The Race Between Education, Technology, and Institutions*

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Abstract

I attempt to synthesize the neoclassical and institutional perspectives on the evolution of wage inequality in the US. I estimate an extended version of the canonical model incorporating the real minimum wage and show that changes in supply (education), demand (technology), and the minimum wage (institutions) each play central roles in shaping the evolution of the U.S. college premium between 1963 and 2017. I generalize the model’s theoretical foundation to include monopsony power, minimum wages, and unemployment; derive the reduced-form estimating equation; and show that its coefficients have simple theoretical interpretations. Finally, I use the model to quantify the impact of changes in the minimum wage on real wages and broader patterns of inequality across more disaggregated groups.

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1 Introduction

Why has U.S. inequality changed over time? The voluminous literature studying changes in the returns to skills and the evolution of earnings inequality in the United States can be separated into two essentially distinct strands. The neoclassical perspective highlights the roles of labor supply and demand, viewing the evolution of inequality as the outcome of “The Race Between Education and Technology,” in the words of Goldin and Katz (2008). An alternative perspective emphasizes changes in labor-market institutions, viewing the evolution of inequality as the outcome of “trends in the minimum wage, and other factors such as declining unionization,” in the words of Card and DiNardo (2002). While most economists view labor supply and demand and labor-market institutions as important drivers of inequality, these literatures have remained largely distinct in part because no simple framework synthesizes the two. This paper attempts to fill that gap.

The canonical model—introduced by Tinbergen (1974), operationalized by Katz and Murphy (1992), and named by Acemoglu and Autor (2011)—provides the central organizing framework for the neoclassical literature. It straightforwardly relates relative wages of more and less educated workers to their relative supply and demand. Empirically, it has proven remarkably successful. I generalize this theoretical model to incorporate labor market institutions—monopsony power and minimum wages—and unemployment while maintaining its simplicity and tractability. I demonstrate that the extended model generates an estimating equation that is almost identical to the canonical model except for the presence of one additional term: the real minimum wage. I also show that the reduced-form coefficients of the extended estimating equation have relatively simple theoretical interpretations.

Incorporating institutional features into the canonical model alleviates certain of its widely-known shortcomings. For example, empirically I document that the out-of-sample fit of the model including the real minimum wage is improved relative to the model without. And quantitatively, the extended model generates declines in real wages of low-education workers without a corresponding reduction in technology in the 1980s and non-monotone changes in wages across the income distribution starting in the early 1990s. Most importantly, I document empirically that changes in supply, demand, and the minimum wage each play central roles in shaping the evolution of the U.S. college premium between 1963 and 2017.

In Section 2, I estimate an extended version of the canonical model incorporating the real minimum wage, using data spanning 1963 to 2017. I measure the real minimum wage using either the federal, Fair Labor Standards Acts (FLSA), minimum wage or a weighted
average of state-specific effective real minimum wages. I define skilled workers as those with four or more years of college education, composition adjust the relative wage of skilled to unskilled workers, and construct their relative supply as the dual to exactly match their relative labor incomes in each year.

Estimated over the full sample period, I find that the elasticity of the skill premium with respect to the real minimum wage is approximately −0.14 in my baseline. The coefficient estimate and its significance are robust to incorporating high-order polynomials of time and instrumenting for relative supply. The baseline point estimate implies that the 27% decline in the real minimum wage between 1979 and 1989 caused a 3.7% increase in the skill premium over this time period, which is about three-tenths of the observed increase. I find that the elasticity of the skill premium with respect to the relative supply of college is approximately −0.53. This implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to its growth rate between 1969 and 1979, raised the skill premium by 6.4% in the latter period, which is almost half of the observed increase. Empirically, supply, demand, and the minimum wage each play important roles in the evolution of the U.S. skill premium in the 1980s and, more generally, across the full sample of years.

Introducing the real minimum wage additionally helps ameliorate a well-known issue: the out-of-sample fit of the canonical model is poor. In particular, the canonical model estimated on data spanning 1963 - 1987 (the set of years used in the seminal work of Katz and Murphy, 1992) predicts substantially more rapid increases in the skill premium than actually occur in the data thereafter. Incorporating the real minimum wage mitigates this issue because its rise in the later period reduces the predicted growth of the skill premium.

In Section 3, I present a generalized version of the canonical model that microfounds the main regression analysis of Section 2 and provides structural interpretations of the reduced-form coefficients. My objective is to maintain the simplicity and tractability of the original framework. In addition to supply and demand, the model incorporates monopsony power, minimum wages, and unemployment. As in the canonical model, the aggregate production function combines the output of low- and high-skill output and exhibits constant returns to scale. Unlike the canonical model, firms have monopsony power in the labor market. Each worker is characterized not only by an arbitrary worker-specific productivity, but also by an arbitrary vector of amenity values across firms. In equilibrium, if a given worker is employed she works for the firm with which she is best matched. In the absence of a minimum wage, this firm would pay her less than the value of her marginal product, with her wage markdown (the ratio of her value marginal prod-
uct to her wage) determined by the ratio of her best and second-best match qualities. The model additionally incorporates a minimum wage. In the presence of a positive minimum wage, the lowest productivity workers may be unemployed: a worker is unemployed if her value marginal product is below the minimum wage. The minimum wage additionally constrains firms’ monopsony power: if employed, a worker’s wage is the maximum of the minimum wage and the wage the firm would have paid in its absence.

Theoretically, I consider the impact of changes in the productivities and supplies of low- and high-skill workers, as in the canonical model, as well changes in the real minimum wage. Without imposing any restrictions on worker productivities, worker-firm-specific match qualities, or the aggregate production function, I express changes in the skill premium, to a first-order approximation, as a function of these shocks. The expression is similar to the canonical model except for the inclusion of one additional term: the real minimum wage.\(^1\) I additionally provide structural interpretations of the reduced-form parameters in this equation. In slightly simplified versions of the model, these interpretations are particularly clear.

For example, if the minimum wage has no effect on unemployment locally, then the theory predicts that the impact of a change in the minimum wage on the full distribution of income (or the distribution within any arbitrary group of workers) is exactly as assumed in the famous empirical decomposition of DiNardo et al. (1996).\(^2\) This further implies that the elasticity of the average wage across an arbitrary group of workers with respect to the real minimum wage is given by the share of that group’s salary income earned by minimum wage workers. Hence, the elasticity of the skill premium with respect to the real minimum wage is simply the share of labor income earned by workers at the minimum wage among high- minus the same share among low-skill workers.

Finally, in Section 4 I use the analytic solution for real wages from the model together with hourly wage data between 1979 and 2018 from the CPS Merged Outgoing Rotation Groups to study the quantitative implications of changes in the minimum wage. First, I compare my baseline estimate of the elasticity of the skill premium with respect to the real minimum wage of almost \(-0.14\) with my model’s prediction for this elasticity under the assumption that the minimum wage has no impact on employment. If I assume that minimum wage earners are those reporting hourly wages no larger than the national minimum wage, then my model predicts a coefficient of only \(-0.02\). In order for the

\(^1\)To a first-order approximation, the change in the log of the skill premium must be a linear function of the underlying shocks. Because the aggregate production function exhibits constant returns to scale, the ratio of high-to-low skill labor is a sufficient statistic for each (as in the canonical model).

\(^2\)This result holds regardless of the number of skills (two in the canonical model) and the aggregate production function (CES in the canonical model) that combines them.
estimate to line up well with the theory, I must assume that workers earning up to approximately 54% above the minimum wage (up to the 23rd percentile of the hourly wage distribution) are minimum wage earners. This is a strong assumption, yet broadly consistent with results on measurement error potentially explaining spillover effects of the minimum wage for workers earning above the minimum wage in Autor et al. (2016). I maintain this assumption in the remainder of my quantification.

Second, I show that changes in minimum wages generate large declines in real wages between 1979 and 1993 for male high school dropouts and small declines for male high school graduates and those with some college. Third, I show that the evolution of the minimum wage can explain about a third of the difference between the 1979 - 1993 and 1996 - 2017 periods in the evolution of inequality across males who not complete high school relative to high school graduates. The first period is characterized by a decline in the real minimum wage and an increase in inequality across these lowest education levels whereas the second period is characterized by an increase in the real minimum wage and a decline in inequality across these lowest education levels. Given that the minimum wage has essentially no effect on higher education groups, this implies that the evolution of the minimum wage helps explain why the earlier period features pervasive increases in inequality whereas the latter period features wage polarization.

Additional Literature. Autor et al. (2008) estimate a related reduced-form regression including the real minimum wage and find that its coefficient is insignificant. As I describe in the Empirical Appendix, the main difference between their specification and mine comes from how wages and supplies are measured. I measure them as standard price and quantity indices. Following Katz and Murphy (1992) and many others, they measure wages and supplies on different worker samples whereas I measure them on common samples; they partially allocate some groups of workers to college and non-college whereas I do not; and my wages and supplies are consistent with observed college and non-college incomes in each year. As a consequence, I find that the coefficient on the real minimum wage is always significant, even when including flexible polynomials of time. According to my empirical results, relative supply growth fluctuations and trend demand growth remain crucial drivers of the skill premium, consistent with Autor et al. (2008), but so too are changes in the real minimum wage.

A growing literature extends the neoclassical approach to explicitly study the allocative effects of the minimum wage. An alternative hypothesis compatible with my model is that increases in the minimum wage increase unemployment, which would further reduce the skill premium. Given small employment response estimates in the literature (see, e.g., Dube et al., 2010) I choose to entertain instead the possibility of measurement error in hourly wages.
tion of labor (and potentially capital) across heterogeneous tasks (e.g., Caselli and Coleman, 2001; Autor et al., 2003; Autor and Dorn, 2013; Buera et al., 2015; Burstein et al., 2019; Acemoglu and Restrepo, 2020; Oberfield and Raval, 2021). This approach has the added benefit of linking changes in inequality with changes in labor (and capital) allocations across sectors, occupations, and/or firms and broadening the nature of technical change; my work largely abstracts from these changes in allocations and maintains a factor-biased perspective on technical change. Complementing this predominantly neoclassical literature is a vast body of work studying the roles of labor-market institutions for shaping inequality. Much of the focus is on minimum wages (e.g., DiNardo et al., 1996; Lee, 1999; Card and DiNardo, 2002; Dube, 2019) and monopsony power (see Manning, 2013 for a summary). My results on the impact of changes in the minimum wage on inequality in the 1980s are broadly consistent with those of DiNardo et al. (1996) and Autor et al. (2016). Relative to much of this literature, I do not leverage spatial variation, given my goal of deviating minimally from the canonical model. My quantitative exercises are broadly related to a growing macro-labor literature studying the implications of minimum wages in quantitative models of the labor market (e.g., Haanwinckel, 2020; Engbom and Moser, 2021; Ahlfeldt et al., 2022; Berger et al., 2022; Hurst et al., 2022). Relative to each of these literatures, I synthesize the neoclassical and institutional approaches and provide a unified analysis of the evolution of inequality, both at an aggregate level (the skill premium) and a disaggregate level (for any arbitrary grouping of workers).

This allows me to reach a number of conclusions, including: (i) the joint evolution of observable supplies and the real minimum wage, together with a linear evolution of skill-biased technology, each play important roles in shaping the evolution of the U.S. skill premium; (ii) the minimum wage plays an important role in additionally explaining changes in real wages (e.g., real wage declines in the 1980s for high school dropouts and high school graduates) and more disaggregated measures of inequality (e.g., wage polarization beginning in the 1990s); (iii) and incorporating the real minimum wage implies that the evolutions of relative demand and relative college supply play not only a central role in shaping the college premium (as previously noted by the vast literature) but also that this relationship is more stable over time than previously understood. Of course, no synthesis can be exhaustive. Among other omissions, I abstract from unions and changes in unionization rates (Farber et al., 2021). Similarly, while my theory has clear implications for the impact of factual and counterfactual changes in the minimum wage across the full distribution of earnings, I have chosen to focus my quantitative exercises on various dimensions of between-group inequality and group-level average real wages.
2 Empirics

I consider regressions of the form

$$\log \left( \frac{w_{ht}}{w_{lt}} \right) = \alpha + \beta_m \log m_t + \beta_L \log \left( \frac{\text{Supply}_{ht}}{\text{Supply}_{lt}} \right) + \gamma_1 t + \left[ \gamma_2 t^2 + \gamma_3 t^3 + \ldots \right] + \iota_t$$  \hspace{1cm} (1)

Here, $\log w_{ht}$ and $\log w_{lt}$ are measures of average log wages of high- and low-education workers; $\log \text{Supply}_{ht}$ and $\log \text{Supply}_{lt}$ are measures of high- and low-education labor supply; $m_t$ is a measure of the real minimum wage; and $t$ is time with $t = 1$ in the first sample year.

Measurement. Here I describe how I measure each variable; see the Empirical Appendix for details. I restrict my sample to the working-age population of those between 16 and 64 years old and define high-education workers as those with 16 or more years of education and low-education workers are those with fewer than 16 years of education.\(^4\) I construct the composition-adjusted college premium by first measuring the average log hourly wage within each of 180 groups (defined by the intersection of 9 age bins, 2 genders, 2 races, and 5 education levels) in year $t$ and then averaging across those groups with college education and those groups without using time-invariant weights. I refer to the relative wage as both the college premium and the skill premium. I measure the supply of college and non-college workers as the dual of these wages, such that the product of composition-adjusted supply and wages equals the observed total income of college and non-college workers in each year. I measure these variables using the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017.

\(^4\)Results are broadly robust defining high-education workers as those with some college and above and low-education workers as those with high school degrees and below.
Figure 1: Relative Wages, Relative Supplies, and the Minimum Wage

Notes: Panels (a) and (b) display the composition-adjusted college premium and relative supply of college. Panel (c) displays the real federal (FLSA) minimum wage and the real national minimum wage series, which averages effective minimum wages across states; both series are deflated by the GDP deflator. All series are normalized to zero in 1963.

I measure the national minimum wage in year $t$ two ways. In one approach, I use the federal (FLSA) minimum wage. In my baseline, I use the average effective minimum wage across states, using time-invariant weights. I deflate each series using the GDP deflator. I refer to my baseline measure as the real national minimum wage and to my alternative measure as the real federal (or FLSA) minimum wage.

Panels (a) and (b) of Figure 1 display the college premium and relative supply of college workers, both normalized to zero in 1963. The particularly steep rise in the college premium starting in the early 1980s coincides with a decline in the growth rate of the relative supply of college workers, a fact first emphasized in Katz and Murphy (1992). Panel (c) of Figure 1 displays my two real minimum wage series, each normalized to zero in 1963. The real national and FLSA minimum wage series move in lockstep until the late 1980s and diverge thereafter as more states set minimum wages above the federal level. Both real minimum wage series decline dramatically in the 1980s, as fixed nominal minimum wages are eroded by inflation. Finally, there is substantial time variation in both real minimum wage series, which remain even after residualizing on a high-dimensional polynomial of time.
Results. Before providing estimation results, I display the variation in the data that identifies the parameters of interest. Panel (a) of Figure 2 displays the college premium and relative supply of college, each residualized of the real minimum wage and a linear time trend. Panel (b) displays the composition-adjusted college premium and real national minimum wage, each residualized of the relative supply of college and a linear time trend. The variation in Panel (a) identifies $\beta_L$ and the variation in Panel (b) identifies $\beta_m$ when estimating regression (1) using OLS. There is a striking negative relationship in each panel. These qualitative patterns are robust to residualizing on high-dimensional polynomials of time.

Column (a) of Table 1 displays results of estimating regression (1) using OLS including a linear time trend and omitting the real national minimum wage on the sample 1963 - 1987. This is the specification and sample years included in the seminal work of Katz and Murphy (1992). Column (b) replicates this analysis, but using the full sample of 1963 - 2017. Consistent with past work, estimates are unstable across samples. The growth rate of skill-biased technical change $\gamma$ falls from 2.36% per year to 1.81%; and the coefficient on relative supply $\beta_L$ rises from $-0.596$ to $-0.441$ when changing the sample from the Katz and Murphy years to the full sample; in spite of slight differences in data cleaning, these results are very similar to Acemoglu and Autor (2011). The model estimated on the 1963 - 1987 sample systematically deviates from the data thereafter, predicting a sharper rise in the college premium than actually occurs.

Introducing the real minimum wage goes some way towards fixing this well-known
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<td>(0.082)</td>
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**Table 1: Regression Models for the College/High School Log Wage Gap**

*Notes:* Results of estimating (1) using OLS. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real national mw.” and “Real federal mw.” are the logs of the real national (averaging effective minimum wages across states) and real federal (FLSA) minimum wages. The sample is 1963-1987 in columns (a) and (b) and 1963-2017 elsewhere. “Time Polynom.” refers to the degree of the polynomial of time; the coefficient on the linear trend on “Time” is omitted from the table whenever this polynomial is of degree 2 or more.
issue. Columns (c) and (d) replicate Columns (a) and (b)—including a linear time trend and estimating on the 1963 - 1987 sample in Column (c) and the full sample in Column (d)—but additionally include the value of the real national minimum wage. Including the national minimum wage leads to stable parameter estimates across samples, a result that is robust to varying the sample years. The growth rate of skill-biased technical change \( \gamma \) and the coefficient on relative supply \( \beta_L \) are 2.22\% and \(-0.594\) when estimated on the 1963 - 1987 sample and are 2.11\% and \(-0.556\) when estimated on the full sample.

Columns (e) - (g) include higher-dimensional polynomials of time. Column (f), which includes a cubic of time, is my baseline specification simply because it yields the most conservative elasticity of the skill premium with respect to the minimum wage. Column (g) replicates my baseline specification, but replacing the real national minimum wage (which averages effective minimum wages across states) with the federal (FLSA) minimum wage; results are largely robust.

The estimates in Table 1 highlight the importance of supply (education), demand (technology), and the minimum wage (institutions) in shaping the evolution of the U.S. skill premium. In the baseline specification in Column (f), the elasticity of the skill premium with respect to relative supply is \(-0.531\) and with respect to the real national minimum wage is \(-0.136\). The second elasticity implies that the 27\% decline in the real national minimum wage between 1979 and 1989 caused a 3.7\% increase in the skill premium over this time period, which is almost three tenths of the observed 13.4\% increase. The first elasticity implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to the growth rate between 1969 and 1979, raised the skill premium by approximately 6.4\% in the latter period, which is almost one half of the observed increase. All three forces—supply, demand, and the minimum wage—play important roles not only in this decade, but also throughout the sample.

Appendix Table A.1 extends this analysis including progressively higher-dimensional polynomials of time, up to an eighth degree; results on the impact of the minimum wage are largely robust. So are results from a double-lasso analysis. Appendix Table A.2 replicates Appendix Table A.1, but defining high-education workers as those with some college and up and defining low education workers as those with high school degrees and below; results on the impact of the minimum wage are largely robust.

Finally, my measure of relative supply—as well as the standard measure in the literature—depends on (efficiency unit) hours worked. Whereas it may be reasonable to assume that population education levels do not respond to deviations from a time trend (whether linear, quadratic, cubic...) in relative demand for high- and low-education workers, the same need not be true for hours worked. Since these deviations in relative de-
mand enter the residual in equation (1) both in the canonical model and in my extended model in Section 3, I also instrument for relative supply using the relative population of the college educated. Appendix Table A.3 displays results of estimating the reduced-form and two-stage least squares specifications. The first stage is strong and instrumenting leaves the results broadly unchanged.\(^5\)

In summary, introducing the real minimum wage into the canonical model’s estimating equation demonstrates that supply, demand, and the minimum wage each play central roles in shaping the evolution of the U.S. skill premium.

3 Extending the Canonical Model

My objectives are threefold: (i) to provide a simple extension of the theoretical framework referred to as the canonical model that incorporates monopsony power, minimum wages, and unemployment; (ii) to derive equation (1) estimated above in a general formulation; and (iii) to study simple examples in which the resulting reduced-form coefficients have particularly clear and intuitive structural interpretations.

Setup. The economy features two labor skills indexed by \(s\), high \(h\) and low \(\ell\), and two types of firm, one hiring only high- and the other hiring only low-skilled labor.\(^6\) There is a continuum of skill \(s\) labor, indexed by \(\omega \in \Omega_s\), with measure \(L_s \equiv |\Omega_s|\), and there are \(N_s \geq 2\) skill \(s\) firms, indexed by \(n \in N_s\). Each skill \(s\) worker is characterized by a vector \(\epsilon_\omega\) with \(N_s\) elements \(\epsilon_{\omega n} > 0\), where \(1/\epsilon_{\omega n}\) denotes a worker-firm-specific amenity or compensating differential. Each worker is also characterized by a scalar \(\phi_{\omega} > 0\) number of efficiency units. I impose no restrictions on the joint distribution of \(\phi_{\omega}\) and the vector \(\epsilon_\omega\).

High- and low-skill workers are imperfect substitutes in the production of aggregate output, \(Y\), which is produced by combining skilled and unskilled output according to a

\(^5\)Conditional on including a cubic polynomial of time, the first-stage suggests no effect of the real national minimum wage on relative supply. This result is relevant to the structural interpretation of the reduced-form coefficients reported above—according to the model of Section 3—and broadly consistent with the empirical literature; see, e.g., Dube et al. (2010).

\(^6\)The framework can easily be extended to incorporate many skill groups, as in extensions of the canonical model such as Card and Lemieux (2001), as shown in the Theoretical Appendix. If there is no (un)employment effects of the minimum wage, then the impact of the minimum wage across arbitrary groups of workers is invariant to the number of assumed labor skills and the functional form of the aggregate production (two issues that are crucially important in the analysis of the impact of changes in labor supply, as discussed in Card, 2012).
constant returns to scale production function,

\[ Y = Y(Y_\ell, Y_h) \]

Skill \( s \) output, \( Y_s \), is the sum of output across firms, \( Y_s = \sum_{n \in N_s} Y_{sn} \), where \( Y_{sn} \) is the output of firm \( n \). Firm \( n \)'s output is \( Y_{sn} = A_s \sum_{\omega \in \Omega_{sn}} \phi_\omega \) where \( A_s > 0 \) is skill-specific productivity common across all firms and \( \Omega_{sn} \) is the set of skill \( s \) workers employed by firm \( n \). The price of the final good is the numeraire and \( P_s \) is the endogenous real price of each unit of \( Y_s \).

Given a vector of real wages offered to worker \( \omega \), one \( w_{\omega n} \) for each firm \( n \), her utility if working for firm \( n \) is given by \( w_{\omega n}/\epsilon_{\omega n} \). The exogenous real minimum wage is \( m \geq 0 \). Firm \( n \) can employ worker \( \omega \) only if \( w_{\omega n} \geq m \).

All skill \( s \) firms observe \( \epsilon_\omega \) and \( \phi_\omega \) for all workers and simultaneously make take-it-or-leave-it wage offers to each worker. Each worker then chooses to work for the firm that maximizes her utility, conditional on that wage being no less than the minimum wage. If no wage offer is as large as the minimum wage, \( \max_n \{w_{\omega n}\} < m \), she is unemployed.

The model reduces to the canonical model under three restrictions: (i) if \( \epsilon_{\omega n} = \epsilon_{\omega n'} \) for all \( \omega \) and each \( n, n' \); (ii) the minimum wage is set to zero, \( m = 0 \); and (iii) the aggregate production function takes the constant elasticity of substitution form.

**Equilibrium.** In general equilibrium, skill prices are given by \( P_s = \partial Y / \partial Y_s \). Equilibrium characterization then boils down to determining the partial equilibrium allocation of workers across firms and worker wages for a given skill price \( P_s \). I turn to this now.

Define

\[ n_1(\omega) \equiv \arg \min_{n \in N_s} \{\epsilon_{\omega n}\} \quad \text{and} \quad n_2(\omega) \equiv \arg \min_{n \in N_s/n_1(\omega)} \{\epsilon_{\omega n}\} \]

as the firms that are the best and second-best matches for worker \( \omega \) and define the scalar

\[ \epsilon_\omega \equiv \epsilon_{n_1(\omega)\omega}/\epsilon_{n_2(\omega)\omega} \leq 1 \]

Characterizing equilibrium requires only a joint density over the two scalars, \( \epsilon \leq 1 \) and \( \phi \geq 0 \), rather than over all values of the vector \( \epsilon \) and \( \phi \). Let \( g_s(\epsilon, \phi) \) be the joint density of \( \epsilon \) and \( \phi \) for skill \( s \). In what follows, I impose the technical restriction that the marginal distribution of \( \phi \) has no mass points.

A worker \( \omega \in \Omega_s \) is employed if her value marginal product \( P_s A_s \phi_\omega \) is above the minimum wage \( m \) and unemployed if it is below. Under the assumption that there are no
mass points in the marginal distribution of \( \phi \), skill \( s \) output is

\[
Y_s = L_s A_s \int_0^1 \left( \int_{m_s}^{\infty} \phi g_s(\epsilon, \phi) d\phi \right) d\epsilon
\]

(2)

where I have defined \( m_s \equiv m / (P_s A_s) \), so that worker \( \omega \) is employed if \( \phi_\omega > m_s \).

In any SPNE, if \( \omega \in \Omega_s \) is employed, then she works for the firm \( n_1(\omega) \) with which she is best matched. And in any SPNE in weakly undominated strategies, firm \( n_1(\omega) \) either pays her the lowest wage at which \( \omega \) weakly prefers \( n_1(\omega) \) to \( n_2(\omega) \) when \( n_2(\omega) \) is offering a wage equal to the worker’s value marginal product or, if this is below the minimum wage, it pays the minimum wage.\(^7\) An employed worker \( \omega \) earns a wage \( w_\omega \) given by

\[
w_\omega = \max \{ A_s P_s \phi_s \epsilon_\omega, m \}
\]

(3)

**Summary of cross sectional results.** In the absence of a minimum wage, \( m = 0 \), all workers are employed and each worker’s wage is determined by her value marginal product and the quality of her second best match relative to her best: \( w_\omega = A_s P_s \phi_s \epsilon_\omega \). In this case, each worker’s wage markdown, i.e. her value marginal product divided by her wage, is given by \( 1/\epsilon_\omega \geq 1 \).

The equilibrium differs in two respects in the presence of a positive minimum wage. First, some workers may be unemployed. If the minimum wage creates unemployment, it does so only for the least productive workers, whose value marginal product is below the minimum wage, \( \phi_\omega < m_s \). Second, the minimum wage may also raise wages for some workers, with wages now being the maximum of \( A_s P_s \phi_s \epsilon_\omega \) and \( m \), as shown in equation (3). The minimum wage raises a worker’s wage if it constrains the wage markdown she faces, which is now the minimum of \( 1/\epsilon_\omega \) and \( \phi_s / m_s \).

Finally, a worker earns the minimum wage if \( \phi_\omega \in [m_s, m_s / \epsilon_\omega] \). Such a worker either has a low productivity (although higher than any unemployed worker) or a relatively high amenity value in her best- relative to her second-best matched firm.

----

\(^7\)In the SPNE in weakly undominated strategies, firm \( n_1(\omega) \) offers the wage \( w_{\omega f} = w_\omega \) and every other firm \( n \in N_f \setminus n_1(\omega) \) offers a wage equal to the worker’s value marginal product. If firm \( n_1(\omega) \) offers a wage below \( A_f P_f \phi_\omega \epsilon_\omega \) and above \( m \), then firm \( n_2(\omega) \) can profitably hire \( \omega \) away from firm \( n_1(\omega) \) by offering a slightly higher wage. If firm \( n_1(\omega) \) offers a wage above \( A_f P_f \phi_\omega \epsilon_\omega \) and above \( m \), then it can increase its profit by slightly reducing its wage offer, conditional on other firms not playing weakly dominated strategies. Finally, there is no equilibrium in which firms follow their strategies and workers mix with positive probability over choosing any firm but \( n_1(\omega) \), since in this case the best matched firm would have an incentive to slightly increase its wage.
Changes in average wages. I am interested in the average wage paid to employed workers with skill $s$, which I denote by $\bar{w}_s$. Given the unemployment cutoff and the determination of worker wages, the average wage can be expressed as

$$\bar{w}_s = \mu_s m + (1 - \mu_s) P_s A_s \mathbb{E} [\phi e | \phi e > m_s]$$ (4)

where $P_s A_s \mathbb{E} [\phi e | \phi e > m_s]$ is the average wage of those earning above the minimum wage and where

$$\mu_s \equiv \Pr [\phi e \leq m_s | \phi \geq m_s]$$ (5)

is the share of skill $s$ workers with wage equated to the real minimum wage ($\phi e \leq m_s$) conditional on being employed ($\phi \geq m_s$).

I now introduce time $t$ subscripts, so that I can make an assumption on the evolution of factor-biased productivities equivalent to that in the canonical model. Suppose that $\log A_{st}$ follows

$$\log A_{st} = \alpha_s + \gamma_s t + \iota_{st}$$

where $\gamma_s$ denotes the rate of the skill $s$ specific time trend of technical change and $\iota_{st}$ denotes zero mean i.i.d. deviations from this trend. Under this assumption, I obtain the following result about how the skill premium responds to changes in the minimum wage $m_t$, the supply of more skilled workers $L_{ht}$, the supply of less skilled workers $L_{lt}$, the productivity of more skilled workers $A_{ht}$, and the productivity of less skilled workers $A_{lt}$, to a first-order approximation. In this and all results, I hold fixed the distribution $g$.

**Proposition 1.** The log derivative of the skill premium can be expressed as

$$d \log \left( \frac{\bar{w}_{ht}}{\bar{w}_{lt}} \right) = \beta_m d \log m_t + \beta_L d \log \left( \frac{L_{ht}}{L_{lt}} \right) + \beta_A dt + \iota_t$$ (6)

The proof of Proposition 1 and the solution for the $\beta$ parameters are provided in the Theoretical Appendix. By the definition of a first-order approximation, changes in average wages can be expressed as a linear function of the underlying shocks. The only additional result in Proposition 1, besides the solutions for the $\beta$ coefficients provided in the Theoretical Appendix, is that changes in average wages depend only on the ratio of the supply of skilled to unskilled workers; this follows because the aggregate production function exhibits constant returns to scale.

The $\beta_m$ and $\beta_L$ coefficients in equation (6) are particularly simple in two special examples, to which I now turn. I first consider an example in which the skill premium in the general model simplifies to that in the canonical model.
Example 1 (Canonical Model). Suppose that $m_t = 0$ for all $t$ and that the aggregate production function takes a constant elasticity form, with elasticity of substitution $\eta$.

Example 1 is a generalization of the canonical model featuring monopsony power. Yet, with the minimum wage set to zero and no changes in monopsony power over time, the implied skill premium at any moment in time can be expressed as in the canonical model. In particular, I obtain the following corollary of Proposition 1.

**Corollary 1.** In Example 1, the skill premium can be expressed as

$$\log \left( \frac{w_{ht}}{w_{\ell t}} \right) = \alpha - \frac{1}{\eta} \log \left( \frac{L_{ht}}{L_{\ell t}} \right) + \frac{\eta - 1}{\eta} \left( \alpha_h - \alpha_\ell \right) t + \iota_t \quad (7)$$

I next turn to the more interesting example, in which there is a positive minimum wage, yet still no unemployment.

Example 2 (Minimum Wage Without Unemployment). Suppose that $\inf \phi_\omega > m_{st}$ for all $\omega \in \Omega_s$ and for $s = h, \ell$.

Example 2 is a generalization of the canonical model featuring monopsony power, a minimum wage, and a general aggregate production function. The restriction imposed in Example 2 implies that marginal changes in the minimum wage have no impact on unemployment. Now, the skill premium depends on the minimum wage, as shown in the following corollary of Proposition 1.

**Corollary 2.** In Example 2, the log derivative of the skill premium can be expressed as in equation (6) where

$$\beta_m = \sigma_h - \sigma_\ell \quad (8)$$

$$\beta_L = -\frac{1}{\eta} \left( 1 - \sigma_h \frac{P_\ell Y_\ell}{Y} - \sigma_\ell \frac{P_h Y_h}{Y} \right) \quad (9)$$

where $\eta$ is the local elasticity of substitution in the aggregate production function given full employment and $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill $s$ income earned by workers at the minimum wage.

Consider first a change in the minimum wage $m$. The direct effect (i.e., holding fixed the set of workers who are employed and the set earning the minimum wage) of a small increase in the minimum wage $m$ is to raise the wage of minimum wage earners proportionately. The partial elasticity of $w_s$ with respect to $m$ is then $\partial \log \bar{w}_s / \partial \log m = \sigma_s$, where $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill $s$ income earned by workers at the minimum wage. The minimum wage also affects the set of employed workers earning the minimum wage; this
effect, however, is generically (i.e. in Example 2 and more generally) of second order. Finally, a change in the minimum wage also affects average wages via changes in the set of workers who are employed; however, this channel is inactive by assumption in Example 2. Hence, the coefficient $\beta_m$ in Corollary 2 reflects only the direct effect of changes in the minimum wage. As long as the share of income earned by workers at the minimum wage is lower for higher than lower skill workers, $\sigma_h < \sigma_\ell$, an increase in the minimum wage will reduce the skill premium.

Consider next a change in the relative supply of more- to less-skilled labor. This matters through its impact on the relative price $P_h/P_\ell$, where the elasticity of $P_h/P_\ell$ with respect to $L_h/L_\ell$ is given by $-1/\eta$, as in the canonical model. Unlike the canonical model, however, not all worker wages respond to changes in the value marginal product of labor. Instead, only the incomes of workers earning above the minimum wage respond. Hence, the elasticity of the average wage of skill $s$ with respect to $P_s$ is now $1 - \sigma_s$, which—together with the solutions for changes in $P_\ell$ and $P_h$—yields $\beta_L$ in Corollary 2.

Finally, in general—when changes in the minimum wage potentially affect unemployment—the $\beta$ coefficients in equation (6) depend in addition on the share of workers who are marginally employed (such that a small increase in the minimum wage causes them to move into unemployment), the share of employed workers, and the share of workers earning the minimum wage; see the Theoretical Appendix.

4 Some Simple Quantitative Results

Finally, I use analytic solutions from the model to study the quantitative implications of changes in the minimum wage. In this section I use the simplified version of the model of Example 2, in which changes in the minimum wage do not affect unemployment. In this case, the elasticity of the average wage of any arbitrary group of workers with respect to the minimum wage is simply the share of that group’s income earned by workers earning no higher than the minimum wage; as described in footnote 6 and proven in the Theoretical Appendix, in this case theoretical results on the impact of the minimum wage on real wages are robust to incorporating arbitrarily many skills and any aggregate production function. I measure both average wages and the share of earnings by workers earning no higher than the minimum wage using data between 1979 and 2018 from the CPS Merged Outgoing Rotation Groups (MORG).

I clean the MORG CPS data following closely the approach in Lemieux (2006). See the Empirical Appendix for details.
all sample years rather than picking a particular year around which to take the first-order approximation.

First, I study the mapping between my model’s predictions and my baseline estimates in Column (f) of Table 1, where the elasticity of the skill premium with respect to the real minimum wage is $-0.136$. According to Corollary 2, if changes in the minimum wage do not affect unemployment then the elasticity of the skill premium with respect to the minimum wage equals the share of labor income earned by minimum wage earners (or below) among the college-educated workforce minus the same share among the non-college-educated workforce. Averaging these shares across all years yields a predicted coefficient of only $-0.02$. The model predicts that the skill premium is substantially less responsive to the minimum wage than in the data. There are two simple “fixes” to this discrepancy. The first is to allow for minimum wages to generate unemployment. This case is already considered in the model and the implications are derived explicitly in the Theoretical Appendix. However, matching the observed coefficient of $-0.136$ would require counterfactually large unemployment effects. Instead, I allow for the possibility that workers earning above the minimum wage in the data are actually minimum-wage earners in practice. This is, of course, an imperfect theoretical fix to the problem that the model predicts no wage spillover effects in response to changes in the minimum wage; nevertheless, it is broadly consistent with evidence in Autor et al. (2016) on the potential importance of measurement error in generating empirical wage spillover effects. In order for the empirical estimate of $-0.136$ to line up well with the theory, I assume that workers earning up to approximately 54% above the minimum wage are minimum wage earners (averaging across years, this implies that workers are minimum wage earners up to the 23rd percentile of the hourly wage distribution). I make this assumption in what follows.

The first question I use the model to answer is: To what extent can the declining real minimum wage explain falling real wages among low-education male workers between 1979 and 1993? Since the share of income earned by minimum wage workers (averaged across years) is 27.8%, 11.3%, and 8.1% among male high school dropouts, high school graduates, and those with some college, the 18.3% decline in the real minimum wage over this time period causes a decline in real wages of 5.1%, 2.1%, and 1.5%, respectively.\(^9\)

The second question I answer using the model is: To what extent do changes in the evolution of the minimum wage between the 1979 - 1993 and 1996 - 2017 periods explain differences in these periods in the evolution of inequality across male high school

\(^9\)If I assume instead that workers earning up to approximately 27% above the minimum wage are minimum wage earners, then the shares fall to 14.8%, 5.0%, and 3.8%; the resulting real wage declines are about half as large.
dropouts relative to high school graduates? To answer this question, I use my model to solve for the counterfactual changes in these measures of inequality in each period if the real minimum wage had remained unchanged. The 1979 - 1993 period is characterized by a decline in the real minimum wage of 18.3% and an increase in inequality, with high school dropout wages falling by 11.4% relative to high school graduate wages. Both the real minimum wage and inequality at the bottom of the education distribution move in opposite directions in the 1996 - 2017 period: the real minimum wage rises by 26.2% and male high school dropout wages rise by 5.9% relative to high school graduates. Since the share of income earned by minimum wage workers (averaged across years) is 27.8% and 11.3% in these two groups, I find that high school dropout relative to high school graduate male wages would have fallen by 8.4% instead of 11.4% in the first period and would have risen by 1.5% instead of 5.9% in the latter period. More than two-fifths \((2/5 \approx 1 - (8.4 + 1.5)/(11.4 + 5.9))\) of the differential evolution of inequality across these groups in the earlier and latter periods is then explained by the differential pattern of real minimum wage changes across the periods. This suggests that the pattern of pervasive increases in inequality in the earlier period and wage polarization in the latter period are partially explained by differences in observed changes in the minimum wage (which in my model have little to no effect on higher education groups and, therefore, leave upper tail inequality unaffected).

In sum, under the strong assumption that workers earning up to 54% above the minimum wage (up to the 23rd percentile of the hourly wage distribution) are actually minimum wage earners, observed declines in the real minimum wage have had substantial negative effects on real wages and changes in the evolution of the real minimum wage have had substantial effects on divergent patterns of changes in inequality at the bottom of the education distribution over time.

5 Conclusions

While most economists view each of labor supply, labor demand, and labor-market institutions as an important driver of inequality, these literatures have remained distinct in part because no simple framework synthesizes the two. In this paper I have attempted such a synthesis.

Empirically, I have shown that changes in supply, demand, and the real minimum wage each play important roles in generating the observed evolution of the college premium in the U.S. between 1963 and 2017. I have also documented that incorporating the

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\(^{10}\)I drop 1994 and 1995 because of missing imputation flags in all of 1994 and most of 1995.
real minimum wage into the empirical implementation of the canonical model improves its out-of-sample fit.

Theoretically, I have provided a generalization of the canonical model featuring monopsony power, minimum wages, and unemployment that maintains the simplicity and tractability of the purely neoclassical model. The model generates a simple estimating equation similar to the canonical model, except it additionally incorporates the real minimum wage. Because the reduced-form coefficients of the extended estimating equation have relatively simple theoretical interpretations, it is straightforward to quantify the impact of changes in the minimum wage for real wages and inequality across highly disaggregated groups of workers.

Models are simplifications of reality. My goal in constructing this model was to expand the insights of the canonical model to shed light on the impact of institutional changes, placing institutional changes on the same footing as changes in supply and demand, while maintaining the overall structure of the model’s empirical implementation. In many respects this endeavor was successful. Empirically, I shed light on past trends (providing insights on the relative importance of minimum wages, labor supply, and labor demand for the historical evolution of the U.S. college premium) and provide insight into the impacts of counterfactual changes in the minimum wage (showing that the model’s out-of-sample fit is good). The model is also able to provide answers to quantitative questions, shedding light on declining real wages and patterns of wage polarization. However, the model is less successful in one important respect: the empirical elasticity of the U.S. college premium with respect to the real minimum wage is higher than predicted by the model (in the absence of counterfactually large unemployment effects) because the model implies no wage spillover effects of the minimum wage. Future work should investigate whether alternative models featuring such spillovers—such as search and matching—can jointly match the elasticity of the skill premium with respect to the real minimum wage identified in this paper and the wage spillover effects identified elsewhere, without introducing strong assumptions on measurement error.
References


A Empirical Appendix

Basic Processing of the March CPS Data I use the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017, for workers age 16 to 64 during the earnings year. I drop respondents with missing schooling, missing or negative earnings, with missing weeks worked. I additionally drop those who are self employed or engage in unpaid family work and anyone with allocated earnings. Following Autor et al. (2008) I multiply top-coded earnings by 1.5.

In composition adjusting, I bin workers into one of 180 groups, denoted by $g$, defined by the intersection of 9 age bins, 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training). The lowest three educations—high school dropouts, those with a high school degree, and those with some college—are allocated to non-college; the highest two educations—college graduates and graduate training—are allocated to college.

Constructing Wages and Supplies In each year and for each of the 180 groups I construct total hours worked and total wage and salary income (using sample weights) and, from this, the average wage of each group-year pair, $w_{gt}$ for group $g$. Within each year I average across the log wages of all groups with at least a college degree (denoting the set of these groups by $G_h$) and, separately, across all groups without (denoting the set of these groups by $G_τ$) using time-invariant weights. For instance, for college-educated workers, I have

$$\log w_{ht} = \sum_{g \in G_h} \omega_g \log w_{gt}$$

where $\omega_g$ is the time-invariant weight on group $g$. These weights are constructed using the average across years of the share of hours worked of each group $g$ within the set of college groups $G_h$ and, separately, within the set of non-college groups $G_τ$. The resulting averages are the composition-adjusted wages $\log w_{ht}$ and $\log w_{τt}$ used in the analysis. I construct composition-adjusted supplies of college and non-college workers as the dual of these wages. In particular, I set $\log Supply_{ht} = \log Inc_{ht} - \log w_{ht}$ where $Inc_{ht}$ is the total income of college-educated workers in raw data in year $t$; and I do the same for $\log Supply_{τt}$. Finally, my instrument used in the analysis displayed in Table A.3 is simply the logarithm of the number of college-educated relative to non-college educated in my

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11Up to and including 1991, I use the highest grade of school completed; I define college complete as having finished the fourth year of college and graduate training as having more than four years of college. Starting in 1992 I use degree completion, assigning associate’s degrees to some college.
### Table A.1: Regression Models for the College/High School Log Wage Gap

<table>
<thead>
<tr>
<th></th>
<th>1963-2017</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.556</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
</tr>
<tr>
<td>Real national mw.</td>
<td>-0.195</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Time Polynom.</td>
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</tr>
<tr>
<td>Observations</td>
<td>55</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.968</td>
</tr>
</tbody>
</table>

Notes: The estimating equation is (1) and the sample is 1963-2017 in all columns. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real min. wage” is the log of the real national minimum wage. “Time Polynom.” refers to the degree of the polynomial of time.

My approach to constructing wages and supplies is the standard approach to constructing consistent price and quantity indices. There are three differences in my approach from typical approaches.

First, I measure the wage and the supply of college (and non-college) labor on a common set of groups. It is standard practice in the literature to measure the non-college wage using only the subset of non-college groups with exactly a high school degree (12 years of completed schooling) and to measure the non-college supply using all workers who do not have a high school diploma, all workers with exactly a high school degree, and half the hours of workers with some college. Similarly, it is standard practice to measure the college wage using only those with a college degree or above, but to measure their supply including half the hours of those with some college. Second, my measure of supply is the dual of the wage, so that the product of the supply of college hours and the hourly wage of college labor exactly equals college income in the data in each year; and the same is true of non-college hours and wages. Third, I do not partially allocate any groups across college and non-college. Instead, I allocate each group either to college or to non-college, but not to both.

**Constructing the real national minimum wage** In some analyses, I use the federal FLSA minimum wage. In my baseline, I use a weighted average across states of effective minimum wages. I construct this as follows.
In 1974-2012, I use data from Autor et al. (2016) to measure state-year minimum wages. For each month and state, I define the relevant effective minimum wage as the maximum of the legislated state and federal (FLSA) minimum wages. For each state and year I then average effective minimum wages across months to construct the state-year effective minimum wage. For the remaining years, I measure annual federal minimum wages using data from the Department of Labor (averaging across months) and I measure annual legislated state minimum wages using data from FRED. For each state-year, I take the maximum of these annual numbers to obtain the effective minimum wage. After constructing this state-year series of annual effective minimum wages, I average across states in each year using fixed weights. I deflate this national, annual, effective minimum wage series using the GDP deflator and refer to this as the national real minimum wage. When using federal minimum wages instead, I also deflate using the GDP deflator.

**Robustness of Table 1** Table A.1 displays results of estimating equation (1) including progressively higher polynomials of time. The minimum wage is negative and significant in all specifications, up to and including an eighth-degree polynomial of time. Results from a double-lasso analysis are consistent with my baseline result: both supply and the minimum wage are negative and significant.

Table A.2 displays robustness in which I redefine high and low education workers, allocating the top three education levels to high education (some college and above) and the bottom two education levels to low education (high school graduate and below). The
The coefficient on the real minimum wage is negative and significant in all specifications up to and including an eighth-degree polynomial of time, with one exception: it is insignificant in column (c), which includes a third-degree polynomial of time.\footnote{The same pattern applies when using Autor et al.’s (2008) data. The coefficient on the real minimum wage is negative and significant and similar to my baseline estimate across all specifications up to and including a sextic polynomial of time (given the shorter time period, neither the real minimum wage nor relative supply is significantly different from zero with a seventh degree polynomial of time) except for the case of a third-degree polynomial of time, in which case the coefficient on the real minimum wage is insignificant.} As in the baseline case, results from a double-lasso analysis are consistent with my baseline result.

**Basic Processing of the Merged Outgoing Rotation Groups CPS Data** I use statistics from the Current Population Survey Merged Outgoing Rotation Groups (MORG CPS) my quantification, but not my estimation. I use the MORG CPS from 1979 to 2018, which reflects current wages. Because of missing imputation flags—the CPS did not flag workers with missing wages—in all of 1994 and up to September in 1995, I do not include these two years. As above, I restrict attention to worker ages 16 to 64.

In the processing the files, I broadly follow the approaches of Lemieux (2006) and Autor et al. (2008), using hourly wages for workers paid by the hour and using usual weekly earnings divided by hours worked last week for non-hourly workers. I multiply top-coded constructed hourly wages by 1.5. I drop respondents with allocated earnings flags. I identify and drop non-flagged allocated observations between 1989 and 1993 using the unedited earnings values.

In constructing the distribution of wages—which I only use in reporting the share of the distribution earning no more than the minimum wage (or no more than some multiple of the minimum wage)—I use the product of earnings weights and hours worked. I am therefore constructing the distribution of hourly wages across hours worked rather than across workers.

### B Theoretical Appendix

#### B.1 Partial Equilibrium

To understand the impact of changes in the economic environment on the average skill $s$ wage, I begin in partial equilibrium, taking as given changes in skill prices $P_s$. In this case, I allow for arbitrarily many skill groups and only restrict attention to two in general equilibrium.
### Table A.3: Regression Models for the College/High School Log Wage Gap

<table>
<thead>
<tr>
<th></th>
<th>Reduced form</th>
<th>Two-stage least squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>Relative supply</td>
<td>-0.583</td>
<td>-0.993</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(0.133)</td>
</tr>
<tr>
<td>Real national mw.</td>
<td>-0.316</td>
<td>-0.269</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Time</td>
<td>0.023</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.035</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Time Polynom.</td>
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<td>2</td>
</tr>
<tr>
<td>Observations</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.941</td>
<td>0.953</td>
</tr>
<tr>
<td>First-stage F</td>
<td>134</td>
<td>89</td>
</tr>
</tbody>
</table>

Notes: The estimating equation is (1). Columns (a)-(c) report reduced-form results and Columns (d)-(f) report 2SLS results. The instrument is the log of the working-age population with college degrees or above relative to without. “First-stage F” reports the Cragg-Donald Wald F statistic. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real min. wage” is the log of the real national minimum wage. “Time Polynom.” refers to the degree of the polynomial of time; the coefficient on the linear trend on “Time” is omitted from the table whenever this polynomial is of degree 2 or more.
Consider first a change in the minimum wage $m$. The direct effect (i.e., holding fixed the set of workers who are employed and the set earning the minimum wage) of a small increase in the minimum wage $m$ is to raise the wage of minimum wage earners proportionately. The partial elasticity of $\bar{w}_s$ with respect to $m$ is then $\partial \log \bar{w}_s / \partial \log m = \sigma_s$, where $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill $s$ income earned by workers at the minimum wage.

In partial equilibrium there are also two indirect effects of a small increase in the minimum wage. First, an increase the minimum wage may induce some workers to move from earning above the minimum wage to earning the minimum wage. However, this effect is of second order in the absence of mass points in the join distribution of $\epsilon$ and $\phi$, since these workers were already earning wages arbitrarily close to the minimum wage. Second, an increase in the minimum wage may also induce some minimum wage workers to move from employment to unemployment, thereby raising the average wage. The change in the average wage $\bar{w}_s$ induced by an increase in the minimum wage $m$ though its effect on unemployment depends on the impact on the share of employed workers earning the minimum wage $\mu_s$—this is the probability that a worker is on the margin of employment and unemployment, which I denote by

$$\chi_s \equiv \int_0^1 g(\epsilon, \phi = m_s) d\epsilon$$

divided by the probability a worker is employed, which I denote by

$$e_s \equiv \int_{\mu_s}^{\infty} \int_0^1 g(\epsilon, \phi) d\epsilon d\phi$$

—and the difference between the minimum wage and the average wage of workers earning above the minimum wage—which can be expressed as the percentage difference between the share of employment $\mu_s$ and the share of earnings $\sigma_s \leq \mu_s$ of workers at the minimum wage, $(\mu_s - \sigma_s) / \mu_s$.

The intuition for the impact on $\bar{w}_s$ of a small increase in $P_s A_s$ is equivalent. The direct effect of a small increase in $P_s A_s$ is to raise the wage of workers earning above the minimum wage proportionately: $\partial \log \bar{w}_s / \partial \log (P_s A_s) = 1 - \sigma_s$. The indirect effect of a small increase in $P_s A_s$ is to move workers from unemployment into employment, since workers are employed if $w_s P_s A_s > m$; hence, the indirect effect of a small increase in $P_s A_s$ is equivalent to the indirect effect of a proportionate reduction in $m$. The following proposition summarizes this discussion.

**Proposition 2.** The log derivative of the average skill $s$ wage with respect to given changes in $P_s$,
\( A_s \), and \( m \) is given by

\[
d \log w_s = \sigma_s d \log m + (1 - \sigma_s) d \log (P_s A_s) + \frac{\chi_s (\mu_s - \sigma_s)}{e_s \mu_s} m_s (d \log m - d \log (P_s A_s))
\]

(B.1)

where \( \mu_s \) and \( \sigma_s \) are the shares of employment and income earned by workers at the minimum wage, \( e_s \) is the share of workers who are employed, and \( \chi_s \) is the probability that a worker is marginally employed.

**B.2 General Equilibrium**

Of course, the skill price \( P_s \) is itself a function of changes in the minimum wage \( m \), productivities \( A_\ell \) and \( A_h \), and factor supplies \( L_\ell \) and \( L_h \). Let \( \eta \) denote the local elasticity of substitution between \( Y_\ell \) and \( Y_h \) in the aggregate production function,

\[
\eta \equiv -\frac{d \log (Y_s / Y_s')}{d \log (P_s / P_s')}
\]

Then

\[
d \log P_s = \frac{P_s Y_s' / Y}{\eta + \xi_h m_h} \sum_{j=s,s'} \left( I_{j=s'} - I_{j=s} \right) \left[ d \log L_j + (1 + \xi_j m_j) d \log A_j - \xi_j m_j d \log m \right]
\]

(B.2)

where

\[
\xi_s \equiv \frac{m_s \chi_s}{\int_0^1 \left( \int_{m_s}^\infty \phi g(\epsilon, \phi) d\phi \right) d\epsilon}
\]

is the share of output \( Y_s \) lost (or gained) when the cutoff \( m_s \) marginally increases (or decreases) and \( I_{j=s} \) is an indicator function that takes the value 1 if \( j = s \) and 0 otherwise. Since \( Y \) is constant returns to scale, the change in the ratio of \( L_h \) and \( L_\ell \) is a sufficient statistic for the separate impacts of changes in \( L_h \) and \( L_\ell \) on skill prices, the average wage of each skill, and therefore the skill premium, as shown in equation (B.2).

Finally, combining equations (B.1) and (B.2) yields the solution for \( \beta_m \) and \( \beta_L \) in Proposition 1. In particular, we have

\[
\beta_m \equiv \rho_\ell - \rho_h - \frac{(\xi_\ell m_\ell - \xi_h m_h)}{\eta + \xi_h m_h} \left( \rho_h \frac{P_\ell Y_\ell}{Y} + \rho_\ell \frac{P_h Y_h}{Y} \right)
\]

(B.3)

and

\[
\beta_L \equiv -\frac{1}{\eta + \xi_h m_h} \left( \rho_h \frac{P_\ell Y_\ell}{Y} + \rho_\ell \frac{P_h Y_h}{Y} \right)
\]

(B.4)
where I have defined

\[ \rho_s \equiv (1 - \sigma_s) - \frac{\chi_s (\mu_s - \sigma_s)}{e_s \mu_s} m_s \]

which is simply the elasticity of the average wage of group s with respect to \( P_s A_s \), as shown in equation (B.1). In Example 2, we have \( \zeta_s = \chi_s = 0 \) for both s. In this case, equations (B.3) and (B.4) simplify to equations (8) and (9), as shown in Corollary 2. Finally, in Example 2, the impact of changes in the minimum wage on relative wages across any two skill groups is invariant to the number of skill groups assumed in the analysis.