The Race Between Education, Technology, and the Minimum Wage

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Where we’re going

- **What is the dynamic impact of the minimum wage on inequality?**

- Theory: inequality effects *grow over time*

- Empirics: document these dynamic effects

- **The impact of the minimum wage on inequality grows over time, with the effect more than doubling over two years**
How we get there

1 Motivation (The Race Between...): In national time series, the real minimum wage helps shape the evolution of U.S. college premium + partially resolves a “puzzle”

2 Theory: Job-ladder model with many skills (or groups)
   - On impact $\uparrow \text{mw} \Rightarrow \uparrow$ in wages for those individuals bound by it \textit{(direct effect)}
   - Over time, workers move up the job ladder $\Rightarrow$ magnified effect \textit{(indirect effect)}
   - Elasticity of the average wage of a skill group wrt mw $\uparrow$ in share of wage income bound by mw and grows over time \textit{(combo of effects)}
   - Theory embeds into any aggregate production function combining skill outputs:

3 Empirics: Using state-and-group-level data, document that the elasticity of the state $\times$ group average wage w.r.t. the minimum wage
   - is $\uparrow$ in share of wage income earned at the mw on impact \textit{(direct effect)}
   - this difference in elasticities $\uparrow$ by a factor of $> 2$ over 2 years \textit{(indirect effect)}
     - quantitative elasticity consistent w/ national elasticity from “The Race...”
     - $\downarrow$ in real wage of HSD in 1980s and early 1990s caused by $\downarrow$ real minimum wage
Contributions

- **Canonical model** (Tinbergen 74, Katz and Murphy 92, ...), including minimum wage (Autor, Katz, Kearney 08)
  - While supply and demand remain crucial, so too is the minimum wage
  - Minimum wage helps resolve the apparent rapid slowing of SBTC in the 1990s

- **Impact of minimum wage on inequality**
  Meyer, Wise, 83; DiNardo et al., 96; Lee, 99; Card, DiNardo, 02; Autor ea., 16; Cengiz ea. 19; Dube, 19; Fortin ea. 21; Chen, Teulings 22; ...
  - Direct effect dominates in short run, but indirect effect grows over time
  - Identified within worker groups (i.e. for fixed observable characteristics)

- **Growing macro-labor literature of monopsony using quantitative models**
  Haanwinckel, 20; Engbom, Moser, 21; Ahlfeldt et al., 22; Berger et al., 22; Hurst et al., 22; Trottner, 22; ...
  - Job-ladder model like EM, but focus on dynamics
  - Dynamics like HKPW, but driven by job ladder rather than putty-clay capital
Motivation:
National time-series variation
I consider (for now atheoretical) regressions of the form

$$\log \left( \frac{w_{ht}}{w_{\ell t}} \right) = \alpha + \nu_m \log m_t + \nu_L \log \left( \frac{Supply_{ht}}{Supply_{\ell t}} \right) + \gamma_1 t + [...] + \iota_t$$

national time-series, $t$, variation across college and non-college workers, $h$ and $\ell$

- $\log w_{ht}$ and $\log w_{\ell t}$ are measures of average log wages
- $\log Supply_{ht}$ and $\log Supply_{\ell t}$ are measures of labor supply (hours worked)
- $m_t$ is a measure of the real minimum wage at the national level

Measurement

- **Supply and wages** are composition adjusted (March CPS 1964 - 2017 spanning working years 1963 - 2016)
- **Instrument for supply** is composition-adjusted population (March CPS)
- **National real minimum wage** (Cengiz et al. (2019), DOL, FRED, March CPS)
  - For each state use the max of the state and national statutory minima
  - ... then average across states using time-invariant weights
  - ... and apply the GDP deflator
Data

- College premium
- Population
- Hours
- Baseline
- Federal

Graphs showing trends over time from 1960 to 2020.
Result 1: out-of-sample fit (2SLS)

College premium

Predicted w/out mw
Result I: out-of-sample fit (2SLS)
Residualized data
Using the population-based measure of relative college supply
Regression Models for the College Wage Premium

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<tr>
<td>Relative supply of college workers</td>
<td>-0.632</td>
<td>-0.703</td>
<td>-0.608</td>
<td>-0.619</td>
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<td>(0.069)</td>
<td>(0.077)</td>
<td>(0.104)</td>
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<td>Real minimum wage</td>
<td>-0.220</td>
<td>-0.199</td>
<td>-0.133</td>
<td>-0.129</td>
<td>-0.132</td>
<td>-0.161</td>
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<td>(0.048)</td>
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<tr>
<td>Real federal minimum wage</td>
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<td>-0.161</td>
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<td>Time Polynomial</td>
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The extended canonical model including polynomials of time up to degree $j$ in column $j$. Estimated using 2SLS, instrumenting for hours-based supply using population-based measure. Robust standard errors are shown in parentheses.

**Sizable elasticity of the college premium wrt the real minimum wage**

- e.g., 26% ↓ in real minimum wage 1979 – 89 $\Rightarrow$ 2.7 – 5.7% ↑ in national college premium
Summary + robustness

- **Summary:**
  1. relative supply growth fluctuations + trend demand growth crucial drivers of college premium, but changes in real minimum wage are also important
  2. less dramatic slowing of SBTC (more generally, improved out-of-sample fit)

- **Sensitivity and additional results:**
  - ... using two alternative measures of relative supply
    - Tables with estimated elasticities
    - Figures with out-of-sample fit
  - ... using Autor, Katz, and Kearney (2008) data
  - ... separately for college and non-college workers
Theory
Supply and demand:

- Exogenous supply $L_{st}$ of homogeneous skill $s = 1, \ldots, S$ workers
- Aggregate production function combining skill output with skill-time-specific productivity $A_{st}$ shaping relative demand
Framework

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- **Job ladder:**
  - Zero discount factor: analysis of transitions to aggregate shocks
  - Worker can be employed or unemployed with exogenous separation ($\delta_s$) and job-finding rates ($\lambda_{su}$ unemployed and $\lambda_{se}$ employed)
  - Bilateral generalized Nash bargaining btw new worker-firm match ($\beta_s$ = worker weight) over fixed real wage w/ current job as worker outside option
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**Real minimum wage:** $m_t$
- Impose no employment effect of mw ($m_t < VMPL_{st}$), but will generalize
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- **Real minimum wage:** $m_t$
  - Impose no employment effect of mw ($m_t < VMPL_{st}$), but will generalize

- Discrete time: within $t$, separation shocks occur first, then new matches realized for those workers who did not separate in $t$
Steady-state characterization
Suppose “binding” mw for given s (for exposition only)

- Wage ladder across “rungs”
- First rung is the minimum wage...
- ... and move up over time (if no separation shock)
- Average wage an average of mw and VMPL

\[
(1-\beta)m + \beta Ps
\]

unemployment
rung
1
wage
∞

\[
\text{VMPL}_s
\]
Proposition

Consider an economy in steady state at date 0 that faces a small one-time change from \( m \) to \( m' > m \). For any skill \( s \) that was bound by \( m \):

1. For all \( t \geq 0 \) two job ladders coexist, with first rungs \( m \) (old) and \( m' \) (new)
2. The share on each rung \( j \) summed across the two ladders is constant across \( t \)
3. The share on rung \( j \) of the new ladder weakly increases in \( t \)
4. At each rung \( j \), wage on the new job ladder \( > \) than on the old one
5. The elasticity of the average wage wrt the minimum wage rises in \( t \)
6. On impact, this elasticity equals the share of income earned at \( m \) (the “bite”)

visualization
minimum wage decline
proof of 2
Transitions implication

- $T$-period elasticity of any group’s average wage wrt to a one-time increase in the real mw from $m$ to $m' > m$ at $t = 0$ (impulse response)

$$\log \left( \frac{w_{T,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) \equiv M_{T,s} \times b_{-1,s}$$

$T$ period elasticity of average wage wrt $m$

decomposed into initial minimum wage “Bite” + “Magnification” elasticity

- $b_{-1,s}$ is the pre-shock share of wage income earned at the mw

- $M_{T,s}$ is the “Magnification elasticity”
  - $M_{0,s} = 1$
  - $dM_{T,s}/dT > 0$
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  - $M_{0,s} = 1$
  - $dM_{T,s}/dT > 0$

- If $\delta$, $\lambda_u$, $\lambda_e$, $\beta$ common across $s$ ⇒ elasticity of college premium (or any relative wage)

$$\log \left( \frac{w_{T,h}/w_{T,\ell}}{w_{-1,h}/w_{-1,\ell}} \right) / \log \left( \frac{m'}{m} \right) \equiv M_T \times (b_{-1,h} - b_{-1,\ell})$$
Across steady state effects of changes in the real mw, supply, demand

- **Average wage of skill** \( s \) **for given changes in VMPL**

\[
d \log w_s = M_s b_s \partial \log m + (1 - M_s b_s) \partial \log VMPL_s
\]

where VMPL\(_s\) is real and \( M_s \) is the steady-state magnification elasticity

\[
M_s \equiv \lim_{D \to \infty} M_{D,s} = \frac{\delta_s + (1 - \delta_s)\lambda_{se}}{\delta_s + \beta_s(1 - \delta_s)\lambda_{se}} > 1
\]

- **And finally, solving for changes in VMPL**
  - **Given assumption** \( m \) doesn’t affect unemployment (\( m < VMPL_s \) for all \( s \))
  - **VMPL\(_s\)** same as in competitive model w/ same aggregate production function except
    - Replace \( L_{st} \) with \((1 - u_s)L_{st}\)
    - Hence, w/ 2 skills + CES production function + linear rates of growth of \( A_{st} \):
      
      “Race between education, technology, and the minimum wage”

- **Distribution of wages for skill** \( s \)
Empirical Approach
From theory to estimation

- Theory (omit $s$): if one-time permanent $m \uparrow$ in any period btw $t - T$ and $t$ then

  \[
  \log \frac{w_t}{w_{t-T}} = \sum_{j=0}^{T-1} M_j b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}
  \]

- This is a distributed lag model where the lag weights equal the magnification elasticities
From theory to estimation

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- This is a distributed lag model where the lag weights equal the magnification elasticities

- Theory: allowing the one-time $m \uparrow$ to occur earlier, btw $t - T'$ and $t$ for $T' > T$ then

  $$\log \frac{w_t}{w_{t-T}} = \sum_{j=0}^{T-1} M_j b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}} + \sum_{j=T}^{T'-1} (M_j - M_{j-T}) b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$

- Now lag weights equal the magnification elasticities only over the $t - T$ to $t$ period; otherwise lag weights are smaller than the magnification elasticities
From theory to estimation

- **Theory (omit s):** if one-time permanent $m \uparrow$ in any period btw $t-T$ and $t$ then

  \[
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  \]

  Now lag weights equal the magnification elasticities only over the $t-T$ to $t$ period; otherwise lag weights are smaller than the magnification elasticities

- In practice, $m$ is changing in every period in every empirical context
  - Apply versions of previous formula in presence of many changes
  - And run same regressions in model-generated data
Mapping to the data

- **Time** $t$: half year periods (m1-6, m7-12) between 1979m1 – 2016m12

- **Skills** $s$: labor groups $g \times$ regions $r$
  - 100 labor groups (5 age bins $\times$ 2 genders $\times$ 2 races $\times$ 5 educations)
  - 50 regions (U.S. states)
  - Minimum wages are $r, t$ specific
  - Wages and minimum wage bites are $g, r, t$ specific

- I study disaggregate outcomes as in the theory, rather than aggregating up and composition adjusting
Specification augmented in five ways (relative to theory)

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + FE_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

1. Incorporate leads \((j < 0)\) in addition to lags \((j \geq 0)\)
2. Estimate lag weights \(\mu_j\) that don’t depend on worker characteristics
3. Omit changes in minimum wages that occurred more than 5 years before \(t\)
Specification augmented in five ways (relative to theory)

\[
\log \frac{W_{g,r,t}}{W_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{E}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

4 Instrument replaces \( b_{g,r,t-j-1} \) in interactions with its median value \( b_{g,r} \)

- \( b_{g,r,t} \) is measured with error
- this ME can be correlated w/ ME in dependent variable
  - as pointed out in related context by Autor et al., 2016
Specification augmented in five ways (relative to theory)

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\]

Include two additional controls \( FE_t \times b_{g,r} \) and \( \alpha_t \)

- Treatment fits into “shock-exposure” framework of Borusyak and Hull (2023)
- Specification may suffer from OVB. For example:
  - In periods experiencing \( m \downarrow \) (e.g., 1980s), treatment – correlated w/ bite
  - Bite can be correlated with other shocks in residual, e.g., SBTC, which raises wages of groups with lower bites
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\]

5. Include two additional controls \( F_{E_t} \times b_{g,r} \) and \( \alpha_t \)

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- Avoid by controlling for \( E[treatment] \)
- \( F_{E_t} \times b_{g,r} \) controls for \( E[treatment] \) under assumption of an arbitrary time-varying national expectation of the change in the real minimum wage
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  - Bite can be correlated with other shocks in residual, e.g., SBTC, which raises wages of groups with lower bites
- Avoid by controlling for $E[treatment]$
  - $\text{FE}_t \times b_{g,r}$ controls for $E[treatment]$ under assumption of an arbitrary time-varying national expectation of the change in the real minimum wage
- Additional benefits:
  - Control absorbs changes in inflation: identical RF results using nominal mw.
  - “Design-based” approach avoids negative ex-ante weights (BH, 2024)
Data

- NBER Merged Outgoing Rotation Group of the CPS (1979 – 2016)
  - Drop 1994 and 1995m1–m8: missing allocation flags
  - End 2016m12 before many municipalities begin setting their own mws; e.g.,
    - NYC + Nassau, Suffolk, and Westchester counties on 12/31/2016
    - Minneapolis, MN in 2018
    - Los Angeles $0.50 above for large businesses on 7/1/16
    - San Diego $0.75 above on 1/1/15
  - Measure $b_{g,r,t}$ defining mw worker as those with wage $\leq 1.05$ mw

- Minimum wage data from Vaghul and Zipperer (2016), Cengiz et al. (2019)
  - Use maximum nominal mw in state-period
  - Deflate by maximum monthly GDP deflator in period

- 5,000 $(g, r)$ pairs $\approx 60,000$ obs. per $t \Rightarrow \approx 12$ obs. per $(g, r, t)$
  - Winsorize wage at 2nd percentile within each $(g, r)$
  - Weigh by product of $(g, r)$ work hours in $t$ and $t - T$ divided by their sum
  - Use balanced sample: $(g, r)$ with no missing wage data across $t$
Additional details

- Cluster standard errors by state

- All results in figures, which...
  - ... convert pre-trends to their more typical “levels” form
    - negative of coefficients for $j < 0$
    - see Roth (2024): “Interpreting Event-Studies...”
  - ... display averaged annual effects, except for impact effect
    - for period $j \geq 1$, display $(\mu_{j-1} + \mu_j)/2 +$ corresponding 95% confidence interval
    - for period $j < 0$, display $(\mu_j + \mu_{j-1})/2 +$ corresponding 95% confidence interval
    - for period $j = 0$, display $\mu_0 +$ corresponding 95% confidence interval
Results
Outline of results

1. 2SLS specification in model-generated data
2. Reduced-form specification
3. 2SLS specification
4. Robustness of 2SLS specification
5. Implications
Choose model parameters

- Externally to direct survey evidence (Hall and Mueller, 2018)
  - Converting from weekly to bi-annual: $\gamma_u \approx 0.79$, $\gamma_e = \gamma_u / 2$, and $\delta \approx 0.10$
- $\beta = 0.25$ to obtain long-run magnification elasticity of 2.4 (using analytics)
- Choose 5,000 values of $VMPL_{g,r}$ targeting average real wages
Choose model parameters

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Quantify impacts of changes in minimum wages

- Start from a steady state in 1979m1 – m6
- Feed in observed changes in real minimum wages in every state, period
- Estimate baseline 2SLS specification using model-generated data
2SLS specification using model-generated data

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{P}E_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

Notes: Point estimate for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

- Estimates rise from 1 to 2.1 over three years, given magnification elasticity of 2.4
RF specification

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{F} \mathbb{E}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \( \mu_0 \), linear combination of \( (\mu_{j-1} + \mu_j)/2 \) for \( j \geq 1 \) and of \( -(\mu_j + \mu_{j-1})/2 \) for \( j < 0 \)

- Qualitative pattern as in model-generated data
- No evidence of pre-existing differential trends before changes in \( m \)
RF specification: Sensitivity

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \text{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

- Baseline: mw workers those with wages \( \leq 1.05 \times mw \)
  - mw workers those with wages \( \leq 1.00 \times mw \)
  - mw workers those with wages \( \leq 1.10 \times mw \)
  - mw workers those with wages \( \leq 1.15 \times mw \)

- Baseline uses all groups
  - Only for groups without college degrees
  - Separately by gender
  - Exclude final 6 sample years (w/ sub-state mws)
  - Unbalanced panel of \((g, r)\)
2SLS specification

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{E}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
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- Lag weight \( \approx 0.5 \) on impact, ↑ to 1 in one year, and peaks at \( > 2.2 \) over two years
- Conclude a magnification elasticity of approximately 2.4
2SLS specification: Sensitivity

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + FE_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

- **Baseline:** mw workers those with wages \( \leq 1.05 \times mw \)
  - mw workers those with wages \( \leq 1.00 \times mw \)
  - mw workers those with wages \( \leq 1.10 \times mw \)
  - mw workers those with wages \( \leq 1.15 \times mw \)

- Baseline uses all groups
  - Only for groups without college degrees
  - Separately by gender
  - Exclude final 6 sample years (w/ sub-state mws)
  - Unbalanced panel of \((g,r)\)

- Assume \(t\)- and \(r\)- specific components of \(m\) changes (control for \(FE_r b_{g,r}\))

- Incorporate an \((r, t)\) fixed effect

- Incorporate a \((g, t)\) fixed effect
Implication I: Revisiting the college premium

College premium elasticity with respect to $m$

- Long-run national elasticity of college premium wrt to the minimum wage of
  \[ M \times (b_h - b_\ell) \]

- Estimates suggest a long-run magnification elasticity of 2.4

- Average value across all $(g, r, t)$ of $b_{g,r,t}$ is
  - for college $g$: 1.8%
  - for non-college $g$: 7.6%

- College premium elasticity wrt to $m$ in the range of $2.4 \times (-0.058) \approx -0.14$
  - In the middle of the range of the national estimates

Both national and disaggregated state $\times$ group estimates imply similar and sizable elasticities of the college premium with respect to the real minimum wage.
Implication II: Real wages (and their decline)

Real wage elasticity with respect to $m$

- Real wages of low-education workers declined dramatically in 1980s into early 1990s
- This decline is impossible in “The Race Between Education and Technology” without a ↓ in productivity (Acemoglu and Restrepo, 2020)
- Possible in “The Race Between Education, Technology, and the Minimum Wage”

- Consider those without completed high school (HSD)
  - 26% decline of real mw between 1979 and 1989
  - Average value across all $(g, r, t)$ of $b_{g,r,t}$ among HSD is 13.9%
  - $\downarrow m \Rightarrow \downarrow 8.7\% \ (\approx 0.26 \times 0.139 \times 2.4)$ in HSD real wage

$\downarrow$ real minimum wage explains entirely of $\downarrow$ real wage of HSD btw 1979 – 1992
Model limitations

What might these results suggest the theory lacks?

- In the data, coefficients begin to ↓ one period early (compared to in the model)
  - Something is either pushing up wages higher in the wage distribution over time
  - ... or pushing down wages lower in the wage distribution over time

- Prominent possibilities:
  1. Fairness or efficiency wage concerns ↑ wages higher up distribution
     - e.g., Grossman (1983)
  2. Technical change ↑ demand for higher-wage and/or ↓ demand for lower-wage groups
     - results apply with $g, t$ fixed effects: localized demand changes
     - e.g., Hurst et al. (2022)
Conclusions

What is the impact of the mw on inequality?

- **Empirical motivation:** two new facts in the national time series
  - minimum wage helps shape U.S. college wage premium
  - incorporating mw improves fit of “The Race” + reduces trend break in SBTC

- **Theoretically:**
  - on impact, $\uparrow$ mw raises wages more for groups more bound by it
  - over time, this difference in wage elasticities rises due to indirect effects

- **Empirically:** Find evidence consistent with these dynamic predictions
  - using state and group level data
  - holding the composition of workers fixed
  - with magnification elasticity $> 2$ after $\approx 2$ years
  - quantitatively consistent w/ national-time series estimates
  - $\downarrow m \Rightarrow$ all of $\downarrow$ real wage of HSD in 1980s and early 1990s
Empirical Appendix
### Regression Models for the College Wage Premium

Using *dual* of composition-adjusted changes in wages

Instrumenting with *efficiency-unit populations*

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td>Relative supply of college workers</td>
<td>-0.584</td>
<td>-0.724</td>
<td>-0.745</td>
<td>-0.668</td>
<td>-0.518</td>
<td>-0.512</td>
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<td></td>
<td>(0.063)</td>
<td>(0.085)</td>
<td>(0.118)</td>
<td>(0.124)</td>
<td>(0.175)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Real minimum wage</td>
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<td>-0.158</td>
<td>-0.171</td>
<td>-0.186</td>
<td>-0.175</td>
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<tr>
<td></td>
<td>(0.044)</td>
<td>(0.054)</td>
<td>(0.060)</td>
<td>(0.063)</td>
<td>(0.052)</td>
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<tr>
<td>Real federal minimum wage</td>
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<td></td>
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<td>(0.039)</td>
<td></td>
</tr>
<tr>
<td>Time</td>
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<td></td>
<td></td>
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<td>0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
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<td>(0.001)</td>
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<tr>
<td>Time Polynomial</td>
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</table>
Regression Models for the College Wage Premium  
Reduced-form specification

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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative supply of college workers</td>
<td>$-0.555$</td>
<td>$-0.816$</td>
<td>$-0.632$</td>
<td>$-0.692$</td>
<td>$-0.356$</td>
<td>$-0.464$</td>
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<tr>
<td></td>
<td>$(0.060)$</td>
<td>$(0.083)$</td>
<td>$(0.101)$</td>
<td>$(0.110)$</td>
<td>$(0.106)$</td>
<td>$(0.060)$</td>
</tr>
<tr>
<td>Real minimum wage</td>
<td>$-0.307$</td>
<td>$-0.257$</td>
<td>$-0.132$</td>
<td>$-0.115$</td>
<td>$-0.126$</td>
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</tr>
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<td>$(0.053)$</td>
<td>$(0.058)$</td>
<td>$(0.047)$</td>
<td>$(0.058)$</td>
<td>$(0.038)$</td>
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<td>Time</td>
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<td>$(0.001)$</td>
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<tr>
<td>Time Polynomial</td>
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<td>3</td>
<td>4</td>
<td>5</td>
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</tbody>
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Results: predicted college premium alternative supply

Using dual of composition-adjusted changes in wages
Instrumenting with efficiency-unit populations

College premium
Predicted w/out mw
Predicted with mw
Results: predicted college premium reduced form

Reduced-form specification

College premium
Predicted w/out mw
Predicted with mw

1960 1980 2000 2020
College premium
Predicted w/out mw
Predicted with mw
### Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)

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<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
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<tr>
<td>Relative supply of college workers</td>
<td>-0.431</td>
<td>-0.607</td>
<td>-0.612</td>
<td>-0.216</td>
<td>0.013</td>
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<td></td>
<td>(0.051)</td>
<td>(0.077)</td>
<td>(0.091)</td>
<td>(0.113)</td>
<td>(0.104)</td>
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<tr>
<td>Minimum wage</td>
<td>-0.112</td>
<td>-0.108</td>
<td>-0.064</td>
<td>-0.174</td>
<td>-0.123</td>
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<td>(0.049)</td>
<td>(0.049)</td>
<td>(0.048)</td>
<td>(0.052)</td>
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<table>
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<tr>
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<th>Using my baseline real minimum wage</th>
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<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>Relative supply of college workers</td>
<td>-0.459</td>
<td>-0.605</td>
<td>-0.610</td>
<td>-0.244</td>
<td>-0.019</td>
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<td>(0.051)</td>
<td>(0.078)</td>
<td>(0.089)</td>
<td>(0.108)</td>
<td>(0.106)</td>
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<tr>
<td>Minimum wage</td>
<td>-0.150</td>
<td>-0.139</td>
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<td>-0.184</td>
<td>-0.119</td>
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<td>(0.050)</td>
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</tbody>
</table>

| Time Polynomial      | 1                     | 2                     | 3                     | 4                     | 5                     |
| Observations         | 43                    | 43                    | 43                    | 43                    | 43                    |

Impact of minimum wage is at least as robust as impact of supply.
Replace $\log \frac{w_{ht}}{w_{lt}}$ with $\log w_{ht}$ and with $\log w_{lt}$

### Regression Models for the College and Non-College Wages

<table>
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<tr>
<th></th>
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<td>(High)</td>
<td>(Low)</td>
<td>(Premium)</td>
<td>(High)</td>
<td>(Low)</td>
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<td>Relative supply</td>
<td>-0.632</td>
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<td>-1.029</td>
<td>-0.327</td>
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<td>(0.069)</td>
<td>(0.127)</td>
<td>(0.123)</td>
<td>(0.077)</td>
<td>(0.140)</td>
<td>(0.139)</td>
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<tr>
<td>Real minimum wage</td>
<td>-0.220</td>
<td>-0.104</td>
<td>0.117</td>
<td>-0.199</td>
<td>0.083</td>
<td>0.282</td>
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<tr>
<td></td>
<td>(0.048)</td>
<td>(0.059)</td>
<td>(0.058)</td>
<td>(0.059)</td>
<td>(0.077)</td>
<td>(0.066)</td>
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<tr>
<td>Time</td>
<td>0.021</td>
<td>0.020</td>
<td>-0.002</td>
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</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

↑ $mw \Rightarrow ↑$ non-college average wage with no robust impact on college wage
RF specification: Non-college sample

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(- (\mu_j + \mu_{j-1})/2\) for \(j < 0\)
RF specification: Female sample

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \( \mu_0 \), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \( j \geq 1 \) and of \(-(\mu_j + \mu_{j-1})/2\) for \( j < 0 \)
RF specification: Male sample

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)
RF specification: 1979-2010 sample

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)
RF specification: Unbalanced sample

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(- (\mu_j + \mu_{j-1})/2\) for \(j < 0\).
RF specification: mw cutoff of 1.00

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \( \mu_0 \), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \( j \geq 1 \) and of \(-(\mu_j + \mu_{j-1})/2\) for \( j < 0 \)
RF specification: mw cutoff of 1.10

\[
\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $\frac{(\mu_{j-1} + \mu_j)}{2}$ for $j \geq 1$ and of $\frac{-(\mu_j + \mu_{j-1})}{2}$ for $j < 0$
RF specification: mw cutoff of 1.15

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times \text{bite}_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$

**Notes:** Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: Non-college sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \text{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

**Notes:** Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: Female sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{E}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: Male sample

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \text{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)
2SLS specification: 1979-2010 sample

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \( \mu_0 \), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \( j \geq 1 \) and of \(-(\mu_j + \mu_{j-1})/2\) for \( j < 0 \)
2SLS specification: Unbalanced sample

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{E}_{t} b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: mw cutoff of 1.00

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \text{FE} b_{g,r} + \alpha_t + \epsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)

Lag weights

Event time (in half years)
2SLS specification: mw cutoff of 1.10

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{E} b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)
2SLS specification: mw cutoff of 1.15

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + F\bar{E}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: An additional control

Assuming changes in the mw are $i.i.d.$ + a $t$-specific component + an $r$-specific component

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \mathbb{FE}_r b_{g,r} + \alpha_t + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
2SLS specification: An additional control

Assuming changes in the mw are \textit{i.i.d.} + a \textit{t}-specific component + include an \((r, t)\) fixed effect

\[
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{E}_t b_{g,r} + \alpha_{r,t} + \varepsilon_{g,r,t}
\]

Notes: Point estimate and 95% CI for \(\mu_0\), linear combination of \((\mu_{j-1} + \mu_j)/2\) for \(j \geq 1\) and of \(-(\mu_j + \mu_{j-1})/2\) for \(j < 0\)

Back to baseline robustness
2SLS specification: An additional control

Assuming changes in the mw are i.i.d. + a $t$-specific component + include a $(g, t)$ fixed effect

$$
\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \text{FE}_t b_{g,r} + \alpha_{g,t} + \varepsilon_{g,r,t}
$$

Notes: Point estimate and 95% CI for $\mu_0$, linear combination of $(\mu_{j-1} + \mu_j)/2$ for $j \geq 1$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$
Theoretical Appendix
Proposition

Consider an economy in steady state at date 0 that faces a small one-time change from \( m \) to \( m' > m \). Then for any skill \( s \) that was bound by \( m \):

1. For all \( t \geq 0 \) two job ladders coexist, with first rungs \( m \) (old) and \( m' \) (new)
2. The share on each rung \( j \) summed across the two ladders is constant across \( t \)
3. The share on rung \( j \) of the new ladder rises (weakly) in \( t \)
4. At each rung \( j \), wage on the new job ladder is higher than on the old one
5. The elasticity of the average wage wrt the minimum wage rises in \( t \)
6. On impact, this elasticity equals the share of income earned at \( m \) (the “bind”)

And, if \( m' < m \), then

- 1, 2, and 3 are identical
- 4 and 5 are reversed
- 6: the instantaneous elasticity of the average wage = 0
Transitions (for one $s$ bound by $m$)

- Economy in steady state at date 0

pre shock
$t = -1$
Transitions (for one $s$ bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

On impact, the first rung disappears on the original ladder and appears, higher, on the new ladder (direct effect)

\[
\log \left( \frac{w_{0^+,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) \equiv b_{-1,s}
\]

share of wage income earned at $mw$ before shock

instantaneous elasticity
Transitions (for one s bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

One period later, the second rung on the original ladder starts emptying out as the second rung on the new ladder starts filling in (small indirect effect)

$$\log \left( \frac{w_{1,s}}{w_{-1,s}} \right) / \log \left( \frac{m'}{m} \right) > b_{-1,s}$$
Transitions (for one $s$ bound by $m$)

- Economy in steady state at date 0; a small one-time change to $m' > m$

- ... converging to the new steady state, w/ all on the new job ladder, with the importance of the indirect effect growing each period

\[
\log \left( \frac{w_{t',s}}{w_{t-1,s}} \right) / \log \left( \frac{m'}{m} \right) \geq \log \left( \frac{w_{t,s}}{w_{t-1,s}} \right) / \log \left( \frac{m'}{m} \right) \quad \text{for all} \quad t' > t \geq 0
\]
Transition proof (for given $s$) for a small $↑m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g'_{t,j}$ rung $j$ shares on original, new ladders at $t$
Transition proof (for given $s$) for a small $\uparrow m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g'_{t,j}$ rung $j$ shares on original, new ladders at $t$

- At $t = 0$: first rung + unemployment fully reallocate (and nothing else)
  - $\Rightarrow g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$
Transition proof (for given $s$) for a small $\uparrow m$ at $t = 0$

- $g_j$ rung $j$ share in SS; $g_{t,j}$ and $g'_{t,j}$ rung $j$ shares on original, new ladders at $t$

- At $t = 0$: first rung + unemployment fully reallocate (and nothing else)
  - $\Rightarrow g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$

- For some $t \geq 0$: suppose $g_{t,j} + g'_{t,j} = g_j$ for all $j \geq 0$
  - $\Rightarrow$ date $t + 1$ and for any $j > 1$, we have

\[
\begin{align*}
g_{t+1,j} &= g_{t,j}(1 - \delta)(1 - \lambda_e) + g_{t,j-1}(1 - \delta)\lambda_e \\
g'_{t+1,j} &= g'_{t,j}(1 - \delta)(1 - \lambda_e) + g'_{t,j-1}(1 - \delta)\lambda_e
\end{align*}
\]

and if $j = 1$ replace $(1 - \delta)\lambda_e$ w/ $\lambda_u$ [and note that $g_{t,0} = 0$]

- Summing these expressions and using $g_{t,j} + g'_{t,j} = g_j$ yields

\[
g_{t+1,j} + g'_{t+1,j} = g_j(1 - \delta)(1 - \lambda_e) + g_{j-1}(1 - \delta)\lambda_e = g_j
\]

where the final equality follows from the steady-state derivation of $g_j$
Distribution and the average wage

- Density $g_s(w_j)$ satisfies
  \[
  [\delta_s + (1 - \delta_s)\lambda_{se}] g_s(w_{1,s}) = \lambda_{su} g_s(w_{0,s})
  
  [\delta_s + (1 - \delta_s)\lambda_{se}] g_s(w_{j+1,s}) = (1 - \delta_s)\lambda_{se} g_s(w_{j,s}) \quad \text{for} \quad j \geq 1
  \]

- Unemployment rate
  \[
  g_s(w_{0,s}) = \frac{\delta_s}{\delta_s + \lambda_{su}}
  \]

- Share at each rung
  \[
  g_s(w_{j,s}) = \left( \frac{(1 - \delta_s)\lambda_{se}}{\delta_s + (1 - \delta_s)\lambda_{se}} \right)^{j-1} \frac{\lambda_{su}}{\delta_s + (1 - \delta_s)\lambda_{se}} \frac{\delta_s}{\delta_s + \lambda_{su}} \quad \text{for} \quad j \geq 1
  \]

- Average wage $w_s \equiv \frac{1}{1 - g_s(w_{0,s})} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$ among the employed
  \[
  w_s = \frac{\delta_s}{\delta_s + \beta_s (1 - \delta_s)\lambda_{se}} m + \left( 1 - \frac{\delta_s}{\delta_s + \beta_s (1 - \delta_s)\lambda_{se}} \right) P_s
  \]
Equation (2.10) in van den Berg and Ridder (1998), eqm earnings density

\[ g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \text{ for all } w \in [m, w_{\max}] \]

with maximum wage

\[ w_{\max} \equiv \left( \frac{\delta}{\delta + \lambda_e} \right)^2 m + \left(1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P \]

Average wage is then

\[ w = \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left(1 - \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} \right) P \]

weighted avg of \( m \) and \( P \) as in baseline model, but weights depend on \( m \)
Across steady state effects of changes in the real mw, supply, demand

- **Distribution of wages for skill** $s$ (whether or not $m$ affects unemployment)

\[ W_s(c) < W_s(c') \Rightarrow \frac{d \left[ \frac{W_s(c)}{W_s(c')} \right]}{dm} > 0 \]

where $W_s(c)$ is wage at percentile $c$ of employed skill $s$ workers
Burdett and Mortensen (1998) + binding minimum wage

- $W_c(m)$: wage at centile $c \in [0, 100]$

\[
W_c(m) = P - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2
\]

- Hence

\[
\frac{W_c'(m)}{W_c(m)} = \frac{P - (P - m) \left( \frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{p - (P - m) \left( \frac{100\delta}{c\lambda_e + 100\delta} \right)^2}
\]

- Differentiating with respect to $m$ yields

\[
\frac{d \left[ W_c'(m)/W_c(m) \right]}{dm} < 0 \iff c' > c
\]

- As in baseline model, $W_c(m)$ is log-submodular in $(c, m)$

- This result has been shown quantitatively in Engbom and Moser (2021)
Incorporating Supply and Demand

- Given focus on canonical model, impose those assumptions
  - + assumptions s.t. $\beta_m = \beta_{ms}$

- Then across steady states (to a first-order approximation)

$$\log \left( \frac{w_{ht}}{w_{\ell t}} \right) = \beta_m (b_{ht} - b_{\ell t}) \log m_t - \beta_L \log \left( \frac{L_{ht}}{L_{\ell t}} \right) + \beta_A t + \nu_t$$

- “Race between education, technology, and the minimum wage”
  - Extended canonical model
Comparative statics across steady states: $\beta_L$

- Elasticity wrt relative supply [if both $s$ bound by mw]

$$
\beta_L \equiv \frac{1}{\eta} (1 - \beta_m b_\ell) \frac{P_h(1 - u_h) L_h}{Y} + \frac{1}{\eta} (1 - \beta_m b_h) \frac{P_\ell(1 - u_\ell) L_\ell}{Y}
$$

- When $b_s = 0$ for both $s$, this is just $1/\eta$ as in canonical model