we solve for a small-open-economy equilibrium in which the probability of successfully finding a job is common across sectors for a given labor group. In this equilibrium, we solve for the probability of a worker in group g successfully obtaining employment given participation in the labor force (one minus the unemployment rate) denoted by E_g, the average wage per hour worked denoted by W_g, hours worked per employee denoted by H_g, and expected utility denoted by U_g; we also solve the impact of sectoral price changes on each of these outcomes. Under a set of restrictions, we show that changes in each outcome $K \in \{E, W, H, U\}$, can be expressed as

$$d \ln K_g = \left(\rho_g^K / \iota_g\right) d \ln \Phi_g - \rho_g^K d \ln P_g$$

where ι_g and ρ_g^K are functions of structural elasticities and where $d \ln \Phi_g$ is a production-side trade shock (and $d \ln P_g$ is a consumption-side trade shock) that is a weighted average of sectoral price changes, with weights given by the share of income group g earned in (and share of expenditure group g allocated to) each sector s before the price shock.

We prove analytically that if a labor group is disproportionately employed in sectors with relative price declines (in a sense made precise in Proposition 3), then it will experience a relative increase in unemployment and non-participation and a decrease in average wages and welfare. These results extend Stolper-Sameulson-style logic not only to high-dimensional environments (as in Costinot and Vogel (2010) for wages) but also to a much broader range of labor-market adjustments.

Literature and contribution. Our paper's objective of incorporating non-wage margins of adjustment into high-dimensional environments is motivated by a vast empirical literature—e.g. many regions in Topalova (2010), Autor et al. (2013), and Kovak (2013)—that emphasizes the relative importance of non-wage margins of adjustment. For instance, Autor et al. (2013) argue that their "results suggest that the predominant focus of the previous literature on wages misses important aspects of labor-market adjustments to trade. We find that local labor markets that are exposed to... China's rising competitiveness experience increased unemployment, decreased labor-force participation, and increased use of disability and other transfer benefits, as well as lower wages."

In addition, our theoretical results connect closely with the empirical specifications in this literature: the measure of our production-side trade shock, $d \ln \Phi_g$, is equivalent to a shift-share or Bartik-style measure of exposure leveraged in the empirical literature. As we describe in Section 5.2, this connection implies that reduced-form estimates on the responses of multiple margins can be appropriately aggregated to identify relative welfare impacts of trade shocks across groups.

In this respect, our paper is complementary to Kim and Vogel (2020), which estimates the impact of a particular trade shock on relative welfare across U.S. commuting zones, measuring a welfare-change sufficient statistic derived from a substantially more general framework that dispenses with the many functional form restrictions imposed here. Relative to Kim and Vogel (2020), the present paper's structure allows us to solve for the implications of counterfactual shocks.

Our structural approach is closely related to a growing quantitative trade literature using assignment models to study the impact of trade on wage inequality either at the national level—see e.g. Lee (2017) and Burstein et al. (2019)—or across local labor markets—see e.g. Adão (2015) and Galle et al. (2017). Into this environment, Adão et al. (2018) and Caliendo et al. (2019) introduce a labor-leisure choice. We extend these models to incorporate both an intensive and extensive margin of labor supply adjustment as well as frictional unemployment and provide analytic comparative static results to shed light on this literature's quantitative conclusions.

Our analytic results highlight the theoretical relevance of Stolper-Sameulson-style insights across a range of margins of adjustment. These results clarify that empirical findings of relatively small wage effects of trade shocks—which have been interpreted as evidence against the practical relevance of Stolper-Sameulson-style mechanisms—are potentially consistent with large effects transmitted through non-wage margins of adjustment. In this respect, our work answers the call of Davis and Mishra (2007), who write that "We shouldn't ignore [our toy theories], discard them, or least of all mutilate them. But we do need to ask what the deep lesson is to be learned from the simple models and how one should go about using this insight in a more complex setting."

Our theory builds most directly on Costinot and Vogel (2010) and Davidson et al. (1999). We use the tools and techniques developed in Costinot and Vogel (2010), which provides analytic results on factor allocation and wages in high-dimensional environments. These results apply directly in modern quantitative environments; as reviewed in Costinot and Vogel (2015). We extend Stolper-Samuelson insights to additional margins of adjustment as in Davidson et al. (1999), which introduces frictional unemployment into a two-by-two Heckscher-Ohlin-like model.

2 Model

Consider an economy populated by a continuum of agents separated into finitely many disjoint groups indexed by g, each of which corresponds to the intersection of various observable characteristics, such as the region in which an agent lives and her education, gender, nativity, etc. The set of group g agents is denoted Ω_g, which we treat as exogenous, and its mass is denoted $N_g = |\Omega_g|$.

Agents can either be employed in one of S sectors, unemployed, or out of the labor force. We index each possibility by s, with $s \in \{1, ..., S\}$ indicating each of the S sectors in which an agent can be employed, $s = u$ indicating unemployment, and $s = 0$ indicating non-participation in the labor force.

Each agent chooses to apply to the $s \in \{0, 1, ..., S\}$ that maximizes her expected utility. Agents do not apply to be unemployed; however, an agent who applies to $s \in \{1, ..., S\}$

may become employed in s or unemployed. We denote by $E_{gs} \in [0,1]$ the probability of successfully obtaining employment for an agent $\omega \in \Omega_g$ who applies to s. Of course, agents who choose not to participate in the labor force cannot become unemployed, so $E_{g0} = 1$ for all ω. We say that an agent "works" in s even if $s = u$, in which case the agent is unemployed.

The utility of an agent $\omega \in \Omega_g$ who consumes C units of the final good and supplies H hours of labor is

$$U(C, H; g) = \zeta_g C - \frac{H^{1+v_g}}{1+v_g}, \tag{1}$$

where $\zeta_g, v_g > 0$. As will become apparent in Section 3, the parameter v_g shapes both the uncompensated (Marshallian) and the compensated (Hicksian) intensive-margin labor supply elasticities, which are both given by $1/v_g$.[1] The price index for final goods consumption is denoted P_g. We make no explicit assumptions on the consumption aggregator other than homotheticity; in particular, we allow preferences across goods to differ arbitrarily across groups. In our comparative statics feeding in small shocks to prices, $d \ln P_g$ can be measured simply as the weighted average of sectoral price changes, with weights given by initial group-g expenditure shares (in the absence of product entry and exit). That is, $d \ln P_g$ is measured using a Laspeyres price index, which is locally identical to a Paasche price index.

Production and revenue. We assume that all agents—whether working in a sector, not participating in the labor force, or unemployed—produce output in the respective $s \in \{u, 0, 1, ..., S\}$. Both the unemployed and those not participating in the labor force operate a home production technology.

An agent $\omega \in \Omega_g$ working in $s \in \{u, 0, 1, ..., S\}$ produces

$$y_{\omega s} = A^Y_{gs} \varepsilon_{\omega s} H_{\omega s} \tag{2}$$

units of s output, where $A^Y_{gs} \geq 0$ is the systematic component of productivity common to all group g workers working in s, $\varepsilon_{\omega s}$ is the idiosyncratic component of productivity specific to agent ω if working in s, and $H_{\omega s}$ is the hours agent ω chooses to work.[2]

[1]The uncompensated and compensated labor supply elasticities are equal given lack of income effects in (1). The lack of income effects in (1) is a restriction on the intensive margin of labor supply alone, as we model the extensive margin of labor supply separately through a participation choice. We leave to future work relaxing this strong restriction on the intensive margin.

[2]We are assuming that workers are perfect substitutes in the production of all goods. Very similar results hold in a model in which production in each sector is a Cobb Douglas combination of labor groups.

The joint distribution of $\{\varepsilon_{\omega s}\}_{s=0}^{S}$, denoted by

$$G(\varepsilon_0, ..., \varepsilon_S; g) \equiv \Pr(\varepsilon_{\omega 0} \leq \varepsilon_0, ..., \varepsilon_{\omega S} \leq \varepsilon_S | \omega \in \Omega_g)$$

is assumed to be

$$G(\varepsilon_0, ..., \varepsilon_S; g) = \exp\left[-\varepsilon_0^{-\iota_g} - \left(\sum_{s=1}^{S} \varepsilon_s^{-\iota_g/(1-\kappa_g)}\right)^{(1-\kappa_g)}\right] \tag{3}$$

where $\iota_g > (1 + v_g)/v_g$ and $\kappa_g < 1$. As will become apparent in Section 3, ι_g and κ_g shape the elasticities of relative labor supply across sectors and of the extensive margin of labor supply. We assume that an agent's idiosyncratic productivity if out of the labor force equals her idiosyncratic productivity if unemployed, $\varepsilon_{\omega u} = \varepsilon_{\omega 0}$, since in both cases she operates the home production technology.

The economy is small and open, taking the nominal price of output, p_s, in each sector, $s \in \{1, ..., S\}$, as given. We assume that the nominal return per unit of output in home production is given by $p_0 P_g^{\psi}$, where $\psi \in [0, 1]$ and where p_0 is fixed. The parameter ψ allows us to consider two polar cases (and all cases intermediate). When $\psi = 1$, the real return per unit of home-production output is fixed at p_0, which does not vary with the price index. On the other hand, when $\psi = 0$, the real return is p_0/P_g, which moves one-for-one with the price index. For any value $\psi \in (0, 1)$, the real return moves with the price index, but less than one-for-one.[3] Finally, we assume that unemployed agents operate a (potentially less productive) version of the home production technology, with systematic productivity A_{gu}^{Y} and nominal value $p_0 P_g^{\psi}$.[4]

Labor market frictions and factor market clearing. Whereas agents produce alone when out of the labor force or when unemployed, production within a sector $s \in \{1, ..., S\}$ requires a worker to be matched with a firm. Search is directed: a firm chooses the sector in which to post a vacancy and the group to target; and each worker chooses the s in which to apply for a job.

The constant real cost of posting each vacancy targeted at agents in group g is given

[3]See Footnote 8 for the implications of ψ for labor force participation.

[4]We can treat $p_0 P_g^{\psi}$ as financed, in total or in part, by the government. In this case, we must specify a government budget constraint (which may either be balanced period-by-period or intertemporally in the case of international borrowing) and a tax. If we assume that the government raises revenue using an ad valorem production tax that is common across all goods, then our equilibrium characterization and all of our comparative static results continue to hold; however, in this case we have one additional equation that determines the level of the tax that balances the relevant budget constraint.

by $F_g > 0$ in each sector.[5] There is free entry, so (risk-neutral) firms post vacancies in each gs pair until expected profits from a new posting are zero, conditional on any gs vacancies being posted. Given a number of vacancies directed at gs, denoted by V_{gs}, and a number of job applicants, denoted by N_{gs}, the number of successful matches is determined by the matching function

$$M_{gs}\left(V_{gs}, N_{gs}\right) = A_g^M V_{gs}^{\alpha_g} N_{gs}^{1-\alpha_g}, \tag{4}$$

where $A_g^M > 0$ is the productivity of the matching function. Firms choose how many vacancies to post taking as given market tightness, $\theta_{gs} \equiv V_{gs}/N_{gs}$, and prices. The probability that an applicant finds a job is $E_{gs} \equiv M_{gs}/N_{gs} = A_g^M \theta_{gs}^{\alpha_g}$.[6] Labor-market clearing requires

$$N_g = \sum_{s=0}^{S} N_{gs} \tag{5}$$

We model the interaction between the firm and worker as being subject to a hold-up problem. After matching, the worker unilaterally chooses hours, then output is produced, and finally the worker and firm bargain over surplus. We assume a generalized Nash bargaining solution where the worker's weight is β_g. The firm's vacancy cost is sunk; hence, it's outside option at this stage is zero. The worker's hours choice is also sunk and production has already occurred, so that she cannot avail herself of unemployment or home production; hence, her outside option at this stage is also zero.

Timing. Each entrepreneur knows all nominal prices, p_s, the price index, P_g, and market tightness. Each agent knows all parameters, including the realization of the $\varepsilon_{\omega s}$ vector, all nominal prices, p_s, the price index, P_g, and the probability of successfully finding employment in each s, E_{gs}. At the same time that firms post vacancies, each agent applies to the $s \in \{0, 1, ..., S\}$ that maximizes her expected utility. Subsequently, each worker either successfully obtains employment in the s to which she applied or becomes unemployed. After the realization of her employment probability, she chooses her work hours and, if she is employed in a sector $s \in \{1, ..., S\}$, then bargains with her firm.

[5] Denominating vacancy posting costs in terms of the final consumption good implies that this activity is not intensive in the labor of any groups. If posting costs are denominated in labor, then the impact of a shock on inequality across groups depends on the factor intensity of this activity and the impact of the shock on posting; see e.g. Grossman and Helpman (2018) for a related analysis of innovation.

[6] As is well understood in the search-and-matching literature, the Cobb Douglas functional form in (4) does not restrict E_{gs} to satisfy $E_{gs} \in [0, 1]$. We do not impose $E_{gs} \in [0, 1]$ as an additional constraint. We could always choose A_g^M such that $E_{gs} \in (0, 1)$ in a given equilibrium, in which case our comparative static results would then apply for small shocks.

3 Equilibrium

Firm choices. We assume that firms post vacancies in all gs pairs, a condition that is satisfied in equilibrium if $A^Y_{gs} > 0$, $A^M_g > 0$, and $N_g > 0$ for all g and $s \in \{1, ..., S\}$. Generalized Nash bargaining implies that any agent working in $s \in \{1, ..., S\}$ earns a wage that is a share β_g of revenue. Given this wage determination, in any gs pair with positive vacancies posted, the zero-profit condition is given by

$$(1 - \beta_g) p_s Y_{gs} / P_g = F_g V_{gs},$$

where $F_g V_{gs}$ is the total real cost of vacancy posting, $p_s Y_{gs} / P_g$ is total real revenue (Y_{gs} is output of group g in sector s), and firms' share of this revenue is $1 - \beta_g$. The zero profit condition above and the matching function defined in (4) yield a solution for the nominal value of output per match as a function of market tightness and the price index,

$$\frac{p_s Y_{gs}}{M_{gs}} = \theta_{gs}^{1-\alpha_g} \frac{P_g}{A^M_g} \frac{F_g}{1 - \beta_g} \tag{6}$$

Worker choices. Consider an agent $\omega \in \Omega_g$. Conditional on working in s, she faces a real hourly wage of $A^Y_{gs} \varepsilon_{\omega s} \tilde{p}_{gs} / P_g$, where $\tilde{p}_{gs} = \beta_g p_s$ for all $s \in \{1, ..., S\}$ and $\tilde{p}_{g0} = p_0 P_g^{\psi}$. If she works in s, she chooses the number of hours to work to maximize utility, (1), yielding

$$H_{\omega s} = \left(\varepsilon_{\omega s} \zeta_g A^Y_{gs} \tilde{p}_{gs} \Big/ P_g \right)^{1/v_g} \tag{7}$$

where $1/v_g$ is the compensated (and uncompensated) intensive-margin labor supply elasticity.

Denote by $V_{\omega s}$ her (indirect) utility if working in s. Equations (1) and (7) imply

$$V_{\omega s} = \frac{v_g}{1 + v_g} \left(\varepsilon_{\omega s} \zeta_g A^Y_{gs} \tilde{p}_{gs} \Big/ P_g \right)^{(1+v_g)/v_g}$$

If ω applies to s, her expected utility is given by $E_{gs} V_{\omega s} + (1 - E_{gs}) V_{\omega u}$. Agent ω chooses to apply to the $s \in \{0, 1, ..., S\}$ that maximizes her expected utility: she applies to s if and only if[7]

$$E_{gs} V_{\omega s} + (1 - E_{gs}) V_{\omega u} > \max_{s' \in \{0,1,...,S\}, s' \neq s} \left\{ E_{gs'} V_{\omega s'} + (1 - E_{gs'}) V_{\omega u} \right\}$$

[7]Given our distributional assumption, for almost all agents there is a unique choice that maximizes expected utility. Hence, we use $>$ rather than $\geq$.

Worker choices with common market tightness. Consider an equilibrium in which market tightness for a given g is common across sectors: $\theta_g = \theta_{gs}$ for all $s \in \{1, ..., S\}$. Common market tightness implies $E_g = E_{gs}$ for all $s \in \{1, ..., S\}$.

In such an equilibrium, agent ω prefers one sector s to another sector s' if and only if $V_{\omega s} > V_{\omega s'}$. She prefers home production to sector s if and only if $\frac{1}{E_g} V_{\omega 0} - \frac{1-E_g}{E_g} V_{\omega u} > V_{\omega s}$. More compactly, she chooses s if and only if

$$w_{gs} \varepsilon_{\omega s} > \max_{s' \neq s} \left\{ w_{gs'} \varepsilon_{\omega s'} \right\}$$

where we define

$$w_{gs} \equiv \begin{cases} A^Y_{gs} \beta_g p_s & \text{if } s \in \{1, ..., S\} \\ \mathbb{I}_{(\tilde{w}_{g0} > 0)} \tilde{w}_{g0}^{\frac{v_g}{1+v_g}} p_0 P_g^{\psi} & \text{if } s = 0 \end{cases}$$

$\tilde{w}_{g0} \equiv \frac{1}{E_g} \left(A^Y_{g0} \right)^{\frac{1+v_g}{v_g}} + \frac{E_g - 1}{E_g} \left(A^Y_{gu} \right)^{\frac{1+v_g}{v_g}}$, and $\mathbb{I}_{(\tilde{w}_{g0} > 0)}$ is an indicator function that equals one if $\tilde{w}_{g0} > 0$. Here, w_{gs} is the nominal wage per efficiency-unit-hour in each sector. And w_{g0} is either zero, if $\tilde{w}_{g0} \leq 0$, or a CES combination of the nominal wages per efficiency-unit-hour if not participating in the labor force and if unemployed. If $\tilde{w}_{g0} < 0$, its particular value has no effect on choices, since in this case all agents would prefer unemployment to non participation (a necessary condition for it to be negative is that agents are more productive when unemployed than when out of the labor force) so all agents would choose to participate in the labor force.[8]

Given our distributional assumption on the vector $\{\varepsilon_{\omega s}\}_{s=0}^{S}$ in (3), the probability that ω applies to $s \in \{0, 1, ..., S\}$ is given by

$$\pi_{gs} = \frac{1}{\Phi_g} \left(\sum_{s' \in \mathcal{S}(s)} w_{gs'}^{\frac{\iota_g}{1-\kappa_g}} \right)^{-\kappa_g} w_{gs}^{\frac{\iota_g}{1-\kappa_g}} \tag{8}$$

where $\mathcal{S}(s)$ indicates the set of all choices in either non-participation or participation, depending on whether s is non-participation ($s = 0$) or a sector ($s \in \{1, ..., S\}$),

$$\mathcal{S}(s) = \begin{cases} \{1, ..., S\} & \text{if } s \in \{1, ..., S\} \\ \{0\} & \text{if } s = 0 \end{cases}$$

[8]It is apparent that if $\psi > 0$ then a decrease in the price index—holding p_s and E_g fixed—increases the incentive for an individual agent out of the labor force to switch into s.

and where

$$\Phi_g \equiv \left(w_{g0}^{\frac{\iota_g}{1-\kappa_g}}\right)^{1-\kappa_g} + \left(\sum_{s\in\{1,\dots,S\}} w_{gs}^{\frac{\iota_g}{1-\kappa_g}}\right)^{1-\kappa_g} \tag{9}$$

measures group g's production-side exposure to price changes, as will become clear in Section 4.

Of course, (8) can be re-expressed as

$$\pi_{gs} = \pi_g^{\mathcal{S}(s)} \pi_{gs}^{\mathcal{S}(s)}$$

where

$$\pi_g^{\mathcal{S}(s)} = \frac{1}{\Phi_g}\left(\sum_{s'\in\mathcal{S}(s)} w_{gs'}^{\frac{\iota_g}{1-\kappa_g}}\right)^{1-\kappa_g} \tag{10}$$

is the probability that agents in group g choose to participate in the labor force if $s \in \{1, \dots, S\}$ or not to participate if $s = 0$, and where

$$\pi_{gs}^{\mathcal{S}(s)} = \frac{w_{gs}^{\frac{\iota_g}{1-\kappa_g}}}{\sum_{s'\in\mathcal{S}(s)} w_{gs'}^{\frac{\iota_g}{1-\kappa_g}}} \tag{11}$$

is the probability ω chooses sector s conditional on participating in the labor force and is one if ω is not participating in the labor force. According to (11), the partial elasticity of labor supply across sectors conditional on participation is given by $\iota_g/(1-\kappa_g)$: if the price of output in sector s rises by one percent, then—holding fixed the denominator of $\pi_{gs}^{\mathcal{S}(s)}$—the share of agents participating in the labor force who apply to s would rise by $\iota_g/(1-\kappa_g)$ percent. According to (10), the partial extensive-margin labor supply elasticity is ι_g: if the price of output in all sectors rose by one percent, then—holding fixed the denominator of $\pi_g^{\mathcal{S}(s)}$—the share of agents participating in the labor force would rise by ι_g percent.

Combining (2) and (7) yields the value of ω's output if working in sector $s \in \{1, \dots, S\}$,

$$p_s y_{\omega s} = p_s \left(A_{gs}^{Y} \varepsilon_{\omega s}\right)^{\frac{1+v_g}{v_g}} \left(\zeta_g \tilde{p}_{gs} \big/ P_g\right)^{\frac{1}{v_g}} \tag{12}$$

To calculate the value of output across all agents working in s, we must integrate across all such agents taking into account that these agents self select into s.[9] Denote by

$$\mathbb{E}[\varepsilon_{\omega s}^{b}|s] \equiv \mathbb{E}\left[\varepsilon_{\omega s}^{b}\middle| w_{gs}\varepsilon_{\omega s} > \max_{s'\neq s}\left\{w_{gs'}\varepsilon_{\omega s'}\right\}\right]$$

the expectation of $\varepsilon_{\omega s}^{b}$ conditional on choosing option s for an arbitrary value of $b \in (0, \iota_g)$. Given our distributional assumption on the vector $\{\varepsilon_{\omega s}\}_{s=0}^{S}$, we have

$$\mathbb{E}[\varepsilon_{\omega s}^{b}|s] = \Gamma\left(1 - \frac{b}{\iota_g}\right)\Phi_g^{\frac{b}{\iota_g}} w_{gs}^{-b} \tag{13}$$

where $\Gamma(\cdot)$ is the gamma function. Equations (12) and (13) yield

$$\frac{p_s Y_{gs}}{M_{gs}} = \gamma_{g1}\frac{1}{\beta_g}\left(\frac{\zeta_g}{P_g}\right)^{\frac{1}{v_g}}\Phi_g^{\frac{1+v_g}{v_g\iota_g}} \tag{14}$$

where $\gamma_{g1} \equiv \Gamma\left(1 - (1+v_g)/(v_g\iota_g)\right)$ and where we used $\mathbb{E}[y_{\omega s}|s] = Y_{gs}/M_{gs}$. According to (14), in any equilibrium in which market tightness is common across sectors, worker choices imply that the value of output per worker is also common across sectors.

Equilibrium characterization with common market tightness across sectors. Combining (14)—which was derived from worker optimization under the assumption that $\theta_g = \theta_{gs}$ for all $s \in \{1, ..., S\}$—with equation (6)—which was derived from firm optimization without imposing the assumption of common market tightness—we obtain a sector-specific solution for market tightness

$$\theta_{gs}^{1-\alpha_g} = \gamma_{g1}\frac{1-\beta_g}{\beta_g}\frac{A_g^M}{F_g}\zeta_g^{\frac{1}{v_g}}\left(\Phi_g^{1/\iota_g}/P_g\right)^{\frac{1+v_g}{v_g}} \tag{15}$$

consistent with firm optimization. The previous expression is common across sectors, satisfying $\theta_{gs} = \theta_g$.[10]

Combining optimal hours worked, (7), with (13) yields a solution for hours worked per employee that is common across sectors,

$$H_g = \gamma_{g2}\zeta_g^{\frac{1}{v_g}}\left(\Phi_g^{\frac{1}{\iota_g}}/P_g\right)^{\frac{1}{v_g}} \tag{16}$$

[9] It is sufficient to measure the average output across workers who apply to s, rather than who work in s, since unemployment is random.

[10] This condition highlights the role of the assumptions that A_g^M, β_g, F_g, and α_g do not vary across sectors.

where $\gamma_{g2} \equiv \Gamma\left(1 - 1/(v_g \iota_g)\right)$. The average real wage per hour worked, denoted by W_g, is also common across sectors. It equals total real labor income, $\beta_g p_s Y_{gs}/P_g$, divided by total hours worked, $M_{gs} H_g$. Equations (14) and (16) yield

$$W_g = \frac{\gamma_{g1}}{\gamma_{g2}} \left(\Phi_g^{\frac{1}{\iota_g}} / P_g\right) \tag{17}$$

The probability of successful employment, denoted by $E_g \equiv E_{gs}$ for all $s \in \{1, ..., S\}$, can be calculated using (15) as

$$E_g = A_g^M \left[\gamma_{g1} \frac{1-\beta_g}{\beta_g} \frac{A_g^M}{F_g} \zeta_g^{\frac{1}{v_g}}\right]^{\frac{\alpha_g}{1-\alpha_g}} \left(\Phi_g^{\frac{1}{\iota_g}} / P_g\right)^{\frac{1+v_g}{v_g} \frac{\alpha_g}{1-\alpha_g}} \tag{18}$$

Proposition 1 summarizes these results.

Proposition 1. *In any equilibrium in which $\theta_g = \theta_{gs}$ in all sectors, the probability of successfully obtaining employment given participation in the labor force and the average wage per hour worked and hours worked per employee in any sector are given by*

$$K_g = \chi_g^K \left(\Phi_g^{\frac{1}{\iota_g}} / P_g\right)^{\rho_g^K} \quad \textit{for} \quad K \in \{W, H, E\} \tag{19}$$

where the χ_g^K terms are positive constants, $\rho_g^W \equiv 1$, $\rho_g^H \equiv \frac{1}{v_g}$, and $\rho_g^E \equiv \frac{\alpha_g}{1-\alpha_g} \frac{1+v_g}{v_g}$. In such an equilibrium, the labor force participation rate, $L_g \equiv \sum_{s=1}^S \pi_{gs}$, is

$$L_g = \frac{1}{\Phi_g} \left(\sum_{s=1}^S \left(A_{gs}^Y \beta_g p_s\right)^{\frac{\iota_g}{1-\kappa_g}}\right)^{1-\kappa_g} \tag{20}$$

According to Proposition 1, the variables Φ_g and P_g are sufficient statistics that fully characterize the impact of sectoral prices on W_g, H_g, and E_g. The result that each of these margins of adjustment is an iso-elastic function of Φ_g and P_g follows from our many iso-elastic functional form assumptions: on the matching function in (4), the intensive margin of labor supply embedded in (1) and solved for in (7), and both the extensive margin of labor supply as well as labor supply across sectors embedded within (3) and solved for in (10) and (11).[11]

We now turn to solving for the expectation of welfare, denoted U_g, for an arbitrary $\omega \in \Omega_g$ before the realization of the vector $\varepsilon_{\omega s}$ and before the realization of employment

[11]Labor force participation, (20), is itself an iso-elastic function of sectoral prices.

for those participating in the labor force. We solve for U_g under a particular assumption:

Assumption 1. Either (i) the productivity of non-participation is zero, $A^Y_{g0} = 0$, or (ii) the productivity of unemployment is zero, $A^Y_{gu} = 0$.

In case (i), all agents choose to participate in the labor force. In case (ii), the income of the unemployed is zero. In either case, w_{g0} simplifies dramatically.

Expected utility conditional on applying to $s \in \{0, 1, ..., S\}$ can be expressed as

$$\mathbb{E}[V_\omega | s] = E_{gs} \frac{v_g}{1 + v_g} \left(\zeta_g A^Y_{gs} \tilde{p}_{gs} \frac{1}{P_g} \right)^{\frac{1+v_g}{v_g}} \mathbb{E}\left[\varepsilon_{\omega s}^{\frac{1+v_g}{v_g}} | s \right]$$

The previous expression, (13), and (19) yield the following Lemma.

Lemma 1. *If Assumption 1 is satisfied, then in any equilibrium in which $\theta_{gs} = \theta_g$ across all sectors, expected welfare is*

$$U_g = \frac{v_g}{1 + v_g} \zeta_g W_g H_g E_g \tag{21}$$

Does an equilibrium exist in which market tightness is common across sectors? The solution for Φ_g in (9) is implicit, since Φ_g depends on E_g through w_{g0}, and E_g itself depends on Φ_g according to (19). However, in the special case in which $A^Y_{g0} = 0$—case (i) of Assumption 1—there exists a unique equilibrium in which market tightness is common across sectors. In this case, $w_{g0} = 0$ so that the solution for Φ_g in (9) does not depend on E_g and is uniquely given by

$$\Phi_g = \left(\sum_{s \in \{1,...,S\}} \left(A^Y_{gs} \beta_g p_s \right)^{\frac{\iota_g}{1-\kappa_g}} \right)^{1-\kappa_g}$$

4 Comparative static results

What are the implications of changes in sectoral prices for each margin of adjustment and for welfare? Adjustments in each—and welfare under Assumption 1—are fully characterized by $d \ln \Phi_g$, $d \ln P_g$, the fixed elasticities and initial shares.

Hence, solving for changes in each margin boils down to solving for $d \ln \Phi_g$ and $d \ln P_g$. It is well-known that $d \ln P_g$ is a weighted average of sectoral price changes with weights given by initial expenditure shares of group g, which we treat as exogenous given that we do not model the consumption aggregator. Given that we have nothing to add to this well-known result, we will not express $d \ln P_g$ in terms of the changes in sectoral prices in what follows.

How does $d\ln\Phi_g$ depend on sectoral price changes? In general, we have

$$d\ln\Phi_g = \iota_g(1-L_g)d\ln w_{g0} + \iota_g L_g \sum_{s=1}^{S} \pi_{gs}^{\mathcal{S}(s)} d\ln p_s \tag{22}$$

In what follows, we impose Assumption 1, in which case $d\ln w_{g0}$ and, therefore, $d\ln\Phi_g$, are particularly simple. Consider case (ii) of Assumption 1: $A_{gu}^{Y} = 0$. In this case, $w_{g0} = E_g^{-v_g/(1+v_g)} A_{g0}^{Y} p_0 P_g^{\psi}$, which combined with the solution for E_g in (19) yields

$$\iota_g d\ln w_{g0} = -\frac{\alpha_g}{1-\alpha_g} d\ln\Phi_g + \left(\frac{\alpha_g}{1-\alpha_g} + \psi\right) \iota_g d\ln P_g$$

The previous expression and (22) yield

$$d\ln\Phi_g = \frac{\iota_g(1-\alpha_g)}{1-\alpha_g L_g}\left[(1-L_g)\left(\frac{\alpha_g}{1-\alpha_g}+\psi\right) d\ln P_g + L_g \sum_{s=1}^{S} \pi_{gs}^{\mathcal{S}(s)} d\ln p_s\right]$$

The previous expression also holds in case (i) of Assumption 1, although it is further simplified since $L_g = 1$ in that case. The previous expression, (19), (20), and (21) yield the following proposition.

Proposition 2. *In any equilibrium in which $\theta_g = \theta_{gs}$ in all sectors, under Assumption 1 we have*

$$d\ln K_g = \frac{\rho_g^K(1-\alpha_g)}{1-\alpha_g L_g}\left[L_g \sum_{s=1}^{S} \pi_{gs}^{\mathcal{S}(s)} d\ln p_s - \left(1-\psi(1-L_g)\right) d\ln P_g\right] \tag{23}$$

for any $K \in \{W, H, E, U\}$, where $\rho_g^U \equiv \rho_g^W + \rho_g^H + \rho_g^E$; we also have

$$d\ln L_g = \frac{(1-L_g)\iota_g}{1-\alpha_g L_g}\left[\sum_{s=1}^{S} \pi_{gs}^{\mathcal{S}(s)} d\ln p_s - (\psi + \alpha_g - \alpha_g\psi) d\ln P_g\right] \tag{24}$$

Indeed, when the underlying elasticities and price indices are common across groups, and the labor force participation rate is equated across groups in the initial equilibrium—as in Assumption 2—we obtain particularly stark results.

Assumption 2. $\theta_g = \theta_{gs}$ in all sectors; $\alpha = \alpha_g$, $v = v_g$, $\iota = \iota_g$, and $P = P_g$ for all g; and in the initial equilibrium $L = L_g$ for all g.

Under Assumptions 1 and 2, the differential change in any $K \in \{W, H, E, L, U\}$ across two groups is given by

$$d\ln K_{g'} - d\ln K_g = \frac{\rho^K(1-\alpha)L}{1-\alpha L}\sum_{s=1}^{S}\left(\pi_{g's}^{\mathcal{S}(s)} - \pi_{gs}^{\mathcal{S}(s)}\right) d\ln p_s \tag{25}$$

where $\rho^L \equiv \iota(1-L)/((1-\alpha)L)$. Equation (25) directly implies an extension of canonical results in the trade literature on the effects of price changes on wages for two groups—related to the Slolper-Samuelson theorem—to higher dimensional comparisons with arbitrarily many groups (as in Costinot and Vogel, 2010) but also to a broad range of adjustment margins.

Proposition 3. *Under Assumptions 1 and 2, if $d\ln p_s$ is increasing in s and $\sum_{s=1}^{s'}\pi_{g's}^{\mathcal{S}(s)} \leq \sum_{s=1}^{s'}\pi_{gs}^{\mathcal{S}(s)}$ for all $s' \in \{1, ..., S\}$, then $d\ln K_{g'} \geq d\ln K_g$ for $K \in \{W, H, E, L, U\}$.*

According to Proposition 3, if group g' is disproportionately employed in high s sectors compared to group g (in the sense of first-order stochastic dominance), then a shock that increases the relative prices of high s sectors must increase the average wage, average hours worked per employee, the employment rate, labor force participation, and utility of group g' relative to group g.

Finally, using (11) there is a simple sufficient condition for group g' to be disproportionately employed in high s sectors compared to group g (in the sense of first-order stochastic dominance): $A_{g's'}^Y A_{gs}^Y \geq A_{g's}^Y A_{gs'}^Y$ for all sectors $s' \geq s$.

5 Intuition and Discussion

5.1 Intuition

Here we provide intuition for the qualitative and quantitative structure our comparative static results, focusing in particular on case (i) of Assumption 1, in which the labor force participation rate is one and (23) simplifies to[12]

$$d\ln K_g = \rho_g^K\left[\sum_{s=1}^{S}\pi_{gs}^{\mathcal{S}(s)} d\ln p_s - d\ln P_g\right] \quad \text{for} \quad K \in \{W, H, E, U\} \tag{26}$$

The wage comparative static result follows from the envelope condition. In response to a small change in sector prices, the average g wage may change through two channels. First, each infra-marginal worker who does not switch sectors will experience a

[12] Intuition in case (ii) is very similar.

change in real wage exactly equal to the change in her sector's real price, $d \ln p_s - d \ln P_g$. Second, workers switch across sectors. However, since a switcher is indifferent between sectors and there are no compensating differentials or switching costs, switching has only a second-order effect on wages. Hence, only the first channel has a first-order effect on wages. Averaging wage changes across workers within g then yields the weighted average of real sector price changes, , $d \ln p_s - d \ln P_g$, weighted by the g-specific income share across sectors in the original equilibrium. This explains both the structure of the relative wage result in (26) as well as the fact that the elasticity, ρ_g^W, is exactly equal to one.

Taking these wage effects as given, if the labor supply elasticity is $1/v_g$—as implied by (1)—then the response of average hours worked across g workers is simply $\rho_g^H = 1/v_g$ times the response of average wages.[13]

Consider the response of frictional unemployment. Changes in employment rates are induced by changes in firm vacancy choices, and all else equal firms post more vacancies when a sector's price rises. This explains why $\rho_g^E = \frac{\alpha_g}{1-\alpha_g} \frac{1+v_g}{v_g}$ is increasing in the elasticity of matches to vacancies, α_g. The incentive for firms to post vacancies in a given gs rises with the price of the sector for two reasons. First, a higher price generates more revenue at fixed hours worked. Second, a higher price increases hours worked of each individual worker, which raises revenue further. This explains why ρ_g^E is also increasing in the labor supply elasticity, $1/v_g$.

Finally, (26) also demonstrates how production-side, $\sum_{s=1}^S \pi_{gs}^{\mathcal{S}(s)} d \ln p_s$, and consumption-side, $d \ln P_g$, exposure jointly shape changes in each margin of adjustment and welfare. As is well known, see e.g. Porto (2006), reductions in prices benefit g through consumption-side exposure whereas they harm g through production-side exposure. Equation (26) clarifies exactly how these two sources of exposure shape the ln change in each margin and utility: $d \ln K_g$ is equal to ρ_g^K (a combination of structural elasticities) times the sum across sectors of the product of each sector's price change and the difference in the share of income earned and expenditure allocated to the sector in the initial equilibrium.

5.2 Discussion

Our theoretical results provide a mapping from reduced-form empirical findings on the responsiveness of multiple margins of adjustment to changes in welfare consistent with these margins of adjustment. Consider the empirical exercises in, e.g., Autor et al. (2013). Their measure of the China shock is $\sum_{s=1}^S \pi_{gs}^{\mathcal{S}(s)} (\Delta M_s / L_s)$, where $\Delta M_s / L_s$ is the change in

[13] Selection into unemployment does not affect the wage or hours margins, since unemployed workers are randomly selected (within each g).

U.S. imports from China in sector s divided by initial U.S. sector s employment and where g denotes U.S. commuting zones. This is identical to our theoretical object $d \ln \Phi_g$ in case (i) of Assumption 1, except $\Delta M_s / L_s$ proxies in the data for $d \ln p_s$ in our theory. Hence, regressions of the form run in Autor et al. (2013) identify a combination of our structural elasticities (ρ^W, ρ^H, and ρ^E) and the U.S. producer price impact of changes in imports per worker (the parameter τ in $d \ln p_s = \tau \Delta M_s / L_s$).[14] Given the estimated coefficients from regressions on changes in ln wages, ln hours worked per employee, and the ln of one minus the unemployment rate, one can then construct a measure of the causal change in relative welfare across U.S. commuting zones that is consistent with our theory, using $\rho^U = \rho^W + \rho^H + \rho^E$.[15]

In addition, our theoretical work also facilitates counterfactual analysis. Depending on how one defines a group, g, Proposition 2 can be used to solve analytically for changes in each margin of adjustment and welfare at various levels of aggregation: across regions (the dimension focused on by, e.g., Kovak, 2013; Galle et al., 2017; and Adão et al., 2018), across workers with different education, age, gender, and/or race at the national level (the dimension focused on by, e.g., Lee, 2017 and Burstein et al., 2019), across workers with different education levels within regions (the dimension focused on by, e.g., Dix-Carneiro and Kovak, 2015), across workers with the same education levels across regions, etc.

Finally, under our small open economy assumption and for a given choice of g, the data required to conduct counterfactuals is minimal: initial income shares ($\pi_{gs}^{\mathcal{S}(s)}$), initial labor force participation (L_g), changes in sectoral prices ($d \ln p_s$), changes in price indices, and structural elasticities.

Of course, the small open economy assumption is often inappropriate, in which case the impact of a particular change in the economic environment (e.g. trade liberalization) on sectoral prices must be solved. This requires an additional set of assumptions (e.g. a gravity model of trade within each sector as in many of the quantitative papers cited above). In this case, changes in producer and consumer prices may differ; here, $\sum_{s=1}^{S} \pi_{gs}^{\mathcal{S}(s)} d \ln p_s$ should be defined using changes in local producer prices whereas $d \ln P_g$ should be defined using changes in local consumer prices.

[14]This requires versions of their regression in which dependent variables are consistent with our theoretical margins of adjustment.

[15]This requires the value of τ, which can be recovered from the regression on changes in ln wages, since $\rho^W = 1$.

6 Conclusions

We develop a framework to analyze the impact of trade shocks on a wide range of labor-market adjustment margins in economies with a large number of sectors and labor groups. We provide analytic results characterizing equilibria and showing how changes in sectoral prices affect each margin of adjustment as well as welfare for each labor group.

We hope that our framework and results will be useful to guide future empirical and quantitative investigations of the effect of trade shocks on labor-markets as well as to provide welfare interpretations of the coefficients in reduced-form empirical work.

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