Horizontal and Vertical Differentiation

The point of the extension provided in Section IV was to show that the key results obtained in Section III all survive in a more general model in which differentiation is not only horizontal—as in Section III—but also vertical. This is true. But in the extended model, theses results require an additional parametric restriction not explained in the paper. In this Note I provide the required additional parametric restriction on consumer preferences in the extended version of the model presented in Section IV.

In Section IV, I assumed that a consumer located at point z buying a unit of output from firm i derives utility

$$u\left(z,i\right)=v-1+q_i^{\gamma}-p_i-tD\left(z,i\right),$$

where q_i is the quality of firm i's output and $\gamma\in[0,1)$. Throughout the section I imposed no additional restrictions on γ.

However, γ must be sufficiently close to zero. That is, the equilibrium presented in Proposition 3 exists (and is unique with $\tau>0$, as stated in Proposition 4) if γ is sufficiently close to zero. On the other hand, if γ is sufficiently large, then some firm may have an incentive to increase its quality by a discreet amount and undercut at least one of its neighbors.

The proof of Proposition 3, in Appendix A, extended the proof of Proposition 1 to show that *if no firm is undercut*, then the quality of each firm is as prescribed in Proposition 3. I never showed, however, that for any $\gamma\in[0,1)$, if all firms $j\neq i$ follow the equilibrium strategies prescribed by Proposition 3, then firm i has no incentive to deviate and undercut at least one of its competitors. If γ is sufficiently close to zero, nevertheless, the strategies prescribed in Proposition 3 would constitute an equilibrium and the uniqueness result of Proposition 4 would hold. This follows from a continuity argument.

Propositions 3 and 4 should include this parametric restriction on γ.

The beginning of Proposition 3 should be stated as follows (with bold to mark changes):

Proposition 3. For any set of parameters $\theta\equiv(n,t,\tau,L)$, $k\geq 0$, and $c>0$ there exists a $\mathbf{\Gamma>0}$, $\phi>0$, and $\varphi>0$ such that if $k_i\times c_i\in[k,k+\phi]\times[c,c+\varphi]$ for all i **and if** $\boldsymbol{\gamma}<\Gamma$, then there is a non-empty set $O^{*\prime}\subseteq\Omega^{n\prime}$ such that any $\boldsymbol{\omega}\in O^{*\prime}$ is a SPNE. $O^{*\prime}$ has the following properties...

Proposition 4 should be stated as follows (with bold to mark changes):

Proposition 4. Suppose $\tau>0$. For any set of parameters $\theta\equiv(n,t,\tau,L)$, $k\geq 0$, and $c>0$ there exists a $\mathbf{\Gamma>0}$, $\phi>0$, and $\varphi>0$ such that if $k_i\times c_i\in[k,k+\phi]\times[c,c+\varphi]$ for all i **and if** $\boldsymbol{\gamma<\Gamma}$, then $\boldsymbol{\omega}$ is a strict SPNE if and only if $\boldsymbol{\omega}\in O^{*\prime}$.

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