The Race Between Education, Technology, and the Minimum Wage

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Where we’re going

- What is the dynamic impact of the minimum wage on inequality (and, specifically, the college premium)?

- Theory: effect is small on impact and grows over time

- Empirics: document these dynamic effects

- Changing minimum wages have small effects on impact. Over longer horizons, however, these effects become sizable
How we get there

1 Motivation (The Race Between...): In national time series, the real minimum wage shapes the evolution of U.S. college premium and resolves a well-known “puzzle”

2 Theory: Job-ladder model with many skills
   - On impact $\uparrow$ m.w. $\Rightarrow$ $\uparrow$ in wages for those bound by it only \textit{(direct effect)}
   - Over time, workers move up the job ladder $\Rightarrow$ magnified effect \textit{(indirect effect)}
   - Incorporating labor supply, demand: extended canonical estimating equation

3 Main Results: Estimation + model calibration using state-level data
   - Elasticity of college premium with respect to m.w. grows 2 - 3x over time

4 Verify mechanism: Document dynamic impact of m.w. over full wage distribution
   - Over time, impact grows at the bottom and moves up the distribution
Contributions

- **Canonical model** (Tinbergen 74, Katz and Murphy 92, ...) **including minimum wage** (Autor, Katz, Kearney 08)
  - While supply and demand remain crucial, so too is the minimum wage
  - Minimum wage helps resolve the (apparent) rapid slowing SBTC in the 1990s

- **Impact of minimum wage on inequality**
  - Meyer, Wise, 83; DiNardo et al., 96; Lee, 99; Card, DiNardo, 02; *Autor ea.*, 16; *Cengiz ea.* 19; Dube, 19; Fortin ea. 21; Chen, Teulings 22; ...
  - Direct effect dominates in short run, but indirect effect grows over time
  - Effects move up wage distribution over time

- **Growing macro-labor literature of monopsony using quantitative models**
  - Haanwinckel, 20; *Engbom*, Moser, 21; Ahlfeldt et al., 22; Berger et al., 22; *Hurst et al.*, 22; Trottner, 22; ...
  - Job-ladder model like EM, but focus on dynamics
  - Dynamics like HKPW, but driven by job ladder rather than putty-clay capital
Motivation:
National time-series variation
Canonical model + minimum wage

I consider (for now atheoretical) regressions of the form

$$\log \left( \frac{w_{ht}}{w_{ℓt}} \right) = \alpha + \beta_m \log m_t + \beta_L \log \left( \frac{Supply_{ht}}{Supply_{ℓt}} \right) + \gamma_1 t + [...] + \iota_t$$

using national time-series, $t$, variation across two groups of workers, $h$ and $ℓ$

- $\log w_{ht}$ and $\log w_{ℓt}$ are measures of average log wages
- $\log Supply_{ht}$ and $\log Supply_{ℓt}$ are measures of labor supply
- $m_t$ is a measure of the real minimum wage at the national level

Measurement

- Supply and wages are composition adjusted
- National real minimum wage
  - For each state use the max of the state and national statutory minima
  - ... then average across states using time-invariant weights
  - ... and apply the GDP deflator

March CPS data spanning working years 1963 - 2017
Results: predicted college premium (estimated on 63-87)

College premium

Predicted w/out mw

Predicted with mw
### Regression Models for the College Wage Premium

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</thead>
<tbody>
<tr>
<td>Relative supply</td>
<td>-0.596 (0.116)</td>
<td>-0.441 (0.044)</td>
<td>-0.595 (0.137)</td>
<td>-0.557 (0.054)</td>
<td>-0.601 (0.052)</td>
<td>-0.531 (0.102)</td>
<td>-0.510 (0.049)</td>
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<tr>
<td>Real m.w.</td>
<td>-0.236 (0.091)</td>
<td></td>
<td>-0.196 (0.040)</td>
<td>-0.160 (0.059)</td>
<td>-0.136 (0.055)</td>
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<tr>
<td>Real FLSA m.w.</td>
<td></td>
<td></td>
<td></td>
<td>-0.162 (0.039)</td>
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<tr>
<td>Time</td>
<td>0.024 (0.005)</td>
<td>0.018 (0.001)</td>
<td>0.022 (0.006)</td>
<td>0.021 (0.001)</td>
<td>0.020 (0.001)</td>
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<tr>
<td>Constant</td>
<td>0.013 (0.013)</td>
<td>0.024 (0.007)</td>
<td>0.034 (0.021)</td>
<td>0.032 (0.009)</td>
<td>0.021 (0.014)</td>
<td>0.035 (0.029)</td>
<td>0.032 (0.009)</td>
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<td>Time Polynom.</td>
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<td>1</td>
<td>2</td>
<td>3</td>
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<td>Observations</td>
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<td>55</td>
<td>25</td>
<td>55</td>
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<tr>
<td>R-squared</td>
<td>0.324</td>
<td>0.950</td>
<td>0.517</td>
<td>0.968</td>
<td>0.969</td>
<td>0.969</td>
<td>0.963</td>
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</tbody>
</table>

**Table 1:** Regression Models for the College Wage Premium  
*Notes:* Results of estimating (1) using OLS. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real m.w.” and “Real FLSA m.w.” are the logs of the real minimum wage and real FLSA minimum wage. The sample is 1963-1987 in columns (a) and (b) and 1963-2017 elsewhere. “Time Polynom.” refers to the degree of the polynomial of time; the coefficient on the linear trend on “Time” is omitted from the table whenever this polynomial is of degree 2 or greater.

- **Sizable elasticity of the college premium wrt the real minimum wage**
  - e.g., 27% ↓ in real minimum wage 1979 – 89 ⇒ 3.7 – 5.3% ↑ in college premium
Summary + robustness

**Summary:**

1. Relative supply growth fluctuations + trend demand growth crucial drivers of college premium, but changes in real minimum wage are also important.
2. Less dramatic slowing of SBTC (more generally, improved out-of-sample fit).

**Sensitivity:**

- ... higher-dimensional polynomials of time
- ... using two alternative measures of relative supply
- ... using Autor, Katz, and Kearney (2008) data
Theory
Supply and demand:
- Exogenous supply $L_{st}$ of homogeneous skill $s = 1, \ldots, S$ workers
- Skill-time-specific productivity $A_{st}$ shaping relative demand
Framework

- **Supply and demand:**
  - Exogenous supply $L_{st}$ of homogeneous skill $s = 1, \ldots, S$ workers
  - Skill-time-specific productivity $A_{st}$ shaping relative demand

- **Job ladder:**
  - Zero discount factor: analysis of transitions to sequence of aggregate shocks
  - Worker can be employed or unemployed with exogenous separation ($\delta_s$) and job-finding rates ($\lambda_{su}$ unemployed and $\lambda_{se}$ employed)
  - Generalized Nash bargaining btw new worker-firm match ($\beta_s = \text{worker weight}$) over fixed real wage
  - Worker outside option is unemployment benefit or wage in current match
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- **Real minimum wage:** $m_t$
  - Impose binding mw for given $s$ to avoid taxonomy (only in description)
  - Impose no employment effect of mw ($m_t < VMPL_{st}$), but can be generalized
Framework

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- Discrete time: within $t$, separation shocks occur first, then new matches realized for those workers who did not separate in $t$
Steady-state characterization

- Wage ladder across “rungs”
- First rung is the minimum wage...
- ... and move up over time (if no separation shock)
Comparative statics across steady states

Across steady-state effects of small change in real minimum wage

Elasticity is \( \frac{d \log w_s}{d \log m} = \beta_{ms} \text{share}_s \) where

\[
\beta_{ms} \equiv \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} > 1
\]

let \( W_s(c) \) be wage at percentile \( c \) of employed skill \( s \) workers

\[
W_s(c) < W_s(c') \Rightarrow \frac{d [W_s(c)/W_s(c')]}{dm} > 0
\]
Transitions

- Economy in steady state at date 0
Transitions

- Economy in steady state at date 0; a small one-time change to $m' > m$

- In all future (finite) dates:
  1. two ladders co-exist: original (pre-shock steady state) and new
  2. each rung $j$ on new ladder is “higher” (higher wage) than on old ladder
  3. share on each rung (summed across ladders) is constant across time
Transitions

- Economy in steady state at date 0; a small one-time change to $m' > m$

On impact, the first rung disappears on the original ladder and appears, higher, on the new ladder (direct effect)

\[
\log \left( \frac{w_{s0^+}}{w_{s0^-}} \right) / \log \left( \frac{m'}{m} \right) \equiv \frac{\text{share}_{s0^-}}{\text{instantaneous elasticity}}
\]

share of wage income earned at m.w. before shock
Economy in steady state at date 0; a small one-time change to $m' > m$

One period later, the second rung on the original ladder starts emptying out as the second rung on the new ladder starts filling in (small indirect effect)

$$\log \left( \frac{w_{s1}}{w_{s0^-}} \right) / \log \left( \frac{m'}{m} \right) > share_{s0^-}$$
Economy in steady state at date 0; a small one-time change to \( m' > m \)

... converging to the new steady state, w/ all on the new job ladder, with the importance of the indirect effect growing each period

\[
\frac{\log \left( \frac{W_{st'}}{W_{s0^-}} \right)}{\log \left( \frac{m'}{m} \right)} > \frac{\log \left( \frac{W_{st}}{W_{s0^-}} \right)}{\log \left( \frac{m'}{m} \right)} \quad \text{for all} \quad t' > t \geq 0
\]
Decompose the $D$-period elasticity of the college premium wrt to a change in the real m.w. into

$$\log \left( \frac{w_{hD}}{w_{\ell D}} / \frac{w_{h0^-}}{w_{\ell0^-}} \right) / \log \left( \frac{m'}{m} \right) \equiv \beta_{m,D} \times b_{0^-}$$

the initial minimum wage bite and the magnification elasticity

- $b_{0^-} \equiv \text{share}_{h0^-} - \text{share}_{\ell0^-}$ is the minimum wage bite
- $\beta_{m,0} = 1$
- $d\beta_{m,D}/dD > 0$ under some conditions
Brining in Supply and Demand

- $d \log P_s$ the same as in neoclassical model, replacing $L_{st}$ with $(1 - u_s)L_{st}$

- Given focus on canonical model, impose those assumptions

- Then across steady states (to a first-order approximation)

$$
d \log \left( \frac{w_{ht}}{w_{lt}} \right) = \beta_{m,\infty} b d \log m_t - \beta_{L,\infty} d \log \left( \frac{L_{ht}}{L_{lt}} \right) + \beta_{A,\infty} t + \iota_t
$$

where $b \equiv \text{share}_{ht} - \text{share}_{lt}$ is the “minimum wage bite”

- “Race between education, technology, and the minimum wage”
  - Extended canonical model
Empirics leveraging state variation
I: College premium
Baseline specification

- Bring extended canonical model to data using many regions $r$ and dates $t$
- Suppose skill $s$ output freely traded across regions: $P_{st} = P_{srt}/A_{srt} \forall r$

$$\Delta_D \log \left( \frac{w_{hrt}}{w_{lrt}} \right) = \gamma_{t,D} + \gamma_{m,D} b_{rt} \Delta_D \log m_{rt} + […] + \iota_{rt,D}$$

- $r$ indexes region (fifty states) and $t$ time (1979 – 2018 excluding 94 and 95)
- $\Delta_D x_t \equiv x_{t+D} - x_t$ is the $D$-year difference for any $x_t$
- $b_{rt}$ is the share of income earned at mw. for high - low education in $r$ at $t$
- $m_{rt}$ is real minimum wage in state $r$ in year $t$
- $\gamma_{t,D}$ absorbs the (national) change in the relative skill price
  - robustness 1: state-specific measures of relative supply changes
  - robustness 2: state FE controlling for linear deviations from national SBTC
  - etc.
- Change in regional wages common across skills cancels out of triple difference
Baseline specification: Instrument

$$\Delta_D \log \left( \frac{W_{hrt}}{W_{l\ell rt}} \right) = \gamma_{t,D} + \gamma_{m,D} b_{rt} \Delta_D \log m_{rt} + [...] + \iota_{D,rt}$$

- **Issue 1**: $\Delta_D \log m_{rt}$ cumulates across many changes over time
- **Issue 2**: $b_{rt}$ measured with error, likely correlated w/ dependent variable
- **Instrument**: $b_{rt-1}(\log m_{rt+1} - \log m_{rt})$
  - lagged bite in the spirit of Autor, Manning, Smith (2016)
  - one-year change in mw in spirit of local projection (e.g. Jorda, 2005)
Baseline specification: Instrument

\[
\Delta_D \log \left( \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_{t,D} + \gamma_{m,D} b_{rt} \Delta_D \log m_{rt} + [...] + \nu_{D,rt}
\]

- **Issue 1:** \( \Delta_D \log m_{rt} \) cumulates across many changes over time

- **Issue 2:** \( b_{rt} \) measured with error, likely correlated w/ dependent variable

- **Instrument:**
  \[ b_{rt-1} (\log m_{rt+1} - \log m_{rt}) \]
  - lagged bite in the spirit of Autor, Manning, Smith (2016)
  - one-year change in mw in spirit of local projection (e.g. Jorda, 2005)

- **Data:** CPS Merged Outgoing Rotation Groups

- Define workers as earning \( m_{rt} \) if their wages are \( \leq 1.15 \times m_{rt} \)

- Cluster standard errors by state and weigh each state by avg across years of its share of national population
Baseline specification: Reduced form

\[
\log \left( \frac{W_{\text{hrt}+D}}{W_{\text{lr}rt+D}} / \frac{W_{\text{hrt}}}{W_{\text{lr}rt}} \right) = \gamma_{t,D}^{RF} + \gamma_{m,D}^{RF} b_{rt-1} \log \left( \frac{m_{rt+1}}{m_{rt}} \right) + \iota_{D,rt}^{RF}
\]

Notes: Point estimates and 90% CIs from estimating RF specification separately by time difference

- **Point estimates grow (to \( \approx 5.2 \) in 3-year differences)**
- **No evidence of pre-existing trends**
Baseline specification: First stage

\[ b_{rt} \log \left( \frac{m_{rt+D}}{m_{rt}} \right) = \gamma_{t,D}^{FS} + \gamma_{m,D}^{FS} b_{rt-1} \log \left( \frac{m_{rt+1}}{m_{rt}} \right) + \iota_{D,rt}^{FS} \]

Notes: Point estimates and 90% CIs from estimating first-stage separately by time difference

- **Subsequent ↑ in first stage ⇒ some of later ↑ in RF coefficients**
Baseline specification: 2SLS

\[
\log \left( \frac{W_{hrt+D}}{W_{lrt+D}} / \frac{W_{hrt}}{W_{lrt}} \right) = \gamma_{t,D} + \gamma_{m,D} \left[ b_{rt} \log \left( \frac{m_{rt+D}}{m_{rt}} \right) \right] + \iota_{D,rt}
\]

Notes: Point estimates and 90% CIs from estimating baseline specification separately by time difference. First-stage KP F $> 16$ in each regression.

- **Point estimates grow (to $\approx 3.8$ in 3-year differences)**
Baseline specification: Sensitivity

Annual specification sensitivity

- incorporating state-specific supply
- controlling for the bite
- nominal minimum wage increases
- state-specific linear deviation of SBTC from national rate

Quarterly specification
Alternative specification: (finite) distributed lag

- **Distributed lag model** identifies impact on one-year change in college premium of contemporaneous + lagged one-year changes in the real minimum wage

## 1. Unrestricted distributed lag model

\[
\Delta_1 \log \left( \frac{w_{hrt}}{w_{\ell rt}} \right) = \gamma_t + \sum_{j=0}^{D} \gamma_{m,j} b_{r,t-j} \Delta_1 \log m_{r,t-j} + [...] + \iota_{rt}
\]

- Instruments are \( b_{r,t-(D+1)} \Delta_1 \log m_{r,t-j} \) for each \( j \)

- Long-run magnification elasticity\(^*\) is \( \sum_{j=0}^{D} \gamma_{m,j} \)

\(^*\) (partial effect, holding future mw. bites fixed)
Alternative specification: (finite) distributed lag

- **Distributed lag model** identifies impact on the one-year change in the college premium of lagged one-year changes in the real minimum wage.

- **Restricted** distributed lag model imposing that $\gamma$s decline **linearly** from $\gamma_{m,0}$ to zero in $D + 1$ lags, i.e.,

  $$\gamma_{m,j} = \frac{D + 1 - j}{D + 1} \gamma_{m,0} \quad \text{for} \ j = 1, \ldots, D$$

  yields

  $$\Delta_1 \log \left( \frac{W_{hrt}}{W_{lrt}} \right) = \gamma_t + \gamma_{m,0} \sum_{j=0}^{D} \frac{D + 1 - j}{D + 1} b_{r,t-j} \Delta_1 \log m_{r,t-j} + [...] + \iota_{rt}$$

- Instrument is $\sum_{j=0}^{D} \frac{D + 1 - j}{D + 1} b_{r,t-(D+1)} \Delta_1 \log m_{r,t-j}$

- Long-run magnification elasticity* is $\sum_{j=0}^{D} \gamma_{m,j} = \gamma_{m,0} \left( 1 + \frac{D}{2} \right)$

* (partial effect, holding future mw. bites fixed)
Unrestricted distributed lag

\[ \Delta_1 \log \left( \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_t + \sum_{j=0}^{D} \gamma_{m,j} b_{r,t-j} \Delta_1 \log m_{r,t-j} + [...] + \iota_{rt} \]

Notes: Point estimates and 90% CIs

- Noisy estimates that fall from about 1.6 to zero over three years
- 3-year magnification elasticity of 3.1 (standard error 1.3)
  - Slightly ↓ than long-difference specification
  - As expected given first-stage results
Restricted distributed lag

\[ \Delta_1 \log \left( \frac{w_{hrt}}{w_{\ell rt}} \right) = \gamma_t + \gamma_{m,0} \sum_{j=0}^{D} \frac{D + 1 - j}{D + 1} b_{r,t-j} \Delta_1 \log m_{r,t-j} + [...] + \nu_{rt} \]

- \( \hat{\gamma}_{m,0} = 1.7 \) (standard error 0.51)
- \( \Rightarrow \) 3-year magnification elasticity of 3.4 (standard error 1.0)
  - Again, slightly lower than long-difference specification
Empirics leveraging state variation
II: Wage distribution
Wage distribution

- Empirical results consistent w/ prediction of slow-developing wage spillovers
  - But not conclusive on these spillovers

- Theory predicts that an increase in the minimum wage
  1. first affects wage centiles bound by it
  2. slowly affects wage centiles above
Empirical results consistent w/ prediction of slow-developing wage spillovers
  ▶ But not conclusive on these spillovers

Theory predicts that an increase in the minimum wage
  1 first affects wage centiles bound by it
  2 slowly affects wage centiles above

Estimate elasticity of wage at each centile of the wage distribution wrt real mw as a function of distance from mw (separately by time difference $D$)

$$\Delta_D \log W_{crt} = \sum_{p=0}^{K} \beta_D^p \left[ \log(W_{crt-1}) - \log(m_{rt-1}) \right]^p \Delta_1 m_{rt} + FE + \iota_{D,.crt}$$

▶ $W_{crt}$ is the wage at centile $c$ of the wage distribution in region $r$ at time $t$
▶ $\Delta_D \log W_{crt} \equiv \log W_{crt+D} - \log W_{crt}$
▶ Cluster by state, weigh states by average share of national hours worked
Wage distribution: one-year difference

Notes: Point estimates and 90% CIs. Fixed effect: year. Estimated on the sample of centiles 2-95. Plotted at the 1st, 5th, 10th, ..., 95th centiles of the sample gap between wage centile and mw.

- Sizable elasticity near the mw, dissipates a bit above 50% above mw.
- Results very similar for sample of men and women
  - Gender differences in Lee (1999) and Autor et al. (2016) appear to arise from differences in distributions, not elasticities
Wage distribution: one- vs. two-year difference

Notes: Point estimates and 90% CIs. Fixed effect: year. Estimated on the sample of centiles 2-95. Plotted at the 1st, 5th, 10th, ..., 95th centiles of the sample gap between wage centile and mw.

- Effect grows substantially with time
- Could this result from time aggregation?
Wage distribution: various quarterly differences

1 and 2-quarter differences

1 and 4-quarter differences

1 and 6-quarter differences

1 and 10-quarter differences
Conclusions

What is the impact of the m.w. on the inequality?

- Minimum wage helps shape U.S. college wage premium and its variation across states, with effects growing over time
- Impacts of minimum wage grow and move up distribution over time

Open question: Are the greater spillover effects of the m.w.

- in Brazil (Engbom and Moser, 2022)
- in the 1980s in the U.S. (Fortin, Lemieux, and Lloyd, 2021)

caused by dynamic effects + monotonic changes in m.w. over time?
Empirical Appendix
### Regression Models for the College Wage Premium

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<tr>
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<th>1963-2017</th>
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<tr>
<td></td>
<td>(a)</td>
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<tr>
<td>Relative supply</td>
<td>-0.556</td>
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<td>(0.054)</td>
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<tr>
<td>Real m.w.</td>
<td>-0.197</td>
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<td></td>
<td>(0.041)</td>
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Using composition-adjusted changes in *efficiency-unit hours worked*

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<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
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<td>-0.519</td>
<td>-0.439</td>
<td>-0.575</td>
<td>-0.575</td>
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<td>(0.109)</td>
<td>(0.047)</td>
<td>(0.112)</td>
<td>(0.057)</td>
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<tr>
<td><strong>Real m.w.</strong></td>
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<tr>
<td></td>
<td>-0.271</td>
<td>-0.212</td>
<td>-0.178</td>
<td>-0.134</td>
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<td></td>
<td>(0.089)</td>
<td>(0.042)</td>
<td>(0.060)</td>
<td>(0.055)</td>
</tr>
<tr>
<td><strong>Real FLSA m.w.</strong></td>
<td>-0.172</td>
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<td></td>
<td>(0.041)</td>
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<tr>
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<td>0.018</td>
<td>0.023</td>
<td>0.040</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.007)</td>
<td>(0.020)</td>
<td>(0.009)</td>
</tr>
<tr>
<td><strong>Time Polynom.</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>25</td>
<td>55</td>
<td>25</td>
<td>55</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.282</td>
<td>0.946</td>
<td>0.534</td>
<td>0.965</td>
</tr>
</tbody>
</table>
Regression Models for the College Wage Premium Using composition-adjusted changes in efficiency-unit populations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative supply</td>
<td>-0.786 (0.167)</td>
<td>-0.317 (0.066)</td>
<td>-0.798 (0.152)</td>
<td>-0.594 (0.083)</td>
<td>-0.804 (0.097)</td>
<td>-0.573 (0.140)</td>
<td>-0.491 (0.078)</td>
</tr>
<tr>
<td>Real m.w.</td>
<td>-0.247 (0.078)</td>
<td>-0.302 (0.058)</td>
<td>-0.270 (0.065)</td>
<td>-0.135 (0.055)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real FLSA m.w.</td>
<td></td>
<td></td>
<td></td>
<td>-0.235 (0.054)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>0.026 (0.006)</td>
<td>0.014 (0.002)</td>
<td>0.025 (0.005)</td>
<td>0.020 (0.002)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.001 (0.019)</td>
<td>0.006 (0.009)</td>
<td>0.023 (0.023)</td>
<td>0.022 (0.009)</td>
<td>-0.006 (0.018)</td>
<td>0.052 (0.032)</td>
<td>0.021 (0.009)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Observations</td>
<td>25</td>
<td>55</td>
<td>25</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.423</td>
<td>0.909</td>
<td>0.633</td>
<td>0.936</td>
<td>0.940</td>
<td>0.964</td>
<td>0.930</td>
</tr>
</tbody>
</table>
Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)

<table>
<thead>
<tr>
<th></th>
<th>1963-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.431</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
</tr>
<tr>
<td>Minimum wage</td>
<td>-0.113</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.253</td>
</tr>
<tr>
<td></td>
<td>(0.083)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
</tr>
<tr>
<td>Observations</td>
<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.942</td>
</tr>
</tbody>
</table>

Notes: This table estimates using relative wages, relative supply, and the nominal minimum wage from Autor et al. (2008). I deflate their nominal minimum wage using the GDP deflator from FRED rather than the GDP deflator in their replication package. Robust standard errors are reported.

The coefficient on the real minimum wage is negative, significant, and similar to my baseline estimate across all specifications up to and including a sextic polynomial of time except for the case of a third-degree polynomial of time in column (c), in which case the coefficient on the real minimum wage is insignificant and relatively small.

Impact of minimum wage is at least as robust as impact of supply
Robustness: Using data from AKK but alternative mw

Constructing the minimum wage using data across states

Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)...
... but replacing FLSA mw. with average across states

<table>
<thead>
<tr>
<th></th>
<th>1963-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.437</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
</tr>
<tr>
<td>Minimum wage</td>
<td>-0.144</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.310</td>
</tr>
<tr>
<td></td>
<td>(0.090)</td>
</tr>
<tr>
<td>Time Polynom.</td>
<td>1</td>
</tr>
<tr>
<td>Observations</td>
<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.942</td>
</tr>
</tbody>
</table>

Slight increase in elasticity wrt real minimum wage
National: separate regressions by education

Replace \( d \log \frac{w_{ht}}{w_{\ell t}} \) with \( d \log w_{ht} \) and with \(-d \log w_{\ell t}\)

### Regression Models for the College and Non-College Wages

<table>
<thead>
<tr>
<th></th>
<th>College premium</th>
<th>College wage</th>
<th>Non-college wage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>Real m.w.</td>
<td>-0.197</td>
<td>-0.079</td>
<td>0.118</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.059)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>Observations</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
</tbody>
</table>

Single difference requires stronger assumptions to interpret than double diff.

Minimum wage associated with rising non-college average wage, but also falling college wage
State robustness: RF specification

$$\log \left( \frac{W_{hrt+D}}{W_{\ell rt+D}} / \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt-1} \log \left( \frac{m_{rt+1}}{m_{rt}} \right) + \iota_{D,rt}$$

Notes: Point estimate and 90% CIs from estimating reduced-form specification separately by time difference.
State robustness: supply #1

Supply as dual of wages, to match income in state $r$ and time $t$ in the data.

Notes: Point estimate and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate.
State robustness: supply #2

Supply as *composition-adjusted measure of efficiency-unit hours worked*

Notes: Point estimate • and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate ◇.
Supply as composition-adjusted measure of efficiency-unit populations

Notes: Point estimate and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate.
State robustness: control for the bite $b_{rt-1}$

$$\Delta_D \log \left( \frac{w_{hrt}}{w_{lrt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt} \Delta_D \log m_{rt} + \gamma_D b_{rt} + \iota_D,rt$$

and instrument for the bite using its lagged value (all SW F stats $> 70$)

Notes: Point estimate $\bullet$ and 90% CIs from estimating above specification separately by time difference. Baseline estimate $\diamond$.
State robustness: increases in nominal minimum wages

\[ \Delta_D \log \left( \frac{W_{ht}}{W_{lt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt} \Delta_D \log m_{rt} + \gamma_{D,Index} b_{rt} \Delta_D \log Index_t + \iota_{D,rt} \]

- \( \text{Index}_t \) is the price index

- Second instrument is \( b_{rt-1} (\log \text{Index}_{t+1} - \log \text{Index}_t) \) (all SW F stats > 20)

Notes: Point estimate and 90% CIs from estimating above specification separately by time difference. Baseline estimate.
State robustness: state fixed effects

Absorbing linear deviations from national rate of SBTC

Notes: Point estimate • and 90% CIs from including a state FE in estimating equation separately by time difference. Baseline estimate ⧫.
Separate state-level regressions by education

Estimating separately by education, so that baseline = sum of estimates

Notes: Baseline estimates ○. Coefficients estimated on college wage X. Coefficients estimated on negative of non-college wage ♦.

Double difference requires stronger assumptions to interpret than triple diff. For instance, if m.w. increases occur in booms but not busts...
Quarterly data baseline 2SLS specification

\[
\log\left(\frac{W_{hrt+D}}{W_{ℓrt+D}} / \frac{W_{hrt}}{W_{ℓrt}}\right) = \gamma_{t,D} + \gamma_{m,D} \left[ b_{rt} \log\left(\frac{m_{rt+D}}{m_{rt}}\right) \right] + \iota_{D,rt}
\]

Notes: Point estimates and 90% CIs from estimating baseline specification separately by time difference using quarterly data

- Point estimates grow (to \( \approx 2.3 \) in 9-quarter differences)
Relation to national estimates

- **Comparison**: National specification in levels + m.w. not interacted with bite
- **In levels**: interact national m.w. with its average bite (-0.046)
  - Equivalent to dividing coefficient by average bite
  - National estimates btw −0.14 and −0.2 ⇒ magnification elasticity btw 3 and 4
  - National estimates broadly consistent with state-level estimates
Theoretical Appendix
Burdett and Mortensen (1998) + binding minimum wage

- Equation (2.10) in van den Berg and Ridder (1998), eqm earnings density

\[ g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \quad \text{for all } w \in [m, w_{\text{max}}] \]

with maximum wage

\[ w_{\text{max}} \equiv \left( \frac{\delta}{\delta + \lambda_e} \right)^2 m + \left( 1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P \]

- Average wage is then

\[ w = \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left( 1 - \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} \right) P \]

weighted avg of \( m \) and \( P \) as in baseline model, but weights depend on \( m \)
• $W_c(m)$: wage at centile $c \in [0, 100]$

$$W_c(m) = P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2$$

• Hence

$$\frac{W_c'(m)}{W_c(m)} = \frac{P - (P - m) \left( \frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{p - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2}$$

• Differentiating with respect to $m$ yields

$$\frac{d}{dm} \left[ \frac{W_c'(m)}{W_c(m)} \right] < 0 \iff c' > c$$

• As in baseline model, $W_c(m)$ is log-submodular in $(c, m)$
Distribution and the average wage

- Density \( g_s(w_j) \) satisfies

\[
\left[ \delta_s + (1 - \delta_s) \gamma_{se} \right] g_s(w_{1,s}) = \gamma_{su} g_s(w_{0,s})
\]

\[
\left[ \delta_s + (1 - \delta_s) \gamma_{se} \right] g_s(w_{j+1,s}) = (1 - \delta_s) \gamma_{se} g_s(w_{j,s}) \quad \text{for} \quad j \geq 1
\]

- Unemployment rate

\[
g_s(w_0) = \frac{\delta_s}{\delta_s + \gamma_{su}}
\]

- Share at each rung

\[
g_s(w_j) = \left( \frac{(1 - \delta_s) \gamma_{se}}{\delta_s + (1 - \delta_s) \gamma_{se}} \right)^{j-1} \frac{\gamma_{su}}{\delta_s + (1 - \delta_s) \gamma_{se}} \frac{\delta_s}{\delta_s + \gamma_{su}} \quad \text{for} \quad j \geq 1
\]

- Average wage \( w_s \equiv \frac{1}{1 - g_s(w_0)} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s}) \) among the employed

\[
w_s = \frac{\delta_s}{\delta_s + \beta_s (1 - \delta_s) \gamma_{se}} m + \left( 1 - \frac{\delta_s}{\delta_s + \beta_s (1 - \delta_s) \gamma_{se}} \right) P_s
\]
Comparative statics across steady states: $\beta_L$

- Elasticity wrt relative supply

$$\beta_{L,\infty} \equiv \frac{1}{\eta} (1 - \beta_m share_{\ell}) \frac{P_h(1 - u_h)L_h}{Y} + \frac{1}{\eta} (1 - \beta_m share_h) \frac{P_{\ell}(1 - u_{\ell})L_{\ell}}{Y}$$

- When $share_s = 0$ for both $s$, this is just $1/\eta$ as in canonical model
Transitions
Those on original job ladder, assuming $m'$ eliminates only first rung

\[ g_{0j} = \begin{cases} 
0 & \text{if } j = 1 \\
g(w_j) & \text{otherwise}
\end{cases} \]

And at any date $t \geq 1$

\[ g_{tj} = g_{t-1,j-1}(1 - \delta)\gamma_e + g_{t-1,j}(1 - \delta)(1 - \gamma_e) \quad \text{for } j > 1 \]

At any date $t \geq 0$ and for any $j \geq 1$

\[ g_{tj} = \begin{cases} 
0 & \text{if } j = 1 \\
\tilde{g}_{tj} & \text{if } 2 \leq j \leq t + 1 \\
g(w_j) & \text{if } j > t + 1
\end{cases} \]

where

\[ \tilde{g}_{tj} \equiv \sum_{k=2}^{j} \binom{t+1-k}{j-k} g(w_k)(1 - \delta)^{2+t-k}(1 - \gamma_e)^{2+t-j-\gamma_e} \]
Transitions
Those on new ladder, assuming \( m' \) eliminates only first rung

\[
g'_{0j} = \begin{cases} 
g(m) & \text{if } j = 1 \\
0 & \text{otherwise} 
\end{cases}
\]

And at any date \( t \geq 1 \)

\[
g'_{tj} = \begin{cases} 
u \gamma_u + g'_{t-1,1}(1 - \delta)(1 - \gamma_e) & \text{if } j = 1 \\
g'_{t-1,j-1}(1 - \delta)\gamma_e + g'_{t-1,j}(1 - \delta)(1 - \gamma_e) & \text{if } 1 < j \leq t + 1 \\
0 & \text{if } j > t + 1 
\end{cases}
\]

At any date \( t \geq 0 \) and for any \( j \geq 1 \)

\[
g'_{tj} = \begin{cases} 
\tilde{g}'_{tj} & \text{if } j \leq t + 1 \\
0 & \text{if } j > t + 1 
\end{cases}
\]

\[
\tilde{g}'_{tj} = \sum_{t' > 0}^{t + 1 - j} \binom{t - t'}{j - 1} u \gamma_u (1 - \delta)^{t-t'}(1 - \gamma_e)^{t-t'}-(j-1)\gamma_e^{j-1} \\
+ \binom{t}{j - 1} g(m)(1 - \delta)^{t}(1 - \gamma_e)^{t-(j-1)}\gamma_e^{j-1}
\]