

**Online Appendix for**

**“The Race Between Education, Technology, and the Minimum Wage”**

Jonathan Vogel

<b>A Empirical Appendix</b>	<b>A.1</b>
A.1 National time-series analysis . . . . .	A.1
A.2 State, labor group, and time analysis . . . . .	A.7
<b>B Theoretical appendix</b>	<b>A.14</b>
B.1 Details of the steady-state analysis . . . . .	A.14
B.2 Proof of Proposition 2 . . . . .	A.17
B.3 Wage-posting model . . . . .	A.20

# Appendix A Empirical Appendix

## A.1 National time-series analysis

**Basic processing of the March CPS data.** I use the March Annual Demographic Files of the Current Population Survey from 1964 to 2017, which report earnings from 1963 to 2016. Thus, throughout when I refer to any year, I am using data from the following year’s March CPS.

I restrict my sample to workers age 16 to 64 during the earnings year. I drop respondents with missing schooling, missing or negative earnings, missing weeks worked, or with a negative weight. I additionally drop those who are self employed or engage in unpaid family work and anyone with allocated earnings. Finally, I drop respondents who are part of the 3/8 redesign in the 2014 ASEC sample. Following Autor, Katz and Kearney (2008) I multiply top-coded earnings by 1.5.

In composition adjusting, I bin workers into one of 100 groups, denoted by  $g$ , defined by the interaction of 5 age bins (16-25, 26-35, 36-45, 46-55, and 56-64), 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training).<sup>1</sup> The lowest three educations—high school dropouts, those with a high school degree, and those with some college—are allocated to non-college; the highest two educations—college graduates and graduate training—are allocated to college.

**Constructing Wages and Supplies.** In each year  $t$  and for each of the 100 groups  $g$  I construct total hours worked and total wage and salary income (using sample weights) and, from this, the average wage of each group-year pair,  $w_{gt}$  for group  $g$ . Within each year I average across the logarithm of the wages of all groups with at least a college degree (denoting the set of these groups by  $\mathcal{G}_h$ ) and, separately, across all groups without (denoting the set of these groups by  $\mathcal{G}_\ell$ ) using time-invariant weights. For instance, for college-educated workers, I have

$$\log w_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g \log w_{gt}$$

where  $\omega_g$  is the time-invariant weight applied to group  $g$ . These weights are constructed using the average across years of the share of hours worked of each group  $g$  within the set

---

<sup>1</sup>Up to and including 1991, I use the highest grade of school completed; I define college complete as having finished the fourth year of college and graduate training as having more than four years of college. Starting in 1992 I use degree completion, assigning associate’s degrees to some college.

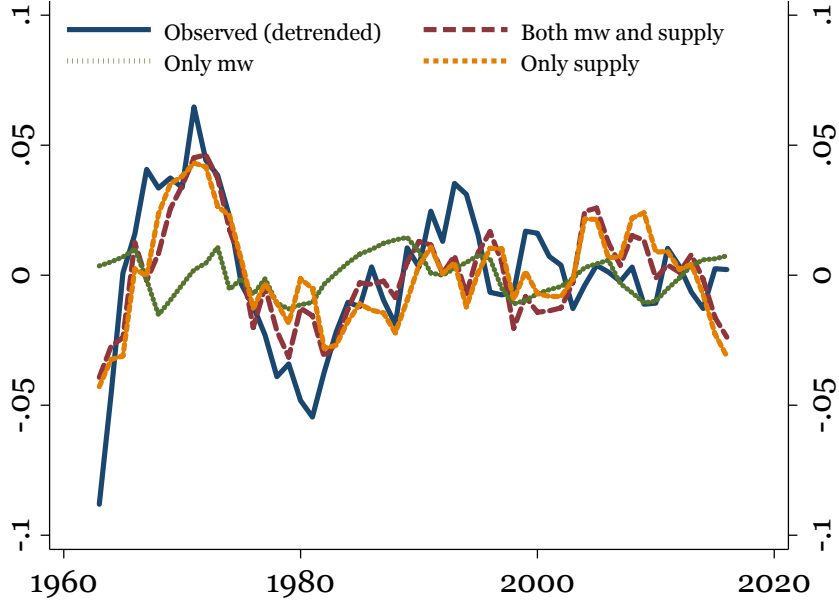


Figure A.I: Detrended National College Premium and the Predicted Impact of the Minimum Wage and Supply

Notes: “Observed” is the observed national college wage premium detrended using a cubic polynomial of time. To construct the remaining plots, I regress the detrended college premium on a constant, detrended supply of college workers, and detrended real minimum wage (each using a cubic polynomial). “Both mw and supply” plots the predicted value of the detrended college premium. “Only mw” and “Only supply” plot the predictions of the detrended college premium using the estimated minimum wage (omitting supply) or estimated supply (omitting the minimum wage) coefficient.

of college groups  $\mathcal{G}_h$  and, separately, within the set of non-college groups  $\mathcal{G}_\ell$ .<sup>2</sup> The resulting averages are the composition-adjusted wages  $\log w_{ht}$  and  $\log w_{\ell t}$  used in the analysis.

I construct three measures of supply of college and non-college workers. First, I measure supply using a composition-adjusted measure of changes in efficiency-unit hours worked. The hours-based measure is constructed as follows. In the first step, for each year and college group ( $\mathcal{G}_h$  and  $\mathcal{G}_\ell$ ), I construct a composition-adjusted weighted average wage, where the fixed-over-time weights are identical to those used in the construction of composition-adjusted wages,  $\omega_g$ , as follows (I focus on  $h$  in what follows):  $\tilde{w}_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g w_{gt}$ . In the second step, I then divide the average wage of each labor group in that year-college pair by the average across all labor groups (in the corresponding year-college pair) created in the first step:  $\tilde{w}_{gt} = w_{gt} / \tilde{w}_{ht}$  for  $g \in \mathcal{G}_h$ . This provides a year-specific measure of the relative wage of each group within the college-educated and, separately, within the non-college-educated. In the third step, I take an average across years of this

<sup>2</sup>I drop any group  $g$  that has no observations in at least one year. Keeping such a group yields a composition-adjusted wage (for the corresponding education group of either college or non-college workers) in which the total weights sum to less than one in any year in which that group is not observed.

relative wage within each labor group:  $\tilde{w}_g = (1/T) \sum_t \tilde{w}_{gt}$ . This average across years is a measure of the average efficiency units supplied by each hour of labor of this labor group relative to the average labor group in the same college group. In the fourth step, I take a weighted average across all labor groups in the year-college pair of the product of observed hours worked ( $hours_{gt}$ ) and the time-invariant efficiency units supplied by each hour of labor, using the same weights as in the construction of composition-adjusted wages:  $\sum_{g \in \mathcal{G}_h} hours_{gt} \tilde{w}_g \omega_g$ .<sup>3</sup> Finally, I take the logarithm of this composition-adjusted weighted average of efficiency-unit hours worked. I refer to this as the hours-based measure of supply.

Second, I measure supply using a composition-adjusted measure of changes in efficiency-unit populations. Here, I follow the same first three steps as in the construction of the hours-worked measure of supply. In the fourth step, I take a weighted average across all labor groups in the year-college pair of the product of populations (rather than hours worked) and the time-invariant efficiency units supplied by each hour of labor, using the same weights as in the construction of composition-adjusted wages. The measure of the population of each labor group in each year is constructed before dropping respondents with allocated, missing, or negative earnings and before dropping respondents who are self employed or engage in unpaid family work. I refer to this as the population-based measure of supply.

My final measure of supply, used only in Table A.II, is the dual of the composition-adjusted wage measures. In particular, I set  $\log Supply_{ht} = \log Inc_{ht} - \log w_{ht}$  where  $Inc_{ht}$  is the total income of college-educated workers in raw (weighted) data in year  $t$ . I similarly construct  $\log Supply_{\ell t}$ .

**Constructing real minimum wages.** I construct annual real minimum wages  $m_t$  as follows. In 1974-2016, I use data from Cengiz et al. (2019) to measure state-year minimum wages,  $\tilde{m}_{rt}$ . For each state and quarter, Cengiz et al. (2019) report the minimum, mean, and maximum effective nominal minimum wage (the maximum of the legislated state and federal minimum wages). For each state and year, I take the average across quarters within the year of the average within-quarter minimum wage.

For the years 1963-1973 I obtain the federal (FLSA) minimum wage from the Department of Labor and state minimum wages from FRED. For each state, I use the maximum of the state's minimum wage and the federal minimum wage to measure  $\tilde{m}_{rt}$ .

To construct the national time series of real minimum wages, I additionally take the following steps. After constructing the state-year series of annual nominal minimum wages

---

<sup>3</sup>This variable's units are irrelevant, since I take its logarithm and include a constant in the regression.

I average across states in each year using fixed-across-time weights. These weights are defined as follows. For the years in which the March CPS contains 51 'states' (including Washington DC), I construct the share of the national population in each of the states. For each state, I take an average across all years of this share and use this as the fixed weight for each state. Finally, I deflate the resulting nominal minimum wage using the GDP deflator from FRED.

**Alternative measures of supply in Table I.** Here, I replicate the first six columns of Table I using alternative measures of the relative supply of college workers. Table A.I displays results of estimating the reduced-form specification, using OLS and defining the relative supply of college workers using the population-based measure. Table A.II displays results using 2SLS as in the baseline, but replacing the hours-worked-based measure of the relative supply of college workers with the supply measure constructed using the dual of wages.

In both tables, the estimated coefficient on the real minimum wage remains negative and significant across all specifications. Moreover, the estimated coefficients in the 1963 – 1987 period are more similar to those in the full sample period in the extended canonical model in columns (3) and (4) than in the canonical model in column (1) and (2). However, in the case in which the relative supply of college workers is measured using the dual of wages, estimates still change substantially across sample periods.

	1963-1987	1963-2016	1963-1987	1963-2016		
	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.806 (0.134)	-0.317 (0.058)	-0.737 (0.130)	-0.555 (0.060)	-0.816 (0.083)	-0.632 (0.101)
Real minimum wage			-0.221 (0.068)	-0.307 (0.053)	-0.257 (0.058)	-0.132 (0.047)
Time	0.027 (0.005)	0.014 (0.001)	0.024 (0.005)	0.019 (0.001)		
Constant	-0.008 (0.017)	0.007 (0.009)	0.008 (0.017)	0.010 (0.009)	-0.027 (0.016)	0.035 (0.027)
Time Polynomial	1	1	1	1	2	3
Observations	25	54	25	54	54	54
R-squared	0.532	0.912	0.694	0.941	0.949	0.972

Table A.I: Reduced-Form Regression Models for the National College Premium

*Notes:* Replicating the first six columns of Table I estimating the reduced-form specification of equation (1) in columns (1) and (2) and of equation (2) in columns (3) – (6), replacing the hours-worked measure of the relative supply of college workers with the population-based measure and estimating using OLS.

	1963-1987	1963-2016	1963-1987	1963-2016		
	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-1.166 (0.302)	-0.388 (0.055)	-1.059 (0.243)	-0.584 (0.063)	-0.724 (0.085)	-0.745 (0.118)
Real minimum wage			-0.236 (0.094)	-0.207 (0.044)	-0.158 (0.054)	-0.171 (0.060)
Time	0.045 (0.011)	0.017 (0.001)	0.040 (0.009)	0.022 (0.002)		
Constant	-0.021 (0.021)	0.018 (0.008)	-0.003 (0.021)	0.024 (0.008)	0.009 (0.012)	0.002 (0.022)
Time Polynomial	1	1	1	1	2	3
Observations	25	54	25	54	54	54
R-squared	0.048	0.948	0.328	0.967	0.966	0.965

Table A.II: Regression Models for the National College Premium: Dual Measure of Supply

*Notes:* Replicating the first six columns of Table I, but using the dual of wages to measure the relative supply of college workers. See the Notes of Table I for details.

	(1)	(2)	(3)	(4)	(5)
Relative supply of college workers	-0.632 (0.069)	-0.703 (0.077)	-0.608 (0.104)	-0.619 (0.119)	-0.387 (0.134)
Real minimum wage	-0.220 (0.048)	-0.199 (0.059)	-0.133 (0.052)	-0.129 (0.064)	-0.132 (0.046)
Time	0.021 (0.002)				
Time Polynomial	1	2	3	4	5

Table A.III: Regression Models for the National College Wage Premium: Higher Dimensional Polynomials of Time

*Notes:* Columns (1), (2), and (3) in this table replicate columns (4), (5), and (6) of Table I exactly. Columns (4) and (5) in this table include higher-dimensional polynomials of time, with column  $j$  including polynomials of time up to and including degree  $j$ .

**Comparison of results with Autor, Katz and Kearney (2008).** Table A.III displays estimates of the extended canonical model in equation (2) using my data and including higher dimensional polynomials of time than displayed in Table I. Across all specifications, the coefficient on the real minimum wage remains negative and statistically significantly different from zero. Moreover, the coefficient stabilizes at  $-0.13$  across the third-, fourth-, and fifth-degree polynomial specifications.

The top panel of Table A.IV estimates equation (2) with a linear (column 1) to a quintic (column 5) polynomial of time, as in columns 1 – 5 of Table A.III, but using data from the replication package for Autor, Katz and Kearney (2008) (although I use the GDP deflator from FRED rather than the deflator in their replication package). Outside of the specification with a cubic time trend, results that can be obtained using their data are similar to my baseline national results displayed in Tables I and A.III.

The bottom panel of Table A.IV replicates the top panel, but replaces the nominal minimum wage from Autor, Katz and Kearney (2008), which is the federal minimum wage, using my baseline measure of the real minimum wage, which averages minimum wages across states. The coefficient on the real minimum wage tends to be larger in the bottom panel of Table A.IV than the top panel. This is similar to the comparison between columns (4) and (7) in my Table I. As there, using the federal minimum wage introduces bias, since it doesn't apply in many states, especially starting in the late 1990s.

**Separate elasticities of college and non-college wages.** In Table A.V I decompose the estimated elasticities of the college premium into separate elasticities of college and non-college real wages. To do so, I estimate equation (2) replacing  $\log(w_{ht}/w_{\ell t})$  with  $\log w_{ht}$  and, separately, with  $\log w_{\ell t}$ . The elasticity of the college real wage minus the elasticity



	Using AKK data				
	(1)	(2)	(3)	(4)	(5)
Relative supply of college workers	-0.431 (0.051)	-0.607 (0.077)	-0.612 (0.091)	-0.216 (0.113)	0.013 (0.104)
Real minimum wage	-0.112 (0.049)	-0.108 (0.049)	-0.064 (0.048)	-0.174 (0.052)	-0.123 (0.040)
	Using my baseline real minimum wage				
	(1)	(2)	(3)	(4)	(5)
Relative supply of college workers	-0.459 (0.051)	-0.605 (0.078)	-0.610 (0.089)	-0.244 (0.108)	-0.019 (0.106)
Real minimum wage	-0.150 (0.051)	-0.139 (0.053)	-0.087 (0.057)	-0.184 (0.059)	-0.119 (0.050)
Time Polynomial	1	2	3	4	5
Observations	43	43	43	43	43

Table A.IV: Regression Analysis for the National College Premium: Using Data From the Replication Package for Autor, Katz and Kearney (2008)

*Notes:* The top panel displays results of estimating (2) using the college premium, the relative supply of college workers, and the nominal minimum wage from Autor, Katz and Kearney (2008). I deflate their nominal minimum wage using the GDP deflator from FRED rather than deflator in their replication package. The bottom panel replicates this exercise but uses my baseline measure of the real minimum wage. Robust standard errors are reported in parentheses.

of the non-college real wage equals the elasticity of the college premium. I do so for the specifications that include a linear time trend, an additional quadratic time trend, and an additional cubic time trend, corresponding to columns (4) – (6) of Table I.

Across specifications, a higher minimum wage raises the average real wage of workers without a college education,  $w_{lt}$ ; and this effect is statistically significant in each specification. On the other hand, the impact of the minimum wage on the average real wage of college-educated workers,  $w_{ht}$ , is not robust across specifications.

## A.2 State, labor group, and time analysis

**Basic processing of the Merged Outgoing Rotation Groups CPS data.** I use the Current Population Survey Merged Outgoing Rotation Groups (MORG CPS) from 1979 to 2016 in the state-level estimation. The MORG CPS reflects current wages. Because of missing imputation flags—the CPS did not flag workers with missing wages—in all of 1994 and the first eight months of 1995, I do not include 1994 or the first half of 1995; and I measure the second half of 1995 using wage data only from September – December. As in the na-

	Linear			Quadratic			Cubic		
	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)
Relative supply	-0.632 (0.069)	-0.414 (0.127)	0.218 (0.123)	-0.703 (0.077)	-1.029 (0.140)	-0.327 (0.139)	-0.608 (0.104)	-1.117 (0.158)	-0.509 (0.103)
Real minimum wage	-0.220 (0.048)	-0.104 (0.059)	0.117 (0.058)	-0.199 (0.059)	0.083 (0.077)	0.282 (0.066)	-0.133 (0.052)	0.022 (0.091)	0.155 (0.063)
Time	0.021 (0.002)	0.020 (0.003)	-0.002 (0.002)						

Table A.V: Separate Regression Models for National College and Non-College Real Wages

*Notes:* The three columns labeled “Premium” replicate columns (4), (5), and (6) of Table I exactly. The three columns labeled “High” replicate these columns but replacing the dependent variable with the log of the college wage whereas the columns labeled “Low” do the same but with the non-college wage. Each estimate in the “High” column minus the corresponding estimate in the “Low” column equals the estimate in the “Premium” column. See the Notes of Table I for additional details.

tional analysis, I restrict attention to workers aged 16 to 64. As in the national analysis described in Section A.1, I bin workers into one of 100 groups, denoted by  $g$ , defined by the interaction of 5 age bins (16-25, 26-35, 36-45, 46-55, and 56-64), 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training).

In processing the files, I broadly follow the approaches of Lemieux (2006) and Autor, Katz and Kearney (2008), using hourly wages for workers paid by the hour and using usual weekly earnings divided by hours worked last week for non-hourly workers. I multiply top-coded constructed hourly wages by 1.5. I drop respondents with allocated earnings flags.

**Constructing real minimum wages.** I construct bi-annual state-specific real minimum wages as follows. I use data from Cengiz et al. (2019) to construct bi-annual state-specific nominal minimum wages. For each state and quarter, Cengiz et al. (2019) report the minimum, mean, and maximum effective nominal minimum wage (the maximum of the of the legislated state and federal minimum wages). For each state and half-year, I take the maximum across quarters within the half year of the maximum minimum wage within each quarter. Finally, I deflate using the GDP deflator from FRED, using the maximum value of the GDP deflator across months within the half year.

**Summary statistics.** Figure A.II displays summary statistics of the inputs used in estimating regression (10). Table A.VI provides additional statistics on the minimum wage bite across all  $g, r, t$  triplets (among the balanced estimation sample of  $g, r$  pairs), as well as among the subsamples of workers who are college educated, non-college educated, and

do not have a high school degree.

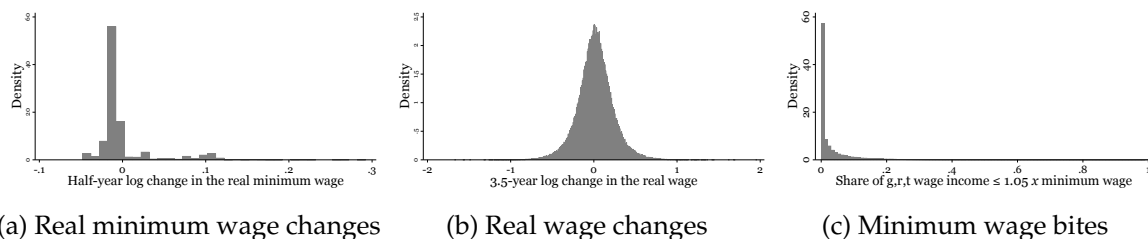


Figure A.II: Distributions of changes in real minimum wages and real wages and the distribution of minimum wage bites

*Notes:* Panel (a) displays the distribution across states and periods in half-year changes in the real minimum wage. Panel (b) displays the distribution across labor groups, states, and periods in 3.5-year changes in the real minimum wage. Panel (c) displays the distribution across labor groups, states, and periods in the minimum wage bite. Panels (b) and (c) use the balanced sample of  $g, r$  pairs.

	Full Population	College-Educated	Non-College Educated	Without a High School Degree
<b>Mean</b>	0.039	0.006	0.050	0.135
<b>Standard Deviation</b>	0.076	0.016	0.085	0.151

Table A.VI: Minimum Wage Bite Summary Statistics

*Notes:* The first column displays the mean and standard deviation of the minimum wage bite across all  $g, r, t$  triplets among the balanced estimation sample of  $g, r$  pairs. The second, third, and fourth columns replicate this on the subsamples of workers who are college educated, non-college educated, and do not have a high school degree.

**Additional state-group-time-level results.** In this section I display additional empirical results described in Section 5. Figure A.III displays results from estimating the reduced-form specification of equation (10).

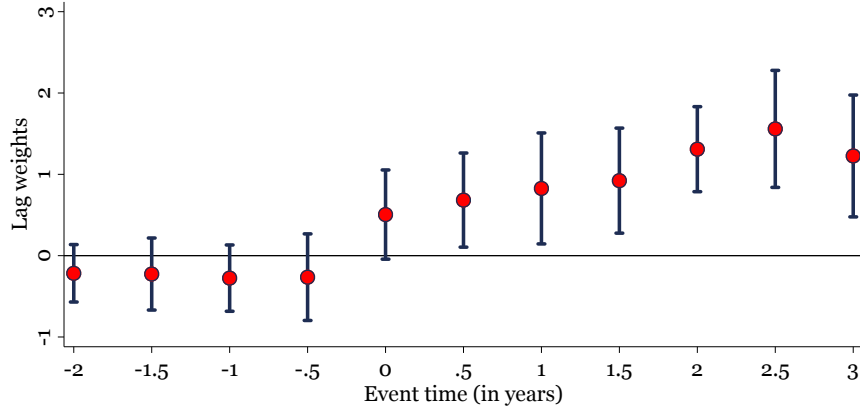


Figure A.III: Reduced-Form Specification of Equation (10)

Notes: Reduced-form estimates of regression (10), in which  $b_{g,r} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$  replaces  $b_{g,r,t-j-1} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$ , and in which the regression is estimated using OLS. Results display  $\mu_0$  and the linear combinations of  $(\mu_{j-1} + \mu_j)/2$  for  $j > 0$  and of  $-(\mu_j + \mu_{j-1})/2$  for  $j < 0$ , with corresponding 90% confidence intervals; robust standard errors are clustered by state.

Figures A.IV and A.V consider other sample restrictions on worker groups, with Figure A.IV focusing on male and, separately, non-white workers and with Figure A.V focusing on college-educated and, separately, older workers.

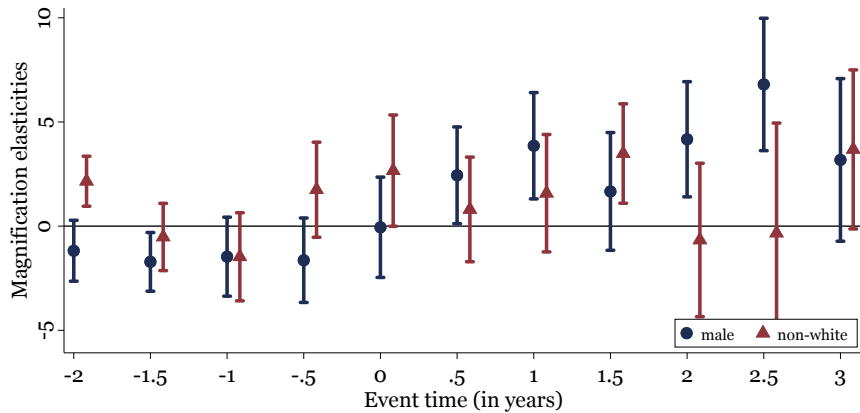


Figure A.IV: Time-Varying Magnification Elasticities: Additional Heterogeneity Analysis (Male and Non-White Workers)

Notes: Replicating Figure IV separately on the samples of male and non-white workers.

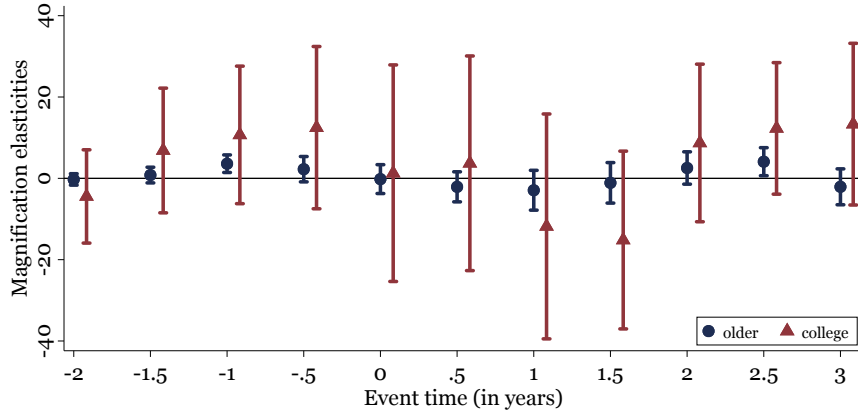


Figure A.V: Time-Varying Magnification Elasticities: Additional Heterogeneity Analysis (Older and College-educated Workers)

Notes: Replicating Figure IV separately on the samples of older (aged  $\geq 36$ ) and college workers.

As the wage cutoff rises (for example), more workers are included in the set of minimum wage workers and the value of the minimum wage bite,  $b_{g,r,t}$ , rises mechanically. If all minimum wage bites rise proportionately, then the lag weight estimates fall proportionately, but the overall elasticity of the real wage with respect to the real minimum wage is unaffected. In practice, of course, it is not the case that all minimum wage bites are proportionately affected. Figure A.VI shows that this logic applies, with higher magnification elasticities when using a wage cutoff of 1.00 and lower magnification elasticities when using a cutoff of 1.10.

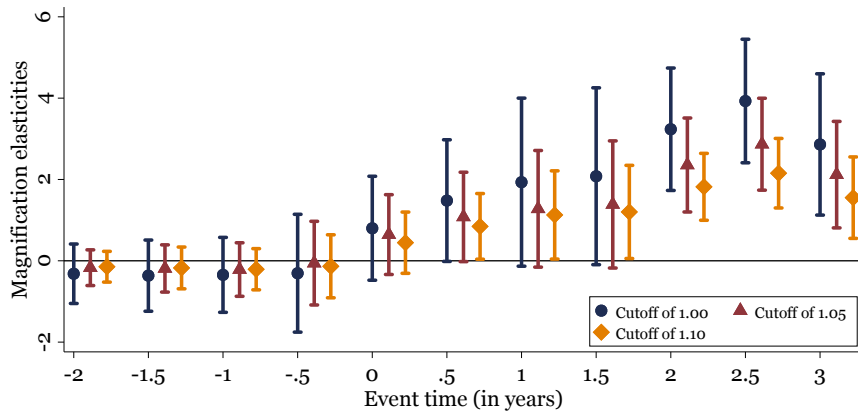


Figure A.VI: Time-Varying Magnification Elasticities: Alternative Minimum Wage Cutoffs

Notes: Replicating Figure IV defining workers as earning the minimum wage if earning a wage no greater than 1.00, 1.05 (as in Figure IV), or 1.10 times the minimum wage.

The baseline estimation sample is balanced across time periods: I restrict the sample to those  $(g, r)$  pairs for which there are observations in every  $t$  in the estimation sample. Figure A.VII shows that results are very similar when I do not impose this restriction on sample inclusion.

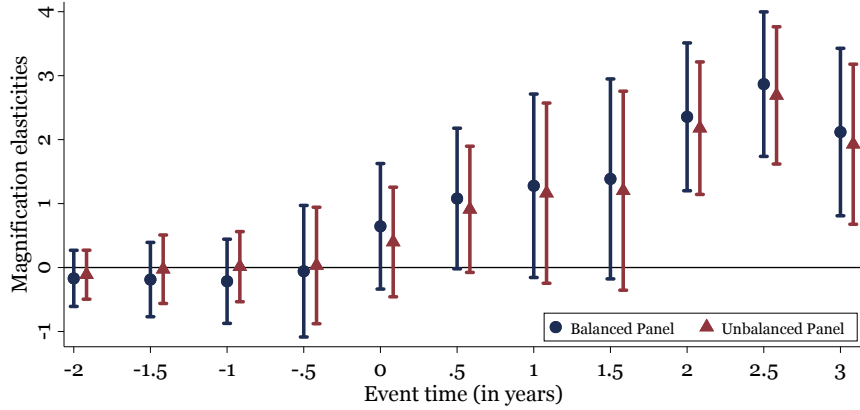


Figure A.VII: Time-Varying Magnification Elasticities: Unbalanced Panel

Notes: Replicating Figure IV including all  $(g, r)$  pairs (i.e., without restricting to those  $(g, r)$  pairs with for which there are observations in every  $t$ ).

In the baseline, the dependent variable is a 3.5-year time difference in real wages. Figure A.VIII displays results using a 3-year time difference whereas Figure A.IX displays results using a 4-year time difference.

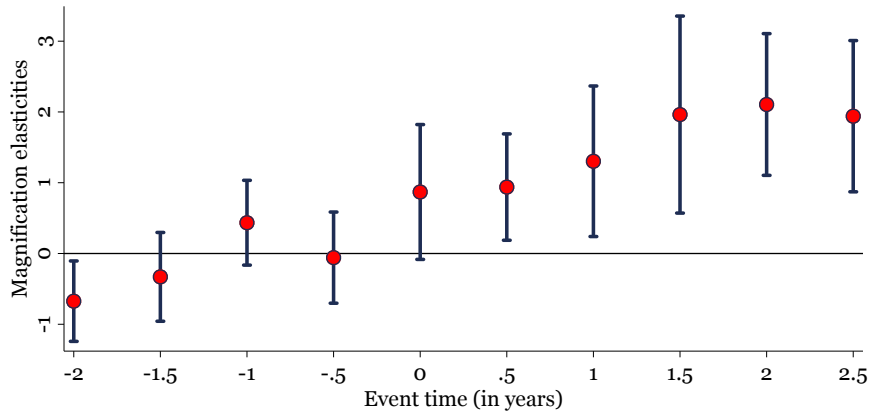


Figure A.VIII: Time-Varying Magnification Elasticities: Shorter Time Difference

Notes: Replicating Figure IV using a 3-year time difference in the dependent variable.

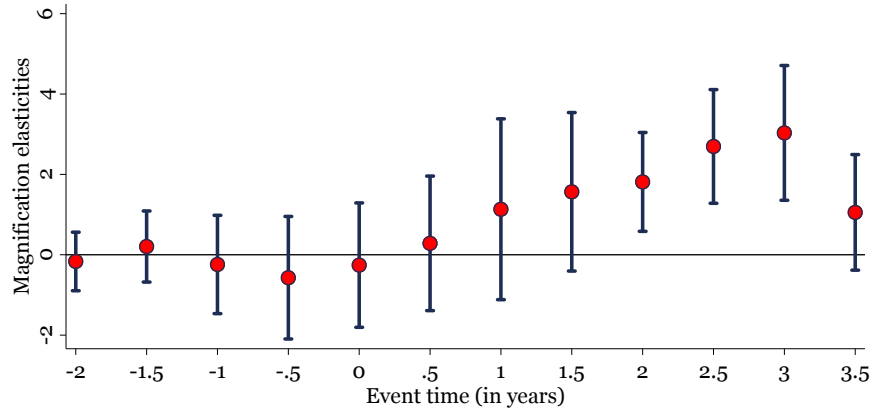


Figure A.IX: Time-Varying Magnification Elasticities: Longer Time Difference

Notes: Replicating Figure IV using a 4-year time difference in the dependent variable.

## Appendix B Theoretical appendix

### B.1 Details of the steady-state analysis

These results are proven under the stated assumption that  $m < P_s$ .

**Steady-state wage ladder.** Consider a skill  $s$  for which the minimum wage is binding:  $m \geq (1 - \beta_s) v_s + \beta_s P_s$ . The wage on the first rung of the job ladder is  $w_{1,s} = m$ . Starting from any rung  $j \geq 1$ , the wage of the next rung  $j + 1$  on the job ladder is  $w_{j+1,s} = (1 - \beta_s) w_{j,s} + \beta_s P_s$  for  $j \geq 1$ . This recursive system yields the following explicit solution for the wage at each rung

$$w_{j,s} = (1 - \beta_s)^{j-1} m + \beta_s P_s \sum_{k=0}^{j-2} (1 - \beta_s)^k \text{ for all } j > 1 \text{ if } m \geq (1 - \beta_s) v_s + \beta_s P_s \quad (\text{A1})$$

Next, consider a skill  $s$  for which the minimum wage is not binding:  $m < (1 - \beta_s) v_s + \beta_s P_s$ . The wage on the first rung of the job ladder is  $w_{1,s} = (1 - \beta_s) v_s + \beta_s P_s$ . Starting from any rung  $j \geq 1$ , the wage of the next rung  $j + 1$  on the job ladder is simply  $w_{j+1,s} = (1 - \beta_s) w_{j,s} + \beta_s P_s$  for  $j \geq 1$ . This recursive system yields the following explicit solution for the wage at each rung

$$w_{j,s} = (1 - \beta_s)^j v_s + \beta_s P_s \sum_{k=0}^{j-1} (1 - \beta_s)^k \text{ for all } j \geq 1 \text{ if } m < (1 - \beta_s) v_s + \beta_s P_s \quad (\text{A2})$$

**Steady-state distribution across the job ladder.** Solving the recursive system displayed in equation (3) and using the fact that these densities must sum to one across all  $j = 0, \dots, \infty$ , yields both

$$u_s = \frac{\delta_s}{\delta_s + \gamma_{su}} \quad (\text{A3})$$

and

$$g_s(w_j) = \left( \frac{(1 - \delta_s) \gamma_{se}}{\delta_s + (1 - \delta_s) \gamma_{se}} \right)^{j-1} \frac{\gamma_{su}}{\delta_s + (1 - \delta_s) \gamma_{se}} \frac{\delta_s}{\delta_s + \gamma_{su}} \text{ for } j \geq 1 \quad (\text{A4})$$

**Steady-state average wage.** The average wage of the employed is given by

$$w_s \equiv \frac{1}{1 - u_s} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$$



Substituting in from equations (A3) and (A4) yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s)\gamma_{se}} \sum_{i \geq 0} w_{i+1,s} \left( \frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right)^i \quad (\text{A5})$$

First consider a skill  $s$  for which the minimum wage is binding:  $m \geq (1 - \beta_s)v_s + \beta_s P_s$ . Substituting into equation (A5) both  $w_{1,s} = m$  and equation (A1) yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s)\gamma_{se}} \left\{ m \sum_{i \geq 0} \left[ \frac{(1 - \delta_s)(1 - \beta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i + \beta_s P_s \sum_{i \geq 1} \left[ \sum_{k=0}^{i-1} (1 - \beta_s)^k \right] \left[ \frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i \right\}$$

Simplifying the above expression yields equation (4) in Section 3.2.

Next, consider a skill  $s$  for which the minimum wage is not binding:  $m < (1 - \beta_s)v_s + \beta_s P_s$ . Substituting into equation (A5) both  $w_{1,s} = (1 - \beta_s)v_s + \beta_s P_s$  and equation (A2) yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s)\gamma_{se}} \left\{ v_s(1 - \beta_s) \sum_{i \geq 0} \left[ \frac{(1 - \delta_s)(1 - \beta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i + P_s \beta_s \sum_{i \geq 0} \left[ \sum_{k=0}^i (1 - \beta_s)^k \right] \left[ \frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i \right\}$$

Simplifying the above expression yields

$$w_s = \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} v_s + \left( 1 - \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \right) P_s \quad (\text{A6})$$

Equation (4) is a weighed average of  $m$  and  $P_s$  and equation (A6) is a weighted average of  $v_s$  and  $P_s$ . The weight on  $v_s$  in equation (A6) is  $(1 - \beta_s)$  times the weight on  $m$  in equation (4). The reason equation (4) places a higher weight on  $m$  than equation (A6) places on  $v_s$  is that the wage on the first rung of the job ladder associated with equation (4) is  $m$ , whereas the wage on the first rung of of the job ladder associated with equation (A6) is  $(1 - \beta_s)v_s + \beta_s P_s$ .

**Across steady state comparative statics for given changes in  $P_s$ .** Here, I consider the impact of given changes in  $m$  and  $P_s$ .

First, consider the case in which the minimum wage binds for skill  $s$ . In this case,

equation (4) implies

$$d \log w_s = \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \frac{m}{w_s} d \log m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}}\right) \frac{P_s}{w_s} d \log P_s$$

Equation (4), the definition of  $b_s = m g_s(m) [(1 - u_s)w_s]^{-1}$ , and the solutions for  $u_s$  and  $g_s(m)$  in equations (A3) and (A4) imply

$$\frac{m}{w_s} = b_s \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s}$$

and

$$\frac{P_s}{w_s} = \frac{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}}{\beta_s(1 - \delta_s)\gamma_{se}} \times \left(1 - \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s\right)$$

The three previous expressions imply

$$d \log w_s = \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s d \log m + \left(1 - \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s\right) d \log P_s$$

This equation is the same as equation (5) given the definition of  $M_s$  in equation (6).

Next, consider the case in which the minimum wage binds for skill  $s$ . In this case, equation (A6) implies

$$d \log w_s = \left(1 - \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \frac{v_s}{w_s}\right) d \log P_s \quad (\text{A7})$$

where  $v_s/w_s$  is a measure of the unemployment replacement rate for skill  $s$  (the ratio of unemployment real income to the average real wage).

**Determining changes in  $P_s$  across steady states and the extended canonical model.** Since final good producers—who combine the output of firms producing skill-specific output—are always on their labor demand curves, value marginal products of labor will be determined exactly as in a neoclassical model featuring the same aggregate production function, but with employment  $\tilde{L}_s = (1 - u_s)L_s$  replacing labor supply  $L_s$ . Here, I assume that the aggregate production function is given by

$$Y = \left\{ \left(A_h \tilde{L}_h\right)^{\frac{\eta-1}{\eta}} + \left(A_\ell \tilde{L}_\ell\right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \quad (\text{A8})$$

where each employed worker produces  $A_s$  units of output. The value marginal product of  $s$  labor is  $P_s = \partial Y / \partial \tilde{L}_s$ . In what follows, I maintain the assumption that  $m < P_s$  for all  $s$ , so that the unemployment rate is fixed. All derivations that follow are, therefore, standard.

Equation (A8) then implies

$$P_s = Y^{\frac{1}{\eta}} A_s^{\frac{\eta-1}{\eta}} \tilde{L}_s^{\frac{-1}{\eta}}$$

$$d \log Y = \frac{1}{Y} \sum_{s \in \{h, \ell\}} Y^{\frac{1}{\eta}} (A_s \tilde{L}_s)^{\frac{\eta-1}{\eta}} (d \log A_s + d \log L_s)$$

where I imposed  $du_s = 0$  in the second expression. The previous expressions yield

$$d \log Y = \left[ \frac{P_h \tilde{L}_h}{Y} (d \log A_h + d \log L_h) + \frac{P_\ell \tilde{L}_\ell}{Y} (d \log A_\ell + d \log L_\ell) \right]$$

$$d \log P_s = \frac{1}{\eta} d \log Y + d \log A_s - \frac{1}{\eta} (d \log A_s + d \log L_s)$$

which in turn imply

$$d \log P_s = \frac{1}{\eta} \frac{P_{s'} \tilde{L}_{s'}}{Y} \left( d \log \frac{A_{s'}}{A_s} + d \log \frac{L_{s'}}{L_s} \right) + d \log A_s \quad \text{for } s' \neq s \quad (\text{A9})$$

I now use the previous result to prove Proposition 1.

**Proof of Proposition 1.** Suppose that across steady states, represented by  $t$ , there is linear growth in  $A_{ht}$  and  $A_{\ell t}$ , with mean zero deviations. Additionally suppose that  $M_h = M_\ell$  (a sufficient condition for which is  $\delta_h = \delta_\ell$ ,  $\gamma_{he} = \gamma_{\ell e}$ , and  $\beta_h = \beta_\ell$ ). Then combining equation (A9) with equations (5) and/or (A7)—depending on whether the minimum wage binds the wages of  $h$  and/or  $\ell$ —yields (2), where the residual is a function of the deviations of the rate of skill-biased technical change from the linear trend. In equation (2), the coefficient  $v_m$  on the real minimum wage is  $M(b_h - b_\ell)$ .<sup>4</sup>□

If the minimum wage only binds for one skill, then parameters needn't satisfy  $M_h = M_\ell$ , since only the value of  $M_s$  for the skill group bound by the minimum wage is relevant.

## B.2 Proof of Proposition 2

I focus on a single skill  $s$  that is bound by the minimum wage,  $m \geq (1 - \beta_s) v_s + \beta_s P_s$ , and omit skill subscripts where possible. I consider a one-time increase in the real minimum wage, starting from a steady state, from  $m$  to  $m' < (1 - \beta)m + \beta P$  that occurs during date 0. This constraint on  $m'$  implies that the second rung of the initial job ladder is not constrained by the new minimum wage.

---

<sup>4</sup>The particular values of  $v_L$  and  $v_{A_s}$  in equation (2) also depend on which skills are bound by the minimum wage.

**Proof of Results (i) and (ii) of Proposition 2:** Result (i) is proven below the proposition. The wage ladder associated with  $m$  (original) and the wage ladder associated with  $m'$  (new) are denoted by  $w_j$  and  $w'_j$ , respectively. Since  $s$  is bound by  $m$ , the wage on the first rung of each ladder is given by  $w_1 = m$  and  $w'_1 = m'$ , respectively. Wages  $w_j$  and  $w'_j$  for each  $j > 1$  are then given by equation (A1), using minimum wage  $m$  in the original wage ladder and  $m'$  in the new wage ladder. Because the wage on rung  $j$  is increasing in the minimum wage in equation (A1), Result (ii) of Proposition 2 directly applies:  $w'_j > w_j$  for each  $j \geq 1$ .  $\square$

**Proof of Result (iii) of Proposition 2:** Let  $g_{t,j}$  and  $g'_{t,j}$  denote the shares of all workers at time  $t \geq 0$  employed on rung  $j$  of the wage ladders associated with  $m$  and  $m'$ , respectively. Let  $g_j$  denote the share of all workers on rung  $j$  in the steady state before the shock. I now show that  $g_{t,j} + g'_{t,j} = g_j$  for all  $j$  and any  $t \geq 0$ .

Consider  $t = 0$ , just after the shock. The assumption that  $m' < (1 - \beta)m + \beta P$  implies that  $g'_{0,j} = 0$  and  $g_{0,j} = g_j$  for any rung  $j \geq 2$ . The additional assumption that  $m' > m$  implies  $g'_{0,1} = g_1$  and  $g_{0,1} = 0$ . Since all unemployed exit unemployment on the new job ladder, I assign the unemployed to the new job ladder in all dates. And since  $m, m' < P$ , the unemployment rate is unaffected on impact. Hence,  $g'_{0,0} = g_0$  and  $g_{0,0} = 0$ . In summary, just after the shock  $g_{0,j} + g'_{0,j} = g_j$  for all  $j$ .

Now suppose that  $g_{t,j} + g'_{t,j} = g_j$  for all  $j$  for some  $t \geq 0$ . Then at date  $t + 1$  and for any  $j > 1$ , we have

$$\begin{aligned} g_{t+1,j} &= g_{t,j}(1 - \delta)(1 - \gamma_e) + g_{t,j-1}(1 - \delta)\gamma_e \\ g'_{t+1,j} &= g'_{t,j}(1 - \delta)(1 - \gamma_e) + g'_{t,j-1}(1 - \delta)\gamma_e \end{aligned}$$

Summing these expressions yields

$$g_{t+1,j} + g'_{t+1,j} = g_j(1 - \delta)(1 - \gamma_e) + g_{j-1}(1 - \delta)\gamma_e = g_j$$

where the first equality follows from  $g_{t,j} + g'_{t,j} = g_j$  at date  $t$  and the second equality follows from the steady-state derivation of  $g_j$ . The same applies for  $j = 1$ , except the term  $(1 - \delta)\gamma_e$  must everywhere be replaced by  $\lambda_u$ . Since  $\sum_{j=0}^{\infty} (g_{t,j} + g'_{t,j}) = 1$ , this implies  $g'_{t+1,0} = g_0$ . Hence, part (ii) of Proposition 2 is proven by induction.  $\square$

**Proof of Result (iv) of Proposition 2:** To obtain a contradiction, suppose that there exists a  $(t, j)$  pair satisfying  $g_{t+1,j} - g_{t,j} > 0$ . Such a  $(t, j)$  pair must satisfy  $j > 1$ , since  $g_{t,1} = 0$  for all  $t \geq 0$ . If  $j > 1$  and  $t \geq 0$  then

$$g_{t+1,j} = xg_{t,j} + yg_{t,j-1} \tag{A10}$$

where I have defined  $x \equiv (1 - \delta)(1 - \gamma_e)$  as the probability a worker who starts on any rung  $j \geq 1$  on the original ladder remains there the following period and  $y \equiv (1 - \delta)\gamma_e$  as the probability that a worker who starts on any rung  $j \geq 1$  moves up to rung  $j + 1$  in the following period. Hence, for  $j > 1$  and  $t \geq 0$ ,  $g_{t+1,j} - g_{t,j} > 0$  is equivalent to

$$(1 - x)g_{t,j} < yg_{t,j-1}$$

If  $t = 0$ , the previous expression is equivalent to  $g_j(1 - x) < g_{j-1}y$ , where  $g_j$  denotes the steady-state share; this contradicts the steady-state relationship  $g_j(1 - x) = g_{j-1}y$ . If  $t > 0$ , then  $g_{t,j}$  can be substituted out of the previous expression using equation (A10), with each time period lagged once, to obtain

$$y [g_{t,j-1} - g_{t-1,j-1}] > x [(1 - x)g_{t-1,j} - yg_{t-1,j-1}] \quad (\text{A11})$$

If the right-hand side of equation (A11) is weakly positive, then the left-hand-side of equation (A11) must be strictly positive, which requires that  $g_{t,j-1} - g_{t-1,j-1} > 0$ . This is contradicted if  $j = 2$  (since  $g_{t,1} = 0$  for all  $t \geq 0$ ) or if  $t = 1$  (since  $g_{1,j} = g_{2,j} = g_j$  for all  $j > 2$ ). If, on the other hand, the right-hand side of equation (A11) is strictly negative,  $(1 - x)g_{t-1,j} < yg_{t-1,j-1}$ , then substituting out  $xg_{t-1,j} = g_{t,j} - yg_{t-1,j-1}$  using equation (A10), yields  $g_{t,j} - g_{t-1,j} > 0$ . This is contradicted if  $t = 1$  (since  $g_{1,j} = g_{0,j} = g_j$  for all  $j > 2$ ).

If no contradiction is reached, the same process restarts. However, now instead of assuming  $g_{t+1,j} - g_{t,j} > 0$ , either both  $t$  and  $j$  are reduced by one ( $g_{t,j-1} - g_{t-1,j-1} > 0$ ) if the right-hand side of equation (A11) is weakly positive or only  $t$  is reduced by one ( $g_{t,j} - g_{t-1,j} > 0$ ) if the right-hand side of equation (A11) is strictly negative. This argument proceeds until a contradiction is reached.  $\square$

**Proof of Result (v) of Proposition 2:** On impact, the wage on the first rung on the old job ladder,  $m$ , becomes constrained by the new, higher minimum wage. Because surplus in each match remains positive, every worker earning  $m$  moves instantly to the first rung of the new ladder,  $m'$ . The increase in the minimum wage does not constrain the second rung of the initial wage ladder (given the assumption that  $m' < (1 - \beta)m + \beta P$ ), so no other workers are affected. This implies that the instantaneous, or impact elasticity of the average wage of skill  $s$  equals  $b_s$ .  $\square$

**Proof of Result (vi) of Proposition 2:** The average wage at time  $t$  is

$$w_t \equiv \frac{1}{1 - u_s} \sum_{j \geq 1} (w_j g_{t,j} + w'_j g'_{t,j})$$

Results (ii), (iii), and (iv) imply that  $w_j g_{t,j} + w_j' g_{t,j}'$  is weakly increasing in  $t$  for each  $j$ , with a strict inequality for at least one value of  $j$ .  $\square$

### B.3 Wage-posting model

In the baseline model, workers and firms bargain over wages. I assume that agents are not forward looking to facilitate solving the transition in response to a sequence of aggregate shocks; and the model counterfactually predicts that expected job duration is independent of the worker's wage. Here, I show that steady-state results are broadly similar in the canonical wage-posting model of Burdett and Mortensen (1998) with homogeneous workers (within a skill) and firms, extended to include a minimum wage, as in van den Berg and Ridder (1998). In this framework, agents are forward looking and job duration is increasing in the worker's wage. I consider the case of a single skill  $s$  (omitting  $s$  subscripts), since this is sufficient to show which results are robust, and I focus on steady states, since solving for the transition to an aggregate shock is not straightforward.

According to equation (2.10) in van den Berg and Ridder (1998), the equilibrium earnings density is

$$g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \text{ for all } w \in [m, w_{\max}]$$

under the assumption that the minimum wage  $m$  is binding (an assumption I make throughout), where

$$w_{\max} \equiv \left( \frac{\delta}{\delta + \lambda_e} \right)^2 m + \left( 1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P$$

In these expressions, I have used my notation:  $P$  is the value marginal product of labor,  $\delta$  is the exogenous separation rate,  $\lambda_e$  is the rate at which workers receive offers when employed, and  $w_{\max}$  is the (endogenous) supremum of offered wages. Integrating this yields the probability a worker earns less than  $w$ ,

$$G(w) = \frac{\delta}{\lambda_e} \left( \frac{P - m}{P - w} \right)^{1/2} - \frac{\delta}{\lambda_e} \text{ for all } w \in [m, w_{\max}]$$

which is the earnings distribution (conditional on employment).

**Impact of a change in the minimum wage on the distribution of wages.** Define  $W_c(m)$  to be the wage at centile  $c \in [0, 100]$ . The above distribution of wages implies

$$\frac{c}{100} = \frac{\delta}{\lambda_e} \left( \frac{P - m}{P - W_c(m)} \right)^{1/2} - \frac{\delta}{\lambda_e}$$

The previous expression can be inverted to obtain an explicit solution for the wage at centile  $c$  as a function of the minimum wage  $m$ ,

$$W_c(m) = P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2$$

This implies

$$\frac{W_{c'}(m)}{W_c(m)} = \frac{P - (P - m) \left( \frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2}$$

Differentiating with respect to  $m$  yields

$$\frac{d [W_{c'}(m)/W_c(m)]}{dm} < 0 \iff c' > c$$

Hence, as in my baseline model, an increase in the minimum wage increases the relative wage of centile  $c$  to centile  $c'$  for any  $W(c') > W(c)$ . That is,  $W_c(m)$  is log sub-modular in  $c, m$  in both models. This result has been previously documented quantitatively by Engbom and Moser (2022).

**The average wage.** The average wage is given by  $w = \int_m^{w_{\max}} w g(w) dw$ . Using a change of variables, the average wage can alternatively be expressed using the  $W_c(m)$  function (the wage at centile  $c$ ), noting that the distribution across centiles is uniform (by definition) with density  $\tilde{g}(c) = (w_{\max} - m)^{-1}$ . This density can be expressed as

$$\tilde{g}(c) = \left( 1 - \frac{\delta}{\delta + \lambda_e} \right)^{-2} (P - m)^{-1}$$

Using this formulation, I obtain

$$w = \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left( 1 - \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} \right) P$$

The previous expression gives the average wage. As in the baseline model, see equation (4), the average wage can be expressed as a weighted average of the minimum wage  $m$  and the value marginal product of labor,  $P$ . Unlike the baseline model, the weights themselves depend on the minimum wage.

**Key differences across models for my analysis.** From the perspective of my analysis, there are two key distinctions between this model and my baseline model. First, studying the transition analytically is straightforward in my baseline model whereas solving even quantitatively for the transition in response to an aggregate shock (changing the minimum

wage) in Burdett and Mortensen (1998) is more difficult. Second, in my baseline model—as in the data—a mass of workers earn exactly the minimum wage; in the Burdett and Mortensen (1998) model there are no mass points in the wage distribution. Hence, in response to a marginal increase in the minimum wage, the direct effect in my baseline model is positive whereas it is zero here.<sup>5</sup>

---

<sup>5</sup>Related to both points: Engbom and Moser (2022) studies the impact of changes in minimum wages in the Burdett and Mortensen (1998) model, focusing exclusively on the steady state, and also extends the model to incorporate a mass point at the minimum wage.



## References

- Acemoglu, Daron and David Autor**, “Skills, Tasks and Technologies: Implications for Employment and Earnings,” in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Vol. 4 of *Handbook of Labor Economics*, Elsevier, 2011, chapter 12, pp. 1043–1171.
- Ahlfeldt, Gabriel M., Duncan Roth, and Tobias Seidel**, “Optimal Minimum Wages,” *mimeo LSE*, 2022.
- Autor, David, Claudia Goldin, and Lawrence F. Katz**, “Extending the Race between Education and Technology,” *AEA Papers and Proceedings*, May 2020, 110, 347–51.
- Autor, David H., Alan Manning, and Christopher L. Smith**, “The Contribution of the Minimum Wage to US Wage Inequality over Three Decades: A Reassessment,” *American Economic Journal: Applied Economics*, January 2016, 8 (1), 58–99.
- , **Lawrence F. Katz, and Melissa S. Kearney**, “Trends in U.S. Wage Inequality: Revising the Revisionists,” *Review of Economics and Statistics*, 2008, 90 (2), 300–323.
- Berger, David W., Kyle F. Herkenhoff, and Simon Mongey**, “Minimum Wages, Efficiency and Welfare,” NBER Working Papers 29662, National Bureau of Economic Research, Inc January 2022.
- Borusyak, Kirill and Peter Hull**, “Nonrandom Exposure to Exogenous Shocks,” *Econometrica*, November 2023, 91 (6), 2155–2185.
- and – , “Negative Weights are no Concern in Design-Based Specifications,” NBER Working Papers 32017, National Bureau of Economic Research, Inc January 2024.
- Burdett, Kenneth and Dale T. Mortensen**, “Wage Differentials, Employer Size, and Unemployment,” *International Economic Review*, 1998, 39 (2), 257–273.
- Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin**, “Wage Bargaining with On-the-Job Search: Theory and Evidence,” *Econometrica*, March 2006, 74 (2), 323–364.
- Card, David and John E. DiNardo**, “Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles,” *Journal of Labor Economics*, October 2002, 20 (4), 733–783.
- and **Thomas Lemieux**, “Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis\*,” *The Quarterly Journal of Economics*, May 2001, 116 (2), 705–746.
- Cengiz, Doruk, Arindrajit Dube, Attila Lindner, and Ben Zipperer**, “The Effect of Minimum Wages on Low-Wage Jobs,” *The Quarterly Journal of Economics*, 2019, 134 (3), 1405–1454.
- Chen, Yujiang River and Coen N Teulings**, “What is the optimal minimum wage,” CEPR Working Papers DP17026, The Centre for Economic Policy Research February 2022.
- Clemens, Jeffrey and Michael R. Strain**, “The Heterogeneous Effects of Large and Small Minimum Wage Changes: Evidence over the Short and Medium Run Using a Pre-Analysis Plan,” NBER Working Papers 29264, National Bureau of Economic Research, Inc September 2021.
- Derenoncourt, Ellora and Claire Montialoux**, “Minimum Wages and Racial Inequality,” *The Quarterly Journal of Economics*, 2021, 136 (1), 169–228.
- DiNardo, John, Nicole M. Fortin, and Thomas Lemieux**, “Labor Market Institutions and the Distribution of Wages, 1973–1992: A Semiparametric Approach,” *Econometrica*, 1996, 64 (5), 1001–1044.

- Dube, Arindrajit**, “Minimum Wages and the Distribution of Family Incomes,” *American Economic Journal: Applied Economics*, October 2019, 11 (4), 268–304.
- , **Michael Reich, Akash Bhatt, and Denis Sosinskiy**, “Restaurant Employment, Minimum Wages, and Border Discontinuities,” NBER Working Papers 32902, National Bureau of Economic Research, Inc September 2024.
- , **T. William Lester, and Michael Reich**, “Minimum Wage Effects Across State Borders: Estimates Using Contiguous Counties,” *The Review of Economics and Statistics*, November 2010, 92 (4), 945–964.
- Dustmann, Christian, Attila Lindner, Uta Schönberg, Matthias Umkehrer, and Philipp vom Berge**, “Reallocation Effects of the Minimum Wage,” *The Quarterly Journal of Economics*, 08 2021, 137 (1), 267–328.
- Engbom, Niklas and Christian Moser**, “Earnings Inequality and the Minimum Wage: Evidence from Brazil,” *American Economic Review*, 2022, 112 (12), 3803–3847.
- Freeman, Richard**, *The Overeducated American*, Academic Press, 1976.
- Goldin, Claudia and Lawrence F. Katz**, *The Race between Education and Technology*, Harvard University Press, 2008.
- Grossman, Jean Baldwin**, “The Impact of the Minimum Wage on Other Wages,” *The Journal of Human Resources*, 1983, 18 (3), 359–378.
- Haanwinckel, Daniel**, “Supply, Demand, Institutions, and Firms: A Theory of Labor Market Sorting and the Wage Distribution,” *mimeo UCLA*, 2023.
- Hurst, Erik, Patrick J. Kehoe, Elena Pastorino, and Thomas Winberry**, “The Distributional Impact of the Minimum Wage in the Short and Long Run,” NBER Working Papers 30294, National Bureau of Economic Research, Inc July 2022.
- Katz, Lawrence F. and Kevin M. Murphy**, “Changes in Relative Wages, 1963-1987: Supply and Demand Factors,” *Quarterly Journal of Economics*, 1992, 107 (1), 35–78.
- Lee, David S.**, “Wage Inequality in the United States during the 1980s: Rising Dispersion or Falling Minimum Wage?,” *The Quarterly Journal of Economics*, 1999, 114 (3), 977–1023.
- Lemieux, Thomas**, “Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?,” *American Economic Review*, June 2006, 96 (3), 461–498.
- Manning, Alan**, *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press, 2013.
- Mortensen, Dale T.**, *Wage Dispersion: Why Are Similar Workers Paid Differently?*, The MIT Press, 08 2003.
- Postel-Vinay, Fabien and Jean-Marc Robin**, “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity,” *Econometrica*, November 2002, 70 (6), 2295–2350.
- Roth, Jonathan**, “Interpreting Event-Studies from Recent Difference-in-Differences Methods,” Technical Report 2024.
- Shimer, Robert**, “On-the-job search and strategic bargaining,” *European Economic Review*, May 2006, 50 (4), 811–830.
- Teulings, Coen N.**, “The Contribution of Minimum Wages to Increasing Wage Inequality,” *The Economic Journal*, 2003, 113 (490), 801–833.
- Tinbergen, Jan**, “Substitution of Graduate by Other Labour\*,” *Kyklos*, 1974, 27 (2), 217–226.
- Trottner, Fabian**, “Misallocations in Monopsonistic Labor Markets,” *mimeo UCSD*, 2022.
- van den Berg, Gerard J. and Geert Ridder**, “An Empirical Equilibrium Search Model of

the Labor Market," *Econometrica*, 1998, 66 (5), 1183–1221.