

The Race Between Education, Technology, and the Minimum Wage*

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Abstract

What is the impact of the minimum wage on wages and inequality? I develop a theory that nests “The Race Between Education and Technology.” The theory predicts that the impact of changes in minimum wages is initially small but grows over time. I present empirical evidence of these dynamic effects: the elasticity of real wages grows by more than a factor of two over a two-year period. I additionally show that minimum wages help rationalize a considerable decline in real wages of low-education workers in the 1980s and play a role in shaping the evolution of the U.S. college premium.

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1 Introduction

What are the dynamic impacts of minimum wages on real wages and inequality? I present a theory in which the elasticity of real wages with respect to the real minimum wage is small on impact—with an increase in the minimum wage raising only the wages of workers whose wages are below the new minimum—and grows over time. I present empirical evidence consistent with these dynamic effects. And I use the model and empirical evidence to document that minimum wages—together with supply and demand—play a central role in shaping the evolution of the U.S. college wage premium and that declining real minimum wages in the 1980s in the U.S. caused a considerable decline in the real wages of workers without a high school degree.

To motivate the analysis, in Section 2 I show that the real minimum wage helps shape even a very aggregated measure of inequality—the college wage premium—even at the national level.¹ *The Race Between Education and Technology* or the *canonical model*—introduced by Tinbergen (1974) and Freeman (1976), operationalized by Katz and Murphy (1992), and named by Goldin and Katz (2008) and Acemoglu and Autor (2011)—provides the central organizing framework for studying the evolution of the college premium. It straightforwardly relates relative wages of more and less educated workers to their relative supply and the skill bias of demand. I estimate an extended version of the empirical canonical model, incorporating the real minimum wage. I find that the elasticity of the national college premium with respect to the real minimum wage is approximately -0.13 , implying (for example) that the 26% decline in the real minimum wage between 1979 and 1989 increased the college premium by 3.4%, which is about a quarter of its observed increase.

Introducing the real minimum wage additionally alters a standard conclusion in the literature, that the rate of skill-biased technical change has declined dramatically in the 1990s and thereafter. The canonical model estimated on data spanning 1963–1987 (the set of years used in the seminal work of Katz and Murphy, 1992) predicts substantially more rapid increases in the college wage premium than actually occur in the data thereafter. Through the lens of the model, this problem with the model’s out-of-sample fit implies a substantial decline in the rate of skill-biased technical change. This issue is mitigated by incorporating the real minimum wage; its rise in the later period reduces the predicted growth rate of the college premium in the absence of changes in the rate of skill-biased technical change.

In Section 3, I present a generalized version of the canonical model that microfounds

¹I focus in Section 2 on the college premium because its evolution plays a central role in shaping overall U.S. wage inequality. For instance, Autor et al. (2020) find that increased returns to post-secondary schooling generate 57% of the observed increase in wage variance between 1980 and 2017.

the national regression analysis of Section 2; rationalizes its findings of a substantial elasticity of the college premium with respect to the real minimum wage in spite of the small share of workers actually earning the minimum wage; introduces novel predictions on the dynamic implications of changes in the minimum wage on which the results in Section 2 are silent; and guides the subsequent empirical analysis in Sections 4 and 5, which uses disaggregated data and focuses directly on these dynamics. My objective is to maintain the simplicity and tractability of the original framework yet facilitate the study of the dynamic implications—both across steady states and in transitions—of real minimum wages on real wages and inequality. In addition to supply and demand, the model incorporates monopsony power, minimum wages, a job-ladder, dynamics, and unemployment. As in the canonical model, an aggregate constant returns to scale production function combines the output of different skills. Unlike the canonical model, the labor market is frictional. A worker who meets a potential new employer bargains over her wage with her current wage (or unemployment benefit) serving as her outside option.

In equilibrium, a worker exits unemployment at the minimum wage and slowly moves up the job ladder to higher wages as she matches with new employers over time.² In the steady state, the average real wage of a given skill group is a weighted average of the real minimum wage and that group's real value marginal product of labor. I characterize how changes in supply, demand, and the minimum wage affect real wages across steady states. I focus on a baseline case in which the minimum wage has no effect on unemployment, in which case changes in supply and demand shape value marginal products of labor (exactly as in a frictionless and competitive model with the same aggregate production function) whereas changes in the minimum wage affect wage markdowns; I additionally consider the case in which unemployment depends on the minimum wage. I show that, for any aggregate production function and any number of worker skills, the steady-state elasticity of a skill group's average real wage with respect to the real minimum wage can be expressed as the product of the share of that group's wage income earned at the minimum wage (*the minimum wage bite*) and a term that is strictly greater than one, which I refer to as the steady-state *magnification elasticity*. Under the additional assumptions of the traditional canonical model—two skills, a CES aggregate production function, constant factor-biased productivity growth rates—I micro-found the extended canonical model estimating equation of Section 2, to a first-order approximation. Theoretically, the elasticity of the college wage premium with respect to the real minimum wage estimated

²Here, I describe model implications focusing on workers in a skill group that exits unemployment at the minimum wage. In the model, other worker groups may exit unemployment at a wage higher than the minimum wage. The case described here is particularly relevant for lower-education workers in the data.

in Section 2 equals the product of the long-run magnification elasticity and the difference between the minimum wage bite of college-educated and non-college-educated workers.

I also characterize the transition in response to a one-time increase in the real minimum wage. On impact, its increase raises real wages only for workers whose initial wage is bound by the new minimum wage. Hence, on impact the elasticity of a skill group's average real wage equals its minimum wage bite. Over time, however, workers initially at the new minimum wage slowly rise up the new job ladder, and the elasticity of the average real wage rises monotonically over time, with the time-varying magnification elasticity starting at one and converging to its steady-state level.

The model in Section 3 micro-founds and helps interpret the regression results not only in Section 2, but also in Sections 4 and 5, where the analysis is conducted at a more disaggregated level and focuses on dynamics. In Section 4 I show that the model of Section 3 implies that the change in the real wage of any group between period $t - T$ and period t in response to a one-time change in the real minimum wage can be expressed as a distributed lag model in which the independent variables are interactions between the time-varying initial minimum wage bites and lagged one-period changes in the real minimum wage. The lag weights to be estimated are exactly equal to the corresponding time-varying magnification elasticities for lagged shocks that occur between periods $t - T$ and t and are strictly less than the time-varying magnification elasticities for lagged shocks that occur beforehand.

I estimate this regression defining a skill group in the data by the interaction of 100 labor groups (defined by age, gender, race, and education) and 50 regions (the U.S. states) and defining time periods in the data as each half year between 1979 and 2016. Specifically, the dependent variable is the three-year change in the real wage of each of the 5,000 region-group pairs for each time period. I instrument for the independent variables by replacing the time-varying minimum wage bite for a given region and group with its median value across time periods to address issues of measurement error similar to those emphasized in a related context by Autor et al. (2016). Finally, I use a “design-based” specification in the spirit of Borusyak and Hull (2023), controlling for the expectation of treatment under an assumption on the process of real minimum wage changes.

Because the lag weights can only be interpreted directly as estimates of the time-varying magnification elasticities in the special case in which the real minimum wage experiences a one-time change—which is not satisfied in any empirical context given inflation and minimum wage policy—I begin in Section 5 by estimating the specification using model-generated data, feeding into the model all observed changes in real minimum wages in each state. Here, I show that the estimates converge reasonably quickly

towards the true long-run magnification elasticity. I then turn to estimating the same specification in the data. On impact, the magnification elasticity is approximately 0.5. It rises to approximately 1 within the first year, and peaks at over 2 at a two-year time horizon. Through the lens of the theory, this is consistent with a long-run magnification elasticity in the range of 2.4.³

I then consider the implications of this magnification elasticity for the college premium, revisiting the question in Section 2. Model results in Section 3, the average minimum wage bite across workers with and without a college education, and the long-run magnification elasticity of Section 5 together imply an elasticity of the college premium with respect to the real minimum wage of approximately -0.14 , consistent with the estimates in Section 2. From both exercises, I conclude that the elasticity of the college premium with respect to the real minimum wage is sizable.

The theoretical and empirical results in Section 3 and 5 shed light on the impact of minimum wages not only on relative wages, but also on real wage levels. In the U.S., real wages of workers without a high school degree declined dramatically starting in the 1980s and into the early 1990s. Theoretically, this outcome is impossible in the canonical model in the absence of productivity declines for non-college-educated workers. However, in the extended canonical model a sizable decline in the real minimum wage (as occurred in the 1980s) substantially reduces the real wage of any labor group with a large minimum wage bite (as is the case for workers without a high school degree). According to the model and estimated long-run magnification elasticity, the 26% decline in the real minimum wage between 1979 and 1989 caused the entirety of the observed decline in the real average wage of workers without a high school degree between 1979 and 1992.

Additional Literature. In terms of static economic questions, my paper is perhaps most related to Autor et al. (2008), who estimate a regression very similar to my motivating empirics in Section 2 leveraging national-time series variation. They contend that the real minimum wage “does not much alter the central role for relative supply growth fluctuations and trend demand growth in explaining the evolution of the college wage premium” and that “institutional factors are insufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s.” They reach their conclusions because they find that the coefficient on the real minimum wage is negative, significant, and of a similar magnitude to my estimates in one of their two specifications

³I consider sensitivity analyses that fit into four broad categories. I check for heterogeneity in magnification elasticities across worker groups by replicating the baseline empirical analysis separately for workers with different observable characteristics. I drop sample years in which some municipalities apply a minimum wage above the statutory level set by their states. I change the definition of a minimum wage worker used in the construction of the minimum wage bite. And I consider a broader set of controls.

(with a linear time trend) but smaller and insignificantly different from zero in their other specification (with a cubic time trend). I conclude differently. I show that my conclusions differ from theirs for three reasons: I consider a broader set of specifications, I have access to a longer sample, and I measure the real minimum wage using the average of minimum wages across states instead of the federal minimum wage (which introduces attenuation bias). More importantly, my empirical results additionally hold using state-and-worker-group-level variation. Based on these empirical findings, I conclude that relative supply growth fluctuations and trend demand growth remain crucial drivers of the college premium, consistent with [Autor et al. \(2008\)](#), but so too are changes in the real minimum wage. I also conclude that changes in the minimum wage go some ways towards resolving the puzzle posed by slowing trend relative demand for college workers in the 1990s. Finally, I also micro-found the regression they and I estimate.

A vast body of work studies the roles of labor-market institutions for shaping inequality. Much of the focus is on minimum wages (e.g., [DiNardo et al., 1996](#); [Lee, 1999](#); [Card and DiNardo, 2002](#); [Teulings, 2003](#); [Autor et al., 2016](#); [Cengiz et al., 2019](#); [Dube, 2019](#); [Chen and Teulings, 2022](#)) and monopsony power (see [Manning, 2013](#) for a summary). The analysis in my paper differs from this literature in a few ways. First, and most importantly, I focus on the time-varying impact of minimum wages on real wages and inequality whereas this literature does not focus on dynamics. Second, this literature often focuses on the impact of minimum wage changes on wage centiles (or the number of workers in each wage interval), whereas my empirical focus is on the impact of minimum wages on average real wages for each labor group. In this respect, my focus allows me to control explicitly for any potential changes in the observable composition of the labor force that might be correlated with changes in the minimum wage; but this comes at the cost of not studying within-group inequality empirically (because the data is too sparse to analyze changes in the distribution of wages within each labor group and state).⁴ Nevertheless, my short-run theoretical and empirical results are consistent with this literature's empirical results that the direct effect is the dominant mechanism through which the minimum wage affects inequality (see, e.g., [Autor et al., 2016](#); [Cengiz et al., 2019](#)). My theory, however, implies that the indirect effect strengthens over time. In my empirics, the indirect effect is slightly stronger than the direct effect two years after the minimum wage change. Moreover, I show analytically that a job-ladder model can rationalize these

⁴Note that my analysis is neither more nor less aggregated than this literature. I consider the impact of minimum wages on each of 100 labor groups in each state, whereas (e.g.) [Autor et al. \(2016\)](#) study the impact on each centile of the wage distribution relative to the median, which involves a similar amount of data aggregation.

empirical findings both across steady states and in the full transition.⁵

My paper is also related to a recent and fast growing macro-labor literature conducting counterfactual analyses to study the implications of minimum wage policies for inequality, efficiency, and welfare (e.g., [Haanwinckel, 2023](#); [Ahlfeldt et al., 2022](#); [Berger et al., 2022](#); [Hurst et al., 2022](#); [Engbom and Moser, 2022](#); [Trottner, 2022](#)). In contrast, I use my qualitative theory to guide and interpret my empirical analyses of the impact of minimum wages on real wages and wage inequality. In one respect, my analysis is closest to [Haanwinckel \(2023\)](#), who introduces a model of labor supply, labor demand, and the minimum wage to study the impact of changes in the minimum wage on inequality in Brazil. In another respect, my analysis is closest to [Engbom and Moser \(2022\)](#), who use a job-ladder model to study the implications of minimum wages in Brazil. But I am particularly interested in dynamic effects, from which these papers abstract. In this respect, my analysis is most related to [Hurst et al. \(2022\)](#). Their dynamics are driven primarily by putty-clay capital as opposed to job ladders.⁶

2 Motivation: national time series evidence

The Race Between Education and Technology (also known as the the canonical model) provides the central organizing framework for studying the evolution of the college premium. It relates relative wages of more (h) and less (ℓ) educated workers $w_{ht}/w_{\ell t}$ at time t , to their relative supply $Supply_{ht}/Supply_{\ell t}$ and demand $A_{ht}/A_{\ell t}$. Relative demand captures the extent to which technical change is skill biased. In the typical empirical implementation, wages and supplies are observed, whereas the skill bias of technology is unobserved. A linear time trend $\gamma \times t$ proxies for smooth changes in the rate of skill-biased technological change, leading to the following empirical specification

$$\log \left(\frac{w_{ht}}{w_{\ell t}} \right) = \alpha + v_L \log \left(\frac{Supply_{ht}}{Supply_{\ell t}} \right) + \gamma t + \iota_t \quad (1)$$

⁵While I show that a job ladder model can rationalize my empirical findings, I neither claim that only job ladder models are consistent with the data nor do I directly test that the indirect effects I find are generated by movements up the job ladder; such an exercise would require worker-level wage panel data.

⁶[Clemens and Strain \(2021\)](#) also focus on dynamics—but dynamic employment rather than wage effects—and document that medium-run employment effects exceed short-run effects.

I consider an atheoretical (for now) extension of the canonical model that incorporates the value of the real minimum wage, m_t , as follows

$$\log\left(\frac{w_{ht}}{w_{\ell t}}\right) = \alpha + \nu_m \log m_t + \nu_L \log\left(\frac{\text{Supply}_{ht}}{\text{Supply}_{\ell t}}\right) + \gamma t + [\dots] + \iota_t \quad (2)$$

I refer to equation (2) as both *the race between education, technology, and the minimum wage* and, more compactly, as *the extended canonical model*.

Measurement. Here I briefly describe how I measure each variable in equations (1) and (2); Appendix Section A.1 contains details. I restrict my sample to the working-age population of those between 16 and 64 years old and define high-education workers as those with 16 or more years of education (college educated) and low-education workers as those with fewer than 16 years of education (non-college educated). I construct the composition-adjusted college premium by first measuring the log of the average hourly wage within each of 100 groups (defined by the interaction of 5 age bins, 2 genders, 2 races, and 5 education levels) in year t and then averaging across those groups with and those groups without college education using time-invariant weights. I similarly construct the supply of college and non-college workers as a composition-adjusted average of efficiency-unit hours worked across each of the groups with and without college education. Since hours worked in each year t for high- and low-education workers respond to contemporaneous deviations from the linear rate of skill-biased technical change, and since these deviations are included in the residual ι_t , I additionally construct an instrument for relative supply, using composition-adjusted populations across groups with and without college education. I measure these variables using the March Annual Demographic Files of the Current Population Survey from 1964 to 2017, which report earnings from 1963 to 2016.

I measure the real minimum wage in year t as the average minimum wage across states (the maximum of the legislated state and federal minimum wages), using time-invariant weights. In an alternative approach, I use the federal (Fair Labor Standards Act or FLSA) minimum wage. I deflate each series using the GDP deflator. I refer to my baseline measure as the real minimum wage and to my alternative measure as the real federal minimum wage.

Panels (a) and (b) of Figure 1 display the college premium and the two versions of the relative supply of college workers (measured using hours and populations), each normalized to zero in 1963. The particularly steep rise in the college premium starting in the early 1980s coincides with a decline in the growth rate of the relative supply of college workers, a fact first emphasized by Katz and Murphy (1992). Panel (b) additionally displays the

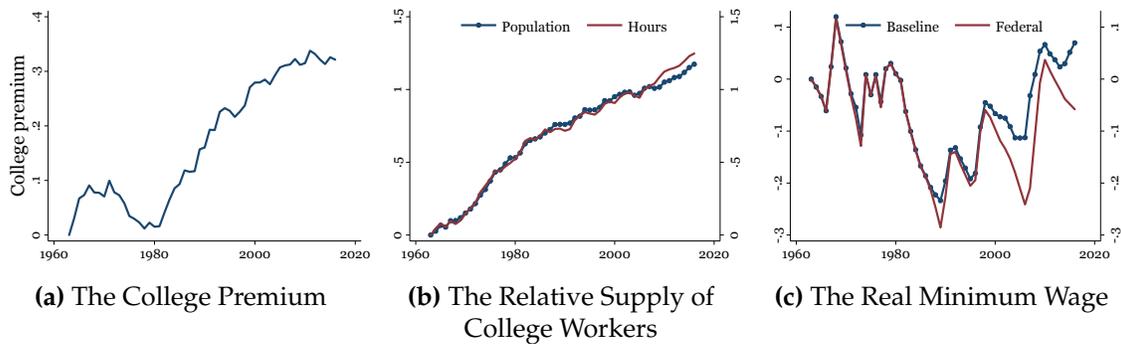


Figure 1: National Relative Wages, Relative Supplies, and Real Minimum Wages

Notes: Panels (a) and (b) display the national composition-adjusted college premium and relative supply of college labor. The relative supply of college labor is measured two ways: hours worked and populations. Panel (c) displays the baseline real minimum wage series, which averages minimum wages across states, and the real federal minimum wage; both series are deflated by the GDP deflator. All series are normalized to zero in 1963.

strong (first-stage) relationship between the hours-worked and population-based measures of the relative supply of college workers. Panel (c) displays the two real minimum wage series (one measured using the average across states and the other using the federal minimum wage), each normalized to zero in 1963. The two series move in lockstep until the late 1980s and diverge thereafter (especially in the 2000s) as more states set minimum wages above the federal level. Both real minimum wage series decline dramatically in the 1980s, as fixed nominal minimum wages are eroded by inflation; the real federal minimum wage experiences sharp swings in the 2000s as well, although these are less dramatic in my baseline series as states raise their nominal minimum wages to dampen the impact of inflation. Finally, there is substantial time variation in both real minimum wage series, which remain even after residualizing on high-dimensional polynomials of time.

Results. I first show that the extended canonical model’s out-of-sample fit improves on the canonical model’s, altering the implied evolution of skill-biased technical change. It is well known that estimating the canonical model on the 1963 – 1987 sample—the years included in the seminal work of [Katz and Murphy \(1992\)](#)—leads to a predicted evolution of the college premium that systematically deviates from the data thereafter, especially in the 2000s. The model predicts a substantially sharper rise in the college premium than actually occurs, as shown in [Figure 2](#); in spite of differences in data cleaning, measurement, and specification, these results are very similar to [Acemoglu and Autor \(2011\)](#), among others. This deviation between the observed growth of the college premium and the model’s prediction suggests a rapid slowdown in the rate of skill-biased technical change in the 2000s. In addition to replicating a known result, [Figure 2](#) displays a new

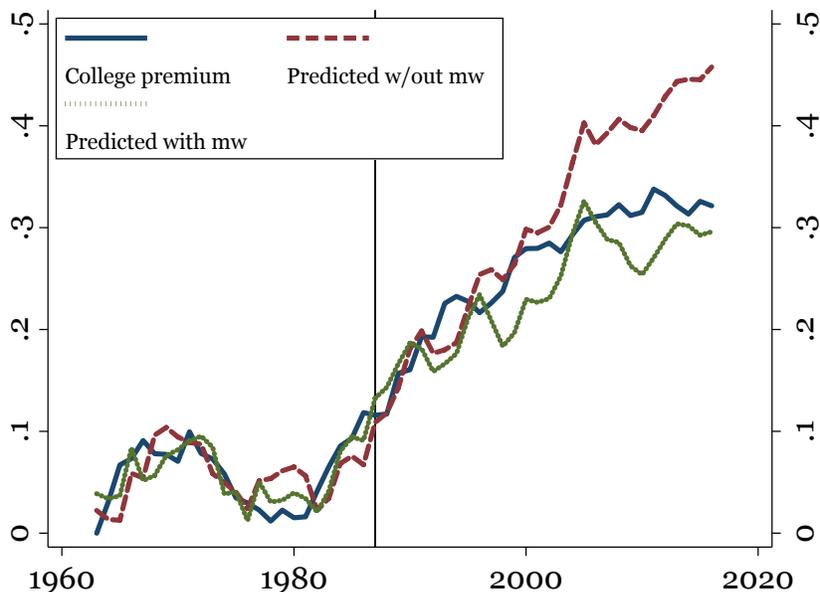
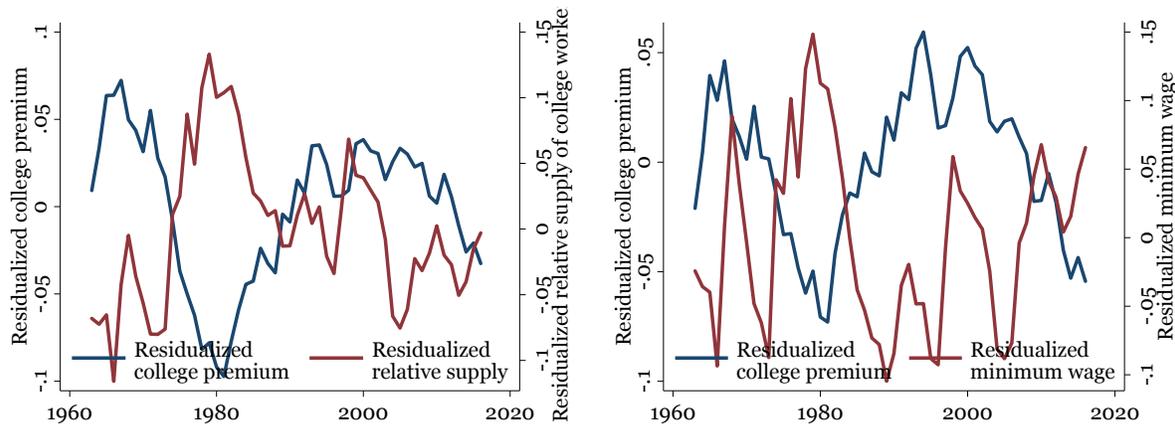


Figure 2: Out-of-Sample Predictions of the Canonical Model and Extended Canonical Model at the National Level

Notes: The observed evolution of the national college premium, in solid blue, and its predicted evolution from estimating equation (1), in dashed red, and equation (2), in dotted green, using 2SLS on the sample of years 1963 – 1987. The vertical line at 1987 indicates the final sample year in the estimation.

result: the extended canonical model’s out-of-sample fit—estimated on the same sample of years—is better. Incorporating the real minimum wage into the race between education and technology reduces the implied slowdown in the rate of skill-biased technical change needed to match the observed college premium. This results from the fact that the real minimum wage is high in the 2000s—especially the baseline series that averages across states, as shown in panel (c) of Figure 1—precisely when the growth rate of the skill premium is low.

I next turn to the in-sample elasticity of the college premium with respect to the relative supply of college labor, the relative demand for college labor, and the real minimum wage. Before providing estimation results, I display the variation in the data that identifies ν_m and ν_L in equation (2). Panel (a) of Figure 3 displays the college premium and relative (population-based) supply of college workers, each residualized of the real minimum wage and a linear time trend. Panel (b) displays the college premium and real minimum wage, each residualized of the relative (population-based) supply of college workers and a linear time trend. The variation Panel (a) identifies ν_L and the variation in Panel (b) identifies ν_m when estimating the reduced-form specification of equation (2), in which the relative supply of college workers is measured using relative populations. There is a clear negative relationship in each panel.



(a) Residualized Relative Supply, College Premium (b) Residualized Minimum Wage, College Premium

Figure 3: Residualized Independent and Dependent Variables at the National Level

Notes: Panel (a) displays the national composition-adjusted college premium and relative supply, both residualized of the real minimum wage and a linear time trend. Panel (b) displays the national composition-adjusted college premium and real minimum wage, both residualized of composition-adjusted relative supply and a linear time trend. In each panel, the relative supply of college labor is measured using the population-based measure.

Column (1) of Table 1 displays results of estimating regression (2) using 2SLS.⁷ Columns (2) – (5) include progressively higher-dimensional polynomials of time. I include these columns to check if allowing for a more flexible evolution of the rate of skill-biased technical change dramatically affects estimated elasticities. Whereas the quantitative estimates do vary with the polynomial of time, the broad conclusions are robust.⁸ Finally, column (6) replicates column (1), but replacing the real minimum wage (which averages minimum wages across states) with the real federal minimum wage. Since the federal minimum wage is a noisy measure of the minimum wage workers actually face, it is perhaps not surprising that the column (6) estimate is attenuated relative to the corresponding estimate in column (1).

The estimates in Table 1 highlight the importance of supply (education), demand (the skill bias of technology), and the real minimum wage in shaping the evolution of the U.S. college premium. The elasticity of the college premium with respect to the real minimum wage over the full sample ranges between -0.13 and -0.22 , settling down to about -0.13 with cubic, quartic, and quintic polynomials of time. The -0.13 elasticity implies that the 26% decline in the real minimum wage between 1979 and 1989 caused a 3.4% increase

⁷The first-stage is strong across all specifications in Table 1, as suggested by panel (b) of Figure 1. The first-stage KP F statistic is no lower than 49 across all columns.

⁸Reassuringly, my baseline empirical results in Section 5 are robust to allowing for arbitrary time-varying national technical change at the level of each of the 100 groups of workers across which I composition adjust in this section.

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.632 (0.069)	-0.703 (0.077)	-0.608 (0.104)	-0.619 (0.119)	-0.387 (0.134)	-0.541 (0.067)
Real minimum wage	-0.220 (0.048)	-0.199 (0.059)	-0.133 (0.052)	-0.129 (0.064)	-0.132 (0.046)	
Real federal minimum wage						-0.161 (0.044)
Time	0.021 (0.002)					0.019 (0.002)
Time Polynomial	1	2	3	4	5	1

Table 1: Regression Models for the National College Wage Premium

Notes: Results of estimating (2) using 2SLS on the sample of years 1963 – 2016. The dependent variable and the “Relative supply of college workers” are the logs of the national composition-adjusted college premium and relative hours worked of college labor. The instrument for the relative hours worked of college labor is the composition-adjusted measure of the relative population of the working-age population with college education. The “Real minimum wage” is the log of the average real minimum wage across states whereas the “Real federal minimum wage” is the version using the federal minimum wage. “Time” is a linear time trend, which is displayed in the columns in which the time trend enters only linearly. The estimation sample is 1963 – 2016 in all columns, resulting in 54 observations. Columns 1-5 differ in the degree of the polynomial of time included in the regression, with column $j \leq 5$ including polynomials up to and including degree j . Robust standard errors are reported in parentheses.

in the college premium over this time period (see Figure A.1 in Appendix Section A.1), which is one quarter of the observed 13.4% increase. The elasticity of the college premium with respect to the relative supply of college workers ranges between -0.39 and -0.7 . The, e.g., -0.61 elasticity estimated in column (3) implies that had the relative supply of college workers grown as fast between 1979 and 1989 as it grew between 1969 and 1979, the college premium would have grown by only 1.6% instead of the observed 13.4% growth between 1979 and 1989. Finally, the incorporation of the real minimum wage does not change the conclusion that skill-biased technical change is a dominant force driving the evolution of the U.S. college premium.

Sensitivity. Here, I briefly describe results of three types of sensitivity exercises. Details are provided in Appendix Section A.1.⁹

First, I check if the previous results are robust to using alternative measures of the relative supply of college workers. The baseline in Table 1 instruments for the hours-worked measure of relative supply with the population-based measure. In Table A.1 in Appendix Section A.1 I estimate the reduced-form specification of equation (2), using OLS

⁹Because I composition adjust wages and supplies, the baseline analysis controls for (observable) changes in the composition of the workforce.

and measuring the relative supply of college workers with the population-based measure. In Table A.2 in Appendix Section A.1 I estimate equation (2) using 2SLS, but replacing the hours-worked measure of the relative supply of college workers with the dual of wages, choosing supply of college and non-college workers such that the product of wages and supplies exactly matches wage income for college and non-college workers in each year. In both cases, results are broadly similar to those displayed in Table 1.

Second, I aim to understand why my conclusions differ, as described in the Introduction, from Autor et al. (2008). Autor et al. (2008) estimate a significant negative coefficient on their measure of the real minimum wage using a linear time trend and an insignificant negative coefficient using a third-degree polynomial of time. My results in Table 1 differ in part because I display results across a broader set of specifications, in part because I have access to a longer time horizon, and in part because I measure the real minimum wage using the average of minimum wages across states instead of the federal minimum wage. I begin, in the top panel of Table A.3 in Appendix Section A.1, by using the data from the replication package for Autor et al. (2008) to show that outside of the specification with a cubic time trend, results that can be obtained using their data are similar to my results. I then show, in the bottom panel of Table A.3, that the coefficient on the real minimum wage tends to be larger when I replace their federal minimum wage with my baseline measure of the minimum wage, similar to the comparison between columns (1) and (6) in my Table 1. As there, using the federal minimum wage introduces attenuation bias, since it doesn't apply in many states, especially starting in the late 1990s.

To this point, I have studied the elasticity of the college premium—rather than the real wage of either education group—with respect to measures of supply, demand, and the real minimum wage, in keeping with the literature focusing on supply and demand. In Table A.4 in Appendix Section A.1 I decompose the estimated elasticities of the college premium into separate elasticities of college and non-college real wages. To do so, I estimate equation (2) replacing $\log(w_{ht}/w_{lt})$ with $\log w_{ht}$ and, separately, with $\log w_{lt}$. Each resulting elasticity of the college real wage minus the corresponding elasticity of the non-college real wage equals the elasticity of the college premium in Table 1. Across each specification (linear, quadratic, and cubic), a higher real minimum wage raises the average real wage of workers without a college education, w_{lt} ; and this effect is statistically significant in each specification. The impact of the real minimum wage on the average real wage of college-educated workers, w_{ht} , is not robust across specifications, varying from negative and statistically significant in the linear specification to positive and statistically insignificant in both the quadratic and cubic specifications. I conclude that increases in the real minimum wage robustly raise non-college-educated workers'

real wages and have ambiguous effects on college-educated workers' real wages.¹⁰

Summary. Supply, demand, and the minimum wage each appear to help shape the evolution of the U.S. college premium. Moreover, the extended canonical model (2) improves on the out-of-sample fit of the canonical model (1), implying a smaller slowdown in the rate of skill-biased technical change in the U.S. in the 2000s.

This motivating evidence on the relevance of real minimum wages for shaping the college premium leaves many questions unanswered. How can the minimum wage have such sizable effects when so few workers earn the minimum wage? If large spillover effects of changes in minimum wages—to workers not earning the minimum wage—are required to generate these elasticity estimates, how is this consistent with past empirical evidence showing stronger direct effects than wage-spillover effects? Finally, is the national time-series empirical strategy sufficiently compelling? I now turn to a model that—together with additional empirical exercises motivated and interpreted by it—helps address these and other questions.

3 Theory

In Section 3.1 I describe a simple job-ladder model with myopic workers and firms that nests the canonical model. In Section 3.2 I characterize the steady state of this model and provide comparative static results on changes in the minimum wage and labor supply and demand across steady states. In Section 3.3 I describe the transition to a one-time change in the economic environment, focusing on a change in the real minimum wage.

3.1 Setup

Time is discrete and indexed by t . The economy features S labor skills and types of firm, each indexed by s , with a type s firm hiring only skill s labor to produce type s output. The exogenous mass of skill s workers is L_{st} . There are many firms of each type, output across type s firms is perfectly substitutable, and each employed skill s worker produces one unit of type s output. Final output, Y_t , is produced by competitive producers. Final output is a constant returns to scale function of the output, Y_{st} , of each type s , $Y_t = Y\left(\{A_{st}Y_{st}\}_{s=1}^S\right)$, where A_{st} is time-varying factor-specific productivity. The price of the final good is the numeraire and P_{st} is the endogenous (real) price of each unit of type s output, so that

¹⁰For non-college-educated workers, these empirical results are consistent with my model's predictions in Section 3. For college-educated workers, the empirical results in the quadratic and cubic specifications are also consistent with my theory, although in the linear specification they are not.

$P_{st}Y_{st}$ is the total revenue of type s firms.

A worker can be employed or unemployed. Each period, a skill s worker-firm match faces an exogenous separation probability $\delta_s \in [0, 1]$, an unemployed skill s worker matches with a firm with probability $\lambda_{su} \in [0, 1]$, and an employed skill s worker meets a new firm with probability $\lambda_{se} \in [0, 1]$. An unemployed skill s worker receives real income v_s . If a worker meets a new firm, the worker and firm engage in generalized (asymmetric) Nash bargaining to determine if the worker moves from her current employer to the new firm and, if so, the worker's wage, which is fixed throughout the life of the contract (subject to the minimum wage, as described below). The worker's bargaining power is $\beta_s \in (0, 1]$. If an unemployed worker meets a firm, her outside option is her unemployment income v_s . If a worker employed at wage w meets a new firm, her outside option is to continue employment in her existing match at wage w .¹¹

Whereas wages are fixed and not renegotiable, they are also subject to a minimum wage, m_t . If the minimum wage rises above an existing wage contract, the wage in that match rises to the minimum wage if the firm and worker find it optimal to maintain the match at this new wage; otherwise, the firm can endogenously fire the worker and the worker can quit to unemployment. I assume that the minimum wage satisfies $m_t < P_{st}$ for each s and t . This upper bound on m_t implies that each worker-firm match generates positive surplus, so firms always find it strictly profitable to hire workers at the minimum wage. This implies that changes in the minimum wage do not affect unemployment. I relax this assumption in Section 3.4.

Within period t , I assume exogenous separation shocks occur first and then new matches are realized for those workers who did not separate in t ; bargaining occurs subject to the minimum wage that prevailed in the previous period. Finally, after matching and bargaining, the new minimum wage is set and employment and worker wages adjust accordingly.¹² I focus on the case in which the discount factor is zero, so that each worker maximizes her current wage and each firm maximizes its current profit. This assumption facilitates the analysis of the transition to aggregate shocks. I derive related steady-state results in a canonical (forward-looking) wage-posting model in Appendix Section B.4.¹³

¹¹I treat the matching probabilities as exogenous, as in, e.g., [Postel-Vinay and Robin \(2002\)](#) and [Cahuc et al. \(2006\)](#); this rules out the possibility that an increase in the minimum wage (which reduces employer profit) might reduce matching probabilities disproportionately more for worker skills that are more likely to earn the minimum wage. I rule out the possibility that a worker can exploit a new job offer to raise her wage with her current employer, consistent with counteroffers being uncommon empirically (see, e.g., [Mortensen, 2003](#)). Finally, unlike [Shimer \(2006\)](#), a worker's outside option is her current wage rather than unemployment; wages are, therefore, not renegotiation proof.

¹²I choose these timing assumptions so that the dynamic implications of the discrete-time model are most similar to those in a continuous-time model.

¹³The model simplifies to the canonical model if $\beta_s = 1$ and $\delta_s = 0$ (in which case worker wages equal

3.2 Steady state

In steady state, the exogenous time-varying parameters L_{st} , A_{st} , and m_t are all time invariant, as are the endogenous price of type s output, P_{st} , and the density of wages across workers within each skill s , which I denote by $g_{st}(w)$. Hence, in what follows, I omit time subscripts and re-introduce them in Section 3.3. I first solve for the steady-state distribution of wages across workers within skill s for a given P_s (and, therefore, for any aggregate production function and any number of skills). I then conduct comparative statics across steady states, again for given changes in P_s . Finally, I show how to solve for P_s given any aggregate production function and derive the extended canonical model estimating equation (2) as an example. Appendix Section B.1 contains details and derivations.

Wages along the job ladder. Suppose that an unemployed skill s worker matches with a firm. If $m \geq (1 - \beta_s) v_s + \beta_s P_s$, then the bargaining outcome is constrained by the minimum wage and the worker's wage is set to m ; in this case, I refer to skill s as being "bound" by the minimum wage. Otherwise, the bargaining outcome is unconstrained and the wage is set to $(1 - \beta_s) v_s + \beta_s P_s$. Each time an employed worker at wage w matches with a new firm, her wage rises to $(1 - \beta_s) w + \beta_s P_s$.

This process implies the existence of a steady-state wage ladder $\{w_{j,s}\}_{j=1}^{\infty}$ for each skill s . The unemployment benefit is what the worker receives when she is below the job ladder, $w_{0,s} = v_s$. The wage on the first "rung" of the job ladder is $w_{1,s} = m$ if s is bound by the minimum wage and is $w_{1,s} = (1 - \beta_s) v_s + \beta_s P_s$ otherwise. The wage on each successive rung $j + 1$ on the job ladder is simply $w_{j+1,s} = (1 - \beta_s) w_{j,s} + \beta_s P_s$ for $j \geq 1$. Consequently, wages rise as a worker moves up rungs of the job ladder, $w_{j+1,s} - w_{j,s} = \beta_s (P_s - w_{j,s}) > 0$, but proportionally less at higher rungs: $d(w_{j+1,s}/w_{j,s})/dw_{j,s} < 0$. In Appendix Section B.1 I solve explicitly for all wages on the job ladder, both in the case in which skill s is and is not bound by the minimum wage.

Distribution of workers across the job ladder and the average wage. In steady-state, the rate at which workers exit each rung of the job ladder equals the rate at which they enter it. The probability that a skill s worker at any wage $w_{j+1,s}$ in period t does not work at this wage in period $t + 1$ is $\delta_s + (1 - \delta_s) \gamma_{se}$, where δ_s is the probability the worker exogenously separates from her firm and, if she does not, γ_{se} is the probability that she matches with a new firm. The probability that a skill s worker begins earning wage $w_{j+1,s}$ at date $t + 1$ is $(1 - \delta_s) \gamma_{se} g_s(w_{j,s})$ if $j > 0$ and is $\gamma_{su} g_s(w_{j,s})$ if $j = 0$, which is the probability that she was one rung below $j + 1$ times the probability that she does not separate (if $j > 0$) and the

their value marginal product of labor and there is no unemployment), there are exactly two skill groups, and the aggregate production function is CES.

probability that she matches. Hence, the steady-state density across rungs can be defined recursively as

$$\begin{aligned} [\delta_s + (1 - \delta_s)\gamma_{se}] g_s(w_{1,s}) &= \gamma_{su} u_s \\ [\delta_s + (1 - \delta_s)\gamma_{se}] g_s(w_{j+1,s}) &= (1 - \delta_s)\gamma_{se} g_s(w_{j,s}) \quad \text{for } j \geq 1 \end{aligned} \quad (3)$$

where $u_s = g_s(w_{0,s})$ is the unemployment rate. Using this recursive system I solve explicitly for the steady-state distribution of workers across the job ladder in Appendix Section B.1. As is apparent from the system of equations in (3), the steady-state distribution of workers across rungs j is invariant to whether or not the real minimum wage binds and its level.

The average real wage among employed skill s workers is $w_s \equiv \frac{1}{1-u_s} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$. Using the explicit solutions for the distribution of workers across the job ladder $g_s(\cdot)$ —which is independent of whether or not the minimum wage binds—and wages along the job ladder $w_{j,s}$ —which depends on whether or not the minimum wage binds—I solve for the average wage in Appendix Section B.1. If skill s is bound by the minimum wage, then the average wage is given by a weighted average of the minimum wage and the value marginal product of labor,

$$w_s = \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}}\right) P_s \quad (4)$$

If, on the other hand, skill s is not bound by the minimum wage, then w_s is a weighted average of v_s and P_s .

Comparative statics across steady states: for given changes in P_s . For any skill s that is not bound by the minimum wage, the partial elasticity (i.e., for a given change in P_s) of the average real wage with respect to the minimum wage is zero. On the other hand, for a skill s that is bound by the minimum wage, the total derivative of w_s is given by

$$d \log w_s = M_s b_s \partial \log m + (1 - M_s b_s) \partial \log P_s \quad (5)$$

The partial elasticity of the average wage with respect to the minimum wage, $M_s b_s$, is the product of two terms, the “magnification elasticity,” denoted by

$$M_s \equiv \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} > 1 \quad (6)$$

and the “bite” of the minimum wage, $b_s \equiv m g_s(m) / [(1 - u_s)w_s]$, which is the share of all

skill s wage income earned by workers earning the minimum wage.

Why do I refer to M_s as the magnification elasticity? Consider the impact of a change in m on the average wage of an arbitrary group of workers under the assumption that the minimum wage has no employment effects (an assumption I am imposing here) and under an additional assumption that an increase in the minimum wage only raises the wages of individual workers whose wages are bound by it (leaving all worker wages above the minimum wage unaffected). Under these assumptions, the elasticity of the average wage is exactly equal to its bite. This channel is what is often referred to as the “direct effect” of minimum wages in the literature. In this job-ladder model, the direct effect is active; however, at least in the long run (i.e., across steady states), there are wage spillovers up the distribution above the minimum wage.

To understand these wage spillovers, let $W_s(c)$ denote the steady-state wage at percentile c of employed skill s workers; and consider a skill s that is bound by the minimum wage. In response to an increase in the minimum wage, $W_s(c)$ rises disproportionately more for lower percentiles,

$$\frac{d [W_s(c)/W_s(c')]}{dm} > 0 \quad \text{for all } W_s(c) < W_s(c')$$

The intuition for this result and its proof are straightforward. The steady-state share of workers on each rung of the job ladder is invariant to the value of the minimum wage. Moreover, in response to an increase in the minimum wage, wages at lower rungs (and, therefore, at lower centiles of the wage distribution) increase disproportionately more.

Finally, in equation (5) the elasticity of the average wage with respect to the value marginal product of labor, P_s , is less than one; this result applies whether or not skill s is bound by the minimum wage. The reason is that the average wage is a weighted average of m and P_s if s is bound by the minimum wage and is a weighted average of v_s and P_s otherwise. Hence, the average wage responds less than one-for-one to changes in P_s .

Comparative statics across steady states: determining changes in P_s and the extended canonical model. For the steady-state analysis, it remains only to determine how changes across steady states in labor supply, labor demand, and the minimum wage affect skill prices, $d \log P_s$. Under the prevailing assumption that $m < P_s$ for all s , the share of workers who are unemployed is fixed across steady states. This implies that changes in skill prices $d \log P_s$ depend only on changes in factor supply and demand. Hence, for a skill s that is bound by the minimum wage, $M_s b_s$ in equation (5) is not only the partial elasticity of the average wage with respect to the real minimum wage, it is also the total elasticity.

This also implies that for any aggregate production function, steady-state skill prices

are determined exactly as in a competitive static model featuring the same aggregate production function, but with the level of supply in the present model, L_s , replaced with $(1 - u_s)L_s$. Endogenous skill prices have been considered theoretically and empirically in a wide class of aggregate production functions, from the canonical model, which features a CES aggregate production function combining only two skill groups, to nested CES models featuring many skill groups as in, e.g., [Card and Lemieux \(2001\)](#), and beyond. Of course, it should be noted that while the determination of steady-state values of P_s are not affected by the job-ladder model, the relationship between the average wage and P_s is, as shown in equation (5).

Given my objective of studying the college premium and extending the estimating equation of the canonical model from equation (1) to (2), in the following proposition I show that imposing the same restrictions on the aggregate production function as in the canonical model yields equation (2); see Appendix Section B.1 for the formal derivation.

Proposition 1. *Suppose the aggregate production function is a CES combination of the output of two skills, h and ℓ , and parameters satisfy $\delta_h = \delta_\ell$, $\gamma_{he} = \gamma_{\ell e}$, and $\beta_h = \beta_\ell$. Then, to a first-order approximation around an initial steady state, the skill premium, $w_{ht}/w_{\ell t}$, is given by equation (2) under the assumption of linear growth in A_{ht} and $A_{\ell t}$ across time t , where $v_m = M(b_h - b_\ell)$ and b_h and b_ℓ are the minimum wage bites in the initial steady state.*

The assumption of common parameters in Proposition 1 is a sufficient condition for common magnification elasticities across skills, $M = M_s$. If the minimum wage only binds for low-skill workers (for example), then this condition is not necessary to obtain the result, since in this case only the magnification elasticity for ℓ is relevant and $v_m = -M_\ell b_\ell$. Of course, under the additional assumption that parameters are common across h and ℓ , this is just a special case of the result in Proposition 1, as $M(b_h - b_\ell) = -M b_\ell$.

3.3 Transition dynamics

Consider an economy in steady state at date $t = 0$ that experiences a small, one-time increase in the real minimum wage from m to $m' > m$. For a skill s that is not bound by the minimum wage, the change in m has no effect. The implications for a skill s that is bound by the minimum wage are summarized by the following proposition.¹⁴

Proposition 2. *Consider an economy in steady state leading into date 0 that faces a small one-time change from m to $m' > m$ satisfying $m' < (1 - \beta_s)m + \beta_s P_s$ for all s . For any skill s that is*

¹⁴If the real minimum wage experiences a small decrease (rather than increase), then Results (i), (iii), (iv), and (vi) of Proposition 2 are unchanged; Result (ii) is reversed, with the wage on rung j of the new job ladder below the wage on the old ladder; and, finally, the impact elasticity is zero in Result (v).

bound by m :

- i. For all $t \geq 0$, two job ladders coexist, with first rungs m (old) and m' (new)
- ii. At each rung j , the wage on the new job ladder is greater than the wage on the old ladder
- iii. The share on each rung j summed across the two ladders is constant across t
- iv. The share on rung j of the new ladder weakly increases in t
- v. On impact, the elasticity of the average wage with respect to m equals b_s
- vi. The elasticity of the average wage with respect to m rises in t , limiting to $M_s b_s$

I prove Proposition 2 in Appendix Section B.2 and describe its intuition here. An employed worker's wage upon forming a new match depends only on her current wage, bargaining power, and value marginal product of labor; it does not depend on the new minimum wage unless her current wage equals the new minimum wage. Hence, any worker who was on the old job ladder and whose wage was not constrained by the new minimum wage remains on the old ladder until she enters unemployment; whereas any worker whose wage was constrained by the new minimum wage or who exited unemployment after its imposition is on the new ladder. This implies the existence of two job ladders after the change in the minimum wage, as stated in Result (i). Result (ii) follows from the fact that wages on steady-state job ladders are increasing functions of the corresponding minimum wage (for any skill s that is bound by the minimum wage).

On impact, the wage on the first rung on the old job ladder, m , becomes constrained by the new, higher minimum wage. Because surplus in each match remains positive, every worker earning m moves instantly to the first rung of the new ladder, m' . The increase in the minimum wage does not constrain the second rung of the old wage ladder or affect unemployment given the assumption that $m' < (1 - \beta_s)m + \beta_s P_s$ for all s , so no other workers are affected on impact. This implies that the instantaneous, or impact elasticity of the average wage of skill s with respect to the minimum wage equals b_s , as stated in Result (v). It also implies that, at least on impact, the share of workers on each rung j summed across the two ladders equals the initial steady-state share. Result (iii) then follows from a proof by induction: if Result (iii) holds for a particular date $t \geq 0$, then it must hold in the subsequent date $t + 1$. Result (iv) is more complicated, but the basic intuition is simple: workers are exiting the old job ladder and entering the new one over time as they exogenously separate from their employers.¹⁵

¹⁵Result (iv) is more complicated because the growth of any rung j on the new ladder depends on the distribution of workers across rungs of the new ladder rather than the share of workers on the new ladder overall. The share of workers on rung j (of the new job ladder) in the previous period determines the number of workers leaving rung j in the present period and the share on rung $j - 1$ in the previous period determines the number of workers entering rung j in the present period.

Finally Result (vi) follows from the previous results: wages on the new job ladder are higher than on the old one (Result ii) and workers are moving from the old to the new ladder over time (Result iv) at a fixed composition of workers across rungs (Result iii). Intuitively, the magnification elasticity grows over time as workers who start at the new minimum wage (either because their wage increased from m to m' on impact or because they exited unemployment after the minimum wage increase) slowly rise up to higher rungs of the new job ladder, each of which is higher than the corresponding rung on the old ladder. In the long run, the elasticity of the average wage with respect to the minimum wage limits to $M_s b_s$, as shown in equation (5), where $M_s > 1$ as shown in equation (6). These results apply for any number of skills and any aggregate production function.

Results (v) and (vi) of Proposition 2 imply that the elasticity of the average real wage of skill s between dates $t = 0$ at date $t = T$ (the impulse response) with respect to a one-time increase in the real minimum wage between dates $t = 0$ and $t = 1$, can be expressed as

$$\log \left(\frac{w_{s,T}}{w_{s,0}} \right) / d \log m \equiv M_{s,T-1} \times b_{s,0} \quad (7)$$

where $b_{s,0}$ is the initial minimum wage bite before the minimum wage increase and where $M_{s,T-1}$ is the $T - 1$ -period magnification elasticity. The magnification elasticity satisfies $M_{s,0} = 1$, $dM_{s,T}/dT > 0$, and $\lim_{T \rightarrow \infty} M_{s,T} = M_s$ from equation (6). Equation (7) applies whether or not the minimum wage binds for s ; but, of course, if it does not bind then the elasticity in equation (7) is zero, since $b_{s,0} = 0$.

These results also have direct implications for the time-varying elasticity of the skill premium. Impose the additional assumption in Proposition 1 that $\delta_h = \delta_\ell$, $\gamma_{he} = \gamma_{\ell e}$, and $\beta_h = \beta_\ell$, so that $M_T = M_{s,T}$ for each s . Then equation (7) directly implies

$$\log \left(\frac{w_{h,T}/w_{\ell,T}}{w_{h,0}/w_{\ell,0}} \right) / d \log m \equiv M_{T-1} \times (b_{h,0} - b_{\ell,0}) \quad (8)$$

On impact, the elasticity of the skill premium with respect to the minimum wage equals the bite of the minimum wage for high- minus for low-skill workers. Over time, this elasticity grows. Equation (8) will help me interpret the estimated elasticity of the college premium with respect to the real minimum wage displayed in Table 1.

3.4 The minimum wage and unemployment

A prevailing assumption throughout the theory presented above is that each worker-firm match generates positive surplus: $m_t < P_{st}$ for all t and s . This implies that changes in the minimum wage do not affect unemployment. Throughout this section, suppose that

$m_t = P_{st}$ for s but not for any $s' \neq s$; and consider all but the knife-edge case in which unemployment is exactly equal to its level when $m_t < P_{st}$.

In this case, the wage ladder collapses for s (and only for s), with all employed s workers earning a real wage equal to m_t . This implies that the elasticity of w_{st} (and of P_{st}) with respect to m_t is then exactly one for skill s . Moreover, changes in the real minimum wage affect unemployment for skill s (and only for skill s). The elasticity of skill s employment with respect to m_t is determined by the elasticity of the residual labor demand curve for skill s induced by the aggregate production function at fixed employment for all other skills. That is, an increase in the real minimum wage raises the wage of s one-for-one and causes a decline in employment of skill s as the economy traces its residual demand curve for s . This is identical to the wage and employment impacts of a change in a binding minimum wage in a competitive labor market.

What is the impact of a change in the real minimum wage on the average wage of other skills? For all $s' \neq s$, equations (4), (5), and (6) continue to hold. However, since changes in m_t affect employment of skill s , they affect the value marginal product of skill s' and, therefore, their average wage $w_{s'}$. Specifically, whereas $M_{s'}b_{s'}$ remains the partial long-run elasticity of the real wage of s' with respect to m , it is no longer the total elasticity, as m also impacts $P_{s'}$. I return to these issues in Section 5.4.

4 Empirical approach

4.1 From theory to estimation

According to equation (7), if there is a small, one-time increase in the real minimum wage that occurs at any point between period $t - T$ and period t , then the change in the average real wage of skill s over this time horizon is given by

$$\log \frac{w_{s,t}}{w_{s,t-T}} = \sum_{j=0}^{T-1} M_{s,j} b_{s,t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}}$$

where, by assumption, the log change in the minimum wage is identically zero in all but one of the periods j in the above summation. The previous expression is a distributed lag model with lag weights that exactly equal the model's skill-specific and time-varying magnification elasticities, $M_{s,j}$. If the single minimum wage change occurred earlier than period $t - T$, then the above expression must be adjusted. According to equation (7), if there is a small, one-time increase in the real minimum wage that occurs at any point between period $t - T'$ and period t , with $T' > T$, then the change in the average real wage

of skill s between $t - T$ and t is given by

$$\log \frac{w_{s,t}}{w_{s,t-T}} = \sum_{j=0}^{T-1} M_{s,j} b_{s,t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}} + \sum_{j=T}^{T'-1} (M_{s,j} - M_{s,j-T}) b_{s,t-j-1} \log \frac{m_{t-j}}{m_{t-j-1}} \quad (9)$$

This is, again, a distributed lag model. The one change from the previous expression is that the lag weights equal the magnification elasticities only for lagged shocks occurring between $t - T$ and t (the time difference of the dependent variable); for further lagged shocks, the lag weights are strictly less than the corresponding magnification elasticities. This is intuitive. Instead of studying the entire impulse response to a one-time change in the real minimum wage, for shocks that are lagged sufficiently I am looking at only the tail-end of the response, between $t - T$ and t , cutting off the initial periods after the shock.

Equation (9) holds exactly in the model in the presence of a one-time increase in the real minimum wage. However, in moving from the model to the data, it is important to remember that reality differs from the model in many respects. Perhaps most importantly, the real minimum wage does not experience a one-time permanent change in any empirical context. Instead, it is always changing, driven by a combination of inflation and minimum wage policy. To understand the extent to which the estimated lag weights correspond to magnification elasticities in the model, I will estimate a version of equation (9) in the data and in model-generated data (where I know the magnification elasticities).

4.2 Specification

In my baseline specification, I estimate the following version of equation (9),

$$\log \frac{w_{g,r,t}}{w_{g,r,t-T}} = \sum_{j=-L}^{T'-1} \mu_j b_{g,r,t-j-1} \log \frac{m_{r,t-j}}{m_{r,t-j-1}} + \kappa_t b_{g,r} + \alpha_t + \varepsilon_{g,r,t} \quad (10)$$

I define time periods t as half years, January – June and July – December of each year between 1979 and 2016. Skill groups s in the model are mapped in the data to the combination of regions r , which correspond to the 50 U.S. states, and labor groups g , which correspond to the same 100 labor groups across which I composition-adjust wages and supplies in Section 2 (defined by the interaction of 5 age bins, 2 genders, 2 races, and 5 education levels). Real minimum wages, $m_{r,t}$, vary across regions and time whereas minimum wage bites, $b_{g,r,t}$, vary across labor groups, regions, and time.

Relative to equation (9), there are five changes in my implementation of equation (10). First, I project changes in real wages not only on lagged changes in real minimum wages,

but also on leads (i.e., the summation begins with $j = -L$ rather than $j = 0$). Second, I estimate lag weights, μ_j that are common across groups; in robustness, I estimate separate specifications for different groups. Third, I omit changes in minimum wages that occurred more than T' periods before period t .

Fourth, in estimating equation (10), I instrument for the interaction between the time-varying bite of the minimum wage and the change in the real minimum wage. I do this because there is measurement error in the minimum wage bite; moreover, measurement error in the time-varying bite will be correlated with measurement error in the average wage in either the numerator or the denominator of the dependent variable for particular values of j in the summation. This concern is very similar to that emphasized in a related context by [Autor et al. \(2016\)](#). I instrument using $b_{g,r} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$, where $b_{g,r}$ is the median value of the time-varying bite for labor group g in region r across periods t .

Fifth, I include two additional controls: a time effect, α_t , and the time-varying impact of the median value of the minimum wage bite, $b_{g,r}$. Treatment in equation (10) is the interaction between shocks to the real minimum wage and the minimum wage bite, which fits into the “shock-exposure” framework of [Borusyak and Hull \(2023\)](#). As they argue, this specification may suffer from omitted variable bias.

In my context, the intuition is simple. In periods in which the real minimum wage tends to fall systematically over time (e.g., the 1980s) the interaction between changes in the real minimum wage and the minimum wage bite will be negatively correlated with the minimum wage bite. But the minimum wage bite could be correlated with other shocks in the residual. For instance, because the minimum wage bite tends to be higher for g, r pairs that are lower skill and the economy experiences skill-biased technical change (pushing up the wages of more skilled groups), expected treatment would be correlated with the impact of skill-biased technical change on real wage growth. Hence, there would be endogeneity.

[Borusyak and Hull \(2023\)](#) show that this type of omitted variable bias can be avoided by re-centering treatment (differencing realized treatment from expected treatment) or controlling for the expectation of treatment, either of which requires assumptions on the distribution of expected minimum wage changes (in my context). The interaction between $b_{g,r}$ and a time effect in equation (10) controls for expected treatment under the assumption that there is an arbitrary time-varying national expectation of the change in the real minimum wage. This addresses the endogeneity concern in my above example.

This control provides additional benefits in my context. In the reduced-form specification, this control absorbs changes in inflation, yielding identical results to an alternative specification replacing real minimum wages with nominal values. Moreover, this

“design-based” approach additionally avoids negative *ex-ante* weights; see, e.g., [Borusyak and Hull \(2024\)](#).¹⁶

In robustness, I additionally control for the interaction between $b_{g,r}$ and state effects, which allows for expected changes in minimum wages to differ arbitrarily across states in a time-invariant way (since, in practice, different states are systematically more or less likely to set minimum wages above the federal level), as well as including additional controls.

I focus on 3-year changes in real wages, setting $T = 6$ in equation (10). I consider the impact of one-period lagged real minimum changes going back 5 years, setting $T' = 10$. And I include two years of one-period lead real minimum wage changes, setting $L = 4$. To allow for correlation across time in the error terms, I cluster standard errors by state. Finally, in all regressions, I weigh each observation by the product of (g, r) workers in t and $t - T$ divided by their sum.

4.3 Data and measurement

Here, I provide an overview of the data and measurement. Appendix Section A.2 contains details. To measure m_{rt} , I use the maximum value of the effective nominal minimum wage in region r in the corresponding half-year period, which I obtain from [Cengiz et al. \(2019\)](#). I deflate this by the maximum monthly GDP deflator in the same half-year period. To measure the bite of the minimum wage, I construct the share of wage income earned by workers at a wage no greater than 1.05 times the minimum wage; I vary this cutoff in robustness.

I measure the minimum wage bite, $b_{g,r,t}$, and average wages, $w_{g,r,t}$, using the CPS Merged Outgoing Rotation Groups (MORG). I switch from the March CPS, which I use in the national analysis in Section 2, both because the MORG CPS includes a larger sample, which is particularly useful when dividing the data across 50 states and 100 labor groups, and because individual wages can be measured with less error (see [Lemieux, 2006](#)), which is especially important for measuring the bite of the minimum wage. These benefits come at the cost of a shorter time frame, starting with 1979 instead of 1963. I additionally drop all of 1994 and the first eight months of 1995 given missing imputation flags; more generally, I clean the MORG CPS data following an approach similar to [Lemieux \(2006\)](#). After cleaning the data, there are approximately 12 (unweighted) worker observations per

¹⁶The result on negative weights in [Borusyak and Hull \(2024\)](#) is proven only for regressions with one treatment variable. I conjecture that the result extends to specifications featuring many treatment variables—as in regression (10)—if these treatments are uncorrelated with each other, after residualizing on controls.

g, r, t (with 100 unique values of g , 50 of r , and 77 of t). In my baseline, I use a balanced panel, only including g, r if there are observations in every t . In robustness I include all g, r . Given outliers, I winsorize average wages at the 2nd percentile within each (g, r) pair.

The final sample year is 2016 because many municipalities begin setting minimum wages above the mandated state level at this point. For instance, New York City, Nassau, Suffolk, and Westchester counties begin setting minimum wages above the New York state level on the very last day of 2016 and Minneapolis begins setting minimum wages above the Minnesota level in 2018. However, some counties were already setting minimum wages above their states' levels beforehand. For instance, San Diego did so throughout 2015 and Los Angeles began doing so in the middle of 2016 (for large businesses). In robustness, I consider ending the analysis after 2010, to minimize the impact of endogeneity from mismeasured minimum wage changes.

5 Results

I first estimate equation (10) using model-generated data in Section 5.1, to show that this empirical approach is informative of the time-varying magnification elasticity in the model. I then turn to the main results in Section 5.2, where I estimate (10) using real-world data and discuss the implications of the empirical results both for the college premium and the real wage of workers without a high school degree. I describe a range of sensitivity analyses for the empirical results in Section 5.3. Finally, in Section 5.4 I discuss forces from which the model of Section 3 abstracts that the empirical results suggest may be important.

In what follows, I display results of estimating equation (10) in figures. In these figures, I display the average of the lag weight estimates μ_j and μ_{j+1} for any period $j > 0$; this smooths estimates across time differences. Similarly, I display the average of $-\mu_j$ and $-\mu_{j-1}$ for any period $j < 0$; this both smooths estimates and also converts pre-trend estimates to their more traditional format, as discussed in Roth (2024).¹⁷ Finally, I display the estimated value of μ_0 (the impact elasticity) without any smoothing.

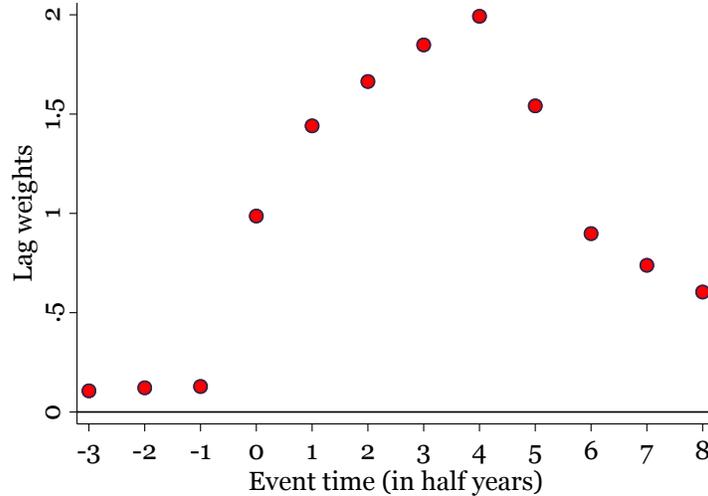


Figure 4: Time-Varying Magnification Elasticity (Estimated in Model-Generated Data)

Notes: Estimates of regression (10) using 2SLS and model-generated data. Results display point estimate for μ_0 , linear combination of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$.

5.1 A first check using model-generated data

I briefly describe how I parameterize the model, with details provided in Appendix Section B.3. I impose common job-ladder parameters γ_u , γ_e , δ , and β across all groups and regions. I choose $\gamma_u = 0.79$ and $\delta = 0.10$ to match direct survey evidence in Hall and Mueller (2018), converting their parameters to the bi-annual frequency. As in Hall and Mueller (2018), I set $\gamma_e = \gamma_u/2$. Given these parameters, I choose $\beta = 0.25$ such that the long-run magnification elasticity is 2.4, using equation (6); I make this choice so that the maximum estimated lag weight is similar in the model-generated and actual data. Finally, I choose separate time-invariant real value marginal product of labors for each of the 5,000 g, r pairs, targeting the corresponding average real wage across the sample period. Starting from a steady state, I feed into the parameterized model observed values of the real minimum wage in each state and period; unlike the analytic results in Section 3.3, I do not impose that changes in real minimum wages are small. Finally, after solving the model in each time period for each g, r pair, I add a small amount of measurement error to worker wages. Using model-generated data, I estimate regression (10) using 2SLS.

Figure 4 displays estimation results. The estimated impact elasticity (the estimate for $j = 0$) is very close to one. This elasticity rises over time and then falls after $j = 4$. Recall that the model predicts that the magnification elasticity is always (weakly) increasing.

¹⁷To understand this choice, suppose that there is a negative estimate for $j < 0$. This would imply that the real wage declines leading into a future increase in the real minimum wage. Displaying the negative value of the estimated lag weight (a positive point associated with this value of $j < 0$) then clarifies that the level of the real wage declines leading into the shock.

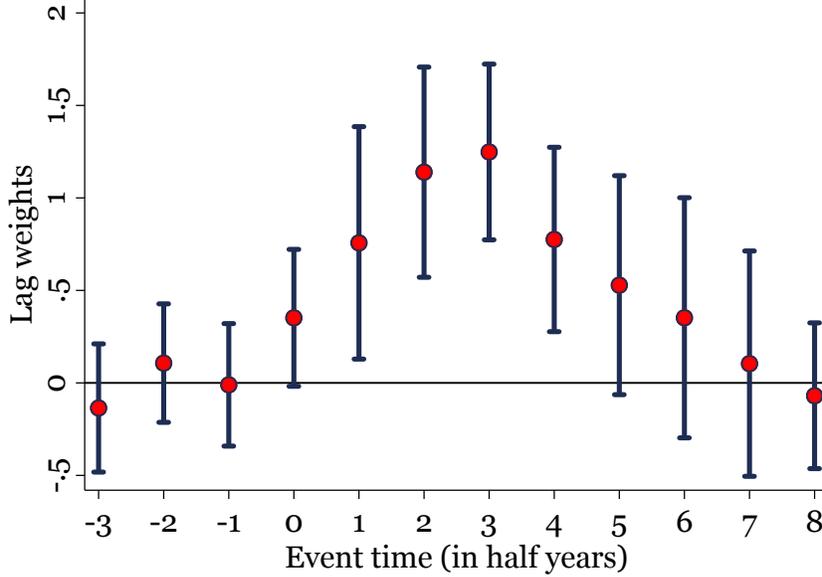


Figure 5: Reduced-Form Specification of Equation (10)

Notes: Reduced-form estimates of regression (10), in which $b_{g,r} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$ replaces $b_{g,r,t-j-1} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$, and in which the regression is estimated using OLS. Results display point estimate for μ_0 , the linear combinations of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$, with corresponding 95% confidence intervals; robust standard errors are clustered by state.

However, for shocks that precede the three-year time change in the wage on the left-hand side of equation (10), the estimated lag weight should be below the magnification elasticity, as shown in equation (9). This is why estimated lag weights decline for sufficiently high values of j .¹⁸ The estimated lag weight converges towards the magnification elasticity (which is, analytically, 2.4 given parameters); over this time horizon, the maximum estimated lag weight is approximately 2.1. I conclude that this empirical approach is informative about the time-varying magnification elasticity in model-generated data, in spite of the fact that the real minimum wage changes between every period in every region.¹⁹

5.2 Results and implications

I begin by estimating the reduced-form specification of equation (10), in which I replace $b_{g,r,t-1} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$ with the instrument $b_{g,r} \log \frac{m_{r,t-j}}{m_{r,t-j-1}}$ and estimate the resulting regression

¹⁸The reason the estimate at $j = 5$ is below the estimate at $j = 4$ is because I smooth estimates, displaying coefficients that are averaged across j and $j + 1$ for each $j > 0$. Without this smoothing, the lag weight would first decline between $j = 5$ and $j = 6$, as expected.

¹⁹Indeed, if I start from a steady state, feed in a small one-time increase in the real minimum wage in one region, and estimate this specification in the resulting model-generated data, I obtain a very similar pattern to the one displayed in Figure 4.

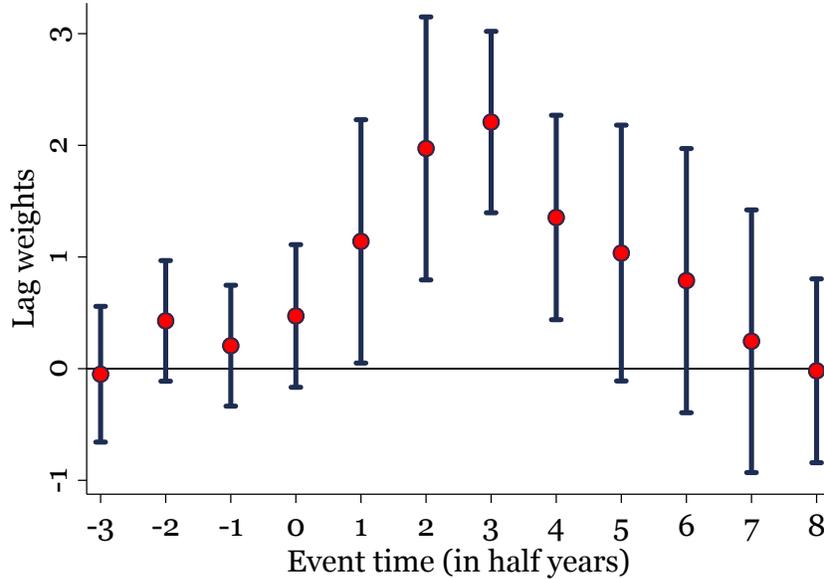


Figure 6: Time-Varying Magnification Elasticity

Notes: Results of estimating regression (10) using 2SLS. Results display point estimate for μ_0 , the linear combinations of $(\mu_j + \mu_{j+1})/2$ for $j > 0$ and of $-(\mu_j + \mu_{j-1})/2$ for $j < 0$, with corresponding 95% confidence intervals; robust standard errors are clustered by state.

using OLS. Figure 5 displays the estimation results, including 95% confidence intervals (using robust standard errors clustered by state). While the point estimates are not directly comparable to those in Figure 4, which is estimated using 2SLS, the qualitative pattern is broadly similar. On impact, the reduced-form coefficient is 0.4 (the estimated coefficient at $j = 0$) and rises to a maximum of 1.5 before declining thereafter. Figure 5 also shows no evidence of pre-existing differential trends before changes in the real minimum wage.

Figure 6 displays the baseline empirical results for the time-varying magnification elasticity, corresponding to the estimates of the lag weights in equation (10) using 2SLS. On impact, the magnification elasticity is approximately 0.5. It rises to approximately 1 within the first year, peaks at over 2 (2.2 in the smoothed estimate shown in Figure 6) over the first two years, and subsequently declines.

These estimates are broadly in line with those in the model-generated data (with exceptions that I discuss in Section 5.4), where the maximum estimated lag weight is approximately 2.1 and the long-run magnification elasticity is 2.4. From this, I conclude that the results in Figure 6 are consistent with a long-run magnification elasticity of approximately 2.4. A long-run magnification elasticity of 2.4 implies that the indirect effect of changes in the minimum wage (the effects on wages above the minimum wage) generates about 58% ($\approx 1 - 1/2.4$) of the overall wage effects (at their peak) of minimum wage

changes.

How do these estimates relate to results in the literature? To my knowledge, this is the first paper to study the time-varying effects of minimum wage changes on inequality. However, like this paper, [Cengiz et al. \(2019\)](#) quantify the direct and indirect effects of changes in the minimum wage on average wages, focusing within a \$4 (deflated to 2016) range around the minimum wage. To do so, they average wage changes over the five years following a minimum wage increase. They find that approximately 60% of the total wage effect of a change in the minimum wage is caused by the direct effect, with 40% caused by wage spillover effects above the minimum wage. This is broadly in line with my results: my finding that the indirect effect generates 58% of the overall wage effects of minimum wage changes is a result about the maximum strength of the indirect effect (which occurs two years after the shock itself), whereas their result is about its average effect over time.

Implications for the college premium and the relation to the national estimates. According to the theoretical result in equation (8), the long-run elasticity of the college premium with respect to the real minimum wage is determined by the product of the long-run magnification elasticity and the difference between the bite of the minimum wage between college and non-college workers. The average value of the minimum wage bite, $b_{g,r,t}$, taken across states r , time t , and all worker groups g with a college education is 1.8% and without a college education is 7.6%. Using a long-run magnification elasticity of 2.4, this implies a long-run elasticity of the college premium with respect to the real minimum wage of approximately $-.14$. This is squarely in the range of the national estimates displayed in Table 1. Hence, the national estimates in Section 2 and the estimates in Figure 6 are consistent. Both imply sizable elasticities of the college premium with respect to the real minimum wage.

Implications for real wages of workers without a high school degree. The combination of the theoretical results in Section 3 and the empirical results in Figure 6 shed light not only on relative wages, but also on real wage levels.

Real wages of workers without a high school degree (HSD) declined dramatically starting in the 1980s and into the early 1990s; see, e.g., [Autor et al. \(2008\)](#). This real wage decline is impossible in the canonical model in the absence of productivity declines for non-college-educated workers (given the observed increase in the relative supply of college-educated workers over this time horizon); see, e.g., [Acemoglu and Restrepo \(2020\)](#).

However, the same is not true of the extended canonical model. Specifically, because the real minimum wage fell during this time horizon and because the bite of the minimum wage is relatively high among HSD workers, it is possible that their real wages declined substantially because of the decline in the real minimum wage. According to equation (7), the long-run implication of the approximately 26% decline in the real minimum wage between 1979 and 1989 equals 26% times the long-run magnification elasticity (of 2.4) times the bite of the minimum wage among HSD groups. The average value of the minimum wage bite, $b_{g,r,t}$, taken across states r , time t , and all HSD worker groups g is approximately 13.9%. Hence, according to the model, the 26% decline in the real minimum wage between 1979 and 1989 caused an approximately 8.7% ($\approx 0.26 \times 0.139 \times 2.4$) decline in the real wage of workers without a high school degree. This is the entirety of the observed decline in their real wages between 1979 and 1992.

5.3 Sensitivity analysis

The results presented in Figures 5 and 6 are robust to a variety of alternative specifications. I consider sensitivity analyses that fit into four broad categories: (i) heterogeneity of magnification elasticities across worker groups, (ii) the inclusion of sample years in which some municipalities apply a minimum wage above the statutory level set by their states, (iii) the definition of a minimum wage worker, and (iv) the set of controls. Here, I discuss sensitivity of the baseline effects displayed in Figure 6, the primary focus of the paper; these figures are displayed in Appendix Section A.3. For the interested reader, sensitivity of Figure 5 can be found in Appendix Section A.4.

The theory in Section 3 explicitly incorporates heterogeneity across worker groups in the elasticity of the real wage with respect to the real minimum wage in two respects: it predicts that the elasticity of the change in a group's real wage with respect to the real minimum wage equals the interaction between that group's minimum wage bite (which differs across groups) and the time-varying magnification elasticity (which depends on potentially group-specific parameters). The empirical specification directly incorporates the first dimension of heterogeneity by interacting the change in the real minimum wage with the observable group-specific minimum wage bite. However, in the specification displayed in Figure 6 I impose a common lag weight across groups. To what extent do these estimates hide heterogeneous magnification elasticities? To answer this question, I replicate the empirical analysis separately by gender (see Figures A.2 and A.3), for workers without a college degree (see Figure A.4), and for a broader sample of workers including those g, r pairs that have missing wage data in at least one t (see Figure A.5). Results

are broadly stable.

Another concern is that the sample ends after 2016, by which point there are certain municipalities with minimum wages above their states' levels; yet my definition of the real minimum wage is state-and-time specific. To check if this measurement error affects results, Figure A.6 replicates Figure 6 on the shorter sample of 1979 – 2010. Again, results are broadly stable.

One concern related to the identification of the lag weights and, therefore, the magnification elasticities, is that they may not be robust to the definition of a minimum wage worker being a worker whose wage is no greater than an arbitrary cutoff (1.05 in the baseline) times the minimum wage.²⁰ As the wage cutoff rises (for example), more workers are included in the set of minimum wage workers and the value of the minimum wage bite, $b_{g,r,t}$, rises mechanically. If all minimum wage bites rise proportionately, then the lag weight estimates fall proportionately, but the overall elasticity of the real wage with respect to the real minimum wage is unaffected. In practice, of course, it is not the case that all minimum wage bites are proportionately affected. Nevertheless, the results of replicating Figure 6 defining a minimum wage worker using a wage cutoff of 1.00, 1.10, and 1.15 times the minimum wage are in line with this intuition; see Figures A.7 – A.9.

Finally, in my baseline analysis I control for a time effect and a time-varying effect of the median value of the minimum wage bite for r, g across all t , $b_{r,g}$. This controls for expected treatment under the assumption that there is an arbitrary time-varying national expectation of the change in the real minimum wage. I additionally control for the interaction between $b_{g,r}$ and state effects, which—in the shock exposure setting of [Borusyak and Hull \(2023\)](#)—allows for expected changes in minimum wages to differ arbitrarily across states in a time-invariant way. Results displayed in Figure A.10 are similar to the baseline results in Figure 6. I alternatively consider two distinct sets of controls in addition to the baseline controls. In Figure A.11 I display results incorporating r, t fixed effects (which absorb the direct effect of changes in the real minimum wage). In Figure A.12 I display results incorporating g, t fixed effects (which absorb national, time-varying, and group-specific technical change and any changes in real value marginal products of labor across groups at the national level). Each of these results is again broadly similar to the baseline results in Figure 6.

²⁰In a different empirical context, [Derenoncourt and Montialoux \(2021\)](#) define minimum wage workers as those earning no more than 1.15 times the minimum wage.

5.4 Discussion

All models are abstractions. I view the model in Section 3 as a useful abstraction because it provides insights into and simple intuition for the dynamic effects of the real minimum wage on real wages and inequality. Here, I first consider where the empirical results in Figure 6—and the many associated sensitivity analyses—deviate from the theory and what these deviations might suggest about important missing dynamic mechanisms. Second, I discuss how changing my baseline assumption that minimum wages do not impact unemployment might affect the interpretation of the empirical results.

Perhaps the main deviation between regression results in model-generated data shown in Figure 4 and in actual data shown in Figure 6 is the timing of the decline in the lag weights. In the model-generated data, the lag weights (which average across lag j and $j + 1$ for all $j > 0$) decline between $j = 4$ and $j = 5$. In the data, the lag weights decline one period earlier. This implies that about two years after the real minimum wage increase, either wages for workers slightly higher in the wage distribution begin to rise or the real wage gains for workers lower in the wage distribution begin to dissipate. There are at least two prominent possibilities. First, fairness or efficiency wage concerns may slowly induce firms to raise worker wages for groups that are less bound by the minimum wage; see, e.g., [Grossman \(1983\)](#). Second, firms may slowly change their technologies in a way that increases demand for higher-wage and/or reduces demand for lower-wage workers; see, e.g., [Hurst et al. \(2022\)](#).²¹

A final empirical concern that I consider is the possibility that the minimum wage affects unemployment. As discussed in Section 3.4, in this case changes in the minimum wage cause changes in value marginal products of labor. In my baseline empirical analysis, these changes in value marginal products of labor are in the residual. Hence, I identify the total effect of changes in the real minimum wage, including their effects via marginal products of labor. If I controlled for changes in value marginal products (or changes in employment induced by the changes in the real minimum wage), then I would instead identify the partial effect. Since the total elasticity is the object of interest, in this respect my approach seems reasonable.

Nevertheless, it might be of independent interest to understand the extent to which the total and partial elasticities differ. Together with the theoretical results Section 3.4, the empirical literature focusing directly on the impact of minimum wages on unemployment

²¹Given that the empirical results hold conditioning on g, t effects, this technical change must be local, which would be consistent with a regional version of [Hurst et al. \(2022\)](#). Related to the issue of firms changing their technologies is the issue of labor reallocating across firms that differ in technology; see, e.g., [Dustmann et al. \(2021\)](#).

can help answer this question. If this impact of minimum wages on employment is non-existent or very small—as in, e.g., [Dube et al. \(2010\)](#) and [Cengiz et al. \(2019\)](#)—then any reasonable elasticity of labor demand will imply non-existent or tiny effects of minimum wages on value marginal products. In this case, estimated total elasticities will be very similar to partial elasticities. On the other hand, if changes in the minimum wage have large effects on unemployment, as might happen for large values of the minimum wage, then partial effects may deviate meaningfully from total effects.²²

6 Conclusions

What are the dynamic impacts of minimum wages on real wages and inequality? I have presented a generalization of the canonical model in which the elasticity of real wages with respect to the real minimum wage is small on impact and grows over time, and I have provided empirical evidence consistent with these dynamic effects. I have used the model and empirical evidence to document that minimum wages—together with supply and demand—play a role in shaping the evolution of the U.S. college wage premium and that declining real minimum wages in the 1980s in the U.S. caused a considerable decline in the real wages of workers without a high school degree.

While my empirical results are consistent with a job-ladder model, without worker-level wage panel data I have not provided direct evidence of the minimum wage affecting wages along the job ladder. I consider this to be an important avenue for future work.

Finally, my analysis suggests an open question. [Engbom and Moser \(2022\)](#) find substantially larger wage spillover effects in Brazil than do [Autor et al. \(2016\)](#) in the U.S. context. Similarly, [Fortin et al. \(2021\)](#) find substantially larger wage spillover effects in the U.S. in the 1980s than in later decades. Why?

These two contexts share in common the fact that real minimum wage changes were monotonic over time, rising in Brazil throughout the period and falling in the U.S. in the 1980s. In the presence of dynamic effects of minimum wage changes, current changes in inequality depend not only on current changes in the minimum wage, but also on past changes. Hence, the impact of changes in the real minimum wage on inequality

²²Finally, note that the possibility that an increase in the minimum wage causes the least productive workers to exit employment is explicitly incorporated in the theory in Section 3.4, as the skills that are pushed into unemployment are precisely those with the lowest wages. If the empirical analysis is conducted after combining worker groups into certain aggregates (such as the college educated), then controlling for the change in the productive composition of these aggregate groups induced by changes in unemployment requires composition adjusting, as in, e.g., [Katz and Murphy \(1992\)](#), [Card and Lemieux \(2001\)](#), and my national analysis in Section 2. In my analysis in this Section, I do not aggregate up, and instead focus on 100 worker groups.

depend not only on the size of the change in the real minimum wage, but also its pattern over time. As an example, a large decline in the real minimum wage occurring over a long period of time (as in the U.S. in the 1980s) will generate a more substantial peak increase in wage inequality than would similarly-sized, but opposite swings in the real minimum wage over the same time horizon. To what extent are the larger wage spillover effects identified in [Engbom and Moser \(2022\)](#) and [Fortin et al. \(2021\)](#) resulting from the monotonicity of the respective real minimum wage changes considered?

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A Empirical Appendix

A.1 National time-series analysis

Basic processing of the March CPS data. I use the March Annual Demographic Files of the Current Population Survey from 1964 to 2017, which report earnings from 1963 to 2016. Thus, throughout when I refer to any year, I am using data from the following year’s March CPS.

I restrict my sample to workers age 16 to 64 during the earnings year. I drop respondents with missing schooling, missing or negative earnings, missing weeks worked, or with a negative weight. I additionally drop those who are self employed or engage in unpaid family work and anyone with allocated earnings. Finally, I drop respondents who are part of the 3/8 redesign in the 2014 ASEC sample. Following [Autor et al. \(2008\)](#) I multiply top-coded earnings by 1.5.

In composition adjusting, I bin workers into one of 100 groups, denoted by g , defined by the interaction of 5 age bins (16-25, 26-35, 36-45, 46-55, and 56-64), 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training).²³ The lowest three educations—high school dropouts, those with a high school degree, and those with some college—are allocated to non-college; the highest two educations—college graduates and graduate training—are allocated to college.

Constructing Wages and Supplies. In each year t and for each of the 100 groups g I construct total hours worked and total wage and salary income (using sample weights) and, from this, the average wage of each group-year pair, w_{gt} for group g . Within each year I average across the logarithm of the wages of all groups with at least a college degree (denoting the set of these groups by \mathcal{G}_h) and, separately, across all groups without (denoting the set of these groups by \mathcal{G}_ℓ) using time-invariant weights. For instance, for college-educated workers, I have

$$\log w_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g \log w_{gt}$$

where ω_g is the time-invariant weight applied to group g . These weights are constructed using the average across years of the share of hours worked of each group g within the

²³Up to and including 1991, I use the highest grade of school completed; I define college complete as having finished the fourth year of college and graduate training as having more than four years of college. Starting in 1992 I use degree completion, assigning associate’s degrees to some college.

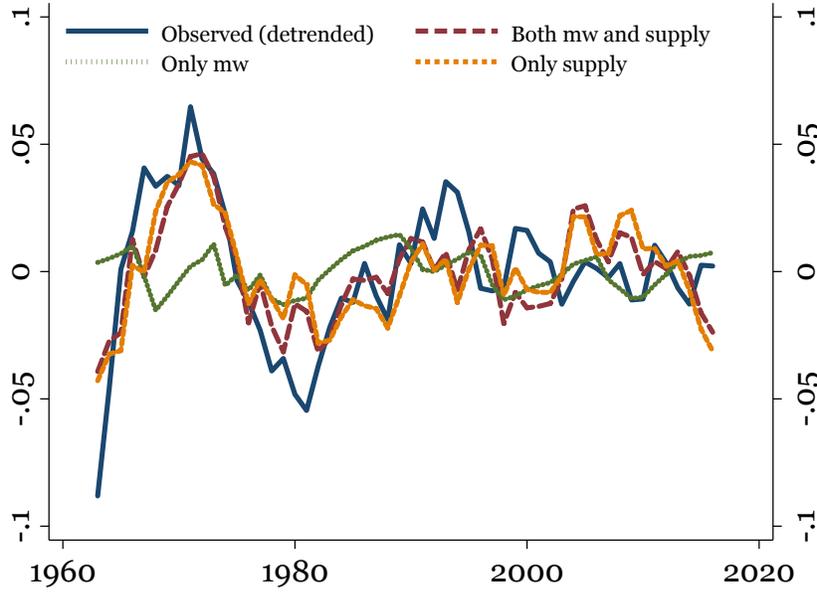


Figure A.1: Detrended National College Premium and the Predicted Impact of the Minimum Wage and Supply

Notes: “Observed” is the observed national college wage premium detrended using a cubic polynomial of time. To construct the remaining plots, I regress the detrended college premium on a constant, detrended supply of college workers, and detrended real minimum wage (each using a cubic polynomial). “Both mw and supply” plots the predicted value of the detrended college premium. “Only mw” and “Only supply” plot the predictions of the detrended college premium using the estimated minimum wage (omitting supply) or estimated supply (omitting the minimum wage) coefficient.

set of college groups \mathcal{G}_h and, separately, within the set of non-college groups \mathcal{G}_ℓ .²⁴ The resulting averages are the composition-adjusted wages $\log w_{ht}$ and $\log w_{\ell t}$ used in the analysis.

I construct three measures of supply of college and non-college workers. First, I measure supply using a composition-adjusted measure of changes in efficiency-unit hours worked. The hours-based measure is constructed as follows. In the first step, for each year and college group (\mathcal{G}_h and \mathcal{G}_ℓ), I construct a composition-adjusted weighted average wage, where the fixed-over-time weights are identical to those used in the construction of composition-adjusted wages, ω_g , as follows (I focus on h in what follows): $\tilde{w}_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g w_{gt}$. In the second step, I then divide the average wage of each labor group in that year-college pair by the average across all labor groups (in the corresponding year-college pair) created in the first step: $\tilde{w}_{gt} = w_{gt} / \tilde{w}_{ht}$ for $g \in \mathcal{G}_h$. This provides a year-specific measure of the relative wage of each group within the college-educated

²⁴I drop any group g that has no observations in at least one year. Keeping such a group yields a composition-adjusted wage (for the corresponding education group of either college or non-college workers) in which the total weights sum to less than one in any year in which that group is not observed.

and, separately, within the non-college-educated. In the third step, I take an average across years of this relative wage within each labor group: $\tilde{w}_g = (1/T) \sum_t \tilde{w}_{gt}$. This average across years is a measure of the average efficiency units supplied by each hour of labor of this labor group relative to the average labor group in the same college group. In the fourth step, I take a weighted average across all labor groups in the year-college pair of the product of observed hours worked ($hours_{gt}$) and the time-invariant efficiency units supplied by each hour of labor, using the same weights as in the construction of composition-adjusted wages: $\sum_{g \in \mathcal{G}_h} hours_{gt} \tilde{w}_g \omega_g$.²⁵ Finally, I take the logarithm of this composition-adjusted weighted average of efficiency-unit hours worked. I refer to this as the hours-based measure of supply.

Second, I measure supply using a composition-adjusted measure of changes in efficiency-unit populations. Here, I follow the same first three steps as in the construction of the hours-worked measure of supply. In the fourth step, I take a weighted average across all labor groups in the year-college pair of the product of populations (rather than hours worked) and the time-invariant efficiency units supplied by each hour of labor, using the same weights as in the construction of composition-adjusted wages. The measure of the population of each labor group in each year is constructed before dropping respondents with allocated, missing, or negative earnings and before dropping respondents who are self employed or engage in unpaid family work. I refer to this as the population-based measure of supply.

My final measure of supply, used only in Table A.1, is the dual of the composition-adjusted wage measures. In particular, I set $\log Supply_{ht} = \log Inc_{ht} - \log w_{ht}$ where Inc_{ht} is the total income of college-educated workers in raw (weighted) data in year t . I similarly construct $\log Supply_{\ell t}$.

Constructing real minimum wages. I construct annual real minimum wages m_t as follows. In 1974-2016, I use data from Cengiz et al. (2019) to measure state-year minimum wages, \tilde{m}_{rt} . For each state and quarter, Cengiz et al. (2019) report the minimum, mean, and maximum effective nominal minimum wage (the maximum of the legislated state and federal minimum wages). For each state and year, I take the average across quarters within the year of the average within-quarter minimum wage.

For the years 1963-1973 I obtain the federal (FLSA) minimum wage from the Department of Labor and state minimum wages from FRED. For each state, I use the maximum of the state's minimum wage and the federal minimum wage to measure \tilde{m}_{rt} .

To construct the national time series of real minimum wages, I additionally take the

²⁵This variable's units are irrelevant, since I take its logarithm and include a constant in the regression.

following steps. After constructing the state-year series of annual nominal minimum wages I average across states in each year using fixed-across-time weights. These weights are defined as follows. For the years in which the March CPS contains 51 ‘states’ (including Washington DC), I construct the share of the national population in each of the states. For each state, I take an average across all years of this share and use this as the fixed weight for each state. Finally, I deflate the resulting nominal minimum wage using the GDP deflator from FRED.

Alternative measures of supply in Table 1. Table A.1 displays results of estimating equation (2) using OLS and defining relative supply of college workers using the population-based measure. Table A.2 displays results of estimating equation (2) using 2SLS, but replacing the hours-worked-based measure of the relative supply of college workers using the supply measure constructed using the dual of wages.

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.555 (0.060)	-0.816 (0.083)	-0.632 (0.101)	-0.692 (0.110)	-0.356 (0.106)	-0.464 (0.060)
Real minimum wage	-0.307 (0.053)	-0.257 (0.058)	-0.132 (0.047)	-0.115 (0.058)	-0.126 (0.038)	
Real federal minimum wage						-0.236 (0.048)
Time	0.019 (0.001)					0.016 (0.001)
Time Polynomial	1	2	3	4	5	1

Table A.1: Reduced-Form Regression Models for the National College Premium

Notes: Estimating equation (2) using OLS and measuring the relative supply of college workers using the population-based measure. See the Notes of Table 1 for details.

	(1)	(2)	(3)	(4)	(5)	(6)
Relative supply of college workers	-0.584 (0.063)	-0.724 (0.085)	-0.745 (0.118)	-0.668 (0.124)	-0.518 (0.175)	-0.512 (0.058)
Real minimum wage	-0.207 (0.044)	-0.158 (0.054)	-0.171 (0.060)	-0.186 (0.063)	-0.175 (0.052)	
Real federal minimum wage						-0.163 (0.039)
Time	0.022 (0.002)					0.020 (0.001)
Time Polynomial	1	2	3	4	5	1

Table A.2: Regression Models for the National College Premium: Dual Measure of Supply
Notes: Replicating Table 1, but using the dual of wages to measure the relative supply of college workers. See the Notes of Table 1 for details.

Comparison of results with Autor et al. (2008). The top panel of Table A.3 estimates equation (2) with a linear (column 1) to a quintic (column 5) polynomial of time, as in columns 1 – 5 of Table 1, but using data from the replication package for Autor et al. (2008) (although I use the GDP deflator from FRED rather than the deflator in their replication package). Outside of the specification with a cubic time trend, results that can be obtained using their data are similar to my baseline national results displayed in Table 1.

The bottom panel of Table A.3 replicates Table A.3, but replaces the nominal minimum wage from Autor et al. (2008), which is the federal minimum wage at the national level, using my baseline measure of the real minimum wage, which averages minimum wages across states. The coefficient on the real minimum wage tends to be larger in the bottom panel of Table A.3 than the top panel. This is similar to the comparison between columns (1) and (6) in my Table 1. As there, using the federal minimum wage introduces attenuation bias, since it doesn't apply in many states, especially starting in the late 1990s.

Separate elasticities of college and non-college wages. In Table A.4 I decompose the estimated elasticities of the college premium into separate elasticities of college and non-college real wages. To do so, I estimate equation (2) replacing $\log(w_{ht}/w_{lt})$ with $\log w_{ht}$ and, separately, with $\log w_{lt}$. The elasticity of the college real wage minus the elasticity of the non-college real wage equals the elasticity of the college premium. I do so for the specifications that include a linear time trend, an additional quadratic time trend, and an additional cubic time trend, corresponding to columns 1 – 3 of Table 1.

Across specifications, a higher minimum wage raises the average real wage of workers

	Using AKK data				
	(a)	(b)	(c)	(d)	(e)
Relative supply of college workers	-0.431 (0.051)	-0.607 (0.077)	-0.612 (0.091)	-0.216 (0.113)	0.013 (0.104)
Minimum wage	-0.112 (0.049)	-0.108 (0.049)	-0.064 (0.048)	-0.174 (0.052)	-0.123 (0.040)
	Using my baseline real minimum wage				
	(a)	(b)	(c)	(d)	(e)
Relative supply of college workers	-0.459 (0.051)	-0.605 (0.078)	-0.610 (0.089)	-0.244 (0.108)	-0.019 (0.106)
Minimum wage	-0.150 (0.051)	-0.139 (0.053)	-0.087 (0.057)	-0.184 (0.059)	-0.119 (0.050)
Time Polynomial	1	2	3	4	5
Observations	43	43	43	43	43

Table A.3: Regression Analysis for the National College Premium (Using Data From the Replication Package for [Autor et al. \(2008\)](#))

Notes: The top panel displays results of estimating (2) using the college premium, the relative supply of college workers, and the nominal minimum wage from [Autor et al. \(2008\)](#). I deflate their nominal minimum wage using the GDP deflator from FRED rather than deflator in their replication package. The bottom panel replicates this exercise but uses my baseline measure of the real minimum wage. Robust standard errors are reported in parentheses.

without a college education, w_{lt} ; and this effect is statistically significant in each specification. On the other hand, the impact of the minimum wage on the average real wage of college-educated workers, w_{ht} , is not robust across specifications.

A.2 State- and labor group-level analysis

Basic processing of the Merged Outgoing Rotation Groups CPS data. I use the Current Population Survey Merged Outgoing Rotation Groups (MORG CPS) from 1979 to 2016 in the state-level estimation. The MORG CPS reflects current wages. Because of missing imputation flags—the CPS did not flag workers with missing wages—in all of 1994 and the first eight months of 1995, I do not include 1994 or the first half of 1995; and I measure the second half of 1995 using wage data only from September – December. As in the national analysis, I restrict attention to worker ages 16 to 64.

In processing the files, I broadly follow the approaches of [Lemieux \(2006\)](#) and [Autor et al. \(2008\)](#), using hourly wages for workers paid by the hour and using usual weekly

	Linear			Quadratic			Cubic		
	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)	(Premium)	(High)	(Low)
Relative supply	-0.632 (0.069)	-0.414 (0.127)	0.218 (0.123)	-0.703 (0.077)	-1.029 (0.140)	-0.327 (0.139)	-0.608 (0.104)	-1.117 (0.158)	-0.509 (0.103)
Real minimum wage	-0.220 (0.048)	-0.104 (0.059)	0.117 (0.058)	-0.199 (0.059)	0.083 (0.077)	0.282 (0.066)	-0.133 (0.052)	0.022 (0.091)	0.155 (0.063)
Time	0.021 (0.002)	0.020 (0.003)	-0.002 (0.002)						

Table A.4: Separate Regression Models for National College and Non-College Real Wages

Notes: The three columns labeled “Premium” replicate columns (1), (2), and (3) of Table 1 exactly. The three columns labeled “High” replicate these columns but replacing the dependent variable with the log of the college wage whereas the columns labeled “Low” do the same but with the non-college wage. Each estimate in the “High” column minus the corresponding estimate in the “Low” column equals the estimate in the “Premium” column. See the Notes of Table 1 for additional details.

earnings divided by hours worked last week for non-hourly workers. I multiply top-coded constructed hourly wages by 1.5. I drop respondents with allocated earnings flags. I identify and drop non-flagged allocated observations between 1989 and 1993 using the unedited earnings values.

Constructing real minimum wages. I construct bi-annual state-specific real minimum wages as follows. I use data from [Cengiz et al. \(2019\)](#) to construct bi-annual state-specific nominal minimum wages. For each state and quarter, [Cengiz et al. \(2019\)](#) report the minimum, mean, and maximum effective nominal minimum wage (the maximum of the of the legislated state and federal minimum wages). For each state and half-year, I take the maximum across quarters within the half year of the maximum minimum wage within each quarter. Finally, I deflate using the GDP deflator from FRED, using the maximum value of the GDP deflator across months within the half year.

A.3 Sensitivity analyses: Baseline 2SLS analysis

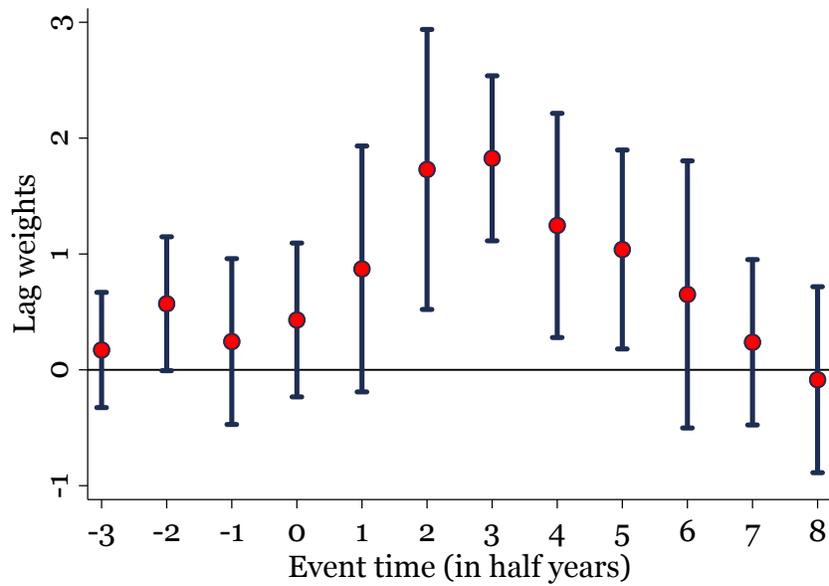


Figure A.2: Sensitivity of Figure 6: Female Sample

Notes: Replication of Figure 6 restricting the sample to worker groups g that are female. See the notes of Figure 6 for details.

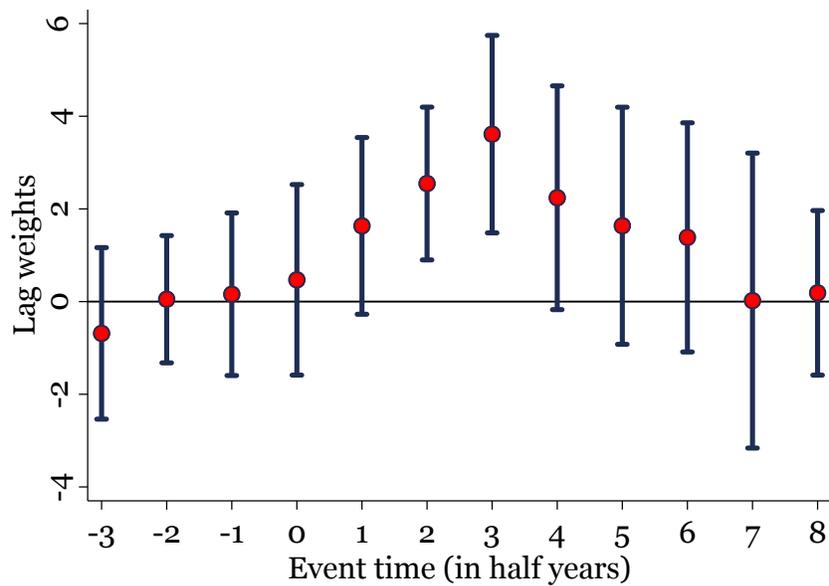


Figure A.3: Sensitivity of Figure 6: Male Sample

Notes: Replication of Figure 6 restricting the sample to worker groups g that are male. See the notes of Figure 6 for details.

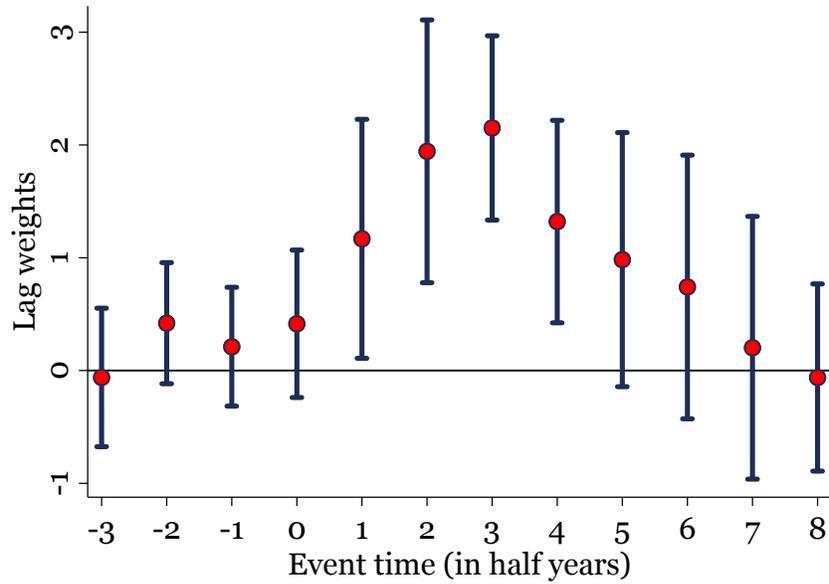


Figure A.4: Sensitivity of Figure 6: Non-College Sample

Notes: Replication of Figure 6 restricting the sample to worker groups g that have less than a completed college education. See the notes of Figure 6 for details.

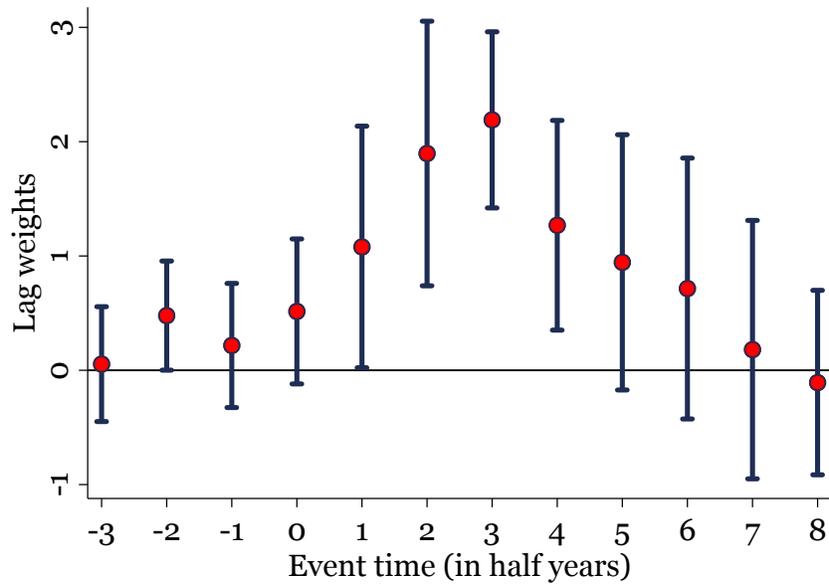


Figure A.5: Sensitivity of Figure 6: Unbalanced Sample

Notes: Replication of Figure 6 expanding the sample from a balanced panel of g, r present in all t to the complete unbalanced panel. See the notes of Figure 6 for details.

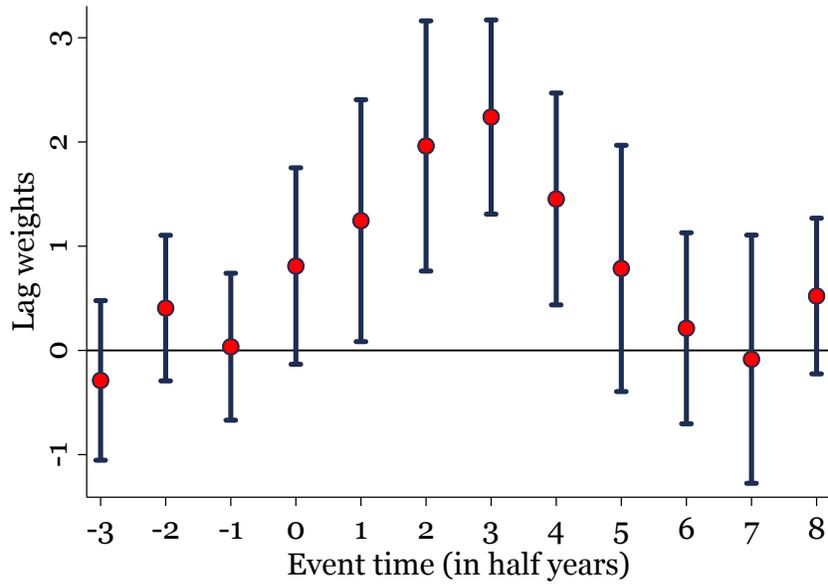


Figure A.6: Sensitivity of Figure 6: 1979 – 2010 Sample

Notes: Replication of Figure 6 restricting the sample to t in the range of 1979 – 2010. See the notes of Figure 6 for details.

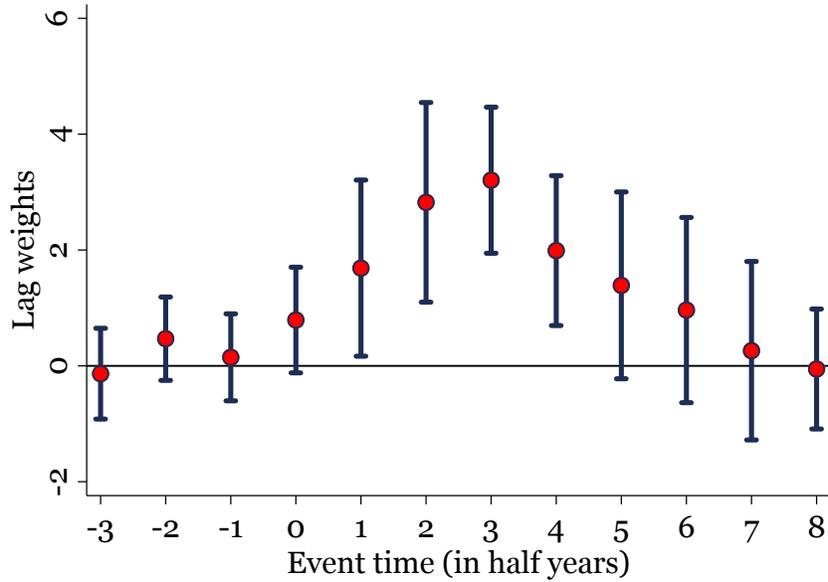


Figure A.7: Sensitivity of Figure 6: Minimum Wage Cutoff 1.00

Notes: Replication of Figure 6 defining a minimum wage earner as a worker earning ≤ 1.00 times the minimum wage. See the notes of Figure 6 for details.

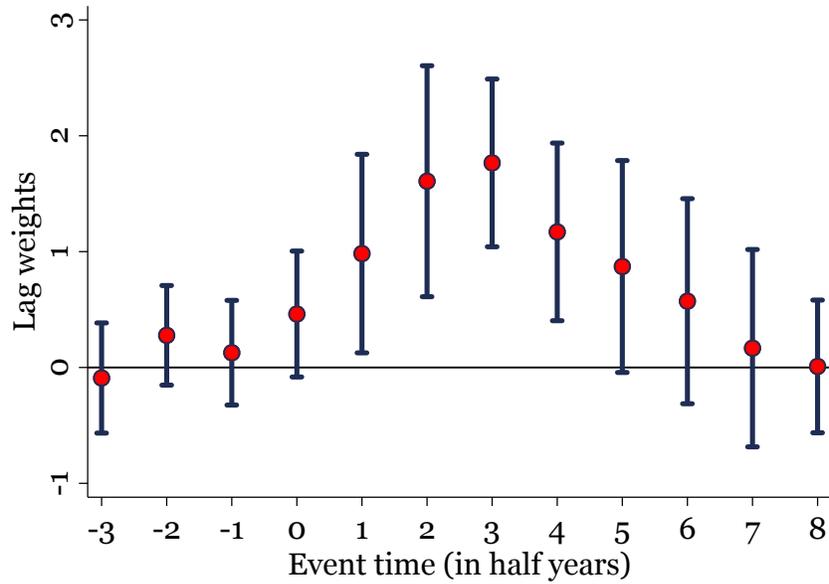


Figure A.8: Sensitivity of Figure 6: Minimum Wage Cutoff 1.10

Notes: Replication of Figure 6 defining a minimum wage earner as a worker earning ≤ 1.10 times the minimum wage. See the notes of Figure 6 for details.

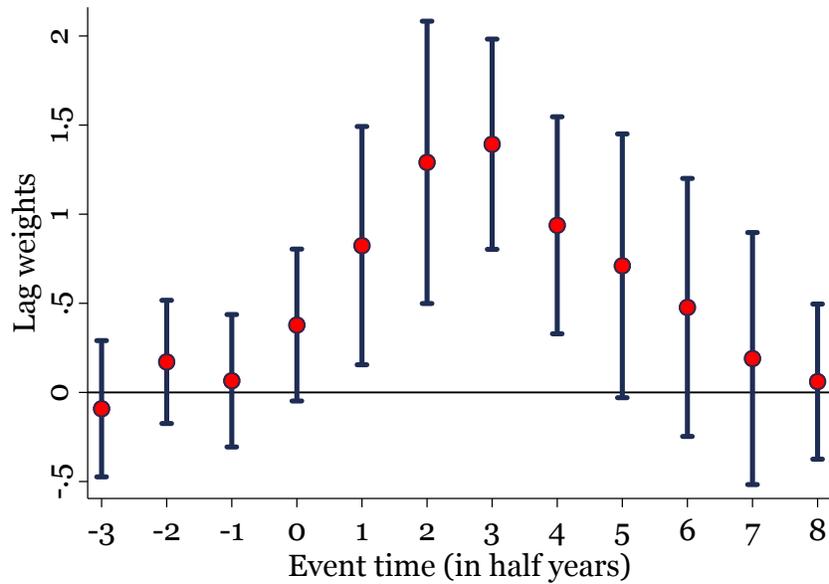


Figure A.9: Sensitivity of Figure 6: Minimum Wage Cutoff 1.15

Notes: Replication of Figure 6 defining a minimum wage earner as a worker earning ≤ 1.15 times the minimum wage. See the notes of Figure 6 for details.

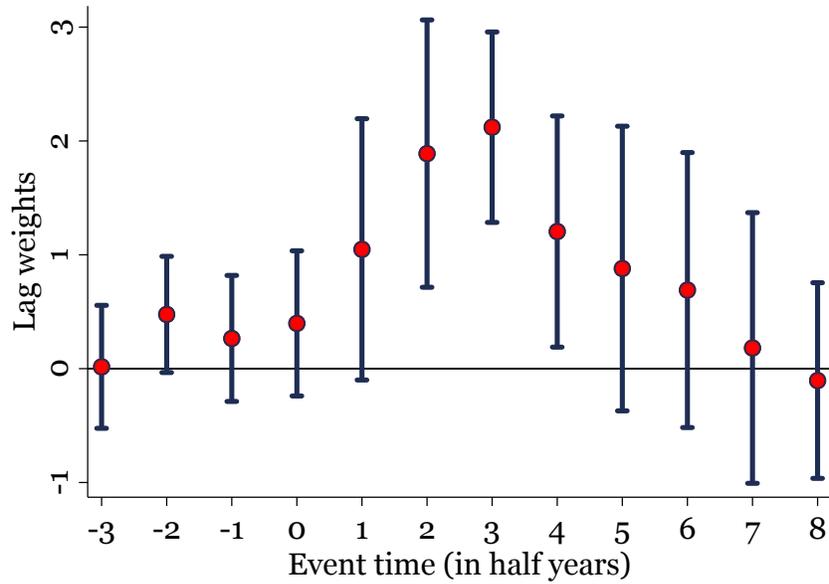


Figure A.10: Sensitivity of Figure 6: Controlling for Region-Specific Effect of $b_{g,r}$
Notes: Replication of Figure 6 additionally controlling for $\kappa_r b_{g,r}$. See the notes of Figure 6 for details.

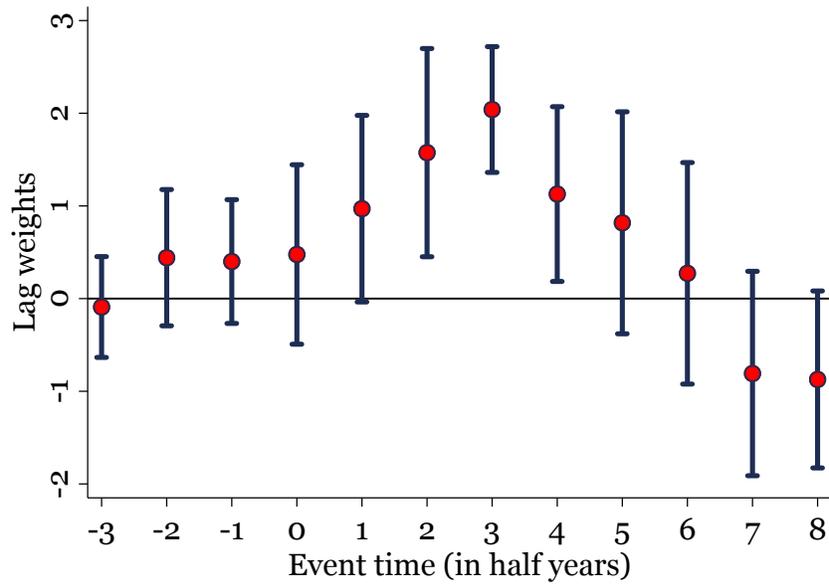


Figure A.11: Sensitivity of Figure 6: Controlling for Region-Time Effects
Notes: Replication of Figure 6 additionally controlling for $\alpha_{r,t}$. See the notes of Figure 6 for details.

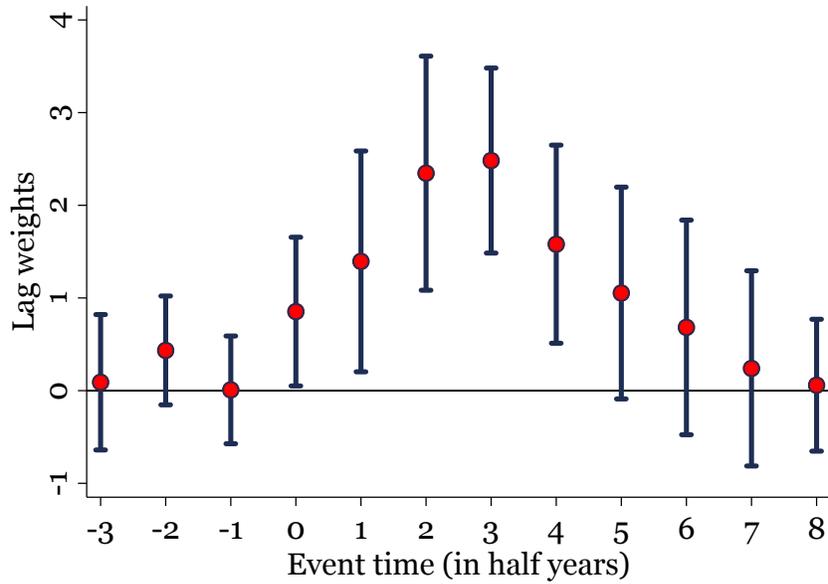


Figure A.12: Sensitivity of Figure 6: Controlling for Group-Time Effects

Notes: Replication of Figure 6 additionally controlling for $\alpha_{g,t}$. See the notes of Figure 6 for details.

A.4 Sensitivity analyses: Reduced-form analysis

To conserve on space, here I display sensitivity analyses of the reduced-form estimation in Figure 5 for a given set of controls.

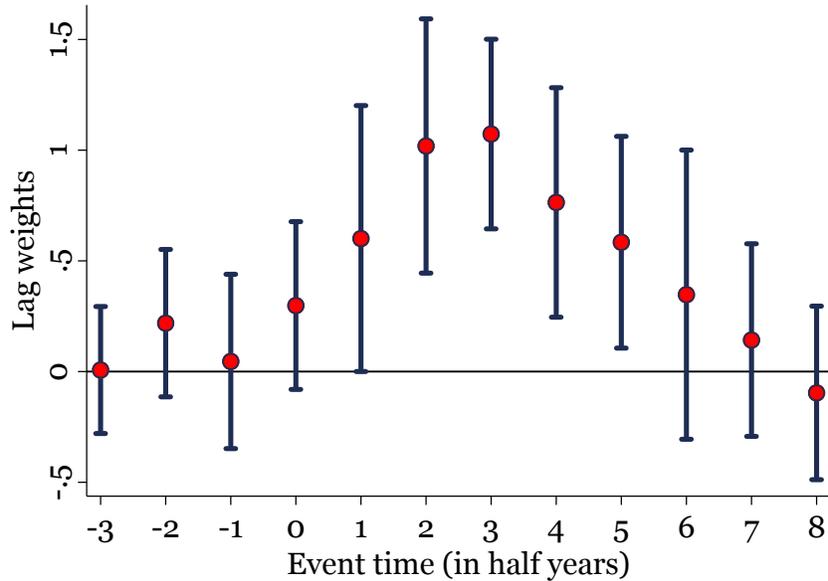


Figure A.13: Sensitivity of Reduced-Form Figure (5): female sample

Notes: Replication of Figure 5 restricting the sample to worker groups g that are female. See the notes of Figure 5 for details.

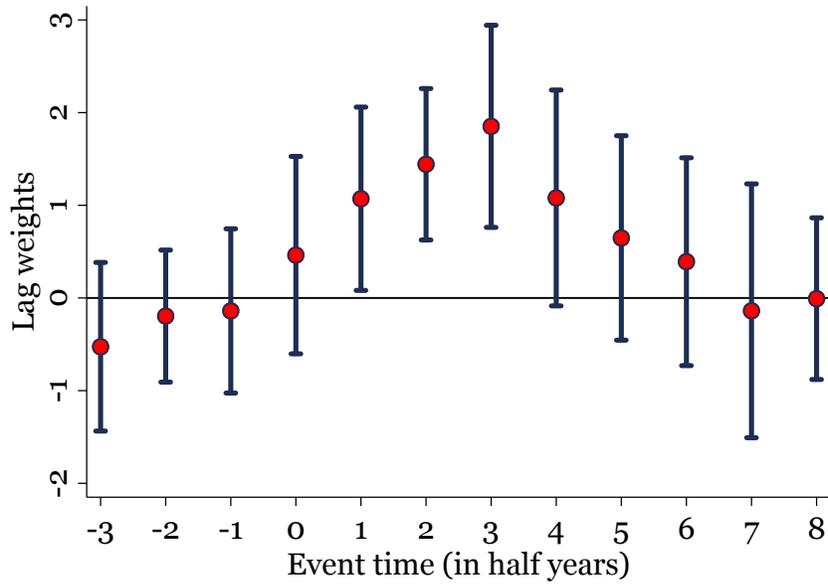


Figure A.14: Sensitivity of Reduced-Form Figure (5): Male Sample

Notes: Replication of Figure 5 restricting the sample to worker groups g that are male. See the notes of Figure 5 for details.

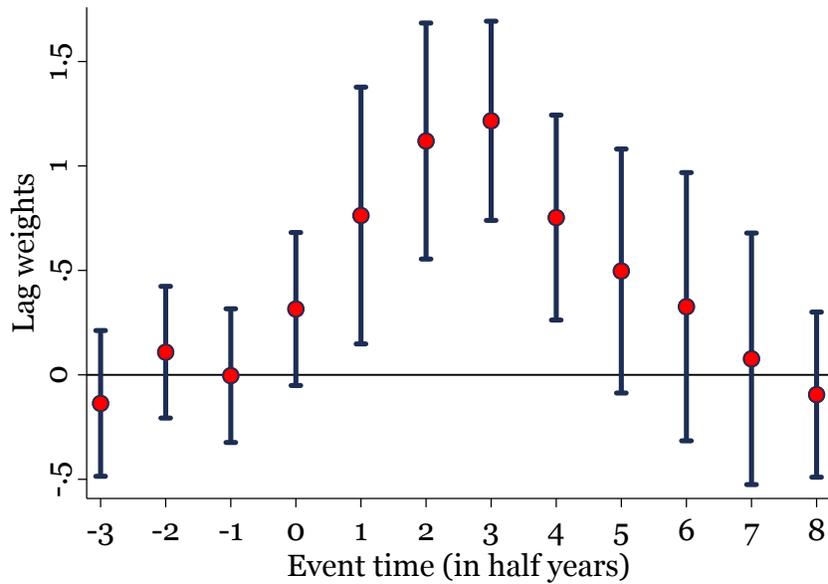


Figure A.15: Sensitivity of Reduced-Form Figure (5): Non-College Sample

Notes: Replication of Figure 5 restricting the sample to worker groups g that have less than a completed college education. See the notes of Figure 5 for details.

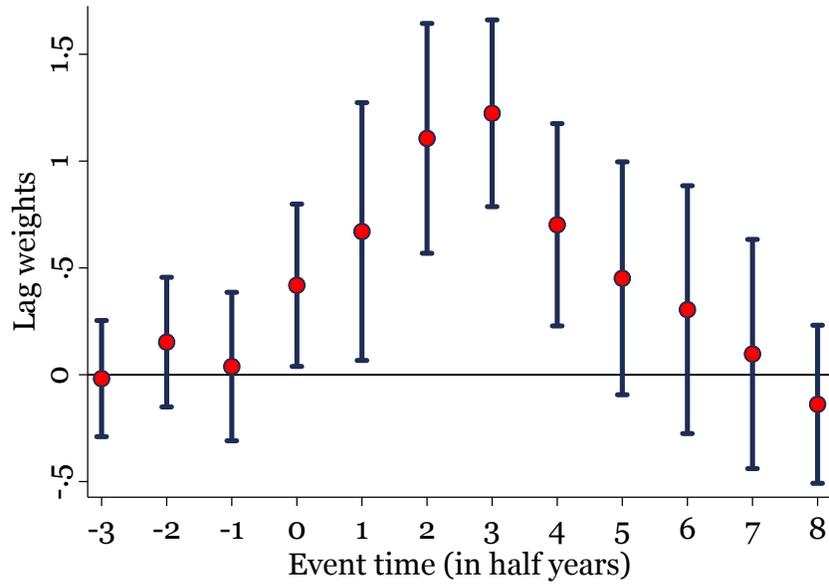


Figure A.16: Sensitivity of Reduced-Form Figure (5): Unbalanced Sample

Notes: Replication of Figure 5 expanding the sample from a balanced panel of g, r present in all t to the complete unbalanced panel. See the notes of Figure 5 for details.

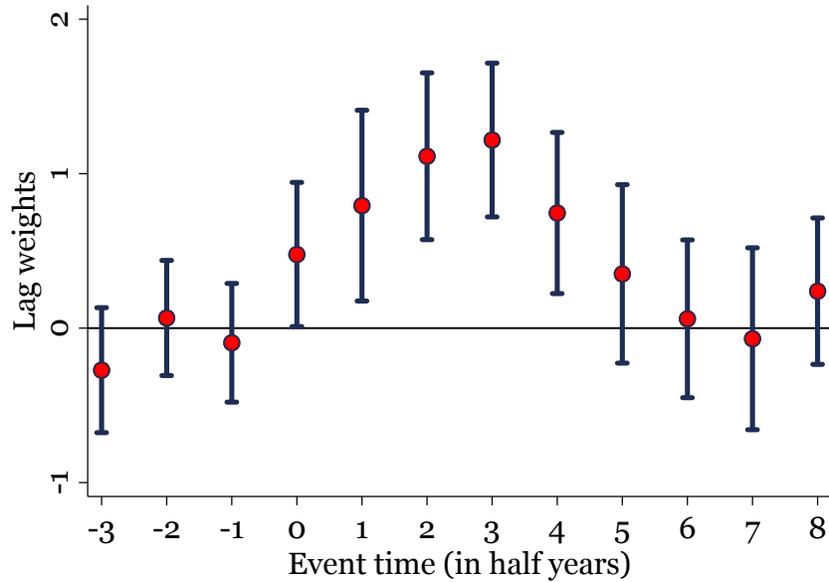


Figure A.17: Sensitivity of Reduced-Form Figure (5): 1979 – 2010 Sample

Notes: Replication of Figure 5 restricting the sample to t in the range of 1979 – 2010. See the notes of Figure 5 for details.

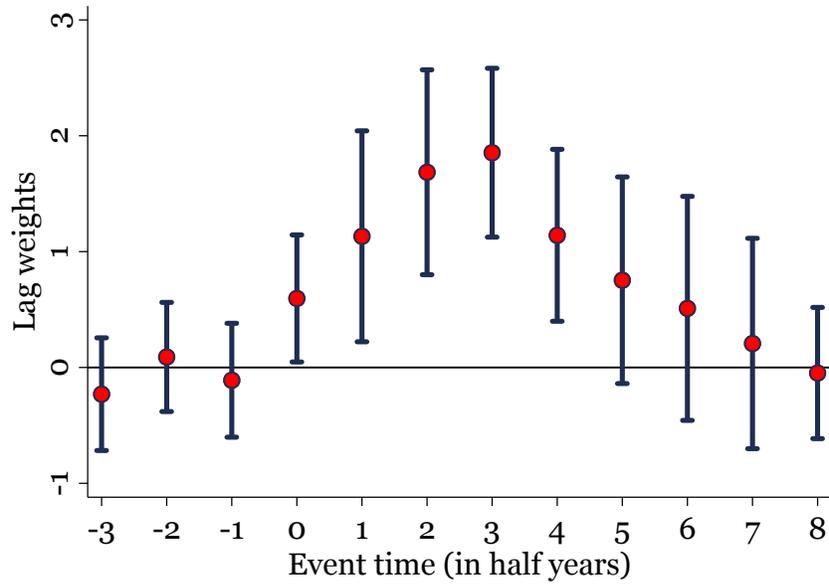


Figure A.18: Sensitivity of Reduced-Form Figure (5): Minimum Wage Cutoff 1.00

Notes: Replication of Figure 5 defining a minimum wage earner as a worker earning ≤ 1.00 times the minimum wage. See the notes of Figure 5 for details.

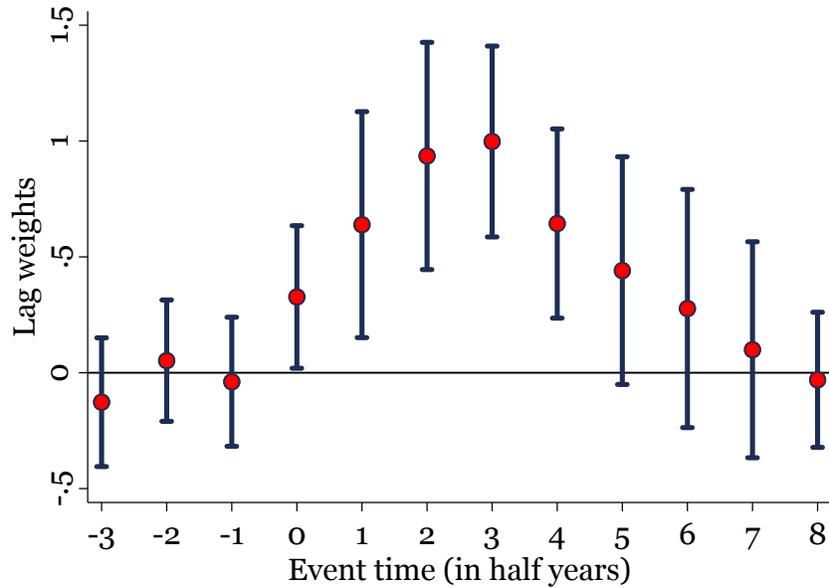


Figure A.19: Sensitivity of Reduced-Form Figure (5): Minimum Wage Cutoff 1.10

Notes: Replication of Figure 5 defining a minimum wage earner as a worker earning ≤ 1.10 times the minimum wage. See the notes of Figure 5 for details.

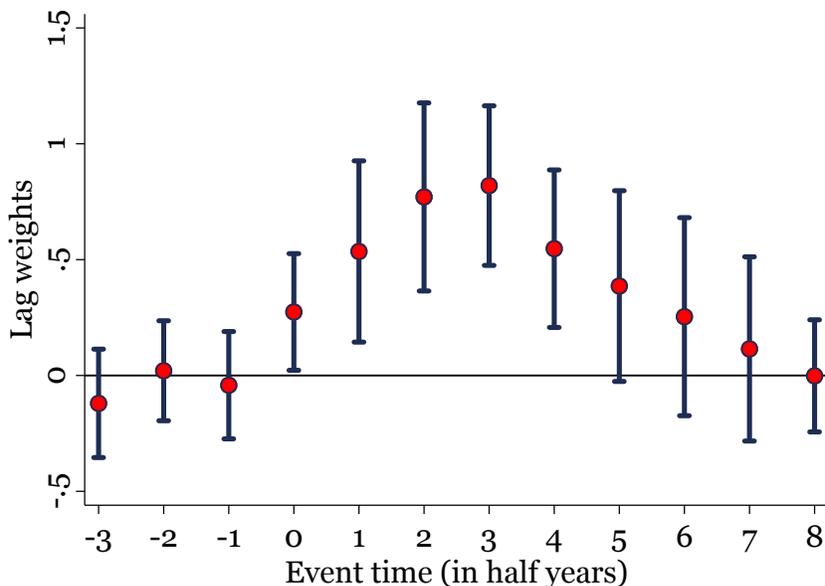


Figure A.20: Sensitivity of Reduced-Form Figure (5): Minimum Wage Cutoff 1.15

Notes: Replication of Figure 5 defining a minimum wage earner as a worker earning ≤ 1.15 times the minimum wage. See the notes of Figure 5 for details.

B Theoretical appendix

B.1 Steady-state details

Steady-state distribution across the job ladder. Solving the recursive system displayed in equation (3) and using the fact that these densities must sum to one across all $j = 0, \dots, \infty$, yields both

$$u_s = \frac{\delta_s}{\delta_s + \gamma_{su}} \quad (\text{B.1})$$

and

$$g_s(w_j) = \left(\frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right)^{j-1} \frac{\gamma_{su}}{\delta_s + (1 - \delta_s)\gamma_{se}} \frac{\delta_s}{\delta_s + \gamma_{su}} \quad \text{for } j \geq 1 \quad (\text{B.2})$$

Steady-state wage ladder Consider a skill s for which the minimum wage is binding: $m \geq (1 - \beta_s)v_s + \beta_s P_s$. The wage on the first rung of the job ladder is $w_{1,s} = m$. Starting from any rung $j \geq 1$, the wage of the next rung $j + 1$ on the job ladder is $w_{j+1,s} = (1 - \beta_s)w_{j,s} + \beta_s P_s$ for $j \geq 1$. This recursive system yields the following explicit solution for the wage at

each rung

$$w_{j,s} = (1 - \beta_s)^{j-1} m + \beta_s P_s \sum_{k=0}^{j-2} (1 - \beta_s)^k \text{ for all } j > 1 \text{ if } m \geq (1 - \beta_s) v_s + \beta_s P_s \quad (\text{B.3})$$

Next, consider a skill s for which the minimum wage is not binding: $m < (1 - \beta_s) v_s + \beta_s P_s$. The wage on the first rung of the job ladder is $w_{1,s} = (1 - \beta_s) v_s + \beta_s P_s$. Starting from any rung $j \geq 1$, the wage of the next rung $j + 1$ on the job ladder is simply $w_{j+1,s} = (1 - \beta_s) w_{j,s} + \beta_s P_s$ for $j \geq 1$. This recursive system yields the following explicit solution for the wage at each rung

$$w_{j,s} = (1 - \beta_s)^j v_s + \beta_s P_s \sum_{k=0}^{j-1} (1 - \beta_s)^k \text{ for all } j \geq 1 \text{ if } m < (1 - \beta_s) v_s + \beta_s P_s \quad (\text{B.4})$$

Steady-state average wage. The average wage of the employed is given by

$$w_s \equiv \frac{1}{1 - u_s} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$$

Substituting in from equations (B.1) and (B.2) yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s) \gamma_{se}} \sum_{i \geq 0} w_{i+1,s} \left(\frac{(1 - \delta_s) \gamma_{se}}{\delta_s + (1 - \delta_s) \gamma_{se}} \right)^i \quad (\text{B.5})$$

First consider a skill s for which the minimum wage is binding: $m \geq (1 - \beta_s) v_s + \beta_s P_s$. Substituting into equation (B.5) both $w_{1,s} = m$ and equation (B.3) yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s) \gamma_{se}} \left\{ m \sum_{i \geq 0} \left[\frac{(1 - \delta_s) (1 - \beta_s) \gamma_{se}}{\delta_s + (1 - \delta_s) \gamma_{se}} \right]^i + \beta_s P_s \sum_{i \geq 1} \left[\sum_{k=0}^{i-1} (1 - \beta_s)^k \right] \left[\frac{(1 - \delta_s) \gamma_{se}}{\delta_s + (1 - \delta_s) \gamma_{se}} \right]^i \right\}$$

Simplifying the above expression yields equation (4) in Section 3.2.

Next, consider a skill s for which the minimum wage is not binding: $m < (1 - \beta_s) v_s + \beta_s P_s$. Substituting into equation (B.5) both $w_{1,s} = (1 - \beta_s) v_s + \beta_s P_s$ and equation (B.4)

yields

$$w_s = \frac{\delta_s}{\delta_s + (1 - \delta_s)\gamma_{se}} \left\{ v_s(1 - \beta_s) \sum_{i \geq 0} \left[\frac{(1 - \delta_s)(1 - \beta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i + P_s \beta_s \sum_{i \geq 0} \left[\sum_{k=0}^i (1 - \beta_s)^k \right] \left[\frac{(1 - \delta_s)\gamma_{se}}{\delta_s + (1 - \delta_s)\gamma_{se}} \right]^i \right\}$$

Simplifying the above expression yields

$$w_s = \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} v_s + \left(1 - \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \right) P_s \quad (\text{B.6})$$

Equation (4) is a weighed average of m and P_s and equation (B.6) is a weighted average of v_s and P_s . The weight on v_s in equation (B.6) is $(1 - \beta_s)$ times the weight on m in equation (4). The reason equation (4) places a higher weight on m than equation (B.6) places on v_s is that the wage on the first rung of the job ladder associated with equation (4) is m , whereas the wage on the first rung of of the job ladder associated with equation (B.6) is $(1 - \beta_s)v_s + \beta_s P_s$.

Across steady-state comparative statics: for given changes in P_s . Here, I consider the impact of given changes in m and P_s .

First, consider the case in which the minimum wage binds for skill s . In this case, equation (4) implies

$$d \log w_s = \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \frac{m}{w_s} d \log m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} \right) \frac{P_s}{w_s} d \log P_s$$

Equation (4), the definition of $b_s = m g_s(m) [(1 - u_s)w_s]^{-1}$, and the solutions for u_s and $g_s(m)$ in equations (B.1) and (B.2) imply

$$\frac{m}{w_s} = b_s \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s}$$

and

$$\frac{P_s}{w_s} = \frac{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}}{\beta_s(1 - \delta_s)\gamma_{se}} \times \left(1 - \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s \right)$$

The three previous expressions imply

$$d \log w_s = \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s d \log m + \left(1 - \frac{\delta_s + (1 - \delta_s)\gamma_{se}}{\delta_s + \beta_s(1 - \delta_s)\gamma_{se}} b_s \right) d \log P_s$$

This equation is the same as equation (5) given the definition of M_s in equation (6).

Next, consider the case in which the minimum wage binds for skill s . In this case, equation (B.6) implies

$$d \log w_s = \left(1 - \frac{\delta_s(1 - \beta_s)}{\delta_s + \beta_s(1 - \delta_s)} \frac{v_s}{w_s} \right) d \log P_s \quad (\text{B.7})$$

where v_s/w_s is a measure of the unemployment replacement rate for skill s (the ratio of unemployment real income to the average real wage).

Comparative statics across steady states: determining changes in P_s and the extended canonical model. I begin by solving explicitly for $d \log P_s$ under the assumption that the aggregate production function is given by

$$Y = \left[(A_h Y_h)^{\frac{\eta-1}{\eta}} + (A_\ell Y_\ell)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (\text{B.8})$$

where $Y_s = (1 - u_s)L_s$. I maintain the assumption that $m < P_s$ for all s . Equation (B.8), $P_s = \partial Y / \partial Y_s$, and $Y_s = (1 - u_s)L_s$ imply

$$P_s = Y^{\frac{1}{\eta}} A_s^{\frac{\eta-1}{\eta}} (1 - u_s)^{-\frac{1}{\eta}} L_s^{-\frac{1}{\eta}}$$

$$d \log Y = \frac{1}{Y} \sum_{s \in \{h, \ell\}} Y^{\frac{1}{\eta}} (A_s (1 - u_s) L_s)^{\frac{\eta-1}{\eta}} (d \log A_s + d \log L_s)$$

The previous expressions yield

$$d \log Y = \left[\frac{P_h Y_h}{Y} (d \log A_h + d \log L_h) + \frac{P_\ell Y_\ell}{Y} (d \log A_\ell + d \log L_\ell) \right]$$

$$d \log P_s = \frac{1}{\eta} d \log Y + d \log A_s - \frac{1}{\eta} (d \log A_s + d \log L_s)$$

which in turn imply

$$d \log P_s = \frac{1}{\eta} \frac{P_{s'} Y_{s'}}{Y} \left(d \log \frac{A_{s'}}{A_s} + d \log \frac{L_{s'}}{L_s} \right) + d \log A_s \quad \text{for } s' \neq s \quad (\text{B.9})$$

I now use the previous result to prove Proposition 1.

Proof of Proposition 1. Suppose that across steady states, represented by t , there is linear growth in A_{ht} and $A_{\ell t}$, with mean zero deviations. Additionally suppose that $M_h = M_\ell$ (a sufficient condition for which is $\delta_h = \delta_\ell$, $\gamma_{he} = \gamma_{\ell e}$, and $\beta_h = \beta_\ell$). Then combining equation (B.9) with equations (5) and/or (B.7)—depending on whether the minimum

wage binds the wages of h and/or ℓ —yields (2), where the residual is a function of the deviations of the rate of skill-biased technical change from the linear trend. In equation (2), the coefficient ν_m on the real minimum wage is $M(b_h - b_\ell)$.²⁶□

If the minimum wage only binds for one skill, then parameters needn't satisfy $M_h = M_\ell$, since only the value of M_s for the skill group bound by the minimum wage is relevant.

B.2 Proof of Proposition 2

I focus on a single skill s that is bound by the minimum wage, $m \geq (1 - \beta_s) v_s + \beta_s P_s$, and omit skill subscripts where possible. I consider a one-time change in the real minimum wage, starting from a steady state, from m to $m' < (1 - \beta)m + \beta P$. This constraint on m' implies that the second rung of the initial job ladder is not constrained by the new minimum wage.

Proof of Results (i) and (ii) of Proposition 2: The wage ladder associated with m (original) and the wage ladder associated with m' (new) are denoted by w_j and w'_j , respectively. Since s is bound by m , the wage on the first rung of each ladder is given by $w_1 = m$ and $w'_1 = m'$, respectively. Wages w_j and w'_j for each $j > 1$ are then given by equation (B.3), using minimum wage m in the original wage ladder and m' in the new wage ladder. Because the wage on rung j is increasing in the minimum wage in equation (B.3), Result (ii) of Proposition 2 directly applies: $w'_j > w_j$ for each $j \geq 1$. □

Proof of Result (iii) of Proposition 2: Let $g_{t,j}$ and $g'_{t,j}$ denote the shares of all workers at time $t \geq 0$ employed on rung j of the wage ladders associated with m and m' , respectively. Let g_j denote the share of all workers on rung j in the steady state before the shock. I now show that $g_{t,j} + g'_{t,j} = g_j$ for all j and any $t \geq 0$.

Consider $t = 0$, just after the shock. The assumption that $m' < (1 - \beta)m + \beta P$ implies that $g'_{0,j} = 0$ and $g_{0,j} = g_j$ for any rung $j \geq 2$. The additional assumption that $m' > m$ implies $g'_{0,1} = g_1$ and $g_{0,1} = 0$. Since all unemployed exit unemployment on the new job ladder, I assign the unemployed to the new job ladder in all dates. And since $m, m' < P$, the unemployment rate is unaffected on impact. Hence, $g'_{0,0} = g_0$ and $g_{0,0} = 0$. In summary, just after the shock $g_{0,j} + g'_{0,j} = g_j$ for all j .

Now suppose that $g_{t,j} + g'_{t,j} = g_j$ for all j for some $t \geq 0$. Then at date $t + 1$ and for

²⁶The particular values of ν_L and ν_{A_s} in equation (2) also depend on which skills are bound by the minimum wage.

any $j > 1$, we have

$$\begin{aligned} g_{t+1,j} &= g_{t,j}(1-\delta)(1-\gamma_e) + g_{t,j-1}(1-\delta)\gamma_e \\ g'_{t+1,j} &= g'_{t,j}(1-\delta)(1-\gamma_e) + g'_{t,j-1}(1-\delta)\gamma_e \end{aligned}$$

Summing these expressions yields

$$g_{t+1,j} + g'_{t+1,j} = g_j(1-\delta)(1-\gamma_e) + g_{j-1}(1-\delta)\gamma_e = g_j$$

where the first equality follows from $g_{t,j} + g'_{t,j} = g_j$ at date t and the second equality follows from the steady-state derivation of g_j . The same applies for $j = 1$, except the term $(1-\delta)\gamma_e$ must everywhere be replaced by λ_u . Since $\sum_{j=0}^{\infty} (g_{t,j} + g'_{t,j}) = 1$, this implies $g'_{t+1,0} = g_0$. Hence, part (ii) of Proposition 2 is proven by induction. \square

Proof of Result (iv) of Proposition 2: To obtain a contradiction, suppose that there exist a (t, j) pair satisfying $g_{t+1,j} - g_{t,j} > 0$. Such a (t, j) pair must satisfy $j > 1$, since $g_{t,1} = 0$ for all $t \geq 0$. If $j > 1$ and $t \geq 0$ then

$$g_{t+1,j} = xg_{t,j} + yg_{t,j-1} \tag{B.10}$$

where I have defined $x \equiv (1-\delta)(1-\gamma_e)$ as the probability a worker who starts on any rung $j \geq 1$ on the original ladder remains there the following period and $y \equiv (1-\delta)\gamma_e$ as the probability that a worker who starts on any rung $j \geq 1$ moves up to rung $j+1$ in the following period. Hence, for $j > 1$ and $t \geq 0$, $g_{t+1,j} - g_{t,j} > 0$ is equivalent to

$$(1-x)g_{t,j} < yg_{t,j-1}$$

If $t = 0$, the previous expression is equivalent to $g_j(1-x) < g_{j-1}y$, where g_j denotes the steady-state share; this contradicts the steady-state relationship $g_j(1-x) = g_{j-1}y$. If $t > 0$, then $g_{t,j}$ can be substituted out of the previous expression using equation (B.10), with each time period lagged once, to obtain

$$y [g_{t,j-1} - g_{t-1,j-1}] > x [(1-x)g_{t-1,j} - yg_{t-1,j-1}] \tag{B.11}$$

If the right-hand side of equation (B.11) is weakly positive, then the left-hand-side of equation (B.11) must be strictly positive, which requires that $g_{t,j-1} - g_{t-1,j-1} > 0$. This is contradicted if $j = 2$ (since $g_{t,1} = 0$ for all $t \geq 0$) or if $t = 1$ (since $g_{1,j} = g_{2,j} = g_j$ for all $j > 2$). If, on the other hand, the right-hand side of equation (B.11) is strictly negative,

$(1 - x)g_{t-1,j} < yg_{t-1,j-1}$, then substituting out $xg_{t-1,j} = g_{t,j} - yg_{t-1,j-1}$ using equation (B.10), yields $g_{t,j} - g_{t-1,j} > 0$. This is contradicted if $t = 1$ (since $g_{1,j} = g_{0,j} = g_j$ for all $j > 2$).

If no contradiction is reached, the same process restarts. However, now instead of assuming $g_{t+1,j} - g_{t,j} > 0$, either both t and j are reduced by one ($g_{t,j-1} - g_{t-1,j-1} > 0$) if the right-hand side of equation (B.11) is weakly positive or only t is reduced by one ($g_{t,j} - g_{t-1,j} > 0$) if the right-hand side of equation (B.11) is strictly negative. This argument proceeds until a contradiction is reached. \square

Proof of Result (v) of Proposition 2: On impact, the wage on the first rung on the old job ladder, m , becomes constrained by the new, higher minimum wage. Because surplus in each match remains positive, every worker earning m moves instantly to the first rung of the new ladder, m' . The increase in the minimum wage does not constrain the second rung of the initial wage ladder (given the assumption that $m' < (1 - \beta)m + \beta P$), so no other workers are affected. This implies that the instantaneous, or impact elasticity of the average wage of skill s equals b_s . \square

Proof of Result (vi) of Proposition 2: The average wage at time t is

$$w_t \equiv \frac{1}{1 - u_s} \sum_{j \geq 1} (w_j g_{t,j} + w'_j g'_{t,j})$$

Results (ii), (iii), and (iv) imply that $w_j g_{t,j} + w'_j g'_{t,j}$ is weakly increasing in t for each j , with a strict inequality for at least one value of j . \square

B.3 Calibration details

Parameter values. I choose values for the following parameters from the job-ladder model: $\gamma_{r,g,u}$, $\gamma_{r,g,e}$, $\delta_{r,g}$, and $\beta_{r,g}$. I set each of these to be common across regions r and groups g and denote them by γ_u , γ_e , δ , and β . Using a panel survey of those looking for a job, Hall and Mueller (2018) find that the weekly rate of job offers for the unemployed is 0.058. Given this value and the observed acceptance rate of job offers from unemployment, they calculate a weekly job-separation rate of 0.0041. I convert each of these weekly rates to bi-annual rates, yielding $\gamma_u = 0.79$ and $\delta = 0.10$. As in Hall and Mueller (2018), I set the job-finding rate from employment equal to half the value from unemployment, $\gamma_e = \gamma_u/2$. I choose $\beta = 0.25$ so that the maximum lag weight estimate in model-generated data displayed in Figure 4 is similar to the maximum lag weight estimate in the actual data displayed in Figure 6. These parameters pin down the long-run magnification elasticity according to equation (6). In particular, I obtain $M = 2.4$.

I additionally choose a time-invariant value of the real value marginal product of labor for each r, g pair, $P_{r,g}$. There are 5,000 such values to choose, corresponding to the 50 regions r and 100 groups g . To do so, I divide r, g pairs into two sets: those for which the median value of the minimum wage bite for r, g across all t , $b_{r,g}$ is “low” and those for which it is “high.” For r, g pairs for which $b_{r,g} < 0.01$, I choose $P_{r,g}$ such that the average real wage of r, g averaged across all t in the data exactly equals the average real real wage of r, g in the model under the assumption that the minimum wage does not bind for r, g . For r, g pairs for which $b_{r,g} \geq 0.01$, I choose $P_{r,g}$ such that the average real wage of r, g averaged across all t in the data equals the steady-state value in the model under the assumption that the average real minimum wage across at t applies. Finally, given these values of $P_{r,g}$, I adjust any values that fall below $\max_t \{m_{r,t}\}$ by raising that value to equal the maximum real minimum wage.

Additional model details. I start the model assuming that the first half of 1979 is a steady state. I then feed in all observed real minimum wages in each state and half year, as described in the text. Unlike in the analytic results, I do not constrain changes in the minimum wage to be positive or small.

After solving the model for each g, r, t triplet, I introduce a small amount of measurement error in wages by multiplying each draw from the wage distribution by a random variable that is uniformly distributed between 0.95 and 1.05. This smooths the resulting distribution of wages. Finally, I measure the bite of the minimum wage for each r, g, t triplet using the resulting wage distribution (with noise) and under the same assumption as in the data, that a worker earns the minimum wage if her wage is ≤ 1.05 times the minimum wage.

B.4 Wage-posting model

In the baseline model, workers and firms bargain over wages. I assume that agents are not forward looking to facilitate solving the transition in response to a sequence of aggregate shocks; and the model counterfactually predicts that expected job duration is independent of the worker’s wage. Here, I show that steady-state results are broadly similar in the canonical wage-posting model of [Burdett and Mortensen \(1998\)](#) with homogeneous workers (within a skill) and firms, extended to include a minimum wage, as in [van den Berg and Ridder \(1998\)](#). In this framework, agents are forward looking and job duration is increasing in the worker’s wage. I consider the case of a single skill s (omitting s subscripts), since this is sufficient to show which results are robust, and I focus on steady states, since solving for the transition to an aggregate shock is not straightforward.

According to equation (2.10) in [van den Berg and Ridder \(1998\)](#), the equilibrium earnings density is

$$g(w) = \frac{\delta (P - m)^{1/2}}{2\lambda_e} (P - w)^{-3/2} \text{ for all } w \in [m, w_{\max}]$$

under the assumption that the minimum wage m is binding (an assumption I make throughout), where

$$w_{\max} \equiv \left(\frac{\delta}{\delta + \lambda_e} \right)^2 m + \left(1 - \frac{\delta}{\delta + \lambda_e} \right)^2 P$$

In these expressions, I have used my notation: P is the value marginal product of labor, δ is the exogenous separation rate, λ_e is the rate at which workers receive offers when employed, and w_{\max} is the (endogenous) supremum of offered wages. Integrating this yields the probability a worker earns less than w ,

$$G(w) = \frac{\delta}{\lambda_e} \left(\frac{P - m}{P - w} \right)^{1/2} - \frac{\delta}{\lambda_e} \text{ for all } w \in [m, w_{\max}]$$

which is the earnings distribution (conditional on employment).

Impact of a change in the minimum wage on the distribution of wages. Define $W_c(m)$ to be the wage at centile $c \in [0, 100]$. The above distribution of wages implies

$$\frac{c}{100} = \frac{\delta}{\lambda_e} \left(\frac{P - m}{P - W_c(m)} \right)^{1/2} - \frac{\delta}{\lambda_e}$$

The previous expression can be inverted to obtain an explicit solution for the wage at centile c as a function of the minimum wage m ,

$$W_c(m) = P - (P - m) \left(\frac{100\delta}{c\lambda_e + 100\delta} \right)^2$$

This implies

$$\frac{W_{c'}(m)}{W_c(m)} = \frac{P - (P - m) \left(\frac{100\delta}{c'\lambda_e + 100\delta} \right)^2}{P - (P - m) \left(\frac{100\delta}{c\lambda_e + 100\delta} \right)^2}$$

Differentiating with respect to m yields

$$\frac{d [W_{c'}(m)/W_c(m)]}{dm} < 0 \iff c' > c$$

Hence, as in my baseline model, an increase in the minimum wage increases the relative wage of centile c to centile c' for any $W(c') > W(c)$. That is, $W_c(m)$ is log sub-modular in c, m in both models. This result has been previously documented quantitatively by [Engbom and Moser \(2022\)](#).

The average wage. The average wage is given by $w = \int_m^{w_{\max}} wg(w)dw$. Using a change of variables, the average wage can alternatively be expressed using the $W_c(m)$ function (the wage at centile c), noting that the distribution across centiles is uniform (by definition) with density $\tilde{g}(c) = (w_{\max} - m)^{-1}$. This density can be expressed as

$$\tilde{g}(c) = \left(1 - \frac{\delta}{\delta + \lambda_e}\right)^{-2} (P - m)^{-1}$$

Using this formulation, I obtain

$$w = \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)}m + \left(1 - \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)}\right)P$$

The previous expression gives the average wage. As in the baseline model, see equation (4), the average wage can be expressed as a weighted average of the minimum wage m and the value marginal product of labor, P . Unlike the baseline model, the weights themselves depend on the minimum wage.

Key differences across models for my analysis. From the perspective of my analysis, there are two key distinctions between this model and my baseline model. First, studying the transition analytically is straightforward in my baseline model whereas solving even quantitatively for the transition in response to an aggregate shock (changing the minimum wage) in [Burdett and Mortensen \(1998\)](#) is more difficult. Second, in my baseline model—as in the data—a mass of workers earn exactly the minimum wage; in the [Burdett and Mortensen \(1998\)](#) model there are no mass points in the wage distribution. Hence, in response to a marginal increase in the minimum wage, the direct effect in my baseline model is positive whereas it is zero here.²⁷

²⁷Related to both points: [Engbom and Moser \(2022\)](#) studies the impact of changes in minimum wages in the [Burdett and Mortensen \(1998\)](#) model, focusing exclusively on the steady state, and also extends the model to incorporate a mass point at the minimum wage.