The Race Between Education, Technology, and the Minimum Wage

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Summer Institute 2023 Labor Studies
Where we’re going

- What is the **dynamic** impact of the minimum wage on inequality (and, specifically, the college premium)?

- Theory: effect is **small on impact** and grows over time

- Empirics: document these **dynamic effects**

- **Changing minimum wages have small effects on impact. Over longer horizons, however, these effects become sizable**
How we get there

1. **Motivation (The Race Between...):** In national time series, the real minimum wage shapes the evolution of U.S. college premium and resolves a well-known “puzzle”

2. **Theory:** Job-ladder model with many skills
   - On impact $\uparrow$ m.w. $\Rightarrow$ $\uparrow$ in wages for those bound by it only $(direct\ effect)$
   - Over time, workers move up the job ladder $\Rightarrow$ magnified effect $(indirect\ effect)$
   - Incorporating labor supply, demand: extended canonical estimating equation

3. **Main Results:** Estimation + model calibration using state-level data
   - Elasticity of college premium with respect to m.w. small on impact; grows over time
   - Changing m.w. explains about 60% of the evolution of national HSD real wage

4. **Verify mechanism:** Document dynamic impact of m.w. over full wage distribution
Motivation:
National time-series variation
I consider (for now atheoretical) regressions of the form

\[
\log \left( \frac{w_{ht}}{w_{\ell t}} \right) = \alpha + \beta_m \log m_t + \beta_L \log \left( \frac{\text{Supply}_{ht}}{\text{Supply}_{\ell t}} \right) + \gamma_1 t + [...] + \iota_t
\]

using national time-series, \( t \), variation across two groups of workers, \( h \) and \( \ell \)

- \( \log w_{ht} \) and \( \log w_{\ell t} \) are measures of average log wages
- \( \log \text{Supply}_{ht} \) and \( \log \text{Supply}_{\ell t} \) are measures of labor supply
- \( m_t \) is a measure of the real minimum wage at the national level

**Measurement**

- **Supply and wages** are composition adjusted
- **National real minimum wage**
  - For each state use the max of the state and national statutory minima
  - ... then average across states using time-invariant weights
  - ... and apply the GDP deflator

**March CPS data spanning working years 1963 - 2017**
Results: predicted college premium (estimated on 63-87)
Results

Regression Models for the College Wage Premium

<table>
<thead>
<tr>
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<td>Real FLSA m.w.</td>
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<td>0.021</td>
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<td>(0.001)</td>
<td>(0.006)</td>
<td>(0.001)</td>
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<td>(0.009)</td>
<td>(0.014)</td>
<td>(0.029)</td>
<td>(0.009)</td>
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<td>Observations</td>
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<td>R-squared</td>
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<td>0.968</td>
<td>0.969</td>
<td>0.969</td>
<td>0.963</td>
</tr>
</tbody>
</table>

Table 1: Regression Models for the College Wage Premium

Notes: Results of estimating (1) using OLS. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real m.w.” and “Real FLSA m.w.” are the logs of the real minimum wage and real FLSA minimum wage. The sample is 1963-1987 in columns (a) and (b) and 1963-2017 elsewhere. “Time Polynom.” refers to the degree of the polynomial of time; the coefficient on the linear trend on “Time” is omitted from the table whenever this polynomial is of degree 2 or greater.

 nuit

Sizable elasticity of the college premium wrt the real minimum wage

- e.g., 27% ↓ in real minimum wage 1979 – 89 $\Rightarrow$ 3.7 – 5.3% ↑ in college premium
Summary + robustness

Summary:

1. relative supply growth fluctuations + trend demand growth crucial drivers of college premium, but changes in real minimum wage are also important

2. less dramatic slowing of SBTC (more generally, improved out-of-sample fit)

Sensitivity:

- ... higher-dimensional polynomials of time
- ... using two alternative measures of relative supply
- ... using Autor, Katz, and Kearney (2008) data
Theory
Framework

- **Supply and demand:**
  - Exogenous supply $L_{st}$ of homogeneous skill $s = 1, \ldots, S$ workers
  - Skill-time-specific productivity $A_{st}$ shaping relative demand
Framework

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  - Zero discount factor: analysis of transitions to sequence of aggregate shocks
  - Worker can be employed or unemployed with exogenous separation and job-finding rates
  - Generalized Nash bargaining btw worker-firm match over fixed real wage
  - Worker outside option is unemployment benefit or wage in current match
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- **Real minimum wage:** $m_t$
  - Impose binding mw for all $s$ to avoid taxonomy
  - Impose no employment effect of mw ($m_t < VMPL_{st}$), but can be generalized
Framework

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- Discrete time: within $t$, separation shocks occur first, then new matches realized for those workers who did not separate in $t$
Steady-state characterization

- Wage ladder across “rungs”
- First rung is the minimum wage...
- ... and move up over time (if no separation shock)

\[
m \frac{dP}{ds} (1-\beta)m + \beta P = \text{unemployment}
\]

unemployment

rung

wage

VMPLs
Transitions

- Economy in steady state at date 0

pre-shock $t \to -0$
Transitions

- Economy in steady state at date 0; a small one-time change to $m' > m$

On impact, the first rung disappears on the original ladder and appears, higher, on the new ladder (direct effect)

$$\log \left( \frac{w_{s0^+}}{w_{s0^-}} \right) \bigg/ \log \left( \frac{m'}{m} \right) \equiv \underbrace{\text{instantaneous elasticity}}_{\text{share of wage income earned at m.w. before shock}}$$
Transitions

- Economy in steady state at date 0; a small one-time change to $m' > m$

- One period later, the second rung on the original ladder starts emptying out as the second rung on the new ladder starts filling in (small indirect effect)

$$\log \left( \frac{w_{s1}}{w_{s0^-}} \right) \bigg/ \log \left( \frac{m'}{m} \right) > \text{share}_{s0^-}$$
Economy in steady state at date 0; a small one-time change to \( m' > m \)

\[
\log \left( \frac{w_{st'}}{w_{s0-}} \right) / \log \left( \frac{m'}{m} \right) > \log \left( \frac{w_{st}}{w_{s0-}} \right) / \log \left( \frac{m'}{m} \right) \quad \text{for all} \quad t' > t \geq 0
\]
Decompose the $D$-period elasticity of the college premium wrt to a change in the real m.w. into

\[
\log \left( \frac{w_{hD}}{w_{\ell D}} / \frac{w_{h0-}}{w_{\ell0-}} \right) / \log \left( \frac{m'}{m} \right) \equiv \beta_{m,D} \times b_{0-}
\]

the initial minimum wage bite and the magnification elasticity

- $b_{0-} \equiv \text{share}_{h0-} - \text{share}_{\ell0-}$ is the minimum wage bite
- $\beta_{m,0} = 1$
- $d\beta_{m,D}/dD > 0$ under some conditions
Brining in Supply and Demand

- $d \log P_s$ the same as in neoclassical model, replacing $L_{st}$ with $(1 - u_s)L_{st}$

- Given focus on canonical model, impose those assumptions

- Then across steady states (to a first-order approximation)

\[
d \log \left( \frac{w_{ht}}{w_{lt}} \right) = \beta_{m,\infty} bd \log m_t - \beta_{L,\infty} d \log \left( \frac{L_{ht}}{L_{lt}} \right) + \beta_{A,\infty} t + \nu_t
\]

where $b \equiv share_{ht} - share_{lt}$ is the “minimum wage bite”

- “Race between education, technology, and the minimum wage”
  - Extended canonical model
**Question**: What is the magnification elasticity for a one-time change in the real minimum wage, starting from a steady state?

**Complication**: Impact of mw change depends on *full history* of all changes
  - There is no region in steady state and there is no one-time change

**Approach**: Match “regional extended canonical model” coefficients in actual data and in model-generated data
  - Use the model to answer question
  - Regression results in data broadly robust across a range of sensitivity
Empirics leveraging state variation
Calibration + counterfactual
Regional extended canonical model

- Bring extended canonical model to data using many regions $r$ and dates $t$
- Suppose skill $s$ output freely traded across regions: $P_{st} = P_{srt}/A_{srt} \forall r$

$$\Delta_D \log \left( \frac{w_{hrt}}{w_{\ell rt}} \right) = \gamma_{t,D} + \gamma_{m,D} b_{rt} \Delta_D \log m_{rt} + [...]+ \iota_{rt,D}$$

- $r$ indexes region (fifty states) and $t$ time (1979 – 2018 excluding 94 and 95)
- $\Delta_D x_t \equiv x_{t+D} - x_t$ is the $D$-year difference for any $x_t$
- $b_{rt}$ is the share of income earned at mw. for high - low education in $r$ at $t$
- $m_{rt}$ is real minimum wage in state $r$ in year $t$
- $\gamma_{t,D}$ absorbs the (national) change in the relative skill price
  - robustness 1: state-specific measures of relative supply changes
  - robustness 2: state FE controlling for linear deviations from national SBTC
  - etc.
- Change in regional wages common across skills cancels out of triple difference
$\Delta_D \log \left( \frac{w_{hrt}}{w_{\ell rt}} \right) = \gamma_{t,D} + \gamma_{m,D} b_{rt} \Delta_D \log m_{rt} + [...] + \iota_{rt,D}$

- **Issue 1:** $\Delta_D \log m_{rt}$ cumulates across many changes over time
- **Issue 2:** $b_{rt}$ measured with error, likely correlated w/ dependent variable

**Instrument:**

\[ b_{rt-1}(\log m_{rt+1} - \log m_{rt}) \]

- lagged bite in the spirit of Autor, Manning, Smith (2016)
- one-year change in mw in spirit of local projection (e.g. Jorda, 2005)
\[
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**Data:** CPS Merged Outgoing Rotation Groups

Define workers as earning \(m_{rt}\) if their wages are \(\leq 1.15 \times m_{rt}\)

Cluster standard errors by state and weigh each state by avg across years of its share of national population
Regional extended canonical model estimates

Notes: Point estimates and 90% CIs from estimating baseline specification separately by time difference. First-stage KP $F > 16$ in each regression.

- **Small estimates on impact; point estimates grow over time**
Sensitivity

- reduced-form specification (in spirit of event study)
- incorporating state-specific supply
- state-specific linear deviation of SBTC from national rate
- controlling for the bite
- nominal minimum wage increases
Model calibration

- **Theoretical assumptions:**
  - At time $t$, $m_{rt}$ affects only those exiting unemployment
  - Allow mw not to bind
  - Parameters may vary between high, low education

- **Calibration I:** college educated not bound by m.w.
  - Only $\ell$ parameters matter for outcomes of interest

- **Calibration II:** Time-varying shocks
  - Observed matrix of real mws (across states and years)

- **Calibration III:** Time-invariant parameters
  - Choose state-education-specific real VMPL to match average real wages
  - Choose remaining parameters to match
    - average “bite” of minimum wage ($\approx -0.046$)
    - approximate unemployment of $\ell$ (10%)
    - evolution of empirical estimates when running identical 2SLS regression in model-generated data using analytic expressions as guides (next slide)
Parameter values

- $\gamma_{le} = 0.7$ and $\gamma_{lu} = 0.9$
- $\delta_{\ell} = 0.095$ and $\beta_{\ell} = 0.25$

Moment not targeted:

- Avg markdown $= 1.32$ for $\ell$
- In the range of Berger et al. (2022)
- A bit lower than in Yeh et al. (2022)
Moment not targeted: RF specification

\[
\log \left( \frac{W_{hrt+D}}{W_{lrlt+D}} \right) = \gamma_{t,D} + \beta_{m,D} b_{rt-1} \log \left( \frac{m_{rt+1}}{m_{rt}} \right) + \nu_{rt,D}
\]

- Change in college premium projected on baseline RF specification
- Estimated in the data and model-generated data

Notes: Point estimate and 90% CIs from estimating RF specification in the data. Estimate in model-generated data.
Moment not targeted: Long-difference instrument

- Estimate baseline specification but instrument with $b_{rt-1} \Delta_D \log m_{rt}$
- Estimated in the data and model-generated data

Notes: Point estimate and 90% CIs using long-difference instrument in the data. Estimate in model-generated data.
Main result

- Start from a steady state, feed in a tiny increase in real mw, and solve for time-varying magnification elasticity (○) [red line is limit]

- Time-varying elasticity of college premium is $b \times$ Magnification elasticity
  - In long-run, college premium elasticity wrt $m$ is $\approx 3b$
  - $b \approx -0.046 \Rightarrow$ LR elasticity $\approx -0.13$, consistent w/ national regression
  - Further comparison to national results

- Small (direct only) effects on impact, growing substantially (indirect) over time
Relation to (some) literature

- Autor et al. (2016) find almost no wage spillovers in first differences
- My college premium results are consistent in one-year differences

- Cengiz et al. (2019) find direct effect accounts for 60% of wage changes...
  - ... by averaging estimates over the five years following a mw increase
- Similarly averaging my CF results, I would find the same for college premium
  - Same is true if averaging my empirical estimates (interpreting them as CF results)

- I return to other papers in the conclusions
  - Fortin, Lemieux, and Lloyd (2021)
  - Engbom and Moser (2022)
Additional implication

- Hard for canonical model to reconcile dramatic decline in real wage of HSD

- **Can mw help explain real wage evolution (and its decline) for HSD? Yes.**

- Feed into model all changes in state real mw.s, aggregate avg HSD wages across states (using non-college baseline parameters, HSD-specific VMPL in each state)

- Model prediction accounts for 60% of observed variance

- Calibration used no aggregate data
Comparing counterfactual to estimates

- To what extent does the regression alone answer the question of interest?

- Start from a steady state, feed in a tiny increase in real mw, and solve for time-varying magnification elasticity (●) [red line is limit]

- Compare to estimates in model-generated data given observed changes in mws (◇)
Mechanism: Spillovers
Wage Spillovers

- Empirical results consistent w/ prediction of slow-developing wage spillovers
  - But not conclusive on these spillovers

- Replicate Autor et al. (AMS) first-difference specification

\[
\Delta \log \left( \frac{W_{rt}(c)}{W_{rt}(50)} \right) = \beta_1(c) \Delta \log \left( \frac{m_{rt}}{W_{rt}(50)} \right) + \beta_2(c) \left[ \Delta \log \left( \frac{m_{rt}}{W_{rt}(50)} \right) \right]^2 + \alpha_r + \alpha_c + \varepsilon_{rt}(c)
\]

- But using $D$-year differences for $D = 1, \ldots, 9$ in:
  - dependent and variables
  - instruments ($\log m_{rt}$, $(\log m_{rt})^2$, $\log m_{rt}$ interacted w/ real log median for $r$ across years),
  - weights

- Report marginal effects for selected percentiles

- See paper for results for each $D$ (for all workers, for men, and for women)
## Wage Spillovers: all workers

The table below presents the time difference in years between wage spillovers for various deciles.

<table>
<thead>
<tr>
<th>Time difference in years</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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<tr>
<td>p(5)</td>
<td>0.295</td>
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<td>0.320</td>
<td>0.310</td>
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<td>0.303</td>
<td>0.329</td>
<td>0.352</td>
<td>0.370</td>
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<tr>
<td>(0.063)</td>
<td>(0.062)</td>
<td>(0.053)</td>
<td>(0.046)</td>
<td>(0.045)</td>
<td>(0.043)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.047)</td>
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<tr>
<td>p(10)</td>
<td>0.168</td>
<td>0.199</td>
<td>0.191</td>
<td>0.179</td>
<td>0.159</td>
<td>0.179</td>
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<td>(0.044)</td>
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<td>(0.034)</td>
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<td>(0.033)</td>
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<tr>
<td>p(20)</td>
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<td>0.036</td>
<td>0.046</td>
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<td>(0.028)</td>
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<td>(0.024)</td>
<td>(0.024)</td>
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<tr>
<td>p(30)</td>
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<td>0.009</td>
<td>0.024</td>
<td>0.037</td>
<td>0.036</td>
<td>0.035</td>
<td>0.025</td>
<td>0.026</td>
<td>0.049</td>
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<tr>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.019)</td>
<td>(0.019)</td>
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<td>(0.028)</td>
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<td>p(40)</td>
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<td>(0.011)</td>
<td>(0.015)</td>
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<td>-0.011</td>
<td>0.004</td>
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<td>(0.023)</td>
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<td>(0.020)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.021)</td>
<td>(0.021)</td>
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</tr>
<tr>
<td>p(90)</td>
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<td>0.030</td>
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<td>0.030</td>
<td>0.003</td>
<td>-0.013</td>
<td>-0.024</td>
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<td>-0.007</td>
</tr>
<tr>
<td>(0.034)</td>
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<td>(0.034)</td>
<td>(0.034)</td>
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<td>(0.038)</td>
<td>(0.040)</td>
<td>(0.040)</td>
</tr>
</tbody>
</table>

- \(D = 1\): effects are small and insignificant by the 20th centile
- \(D > 1\): effects grow at bottom and fall at the top
Wage Spillovers: all workers

One-year differences

Nine-year differences

In long differences, coefficients (below median) are larger and almost all significant
Conclusions

What is the impact of the m.w. on the inequality?

- Minimum wage helps shape U.S. college wage premium and its variation across states, with effects growing over time
- Minimum wage helps shape evolution of real HSD wage
- Impacts of minimum wage move up the wage distribution over time

Open question: Are the greater spillover effects of the m.w.

- in Brazil (Engbom and Moser, 2022)
- in the 1980s in the U.S. (Fortin, Lemieux, and Lloyd, 2021)

caused by dynamic effects + monotonic changes in m.w. over time?
Empirical Appendix
## Regression Models for the College Wage Premium

### 1963-2017

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
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<td>Relative supply</td>
<td>-0.557</td>
<td>-0.601</td>
<td>-0.531</td>
<td>-0.477</td>
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<td>0.047</td>
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<td>(0.052)</td>
<td>(0.102)</td>
<td>(0.089)</td>
<td>(0.126)</td>
<td>(0.092)</td>
<td>(0.094)</td>
<td>(0.078)</td>
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<tr>
<td>Real m.w.</td>
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<td>-0.160</td>
<td>-0.136</td>
<td>-0.177</td>
<td>-0.163</td>
<td>-0.118</td>
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<td>(0.040)</td>
<td>(0.059)</td>
<td>(0.055)</td>
<td>(0.062)</td>
<td>(0.041)</td>
<td>(0.030)</td>
<td>(0.030)</td>
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<td>0.035</td>
<td>0.025</td>
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<td>-0.075</td>
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<td>(0.029)</td>
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<td>(0.012)</td>
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<tr>
<td>R-squared</td>
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<td>0.969</td>
<td>0.971</td>
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<td>0.990</td>
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Regression Models for the College Wage Premium
Using composition-adjusted changes in *efficiency-unit hours worked*

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<tr>
<td>Real FLSA m.w.</td>
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<tr>
<td>Time Polynom.</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
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</tr>
<tr>
<td>R-squared</td>
<td>0.282</td>
<td>0.946</td>
<td>0.527</td>
<td>0.965</td>
<td>0.966</td>
<td>0.969</td>
<td>0.960</td>
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</table>
### Regression Models for the College Wage Premium

Using composition-adjusted changes in efficiency-unit populations

<table>
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<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.786</td>
<td>-0.317</td>
<td>-0.800</td>
<td>-0.597</td>
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<tr>
<td></td>
<td>(0.167)</td>
<td>(0.066)</td>
<td>(0.152)</td>
<td>(0.082)</td>
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<td>Real m.w.</td>
<td></td>
<td>-0.245</td>
<td></td>
<td>-0.302</td>
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<tr>
<td></td>
<td></td>
<td>(0.077)</td>
<td></td>
<td>(0.058)</td>
</tr>
<tr>
<td>Real FLSA m.w.</td>
<td></td>
<td></td>
<td></td>
<td>-0.235</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>(0.054)</td>
</tr>
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<td>Time</td>
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<td>0.025</td>
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<td>(0.006)</td>
<td>(0.002)</td>
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<td>(0.019)</td>
<td>(0.009)</td>
<td>(0.023)</td>
<td>(0.009)</td>
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<td>Observations</td>
<td>25</td>
<td>55</td>
<td>25</td>
<td>55</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.423</td>
<td>0.909</td>
<td>0.629</td>
<td>0.936</td>
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</table>

### Notes:
- For alternative supply #2, robustness analysis was conducted using regression models for the college wage premium.
- The models used composition-adjusted changes in efficiency-unit populations.

### Results:
- Relative supply coefficients for 1963-1987 range from -0.786 to -0.800, and for 1963-2017 from -0.597 to -0.491.
- Real m.w. coefficients show a decrease from -0.245 to -0.235 over the same periods.
- Real FLSA m.w. shows a slight decrease from -0.235 to -0.235.
- Time coefficient increases from 0.020 to 0.021.
- Constant values range from 0.001 to 0.022.
- Observations vary from 25 to 55.
- R-squared values range from 0.423 to 0.964.
Regression Models for the College Wage Premium Using Data from AKK Replication Package (1963-2005)

<table>
<thead>
<tr>
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<th>1963-2005</th>
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<tbody>
<tr>
<td></td>
<td>(a)</td>
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<tr>
<td>Relative supply</td>
<td>-0.431</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
</tr>
<tr>
<td>Minimum wage</td>
<td><strong>-0.113</strong></td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.253</td>
</tr>
<tr>
<td></td>
<td>(0.083)</td>
</tr>
<tr>
<td>Time Polynom.</td>
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<td>Observations</td>
<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.942</td>
</tr>
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</table>

Impact of minimum wage is at least as robust as impact of supply.
Robustness: Using data from AKK but alternative mw
Constructing the minimum wage using data across states

Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)...
... but replacing FLSA mw. with average across states

<table>
<thead>
<tr>
<th>1963-2005</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
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<tbody>
<tr>
<td>Relative supply</td>
<td>-0.436</td>
<td>-0.608</td>
<td>-0.612</td>
<td>-0.214</td>
<td>0.015</td>
<td>0.028</td>
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<td>(0.048)</td>
<td>(0.076)</td>
<td>(0.085)</td>
<td>(0.105)</td>
<td>(0.093)</td>
<td>(0.083)</td>
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<tr>
<td>Minimum wage</td>
<td>-0.142</td>
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<td>-0.177</td>
<td>-0.128</td>
<td>-0.123</td>
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<td>(0.050)</td>
<td>(0.050)</td>
<td>(0.049)</td>
<td>(0.051)</td>
<td>(0.040)</td>
<td>(0.041)</td>
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<tr>
<td>Constant</td>
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<td>0.117</td>
<td>0.022</td>
<td>0.530</td>
<td>0.592</td>
<td>0.599</td>
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<tr>
<td></td>
<td>(0.090)</td>
<td>(0.112)</td>
<td>(0.117)</td>
<td>(0.171)</td>
<td>(0.127)</td>
<td>(0.122)</td>
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<td>4</td>
<td>5</td>
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<td>43</td>
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<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.942</td>
<td>0.947</td>
<td>0.954</td>
<td>0.972</td>
<td>0.979</td>
<td>0.979</td>
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</table>

Slight increase in elasticity wrt real minimum wage
Replace $d \log \frac{w_{ht}}{w_{lt}}$ with $d \log w_{ht}$ and with $-d \log w_{lt}$

### Regression Models for the College and Non-College Wages

<table>
<thead>
<tr>
<th></th>
<th>College premium (a)</th>
<th>College wage (b)</th>
<th>Non-college wage (c)</th>
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<tbody>
<tr>
<td>Real m.w.</td>
<td>-0.196 (0.040)</td>
<td>-0.080 (0.059)</td>
<td>0.116 (0.048)</td>
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<tr>
<td>Observations</td>
<td>55</td>
<td>55</td>
<td>55</td>
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</tbody>
</table>

Single difference requires stronger assumptions to interpret than double diff.

**Minimum wage associated with rising non-college average wage, but also falling college wage**
State robustness: RF specification

\[
\log \left( \frac{W_{hrt+D}}{W_{\ell rt+D}} / \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt-1} \log \left( \frac{m_{rt+1}}{m_{rt}} \right) + \nu_{D,rt}
\]

Notes: Point estimate and 90% CIs from estimating reduced-form specification separately by time difference.
Supply as dual of wages, to match income in state $r$ and time $t$ in the data

Notes: Point estimate $\bullet$ and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate $\diamondsuit$. 

Back
State robustness: supply #2

Supply as composition-adjusted measure of efficiency-unit hours worked

Notes: Point estimate and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate.
Supply as *composition-adjusted measure of efficiency-unit populations*

**Notes:** Point estimate • and 90% CIs from estimating specification including state-specific change in relative supply separately by time difference. Baseline estimate ◇.
State robustness: control for the bite $b_{rt-1}$

$$\Delta_D \log \left( \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt} \Delta_D \log m_{rt} + \gamma_D b_{rt} + \nu_D,rt$$

and instrument for the bite using its lagged value (all SW F stats > 70)

Notes: Point estimate • and 90% CIs from estimating above specification separately by time difference. Baseline estimate ◊.
State robustness: increases in nominal minimum wages

\[ \Delta_D \log \left( \frac{W_{hrt}}{W_{\ell rt}} \right) = \gamma_{D,t} + \gamma_{D,m} b_{rt} \Delta_D \log m_{rt} + \gamma_{D,Index} b_{rt} \Delta_D \log Index_t + \nu_{D,rt} \]

- \( Index_t \) is the price index
- Second instrument is \( b_{rt-1} (\log Index_{t+1} - \log Index_t) \) (all SW F stats > 20)

Notes: Point estimate \( \bullet \) and 90\% CIs from estimating above specification separately by time difference. Baseline estimate \( \bullet \).
State robustness: state fixed effects

Absorbing linear deviations from national rate of SBTC

Notes: Point estimate ○ and 90% CIs from including a state FE in estimating equation separately by time difference. Baseline estimate ♦.
Separate state-level regressions by education

Estimating separately by education, so that baseline = sum of estimates

Notes: Baseline estimates ○. Coefficients estimated on college wage X. Coefficients estimated on negative of non-college wage ♦.

Double difference requires stronger assumptions to interpret than triple diff. For instance, if m.w. increases occur in booms but not busts...
Relation to national estimates

- **Comparison**: National specification in levels + m.w. not interacted with bite
- **In levels**: interact national m.w. with its average bite (-0.046)
  - Equivalent to dividing coefficient by average bite
  - National estimates btw $-0.14$ and $-0.2$ ⇒ magnification elasticity btw 3 and 4
  - National estimates broadly consistent with state-level estimates
Theoretical Appendix
Comparative statics across steady states

- Increase from $m$ to $m'$

- $share_s = \text{share of } s \text{ income earned at minimum wage}$

- **Accounting**: If only a direct effect, elasticity of $w_s$ wrt $m$ is $share_s$
Comparative statics across steady states

- Increase from $m$ to $m'$

- $\text{share}_s = \text{share of } s \text{ income earned at minimum wage}$

- **Accounting:** If only a direct effect, elasticity of $w_s$ wrt $m$ is $\text{share}_s$

- **Steady-state effect:** In the long run, all rungs on the wage ladder adjust and more so for lower rungs
  
  1. Elasticity is magnified by $\beta_{ms} \equiv \frac{\delta_s + (1 - \delta_s) \gamma_se}{\delta_s + \beta_s (1 - \delta_s) \gamma_se} > 1$

  \[
  \frac{d \log w_s}{d \log m} = \beta_{ms} \text{share}_s
  \]

  2. Let $W_s(c)$ be wage at percentile $c$ of employed skill $s$ workers

  \[
  W_s(c) < W_s(c') \implies \frac{d \left[ W_s(c)/W_s(c') \right]}{dm} > 0
  \]
Burdett and Mortensen (1998) + binding minimum wage

- Equation (2.10) in van den Berg and Ridder (1998), eqm earnings density

\[ g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \text{ for all } w \in [m, w_{\text{max}}] \]

with maximum wage

\[ w_{\text{max}} \equiv \left( \frac{\delta}{\delta + \lambda_e} \right)^2 m + \left( 1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P \]

- Average wage is then

\[ w = \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left( 1 - \frac{(100\delta)^2}{(w_{\text{max}}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} \right) P \]

weighted avg of \( m \) and \( P \) as in baseline model, but weights depend on \( m \)
Burdett and Mortensen (1998) + binding minimum wage

- $W_c(m)$: wage at centile $c \in [0, 100]$

$$W_c(m) = P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2$$

- Hence

$$\frac{W_c'(m)}{W_c(m)} = \frac{P - (P - m) \left( \frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{p - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2}$$

- Differentiating with respect to $m$ yields

$$\frac{d \left[ W_c'(m)/W_c(m) \right]}{dm} < 0 \iff c' > c$$

- As in baseline model, $W_c(m)$ is log-submodular in $(c, m)$
Distribution and the average wage

- **Density** $g_s(w_j)$ satisfies

\[
\begin{align*}
[\delta_s + (1 - \delta_s)\gamma_{se}] g_s(w_{1,s}) &= \gamma_{su} g_s(w_{0,s}) \\
[\delta_s + (1 - \delta_s)\gamma_{se}] g_s(w_{j+1,s}) &= (1 - \delta_s)\gamma_{se} g_s(w_{j,s}) \quad \text{for} \quad j \geq 1
\end{align*}
\]

- **Unemployment rate**

\[
g_s(w_0) = \frac{\delta_s}{\delta_s + \gamma_{su}}
\]

- **Share at each rung**

\[
g_s(w_j) = \left( \frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right)^{j-1} \frac{\gamma_{su}}{\delta_s + (1 - \delta_s)\gamma_{se}} \frac{\delta_s}{\delta_s + \gamma_{su}} \quad \text{for} \quad j \geq 1
\]

- **Average wage** $w_s \equiv \frac{1}{1 - g_s(w_0)} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$ among the employed

\[
w_s = \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} m + \left( 1 - \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \right) P_s
\]
Comparative statics across steady states: $\beta_L$

- Elasticity wrt relative supply

$$\beta_{L,\infty} \equiv \frac{1}{\eta} (1 - \beta_m share\ell) \frac{P_h(1 - u_h)L_h}{Y} + \frac{1}{\eta} (1 - \beta_m share_h) \frac{P_\ell(1 - u_\ell)L_\ell}{Y}$$

- When $share_s = 0$ for both $s$, this is just $1/\eta$ as in canonical model
Transitions
Those on original job ladder, assuming $m'$ eliminates only first rung

$$g_{0j} = \begin{cases} 
0 & \text{if } j = 1 \\
g(w_j) & \text{otherwise}
\end{cases}$$

And at any date $t \geq 1$

$$g_{tj} = g_{t-1,j-1}(1 - \delta)\gamma + g_{t-1,j}(1 - \delta)(1 - \gamma) \quad \text{for } j > 1$$

At any date $t \geq 0$ and for any $j \geq 1$

$$g_{tj} = \begin{cases} 
0 & \text{if } j = 1 \\
\tilde{g}_{tj} & \text{if } 2 \leq j \leq t + 1 \\
g(w_j) & \text{if } j > t + 1
\end{cases}$$

where

$$\tilde{g}_{tj} = \sum_{k=2}^{j} \binom{t + 1 - k}{j - k} g(w_k)(1 - \delta)^{2 + t - k}(1 - \gamma)^{2 + t - j}\gamma^{j - k}$$
Transitions

Those on new ladder, assuming $m'$ eliminates only first rung

\[ g'_{0j} = \begin{cases} 
  g(m) & \text{if } j = 1 \\
  0 & \text{otherwise}
\end{cases} \]

And at any date $t \geq 1$

\[ g'_{tj} = \begin{cases} 
  u\gamma_u + g'_{t-1,1} (1 - \delta)(1 - \gamma_e) & \text{if } j = 1 \\
  g'_{t-1,j-1} (1 - \delta)\gamma_e + g'_{t-1,j} (1 - \delta)(1 - \gamma_e) & \text{if } 1 < j \leq t + 1 \\
  0 & \text{if } j > t + 1
\end{cases} \]

At any date $t \geq 0$ and for any $j \geq 1$

\[ g'_{tj} = \begin{cases} 
  \tilde{g}'_{tj} & \text{if } j \leq t + 1 \\
  0 & \text{if } j > t + 1
\end{cases} \]

\[ \tilde{g}'_{tj} = \sum_{t' > 0} \left( \begin{array}{c} t \\ j - 1 \end{array} \right) u\gamma_u (1 - \delta)^{t-t'} (1 - \gamma_e)^{t-t'-(j-1)} \gamma_e^{j-1} \\ + \left( \begin{array}{c} t \\ j - 1 \end{array} \right) g(m) (1 - \delta)^{t} (1 - \gamma_e)^{t-(j-1)} \gamma_e^{j-1} \]