The Race Between Education, Technology, and the Minimum Wage

Jonathan Vogel

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Where we’re going

- **What is the dynamic impact of the minimum wage on inequality?**

- **Theory:** effect is small on impact and grows over time

- **Empirics:** document these dynamic effects

- **The impact of the minimum wage on inequality grows over time, with the medium-run effect being \( \approx \) twice the impact effect**
How we get there

1 **Motivation (The Race Between...):** In national time series, the real minimum wage helps shape the evolution of U.S. college premium + partially resolves a “puzzle”

2 **Theory:** Job-ladder model with many skills (or groups)
   - On impact $\uparrow \text{mw} \Rightarrow \uparrow$ in wages for those individuals bound by it *(direct effect)*
   - Over time, workers move up the job ladder $\Rightarrow$ magnified effect *(indirect effect)*
   - Elasticity of the average wage of a skill group wrt mw $\uparrow$ in share of wage income bound by mw and grows over time *(combo of effects)*

3 **Empirics:** Use state-and-group-level data, document that the elasticity of the state $\times$ group average wage w.r.t. the minimum wage
   - is $\uparrow$ in share of wage income earned at the mw on impact *(direct effect)*
   - this difference in elasticities grows by a factor of $\approx 2$ over 2–3 years *(indirect effect)*
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Theory embeds into any aggregate production function combining skill outputs:


Empirics: Use state-and-group-level data, document that the elasticity of the state × group average wage w.r.t. the minimum wage

- is $\uparrow$ in share of wage income earned at the mw on impact (direct effect)
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Contributions

- **Canonical model** (Tinbergen 74, Katz and Murphy 92, ...), including minimum wage (Autor, Katz, Kearney 08)
  - While supply and demand remain crucial, so too is the minimum wage
  - Minimum wage helps resolve the (apparent) rapid slowing SBTC in the 1990s

- **Impact of minimum wage on inequality**
  Meyer, Wise, 83; DiNardo et al., 96; Lee, 99; Card, DiNardo, 02; Autor ea., 16; Cengiz ea. 19; Dube, 19; Fortin ea. 21; Chen, Teulings 22; ...
  - Direct effect dominates in short run, but indirect effect grows over time
  - Identified within worker groups (i.e. for fixed observable characteristics)

- **Growing macro-labor literature of monopsony using quantitative models**
  Haanwinckel, 20; Engbom, Moser, 21; Ahlfeldt et al., 22; Berger et al., 22; Hurst et al., 22; Trottner, 22; ...
  - Job-ladder model like EM, but focus on dynamics
  - Dynamics like HKPW, but driven by job ladder rather than putty-clay capital
Motivation:
National time-series variation
I consider (for now atheoretical) regressions of the form

\[
\log \left( \frac{w_{ht}}{w_{\ell t}} \right) = \alpha + \beta_m \log m_t + \beta_L \log \left( \frac{Supply_{ht}}{Supply_{\ell t}} \right) + \gamma_1 t + [...] + \iota_t
\]

national time-series, \( t \), variation across college and non-college workers, \( h \) and \( \ell \)

- \( \log w_{ht} \) and \( \log w_{\ell t} \) are measures of average log wages
- \( \log Supply_{ht} \) and \( \log Supply_{\ell t} \) are measures of labor supply (hours worked)
- \( m_t \) is a measure of the real minimum wage at the national level

Measurement

- **Supply and wages** are composition adjusted (March CPS 1964 - 2017 spanning working years 1963 - 2016)
- **Instrument for supply** is composition-adjusted population (March CPS)
- **National real minimum wage** (Cengiz et al. (2019), DOL, FRED, March CPS)
  - For each state use the max of the state and national statutory minima
  - ... then average across states using time-invariant weights
  - ... and apply the GDP deflator
Result I: out-of-sample fit (2SLS)
Result I: out-of-sample fit (2SLS)
Residualized data
Using the population-based measure of relative college supply
Result II: in-sample elasticities...
... of the national college wage premium wrt relative supply and the real minimum wage

Regression Models for the College Wage Premium

<table>
<thead>
<tr>
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<th>(1)</th>
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<tbody>
<tr>
<td>Relative supply</td>
<td>-0.632</td>
<td>-0.703</td>
<td>-0.608</td>
<td>-0.619</td>
<td>-0.387</td>
<td>-0.120</td>
<td>-0.117</td>
<td>-0.045</td>
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<tr>
<td></td>
<td>(0.069)</td>
<td>(0.077)</td>
<td>(0.104)</td>
<td>(0.119)</td>
<td>(0.134)</td>
<td>(0.105)</td>
<td>(0.129)</td>
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<tr>
<td>Real m.w.</td>
<td>-0.220</td>
<td>-0.199</td>
<td>-0.133</td>
<td>-0.129</td>
<td>-0.132</td>
<td>-0.103</td>
<td>-0.131</td>
<td>-0.112</td>
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<tr>
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<td>(0.048)</td>
<td>(0.059)</td>
<td>(0.052)</td>
<td>(0.064)</td>
<td>(0.046)</td>
<td>(0.034)</td>
<td>(0.037)</td>
<td>(0.031)</td>
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<td>Time Polynom.</td>
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<td>Observations</td>
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The extended canonical model including polynomials of time up to degree \( j \) in column \( j \). Estimated using 2SLS, instrumenting for hours-based supply using population-based measure. Robust standard errors are shown in parentheses.

- **Sizable elasticity of the college premium wrt the real minimum wage**
  - e.g., 27\% ↓ in real minimum wage 1979 – 89 \( \implies \) 3 – 6\% ↑ in national college premium
Summary + robustness

Summary:
1. Relative supply growth fluctuations + trend demand growth crucial drivers of college premium, but changes in real minimum wage are also important.
2. Less dramatic slowing of SBTC (more generally, improved out-of-sample fit).

Sensitivity and additional results:
- ... using two alternative measures of relative supply
  - Tables with estimated elasticities
  - Figures with out-of-sample fit
- ... using Autor, Katz, and Kearney (2008) data
  - Table using the FLSA real minimum wage
  - Table using the average real minimum wage across states
- ... separately for college and non-college workers
Theory
Framework

Supply and demand:

- Exogenous supply $L_{st}$ of homogeneous skill $s = 1, ..., S$ workers
- Aggregate production function combining skill output with skill-time-specific productivity $A_{st}$ shaping relative demand
Framework

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  - Exogenous supply $L_{st}$ of homogeneous skill $s = 1, \ldots, S$ workers
  - Aggregate production function combining skill output with skill-time-specific productivity $A_{st}$ shaping relative demand

- **Job ladder:**
  - Zero discount factor: analysis of transitions to aggregate shocks
  - Worker can be employed or unemployed with exogenous separation ($\delta_s$) and job-finding rates ($\lambda_{su}$ unemployed and $\lambda_{se}$ employed)
  - Bilateral generalized Nash bargaining btw new worker-firm match ($\beta_s =$ worker weight) over fixed real wage w/ current job as worker outside option

Real minimum wage: $m_t$

Impose no employment effect of $m_t$ ($m_t < VMPL_{st}$), but will generalize

Discrete time: within $t$, separation shocks occur first, then new matches realized for those workers who did not separate in $t$
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- **Real minimum wage:** $m_t$
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- Discrete time: within $t$, separation shocks occur first, then new matches realized for those workers who did not separate in $t$
Steady-state characterization

Suppose “binding” mw for given s (for exposition only)

- Wage ladder across “rungs”
- First rung is the minimum wage...
- ... and move up over time (if no separation shock)
- Average wage an average of mw and VMPL

Details and distribution
Transitions

Proposition

Consider an economy in steady state at date 0 that faces a small one-time change from \( m \) to \( m' > m \). For any skill \( s \) that was bound by \( m \):

1. For all \( t \geq 0 \) two job ladders coexist, with first rungs \( m \) (old) and \( m' \) (new)
2. The share on each rung \( j \) summed across the two ladders is constant across \( t \)
3. The share on rung \( j \) of the new ladder weakly increases in \( t \)
4. At each rung \( j \), wage on the new job ladder > than on the old one
5. The elasticity of the average wage wrt the minimum wage rises in \( t \)
6. On impact, this elasticity equals the share of income earned at \( m \) (the “bite”)
Transitions implication

- \( D \)-period elasticity of any group’s average wage wrt to a one-time increase in the real mw from \( m \) to \( m' > m \) at \( t = 0 \) (impulse response)

\[
\log \left( \frac{w_{D,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) \equiv M_{D,s} \times B_{-1,s}
\]

\( D \) period elasticity of average wage wrt \( m \)

decomposed into initial minimum wage “Bite" + “Magnification” elasticity

- \( B_{-1,s} \) is the pre-shock share of wage income earned at the mw
- \( M_{D,s} \) is the “Magnification elasticity”
  - \( M_{0,s} = 1 \)
  - \( dM_{D,s}/dD > 0 \)
CS across steady states (including unemployment effects)

Across steady state effects of changes in the real mw, supply, demand

- **Average wage of skill** $s$

  \[ d \log w_s = M_s B_s \partial \log m + (1 - M_s B_s) \partial \log \text{VMPL}_s \]

  where VMPL$_s$ is real and $M_s$ is the steady-state Magnification elasticity

  \[ M_s \equiv \lim_{D \to \infty} M_{D,s} = \frac{\delta_s + (1 - \delta_s)\lambda_{se}}{\delta_s + \beta_s(1 - \delta_s)\lambda_{se}} > 1 \]

- **This holds whether or not** $m$ **affects unemployment**
  - $M_s B_s$ is the partial elasticity wrt $m$, holding employment (so VMPL) fixed
  - If mw does not affect employment,
    1. $M_s B_s$ is also the total elasticity wrt $m$
    2. VMPL determined by production function, but replacing $L_s$ with $(1 - u_s)L_s$
       “Race between education, technology, and the minimum wage”

- **Distribution of wages for skill** $s$

The race
Distributional implication
Empirics
From theory to estimation

- Theory (omit s): if one-time permanent $m \uparrow$ btw $t - D$ and $t - D + 1$ then

$$\log \frac{w_t}{w_{t-D}} = M_{D-1} B_{t-D} \log \frac{m_{t-D+1}}{m_{t-D}}$$

which is equivalent to (a distributed lag model where the magnification elasticities are the lag weights)

$$\log \frac{w_t}{w_{t-D}} = \sum_{j=0}^{D-1} M_j B_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$
From theory to estimation

- Theory (omit $s$): if one-time permanent $m \uparrow$ btw $t - D$ and $t - D + 1$ then

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$$\log \frac{w_t}{w_{t-D}} = \sum_{j=0}^{D-1} M_j B_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$

- Theory: if the one-time $m \uparrow$ was earlier, btw $t - D'$ and $t - D' + 1$ for $D' > D$ then

$$\log \frac{w_t}{w_{t-D}} = (M_{D'-1} - M_{D'-D-1})B_{t-D'} \log \frac{m_{t-D'+1}}{m_{t-D'}}$$

which is equivalent to

(a distributed lag model)

$$\log \frac{w_t}{w_{t-D}} = \sum_{j=0}^{D-1} M_j B_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}} + \sum_{j=D}^{D'-1} (M_j - M_{j-D})B_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$
In practice, \( m \) is changing frequently at state and national levels

- Apply previous formulas in presence of many changes

First set \( D = D' \) and estimate

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \epsilon_{t,g,r}
\]

- the median across all periods of the mw bite for \((g, r)\)
- \( D \) lagged (and \( K \) lead) one-period changes in the mw in region \( r \)

defining

- \( g = \) group: intersection of age bin (5), gender (2), race (2), education (5)
- \( r = \) region: 50 US states
- \( t = \) time: half year (m1-6, m7-12) between 1979m1 – 2016m12
- \( D = 6 \) and \( K = 6 \)

winsorizing wage change at 2nd percentile within each \((g, r)\)
Exposure to exogenous shocks...

\[
\log \frac{W_{t,g,r}}{W_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \ldots + \varepsilon_{t,g,r}
\]

- \(bite_{g,r}\) is (potentially not randomly assigned) exposure and the one-period changes in the \(mw\) are the shocks
- Borusyak and Hull (ECMA, 2023) show
  - how OVB may confound this approach
  - and how re-centering (or controlling for expected treatment) addresses this issue
- Use “design-based” approach (which requires different controls under different assumptions on minimum wage change process)
  - If \(mw\) change is \(i.i.d\). across \(r\) and \(t\): \(bite_{g,r}\)
  - If \(mw\) change is \(i.i.d\). across \(r\) w/ \(t\)-specific mean: \(FE_t \times bite_{g,r}\)
    - same estimates as when using nominal \(mw\) (given same controls)

which avoids negative \(ex\ ante\) weights (Borusyak and Hull WP, 2023)
- In some specifications, additionally include \(FE_{t,r}\)
• NBER Merged Outgoing Rotation Group of the CPS (1979 – 2016)
  ▶ Drop 1994 and 1995m1–m8: missing allocation flags
  ▶ End 2016m12 before many municipalities begin setting their own mws; e.g.,
    ★ NYC + Nassau, Suffolk, and Westchester counties on 12/31/2016
    ★ Minneapolis, MN in 2018
    ★ Los Angeles $0.50 above for large businesses on 7/1/16
    ★ San Diego $0.75 above on 1/1/15
  ▶ Measure $\text{bite}_{g,r}$ defining mw worker as those with wage \( \leq 1.05 \text{ mw} \)
• Minimum wage data from Vaghul and Zipperer (2016), Cengiz et al. (2019)
  ▶ Use maximum nominal mw in state-period
  ▶ Deflate by maximum monthly GDP deflator in period
\[
\log \frac{W_{t,g,r}}{W_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \ldots + \varepsilon_{t,g,r}
\]

- Cluster standard errors by state
- Weigh by product of \((g, r)\) work hours in \(t\) and \(t - D\) divided by their sum
- Use balanced sample: \((g, r)\) with no missing wage data across \(t\)
- All results in figures, which display results...
  
  - ... converting pre-trends to their more typical “levels” form
    - negative of coefficients for \(j < 0\)
    - see Roth (2024): “Interpreting Event-Studies…”
  
  - ... displaying averaged *annual* effects, except for impact effect
    - for period \(j \neq 0\), display \((\hat{\mu}_j + \hat{\mu}_{j-1})/2 + \) corresponding 95% confidence interval
    - for period \(j = 0\), display \(\hat{\mu}_0 + \) corresponding 95% confidence interval
Results without controls

$$\log \frac{W_{t,g,r}}{W_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \varepsilon_{t,g,r}$$

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years $j$ and $j-1$, except for the impact estimate $\mu_0$

- Raw correlations: no pre-trends, jump on impact, rising thereafter
- But cannot view this as a causal relationship
Alternative samples and definitions without controls

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]

- **Baseline:** mw workers those with wages $\leq 1.05 \times$ mw
  - mw workers those with wages $\leq 1.00 \times$ mw
  - mw workers those with wages $\leq 1.10 \times$ mw
  - mw workers those with wages $\leq 1.15 \times$ mw

- **Baseline uses all groups**
  - Only for groups without college degrees
  - Separately by gender
  - Exclude final 6 sample years (w/ sub-state mws)
  - Unbalanced panel of \((g, r)\)
Results with control 1

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \xi \text{bite}_{g,r} + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
log \( \frac{w_{t,g,r}}{w_{t-D,g,r}} \) = \(\alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta \text{bite}_{g,r} + \varepsilon_{t,g,r}\)

- **Baseline:** mw workers those with wages \(\leq 1.05 \times \text{mw}\)
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Results with controls: II

\[
\log \frac{w_{t,g,r}}{\log w_{t-D,g,r}} = \sum_{j=1}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

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Alternative samples and definitions with control II

\[
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  - Separately by gender
  - Exclude final 6 sample years (w/ sub-state mws)
  - Unbalanced panel of (g, r)
Results with controls: III

$$\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \varepsilon_{t,g,r}$$

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Alternative samples and definitions with control III

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \epsilon_{t,g,r}
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Extended specification

Previously had 6 period change in wage on 6 lagged shocks. Now include 12 lags

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{12-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
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Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \( j \) and \( j - 1 \), except for the impact estimate \( \mu_0 \)

Expected hump-shaped pattern: impact effect doubles in \( \approx 2.5 \) years then falls to zero
Alternative samples and definitions from extended spec

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{12-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
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Conclusions

What is the impact of the mw on inequality?

- **Empirical motivation:** two new facts in the national time series
  - minimum wage helps shape U.S. college wage premium
  - incorporating mw improves fit of “The race” + reduces trend break in SBTC

- **Theoretically:**
  - on impact, ↑ mw raises wages more for groups more bound by it
  - over time, this wage elasticity wrt the mw rises due to indirect effects

- **Empirically:** Find evidence consistent with these dynamic predictions
  - using state and group level data
  - holding the composition of workers fixed

with magnification elasticity doubling after ≈ 2.5 years
Empirical Appendix
Regression Models for the College Wage Premium
Using dual of composition-adjusted changes in wages
Instrumenting with efficiency-unit populations

<table>
<thead>
<tr>
<th></th>
<th>1963-2016</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.584</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
</tr>
<tr>
<td>Real m.w.</td>
<td>-0.207</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
</tr>
<tr>
<td>Observations</td>
<td>54</td>
</tr>
</tbody>
</table>
### Regression Models for the College Wage Premium
#### Reduced-form specification

<table>
<thead>
<tr>
<th></th>
<th>1963-2016</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
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<tr>
<td>Relative supply</td>
<td>-0.555</td>
<td>-0.816</td>
<td>-0.632</td>
<td>-0.692</td>
<td>-0.356</td>
<td>-0.119</td>
<td>0.001</td>
<td>0.152</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.083)</td>
<td>(0.101)</td>
<td>(0.110)</td>
<td>(0.106)</td>
<td>(0.103)</td>
<td>(0.158)</td>
<td>(0.210)</td>
</tr>
<tr>
<td>Real m.w.</td>
<td>-0.307</td>
<td>-0.257</td>
<td>-0.132</td>
<td>-0.115</td>
<td>-0.126</td>
<td>-0.102</td>
<td>-0.136</td>
<td>-0.120</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.058)</td>
<td>(0.047)</td>
<td>(0.058)</td>
<td>(0.038)</td>
<td>(0.032)</td>
<td>(0.040)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Observations</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
</tbody>
</table>
Results: predicted college premium alternative supply

Using dual of composition-adjusted changes in wages
Instrumenting with efficiency-unit populations
Results: predicted college premium reduced form

Reduced-form specification

- College premium
- Predicted w/out mw
- Predicted with mw

Year: 1960, 1980, 2000, 2020

Graph showing the predicted college premium over time with and without a variable mw.
Impact of minimum wage is at least as robust as impact of supply
Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)...
... but replacing FLSA mw with average across states

<table>
<thead>
<tr>
<th></th>
<th>1963-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.459</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
</tr>
<tr>
<td>Minimum wage</td>
<td>-0.150</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
</tr>
<tr>
<td>Observations</td>
<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.947</td>
</tr>
</tbody>
</table>

Slight increase in elasticity wrt real minimum wage
Replace $\log \frac{w_{ht}}{w_{lt}}$ with $\log w_{ht}$ and with $-\log w_{lt}$.

Regression Models for the College and Non-College Wages

<table>
<thead>
<tr>
<th></th>
<th>College premium (a)</th>
<th>College wage (b)</th>
<th>Non-college wage (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real m.w.</td>
<td>-0.220 (0.048)</td>
<td>-0.104 (0.059)</td>
<td>-0.117 (0.058)</td>
</tr>
<tr>
<td>Observations</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
</tbody>
</table>

$\uparrow mw \Rightarrow \uparrow$ non-college average wage but also $\downarrow$ college wage
\[
\frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]
No controls female sample

$$\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}$$
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times b_{t,g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]
\[ \log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times b_{t,g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r} \]
No controls mw cutoff of 1.10

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
\]
No controls mw cutoff of 1.15

\[
\log \left( \frac{w_{t,g,r}}{w_{t-D,g,r}} \right) = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \left( \frac{m_{t-j+1,r}}{m_{t-j,r}} \right) + \varepsilon_{t,g,r}
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta \text{bite}_{g,r} + \varepsilon_{t,g,r}
\]
Control I female sample

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta \text{bite}_{g,r} + \varepsilon_{t,g,r}
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta \text{bite}_{g,r} + \varepsilon_{t,g,r}
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\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta \text{bite}_{g,r} + \varepsilon_{t,g,r}
\]
Control I mw cutoff of 1.00

\[ \log \frac{W_{t,g,r}}{W_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r} \]
Control I mw cutoff of 1.10

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\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \varepsilon_{t,g,r}
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\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]
Control II female sample

$$\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}$$
Control II male sample

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_t,g,r
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
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\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]
Control II mw cutoff of 1.00

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times b_{t,g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t b_{t,g,r} + \alpha_t + \varepsilon_{t,g,r}
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\]
Control II mw cutoff of 1.15

\[ \log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times bit_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t bit_{g,r} + \alpha_t + \epsilon_{t,g,r} \]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \varepsilon_{t,g,r}
\]
Control III female sample

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]
Control III male sample

\[
\log \frac{W_{t,g,r}}{W_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \epsilon_{t,g,r}
\]
\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times b_{i,g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t b_{i,g,r} + \alpha_{t,r} + \varepsilon_{t,g,r}
\]
Control III unbalanced sample

$$\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D} \mu_j \times b_{t,g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t b_{t,g,r} + \alpha_{t,r} + \varepsilon_{t,g,r}$$
\[ \log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r} \]
Control III mw cutoff of 1.10

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \epsilon_{t,g,r}
\]
Control III mw cutoff of 1.15

\[
\log \frac{w_{t,g,r}}{w_{t-D,g,r}} = \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_{t,r} + \varepsilon_{t,g,r}
\]
Extended specification: mw cutoff of 1.00

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
Extended specification: mw cutoff of 1.10

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_t,g,r
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \( j \) and \( j - 1 \), except for the impact estimate \( \mu_0 \)
Extended specification: mw cutoff of 1.15

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \( j \) and \( j - 1 \), except for the impact estimate \( \mu_0 \).
Extended specification: non-college sample

\[
\log \frac{w_{t, g, r}}{w_{t-6, g, r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g, r} \times \log \frac{m_{t-j+1, r}}{m_{t-j, r}} + \zeta_t \text{bite}_{g, r} + \alpha_t + \varepsilon_{t, g, r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
Extended specification: female sample

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \( j \) and \( j - 1 \), except for the impact estimate \( \mu_0 \)
Extended specification: male sample

\[
\log \frac{W_{t,g,r}}{W_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
Extended specification: 1979-2010 sample

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
Extended specification: unbalanced sample

\[
\log \frac{w_{t,g,r}}{w_{t-6,g,r}} = \sum_{j=-6}^{11} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{t-j+1,r}}{m_{t-j,r}} + \zeta_t \text{bite}_{g,r} + \alpha_t + \varepsilon_{t,g,r}
\]

Notes: Point estimates and 95% CIs averaging coefficients, averaging two half years \(j\) and \(j - 1\), except for the impact estimate \(\mu_0\)
Theoretical Appendix
Proposition

Consider an economy in steady state at date 0 that faces a small one-time change from $m$ to $m' > m$. Then for any skill $s$ that was bound by $m$:

1. For all $t \geq 0$ two job ladders coexist, with first rungs $m$ (old) and $m'$ (new).
2. The share on each rung $j$ summed across the two ladders is constant across $t$.
3. The share on rung $j$ of the new ladder rises (weakly) in $t$.
4. At each rung $j$, wage on the new job ladder is higher than on the old one.
5. The elasticity of the average wage wrt the minimum wage rises in $t$.
6. On impact, this elasticity equals the share of income earned at $m$ (the “bind”).

And, if $m' < m$, then

- 1, 2, and 3 are identical.
- 4 and 5 are reversed.
- 6: the instantaneous elasticity of the average wage $= 0$. 
Transitions (for one $s$ bound by $m$)

- Economy in steady state at date 0

pre shock
$t = -1$
Transitions (for one s bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

On impact, the first rung disappears on the original ladder and appears, higher, on the new ladder (direct effect)

$$\log \left( \frac{w_{0^+,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) \equiv B_{-1,s}$$

share of wage income earned at mw before shock
Transitions (for one s bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

- One period later, the second rung on the original ladder starts emptying out as the second rung on the new ladder starts filling in (small indirect effect)

\[
\log \left( \frac{w_{1,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) > B_{-1,s}
\]
Transitions (for one $s$ bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

... converging to the new steady state, w/ all on the new job ladder, with the importance of the indirect effect growing each period

$$
\log \left( \frac{w_{t',s}}{w_{t-1,s}} \right) / \log \left( \frac{m'}{m} \right) > \log \left( \frac{w_{t,s}}{w_{t-1,s}} \right) / \log \left( \frac{m'}{m} \right) \quad \text{for all} \quad t' > t \geq 0
$$
Transition proof (for given $s$) for a small $\uparrow m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g_{t,j}'$ rung $j$ shares on original, new ladders at $t$

$$
\Rightarrow g_{t,0}, j + g_{t,0}', j = g_j \text{ for all } j \geq 0
$$

For some $t \geq 0$: suppose $g_{t,j}, j + g_{t,j}', j = g_j$ for all $j \geq 0$

$$
\Rightarrow \text{date } t+1 \text{ and for any } j > 1, \text{ we have } g_{t+1,j}, j = g_{t,j} (1 - \delta)(1 - \lambda e) + g_{t-1,j} (1 - \delta) \lambda e$

$g_{t+1,j}', j = g_{t,j}' (1 - \delta)(1 - \lambda e) + g_{t-1,j}' (1 - \delta) \lambda e$

and if $j = 1$ replace $(1 - \delta) \lambda e / \lambda u \text{[and note that }] g_{t,0} = 0$

$$
\text{Summing these expressions yields } g_{t+1,j} + g_{t+1}', j = g_j (1 - \delta)(1 - \lambda e) + g_j - 1 (1 - \delta) \lambda e = g_j
$$

where the final equality follows from the steady-state derivation of $g_j$ back
Transition proof (for given $s$) for a small $\uparrow m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g'_{t,j}$ rung $j$ shares on original, new ladders at $t$

- At $t = 0$: first rung + unemployment fully reallocate (and nothing else)
  - $\Rightarrow g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$
Transition proof (for given $s$) for a small $\uparrow m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g'_{t,j}$ rung $j$ shares on original, new ladders at $t$

At $t = 0$: first rung + unemployment fully reallocate (and nothing else)

$\implies g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$

For some $t \geq 0$: suppose $g_{t,j} + g'_{t,j} = g_j$ for all $j \geq 0$

$\implies$ date $t + 1$ and for any $j > 1$, we have

$g_{t+1,j} = g_{t,j}(1 - \delta)(1 - \lambda_e) + g_{t-1,j-1}(1 - \delta)\lambda_e$

$g'_{t+1,j} = g'_{t,j}(1 - \delta)(1 - \lambda_e) + g'_{t-1,j-1}(1 - \delta)\lambda_e$

and if $j = 1$ replace $(1 - \delta)\lambda_e$ w/ $\lambda_u$ [and note that $g_{t,0} = 0$]

$\implies$ Summing these expressions yields

$g_{t+1,j} + g'_{t+1,j} = g_j(1 - \delta)(1 - \lambda_e) + g_{j-1}(1 - \delta)\lambda_e = g_j$

where the final equality follows from the steady-state derivation of $g_j$
Distribution and the average wage

- Density $g_s(w_j)$ satisfies

$$[\delta_s + (1 - \delta_s)\lambda_{se}] g_s(w_{1,s}) = \lambda_{su} g_s(w_{0,s})$$
$$[\delta_s + (1 - \delta_s)\lambda_{se}] g_s(w_{j+1,s}) = (1 - \delta_s)\lambda_{se} g_s(w_{j,s}) \text{ for } j \geq 1$$

- Unemployment rate

$$g_s(w_{0,s}) = \frac{\delta_s}{\delta_s + \lambda_{su}}$$

- Share at each rung

$$g_s(w_{j,s}) = \left(\frac{(1 - \delta_s)\lambda_{se}}{\delta_s + (1 - \delta_s)\lambda_{se}}\right)^{j-1} \frac{\lambda_{su}}{\delta_s + (1 - \delta_s)\lambda_{se}} \frac{\delta_s}{\delta_s + \lambda_{su}} \text{ for } j \geq 1$$

- Average wage $w_s \equiv \frac{1}{1 - g_s(w_{0,s})} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$ among the employed

$$w_s = \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\lambda_{se}} m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\lambda_{se}}\right) P_s$$
Burdett and Mortensen (1998) + binding minimum wage

- Equation (2.10) in van den Berg and Ridder (1998), eqm earnings density

\[ g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \text{ for all } w \in [m, w_{\text{max}}] \]

with maximum wage

\[ w_{\text{max}} \equiv \left( \frac{\delta}{\delta + \lambda_e} \right)^2 m + \left( 1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P \]

- Average wage is then

\[ w = \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left( 1 - \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} \right) P \]

weighted avg of \( m \) and \( P \) as in baseline model, but weights depend on \( m \)

back to steady state
CS across steady states (including unemployment effects)

Across steady state effects of changes in the real mw, supply, demand

- Distribution of wages for skill $s$ (whether or not $m$ affects unemployment)

\[ W_s(c) < W_s(c') \Rightarrow \frac{d \left[ \frac{W_s(c)}{W_s(c')} \right]}{dm} > 0 \]

where $W_s(c)$ is wage at percentile $c$ of employed skill $s$ workers

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back to CS across steady states  
equivalent result in Burdett and Mortensen (1998)
Burdett and Mortensen (1998) + binding minimum wage

- $W_c(m)$: wage at centile $c \in [0, 100]$

$$W_c(m) = P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2$$

- Hence

$$\frac{W_c'(m)}{W_c(m)} = \frac{P - (P - m) \left( \frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{p - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2}$$

- Differentiating with respect to $m$ yields

$$\frac{d \left[ W_c'(m)/W_c(m) \right]}{dm} < 0 \iff c' > c$$

- As in baseline model, $W_c(m)$ is log-submodular in $(c, m)$

- This result has been shown quantitatively in Engbom and Moser (2021)
Suppose mw doesn’t affect unemployment

\[ d \log VMP L_s \] the same as in neoclassical model, replacing \( L_s \) with \( (1 - u_s)L_s \)

Given focus on canonical model, impose those assumptions

\[ + \text{ assumptions s.t. } \beta = \beta_s \] [see SS Magnification elasticity]

Then across steady states (to a first-order approximation)

\[
\Delta \log \left( \frac{w_{ht}}{w_{Lt}} \right) = \beta Bite \Delta \log m_t - \beta_L \Delta \log \left( \frac{L_{ht}}{L_{Lt}} \right) + \beta_A \Delta t + \iota_t
\]

where \( Bite \equiv B_{ht} - B_{Lt} \)

“Race between education, technology, and the minimum wage”

\[ \text{Extended canonical model} \]
Comparative statics across steady states: $\beta_L$

- Elasticity wrt relative supply

$$\beta_L \equiv \frac{1}{\eta} (1 - \beta_m B_{\ell}) \frac{P_h (1 - u_h) L_h}{Y} + \frac{1}{\eta} (1 - \beta_m B_h) \frac{P_\ell (1 - u_\ell) L_\ell}{Y}$$

- When $B_s = 0$ for both $s$, this is just $1/\eta$ as in canonical model