The Race Between Education, Technology, and the Minimum Wage

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Where we're going

- **What is the dynamic impact of the minimum wage on inequality?**
- Theory: inequality effects grow over time
- Empirics: document these dynamic effects
- **The impact of the minimum wage on inequality grows over time, with the effect more than doubling over two years**

How we get there

1. Motivation (The Race Between...): In national time series, the real minimum wage helps shape the evolution of U.S. college premium + partially resolves a "puzzle"

2. Theory: Job-ladder model with many skills (or groups)
   - On impact $\uparrow$ mw $\Rightarrow$ $\uparrow$ in wages for those individuals bound by it *(direct effect)*
   - Over time, workers move up the job ladder $\Rightarrow$ magnified effect *(indirect effect)*
   - Elasticity of the average wage of a skill group wrt mw $\uparrow$ in share of wage income bound by mw and grows over time *(combo of effects)*
   - Theory embeds into any aggregate production function combining skill outputs:

     "The Race Between Education, Technology, and the Minimum Wage"

3. Empirics: Using state-and-group-level data, document that the elasticity of the state $\times$ group average wage w.r.t. the minimum wage
   - is $\uparrow$ in share of wage income earned at the mw on impact *(direct effect)*
   - this difference in elasticities $\uparrow$ by a factor of > 2 over 2 years *(indirect effect)*
     - ★ quantitative elasticity consistent w/ national elasticity from "The Race..."
     - ★ $\downarrow$ in real wage of HSD in 1980s and early 1990s caused by $\downarrow$ real minimum wage

Contributions

- Canonical model (Tinbergen 74, Katz and Murphy 92, ...) including minimum wage (Autor, Katz, Kearney 08)
  - While supply and demand remain crucial, so too is the minimum wage
  - Minimum wage helps resolve the apparent rapid slowing of SBTC in the 1990s

- Impact of minimum wage on inequality

  Meyer, Wise, 83; DiNardo et al., 96; Lee, 99; Card, DiNardo, 02; Autor ea., 16; Cengiz ea. 19; Dube, 19; Fortin ea. 21; Chen, Teulings 22; ...
  - Direct effect dominates in short run, but indirect effect grows over time
  - Identified within worker groups (i.e. for fixed observable characteristics)

- Growing macro-labor literature of monopsony using quantitative models

  Haanwinckel, 20; Engbom, Moser, 21; Ahlfeldt et al., 22; Berger et al., 22; Hurst et al., 22; Trottner, 22; ...
  - Job-ladder model like EM, but focus on dynamics
  - Dynamics like HKPW, but driven by job ladder rather than putty-clay capital

Motivation:
National time-series variation

Canonical model + minimum wage

- I consider (for now atheoretical) regressions of the form

$$\log\left(\frac{w_{ht}}{w_{\ell t}}\right) = \alpha + \nu_m \log m_t + \nu_L \log\left(\frac{Supply_{ht}}{Supply_{\ell t}}\right) + \gamma_1 t + [...] + \iota_t$$

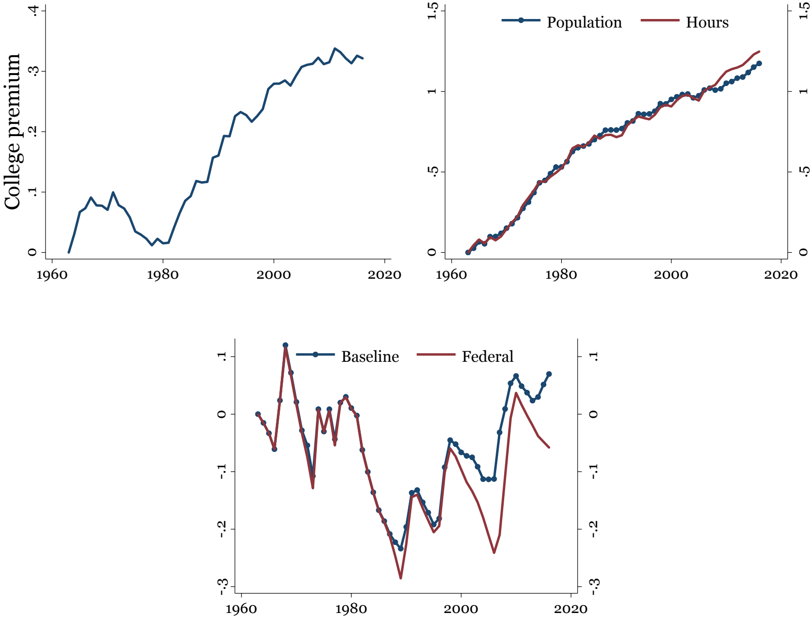
national time-series, t, variation across college and non-college workers, h and ℓ

  - $\log w_{ht}$ and $\log w_{\ell t}$ are measures of average log wages
  - $\log Supply_{ht}$ and $\log Supply_{\ell t}$ are measures of labor supply (hours worked)
  - m_t is a measure of the real minimum wage at the national level

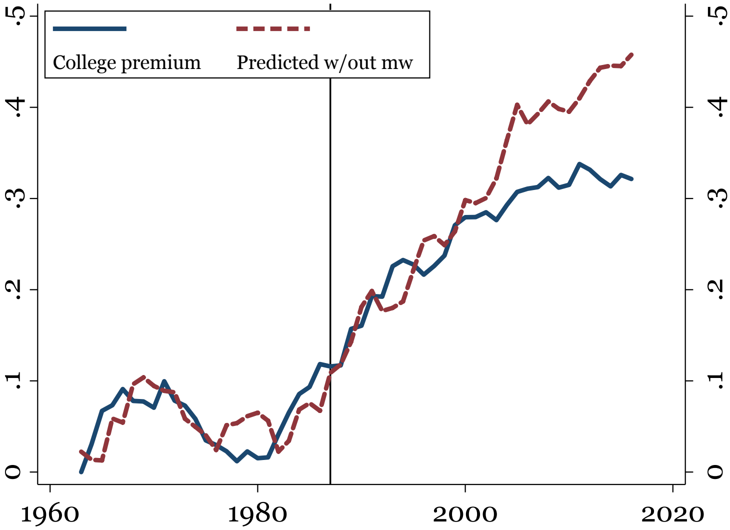
- Measurement

  - Supply and wages are composition adjusted (March CPS 1964 - 2017 spanning working years 1963 - 2016)
  - Instrument for supply is composition-adjusted population (March CPS)
  - National real minimum wage (Cengiz et al. (2019), DOL, FRED, March CPS)
    - ★ For each state use the max of the state and national statutory minima
    - ★ ... then average across states using time-invariant weights
    - ★ ... and apply the GDP deflator

Data


College premium
0
.1
.2
.3
.4
1960
1980
2000
2020
Population
Hours
0
.5
1
1.5
Baseline
Federal
-.3
-.2
-.1
0
.1

Result I: out-of-sample fit (2SLS)



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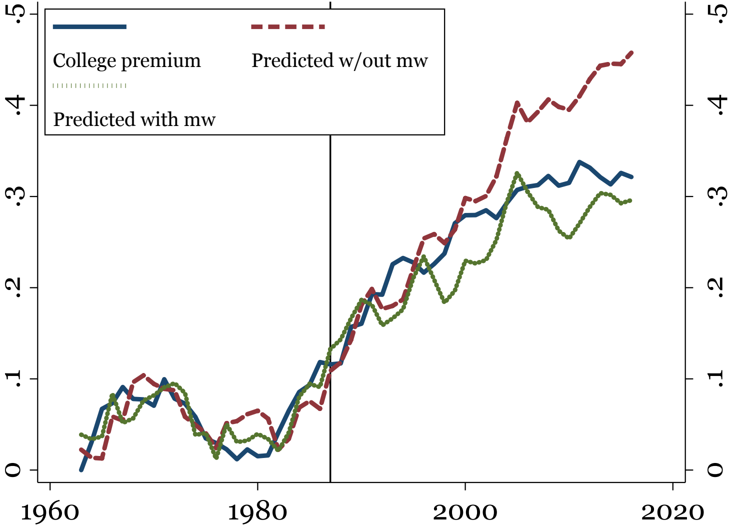
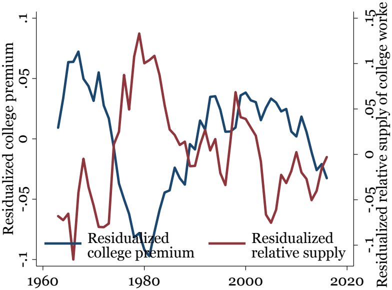
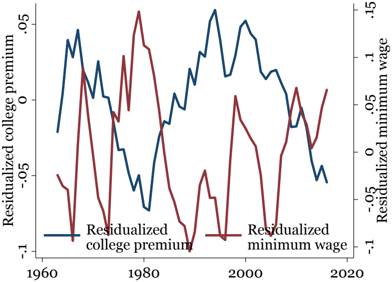


Residualized data

Using the population-based measure of relative college supply





Result II: in-sample elasticities...

... of the national college wage premium wrt relative supply and the real minimum wage

Regression Models for the College Wage Premium

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.632	-0.703	-0.608	-0.619	-0.387	-0.541
	(0.069)	(0.077)	(0.104)	(0.119)	(0.134)	(0.067)
Real minimum wage	-0.220	-0.199	-0.133	-0.129	-0.132	
	(0.048)	(0.059)	(0.052)	(0.064)	(0.046)	
Real federal minimum wage						-0.161
						(0.044)
Time	0.021					0.019
	(0.002)					(0.002)
Time Polynomial	1	2	3	4	5	1

The extended canonical model including polynomials of time up to degree j in column j. Estimated using 2SLS, instrumenting for hours-based supply using population-based measure. Robust standard errors are shown in parentheses.

- Sizable elasticity of the college premium wrt the real minimum wage
  - e.g., 26% $\downarrow$ in real minimum wage 1979 – 89 $\Longrightarrow$ 2.7 – 5.7% $\uparrow$ in national college premium

To baseline empirics

Summary + robustness

- **Summary:**
  1. relative supply growth fluctuations + trend demand growth crucial drivers of college premium, but changes in real minimum wage are also important
  2. less dramatic slowing of SBTC (more generally, improved out-of-sample fit)
- **Sensitivity and additional results:**
  - ... using two alternative measures of relative supply
    - ★ Tables with estimated elasticities go
    - ★ Figures with out-of-sample fit go
  - ... using Autor, Katz, and Kearney (2008) data go
  - ... separately for college and non-college workers go

Theory

Framework

- **Supply and demand:**
  - Exogenous supply L_{st} of homogeneous skill $s = 1, ..., S$ workers
  - Aggregate production function combining skill output with skill-time-specific productivity A_{st} shaping relative demand

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  - Zero discount factor: analysis of **transitions** to aggregate shocks
  - Worker can be employed or unemployed with exogenous separation (δ_s) and job-finding rates (λ_{su} unemployed and λ_{se} employed)
  - Bilateral generalized Nash bargaining btw new worker-firm match ($\beta_s =$ worker weight) over fixed real wage w/ current job as worker outside option

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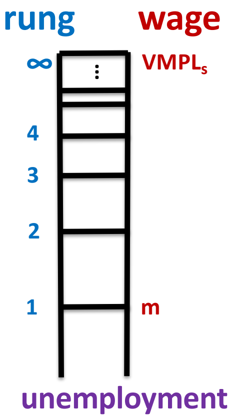
- Discrete time: within t, separation shocks occur first, then new matches realized for those workers who did not separate in t

Steady-state characterization

Suppose "binding" mw for given s (for exposition only)

- Wage ladder across "rungs"
- First rung is the minimum wage...
- ... and move up over time (if no separation shock)
- Average wage an average of mw and VMPL

details and distribution



Transitions

Proposition

Consider an economy in steady state at date 0 that faces a small one-time change from m to $m' > m$. For any skill s that was bound by m:

1. *For all $t \geq 0$ two job ladders coexist, with first rungs m (old) and m' (new)*
2. *The share on each rung j summed across the two ladders is constant across t*
3. *The share on rung j of the new ladder weakly increases in t*
4. *At each rung j, wage on the new job ladder $>$ than on the old one*
5. *The elasticity of the average wage wrt the minimum wage rises in t*
6. *On impact, this elasticity equals the share of income earned at m (the "bite")*

visualization minimum wage decline proof of 2

Transitions implication

- D-period elasticity of any group's average wage wrt to a one-time increase in the real mw from m to $m' > m$ at $t = 0$ (impulse response)

$$\underbrace{\log\left(\frac{w_{D,s}}{w_{-1,s}}\right)\Big/\log\left(\frac{m'}{m}\right)}_{D \text{ period elasticity of average wage wrt } m} \equiv M_{D,s} \times b_{-1,s}$$

decomposed into initial minimum wage "Bite" + "Magnification" elasticity

- $b_{-1,s}$ is the pre-shock share of wage income earned at the mw
- $M_{D,s}$ is the "Magnification elasticity"
  - $M_{0,s} = 1$
  - $dM_{D,s}/dD > 0$

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- If δ, λ_u, λ_e, β common across $s \Rightarrow$ elasticity of college premium (or any relative wage)

$$\log\left(\frac{w_{D,h}/w_{D,\ell}}{w_{-1,h}/w_{-1,\ell}}\right) \Big/ \log\left(\frac{m'}{m}\right) \equiv M_D \times (b_{-1,h} - b_{-1,\ell})$$

CS across steady states

Across steady state effects of changes in the real mw, supply, demand

- **Average wage of skill s for given changes in VMPL**

$$d\log w_s = M_s b_s \partial \log m + (1 - M_s b_s)\,\partial \log VMPL_s$$

  where VMPL_s is real and M_s is the steady-state magnification elasticity

$$M_s \equiv \lim_{D\to\infty} M_{D,s} = \frac{\delta_s + (1-\delta_s)\lambda_{se}}{\delta_s + \beta_s(1-\delta_s)\lambda_{se}} > 1$$

- **And finally, solving for changes in VMPL**
  - Given assumption m doesn't affect unemployment ($m < VMPL_s$ for all s)
  - $VMPL_s$ same as in competitive model w/ same aggregate production function except
    - Replace L_{st} with $(1-u_s)L_{st}$
    - Hence, w/ 2 skills + CES production function + linear rates of growth of A_{st}:

      "Race between education, technology, and the minimum wage"

The race

- **Distribution of wages for skill s**

Distributional implication

Empirical Approach

From theory to estimation

- Theory (omit s): if one-time permanent $m \uparrow$ in any period btw $t - T$ and t then

$$\log \frac{w_t}{w_{t-T}} = \sum_{j=0}^{T-1} M_j b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$

- This is a distributed lag model where the lag weights equal the magnification elasticities

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- Theory: allowing the one-time $m \uparrow$ to occur earlier, btw $t - T'$ and t for $T' > T$ then

$$\log \frac{w_t}{w_{t-T}} = \sum_{j=0}^{T-1} M_j b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}} + \sum_{j=T}^{T'-1} (M_j - M_{j-T}) b_{t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$

- Now lag weights equal the magnification elasticities only over the $t - T$ to t period; otherwise lag weights are smaller than the magnification elasticities

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- In practice, m is changing in every period in every empirical context
  - Apply versions of previous formula in presence of many changes
  - And run same regressions in model-generated data

Mapping to the data

- Time t: half year periods (m1-6, m7-12) between 1979m1 – 2016m12
- Skills s: labor groups g $\times$ regions r
  - 100 labor groups (5 age bins $\times$ 2 genders $\times$ 2 races $\times$ 5 educations)
  - 50 regions (U.S. states)
  - Minimum wages are r, t specific
  - Wages and minimum wage bites are g, r, t specific
- I study disaggregate outcomes as in the theory, rather than aggregating up and composition adjusting

Specification augmented in five ways (relative to theory)

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

1. Incorporate leads ($j < 0$) in addition to lags ($j \geq 0$)
2. Estimate lag weights μ_j that don't depend on worker characteristics
3. Omit changes in minimum wages that occurred more than 5 years before t

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4. Instrument replaces $b_{g,r,t-j-1}$ in interactions with its median value $b_{g,r}$
   - $b_{g,r,t}$ is measured with error
   - this ME can be correlated w/ ME in dependent variable
     - as pointed out in related context by Autor et al., 2016

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5. Include two additional controls $\mathbb{FE}_t \times b_{g,r}$ and α_t
   - Treatment fits into "shock-exposure" framework of Borusyak and Hull (2023)
   - Specification may suffer from OVB. For example:
     - ★ In periods experiencing $m \downarrow$ (e.g., 1980s), treatment $-$ correlated w/ bite
     - ★ Bite can be correlated with other shocks in residual, e.g., SBTC, which raises wages of groups with lower bites

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   - Avoid by controlling for $\mathbb{E}$[treatment]
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   - Additional benefits:
     - Control absorbs changes in inflation: identical RF results using nominal mw.
     - "Design-based" approach avoids negative ex-ante weights (BH, 2024)

Data

- NBER Merged Outgoing Rotation Group of the CPS (1979 – 2016)
  - Drop 1994 and 1995m1–m8: missing allocation flags
  - End 2016m12 before many municipalities begin setting their own mws; e.g.,
    - NYC + Nassau, Suffolk, and Westchester counties on 12/31/2016
    - Minneapolis, MN in 2018
    - Los Angeles \$0.50 above for large businesses on 7/1/16
    - San Diego \$0.75 above on 1/1/15
  - Measure $b_{g,r,t}$ defining mw worker as those with wage $\leq$ 1.05 mw
- Minimum wage data from Vaghul and Zipperer (2016), Cengiz et al. (2019)
  - Use maximum nominal mw in state-period
  - Deflate by maximum monthly GDP deflator in period
- 5,000 (g, r) pairs $+ \approx$ 60,000 obs. per $t \Rightarrow \approx$ 12 obs. per (g, r, t)
  - Winsorize wage at 2nd percentile *within* each (g, r)
  - Weigh by product of (g, r) work hours in t and $t - T$ divided by their sum
  - Use balanced sample: (g, r) with no missing wage data across t

Additional details

- Cluster standard errors by state
- All results in figures, which...
  - ... convert pre-trends to their more typical "levels" form
    - negative of coefficients for $j < 0$
    - see Roth (2024): "Interpreting Event-Studies..."
  - ... display averaged *annual* effects, except for impact effect
    - for period $j > 0$, display $(\mu_j + \mu_{j+1})/2$ + corresponding 95% confidence interval
    - for period $j < 0$, display $(\mu_j + \mu_{j-1})/2$ + corresponding 95% confidence interval
    - for period $j = 0$, display μ_0 + corresponding 95% confidence interval

Results

Outline of results

1. 2SLS specification in model-generated data
2. Reduced-form specification
3. 2SLS specification
4. Robustness of 2SLS specification
5. Implications

Model parameterization and quantitative exercise

- Choose model parameters
  - Externally to direct survey evidence (Hall and Mueller, 2018)
    - Converting from weekly to bi-annual: $\gamma_u \approx 0.79$, $\gamma_e = \gamma_u/2$, and $\delta \approx 0.10$
  - $\beta = 0.25$ to obtain long-run magnification elasticity of 2.4 (using analytics)
  - Choose 5,000 values of $VMPL_{g,r}$ targeting average real wages

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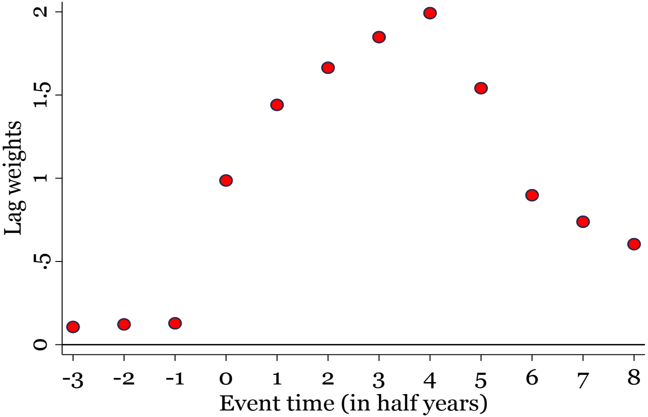
- Quantify impacts of changes in minimum wages
  - Start from a steady state in 1979m1 – m6
  - Feed in observed changes in real minimum wages in every state, period
  - Estimate baseline 2SLS specification using model-generated data

2SLS specification using model-generated data

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

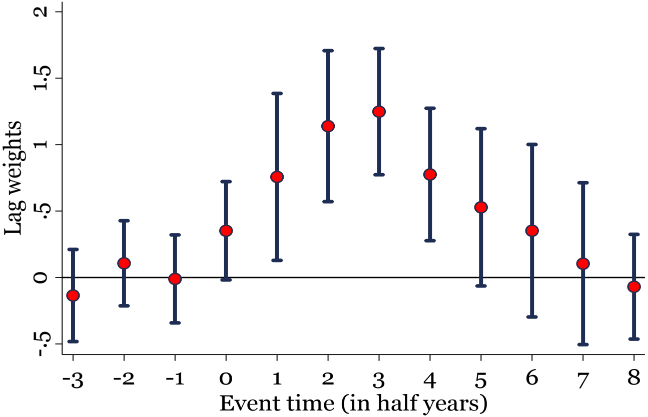


Notes: Point estimate for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

- Estimates rise from 1 to 2.1 over three years, given magnification elasticity of 2.4

RF specification

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



Notes: Point estimate and 95% CI for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

- Qualitative pattern as in model-generated data
- No evidence of pre-existing differential trends before changes in m

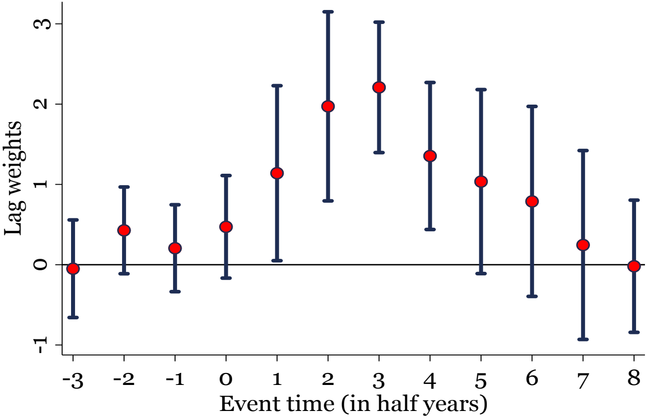
RF specification: Sensitivity

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

- Baseline: mw workers those with wages $\leq 1.05\times$ mw
  - mw workers those with wages $\leq 1.00\times$ mw Go
  - mw workers those with wages $\leq 1.10\times$ mw Go
  - mw workers those with wages $\leq 1.15\times$ mw Go
- Baseline uses all groups
  - Only for groups without college degrees Go
  - Separately by gender Go
  - Exclude final 6 sample years (w/ sub-state mws) Go
  - Unbalanced panel of (g, r) Go

2SLS specification

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



Notes: Point estimate and 95% CI for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

- Lag weight ≈ 0.5 on impact, $\uparrow$ to 1 in one year, and peaks at > 2.2 over two years
- Conclude a magnification elasticity of approximately 2.4

2SLS specification: Sensitivity

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \alpha + \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j-1} \times \log \frac{m_{t-j,r}}{m_{t-j-1,r}} + \mathbb{FE}_t \times b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$

- Baseline: mw workers those with wages $\leq 1.05\times$ mw
  - mw workers those with wages $\leq 1.00\times$ mw — Go
  - mw workers those with wages $\leq 1.10\times$ mw — Go
  - mw workers those with wages $\leq 1.15\times$ mw — Go
- Baseline uses all groups
  - Only for groups without college degrees — Go
  - Separately by gender — Go
  - Exclude final 6 sample years (w/ sub-state mws) — Go
  - Unbalanced panel of (g, r) — Go
- Assume t- and r- specific components of m changes (control for $\mathbb{FE}_r b_{g,r}$) — Go
- Incorporate an (r, t) fixed effect — Go
- Incorporate a (g, t) fixed effect — Go

Implication I: Revisiting the college premium

College premium elasticity with respect to m

- Long-run national elasticity of college premium wrt to the minimum wage of

$$M \times (b_h - b_\ell)$$

- Estimates suggest a long-run magnification elasticity of 2.4
- Average value across all (g, r, t) of $b_{g,r,t}$ is (mw workers those with wages $\leq 1.05\times$ mw)
  - for college g: 1.8%
  - for non-college g: 7.6%
- College premium elasticity wrt to m in the range of $2.4 \times (-0.058) \approx -0.14$
  - In the middle of the range of the national estimates

Go

Both national and disaggregated state $\times$ group estimates imply similar and sizable elasticities of the college premium with respect to the real minimum wage

Implication II: Real wages (and their decline)

Real wage elasticity with respect to m

- Real wages of low-education workers declined dramatically in 1980s into early 1990s
- This decline is impossible in "The Race Between Education and Technology" without a $\downarrow$ in productivity (Acemoglu and Restrepo, 2020)
- Possible in "The Race Between Education, Technology, and the Minimum Wage"

- Consider those without completed high school (HSD)
  - 26% decline of real mw between 1979 and 1989
  - Average value across all (g, r, t) of $b_{g,r,t}$ among HSD is 13.9%
  - $\downarrow m \Rightarrow \downarrow 8.7\%$ ($\approx 0.26 \times 0.139 \times 2.4$) in HSD real wage

$\downarrow$ real minimum wage explains entirety of $\downarrow$ real wage of HSD btw 1979 – 1992

Model limitations

What might these results suggest the theory lacks?

- In the data, coefficients begin to $\downarrow$ one period early (compared to in the model)
  - Something is either pushing up wages higher in the wage distribution over time
  - ... or pushing down wages lower in the wage distribution over time

- Prominent possibilities:
  1. Fairness or efficiency wage concerns $\uparrow$ wages higher up distribution
     - e.g., Grossman (1983)
  2. Technical change $\uparrow$ demand for higher-wage and/or $\downarrow$ demand for lower-wage groups
     - results apply with g, t fixed effects: localized demand changes
     - e.g., Hurst et al. (2022)

Go

Conclusions

What is the impact of the mw on inequality?

- Empirical motivation: two new facts in the national time series
  - minimum wage helps shape U.S. college wage premium
  - incorporating mw improves fit of "The Race" + reduces trend break in SBTC
- Theoretically:
  - on impact, $\uparrow$ mw raises wages more for groups more bound by it
  - over time, this difference in wage elasticities rises due to indirect effects
- Empirically: Find evidence consistent with these dynamic predictions
  - using state and group level data
  - holding the composition of workers fixed

  with magnification elasticity > 2 after ≈ 2 years
  - quantitatively consistent w/ national-time series estimates
  - $\downarrow m \Rightarrow$ all of $\downarrow$ real wage of HSD in 1980s and early 1990s

Empirical Appendix

Robustness: alternative supply #1

Regression Models for the College Wage Premium
Using dual of composition-adjusted changes in wages
Instrumenting with efficiency-unit populations

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.584	-0.724	-0.745	-0.668	-0.518	-0.512
	(0.063)	(0.085)	(0.118)	(0.124)	(0.175)	(0.058)
Real minimum wage	-0.207	-0.158	-0.171	-0.186	-0.175	
	(0.044)	(0.054)	(0.060)	(0.063)	(0.052)	
Real federal minimum wage						-0.163
						(0.039)
Time	0.022					0.020
	(0.002)					(0.001)
Time Polynomial	1	2	3	4	5	1

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Robustness: alternative supply #2

Regression Models for the College Wage Premium

Reduced-form specification

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.555 (0.060)	-0.816 (0.083)	-0.632 (0.101)	-0.692 (0.110)	-0.356 (0.106)	-0.464 (0.060)
Real minimum wage	-0.307 (0.053)	-0.257 (0.058)	-0.132 (0.047)	-0.115 (0.058)	-0.126 (0.038)	
Real federal minimum wage						-0.236 (0.048)
Time	0.019 (0.001)					0.016 (0.001)
Time Polynomial	1	2	3	4	5	1

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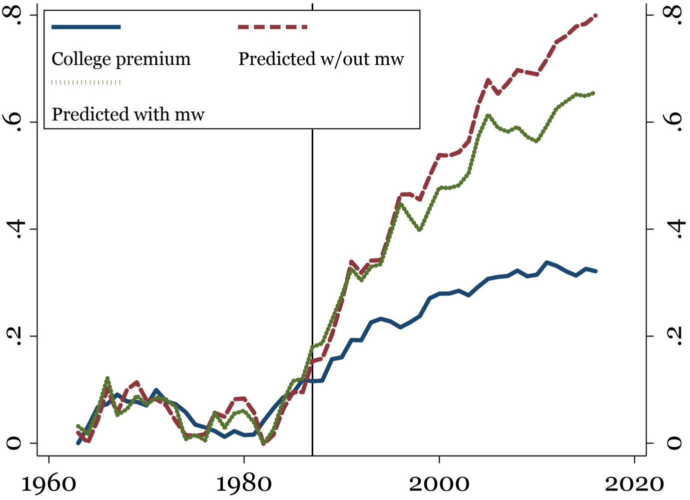
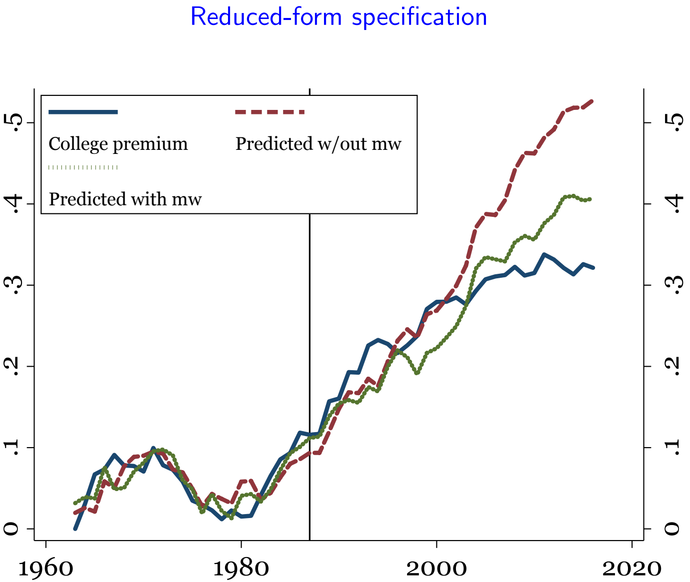
Results: predicted college premium alternative supply

Using dual of composition-adjusted changes in wages
Instrumenting with efficiency-unit populations



Results: predicted college premium reduced form



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Robustness: using data from AKK

Regression Models for the College Wage Premium
Using Data from AKK Replication Package (1963-2005)

	Using AKK data				
	(a)	(b)	(c)	(d)	(e)
Relative supply of college workers	-0.431	-0.607	-0.612	-0.216	0.013
	(0.051)	(0.077)	(0.091)	(0.113)	(0.104)
Minimum wage	-0.112	-0.108	-0.064	-0.174	-0.123
	(0.049)	(0.049)	(0.048)	(0.052)	(0.040)
	Using my baseline real minimum wage				
	(a)	(b)	(c)	(d)	(e)
Relative supply of college workers	-0.459	-0.605	-0.610	-0.244	-0.019
	(0.051)	(0.078)	(0.089)	(0.108)	(0.106)
Minimum wage	-0.150	-0.139	-0.087	-0.184	-0.119
	(0.051)	(0.053)	(0.057)	(0.059)	(0.050)
Time Polynomial	1	2	3	4	5
Observations	43	43	43	43	43

Impact of minimum wage is at least as robust as impact of supply

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National: separate regressions by education

Replace $\log \frac{w_{ht}}{w_{\ell t}}$ with $\log w_{ht}$ and with $\log w_{\ell t}$

Regression Models for the College and Non-College Wages

	Linear			Quadratic			Cubic		
	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)
Relative supply	-0.632	-0.414	0.218	-0.703	-1.029	-0.327	-0.608	-1.117	-0.509
	(0.069)	(0.127)	(0.123)	(0.077)	(0.140)	(0.139)	(0.104)	(0.158)	(0.103)
Real minimum wage	-0.220	-0.104	0.117	-0.199	0.083	0.282	-0.133	0.022	0.155
	(0.048)	(0.059)	(0.058)	(0.059)	(0.077)	(0.066)	(0.052)	(0.091)	(0.063)
Time	0.021	0.020	-0.002						
	(0.002)	(0.003)	(0.002)						

$\uparrow$ mw $\Rightarrow$ $\uparrow$ non-college average wage with no robust impact on college wage

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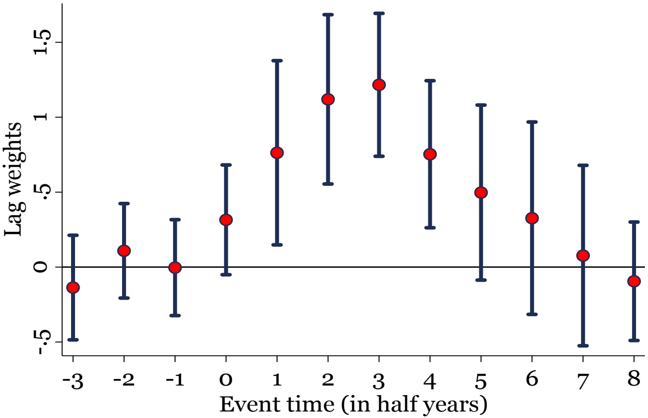
RF specification: Non-college sample

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



Notes: Point estimate and 95% CI for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

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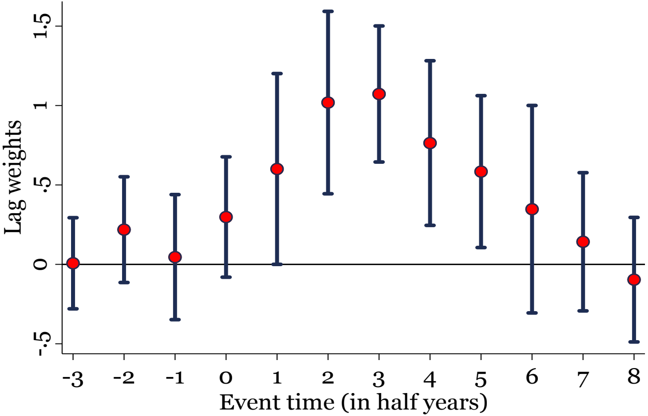
RF specification: Female sample

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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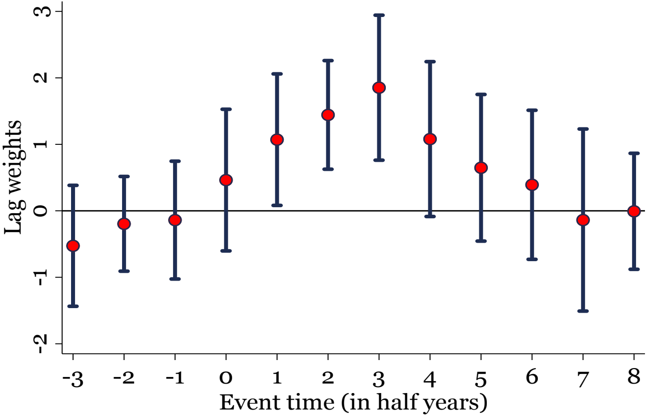
RF specification: Male sample

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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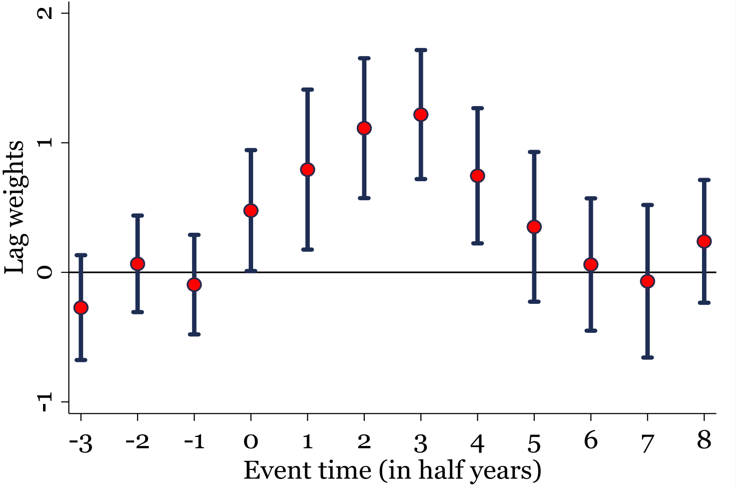
RF specification: 1979-2010 sample

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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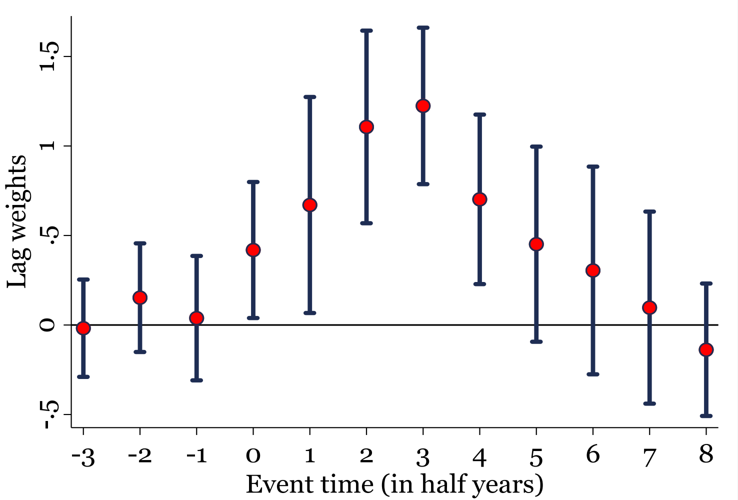
RF specification: Unbalanced sample

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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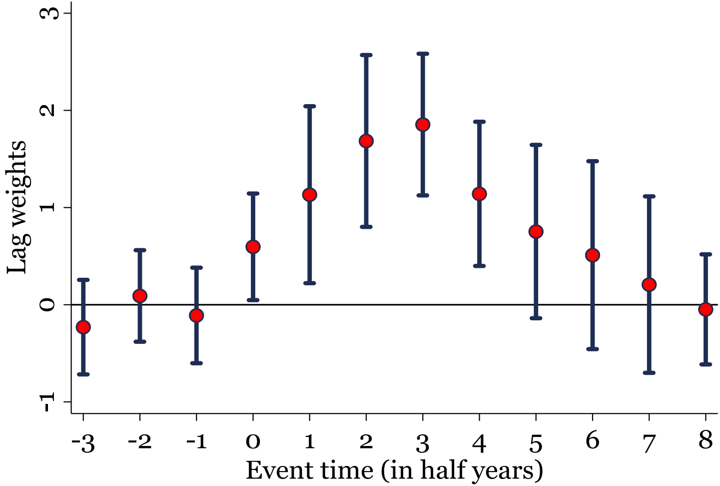
RF specification: mw cutoff of 1.00

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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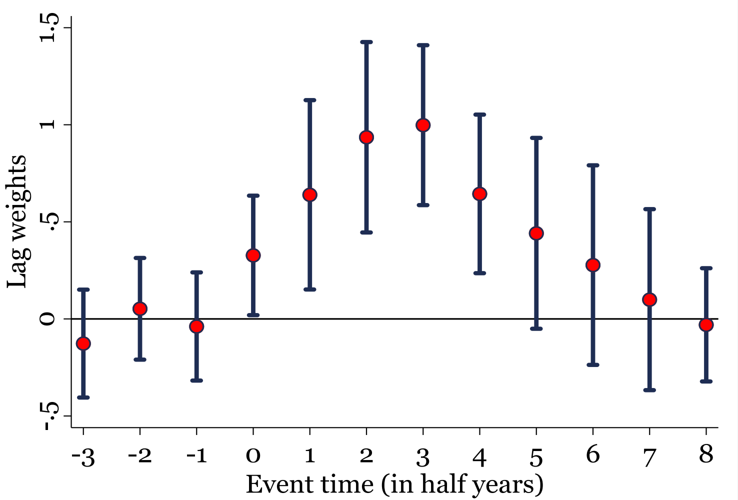
RF specification: mw cutoff of 1.10

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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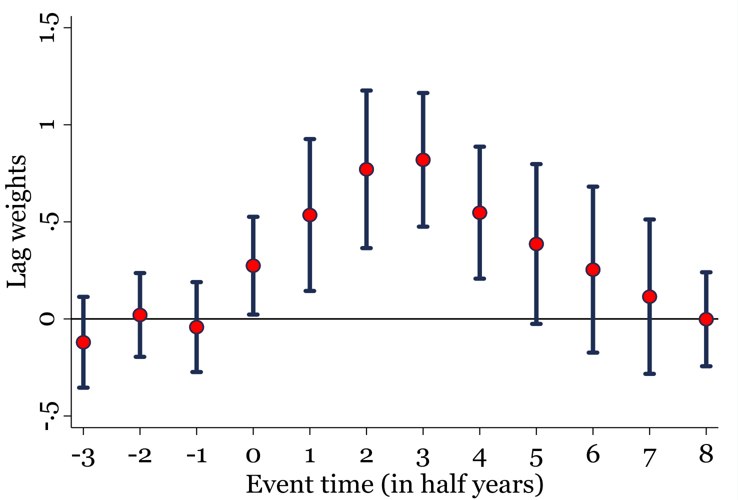
RF specification: mw cutoff of 1.15

$$\log \frac{w_{g,r,t}}{w_{t-D,g,r}} = \alpha + \sum_{j=-K}^{D-1} \mu_j \times bite_{g,r} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \varepsilon_{g,r,t}$$



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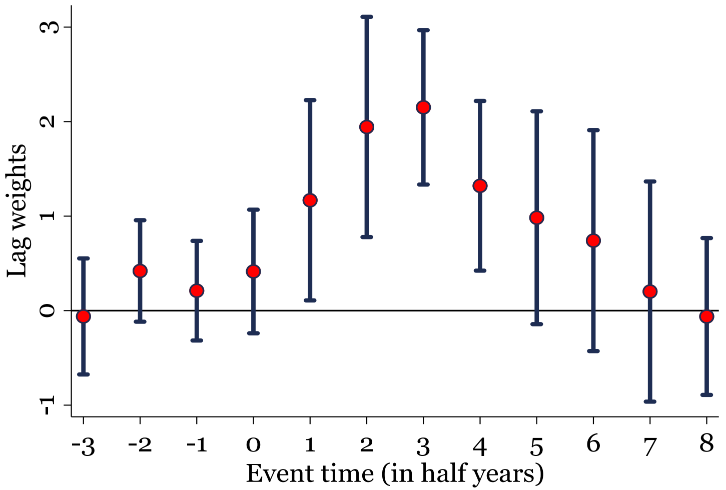
2SLS specification: Non-college sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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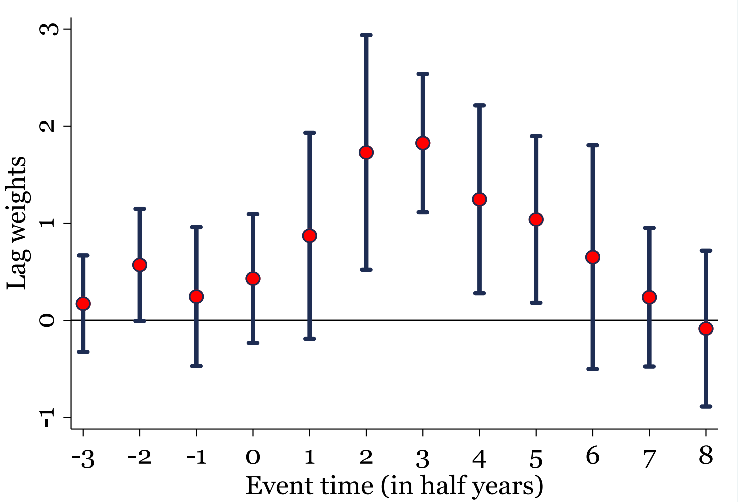
2SLS specification: Female sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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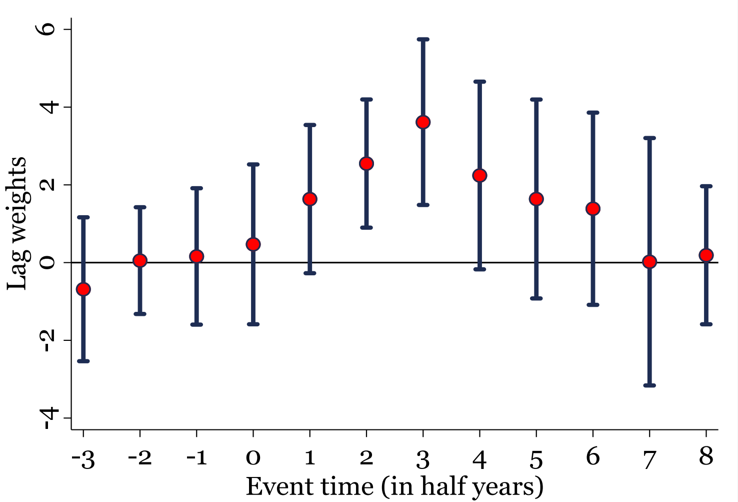
2SLS specification: Male sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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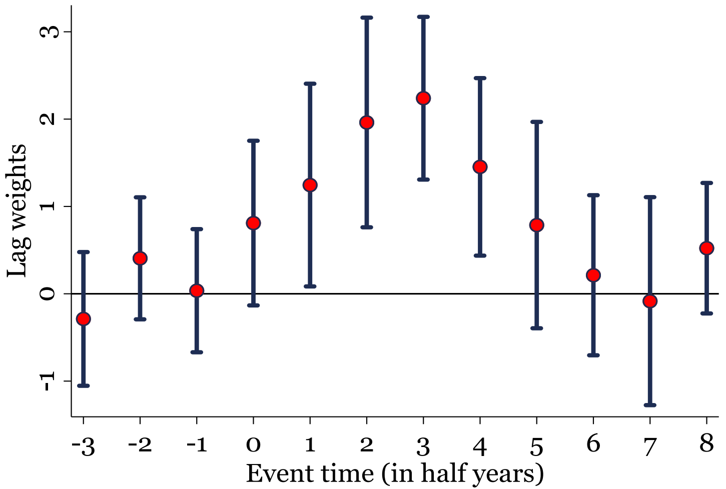
2SLS specification: 1979-2010 sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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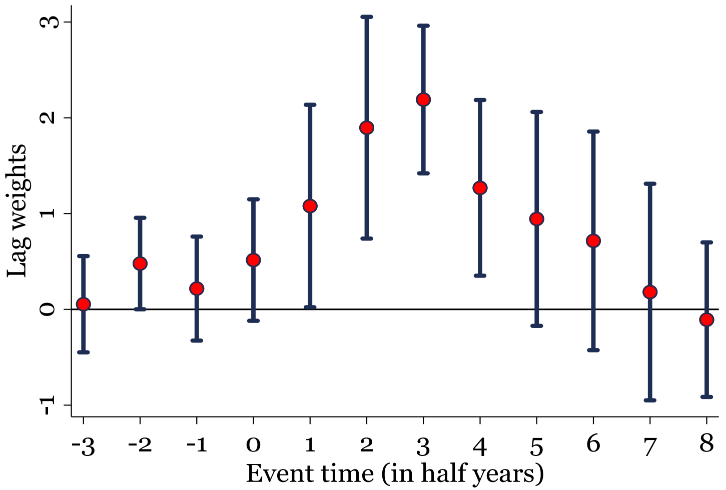
2SLS specification: Unbalanced sample

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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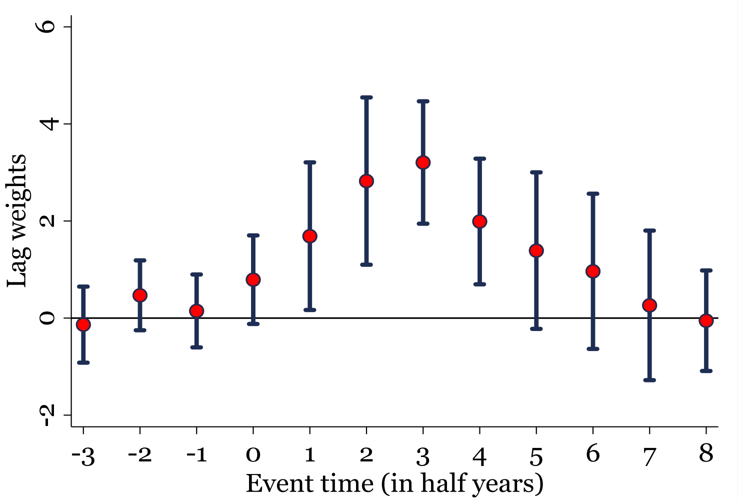
2SLS specification: mw cutoff of 1.00

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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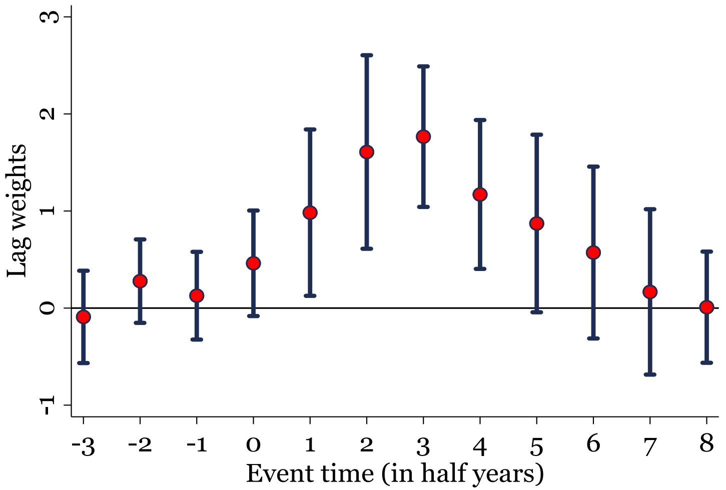
2SLS specification: mw cutoff of 1.10

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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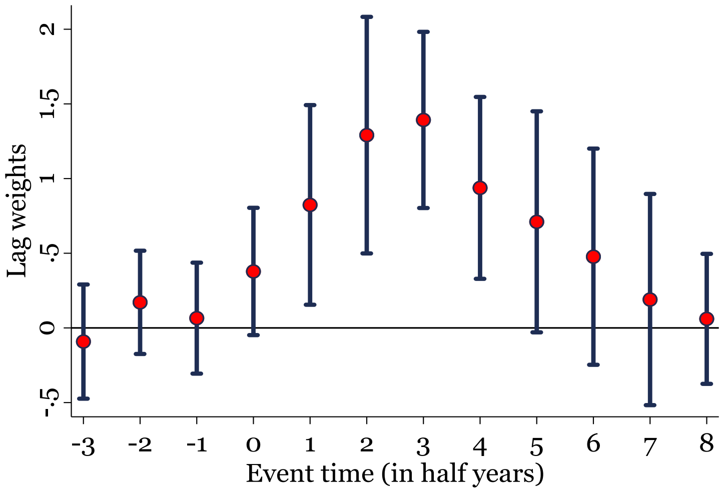
2SLS specification: mw cutoff of 1.15

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



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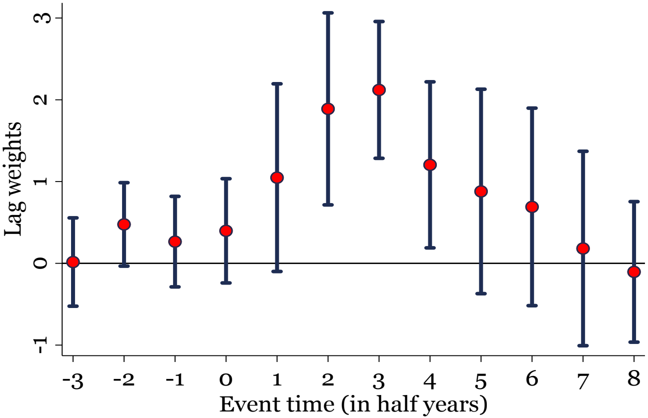
2SLS specification: An additional control

Assuming changes in the mw are *i.i.d.* + a t-specific component + an r-specific component

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \mathbb{FE}_r b_{g,r} + \alpha_t + \varepsilon_{g,r,t}$$



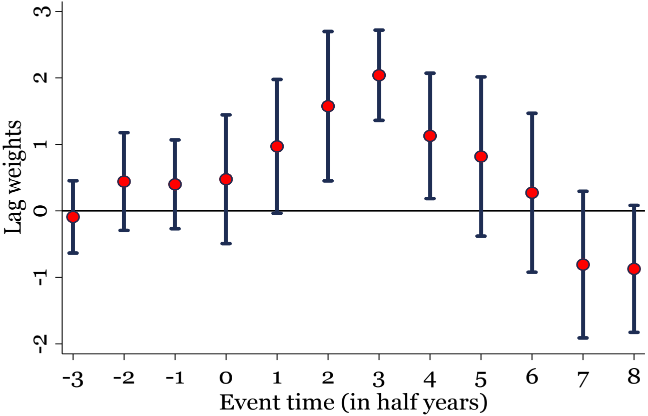
Notes: Point estimate and 95% CI for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

Back to baseline robustness

2SLS specification: An additional control

Assuming changes in the mw are *i.i.d.* + a t-specific component + include an (r, t) fixed effect

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_{r,t} + \varepsilon_{g,r,t}$$



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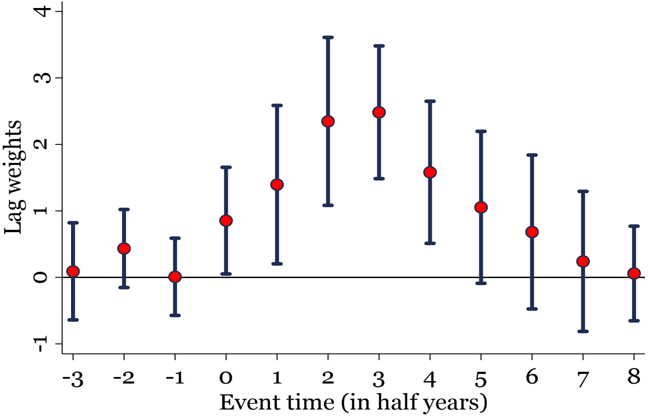
Back to baseline robustness

2SLS specification: An additional control

Assuming changes in the mw are *i.i.d.* + a t-specific component + include a (g,t) fixed effect

$$\log \frac{w_{g,r,t}}{w_{g,r,t-6}} = \sum_{j=-4}^{9} \mu_j \times b_{g,r,t-j} \times \log \frac{m_{r,t-j+1}}{m_{r,t-j}} + \mathbb{FE}_t b_{g,r} + \alpha_{g,t} + \varepsilon_{g,r,t}$$



Notes: Point estimate and 95% CI for μ_0, linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$

Back to Sensitivity

Back to model limitations

Theoretical Appendix

Transitions

Proposition

Consider an economy in steady state at date 0 *that faces a small one-time change from* m *to* $m' > m$*. Then for any skill* s *that was bound by* m*:*

1. *For all* $t \geq 0$ *two job ladders coexist, with first rungs* m *(old) and* m' *(new)*
2. *The share on each rung* j *summed across the two ladders is constant across* t
3. *The share on rung* j *of the new ladder rises (weakly) in* t
4. *At each rung* j*, wage on the new job ladder is higher than on the old one*
5. *The elasticity of the average wage wrt the minimum wage rises in* t
6. *On impact, this elasticity equals the share of income earned at* m *(the "bind")*

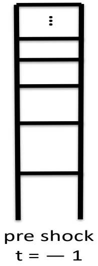
And, if $m' < m$**, then**

- *1, 2, and 3 are identical*
- *4 and 5 are reversed*
- *6: the instantaneous elasticity of the average wage* $= 0$

back

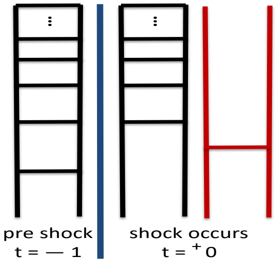
Transitions (for one s bound by m)

- Economy in steady state at date 0



Transitions (for one s bound by m)

- Economy in steady state at date 0; a small one-time change to $m' > m$

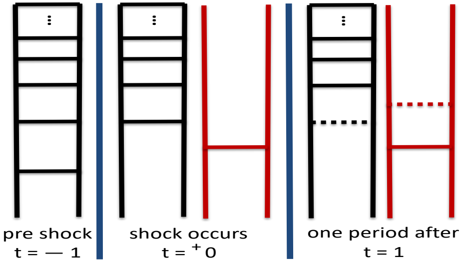


- On impact, the first rung disappears on the original ladder and appears, higher, on the new ladder (direct effect)

$$\underbrace{\log\left(\frac{w_{0^+,s}}{w_{-1,s}}\right) \Big/ \log\left(\frac{m'}{m}\right)}_{\text{instantaneous elasticity}} \equiv \underbrace{b_{-1,s}}_{\text{share of wage income earned at mw before shock}}$$

Transitions (for one s bound by m)

- Economy in steady state at date 0; a small one-time change to $m' > m$

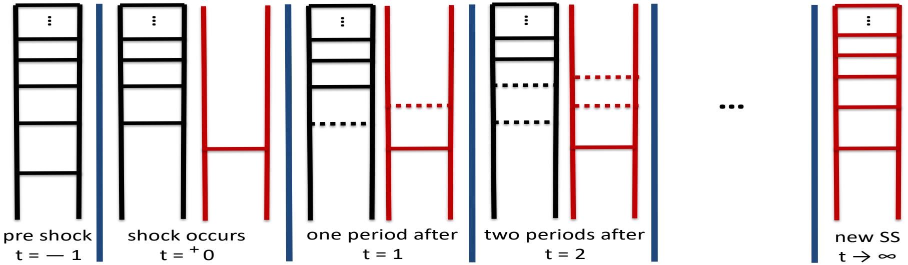


- One period later, the second rung on the original ladder starts emptying out as the second rung on the new ladder starts filling in (small indirect effect)

$$\log\left(\frac{w_{1,s}}{w_{-1,s}}\right) \Big/ \log\left(\frac{m'}{m}\right) > b_{-1,s}$$

Transitions (for one s bound by m)

- Economy in steady state at date 0; a small one-time change to $m' > m$



- ... converging to the new steady state, w/ all on the new job ladder, with the importance of the indirect effect growing each period

$$\log\left(\frac{w_{t',s}}{w_{-1,s}}\right) \Big/ \log\left(\frac{m'}{m}\right) > \log\left(\frac{w_{t,s}}{w_{-1,s}}\right) \Big/ \log\left(\frac{m'}{m}\right) \quad \text{for all} \quad t' > t \geq 0$$

back

Transition proof (for given s) for a small $\uparrow m$ at $t = 0$

- g_j rung j share in SS; $g_{t,j}$ and $g'_{t,j}$ rung j shares on original, new ladders at t

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- At $t = 0$: first rung + unemployment fully reallocate (and nothing else)
  - $\Rightarrow g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$

Transition proof (for given s) for a small $\uparrow m$ at $t = 0$

- g_j rung j share in SS; $g_{t,j}$ and $g'_{t,j}$ rung j shares on original, new ladders at t
- At $t = 0$: first rung + unemployment fully reallocate (and nothing else)
  - $\Rightarrow g_{0,j} + g'_{0,j} = g_j$ for all $j \geq 0$
- For some $t \geq 0$: suppose $g_{t,j} + g'_{t,j} = g_j$ for all $j \geq 0$
  - $\Rightarrow$ date $t + 1$ and for any $j > 1$, we have

$$g_{t+1,j} = g_{t,j}(1-\delta)(1-\lambda_e) + g_{t,j-1}(1-\delta)\lambda_e$$
$$g'_{t+1,j} = g'_{t,j}(1-\delta)(1-\lambda_e) + g'_{t,j-1}(1-\delta)\lambda_e$$

    and if $j = 1$ replace $(1-\delta)\lambda_e$ w/ λ_u [and note that $g_{t,0} = 0$]
  - Summing these expressions and using $g_{t,j} + g'_{t,j} = g_j$ yields

$$g_{t+1,j} + g'_{t+1,j} = g_j(1-\delta)(1-\lambda_e) + g_{j-1}(1-\delta)\lambda_e = g_j$$

    where the final equality follows from the steady-state derivation of g_j

back

Distribution and the average wage

- Density $g_s(w_j)$ satisfies

$$[\delta_s + (1-\delta_s)\lambda_{se}]\, g_s(w_{1,s}) = \lambda_{su} g_s(w_{0,s})$$
$$[\delta_s + (1-\delta_s)\lambda_{se}]\, g_s(w_{j+1,s}) = (1-\delta_s)\lambda_{se} g_s(w_{j,s}) \quad \text{for} \quad j \geq 1$$

- Unemployment rate

$$g_s(w_{0,s}) = \frac{\delta_s}{\delta_s + \lambda_{su}}$$

- Share at each rung

$$g_s(w_{j,s}) = \left(\frac{(1-\delta_s)\lambda_{se}}{\delta_s + (1-\delta_s)\lambda_{se}}\right)^{j-1} \frac{\lambda_{su}}{\delta_s + (1-\delta_s)\lambda_{se}} \frac{\delta_s}{\delta_s + \lambda_{su}} \quad \text{for} \quad j \geq 1$$

- Average wage $w_s \equiv \frac{1}{1-g_s(w_{0,s})} \sum_{j\geq 1} w_{j,s} g_s(w_{j,s})$ among the employed

$$w_s = \frac{\delta_s}{\delta_s + \beta_s(1-\delta_s)\lambda_{se}} m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1-\delta_s)\lambda_{se}}\right) P_s$$

back to steady state

steady state in Burdett and Mortensen (1998)

Burdett and Mortensen (1998) + binding minimum wage

- Equation (2.10) in van den Berg and Ridder (1998), eqm earnings density

$$g(w) = \frac{\delta\,(P-m)^{1/2}}{2\lambda_e}\,(P-w)^{-3/2} \text{ for all } w \in [m, w_{\max}]$$

with maximum wage

$$w_{\max} \equiv \left(\frac{\delta}{\delta+\lambda_e}\right)^2 m + \left(1-\frac{\delta}{\delta+\lambda_e}\right)^2 P$$

- Average wage is then

$$w = \frac{(100\delta)^2}{(w_{\max}\lambda_e+100\delta)(m\lambda_e+100\delta)}\,m + \left(1-\frac{(100\delta)^2}{(w_{\max}\lambda_e+100\delta)(m\lambda_e+100\delta)}\right)P$$

weighted avg of m and P as in baseline model, but weights depend on m

back to steady state

CS across steady states (including unemployment effects)

Across steady state effects of changes in the real mw, supply, demand

- **Distribution of wages for skill** s (whether or not m affects unemployment)

$$W_s(c) < W_s(c') \Rightarrow \frac{d\left[W_s(c)/W_s(c')\right]}{dm} > 0$$

where $W_s(c)$ is wage at percentile c of employed skill s workers

back to CS across steady states

equivalent result in Burdett and Mortensen (1998)

Burdett and Mortensen (1998) + binding minimum wage

- $W_c(m)$: wage at centile $c \in [0, 100]$

$$W_c(m) = P - (P - m)\left(\frac{100\delta}{c\lambda_e + 100\delta}\right)^2$$

- Hence

$$\frac{W_{c'}(m)}{W_c(m)} = \frac{P - (P - m)\left(\frac{100\delta}{c'\lambda_e + 100\delta}\right)^2}{p - (P - m)\left(\frac{100\delta}{c\lambda_e + 100\delta}\right)^2}$$

- Differentiating with respect to m yields

$$\frac{d\left[W_{c'}(m)/W_c(m)\right]}{dm} < 0 \iff c' > c$$

- As in baseline model, $W_c(m)$ is log-submodular in (c, m)
- This result has been shown quantitatively in Engbom and Moser (2021)

back

Incorporating Supply and Demand

- Given focus on canonical model, impose those assumptions
  - $+$ assumptions s.t. $\beta_m = \beta_{ms}$
- Then across steady states (to a first-order approximation)

$$\log\left(\frac{w_{ht}}{w_{\ell t}}\right) = \beta_m(b_{ht} - b_{\ell t})\log m_t - \beta_L \log\left(\frac{L_{ht}}{L_{\ell t}}\right) + \beta_A t + \iota_t$$

- "Race between education, technology, and the minimum wage"
  - Extended canonical model

β_L

back

Comparative statics across steady states: β_L

- Elasticity wrt relative supply [if both s bound by mw]

$$\beta_L \equiv \frac{1}{\eta}\left(1-\beta_m b_\ell\right)\frac{P_h(1-u_h)L_h}{Y}+\frac{1}{\eta}\left(1-\beta_m b_h\right)\frac{P_\ell(1-u_\ell)L_\ell}{Y}$$

- When $b_s = 0$ for both s, this is just $1/\eta$ as in canonical model

back