Spatial Price Discrimination with Heterogeneous Firms

Jonathan Vogel

Columbia and NBER

August 2009

Introduction

Motivation

- Theoretical economists tend to *hold fixed* or *abstract from* product characteristics/firm location
- We know product locations in product characteristics space and firm locations in geography play central role determining price elasticities and therefore
  - economic outcomes
  - responses to policy changes

Introduction

Motivation

- Theoretical economists tend to *hold fixed* or *abstract from* product characteristics/firm location
- We know product locations in product characteristics space and firm locations in geography play central role determining price elasticities and therefore
    - economic outcomes
    - responses to policy changes
- Could argue we abstract from many aspects of reality
- But we know from demand system estimation that product characteristics not of second order

Introduction

Contribution 1

What determines the pattern of location (& firm entry, market share, and profit) in an environment in which heterogeneous firms have the ability to spatially price discriminate?

Introduction

Contribution 1

What determines the pattern of location (& firm entry, market share, and profit) in an environment in which heterogeneous firms have the ability to spatially price discriminate?

- Spatial price discrimination is prevalent:
  - Geographic space: if producer delivers the good or service, e.g. ready-mixed concrete, janitorial services, ...

Introduction

Contribution 1

What determines the pattern of location (& firm entry, market share, and profit) in an environment in which heterogeneous firms have the ability to spatially price discriminate?

- Spatial price discrimination is prevalent:
    - Geographic space: if producer delivers the good or service, e.g. ready-mixed concrete, janitorial services, ...
    - Product characteristic space: if producer tailors the good to the buyer's specs, e.g. differentiated intermediate inputs

Introduction

Contribution 1

What determines the pattern of location (& firm entry, market share, and profit) in an environment in which heterogeneous firms have the ability to spatially price discriminate?

- Spatial price discrimination is prevalent:
  - Geographic space: if producer delivers the good or service, e.g. ready-mixed concrete, janitorial services, ...
  - Product characteristic space: if producer tailors the good to the buyer's specs, e.g. differentiated intermediate inputs
- Not the first to consider spatial price discrimination
  - See e.g. Hoover (1937), Lederer and Hurter (1986), Hamilton, Thisse, and Weskamp (1989), Hamilton, MacLeod, and Thisse (1991), and MacLeod, Norman, and Thisse (1992)
  - These tend to focus on existence

Introduction

Contribution 2

Improves upon empirical content of spatial competition literature in several dimensions

1. Does not impose restrictions on distribution of marginal costs across firms
   - Four-digit SIC industries reviewed in Bartelsman and Doms (2000) have $85^{th} - 15^{th}$ TFP ratios in the range of $2:1$ to $4:1$

Introduction

Contribution 2

Improves upon empirical content of spatial competition literature in several dimensions

1. Does not impose restrictions on distribution of marginal costs across firms
   - Four-digit SIC industries reviewed in Bartelsman and Doms (2000) have $85^{th} - 15^{th}$ TFP ratios in the range of $2:1$ to $4:1$
2. Does not impose restriction on allocation of shipping/customization costs between firms and customers
   - If firms incur costs, consumers can arbitrage away cost differences

Introduction

Contribution 2

Improves upon empirical content of spatial competition literature in several dimensions

1. Does not impose restrictions on distribution of marginal costs across firms
   - Four-digit SIC industries reviewed in Bartelsman and Doms (2000) have $85^{th} - 15^{th}$ TFP ratios in the range of $2:1$ to $4:1$
2. Does not impose restriction on allocation of shipping/customization costs between firms and customers
   - If firms incur costs, consumers can arbitrage away cost differences
3. Includes an entry stage to account for both
   - Within-market location; see e.g. Vogel (2008)
   - Between-market entry and exit decisions; see e.g. Syverson (2004), Jia (2008), and Melitz Ottaviano (2008)

Introduction

Contribution 3

Provides tractable model of location that confirms & extends results in different framework (Vogel 2008)

Introduction

Contribution 3

Provides tractable model of location that confirms & extends results in different framework (Vogel 2008)

- Within a mkt, more productive firms are more isolated, all else equal

Introduction

Contribution 3

Provides tractable model of location that confirms & extends results in different framework (Vogel 2008)

- Within a mkt, more productive firms are more isolated, all else equal
- Firm i equilibrium outcomes depend on firm j's characteristics *only* through a market-level parameter

Introduction

Other related literature

- Very little work investigating how *heterogeneous* firms choose their locations in geographic or product characteristics space *within* a market
  - For symmetric firms, see e.g. Hotelling (1929), d'Aspremont, Gabszewicz, and Thisse (1979), and Lancaster (1979)
  - For heterogeneous firms, see e.g. Vogel (2008)
- A large literature considers entry and exit, abstracting from within market locations, building on Bresnahan and Ries (1990, 1991) and Berry (1992)

Introduction

Important simplifying abstractions

- Single dimensional space
- Uniform demand density w/in a market
- Static game

Setup

Consumers

- A mass 1 of strategic consumers uniformly distributed along a unit circumference

- Consumers buy one unit of a homogeneous good from the lowest price source (reservation value, $v > 0$)

Setup

Firms

- A set N containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$

Setup
Firms

- A set N containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$
- Firms play a three-stage game of complete information
  - Entry stage: to enter incur cost $f > 0$, $f \to 0$
    - $f \to 0$ important for uniqueness result
    - can obtain uniqueness without $f \to 0$ under assumption of ordered & sequential entry
  - Location stage: simultaneously choose locations
  - Price stage: simultaneously choose price schedule, $p_i(z)$ for each location z

Setup

Firms

- A set N containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$
- Firms play a three-stage game of complete information
  - Entry stage: to enter incur cost $f > 0$, $f \to 0$
    - ★ $f \to 0$ important for uniqueness result
    - ★ can obtain uniqueness without $f \to 0$ under assumption of ordered & sequential entry
  - Location stage: simultaneously choose locations
  - Price stage: simultaneously choose price schedule, $p_i(z)$ for each location z
- Firm i located at point η_i selling to point z incurs a *delivered marginal cost* $k_i(\eta_i, z) \equiv c_i + t\|\eta_i - z\|$
  - Throughout talk "transport/customization" cost allocated to firm
  - This is WLOG

Setup

Equilibrium concept

- Focus on (weakly) undominated pure strategy subgame perfect Nash Equilibria: "equilibria"

- Define "equilibrium characterization" as $\{K, \mathbf{x}, \boldsymbol{\pi}\}$
  - $K \subseteq N$ the set of firms that enter the market
  - $\mathbf{x} \in \mathbb{R}^K$ the vector of market shares of the entrants
  - $\boldsymbol{\pi} \in \mathbb{R}^K$ the vector of variable profits of the entrants

- In talk focus on case in which $K > 1$, but allow $K = 1$ in paper

Price stage

Prices solved; equilibrium concept and strategic consumers explained

- Fix # of firms $n \geq 2$, marginal costs $\mathbf{c}$, and locations $\boldsymbol{\eta}$

Price stage

Prices solved; equilibrium concept and strategic consumers explained

- Fix # of firms $n \geq 2$, marginal costs $\mathbf{c}$, and locations $\boldsymbol{\eta}$
- Bertrand comp, identical goods, heterogeneous costs, and a continuum of prices $\Longrightarrow$ two standard technical issues
  1. No pure strategy eqm if consumers not strategic for generic tie breaking rule ($\therefore$ assume strategic consumers)
  2. Continuum of equilibria in which some firm(s) uses weakly dominated strategy ($\therefore$ focus on undominated eqm)

Price stage

Prices solved; equilibrium concept and strategic consumers explained

- Fix # of firms $n \geq 2$, marginal costs $\mathbf{c}$, and locations $\boldsymbol{\eta}$
- Bertrand comp, identical goods, heterogeneous costs, and a continuum of prices $\Longrightarrow$ two standard technical issues
  1. No pure strategy eqm if consumers not strategic for generic tie breaking rule ($\therefore$ assume strategic consumers)
  2. Continuum of equilibria in which some firm(s) uses weakly dominated strategy ($\therefore$ focus on undominated eqm)
- The unique equilibrium price at point z, denoted $p(z)$, is

$$
\begin{aligned}
p(z) &= \min_{i \neq \chi(z)} k_i(\eta_i, z) \\
\text{with } \chi(z) &\equiv \arg \min_{j=1,\dots,n} k_j\left(\eta_j, z\right)
\end{aligned}
$$

Price stage

Prices solved; equilibrium concept and strategic consumers explained

- Fix # of firms $n \geq 2$, marginal costs $\mathbf{c}$, and locations $\boldsymbol{\eta}$
- Bertrand comp, identical goods, heterogeneous costs, and a continuum of prices $\Longrightarrow$ two standard technical issues
    1. No pure strategy eqm if consumers not strategic for generic tie breaking rule ($\therefore$ assume strategic consumers)
    2. Continuum of equilibria in which some firm(s) uses weakly dominated strategy ($\therefore$ focus on undominated eqm)
- The unique equilibrium price at point z, denoted $p(z)$, is

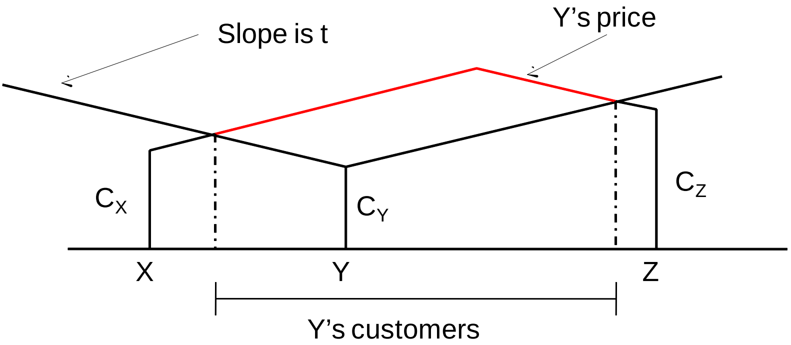
$$p(z) = \min_{i \neq \chi(z)} k_i(\eta_i, z)$$

$$\text{with } \chi(z) \equiv \arg\min_{j=1,\ldots,n} k_j\left(\eta_j, z\right)$$

Given locations, can then solve for mkt shares and profits

Price stage

Graphical representation of prices and sales



Location stage

Notation

- Define $\lambda(K) \equiv \frac{1}{|K|} + \frac{2}{t}\overline{c}(K)$
  - K is a set of $|K|$ firms with average marginal cost $\overline{c}(K)$
  - In eqm $\lambda(K)$ serves as an inverse measure of market toughness (if all firms in K supply positive measure of consumers)

Location stage

Notation

- Define $\lambda(K) \equiv \frac{1}{|K|} + \frac{2}{t}\overline{c}(K)$
  - K is a set of $|K|$ firms with average marginal cost $\overline{c}(K)$
  - In eqm $\lambda(K)$ serves as an inverse measure of market toughness (if all firms in K supply positive measure of consumers)

- Firm n satisfies Condition C relative to K if

$$c_n < \frac{t}{2}\lambda(K)$$

i.e. if

$$c_n - \overline{c}(K) < \frac{t}{2}\frac{1}{|K|}$$

Location stage

Outline of what's to come...

1. Solve for unique equilibrium charaterization **if** all firms $n \in K$ satisfy Condition C relative to K
   - Each firm earns positive var profit

Location stage

Outline of what's to come...

1. Solve for unique equilibrium charaterization **if** all firms $n \in K$ satisfy Condition C relative to K
   - Each firm earns positive var profit

2. Prove that an equilibrium to the subgame exists if **not** all firms $n \in K$ satisfy Condition C relative to K
   - Steps 1 and 2 $\Rightarrow$ an eqm exists to any location-stage subgame

Location stage

Outline of what's to come...

1. Solve for unique equilibrium charaterization **if** all firms $n \in K$ satisfy Condition C relative to K
   - Each firm earns positive var profit
2. Prove that an equilibrium to the subgame exists if **not** all firms $n \in K$ satisfy Condition C relative to K
   - Steps 1 and 2 $\Rightarrow$ an eqm exists to any location-stage subgame
3. If at least one firm does not satisfy Condition C relative to K, then at least one firm earns zero variable profit
   - Steps 1 and 3 and $f > 0 \Rightarrow$ In any eqm to full game, each entrant must satisfy Condition C relative to set of entrants

Location stage

Step 1: Lemma 1

Suppose the set of firms in the location stage is K, $|K| \geq 2$, and each $n \in K$ satisfies Condition C relative to K

Then $\exists$ an eqm to the location-stage subgame. In any such eqm, the distance btw any two neighbors i and $i+1$ is

$$d_{i,i+1}(K) = \lambda(K) - \frac{2}{t}\left(\frac{c_i + c_{i+1}}{2}\right),$$

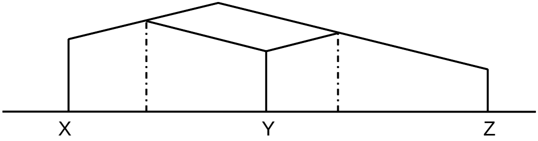
and firm i's mkt share and variable profit are

$$\begin{aligned} x_i(K) &= \lambda(K) - \frac{2}{t}c_i \\ \pi_i(K) &= \frac{t}{2}x_i(K)^2 \end{aligned}$$

Location stage

Step 1: Key property - firms "centered in mkt share"

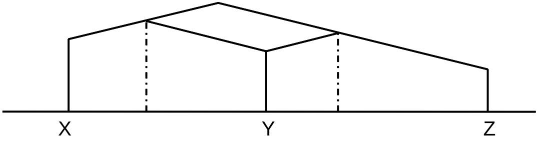
Consider a non-eqm firm Y location btw X and Z:



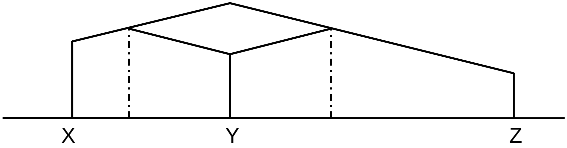
Location stage

Step 1: Key property - firms "centered in mkt share"

Consider a non-eqm firm Y location btw X and Z:



By moving left, Y (i) maintains mkt share, (ii) reduces cost, and (iii) increases revenue



Location stage

Step 1: Implications of "centered" result

Firms "centered in their market shares" yields two key results

1. Firm i's equilibrium outcomes depend on firm j's marginal cost only through j's impact on λ
   - delivered marg cost at boundary customers determines prices, mkt shares, and therefore var profits

Location stage

Step 1: Implications of "centered" result

Firms "centered in their market shares" yields two key results

1. Firm i's equilibrium outcomes depend on firm j's marginal cost only through j's impact on λ
   - delivered marg cost at boundary customers determines prices, mkt shares, and therefore var profits
2. In eqm, there is a unique x_i and π_i for all i

Location stage

Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2}\lambda\left(K\right)$?

Location stage

Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2}\lambda(K)$?

- Suppose that all firms locate as prescribed by Lemma 1 & define $g(z) \equiv \min_{j \in K}\{k_j(z)\}$

Location stage

Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2}\lambda(K)$?

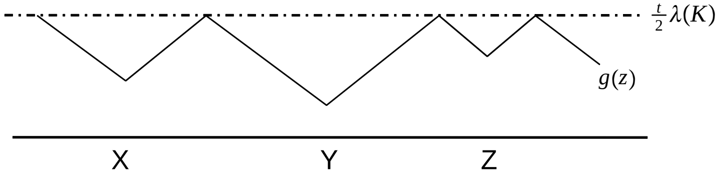
- Suppose that all firms locate as prescribed by Lemma 1 & define $g(z) \equiv \min_{j \in K} \{k_j(z)\}$
- $g(z) = \frac{t}{2}\lambda(K)$ if z is a boundary consumer (b/c firms are "centered")

Location stage

Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2}\lambda(K)$?

- Suppose that all firms locate as prescribed by Lemma 1 & define $g(z) \equiv \min_{j \in K} \{k_j(z)\}$
- $g(z) = \frac{t}{2}\lambda(K)$ if z is a boundary consumer (b/c firms are "centered")
  - $\implies g(z) \leq \frac{t}{2}\lambda(K)$ for all z

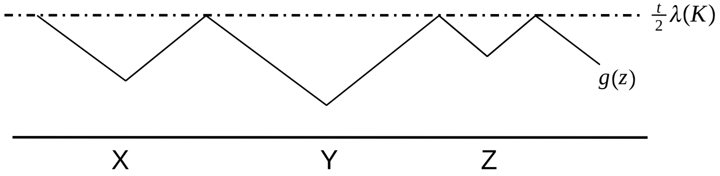


Location stage

Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2}\lambda(K)$?

- Suppose that all firms locate as prescribed by Lemma 1 & define $g(z) \equiv \min_{j \in K} \{k_j(z)\}$
- $g(z) = \frac{t}{2}\lambda(K)$ if z is a boundary consumer (b/c firms are "centered")
    - $\implies g(z) \leq \frac{t}{2}\lambda(K)$ for all z



- If j violates Condition C, then $c_j \geq \frac{t}{2}\lambda(K) \geq g(z)$

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^{*}(K) \equiv \left\{ i \in K \middle| \, c_i < \frac{t}{2}\lambda \left[K^{*}(K) \right] \right\}$$

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^*(K) \equiv \left\{ i \in K \middle| c_i < \frac{t}{2}\lambda\left[K^*(K)\right] \right\}$$

There exists an eqm in which:

- All $i \in K^*(K)$ locate as Prescribed by Lemma 1 if only $K^*(K)$ were in the mkt

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^*(K) \equiv \left\{ i \in K \middle| c_i < \frac{t}{2}\lambda\left[K^*(K)\right] \right\}$$

There exists an eqm in which:

- All $i \in K^*(K)$ locate as Prescribed by Lemma 1 if only $K^*(K)$ were in the mkt
- All $j \in K \backslash K^*(K)$ locate on top of a boundary consumer

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^*(K) \equiv \left\{ i \in K |\, c_i < \frac{t}{2}\lambda\left[K^*(K)\right] \right\}$$

There exists an eqm in which:

- All $i \in K^*(K)$ locate as Prescribed by Lemma 1 if only $K^*(K)$ were in the mkt
- All $j \in K \backslash K^*(K)$ locate on top of a boundary consumer
  - $g(z)$ unaffected by any $j \in K \backslash K^*(K)$

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^*(K) \equiv \left\{ i \in K \middle| \, c_i < \frac{t}{2}\lambda\left[K^*(K)\right] \right\}$$

There exists an eqm in which:

- All $i \in K^*(K)$ locate as Prescribed by Lemma 1 if only $K^*(K)$ were in the mkt
- All $j \in K \backslash K^*(K)$ locate on top of a boundary consumer
  - $g(z)$ unaffected by any $j \in K \backslash K^*(K) \Rightarrow$ no $i \in K^*(K)$ has an incentive to deviate

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^{*}(K) \equiv \left\{ i \in K \middle| \, c_i < \frac{t}{2}\lambda\left[K^{*}(K)\right] \right\}$$

There exists an eqm in which:

- All $i \in K^{*}(K)$ locate as Prescribed by Lemma 1 if only $K^{*}(K)$ were in the mkt
- All $j \in K \backslash K^{*}(K)$ locate on top of a boundary consumer
  - $g(z)$ unaffected by any $j \in K \backslash K^{*}(K) \Rightarrow$ no $i \in K^{*}(K)$ has an incentive to deviate
  - $c_j \geq g(z)$ for all z and all $j \in K \backslash K^{*}(K)$

Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. j does not satisfy Condition C relative to K: Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^*(K) \equiv \left\{ i \in K | \, c_i < \frac{t}{2} \lambda \left[K^*(K) \right] \right\}$$

There exists an eqm in which:

- All $i \in K^*(K)$ locate as Prescribed by Lemma 1 if only $K^*(K)$ were in the mkt
- All $j \in K \backslash K^*(K)$ locate on top of a boundary consumer
  - $g(z)$ unaffected by any $j \in K \backslash K^*(K) \Rightarrow$ no $i \in K^*(K)$ has an incentive to deviate
  - $c_j \geq g(z)$ for all z and all $j \in K \backslash K^*(K) \Rightarrow$ no $j \in K \backslash K^*(K)$ has an incentive to deviate

Location stage

Step 3: If Condition C violated, then at least one firm has zero mkt share

If $K \backslash K^* (K)$ not empty, then in any eqm $\exists$ at least one firm that supplies a mass zero of consumers

Location stage

Step 3: If Condition C violated, then at least one firm has zero mkt share

If $K \backslash K^* (K)$ not empty, then in any eqm $\exists$ at least one firm that supplies a mass zero of consumers

- If all firms have $x_i > 0$, then in any eqm distances, mkt shares, and variable profits given by Lemma 1
  - $\Rightarrow x_i (K) = \lambda (K) - \frac{2}{t} c_i$

Location stage

Step 3: If Condition C violated, then at least one firm has zero mkt share

If $K \backslash K^* (K)$ not empty, then in any eqm $\exists$ at least one firm that supplies a mass zero of consumers

- If all firms have $x_i > 0$, then in any eqm distances, mkt shares, and variable profits given by Lemma 1
  - $\Rightarrow x_i (K) = \lambda (K) - \frac{2}{t} c_i$
- If $K \backslash K^* (K)$ not empty, then $\exists i$ s.t. $x_i (K) \leq 0$, contradiction

Entry Stage

Starting from the full set of pot. entrants N, there is a unique set $K^*(N)$ s.t.

- if $K^*(N)$ enter, then $\pi_i[K^*(N)] > 0$ for all $i \in K(N)$

Entry Stage

Starting from the full set of pot. entrants N, there is a unique set $K^*(N)$ s.t.

- if $K^*(N)$ enter, then $\pi_i[K^*(N)] > 0$ for all $i \in K(N)$
- if any different set of firms, V, enters, either
    1. $\exists j \in V$ s.t. $\pi_j(V) = 0$

Entry Stage

Starting from the full set of pot. entrants N, there is a unique set $K^*(N)$ s.t.

- if $K^*(N)$ enter, then $\pi_i[K^*(N)] > 0$ for all $i \in K(N)$
- if any different set of firms, V, enters, either
    1. $\exists j \in V$ s.t. $\pi_j(V) = 0$
    2. $\exists j \notin V$ s.t. $\pi_j(V \cup j) > 0$

Entry Stage

Starting from the full set of pot. entrants N, there is a unique set $K^*(N)$ s.t.

- if $K^*(N)$ enter, then $\pi_i[K^*(N)] > 0$ for all $i \in K(N)$
- if any different set of firms, V, enters, either
  1. $\exists j \in V$ s.t. $\pi_j(V) = 0$
  2. $\exists j \notin V$ s.t. $\pi_j(V \cup j) > 0$

There exists an $f^*(N) > 0$ such that for all $f < f^*(N)$

- **an equilibrium exists, and**
- **the unique equilibrium characterization is given by $\{K^*(N), \mathbf{x}, \boldsymbol{\pi}\}$, where $d_{i,i+1}$, π_i, and x_i are as given in Lemma 1 w/ $K = K^*(N)$.**

Conclusions

- Provided a tractable model of endogenous product differentiation
  - amenable to extensions, including elastic demand
- Extended spatial competition to better accord with empirical regularities
- Confirmed previous results obtained in a different framework
  - differences in productivity are reflected in location decisions through isolation