Trade shocks and labor market adjustment*

Ryan Kim  
*Johns Hopkins, School of Advanced International Studies*

Jonathan Vogel  
*University of California, Los Angeles*

February 14, 2020

Abstract

We develop a framework to analyze the impact of trade shocks on a wide range of labor market adjustment margins in economies with a large number of sectors and labor groups. We provide analytic results characterizing equilibria and showing how changes in sectoral prices affect each margin of adjustment—average wages, hours worked per worker, the labor force participation rate, the unemployment rate—as well as welfare for each labor group. We show that the simple economics underlying the Stolper-Samuelson theorem applies more generally to adjustment across each of these margins.

*We thank Ariel Burstein, Pablo Fajgelbaum, and Andres Rodriguez-Clare for helpful comments*
1 Introduction

We develop a framework to analyze the impact of trade shocks on a wide range of labor market adjustment margins in economies with a large number of sectors and labor groups. We endogenize the extensive margin of labor supply (labor force participation), the sector in which workers search for a job conditional on labor force participation, the probability of successful employment conditional on job search in a given sector (frictional unemployment), and hours worked and average wages conditional on successful employment. We provide analytic results characterizing equilibria and showing how changes in sectoral prices affect each margin of adjustment as well as welfare for each labor group.

We consider a static assignment model of trade with many sectors and labor groups (where labor groups are defined by observable characteristics such as region, education, gender, etc...) featuring search frictions. Each agent is characterized by a vector of productivities: one in each sector and one in non-participation. Production requires that a worker and a firm match, matching is subject to frictions, and search is directed. An agent in the labor force chooses the sector in which to apply for a job and each firm targets its vacancy at the labor group and sector of its choice. Workers and firms are atomistic, treating the vector of probabilities of successfully matching—with one probability per sector and labor group pair—as exogenous and these probabilities are determined in equilibrium. A worker who successfully matches chooses how many hours to work and the worker’s wage is determined by the outcome of a bargaining process between the firm and worker.

We aim to provide the simplest extension possible of canonical models of international trade. Nevertheless, our framework remains rich. Specifically, on the worker side we choose assumptions to generate a group $g$-specific extensive margin elasticity of labor supply, a second group $g$-specific intensive margin elasticity of labor supply, and a third group $g$-specific labor supply elasticity across sectors conditional on labor force participation. Combining worker and firm optimization, we solve for an equilibrium in which the probability of successfully finding a job is common across sectors for a given labor group.

In this equilibrium, we show that the probability of a worker in group $g$ successfully obtaining employment given participation in the labor force (one minus the unemployment rate) denoted by $E_g$, the average wage per hour worked denoted by $W_g$, hours worked per employee denoted by $H_g$, and average expected utility denoted by $U_g$ are each given by a group $g$-specific constant elasticity function of two equilibrium objects: a group-specific price index of final good consumption, $P_g$, and a separate group-specific aggregator of sectoral prices in which weights are shaped by the group’s productivity in
each sector, $\Phi_s$. Specifically, we characterize an equilibrium in which for each outcome $K \in \{E, W, H, U\}$,

$$K_s = \chi^K_s \left( \Phi_s^{1/\iota_s} / P_s \right)^{\rho^K_s}$$

where $\chi^K_s$ is a positive constant and $\iota_s$ and $\rho^K_s$ are structural elasticities. The equation determining the equilibrium labor force participation rate is similar.

We then show how to solve analytically for changes in these equilibrium outcomes as simple functions of sectoral price changes, initial group $g$ allocations across sectors, and structural elasticities. These results extend Stolper-Samuelson-style logic both to high-dimensional environments (as in Costinot and Vogel (2010) for wages) but also to a much broader range of labor-market adjustments. For instance, we show that if structural elasticities are common for two groups, $g'$ and $g$ that also start with common labor force participation rates, if $g'$ is initially allocated to higher-indexed sectors, and relative sectoral prices rise in higher-indexed sectors, then the average wage, hours worked conditional on employment, the probability of successful employment, and utility rises for $g'$ relative to $g$.

**Literature.** Our paper’s objective of incorporating non-wage margins of adjustment into high-dimensional environments is motivated by a vast empirical literature. This literature studies high-dimensional environments—e.g. many regions in Topalova (2010), Autor et al. (2013), and Kovak (2013)—and emphasizes the relative importance of non-wage margins of adjustment. For instance, Autor et al. (2013) argue that their “results suggest that the predominant focus of the previous literature on wages misses important aspects of labor-market adjustments to trade. We find that local labor markets that are exposed to... China’s rising competitiveness experience increased unemployment, decreased labor-force participation, and increased use of disability and other transfer benefits, as well as lower wages.”

Our paper is complementary to Kim and Vogel (2020), which estimates the impact of a particular trade shock on relative welfare across U.S. commuting zones, measuring a welfare-change sufficient statistic derived from a substantially more general framework that dispenses with the many functional form restrictions imposed here. Relative to Kim and Vogel (2020), the present paper’s structure allows us to solve for the implications of counterfactual shocks.

In this respect, our structural approach is closely related to a growing quantitative trade literature that uses assignment models to study the impact of trade on wage inequality either at the national level—see e.g. Lee (2017) and Burstein et al. (2019)—or across local labor markets—see e.g. Adão (2015) and Galle et al. (2017). Into this environ-
ment, Adão et al. (2018) and Caliendo et al. (2019) introduce a labor-leisure choice. We extend these quantitative models to incorporate both an intensive and extensive margin of labor supply adjustment as well as frictional unemployment; provide analytic comparative static results to shed light on this literature’s quantitative conclusions; and allow for heterogeneous treatment effects across labor groups (via group-specific structural elasticities).

Our analytic results highlight the theoretical relevance of Stolper-Samuelson-style insights across a range of margins of adjustment. These results clarify that empirical findings of relatively small wage effects of trade shocks—which have been interpreted as evidence against the practical relevance of Stolper-Samuelson-style mechanisms—are potentially consistent with large effects transmitted through non-wage margins of adjustment. In this respect, our work answers the call of Davis and Mishra (2007), who write that “We shouldn’t ignore [our toy theories], discard them, or least of all mutilate them. But we do need to ask what the deep lesson is to be learned from the simple models and how one should go about using this insight in a more complex setting.”

Our theory builds most directly on Costinot and Vogel (2010) and Davidson et al. (1999). We use the tools and techniques developed in Costinot and Vogel (2010), which provides analytic results on factor allocation and wages in high-dimensional environments.¹ These results apply directly in modern quantitative environments; as reviewed in Costinot and Vogel (2015). We extend Stolper-Samuelson insights to additional margins of adjustment as in Davidson et al. (1999), which is the paper closest in spirit to our theoretical contribution. Davidson et al. (1999) introduces frictional unemployment into a two-by-two Heckscher-Ohlin-like model, studies the link between trade and the distribution of income, and obtains an extended Stolper-Samuelson-like theorem.

2 Model

We consider an economy populated by a continuum of agents separated into finitely many disjoint groups indexed by $g$, each of which corresponds to the intersection of various observable characteristics, such as the region in which an agent lives and her education, gender, nativity, etc... The set of group $g$ agents is denoted $\Omega_g$, which we treat as exogenous, and its mass is denoted $N_g = |\Omega_g|$.

Agents can either be employed in one of $S$ sectors, in the labor force but unemployed, or

¹Unlike Costinot and Vogel (2010), we treat changes in goods prices as exogenous. Leveraging the approach developed in Costinot and Vogel (2010) to extend our results to endogenous goods prices is straightforward in the special case in which there is no within-group heterogeneity, as in Costinot and Vogel (2010).
or out of the labor force. We index each possibility by $s$, with $s \in \{1,...,S\}$ indicating each of the $S$ sectors in which an agent can be employed, $s = u$ indicating unemployment, and $s = 0$ indicating non-participation in the labor force.

Each agent chooses to apply to the $s \in \{0,1,...,S\}$ that maximizes her expected utility. Agents do not apply to be unemployed; however, an agent who applies to $s \in \{1,...,S\}$ may become employed in $s$ or unemployed. We denote by $E_{gs} \in [0,1]$ the probability of successfully obtaining employment for an agent $\omega \in \Omega_g$ who applies to $s$, which each agent treats as exogenous. Of course, agents who choose not to participate in the labor force cannot become unemployed, so $E_{g0} = 1$ for all $\omega$. We say that an agent “works” in $s$ even if $s = u$, in which case the agent is unemployed.

The utility of an agent $\omega \in \Omega_g$ who consumes $C$ units of the final good and supplies $H$ hours of labor is

$$U(C_\omega, H_\omega; g) = \xi_gC_\omega - \frac{H_\omega^{1+v_g}}{1+v_g},$$

(1)

where $\xi_g, v_g > 0$. As will become apparent in Section 3, the parameter $v_g$ shapes both the uncompensated (Marshallian) and the compensated (Hicksian) intensive-margin labor supply elasticities, which are both given by $1/v_g$. The price index for final goods consumption is denoted $P_g$.

**Production and revenue.** We assume that all agents—whether working in a sector, not participating in the labor force, or unemployed—produce output in the respective $s \in \{u,0,1,...,S\}$. Both the unemployed and those not participating in the labor force operate a home production technology.

An agent $\omega \in \Omega_g$ working in $s \in \{u,0,1,...,S\}$ produces

$$y_{\omega s} = A_{gs}Y_{gs}\epsilon_{\omega s}H_{\omega s},$$

(2)

units of $s$ output, where $A_{gs} \geq 0$ is the systematic component of productivity common to all group $g$ workers working in $s$, $\epsilon_{\omega s}$ is the idiosyncratic component of productivity specific to agent $\omega$ if working in $s$, and $H_{\omega s}$ is the hours agent $\omega$ chooses to work.

The joint distribution of $\{\epsilon_{\omega s}\}_{s=0}^S$, denoted by

$$G(\epsilon_0, ..., \epsilon_S; g) \equiv \Pr(\epsilon_{\omega 0} \leq \epsilon_0, ..., \epsilon_{\omega S} \leq \epsilon_S | \omega \in \Omega_g)$$

\footnote{The uncompensated and compensated labor supply elasticities are equal given lack of income effects in (1).}
is assumed to be

\[ G(\epsilon_0, ..., \epsilon_S; g) = \exp \left[ -\epsilon_0 - \left( \sum_{s=1}^{S} \epsilon_s \right)/(1-\kappa_g) \right] \]

(3)

where \( \eta_g > (1 + \upsilon_g)/\upsilon_g \) and \( \kappa_g < 1 \). As will become apparent in Section 3, the parameters \( \eta_g \) and \( \kappa_g \) shape the elasticity of relative labor supply across sectors, which is given by \( \eta_g/(1-\kappa_g) \); and the parameter \( \eta_g \) is the extensive margin elasticity of labor supply. We assume that an agent’s idiosyncratic productivity if out of the labor force equals her idiosyncratic productivity if in unemployment, \( \epsilon_{\omega u} = \epsilon_{\omega 0} \), since in both cases she operates the home production technology.

The economy is small and open, taking the nominal price of output, \( p_s \), in each sector, \( s \in \{1, ..., S\} \), and in home production, \( p_0 \), as given. We assume that unemployed agents operate a (potentially less productive) version of the home production technology, with systematic productivity \( A_Y^{g u} \) and price \( p_0 \).

**Labor market frictions and factor market clearing.** Whereas agents produce alone when out of the labor force or when unemployed, production within a sector \( s \in \{1, ..., S\} \) requires a worker to be matched with a firm. Search is directed: a firm chooses the sector, \( s \in \{1, ..., S\} \) in which to post a vacancy and the group \( g \) to target; and each worker chooses the \( s \) in which to apply for a job.

The constant real cost of posting each vacancy targeted at agents in group \( g \) is given by \( F_g > 0 \) in each sector. There is free entry, so (risk-neutral) firms post vacancies in each \( gs \) pair until expected profits from a new posting are zero, conditional on any \( gs \) vacancies being posted. Given a number of vacancies directed at \( gs \), denoted by \( V_{gs} \), and a number of job applicants, denoted by \( N_{gs} \), the number of successful matches is determined by the matching function

\[ M_{gs} (V_{gs}, N_{gs}) = A_S^M V_{gs}^{\alpha_g} N_{gs}^{1-\kappa_g}, \]

(4)

where \( A_S^M > 0 \) is the productivity of the matching function. Firms choose how many vacancies to post taking as given market tightness, \( \theta_{gs} \equiv V_{gs}/N_{gs} \), and prices, \( p_s \). The probability that any given \( gs \) vacancy is filled is \( M_{gs}/V_{gs} = A_S^{M \theta_{gs}^{\alpha_g}-1} \). The probability

---

3We can treat \( p_0 A_Y^{g u} \) as financed, in total or in part, by government unemployment insurance. In this case, we must specify a government budget constraint (which may either be balanced period-by-period or intertemporally in the case of international borrowing) and a tax. If we assume that the government raises revenue using an ad valorem production tax that is common across all goods, then our equilibrium characterization and all of our comparative static results continue to hold; however, in this case we have one additional equation that determines the level of the tax that balances the relevant budget constraint.
that an applicant finds a job is then simply $E_{gs} \equiv M_{gs}/N_{gs} = A^M_{gs} \theta_{gs}$.\footnote{As is well understood in the search-and-matching literature, the Cobb Douglas functional form in (4) does not restrict $E_{gs}$ to satisfy $E_{gs} \in [0,1]$. While we do not impose $E_{gs} \in [0,1]$ as an additional constraint, it should be understood that we could always choose $A^M_{gs}$ such that $E_{gs} \in [0,1]$ in a given equilibrium. Moreover, starting from an equilibrium in which $E_{gs} \in (0,1)$, our comparative static results would then apply for small shocks without imposing a constraint on $E_{gs}$.}

The labor-market clearing condition requires that the measure of agents in $\Omega_s$ must equal the measure who apply for positions across all sectors and non-participation,

$$N_s = \sum_{s=0}^{S} N_{gs}$$

We model the interaction between the firm and worker as being subject to a hold-up problem. After matching, the worker unilaterally chooses hours, then output is produced, and finally the worker and firm bargain over surplus. We assume a generalized Nash bargaining solution where the worker’s weight is $\beta_g$. The firm’s vacancy cost is sunk; hence, it’s outside option at this stage is zero. The worker’s hours choice is also sunk and production has already occurred, so that she cannot avail herself of unemployment or home production; hence, her outside option at this stage is also zero.

**Timing.** Each entrepreneur knows all nominal prices, $p_s$, the price index, $P_g$, and market tightness and chooses the number of vacancies to post in each $gs$ pair. Each agent knows all parameters, including the realization of the $\epsilon_{ws}$ vector, all nominal prices, $p_s$, the price index, $P_g$, and the probability of successfully finding employment in each $s$, $E_{gs}$. At the same time that firms post vacancies, each agent applies to the $s \in \{0,1,\ldots,S\}$ that maximizes her expected utility. Subsequently, each worker either successfully obtains employment in the $s$ to which she applied or becomes unemployed. After the realization of her employment probability, she chooses her work hours and, if she is employed in a sector $s \in \{1,\ldots,S\}$, then bargains with her firm.

### 3 Equilibrium

**Firm choices.** Here we assume that firms post vacancies in all $gs$ pairs, a condition that is satisfied in equilibrium if $A^Y_{gs} > 0$, $A^M_{gs} > 0$, and $N_s > 0$ for all $g$ and $s \in \{1,\ldots,S\}$. Generalized Nash bargaining implies that any agent working in $s \in \{1,\ldots,S\}$ earns a wage that is a share $\beta_g$ of revenue. Given this wage determination, in any $gs$ pair with
positive vacancies posted, the zero-profit condition is given by

$$(1 - \beta_g)p_s Y_{gs}/P_g = F_g V_{gs},$$

where $F_g V_{gs}$ is the total real cost of vacancy posting, $p_s Y_{gs}/P_g$ is total real revenue (where $Y_{gs}$ is output of group $g$ in sector $s$), and firms’ share of this revenue is $1 - \beta_g$. The zero profit condition above, the matching function defined in (4), and the definition of market tightness ($\theta_{gs}$) yield a solution for the nominal value of output per match as a function of market tightness and the price index,

$$\frac{p_s Y_{gs}}{M_{gs}} = \theta_{gs}^{1-a_g} \frac{P_g}{A_g \theta_g} F_g$$

(6)

**Worker choices.** Consider an agent $\omega \in \Omega$. Conditional on working in $s$, she faces a real hourly wage of $A^Y_{gs} \epsilon \omega_s \tilde{p}_s/P_g$, where $\tilde{p}_0 = p_0$ and $\tilde{p}_s = \beta_g p_s$ for all $s \in \{1, \ldots, S\}$. If she works in $s$, she chooses the number of hours to work to maximize utility, equation (1), taking as given productivities and prices, yielding

$$H_{\omega s} = \left(\epsilon \omega_s \zeta_g A^Y_{gs} \tilde{p}_s/P_g\right)^{1/v_g}$$

(7)

As stated above, $1/v_g$ is both the compensated and uncompensated intensive-margin labor supply elasticity.

Denote by $V_{\omega s}$ her (indirect) utility if working in $s$. Equations (1) and (7) imply

$$V_{\omega s} = \frac{v_g}{1 + v_g} \left(\epsilon \omega_s \zeta_g A^Y_{gs} \tilde{p}_s/P_g\right)^{(1 + v_g)/v_g}$$

which is simply her real income times $(1 + v_g)\zeta_g/v_g$. Denoting by $E[V_\omega|s]$ her expected utility conditional on applying to $s$, we obtain

$$E[V_\omega|s] = E_g V_{\omega s} + (1 - E_g) V_{\omega u}$$

Agent $\omega$ chooses to apply to the $s \in \{0, 1, \ldots, S\}$ that maximizes her expected utility, so that she applies to $s$ if and only if $E[V_\omega|s] > \max_{s' \in \{0,1,\ldots,S\}, s' \neq s} E[V_\omega|s']$.\(^5\)

**Worker choices with common market tightness.** Consider an equilibrium in which market tightness for a given group $g$ is common across sectors: $\theta_g = \theta_{gs}$ for all $s \in \{1, \ldots, S\}$.

\(^5\)Given our distributional assumption, for almost all agents there is a unique choice that maximizes expected utility. Hence, we use $>$ rather than $\geq$.  

7
Given our assumption that matching productivity, $A_s^M$, is common across sectors, common market tightness implies $E_s = E_s$ for all $s \in \{1, ..., S\}$. In such an equilibrium, agent $\omega$ prefers one sector $s \in \{1, ..., S\}$ to another sector $s' \in \{1, ..., S\}$ if and only if $V_{\omega s} > V_{\omega s'}$. And she prefers home production to a sector $s \in \{1, ..., S\}$ if and only if \[
1 - \frac{1}{E_g} V_{\omega 0} - \frac{1 - E_g}{E_g} V_{\omega u} > V_{\omega s}.
\]
Hence, she chooses $s$ if and only if 
\[
w_{gs} e_{\omega s} > \max_{s' \neq s} \left\{ w_{gs} e_{\omega s'} \right\}
\]
where we define 
\[
w_{gs} \equiv \begin{cases} A_s^Y \beta_s p_s & \text{if } s \in \{1, ..., S\} \\ \max \left\{ 0, \left( \frac{1}{E_g} \left( A_s^Y \right) \frac{1+\nu_s}{1+\nu_s} + \frac{E_s-1}{E_g} \left( A_s^u \right) \frac{1+\nu_s}{1+\nu_s} \right) \frac{\nu_s}{1+\nu_g} \right\} & \text{if } s = 0 \end{cases}
\]
Here, $w_{gs}$ is the nominal wage per efficiency-unit-hour in each sector. And $w_{g0}$ is the maximum of zero and a CES combination of the nominal wages per efficiency-unit-hour if not participating in the labor force and if unemployed. If this CES combination is negative, then its particular value has no effect on choices; hence, $w_{g0}$ is the maximum of zero and the CES combination.\(^6\)

Given our distributional assumption on the vector $\{e_{\omega s}\}_{s=0}^S$ in (3), the probability that $\omega$ applies to $s \in \{0, 1, ..., S\}$ is given by 
\[
\pi_{gs} = \frac{1}{\Phi_g} \left( \sum_{s' \in S(s)} w_{s's}^{1-\kappa_g} \right)^{-\kappa_g} \frac{\nu_g}{w_{gs}^{1-\kappa_g}}
\]
where $S(s)$ indicates the set of all choices in either non-participation or participation, depending on whether $s$ is non-participation ($s = 0$) or a sector ($s \in \{1, ..., S\}$),
\[
S(s) = \begin{cases} \{1, ..., S\} & \text{if } s \in \{1, ..., S\} \\ \{0\} & \text{if } s = 0 \end{cases}
\]
and where
\[
\Phi_g \equiv \left( w_{g0}^{\nu_g} \right)^{1-\kappa_g} + \left( \sum_{s \in \{1, ..., S\}} w_{gs}^{\nu_g} \right)^{1-\kappa_g}
\]
\(^6\)The weight on the wage in unemployment in this CES combination is negative.
is the inclusive value.

Of course, \((8)\) can be re-expressed as

\[
\pi_{gs} = \frac{\pi_{S(s)g}}{\pi_{gs}}
\]

where

\[
\pi_{S(s)g} = \frac{1}{\Phi_{g}} \left( \sum_{s' \in S(s)} w_{gs'}^{1/\gamma_{gs}} \right)^{1-\kappa_{gs}}
\]

is the probability that agents in group \(g\) choose to participate in the labor force if \(s \in \{1, \ldots, S\}\) or not to participate if \(s = 0\), and where

\[
\pi_{gs} = \frac{w_{gs}^{1/\gamma_{gs}}}{\sum_{s' \in S(s)} w_{gs'}^{1/\gamma_{gs}}}
\]

is the probability \(\omega\) chooses sector \(s\) conditional on participating in the labor force and is one if the agent is not participating in the labor force. According to \((11)\), the partial elasticity of labor supply across sectors conditional on participation is given by \(\iota_{g}/(1 - \kappa_{g})\). That is, if the price of output in sector \(s\) rises by one percent, then—holding fixed the denominator of \(\pi_{S(s)g}\)—the share of agents applying to sector \(s\) out of all agents participating in the labor force would rise by \(\iota_{g}/(1 - \kappa_{g})\) percent. According to \((10)\), the partial extensive-margin labor supply elasticity is \(\iota_{g}\). That is, if the price of output in all sectors rose by one percent, then—holding fixed the denominator of \(\pi_{S(s)g}\)—the share of agents participating in the labor force would rise by \(\iota_{g}\) percent.

As we show in the Appendix, \((2), (7), \) and the distributional assumption on \(\epsilon_{gs}\) yield a solution for the nominal value of output per worker in any sector \(s \in \{1, \ldots, S\}\) that is given by

\[
\frac{p_{s}Y_{gs}}{M_{gs}} = \gamma_{1}^{g} \left( \frac{T_{g}}{P_{g}} \right)^{1/\gamma_{g}} \frac{1+\gamma_{g}}{\Phi_{g}^{1+\gamma_{g}}}
\]

where \(\gamma_{1}\) is a constant. According to \((12)\), in any equilibrium in which market tightness is common across sectors, worker choices imply that the value of output per worker is also common across sectors.

**Equilibrium characterization with common market tightness across sectors.** Combining equation \((12)\)—which was derived from worker optimization under the assumption that \(\theta_{g} = \theta_{gs}\) for all \(s \in \{1, \ldots, S\}\)—with equation \((6)\)—which was derived from firm optimization without imposing the assumption of common market tightness across sectors—we
obtain a sector-specific solution for market tightness
\[
\begin{align*}
\theta_{gs}^{1-\alpha_g} &= \gamma_g \frac{1 - \beta_g A^{M}_{gs} \xi_g}{\beta_g F_g} \xi_g \left( \frac{\Phi^{1/\nu_g}_{g}}{P_g^{1/\nu_g}} \right)^{1+\nu_g} \\
\end{align*}
\] (13)
consistent with firm optimization. The previous expression is common across sectors, satisfying \( \theta_{gs} = \theta_g \).\(^7\)

According to the following proposition, if an equilibrium exists in which market tightness is common across sectors, then the probability of successfully matching given labor force participation (i.e. one minus the unemployment rate), the average wage per hour worked conditional on successful employment in a sector, and hours worked per employee conditional on successful employment in a sector are all common across sectors for group \( g \). Denote these outcomes by \( E_g, W_g, \) and \( H_g, \) respectively. And denote by \( L_g \) the labor force participation rate. Proposition 1, which is proven in the Appendix, summarizes the equilibrium values of these outcomes.

**Proposition 1.** In any equilibrium in which \( \theta_g = \theta_{gs} \) in all sectors, the probability of successfully obtaining employment given participation in the labor force and the average wage per hour worked and hours worked per employee in any sector are given by
\[
K_g = \chi^K_g \left( \frac{\Phi^{1/\nu_g}_{g}}{P_g^{1/\nu_g}} \right)^{\rho^K_g}
\text{ for } K \in \{W, H, E\}
\] (14)
where the \( \chi^K_g \) terms are positive constants, \( \rho^K_g = 1, \rho^K_g = \frac{1}{\nu_g}, \) and \( \rho^K_g = \frac{a_g}{1-\alpha_g} \frac{1+\nu_g}{\nu_g}. \) In such an equilibrium, the labor force participation rate is
\[
L_g = \frac{1}{\Phi_g} \left( \sum_{s=1}^{S} \left( A^{Y}_{gs} \beta_g p_s \right)^{\xi_g} \right)^{1-\kappa_g}
\] (15)

According to Proposition 1, the inclusive value \( \Phi_g, \) the aggregate price index \( P_g, \) and the relevant elasticities \( \rho^K_g, \) are sufficient statistics that fully characterize the impact of sectoral prices on real wages, hours worked per employee within each sector, and the probability of successful employment for those in the labor force. The result that each of these margins of adjustment is an iso-elastic function of \( \Phi_g \) and \( P_g \) follows from our many iso-elastic functional form assumptions: on the matching function in (4), the

---

\(^7\)This condition highlights the role of the assumptions that \( A^{M}, \beta_g, F_g, \) and \( a_g \) do not vary across sectors.
intensive margin of labor supply embedded in (1) and solved for in (7), and both the extensive margin of labor supply as well as labor supply across sectors embedded within (3) and solved for in (10) and (11).\footnote{Labor force participation, (15), is itself an iso-elastic function of sectoral prices, where the inclusive value within labor force participation is itself a sufficient statistic for these sectoral prices.}

Denote by $U_g$ the expectation of welfare for an arbitrary $\omega \in \Omega_g$ before the realization of the vector $\varepsilon_{\omega s}$ and before the realization of employment for those participating in the labor force; this is expected welfare behind the veil of ignorance for an agent in group $g$. We solve for $U_g$ under a particular assumption:

**Assumption 1.** Either (i) the productivity of non-participation is zero, $A^Y_{g0} = 0$, or (ii) the productivity of unemployment is zero, $A^Y_{gu} = 0$.

In case (i), all agents choose to participate in the labor force. In case (ii), the income of the unemployed is zero. In either case, $w_{g0}$ simplifies dramatically and we obtain the following lemma.

**Lemma 1.** If Assumption 1 is satisfied, then in any equilibrium in which $\theta_{gs} = \theta_g$ across all sectors, expected welfare is

$$U_g = \frac{\nu_g}{1 + \nu_g} \zeta_g W_s H_s E_g$$

(16)

Does an equilibrium exist in which market tightness is common across sectors? The solution for $\Phi_g$ in (9) is implicit, since $\Phi_g$ depends on $E_g$ through $w_{g0}$, and $E_g$ itself depends on $\Phi_g$ according to (14). However, in the special case in which $A^Y_{g0} = 0$—case (i) of Assumption 1—we have $w_{g0} = 0$. In this case, all agents choose to participate in the labor force and the solution for $\Phi_g$ in (9) does not depend on $E_g$. The solution for $\Phi_g$ is then uniquely given by

$$\Phi_g = \left( \sum_{s \in \{1, \ldots, S\}} (A^Y_{gs} \beta_s)^{\nu_g} \right)^{1 - \kappa_g}$$

In this case, there exists a unique equilibrium in which market tightness is common across sectors.

**4 Comparative static results**

What are the implications of changes in sectoral prices for each margin of adjustment and for welfare? Since $\Phi_g$, $P_g$, and the (fixed) values of $\rho^W_g$, $\rho^H_g$, and $\rho^E_g$ are sufficient statistics
for the average wage per hour worked \((W_g)\), hours worked per employee in the labor force \((H_g)\), and the probability of successfully obtaining employment if in the labor force \((E_g)\), adjustments in each of these margins are fully characterized by \(d \ln \Phi_g\) and \(d \ln P_g\).

Since we have made no assumptions on the final good aggregator, we do not explicitly solve for the change in the price index facing each group, \(d \ln P_g\), and instead treat this as another primitive. Of course, it should be understood that \(d \ln P_g\) is a simple function of the vector \(d \ln p_s\); it is a weighted average of sectoral price changes with weights given by initial expenditure shares of group \(g\).

Hence, solving for changes in each margin boils down to solving for \(d \ln \Phi_g\). In general, we have

\[
d \ln \Phi_g = \iota (1 - L_g) d \ln w_{g0} + \iota_g L_g \sum_{s=1}^{S} \pi_{gs}^{s(s)} d \ln p_s
\]

In what follows, we impose Assumption 1, in which case \(d \ln w_{g0}\) and, therefore, \(d \ln \Phi_g\), are particularly simple. In this case, we obtain the following comparative static results.

**Proposition 2.** In any equilibrium in which \(\theta_g = \theta_{gs}\) in all sectors, under Assumption 1 the change in the probability of successfully obtaining employment given participation in the labor force and in the average wage per hour worked and hours worked per employee in any sector are given by

\[
d \ln K_g = \frac{\rho^K_g}{1 + (1 - L_g)} \left[ L_g \sum_{s=1}^{S} \pi_{gs}^{s(s)} d \ln p_s - d \ln P_g \right]
\]

for \(K \in \{W, H, E\}\)

and the change in utility is given by

\[
d \ln U_g = \frac{\rho^W_g + \rho^H_g + \rho^E_g}{1 + (1 - L_g)} \left( L_g \sum_{s=1}^{S} \pi_{gs}^{s(s)} d \ln p_s - d \ln P_g \right)
\]

In case (i) of Assumption 1, \(L_g = 1\) so that the previous expressions simplify even further. In case (ii) of Assumption 1, we can additionally solve for changes in the labor force participation rate, \(d \ln L_g\).

Depending on how one defines a group, \(g\), Proposition 2 can be used to solve analytically for changes in each margin of adjustment and welfare at various levels of aggregation: across regions (the dimension focused on by, e.g., Kovak, 2013; Galle et al., 2017; and

...
Adão et al., 2018), across workers with different education levels at the national level (the dimension focused on by, e.g., Lee, 2017 and Burstein et al., 2019), across workers with different education levels within regions (the dimension focused on by, e.g., Dix-Carneiro and Kovak, 2015), across workers with the same education levels across regions, etc...

Indeed, when the underlying elasticities and price indices are common across groups and the labor force participation rate is equated across groups in the initial equilibrium—as in Assumption 2—we obtain particularly stark results.

**Assumption 2.** Consider an equilibrium in which \( \theta_g = \theta_{gs} \) in all sectors and assume: \( \alpha = \alpha_g, \nu = \nu_g, \) and \( \iota = \iota_g \) for all \( g; \) labor force participation rates are equated in the initial equilibrium across \( g; \) and the price index is common across group, \( P = P_g. \)

Proposition 3 provides results on relative impacts of price shocks across groups that extend cannonical results in the trade literature on the effects of price changes on wages for two groups—e.g. the Slolper-Samuelson theorem—to higher dimensional comparisons with arbitrarily many groups (as in Costinot and Vogel, 2010) but also to a broad range of adjustment margins.

**Proposition 3.** Under Assumptions 1 and 2, if \( d \ln p_s \) is increasing in \( s \) and \( \pi_{gs}^S(s) \) first-order stochastic dominates \( \pi_{gs}^S(s) \) for \( s \in \{1, ..., S\} \), then \( d \ln K_{g'} \geq d \ln K_g \) for \( K \in \{W, H, E, U\}. \)

According to Proposition 3, if group \( g' \) is disproportionately employed in high \( s \) sectors compared to group \( g \) (in the sense of first-order stochastic dominance), then a shock that increases the relative prices of high \( s \) sectors must increase the average wage, average hours worked per employee, the employment rate, and utility of group \( g' \) relative to group \( g. \)

Finally, under Assumptions 1 and 2, there is a simple sufficient condition for group \( g' \) to be disproportionately employed in high \( s \) sectors compared to group \( g \) (in the sense of first-order stochastic dominance): \( A_{g's}^Y, A_{gs}^Y \geq A_{g's}^Y, A_{gs}^Y \) for all sectors \( s' \geq s. \)

**Intuition.** Here we provide intuition for our comparative static results, focusing in particular on case (i) of Assumption 1, in which the labor force participation rate is one.\(^{10}\) In this case, equation (18) simplifies to

\[
\frac{d \ln K_g}{d \ln p_s} = \rho_g^s \left[ \sum_{s=1}^{S} \pi_{gs}^S(s) d \ln p_s - d \ln P_g \right] \text{ for } K \in \{W, H, E\}
\]

The wage comparative static result follows from the envelope condition. In response to a small change in sector prices, the average \( g \) wage may change for two reasons. First,

---

\(^{10}\)Intuition in case (ii) is obviously very similar, although expressions are less simple.
each infra-marginal worker who does not switch sectors will experience a change in wage exactly equal to the change in her sector’s price. Second, workers switch across sectors. However, since a switcher is indifferent between sectors and there are no compensating differentials or switching costs, switching has only a second-order effect on wages. Averaging wages across workers within \( g \) yields the weighted average of sector price changes, weighted by the \( g \)-specific income share across sectors in the original equilibrium. This explains both the structure of the relative wage result as well as the fact that the elasticity, \( \rho_g^W \), is exactly equal to one.

Taking these wage effects as given, if the labor supply elasticity is \( 1/\upsilon_g \)—as implied by (1)—then the response of average hours worked across \( g \) workers is simply \( \rho_g^H = 1/\upsilon_g \) times the response of average wages.\(^{11}\)

Finally, consider the response of the frictional unemployment margin. While changes in prices may induce agents to switch sectors, to a first order these switches will not affect a labor group’s employment rate—at given vacancies—since the likelihood of successfully finding employment is equalized across sectors. Hence, changes in employment rates are induced by changes in firm vacancy choices, and all else equal firms post more vacancies when a sector’s price rises. This explains why \( \rho_g^E = \frac{\alpha_g}{1-\alpha_g} \frac{1+\upsilon_g}{\upsilon_g} \) is increasing in the elasticity of matches to vacancies, \( \alpha_g \). The incentive for firms to post vacancies in a given \( gs \) rises with the price of the sector for two reasons. First, a higher price generates more revenue at fixed hours worked. Second, a higher price increases hours worked of each individual worker, which raises revenue further. This explains why \( \rho_g^E \) is also increasing in the labor supply elasticity, \( 1/\upsilon_g \).

## 5 Conclusions

We develop a framework to analyze the impact of trade shocks on a wide range of labor market adjustment margins in economies with a large number of sectors and labor groups. We provide analytic results characterizing equilibria and showing how changes in sectoral prices affect each margin of adjustment as well as welfare for each labor group.

While we impose many strong functional-form restrictions, our framework remains rich. For each group it features separate elasticities of the extensive margin of labor supply, the intensive margin of labor supply, and labor supply across sectors conditional on labor force participation. We characterize equilibrium in which each margin of adjustment as well as welfare is a simple function of the group-specific price index and inclusive

\(^{11}\)Selection into unemployment does not affect the wage or hours margins, since unemployed workers are randomly selected within each \( g \) pair.
value as well as group-specific structural elasticities.

We hope that our framework and results will be useful to guide for future empirical and quantitative investigations of the effect of trade shocks on labor markets.

References


KIM, R. AND J. VOGEL (2020): “Trade and welfare (across local labor markets),” Papers, Mimeo, UCLA.


Appendix
Trade shocks and labor market adjustment
Ryan Kim and Jonathan Vogel

Our results build on a simple derivation of the expectation of $\epsilon^b_{\omega s}$ conditional on choosing option $s$ for an arbitrary constant $b \in (0, \iota_g)$. Under the assumption that $E_{gs} = E_g$ for all sectors $s \in \{1, ..., S\}$, an agent $\omega$ chooses $s$ if and only if

$$w_{gs}\epsilon_{\omega s} > \max_{s' \neq s} \left\{ w_{g{s'}}\epsilon_{\omega s'} \right\}$$

where $w_{gs}$ is defined as in the body of the paper. Denote by

$$\mathbb{E}[\epsilon^b_{\omega s} | s] \equiv \mathbb{E} \left[ \epsilon^b_{\omega s} | w_{gs}\epsilon_{\omega s} > \max_{s' \neq s} \left\{ w_{g{s'}}\epsilon_{\omega s'} \right\} \right]$$

the expectation of $\epsilon^b_{\omega s}$ conditional on choosing option $s$ for an arbitrary value of $b \in (0, \iota_g)$. Given our distributional assumption on the vector $\{\epsilon_{\omega s}\}_{s=0}^S$, we have

$$\mathbb{E}[\epsilon^b_{\omega s} | s] = \Gamma \left( 1 - \frac{b}{\iota_g} \right) \Phi^{b/\iota_g}_{g}w_{gs}^{-b}$$

(20)

where $\Gamma(\cdot)$ is the gamma function.

**Deriving (12).** Combining the production function, (2), and optimal hours worked, (7), yields the value of an individual $\omega$’s output if working $s$,

$$p_{s}y_{\omega s} = p_{s} \left( A_{gs}^{Y_{\omega s}} \right)^{1+\nu_g} \left( \zeta_{g}p_{s}/P_{g} \right)^{1/\nu_g}$$

To calculate the value of output across all agents working in $s$, we must integrate across all such agents taking into account that these agents self select into $s$. It is sufficient to measure the average output across workers who apply to $s$, rather than who work in $s$, since unemployment is randomly allocated across agents applying to $s$ within a given group $g$. Combining the previous expression and (20), we obtain

$$p_{s}\mathbb{E}[y_{\omega s} | s] = \gamma_{g1} \left( \zeta_{g}/P_{g} \right)^{1/\nu_g} \frac{1}{\nu_g} \Phi^{1+\nu_g}_{g}$$

where we have defined $\gamma_{g1} \equiv \Gamma \left( 1 - \frac{1+\nu_g}{\nu_g\iota_g} \right)$, which clarifies the role of our restriction that $\iota_g > (1 + \nu_g)/\nu_g$. Noting that $\mathbb{E}[y_{\omega s} | s] = Y_{gs}/M_{gs}$, we obtain (12).
Deriving (14) in Proposition 1. Combining optimal hours worked, (7), with (20) yields

\[ H_g = \mathbb{E}[H_{\omega s} | s \in \{1, ..., S\}] = \gamma g 2^{1/\omega g} \left( \frac{\Phi_g^{1/\omega g}}{P_g} \right)^{1/\omega g} \]  

(21)

where we have defined \( \gamma g 2 \equiv \Gamma \left( 1 - \frac{1}{\omega g} \right) \). The average real wage per hour worked, \( W_g \), equals total real labor income divided by total hours worked. Total real labor income in \( s \) is the number of workers in \( s \), \( N_g s E_g s \), times nominal revenue per worker in \( s \) given in (12), times labor’s share, \( \beta_g \), divided by the aggregate price index, \( P_g \). Total hours in \( s \) is the number of workers in \( s \), \( N_g s E_g s \), times hours per worker in (21). Hence, we have

\[ W_g = \frac{\gamma g 1}{\gamma g 2} \left( \frac{\Phi_g^{1/\omega g}}{P_g} \right) \]  

(22)

The probability of successful employment, defined as \( E_g = E_g s = A_g^M \theta_g^{\alpha_g} \), can be calculated using (13) as

\[ E_g = A_g^M \left[ \gamma g 1 - \beta_g A_g M \frac{1}{F_g} \right]^{\alpha_g \left( \frac{1}{1-\alpha_g} \right)} \left( \frac{\Phi_g^{1/\omega g}}{P_g} \right)^{1/\omega g} \]  

(23)

In Proposition 1, equation (14) follows from (21), (22), and (23), where the constants, \( \chi^K_g \), are given by

\[ \chi^W_g \equiv \gamma g 1 / \gamma g 2 \]
\[ \chi^H_g \equiv \gamma g 2^{1/\omega g} \]
\[ \chi^E_g \equiv A_g^M \left[ \gamma g 1 - \beta_g A_g M \frac{1}{F_g} \right]^{\alpha_g \left( \frac{1}{1-\alpha_g} \right)} \]

Deriving (15) in Proposition 1. The labor force participation rate is defined as \( L_g \equiv \sum_{s=1}^{S} \pi_g s \), which is the sum across all sectors \( s \in \{1, ..., S\} \) of the probability that a randomly selected agent \( \omega \in \Omega_g \) applies to sector \( s \). Combining this definition with (8) yields (15) in Proposition 1.

Deriving (16) in Lemma 1. Let \( \mathbb{E}[U_{\omega s} | s] \) denote the expected utility of an agent who

\[ \text{This is the average across workers of hourly real wages, weighting workers by their hours worked.} \]
applies to \( s \in \{0, 1, \ldots, S\} \), with expectation taken over the random variables \( \varepsilon_{ws} \) and \( E_{gs} \) (where \( E_{g0} = 1 \) and \( E_{gs} = E_{g} \) for all \( s \in \{1, \ldots, S\} \)).

First assume that \( A_{yu}^{Y} = 0 \). Since utility is zero if unemployed with \( A_{yu}^{Y} = 0 \), this expected utility can be expressed as

\[
E[U_{\omega s}|s] = E_{gs} \frac{v_{g}}{1 + v_{g}} \left( \frac{\zeta_{g} A_{yu}^{Y} p_{s}}{\bar{p}_{s}} \right)^{1+\nu_{g}} E_{\varepsilon_{ws}}^{\nu_{g}} | s \]

If the agent applies to a sector \( s \in \{1, \ldots, S\} \), then \( E[U_{\omega s}|s] \) and (20) yield:

\[
E[U_{\omega s}|s \in \{1, \ldots, S\}] = E_{g} \frac{v_{g}}{1 + v_{g}} \left( \frac{\Phi_{1/1g}^{1/1g}}{P_{g}} \right)^{1+\nu_{g}} w_{gs}^{1+\nu_{g}}
\]

where \( w_{g0} = E_{g} \frac{v_{g}}{1+\nu_{g}} A_{g0}^{Y} p_{0} \) under the assumption that \( A_{yu}^{Y} = 0 \). Hence, we obtain

\[
E[U_{\omega s}|s = 0] = E_{g} \frac{v_{g}}{1 + v_{g}} \left( \frac{\Phi_{1/1g}^{1/1g}}{P_{g}} \right)^{1+\nu_{g}} w_{g0}^{1+\nu_{g}}
\]

Expected utility conditional on \( s \in \{0, 1, \ldots, S\} \) being the optimizing choice is then common across all choices \( s \in \{0, 1, \ldots, S\} \). Substituting in for \( E_{g} \) using (14), we obtain

\[
E[U_{\omega s}|s] = E_{g} \frac{v_{g}}{1 + v_{g}} \left( \frac{\Phi_{1/1g}^{1/1g}}{P_{g}} \right)^{1+\nu_{g}} \frac{1}{1+\nu_{g}}
\]

The previous expression and (14) yield (16).

On the other hand, if \( A_{g0}^{Y} = 0 \), then \( w_{g0} = 0 \) and no agent will ever choose non-participation. In this case, the displayed equation solving for \( E[U_{\omega s}|s \in \{1, \ldots, S\}] \) above applies. Combined with (14), this yields (16).

**Deriving (18) and (19) in Proposition 2.** First, consider the special case in which \( A_{g0}^{Y} = 0 \).
In this case $w_{g0} = 0$, which implies both that the labor force participation rate is one, $L_g = 1$, and $d \ln w_{g0} = 0$. Hence, (17) simplifies to

$$d \ln \Phi_g = t_g \sum_{s=1}^{S} \pi_{gs}^{S(s)} d \ln p_s$$

where $\pi_{gs}^{S(s)}$ is the share of employed workers who are in sector $s$. Hence, (14) yields

$$d \ln K_g = \rho^K_g \left( \sum_{s=1}^{S} \pi_{gs}^{S(s)} d \ln p_s - d \ln P_g \right) \quad \text{for} \quad K \in \{W, H, E\}$$

and (16) yields

$$d \ln U_g = \left( \rho^W_g + \rho^H_g + \rho^E_g \right) \left( \sum_{s=1}^{S} \pi_{gs}^{S(s)} d \ln p_s - d \ln P_g \right)$$

both of which are implied by (18) and (19) when $L_g = 1$.

Second, consider the special case in which $A_{gu}^Y = 0$. In this case,

$$w_{g0} = E_{g}^{-\frac{\alpha_g}{1-\alpha_g}} A_{g0}^Y p_0$$

The previous expression and (14) yield

$$t_g d \ln w_{g0} = -\frac{\alpha_g}{1-\alpha_g} d \ln \Phi_g + t \frac{\alpha_g}{1-\alpha_g} d \ln P_g$$

The previous expression and (17) yield

$$d \ln \Phi_g = \frac{t_g}{1 + (1 - L_g) \frac{\alpha_g}{1-\alpha_g}} \left( (1 - L_g) \frac{\alpha_g}{1-\alpha_g} d \ln P_g + L_g \sum_{s=1}^{S} \pi_{gs}^{S(s)} d \ln p_s \right)$$

The previous expression and (14) yield (18) and the previous expression and (16) yield (19).

**Proving Proposition 3.** Here we prove the result for utility. The proofs for the three margins of adjustment are identical. Under Assumptions 1 and 2, (19) holds and simplifies to

$$d \ln U_g = \frac{\rho^W_g + \rho^H_g + \rho^E_g}{1 + (1 - L) \frac{\alpha}{1-a}} \left( L \sum_{s=1}^{S} \pi_{gs}^{S(s)} d \ln p_s - d \ln P \right)$$
The previous expression implies

\[ d \ln U_{g'} - d \ln U_g = c \sum_{s=1}^{\infty} \left( \pi_{g's}^{S(s)} - \pi_{gs}^{S(s)} \right) d \ln p_s \]

where \( c \equiv \left( \frac{W + H + E}{1 + (1-L)S} \right) > 0 \). The previous expression, \( d \ln p_s \) increasing in \( s \), and \( \pi_{g's}^{S(s)} \) first-order stochastic dominates \( \pi_{gs}^{S(s)} \) for \( s \in \{1, ..., S\} \) directly imply \( d \ln U_{g'} > d \ln U_g \).