Contrarian Opinion and Its Predictability: Application to Exchange Rates*

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Abstract

Exchange rates typically exhibit trends that last several quarters. Puzzlingly, at the end of an uptrend(downtrend), the average speculator is overly optimistic(pessimistic). We formalize a novel trend generating mechanism, where this contrarianism arises in equilibrium. News are impounded into exchange-rates via trending(bubbly) paths, which informed-speculators ride until trend-reversals approach. When informed-speculators exit, trends continue because uninformed-speculators trade more optimistically(pessimistically) with hedgers, although trend-reversals may approach shortly. Second, we construct trend-reversal forecasts by applying our model to the Commitments-of-traders speculator position data. Our contrarian directional forecasts achieve 53%-72% success-ratios over 1-to-12-month horizons, outperforming the random-walk for most currency-horizon pairs using the Binomial test and a new test which weights directional forecasts by subsequent exchange-rate changes. Third, we construct a novel contrarian portfolio, which is crash-free: it trades only when our forecasts signal switches in trend direction. Over 1995-2017, our portfolio generates a significantly higher Sharpe-ratio than that of typical momentum strategies.

Keywords: Bubbly Equilibrium, Crash-free Portfolio, Contrarian Opinion, Contrarian Portfolio Strategy, Exchange Rate Trends, Markov-switching Model, Trend Reversal

JEL Classification: F31; F37; G12

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1 Introduction

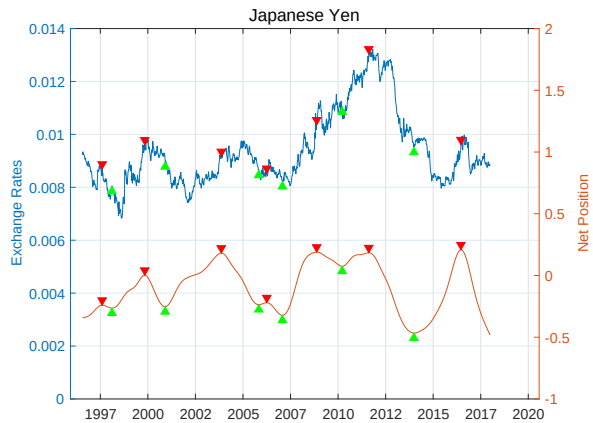
Exchange rates tend to follow *trends* that often last several quarters. Although such trends can be identified in-sample, they are difficult to predict out-of-sample.[1] Associated with these trends is the puzzling contrarian pattern observed in market position data: at the end of an uptrend, the average speculator tends to be overly optimistic, while at the end of a downtrend she is overly pessimistic. Figure 1 exhibits this contrarian pattern, depicting the Yen-US Dollar exchange rate and the speculators' net positions in the Yen from the Commitments-of-traders (COT) report. As we can see, speculators' net positions in the Yen typically exhibit a *maximum* around the end of an uptrend in the Yen and a *minimum* near the end of the Yen's downtrend.

This contrarian pattern can also be observed in other currencies and may be associated with the "principle of contrarian opinion" (PCO): At extremes, what the majority of speculators believes tends to be wrong. When a majority believes the market is going up—by the PCO—it is in fact destined to plunge, and fast. Similarly, when a majority believes the downtrend will continue, an upward reversal is likely to come soon.

Contrarian opinion has a long tradition among practitioners, going back to Lefevre (1923) and Soros (1987). In this paper, we rationalize contrarianism within an exchange rate model and derive a contrarian forecasting strategy, which we then apply to COT speculator-position data. Specifically, this paper makes three contributions to the existing literature. First, we characterize conditions under which the contrarian pattern arises in an economy with three types of traders who may take positions on domestic and foreign bonds: uninformed speculators, informed speculators, and hedgers. Big news, positive or negative, about future interest rate differentials lead informed speculators to take long or short positions in the foreign bond, triggering a trend along which a gap grows between the exchange rate and its long-run expected value. Informed speculators have no incentive to arbitrage the increasing mispricing, instead finding it optimal to ride the trend and exiting only when the gap becomes so large that uninformed speculators may no longer be able to finance their trades, thus making a trend reversal very likely. At this switching time, informed speculators close their long or short positions in the foreign bond and open the converse position.

[1] Engel and Hamilton (1990) show the existence of *long swings* in the Dollar-German Mark, French Franc, and British Pound.

Figure 1: The Yen and Speculators' Net Positions



Note: The upper line plots the Yen-US Dollar exchange rate (i.e., US Dollars per Yen). The lower line plots the hp-filtered speculators' net positions in the Yen, using $\lambda = 50,000$

Afterwards, the trend continues because uninformed speculators trade with hedgers more enthusiastically. Of course, the trend is very likely to reverse soon after the switching time, proving right the contrarian informed speculators and proving wrong the uninformed, momentum-driven speculators.[2]

In order for this contrarian pattern to arise in equilibrium, informed speculators must expect that the exchange rate will move in their favor at the time they will exit their positions. Otherwise, by Tirole's (1982) backwards induction argument, the bubble would unravel. We show that in a positive/negative bubbly equilibrium, along which the foreign currency appreciates/depreciates, the foreign currency necessarily appreciates/depreciates at the switching time. This is because uninformed speculators' demand for the foreign bond is more price elastic than the hedgers' supply, a necessary condition for the existence of bubbly equilibria. Thus, when the informed speculators' demand falls, market clearing may occur only if the foreign currency appreciates.

Second, PCO equilibria imply that the informed speculators' decisions have predictive power over incoming trend reversals. Since the COT data we use is an unknown mixture of informed and uninformed speculators' positions, we estimate the latent entry and exit decisions of informed speculators using a hidden Markov-switching

[2]Similarly, a negative shock to expectations may trigger a downtrend, along which informed speculators are short the foreign bond until a switching time when a melt-down occurs. Shortly thereafter the downtrend is very likely to end.

model. This model allows us to construct leading indicators of trend reversals based on the PCO equilibrium. Based on these indicators, we construct *out-of-sample contrarian directional forecasts* of exchange rates over horizons ranging from 1 to 12 months. Our directional forecasts have a 61% average success ratio across five major currencies vis-a-vis the US dollar over the 1992-2017 period.[3] The forecast success ratios are particularly accurate at the 6-to-12 months horizons, reaching 66.4% for the Yen, 68.6% for the Canadian Dollar, and 67.1% for the Euro.

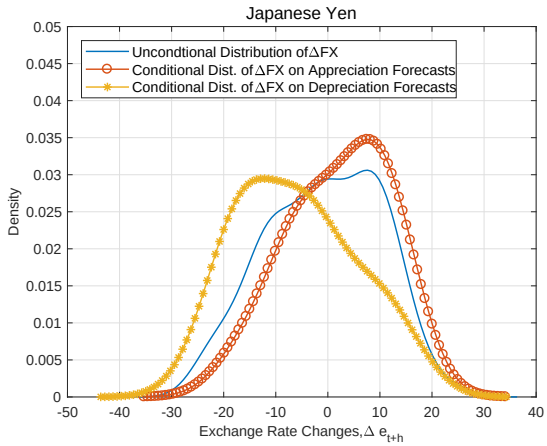
Third, to evaluate whether our directional forecasts successfully predict *big* swings in exchange rates, we propose a new directional forecast-accuracy test. Unlike the traditional directional test, our test weights each directional forecast by the realized exchange rate change, and evaluates whether the weighted directional forecasts outperform the driftless random walk forecasts. At the 9- and 12-month horizons, the weighted directional test rejects the random-walk null in favour of our model across all currencies, controlling for auto-correlation using the long-run variance estimators proposed by Newey and West (1987) and Andrews (1991). The same holds true at the 6-month horizon across all currencies, except the Australian Dollar. At the 1-month and 3-month horizons, the null is rejected in 6 out of the 10 currency-horizon pairs. The standard binomial test generates similar results.[4] The high success ratio of our directional forecasts is reflected in Figure 2, which depicts the densities of one-year ahead changes in the Yen-US Dollar exchange rate. As we can see, the mean of the unconditional density is -0.8%. Meanwhile, the mean of the density conditional on a Yen appreciation (resp. depreciation) forecast is 2.4% (resp. -5.7%).

To show that our model's forecasts are not mere statistical objects, we analyze the connection between our model's signals and major events associated with big swings in the Yen and the Euro. We find that the signals of our model often foresee such events, and on several occasions they indicate when the subsequent exchange rate move may end. Examples include the launch of the quantitative easing programs in Japan (2001), in the USA (2009), and in the Eurozone (2015). Our model exhibits similar

[3] We consider the Euro, Japanese Yen, British Pound, Australian Dollar, and Canadian Dollar at *1m*, *3m*, *6m*, *9m*, and *12m* forecasting horizons.

[4] Throughout this paper, when citing the Newey-West long-run variance (LRV) estimator, we refer to the kernel smoothed LRV estimator proposed in Newey and West (1987) and bandwidth selection rule suggested in Newey and West (1994). When citing the Andrews LRV estimator, we refer to the kernel smoothed LRV estimator using Bartlett kernel and bandwidth selection rule suggested in Andrews (1991). See Section G of the Extended Appendix for the construction of the above LRV estimators.

Figure 2: Kernel Density Function of 1-year Exchange Rate Changes



Note: The solid blue line depicts unconditional kernel density function of 1-year exchange rate changes. The orange (yellow) line with circle (star) markers is the kernel density function of 1-year exchange rate changes conditional on our appreciation (depreciation) forecasts.

signals preceding the Lehman crisis (2008) and the implementation of Abenomics (2013).

We construct a contrarian portfolio based on our directional forecasts. Our contrarian strategy trades only when a trend is likely to switch direction, and otherwise takes no positions. Over the period 1995-2017, our contrarian strategy generates an average Sharpe ratio of 0.86, four times higher than that (0.22) of a typical momentum strategy. This higher Sharpe ratio reflects the positive returns of our strategy during exchange rate momentum crash episodes, such as the Russian financial crisis (1998) and the Lehman bankruptcy (2008). Moreover, our strategy performed well when S&P 500 returns plummeted during the dot-com bubble crash in 2000.

To our knowledge, our model provides a new way of thinking about trends and of rationalizing why contrarianism works. The forecasting strategy, as well as the weighted directional forecast test, are novel contributions. Engel and West (2005) have shown that the poor performance of fundamentals' based models arises because exchange rates may be driven by news about future events. In our model, the current exchange rate actually reacts to news about future events. The novel element is that such news may trigger a bubbly trend, which rational-informed speculators have no incentives to arbitrage away until the end of the trend is nigh.

Our bubbly paths have the same spirit as those of positive-feedback models. In Abreu and Brunnermeier (2003), momentum traders interact with rational but not

perfectly-informed traders, each of whom only becomes aware of the bubble at a random time and does not know when other informed traders also become aware of the bubble. Because of this asymmetry of information across rational traders, they may continue to ride the bubble even after all of them are aware. The price falls when they exit the market. By contrast, our setup has no asymmetric information across informed traders and the foreign currency's price necessarily increases upon their exit, thanks to the PCO mechanism. Barberis et al. (2018) considers fundamental traders and extrapolator traders who put weight both on price momentum and on fundamentals. They show that a bubble arises if there is a sequence of positive shocks to fundamentals. Their focus is on generating increasing volume, rather than forecasting trend reversals. In their model, fundamental traders exit early along a bubble, while in our PCO equilibrium informed speculators ride the bubble until the probability of its end is high. This property allows us to derive an estimation strategy that successfully forecasts incoming trend reversals.

In contrast to much of the literature, our forecasts are contrarian, which naturally follows from the implications of our theoretical model. In the PCO equilibrium, the aggregate speculators' net positions can be expressed as the sum of a Markov switching term and an exchange rate autoregressive term, which correspond to informed and to uninformed speculators, respectively. That is, the speculators' position data follows the process $Y_t = c_{(\kappa_t)} + \theta \Delta S_t + \varepsilon_t$, where the hidden state κ_t indicates whether the informed speculators' net position is long ($\kappa_t = +1$), short ($\kappa_t = -1$) or zero ($\kappa_t = 0$). Given the observations on Y_t and ΔS_t, we estimate $\hat{\kappa}_t$ and $\hat{\theta}$. We then forecast the imminent end of an uptrend if the estimated state switches to $\hat{\kappa}_t = -1$ and there is evidence of momentum along an uptrend, i.e., the estimate $\hat{\theta} > 0$ is significantly different from zero and $\Delta S_t > 0$. Analogously, we forecast an imminent end of a downtrend if the estimated state switches to $\hat{\kappa}_t = +1$ and there is evidence of momentum along a downtrend. Lastly, we forecast the end of a trendless path if we detect a shift in $\hat{\kappa}_t$ to either $+1$ or -1, along a trendless path. Our estimation method relies on the maximum likelihood estimation of the Markov switching model (hereafter MSM) proposed by Hamilton (1989, 1990) and applied by Engel and Hamilton (1990) to exchange rate data. In this paper, we apply a MSM to the speculators' aggregate net position data, rather than to the exchange rate itself. Moreover, rather than a two-state MSM, our model implies three regimes corresponding to the sign of the informed speculators' positions.

By investigating the information content of market participant's positions, our paper is related to Evans and Lyons (2002, 2005, 2012), who argue that order flow contains relevant information about the determinants of exchange rates. Using linear prediction models, they find that order flow forecasts exchange rates successfully up to one-month horizons. In our setup, there is no order flow among informed speculators, as they all have the same information. We instead make contrarian forecasts of incoming trend reversals using net speculator position data. While the order flow data among informed traders used by Evans and Lyons is proprietary, our strategy applies to the less pristine and more noisy data in the COT report, which contains a mixture of informed and uninformed traders, but is public and readily available.[5]

Lastly, our weighted directional test is in the spirit of Elliott and Ito (1999) and may be more relevant than the standard binomial test to situations faced by investors, who look at the profitability associated with their forecasts rather than the share of correct forecasts. As Elliott and Timmermann (2016) point out, the standard binomial test does not capture any notion of profitability.

The structure of the paper is the following. Section 2 presents the model. In Section 3, we apply the forecasting strategy implied by the equilibrium to COT data and present our directional forecasts, the forecasting accuracy tests and the link between our model's signals and major events driving the Euro and the Yen. In Section 4, we construct a contrarian portfolio based on our forecasts, and investigate its profitability using the time-series momentum strategy as comparison. In Section 5, we present the literature review. Section 6 contains the conclusion. The main proofs are contained in the Appendix. Lastly, the Extended Appendix contains auxiliary material and robustness checks of the empirical estimations.

2 Model

We combine a minimal model of exchange rate fluctuations with a positive-feedback asset-pricing model. There are T periods, one good, which is the numeraire, and two

[5]Because COT data is an unknown mixture of informed and uninformed traders' positions, higher/lower positions in the COT report are not necessarily associated with higher/lower future values of foreign currencies and thereby linear models fail to accurately forecast future exchange rate changes over 1 to 12 months horizons. Granger causality tests reveal that across the five currencies, net speculator positions do not Granger-cause exchange rates over 1 to 12 months horizons. See Table H.1 in the Extended Appendix for details.

one-period bonds. The domestic bond pays next period a continuous compounding interest rate i_t, and the foreign bond pays next period a continuous compounding interest rate i_t^f. The domestic bond has a perfectly elastic supply and its price is one. The foreign bond is in zero net supply and we will refer to its price, S_t, as the exchange rate. There are three types of overlapping generations of traders who live two periods and can take short and long positions in the two bonds: informed forward-looking speculators, hedgers, and uniformed momentum-chasing speculators.

Our objective is to forecast exchange rate trends over horizons of several quarters, rather than to forecast high frequency fluctuations or to make multi-year forecasts. To this end we let the terminal time-T exchange rate S_T be an exogenously given random variable and set the interest rate differential $i_t - i_t^f$ to zero for all $t < T$. We then characterize equilibrium responses to shocks to expectations of S_T along which exchange rate trends develop and switches in the sign of the informed speculators' positions predict incoming trend reversals. Estimating such switching times is the basis of our contrarian forecasting strategy.

Informed Speculators. They are risk-neutral rational agents akin to Lefevre (1923)'s manipulators and to De Long et al. (1990) informed rational speculators. At any time t, the young representative informed speculator takes a position q_t^I in foreign bonds and a position $-S_t q_t^I$ in domestic bonds to maximize her next period's expected profits. Her choice set is $q_t^I \in \{-c, 0, c\}$, where c is a real positive number, i.e., she can go long, short or stay off the market.[6] Since $i_t - i_t^f = 0$ for any $t < T$, the young informed speculator's demand for the foreign bond on $t < T$ is:

$$q_t^I(S_t, E_t^I[S_{t+1}]) = C(E_t^I[\Delta S_{t+1}] = \begin{cases} c & \text{if } E_t^I[\Delta S_{t+1}] > 0 \\ 0 & \text{if } E_t^I[\Delta S_{t+1}] = 0 \\ -c & \text{if } E_t^I[\Delta S_{t+1}] < 0 \end{cases}, \tag{1}$$

where $\Delta S_{t+1} = S_{t+1} - S_t$ and $E_t^I[\cdot]$ denotes the conditional expectation given her information at time t. The young representative informed speculator knows the interest rate differential process, but does not know the time-T exchange rate S_T. She knows that the unconditional mean of S_T is μ_Z and observes a noisy signal of S_T that she

[6]The choice set of informed speculators is assumed to be discrete to facilitate the derivation of the bubbly equilibrium. It also makes the characterization of the equilibrium and the PCO clear. It is possible to derive the results under a continuos choice set and risk aversion, but the proof of the results would be much more lengthy.

uses to forecast S_T:

$$Z_t \equiv E_t^I[S_T] = \mu_z + \zeta_t, \tag{2}$$

where the surprise term ζ_t is a random variable.[7] Throughout the paper we will refer to the random variable Z_t as the expected long-run exchange rate or simply as 'the fundamental.'

Hedgers. They are akin to Lefevre (1923)'s investors, that "are in for the long haul," and to the passive investors of De Long et al. (1990). They are risk-averse agents whose expected payoff depends on the time-T value of their profits. They know that $i_t - i_t^f = 0$ for any $t < T$ and observe the informed speculators' expected long-run exchange rate Z_t. Appendix C.1 solves the representative hedger's optimization problem and shows that the demand function is

$$q_t^h(S_t, Z_t) = \phi\left(Z_t - S_t\right), \quad \phi > 0. \tag{3}$$

That is, hedgers short the foreign bond when S_t is greater than Z_t and go long when S_t is lower than Z_t.

Momentum Speculators. They are akin to Lefevre (1923)'s speculators that "buy the trend," and to the positive feedback traders of De Long et al. (1990). They are behavioral agents that do not observe Z_t and believe that exchange rates follow trends. Appendix C.2 describes the problem solved by the representative young momentum speculator and shows that her demand function for the foreign bond is increasing in $S_t - S_{t-1}$ whenever she finds financing and zero otherwise:

$$q_t^m(S_t, S_{t-1}) = \begin{cases} \theta\left(S_t - S_{t-1}\right), & \text{if } L_{t-1} = 0 \text{ and } L_t = 0 \\ 0, & \text{if } L_{t-1} = 1 \text{ or } L_t = 1 \end{cases}, \quad \theta > 0. \tag{4}$$

The binary state variable L_t indicates whether financing is available for momentum speculators at time t ($L_t = 0$) or not ($L_t = 1$). We will refer to a situation in which

[7] There are several ways to derive (2). Consider for instance the following filtering problem. Suppose that S_T is normal with mean μ_z and variance $\sigma_S^2 > 0$. The young informed speculator knows μ_z but does not know $S_T - \mu_z$. At time t, she observes a normal signal y_t with mean μ_y and variance σ_Y^2. Therefore, $E\left[S_T|y_t\right] = E\left[S_T\right] + \rho_{S,Y}\frac{\sigma_S}{\sigma_Y}(y_t - \mu_y)$, where $\rho_{S,Y}$ denotes the correlation coefficient between S_T and y_t. Since $E\left[S_T\right] = \mu_z$, the expression in (2) follows by letting $Z_t = E\left[S_T|y_t\right]$ and $\zeta_t = \rho_{S,Y}\frac{\sigma_S}{\sigma_Y}(y_t - \mu_y)$.

$L_t = 1$ as a *liquidity squeeze.*[8] The probability of a liquidity squeeze next period σ_{t+1} is increasing in the distance between the mean of Z_t and the exchange rate:

$$\sigma_{t+1} \equiv \Pr(L_{t+1} = 1|S_t) = \begin{cases} 0, & \text{if } |S_t - \mu_z| \leq \vartheta \\ \overline{\sigma}, & \text{if } \vartheta < |S_t - \mu_z| \leq \overline{\vartheta} \\ 1 & \text{if } \overline{\vartheta} < S_t - \mu_z, \end{cases} \tag{5}$$

where $\mu_z \equiv E[Z_t]$, $\overline{\vartheta} > \vartheta > 0$ and $\overline{\sigma} \in (0, 1)$ are some fixed constants.[9]

Equation (4) captures the essence of trend-following observed in several asset markets. It implies that if momentum speculators can find financing and S_t has been increasing/decreasing, then their demand for the foreign bond increases/decreases over time. However, if there is a liquidity squeeze, they cannot find financing and their demand becomes zero. According to (5), as the exchange rate diverges from the long-run exchange rate Z_t, it becomes less likely that momentum speculators will be able to find financing.

Equilibrium Concept. We consider competitive equilibria in which each trader submits her demand schedule for the foreign bond, taking exchange rates as given, and the market of the foreign bond clears every period. In order to derive our forecasting rules in the empirical section, we characterize bubbly equilibrium paths, along which the exchange rate follows either a strictly increasing or a strictly decreasing path during a period over which Z_t experiences no shocks.

Definition 1 (Bubbly Path). *Suppose that in the time interval $\{t, \ldots, t'\}$ with $t' > t + 3$, there are no shocks to the long-run expected exchange rate and no liquidity squeeze, i.e., $Z_j = \mu_z$ and $L_j = 0$ at any $j \in \{t, \ldots, t'\}$. Then there is a positive bubbly path on $\{t, \ldots, t'\}$ if $S_j > S_{j'}$ for any $j, j' \in \{t, \ldots, t'\}$ with $j > j'$. Similarly, there is a negative bubbly path on $\{t, \ldots, t'\}$ if $S_j < S_{j'}$ for any $j, j' \in \{t, \ldots, t'\}$ with $j > j'$. We say that there exists a bubbly path on $\{t, \ldots, t'\}$ if there is either a positive bubbly path or a negative bubbly path.*

Equipped with the definition of a bubbly path we now define a bubbly equilibrium.

[8] Even though the carry trade is a zero-cost portfolio, traders are typically required to put down a deposit to meet potential margin calls. Traders typically need to borrow to put down the deposit, and such liquidity may dry from time to time, e.g. Brunnermeier and Pedersen (2008).

[9] The law of motion specified in (5) can be endogeneized by introducing outside lenders. To keep the setup concise we take them as exogenously given.

Definition 2 (**Bubbly Equilibrium**). *A bubbly equilibrium is an exchange rate and liquidity squeeze process* $\{(S_j^*, L_j^*)\}_{j=1}^{\infty}$*, such that: (i) the foreign bond's market clears every period:*

$$q_t^F(S_t^*, Z_t) + q_t^m(S_t^*, S_{t-1}) + q_t^I(S_t^*, E_t^I[S_{t+1}^*]) = 0 \quad \text{if } t \leq T-1 \ , \tag{6}$$

(ii) there is a time interval $\{t, \ldots, t'\}$*, with* $T-1 > t' > t+1$*, on which a bubbly path exists; and (iii) given exchange rates, the liquidity squeeze probability is determined by (5).*

2.1 Discussion of the Setup

The three types of traders in the model are typically present in several asset markets and they have counterparts in the COT report data we will use in our empirical application. These three types of traders are necessary in our argument for rationalizing contrarianism in the presence of informed forward-looking traders. In our model, the informed speculators forecast the future equilibrium exchange rates and initiate bubbly paths when they observe a large enough shock to the expected long-run exchange rate Z_t. The role of momentum speculators is to drive the bubble larger and larger. The hedgers' role is to disconnect the equilibrium trade volumes of informed and momentum speculators. This disconnection allows for the contrarian pattern—whereby informed speculators exit an ongoing trend some time before it reverts—to arise in equilibrium.

The equilibrium bubbly paths will be characterized in terms of deviations of the market clearing exchange rate S_t from Z_t. We show in Proposition 5 that S_t would equal Z_t in an economy with only informed speculators and hedgers, who expect no shocks in the future (i.e., $E_t^I[\zeta_{t+j}] = 0$ for all $j < T-t$). In this case, the market-clearing condition would be $C(E_t^I[\Delta S_{t+1}]) = 0$ for all $t < T$ and market clearing would require that $E_t^I[\Delta S_{t+1}^*] = 0$ for all $t < T$. Thus, the time-t equilibrium exchange rate would be the one expected to prevail at time T, $S_t^* = E_t^I[S_T^*] = Z_t$ for all $t < T$.

Two key properties of the bubbly equilibria we characterize will be quite useful in designing our empirical forecasting strategy. First, a single transitory shock to Z_t can trigger a bubbly path, provided it is large enough. Meanwhile, small shocks to Z_t do not give rise to bubbly paths. Second, if a bubbly equilibrium exists, then along a positive bubbly path, S_t^* must continue going up at the time informed spec-

ulators switch their long positions for short positions.[10] This result implies that backwards induction does not unravel to initial time, and so informed speculators find it profitable to ride the trend for a long time, and exit only when it becomes very likely that momentum traders may not be able to obtain financing anymore. At this switching time τ^*, informed speculators know that the exchange rate is very likely to change direction and revert back to Z_t. Therefore, their entry and exit decisions may act as leading indicators of trend reversals. Hence, the empirical implication of our bubbly equilibria is that if one has time series data which contains a mixture of informed and momentum speculators, then one can estimate the switching times when the uptrend/downtrend is very likely to change direction by identifying times when such speculator position data has a regime change, which indicates a contrarian switch by informed speculators, provided there is evidence of momentum along an uptrend/downtrend.

Lastly, in the empirical section we will consider the US Dollar versus other currencies, and so we will refer to an increase/decrease in the price of the foreign bond, S_t, as a Dollar depreciation/appreciation.

2.2 Bubbly Equilibria

To derive the equilibrium exchange rate, notice that the uncertainty of the economy comes from the random expectational shocks ζ_t to the long-run expected exchange rate Z_t, and the possibility of a liquidity squeeze. The endogenous variables are the demands for the foreign bond, the equilibrium exchange rate S_t^*, and the binary variable L_t^* indicating whether a liquidity squeeze is triggered at time t. The time-t demands for the foreign bond are calculated using $\left\{S_{t-1}, S_t, E_t^I[S_{t+1}], L_{t-1}, L_t\right\}$ together with equations (1), (2), (3), and (4). If no liquidity squeeze is triggered at $t-1$ and t, the market-clearing condition at time t is

$$0 = \theta(S_t^* - S_{t-1}^*) + \phi(Z_t - S_t^*) + C(E_t^I[\Delta S_{t+1}^*]). \tag{7}$$

Otherwise, if a liquidity squeeze is triggered at $t-1$ or t,

$$0 = \phi(Z_t - S_t^*) + C(E_t^I[\Delta S_{t+1}^*]). \tag{8}$$

[10] Analogously, if a negative bubbly path exists, S_t^* must go down at the time informed speculators switch their short positions for long positions.

It follows that for $t \leq T-1$, the equilibrium exchange rate S_t^* satisfies

$$S_t^* = \begin{cases} \frac{\theta S_{t-1}^* - \phi Z_t - C_t^*}{\theta - \phi}, & \text{if } L_{t-1}^* = 0 \text{ and } L_t^* = 0 \\ Z_t + \frac{C_t^*}{\phi}, & \text{if } L_{t-1}^* = 1 \text{ or } L_t^* = 1 \end{cases}, \tag{9}$$

where $C_t^* = C(E_t^I[\Delta S_{t+1}^*])$. The equilibrium exchange rate $\{S_t^*\}_t$ and the law of motion of the liquidity squeeze (4) determine the state variable$\{L_t^*\}_t$. An analysis of (9) reveals the values of θ and ϕ over which bubbly paths may exist.

Lemma 3. *An equilibrium bubbly path exists only if the momentum speculator's demand is more sensitive to current exchange rates than the hedgers' demand. That is:*

$$\theta > \phi. \tag{10}$$

To grasp the intuition of Lemma 3, suppose to the contrary that $\theta \leq \phi$ and that a bubbly path exists. The proof in Appendix B.1 shows that along this hypothetical bubbly path, price changes would satisfy

$$(\theta - \phi)\Delta S_j^* = \theta \Delta S_{j-1}^* - \Delta C_j^* \text{ for any } j \in \{t+2, \ldots, t'\}.$$

Since $\theta > 0$ and by definition, along a positive bubbly path $\Delta S_j^* > 0$ and $\Delta S_{j-1}^* > 0$ for any $j \in \{t+2, \ldots, t'\}$, it follows that the above equation holds under $\theta \leq \phi$ only when $\Delta C_j^* > 0$ for any $j \in \{t+2, \ldots, t'\}$, which is impossible since $t' - t > 3$ and C_j^* can only take three possible values for any j. An analogous argument holds for a negative bubbly path.

Restriction (10) will be key to characterizing the equilibrium paths, and so we will impose this condition throughout the rest of the paper. Both θ and ϕ are latent parameters, so the empirical validity of condition (10) cannot be directly determined. However, given the existence of an equilibrium bubbly path, condition (10) implies that the magnitude of exchange rate changes may increase over time, a condition which is observable.

Lemma 4. *Suppose that an equilibrium bubbly path exists. Then restriction (10) implies that for at least one j, $\Delta S_j^* > \Delta S_{j-1}^*$ along a positive bubbly path, and for at least one j, $\Delta S_j^* < \Delta S_{j-1}^*$ along a negative bubbly path.*

Next, we characterize bubbly equilibria for an economy that is hit by a posi-

tive transitory shock to Z_t.[11] Concretely, we consider the following trajectory of the expectational shock ζ_t in (2)[12]

$$\zeta_t = \begin{cases} z > 0 & \text{if } t = 1 \\ 0 & \text{if } t > 1 \end{cases} . \tag{11}$$

We proceed in two steps. First, Proposition 5 below shows that if a liquidity squeeze were to occur, so that hedgers and informed speculators were the only market participants, then the equilibrium exchange rate would revert to Z_t. Second, in Proposition 7, we characterize equilibrium positive bubbly paths.

Proposition 5. *Consider a time t_0 when a liquidity squeeze is triggered and there is no shock to the fundamental (i.e., $L^*_{t_0} = 1$ and $Z^*_{t_0} = \mu_z$). Then at time t_0, the equilibrium exchange rate is equal to μ_z and informed speculators hold a zero position in the market for the foreign bond, i.e., $S^*_{t_0} = \mu_z$, $C^*_{t_0} = 0$ and $L^*_{t_0+1} = 0$.*

To grasp the intuition of Proposition 5, note that when a liquidity squeeze is triggered at time t_0, the momentum traders' demand is zero and so market clearing condition (8) implies that $S^*_{t_0} = \mu_z + \phi^{-1} C^*_{t_0}$. We show in Subsection B.1 of the Appendix that only a zero demand by informed speculators ($C^*_{t_0} = 0$) can be part of an equilibrium and so we must have $S^*_{t_0} = \mu_z$. Suppose instead that the representative informed speculator expected an appreciation of the foreign currency and chose a long position in the foreign bond ($C^*_{t_0} = c$). Then, regardless of the sign of future informed speculators's positions, the foreign currency cannot be expected to appreciate, i.e., $E^I_{t_0}[\Delta S^*_{t_0+1}] \leq 0$. This clearly contradicts the portfolio policy of informed speculators in (1). An analogous argument holds for $C^*_{t_0} = -c$.

Our contrarian forecasting strategy will exploit two properties of the bubbly equilibrium characterized in Proposition 7 below. First, informed speculators find it optimal to choose long positions in the foreign bond and ride the positive bubble until a time τ^* when a liquidity squeeze becomes very likely. At this time, informed speculators close their long positions in the foreign bond. Second, at time τ^*, the Dollar necessarily depreciates (i.e., S^*_t increases) as momentum speculators increase

[11] In Section E of the Extended Appendix we derive the bubbly equilibria for an economy that is hit by a negative transitory shock to Z_t.

[12] The equilibrium derived under (11) can also be rationalized under more general shock trajectories where the shocks after $t = 1$ are not zero. To ensure the emergence of a bubbly path, the key condition is that the initial shock ζ_1 is large enough, while the subsequent shocks are small.

their demand for the foreign bond. Thus, backwards induction about the eventual fall in S_t^*, does not unravel to initial time. To ensure the emergence of such a bubbly equilibrium path, Proposition 7 imposes the following four parametric restrictions.

Condition 6. *The demand parameters ϕ and θ satisfy (10), the magnitude of the transitory shock satisfies*

$$z > 2c\phi^{-1}, \tag{12}$$

and the parameters that govern the probability of a liquidity squeeze satisfy

$$\overline{\sigma} > \phi\theta^{-1} \text{ and } \vartheta \geq (1-\phi/\theta)^{-1}\left[z + (1+\phi/\theta)c\phi^{-1}\right]. \tag{13}$$

The first restriction, i.e., $\theta > \phi$, is necessary to for a bubbly path to arise (by Lemma 3). The second restriction, (12), requires that the magnitude of the shock at time $t = 1$ be large enough to induce informed speculators to open long positions in the foreign bond, which is the key to initiate a bubbly path. The third restriction, first inequality in (13), makes the probability of the liquidity squeeze $\overline{\sigma}$ large enough, such that at time τ^*, when $S_t^* - \mu_z$ surpasses the threshold ϑ in (5), informed speculators' expectations switch from depreciation to appreciation.[13] This implies that informed speculators reverse their positions at an earlier time than momentum speculators, which is the property of the equilibrium that rationalizes contrarianism. The second inequality in (13), requires that the threshold ϑ be large enough such that the bubbly path lasts more than one period. To see this consider τ_*

$$\tau_* = \max\left\{t \in \mathbb{N} : \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} \leq \vartheta\right\}, \tag{14}$$

where $\mathbb{N}$ denotes the set of natural numbers. As we shall see, the switching time τ^* must equal either $\tau_* + 1$ or $\tau_* + 2$. The second inequality in (13) ensures that τ_* is greater than 2. Lastly, we set terminal time T high enough such that $T > \bar{l}_{\max}$, where

$$\bar{l}_{\max} = \min\left\{t \geq \tau_* + 3 : \frac{z}{(1-\phi/\theta)^{t-2}} + \frac{2c}{\phi(1-\phi/\theta)^{t-\tau_*-2}} - \frac{c}{\phi} > \vartheta\right\}. \tag{15}$$

The quantity $\bar{l}_{\max}$ is an upper bound on the duration of an equilibrium bubbly path (by Proposition 8 below).

[13] If instead $\overline{\sigma} < \phi\theta^{-1}$, informed speculators would not switch the sign of their positions before time l^*, and so their portfolio switch would not generate a leading indicator of a trend reversal.

Proposition 7. *Consider an economy in steady state with $S_0^* = \mu_Z$, that is hit at time $t = 1$ by a transitory positive expectational shock (11). Under Condition 6, a bubbly path is triggered at $t = 1$ lasting until a switching time $\tau^* > 2$, with τ^* equal to either $\tau_* + 1$ or $\tau_* + 2$, where τ_* is defined in (14). The equilibrium exchange rate satisfies*

$$S_t^* = \begin{cases} \mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{t-1}}, & 1 \leq t < \tau^* \\ \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right], & \tau^* \leq t < l^* \end{cases} \tag{16}$$

where l^* denotes the time when the liquidity squeeze is triggered. The demand of the informed speculators satisfies

$$C_t^* = \begin{cases} c, & 1 \leq t < \tau^* \\ -c, & \tau^* \leq t < l^* \end{cases}, \tag{17}$$

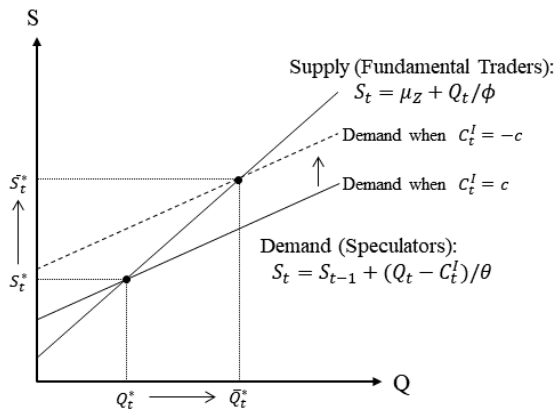
the momentum speculators' demand satisfies

$$q_t^{m^*} = \begin{cases} 0, & t = 1 \\ \frac{z\phi}{(1-\phi/\theta)^{t-1}}, & 2 \leq t < \tau^* \\ \frac{z\phi}{(1-\phi/\theta)^{t-1}} + \frac{2c}{(1-\phi/\theta)^{t-\tau^*+1}} & \tau^* \leq t < l^* \end{cases}, \tag{18}$$

and the hedgers' demand satisfies $q_t^{h^*} = -(C_t^* + q_t^{m^*})$ for $1 \leq t < l^*$.

Remark 2.1. Proposition 7 shows that if the expectational transitory shock is large enough, then informed speculators take long positions in the foreign bond. The resulting Dollar depreciation leads momentum speculators to take long positions in the foreign bond, initiating a bubbly path. Even though S_t^* is above its fundamental value μ_z, informed speculators find it optimal to ride the positive bubble from $t = 1$ to $\tau^* - 1$ because they know momentum speculators will be increasing their long positions: at t they choose $C_t^* = c$ expecting $\Delta S_{t+1}^* > 0$, at $t + 1$ they set $C_{t+1}^* = c$ expecting $\Delta S_{t+2}^* > 0$, and so. However, when S_t^* gets sufficiently far away from μ_z, there is a *switching time* τ^*, when a liquidity squeeze becomes a possibility. Because the probability of a liquidity squeeze is large enough (i.e., $\overline{\sigma} > \phi\theta^{-1}$), at τ^* informed speculators expectations of a Dollar depreciation switch to expected appreciation: $E_{\tau^*}^I\left[\Delta S_{\tau^*+1}^*\right] < 0$. Because informed speculators are forward-looking, at this switching time τ^* they rationally close their long positions and establish short

Figure 3: Principle of Contrarian Opinion



positions in the foreign bond, i.e., they set $C_t^* = -c$ between time τ^* and time l^*, when a liquidity squeeze is triggered. The key point is that at switching time τ^*, the depreciating path necessarily continues, i.e., S_τ^* increases. This ensures that informed speculators do not fear a capital loss and find it optimal to ride the positive bubble from $t = 1$ all the way to $t = \tau^* - 1$. At τ^*, momentum speculators increase their long positions (with hedgers taking the corresponding larger short positions). Thereafter, momentum speculators continue choosing long positions up to liquidity squeeze time l^*, when S_t^* falls back to μ_z.

The equilibrium S_t^* necessarily increases when informed speculators' demand falls at time τ^* because the demand of momentum speculators is more sensitive to the current exchange rate than that of hedgers ($\theta > \phi$), which is a necessary condition for the existence of bubbly equilibria (Lemma 3). To see this consider Figure 3, which depicts market clearing condition (6). The supply schedule $q_t^F = \phi(S_t - \mu_z)$ is the hedgers' net supply function (3), while the demand schedule $Q_t = \theta(S_t - S_{t-1}) + C_t^I$ is the sum of momentum and informed speculators' net demands (1) and (4). Notice that the demand schedule is upward sloping because $\theta > 0$, and it is less steep than the supply schedule because $\theta > \phi$. Now, a fall in the informed speculators' demand from $C_t^I = c$ to $C_t^I = -c$ shifts the demand schedule to the left. Because the supply is steeper than the demand, the equilibrium S_t must increase in response to the fall in C_t^I. If instead S_t^* went down, the momentum speculators' demand for the foreign bond would fall more than the hedgers' supply, and so S_t^* would fall even more. □

Remark 2.2. Proposition 7 rationalizes contrarianism. Near an extreme overval-

uation, when the reversal of a Dollar depreciation trend becomes nigh, uniformed speculators (which are momentum driven) continue accumulating long positions in the foreign asset as the Dollar keeps depreciating, while informed speculators close their long positions. Shortly thereafter the Dollar appreciates back to its fundamental value, proving the majority to be wrong. We refer to this pattern as the principle of contrarian opinion.[14] This contrarian pattern is ensured by the first inequality in (13). □

Remark 2.3. Because $\theta > \phi$, from (16) we see that the equilibrium S_t^* is strictly increasing for $t < \tau^*$ because

$$\Delta S_t^* = \frac{z}{(1-\phi/\theta)^{t-1}} \frac{\phi}{\theta} > 0. \tag{19}$$

Moreover, the magnitude of the exchange rate changes increases strictly over time

$$\Delta S_t^* / \Delta S_{t-1}^* = (1-\phi/\theta)^{-1} > 1. \ \square \tag{20}$$

Remark 2.4. At switching time τ^*, S_t^* increases even faster than before since

$$\Delta S_{\tau^*}^* = \frac{z}{(1-\phi/\theta)^{\tau^*-1}} \frac{\phi}{\theta} + \frac{2c}{\phi} \frac{\phi/\theta}{1-\phi/\theta}. \tag{21}$$

Comparing equations (19) with (21), we can see that there is a jump in the equilibrium exchange rate change at switching time τ^*. Furthermore, at time τ^*, there is a jump in the ratio of the changes of the equilibrium exchange rate:

$$\frac{\Delta S_{\tau^*}^*}{\Delta S_{\tau^*-1}^*} = \frac{1}{1-\phi/\theta} + \frac{c\theta}{\phi^2 z} \frac{\phi/\theta}{(1-\phi/\theta)^{\tau^*-1}}. \tag{22}$$

Despite the larger Dollar depreciation at time τ^*, informed speculators expect the Dollar to start appreciating, i.e., $E_{\tau^*}^I[\Delta S_{\tau^*+1}^*] < 0$. This is because the probability of a liquidity squeeze increases from 0 to at least $\overline{\sigma}$, and the first inequality in (13)

[14] Referring to Jess Livermore's sale of Consolidated Steel, Lefevre (1923), Ch. XXII, explains how through cleverly timed purchases, the informed speculator can create the aura of such interest in an asset that she might move the price higher even while, on a net basis, dispensing of a position in it: "I didn't even do my principal selling on the way down, but on the way up. It was like a dream of Paradise to find an adequate buying power created for you without stirring a finger to bring it about, particularly when you were in a hurry."

ensures $\overline{\sigma}$ is large enough to generate $E^I_{\tau^*}[\Delta S^*_{\tau^*+1}] < 0$.

The increase in volatility at time τ^* resembles the explosive behavior of asset prices typically observed near the so-called top formations that tend to mark the end of price uptrends and the beginning of price downtrends. Investment practitioners refer to these switching times as distribution times because positions are distributed from smart-money to dumb-money.[15] □

Remark 2.5. After switching time τ^*, we have

$$\Delta S^*_t = \frac{2c}{\phi}\frac{\phi/\theta}{(1-\phi/\theta)^{t-\tau^*+1}} + \frac{z\phi/\theta}{(1-\phi/\theta)^{t-1}} \text{ and } \frac{\Delta S^*_t}{\Delta S^*_{t-1}} = \frac{1}{1-\phi/\theta}. \tag{23}$$

As we can see, the jump in the ratio of changes of the equilibrium exchange rate is temporary. It comes back to the level it had before the switching time τ^*. □

Remark 2.6. If there is no further restriction on the latent parameters of the model, the equilibrium switching time τ^* may occur either at $\tau_* + 1$ or $\tau_* + 2$ because during these two periods there are multiple equilibrium exchange rates. For any t with $1 \leq t - \tau_* \leq 2$, a long position by informed speculators ($C^*_t = c$) induces positive expected returns ($E^I_t[\Delta S^*_{t+1}] > 0$), while a short position induces negative expected returns. However, for any $t \geq \tau_* + 3$, expected returns are negative regardless of the informed speculators positions (i.e., ($E^I_t[\Delta S^*_{t+1}] < 0$ for all $C^*_t \in \{-c, 0, c\}$). From Lemmas 13 and 15 in Subsection B.2 of the Appendix, we see that the switching time τ^* is unique, and equal to $\tau_* + 1$, if and only if

$$\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta. \tag{24}$$

The above condition ensures that $S_{\tau_*+1} - \mu_z > \vartheta$ regardless of the informed speculators' demand at time $\tau_* + 1$, which implies that the probability of a liquidity squeeze next period becomes positive and at least $\overline{\sigma}$. In contrast, if (24) does not hold, then τ^* may be either $\tau_* + 1$ or $\tau_* + 2$. □

Lastly, the next proposition shows that there exists an upper bound on the duration of the trend, such that a reversal of the trend occurs before time $\bar{l}_{\max}$ with probability one.

[15] See for instance the accounts of Lefevre (1923) and Soros (1987).

Proposition 8. *The liquidity squeeze will be triggered in finite time, i.e.,* $\tau^* \leq l^* \leq \bar{l}_{\max}$ *where* $\bar{l}_{\max}$ *is a finite integer defined as*

$$\bar{l}_{\max} = \min\left\{ t \geq \tau_* + 3 : \frac{z}{(1-\phi/\theta)^{t-2}} + \frac{2c}{\phi(1-\phi/\theta)^{t-\tau_*-2}} - \frac{c}{\phi} > \overline{\vartheta} \right\}.$$

Moreover, $S_t^* = \mu_z$ *and* $C_t^* = q_t^{m^*} = q_t^{h^*} = 0$ *for* $t = 0$ *and* l^*.

3 Empirical Implications of the Model

As we saw in Section 2, switches in the informed speculators' positions convey useful information about imminent trend reversals. If one had data that included exclusively informed speculators' positions, then one could forecast the incoming trend reversals directly. Unfortunately, publicly available speculator-position data is scarce and noisy, as it is typically a mixture of informed and uninformed market participants' positions. So higher/lower speculators' net positions in the foreign currency are not always associated with a Dollar depreciation/appreciation in the near future. Here is where our theoretical model is handy: If speculator-position data contains a mixture of informed and uninformed speculators, the econometrician can backup useful information about an imminent exchange rate trend reversal by estimating dates when informed speculators switch their positions along paths that satisfy the properties of the PCO equilibrium we have characterized in Proposition 7, Proposition 8, and Corollary 19 in the Extended Appendix. Namely, times when the end of an uptrend is nigh are dates when: (i) informed speculators have switched their long positions into short positions; (ii) S_t^* has been in an uptrend; and (iii) there has been positive momentum. Times when the end of a downtrend is nigh are analogously determined. Along a trendless path, a switch to a long/short position by informed speculators signals the beginning of an uptrend/downtrend.

Concretely, suppose that net speculators' position data is a mixture of informed speculators q_t^I, uninformed momentum-driven speculators q_t^m, and noise ε_t:

$$Y_t = q_t^I + q_t^m + \varepsilon_t, \tag{25}$$

where ε_t is assumed to be i.i.d. normal with mean zero and variance σ_ε^2. Then the

PCO equilibria imply that (25) can be expressed as:

$$Y_t = c_{(\kappa_t)} + \theta \Delta S_t^* + \varepsilon_t, \tag{26}$$

where $\Delta S_t^* = S_t^* - S_{t-1}^*$ is the demand of momentum speculators, and the discrete component $c_{(\kappa_t)}$ is the demand of informed speculators. Depending on the state κ_t, the mean $c_{(\kappa_t)}$ of the net demand of informed speculators may take a positive (net long-position), negative (net short position) or zero value:

$$c_{(\kappa_t)} = \begin{cases} c_{(1)} = c, & \text{if } \kappa_t = 1 \quad (E_t^I \left[\Delta S_{t+1}^*\right] > 0) \\ c_{(0)} = 0, & \text{if } \kappa_t = 0 \quad (E_t^I \left[\Delta S_{t+1}^*\right] = 0) \\ c_{(-1)} = -c, & \text{if } \kappa_t = -1 \ (E_t^I \left[\Delta S_{t+1}^*\right] < 0) \end{cases} . \tag{27}$$

The state variable κ_t is assumed to follow a first-order Markov process with transition matrix $\Pi = [p_{i,j}]_{i,j=-1,0,1}$, where $p_{i,j} = \Pr(\kappa_t = j | \kappa_{t-1} = i)$ is the conditional transition probability that the state is j at time t given that it was i at time $t-1$. The empirical model specified in (26) and (27) is a particular example of the MSM. Our forecasting strategy consists of two steps. First, we estimate the MSM in (26) and find the most likely hidden state $\hat{\kappa}_t$, which allows us to uncover the sign of the informed speculators's conditional expectation about exchange rate changes $E_t^I \left[\Delta S_{t+1}^*\right]$. Second, using $\hat{\kappa}_t$ and the properties of the PCO equilibria, we generate out-of-sample directional forecasts of exchange rate changes over several horizons.

Data. We use speculator position data from the COT report of the Commodity Futures Trading Commission (CFTC). The CFTC requires all large traders to identify themselves as either non-commercial or commercial. Non-commercial and commercial traders may be associated with our model's speculators and hedgers, respectively. We will proxy our model's speculators' positions in the foreign bond Y_t with the foreign currency futures net contracts held by non-commercial traders. Typically, on each Tuesday the CFTC gathers position data and on the Friday of the same week, it publishes the COT report which includes the non-commercial and commercial position data. We consider five major currencies vis-a-vis the US dollar: the Australian dollar (AUD), the Canadian Dollar (CAD), the Euro (EUR), the Japanese yen (JPY), and the British pound (GBP). We use weekly spot exchange rates at the end of trading on Friday, released by the Federal Reserve Board. Our sample begins in September

1992, which is when the CFTC started to release the COT report on a weekly basis, except for the Euro, for which the data begins in January 1999. Our sample ends on the last week of 2017.

Estimation. For each of the five currencies, we estimate the MSM specified in (26) using the EM algorithm proposed by Hamilton (1990) with a rolling sample $\{Y_s, \Delta_k S_s^*\}_{s=r-R+1}^{s=r}$ $(r = R, R+1, \ldots, T)$, where r is the index of the rolling sample, R is the window size, and T is size of the full sample.[16] For each $r \in \{R, R+1, \ldots, T\}$, we use the sample $\{Y_s, \Delta_k S_s^*\}_{s=r-R+1}^{s=r}$ to estimate the unknown parameters in the model (26), and then compute the estimator of the unconditional probability $\xi_{j,s} = \Pr(\kappa_s = j)$ for $j \in \{1, 0, -1\}$ and $s \in \{r-R+1, \ldots, r\}$. Let $\widehat{\xi}_{j,s}$ denote the estimator of $\xi_{j,s}$, i.e., the so called filtered probability. We consider exchange rate changes $\Delta_k S_t^*$ with a lag $k = 4$ weeks and three rolling window sizes: $R = 70$, 90, and 110 weeks respectively.[17,18] To ensure that a global maximum is attained, we consider 150 different initial values of parameters for each estimation.

For each time r, we use the filtered probabilities $(\widehat{\xi}_{-1,r}, \widehat{\xi}_{0,r}, \widehat{\xi}_{1,r})$ to determine the most likely state of the informed speculators' positions at time r $(r = R, R+1, \ldots, T)$ as follows:

$$\widehat{\kappa}_r = \begin{cases} 1, & \text{if } \widehat{\xi}_{1,r} > \max\left\{\widehat{\xi}_{0,r}, \widehat{\xi}_{-1,r}\right\} \\ 0, & \text{if } \widehat{\xi}_{0,r} > \max\left\{\widehat{\xi}_{1,r}, \widehat{\xi}_{-1,r}\right\} \\ -1, & \text{if } \widehat{\xi}_{-1,r} > \max\left\{\widehat{\xi}_{1,r}, \widehat{\xi}_{0,r}\right\} \end{cases}.$$

If the most likely state is $\widehat{\kappa}_r = 1$ (resp. -1), our estimate is that informed speculators take net long positions (resp. net short portions) at time r. Meanwhile, if $\widehat{\kappa}_r = 0$, our estimate is that they take a zero position.[19]

[16] The main advantage of the EM algorithm over direct numerical optimization methods is its robustness with respect to the multiple local maxima problem.

[17] We have used $\Delta_k S_t \equiv S_t - S_{t-k}$, with $k = 4$ rather that $S_t - S_{t-1}$ in estimation and forecasting to better capture momentum effects.

[18] The estimation results are quite robust with respect to the lag k and the size of the rolling window R. In Tables H.2-H.6 (in the Extended Appendix), we report the correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$.

[19] Figure H.2 in Appendix depicts the series of the estimated intercepts $\left(c_{(-1)}, c_{(0)}, c_{(1)}\right)$ with rolling samples. For all currencies, the estimators of $c_{(1)}$ are mostly positive; the estimators of $c_{(-1)}$ are mostly negative; while the estimators of $c_{(0)}$ fluctuate around zero. Figure H.3 in Appendix shows the evolution of the estimated filtered probabilities $(\widehat{\xi}_{-1,r}, \widehat{\xi}_{0,r}, \widehat{\xi}_{1,r})$. Figure H.4 in Appendix depicts the series of the estimators of θ and the lower bounds of the 0.95 confidence intervals in (26) for each currency.

3.1 Directional Forecasts

Given the sequence of most likely states $\{\widehat{\kappa}_r\}_{r=R}^T$, we backup the information of informed speculators as follows. First, by Proposition 7 and Proposition 8, times when an uptrend is likely to end soon $\widehat{\tau}^*_{(-)}$, correspond to dates r when: (i) the most likely state is $\widehat{\kappa}_r = -1$; (ii) S^*_t has been in an uptrend, i.e., $\Delta_m S^*_t = S^*_t - S^*_{t-m} > 0$ over the past $m = 8$ weeks; and (iii) there has been positive momentum, i.e., the estimated coefficient of the momentum traders' demand $\widehat{\theta}_r$ is positive and statistically significant: $\widehat{\theta}_r/\widehat{\sigma}^\theta_r > z_{0.05}$, where $\widehat{\sigma}^\theta_r$ is the estimated standard deviation of $\widehat{\theta}_r$, and $z_{0.05}$ is the 95% quantile of the standard normal distribution. Second, by Corollary 19, times when a downtrend is likely to end soon $\widehat{\tau}^*_{(+)}$, correspond to dates r when: (i) the most likely state is $\widehat{\kappa}_r = 1$; (ii) S^*_t has been in a downtrend $\Delta_m S^*_t = S^*_t - S^*_{t-m} < 0$ over the past $m = 8$ weeks; and (iii) there has been negative momentum: $\widehat{\theta}_r/\widehat{\sigma}^\theta_r > z_{0.05}$. Third, by Proposition 7 and Proposition 8 (resp. Corollary 19 in the Extended Appendix), times when a trendless path is likely to end soon and give way to an uptrend $\widehat{\tau}^*_{(0^+)}$ (resp. downtrend $\widehat{\tau}^*_{(0^-)}$), correspond to dates r when we detect the most likely state shifts to $\widehat{\kappa}_r = 1$(resp. $\widehat{\kappa}_r = -1$) after having been zero over the past 4 weeks.

As we can see, Proposition 7, Proposition 8 and Corollary 19 in the Extended Appendix imply contrarian forecasts. Thus, whenever we detect that $r = \widehat{\tau}^*_{(+)}$ or $r = \widehat{\tau}^*_{(0^+)}$, we predict a trend reversal within a short period and forecast that $S^*_{r+h} - S^*_r > 0$ over the next h weeks, i.e., the Dollar will depreciate (appreciate). Analogously, whenever we detect $r = \widehat{\tau}^*_{(-)}$ or $r = \widehat{\tau}^*_{(0^-)}$, we predict $S^*_{r+h} - S^*_r < 0$ over the next h weeks. That is, our contrarian directional forecasting strategy is

$$D_{r,h} = \begin{cases} -1, & \text{if } r = \widehat{\tau}^*_{(-)} \text{ or } \widehat{\tau}^*_{(0^-)} \\ 1, & \text{if } r = \widehat{\tau}^*_{(+)} \text{ or } \widehat{\tau}^*_{(0^+)} \\ 0, & \text{otherwise} \end{cases}, \tag{28}$$

where $D_{r,h} = -1$(resp. $+1$) means that our directional forecast over an h-period horizon is a decline in S^* (resp. an increase in S^*), i.e., a Dollar appreciation (resp. depreciation). Meanwhile, if $D_{r,h} = 0$, we predict no change.

During every week, we generate out-of-sample exchange rate directional forecasts for five horizons: $h \in \{4, 13, 26, 39, 52\}$ weeks, which correspond to 1, 3, 6, 9 and 12 months forecasting horizons respectively. For each currency i, our directional forecasts

Table 1: Success Ratio of the Directional Forecasts

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.582	0.577	0.617	0.565	0.603
	(201)	(201)	(201)	(200)	(199)
CAD	0.577	0.640	0.720	0.690	0.647
	(175)	(175)	(175)	(174)	(173)
EUR	0.586	0.559	0.699	0.669	0.645
	(145)	(143)	(143)	(142)	(141)
JPY	0.553	0.551	0.614	0.689	0.689
	(208)	(205)	(197)	(190)	(180)
GBP	0.563	0.591	0.534	0.529	0.554
	(208)	(208)	(208)	(208)	(204)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

start in the week after the first estimation of the MSM, and end h weeks prior to the end of our sample. That is, they start in week R and end in week $T-h$. Our sample starts on 10/2/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017. Thus, our directional forecasts start on 01/20/1995 for the Pound; 09/01/1994 for the Canadian Dollar; 04/15/1994 for the Yen and the Australian Dollar; and 05/04/2001 for the Euro.[20]

In all tables and figures, we set the rolling window size $R = 70$ weeks for the Pound; $R = 90$ weeks for the Euro and Canadian Dollar; and $R = 110$ weeks for the Yen and the Australian Dollar.[21] Table 1 exhibits the forecast success ratios at the *1m, 3m, 6m, 9m,* and *12m* horizons. The forecast success ratio is the number of successful forecasts divided by the total number of forecasts. As we can see, we obtain a 61% average forecast success ratio across all currencies over the period October 1992-December 2017. Interestingly, the forecast accuracy is greater at the *6m* to *12m* horizons than at the *1m* to *3m* months horizons. If we confine our attention to *6m*,

[20]The dates of the last forecasts are 12/08/2017 for $h = 4$, 10/06/2017 for $h = 13$, 07/06/2017 for $h = 26$, 04/07/2017 for $h = 39$, and 01/06/2017 for $h = 52$.

[21]See Tables H.7 - H.13 in the Extended Appendix for the directional forecasting results with window sizes $R = 70$, 80, 90, 100, 110, 120, 130 for all five currencies.

9m and *12m* forecasting horizons, the average success ratio is 63.1%. When taken individually, we can see that the success ratios are greater than 52.9% in all cases and that in several cases the success ratio is larger than 60%.

The high success ratios observed in Table 1 are not dominated by either appreciation or depreciation forecasts. Table D.2 in Appendix D reports the success ratios of the appreciation and depreciation forecasts, separately. Moreover, the high forecast success ratios are not limited to specific periods. In most country-horizon pairs, the performance of the directional forecast is stable over the sample period.[22]

3.2 A New Weighted Directional Test

The high forecasting success ratios in Table 1 illustrate the usefulness of the COT data and our contrarian forecasting method in generating directional exchange rate predictions. Two issues remain, however. First, wouldn't flipping a coin result in similar or even better success ratios? To answer this question, we conduct the standard binomial test to evaluates whether our directional forecasts outperform the random walk forecasts. The second and probably more important issue is whether our directional forecasts succeed in predicting big moves in exchange rates. This question captures the spirit of George Soros's observation: It's not whether you're right or wrong, but how much money you make when you're right and how much you lose when you're wrong. To answer this question, we propose a novel directional test that weights each directional forecast by the realized exchange rate change. Our test has the same spirit as the trading rule considered by Elliott and Ito (1999) to evaluate the predictability content of survey forecasts.

In order to motivate such test, consider the following investment rule. At time r, if the h-period-ahead directional forecast is an increase in S, buy 1 unit of the foreign bond at time r and sell it at time $r + h$; if the h-period-ahead directional forecast is a fall in S, sell short 1 unit of the the foreign bond at time r and buy it back at time $r + h$; if the directional forecast is zero, then take no position at time r. We consider two profit measures of the above investment rule. The first is the average

[22]Figures H.5 through H.9 (in the Extended Appendix) exhibit this stability by plotting the evolution of the cumulative forecast success ratios for each forecasting horizon. Approximately after 400 weeks, these ratios converge to a stable level, above 50% in most country-horizon pairs. Initially, however, these ratios fluctuate quite a bit because the sample size is small. For example, in Figure H.8 of the Extended Appendix, the *9m* ahead forecasts for the Japanese Yen initially fluctuate between 60% and 90% before converging to nearly 70%.

profit, which is estimated by

$$T_{a,n} = T_1^{-1} \sum_{r=R}^{T-h} D_{r,h}(S^*_{r+h} - S^*_r), \text{ where } T_1 = T - R - h + 1. \tag{29}$$

Under the random walk assumption, the exchange rate is not predictable since for any $r \geq R$, the conditional expectation of $S^*_{r+h} - S^*_r$, given all the information at time r, is zero. We can use the test statistic in (29) to test the null hypothesis that the average profit of the above investment rule is zero:

$$H_0^1 : E\left[D_{r,h}(S^*_{r+h} - S^*_r)\right] = 0. \tag{30}$$

The alternative hypothesis of (30) is that the weighted sum of our directional forecasts is positive. That is, our forecasts generate expected positive profits:

$$H_1^1 : E\left[D_{r,h}(S^*_{r+h} - S^*_r)\right] > 0. \tag{31}$$

The second measure is the average profit conditioning on the event that a non-zero directional forecast has been made, which is estimated by

$$T_{b,n} = \frac{\sum_{r=R}^{T-h} D_{r,h}(S^*_{r+h} - S^*_r)}{\sum_{r=R}^{T-h} |D_{r,h}|}. \tag{32}$$

Since the exchange rate is not predictable under the random walk assumption, the conditional profit should also be zero. We can use the test statistic in (32) to test the null hypothesis that the conditional profit of the above investment rule is zero:

$$H_0^2 : E\left[D_{r,h}(S^*_{r+h} - S^*_r) \middle| D_{r,h} \neq 0\right] = 0. \tag{33}$$

The alternative hypothesis of (33) is that the weighted conditional sum of our directional forecasts is positive. That is, our forecasts generate expected conditional positive profits:

$$H_1^2 : E\left[D_{r,h}(S^*_{r+h} - S^*_r) \middle| D_{r,h} \neq 0\right] > 0. \tag{34}$$

Let $V_{T_{a,n}}$ denote the consistent estimator of the asymptotic variance of $T_{a,n}$. Then by Slutsky's theorem and the central limit theorem, we deduce that

$$T_1^{1/2} V_{T_{a,n}}^{-1/2} T_{a,n} \to_d N(0,1) \tag{35}$$

under the null hypothesis (30). For the empirical implementation, we construct $V_{T_a,n}$ using two LRV estimators that control for auto-correlation in $\left\{D_{r,h}(S^*_{r+h} - S^*_r)\right\}_{r=R}^{T-h}$ when the random walk assumption does not hold: the Newey-West LRV estimator and the Andrews LRV estimator. By the relation between $T_{a,n}$ and $T_{b,n}$, and asymptotic normality

$$\left(T_1^{-1/2} \sum\nolimits_{r=R}^{T-h} |D_{r,h}|\right) V_{T_{a,n}}^{-1/2} T_{b,n} \to_d N(0,1). \tag{36}$$

Statistical tests of hypotheses (30) and (33) can be conducted using (35) and (36), respectively.

Table 2 presents the values of the $T_{a,n}$ statistic, and its standard errors, for the 5 currencies and the 5 horizons we consider. The null is rejected if the test statistic $T_{a,n}$ is significantly larger than zero. For the one-sided test, a t-value greater than 1.282 implies a 10% significance level. Even after controlling for autocorrelation, over the 6m to 12m horizons, there is very strong evidence of exchange rate predictability in all currencies. As we can see in panel A of Table 2, using the Newey-West LRV estimator, the random-walk null is rejected in favour of our model in 14 out of 15 country-horizon pairs at the 6m, 9m and 12m horizons. Similarly, Panel B shows that using the Andrews LRV estimator to control for autocorrelation, the null is rejected in 13 out of 15 country-horizon pairs. At the 1m and 3m horizons there is weaker evidence of predictability: using the Newey-West LRV estimator, the random walk null is rejected in only 7 out of 10 currency-horizon pairs. This contrast between 1m-3m and 6m-12m forecasts is most sharply observed in the Euro and the Yen.

Comparing Tables 1 and 2 we can see that the high forecast success ratios translate into a rejection of the null of our weighted directional test. However, they are not the same. For example, even though the success ratio of the British Pound at the 9-month horizon is slightly above 50 percent, the null of the weighted directional test is rejected at the 10 percent level after controlling for autocorrelation. In contrast, while the success ratio of the Australian Dollar at the 6-month horizon is above 60 percent, the null of the weighted directional test is not rejected.

Table 3 shows that if we condition on having none-zero directional forecasts, the annualized rate of returns, defined by $T_{b,n}$ in (32), may increase substantially reaching 6.3% for the Euro, 5.3% for the Yen, and 4.9% for the British Pound. Using the Newey-West LRV estimator, the random walk null is rejected in 21 out of 25 currency-horizon pairs. Using the Andrews LRV estimator, the null is rejected in 19 currency-horizon pairs.

Table 2: Weighted Directional Forecasts Test

Currency	Forecasting Horizon (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Newey-West				
AUD	0.063*	0.177**	0.199	0.335*	0.470**
	(0.039)	(0.105)	(0.158)	(0.223)	(0.274)
CAD	0.047*	0.161***	0.253***	0.304***	0.258**
	(0.031)	(0.064)	(0.108)	(0.124)	(0.147)
EUR	0.042	0.159*	0.521***	0.746***	0.629***
	(0.045)	(0.124)	(0.191)	(0.258)	(0.268)
JPY	0.029	0.055	0.439***	0.648***	0.659***
	(0.050)	(0.091)	(0.160)	(0.176)	(0.223)
GBP	0.061*	0.144*	0.221*	0.327*	0.383**
	(0.039)	(0.088)	(0.168)	(0.206)	(0.203)
	Panel B: Andrews				
AUD	0.063*	0.177*	0.199	0.335*	0.470**
	(0.039)	(0.109)	(0.164)	(0.233)	(0.284)
CAD	0.047*	0.161***	0.253**	0.304**	0.258*
	(0.031)	(0.067)	(0.118)	(0.139)	(0.169)
EUR	0.042	0.159	0.521**	0.746**	0.629**
	(0.046)	(0.138)	(0.227)	(0.322)	(0.327)
JPY	0.029	0.055	0.439***	0.648***	0.659***
	(0.051)	(0.095)	(0.178)	(0.195)	(0.249)
GBP	0.061*	0.144*	0.221	0.327*	0.383**
	(0.039)	(0.095)	(0.202)	(0.246)	(0.230)

Note: Panel A reports the test results using the Newey-West LRV estimator to control for autocorrelation. Panel B reports the test results using the Andrews LRV estimator to control for autocorrelation. standard errors are in parentheses. We consider an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The information on our directional forecasts is described in the note to Table 1.

The high success ratios of our directional forecasts are reflected in Table D.1 in the Appendix, which reports the means of one-year ahead changes in the exchange rates. For all currencies, the mean exchange rate change conditional on an appreciation(resp. depreciation) forecast is greater(resp. smaller) than the unconditional mean.

Table 3: Annualized Returns of the Directional Forecasts

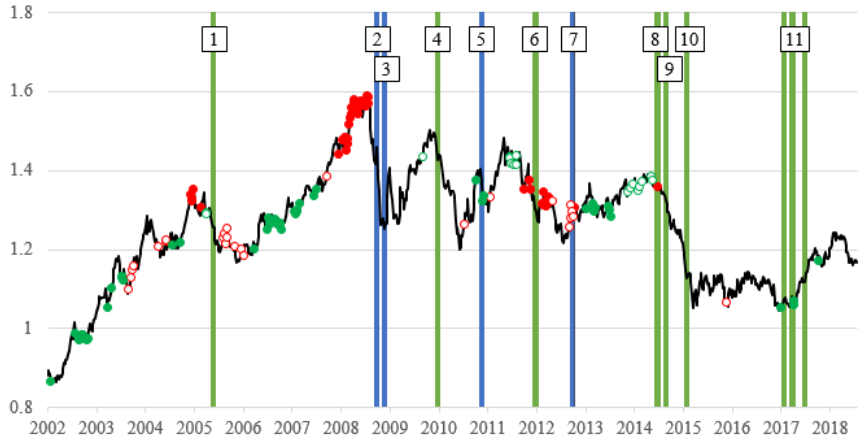
Currency	Forecasting Horizon (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Newey-West				
AUD	4.914*	4.206**	2.338	2.609*	2.723**
	(3.079)	(2.488)	(1.851)	(1.738)	(1.588)
CAD	4.320*	4.463***	3.486***	2.765***	1.748**
	(2.809)	(1.772)	(1.494)	(1.129)	(0.997)
EUR	3.376	3.975*	6.359***	6.021***	3.776***
	(3.644)	(3.082)	(2.330)	(2.078)	(1.610)
JPY	2.188	1.277	5.260***	5.305***	4.224***
	(3.764)	(2.122)	(1.913)	(1.441)	(1.428)
GBP	4.696*	3.412*	2.594*	2.533*	2.242**
	(2.995)	(2.084)	(1.967)	(1.591)	(1.190)
	Panel B: Andrews				
AUD	4.914*	4.206*	2.338	2.609*	2.723**
	(3.051)	(2.592)	(1.921)	(1.811)	(1.649)
CAD	4.320*	4.463***	3.486**	2.765**	1.748*
	(2.809)	(1.860)	(1.627)	(1.267)	(1.145)
EUR	3.376	3.975	6.359**	6.021**	3.776**
	(3.701)	(3.441)	(2.765)	(2.598)	(1.965)
JPY	2.188	1.277	5.260***	5.305***	4.224***
	(3.819)	(2.211)	(2.137)	(1.593)	(1.593)
GBP	4.696*	3.412*	2.594	2.533*	2.242**
	(3.008)	(2.251)	(2.364)	(1.903)	(1.348)

Note: Panel A reports the test results using the Newey-West LRV estimator to control for autocorrelation. Panel B reports the test results using the Andrews LRV estimator to control for autocorrelation. standard errors are in parentheses. We consider an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The information on our directional forecasts is described in the note to Table 1.

3.3 Model Forecasts and Major Events

In order to investigate whether our model's forecasts are not mere statistical objects, we investigate whether major events, identified in the literature as driving forces behind big swings in the Euro and the Yen, are linked to our model's signals. Figures 4 and 5 depict the 9-month ($h = 39$) ahead forecasts using strategy $D_{r,h}$ in (28) for the Euro and the Yen against the US Dollar. The dates of major events are marked

Figure 4: Success and Failures of 9-month Ahead Directional Forecasts of Euro



#	Event Date	Signal Date	Event	Reference
1	May 2005	Nov 2004	European Constitution referendums	OECD (2005)
2	Sep 2008	Jan 2008	Lehman bankruptcy	
3	Nov 2008	Jan 2008	QE1 by US Fed	Rogers et al. (2018)
4	Dec 2009	-	Greek crisis	Kräussl et al. (2016)
5	Nov 2010	Oct 2010	QE2 by US Fed	Rogers et al. (2018)
6	Dec 2011	Sep 2011	LTRO by ECB	
7	Sep 2012	Jan 2013	OMT by ECB, QE3 by US Fed	Rogers et al. (2018)
8	Jun 2014	Jun 2014	QE, TLTRO by ECB	Hutchinson and Smets (2017), Dell'Ariccia et al. (2018)
9	Aug 2014	Jun 2014	Draghi's QE speech	
10	Jan 2015	-	QE announcement by ECB	Hutchinson and Smets (2017), Dell'Ariccia et al. (2018)
11	Jan 2017	Dec 2016	Political elections in euro area	Gerlach (2018)

Note: The figure illustrates the performance of 9-month ahead directional forecasts. Green (red) dots depict the time when an appreciation (depreciation) forecast has been made. If a forecast turned out to correctly predict the direction of movements of the currency, then the dot is filled. If the forecast is wrong, the dot is empty.

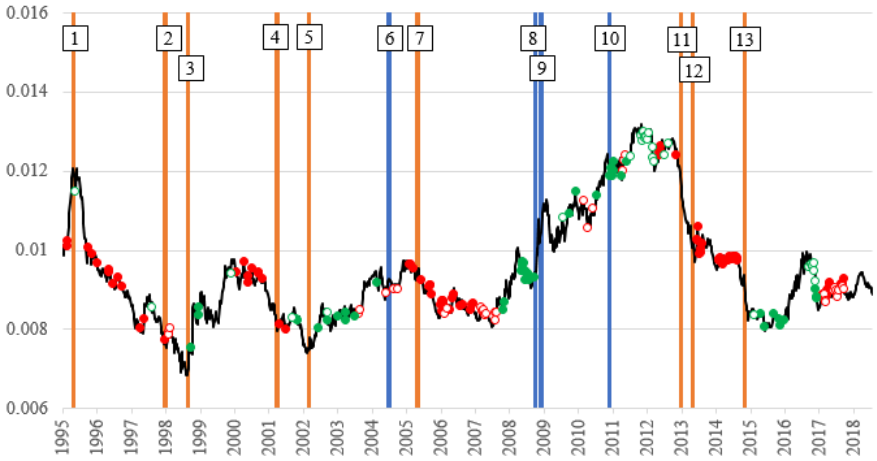
with lines (green for the Eurozone events, blue for the US events, and orange for Japan events). The green balls depict Euro (or Yen) appreciation forecasts, and the red balls depict Euro (or Yen) depreciation forecasts. The full balls are the forecasts that turned out to be successful, while the empty balls are unsuccessful forecasts.[23]

As we can see in Figures 4 and 5, the signals of our model often succeed in foreseeing major events, and in several occasions indicate when the subsequent exchange rate move may end. The informed speculators in our model seem to be able to interpret the effects of major events on exchange rates.

Our forecasting strategy has tracked the tumultuous period for the Eurozone be-

[23]The results for the other currencies are plotted in Figure D.1 in the Appendix.

Figure 5: Success and Failures of 9-month Ahead Directional Forecasts of Japanese Yen



#	Event Date	Signal Date	Event	Reference
1	Apr 1995	Feb 1995	FX interventions by Japan and US	Obstfeld (2009)
2	Late 1997	May 1997	Japanese banking crisis	Obstfeld (2009)
3	Aug 1998	Sep 1998	Russian financial crisis	Cai et al. (2001)
4	Mar 2001	Oct 2000	QE by BOJ	Kimura and Small (2006)
5	Feb 2002	Oct 2001	Japanese deflation	Obstfeld (2009)
6	Jun 2004	May 2004	Tightening MP by US Fed	Obstfeld (2009)
7	Apr 2005	Mar 2005	Japanese long-term real interest rates fall	Obstfeld (2009)
8	Sep 2008	Apr 2008	Lehman bankruptcy	
9	Nov 2008	Apr 2008	QE1 by US Fed	Rogers et al. (2018)
10	Nov 2010	Nov 2010	QE2 by US Fed	Rogers et al. (2018)
11	Dec 2012	Oct 2012	Abenomics launched	Hausman and Wieland (2014)
12	Apr 2013	Jun 2013	QQE introduced by BOJ	Hausman and Wieland (2014)
13	Oct 2014	Aug 2014	QQE accelerated	Hausman and Wieland (2015)

Note: The figure illustrates the performance of 9-month ahead directional forecasts. Green (red) dots depict the time when an appreciation (depreciation) forecast has been made. If a forecast turned out to correctly predict the direction of movements of the currency, then the dot is filled. If the forecast is wrong, the dot is empty.

tween 2008-2016 fairly well. First, after exhibiting only bullish signals for three years, in January 2008 our forecasting strategy started to exhibit a series of bearish signals. Tellingly, these successful bearish signals preceded the Lehman crisis of September 2008. Second, following the announcement of quantitative easing by the US Fed, the model issued a bullish signal in August 2009. Although the Euro appreciated for a few months, the forecast of an appreciation over a 9-month horizon ultimately proved to be wrong. Perhaps this is because the Greek crisis erupted in December 2009, an event around which our model does not exhibit any bearish signal.[24] Third,

[24] Kräussl et al. (2016) investigated the impacts of Greek crisis on the Euro exchange rates and

in November 2010, the US Fed announced the second phase of quantitative easing (Rogers et al. (2018)). We observe successful bullish signals in October 2010 before the program's announcement and slightly after. Fourth, in December 2011, to stem the Eurozone banking crisis, the ECB announced the Long Term Refinancing Operations (LTRO) program; it is similarly preceded and followed by successful bearish signals in September 2011. Fifth, the deepening of the crisis compelled Mario Draghi, president of the ECB, to make his "whatever it takes" statement in the summer of 2012, as well as the ECB's announcement of the Outright Monetary Transactions (OMT) program in September of the same year. Our model exhibits a series of wrong bearish signals in September 2012. Perhaps this is because bad news for the Euro were dominated by the QE3 of the US Fed. Shortly thereafter, in January 2013 our model exhibits a series of successful bullish signals. Sixth, in a speech at Jackson Hole in August 2014 Draghi promised to initiate a QE program, a big surprise to onlookers.[25] The ECB formally announced its QE decision in January 2015, beginning buying in March. Our strategy issued a bearish signal two months before Draghi's speech, in June 2014. Interestingly, most of the Euro's downward 20% move occurred between the time of our signal in June 2014 and January 2015 ECB's announcement of QE (15%). Following the launch of QE in March 2015, our model exhibits a bearish signal. Starting in early 2016, while QE continued to be active, our model exhibits a series of bullish signals, which correctly signal the end of the Euro's downward move (Dell'Ariccia et al. (2018)).

The signals of our model seem also to be consistent with the effects of political events on the Euro. Our model sent successful bearish signals in November 2004 prior to the rejection of the treaty establishing a constitution for Europe in early 2005, which was associated with the subsequent Euro depreciation (OECD (2005)). More recently, it sent signals of the Euro up-turn in December 2016 prior to elections in the Netherlands and France in March and May 2017, respectively. Euro-skeptic politicians lost in these elections, which contributed to the subsequent Euro appreciation (Gerlach (2018)).

Regarding the Yen, in February 1995, our model sent successful signals of the Yen's incoming downtrend, prior to a series of joint foreign exchange market interventions

found that the crisis contributed to the subsequent Euro depreciation.

[25] Draghi's speech was made in a context where the 3-year loans made under the LTRO program were coming due, and there were fears of deflation.

by the US, Japanese and German authorities to support the US dollar, which started in April 1995 (Obstfeld (2009)). Second, the Japanese Banking crisis of late 1997 is preceded by the model's successful bearish signals in May 1997 (Obstfeld (2009)). Third, the LTCM and Russian crises in 1998 were associated with the unwinding of the yen carry trade (Cai et al. (2001)). Our model bullish signal correctly marked the start of the resulting Yen appreciation. Fourth, the launch of QE by the Bank of Japan in March 2001 was preceded by our strategy's October 2000 bearish signal in the Yen. Then, in October 2001, while the first QE was still underway, our strategy correctly signaled the end of the Yen's downward move induced by QE (Kimura and Small (2006)). Fifth, in 2002, the model's bullish signals captured the Yen appreciation associated with the increase in real interest rates, caused by a deepening of Japanese deflation concerns (Obstfeld (2009)). Sixth, in April 2005, Japanese deflation concerns began to lessen and long-term real interest rates started to fall. The Yen reversed to a downtrend as a result (Obstfeld (2009)). Beginning in March 2005, our model issued successful bearish signals along the Yen's downtrend all the way to mid-2007. Seventh, the US 2008 financial crisis was associated with a Yen appreciation that lasted until 2012. The model correctly signalled the impending uptrend in the Yen: After having issued only bearish signals for two years, it started to issue bullish signals in April 2008. More recently, in October 2012 and in mid-2013, our strategy forecasted the incoming Yen's downtrend, prior to the implementation of Abenomics in December 2012 and prior to the acceleration of quantitative and qualitative easing (QQE) in October 2014, respectively. These two unconventional monetary policies were associated with the subsequent Yen depreciation (Hausman and Wieland (2014)) and (Hausman and Wieland (2015)).

4 Contrarian Trading Strategy and its Profitability

Here, we construct a "PCO portfolio" based on our contrarian directional forecasts and compare its profitability with that of time-series momentum strategies, such as those used by Burnside et al. (2011) and Moskowitz et al. (2012). We use as building blocks trading strategies applied to single currencies. Let $D^i_{t,h}$ be the h week ahead directional forecast at time t for currency i, defined in (28). Whenever our directional forecasts predict currency i appreciation over the next h weeks (i.e., $D^i_{t,h} = 1$), we open a long position in currency i and hold it over the next h weeks. Analogously,

whenever our directional forecasts predict currency i depreciation over the next h weeks (i.e., $D^i_{t,h} = -1$), we open a short position in currency i and hold it over the next h weeks.

Since holding-period h is always greater than one week, the size of our bet is not constant, but recursively changes over time. In order to deal with this problem, we follow the methodology used by Jegadeesh and Titman (1993) and Moskowitz et al. (2012). That is, at time t, we invest h^{-1} dollars in a long or short position in currency i when we receive a non-zero signal from our directional forecasts $D^i_{t,h}$. If we follow this trading rule, the time-t position in currency i is the sum of all active positions that have been taken based on the signals $D^i_{t-h+1,h}, \cdots, D^i_{t,h}$ over the past h weeks, i.e., $h^{-1}\sum_{k=0}^{h-1} D^i_{t-k,h}$. Thus, the annualized excess return of the PCO trade with holding-period h for currency i at time $t+1$ (denoted as $r^{PCO,i}_{t+1,h}$) is

$$r^{PCO,i}_{t+1,h} = \left[h^{-1}\sum_{k=0}^{h-1} D^i_{t-k,h}\right] \times er^i_{t+1}, \quad er^i_{t+1} = \Delta S^i_{t+1} - r^{rf}_t, \tag{37}$$

where er^i_{t+1} is the annualized excess return of currency i from time t to time $t+1$, ΔS^i_{t+1} is the annualized log exchange rate change of currency i from time t to time $t+1$ and r^{rf}_t is the risk-free interest rate, which is proxied by the 3-month Treasury bill rate from the Federal Reserve Economic Data (FRED).[26] Next, we define the annualized excess return of the PCO portfolio of n_t currencies with holding-period h at time $t+1$ ($r^{PCO,\,portfolio}_{t+1,h}$) as follows:

$$r^{PCO,\,portfolio}_{t+1,h} = n_t^{-1}\sum_{i=1}^{n_t} r^{PCO,i}_{t+1,h}, \tag{38}$$

where $r^{PCO,i}_{t+1,h}$ is defined in (37). Our trades start on the same day when our directional forecasts begin: 01/20/1995 for the Pound; 09/01/1994 for the Canadian Dollar; 04/15/1994 for the Yen and the Australian Dollar; and 05/04/2001 for the Euro. Our trades end on the last week of 2017.

We compare the performance of our PCO portfolio to a typical *time series-momentum strategy*, $TSM(a,h)$, that goes long (or short) and holds the position

[26] In Appendix D.3, we report results for the case in which excess returns are defined as $er^i_{t+1} = \Delta S^i_{t+1} - (i^{US}_t - i^j_t)$, where i^j_t denotes the 3-month interest rate in country j. The excess returns and Sharpe ratio have the same pattern as those we report here for (37).

Table 4: Annualized Excess Returns and Sharpe Ratios

Currency	Holding Periods (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Equally-weighted Portfolio				
PCO Strategy	0.921	0.785	0.892	0.845	0.655
	(0.692)	(0.825)	(0.964)	(0.948)	(0.872)
$TSM(12, h)$	2.123	1.714	1.368	1.206	1.176
	(0.334)	(0.296)	(0.253)	(0.236)	(0.243)
	Panel B: Volatility-adjusted Portfolio				
PCO Strategy	1.572	1.353	1.356	1.250	0.900
	(0.645)	(0.853)	(1.065)	(1.142)	(0.962)
$TSM(12, h)$	3.805	3.354	2.132	1.474	1.236
	(0.515)	(0.506)	(0.345)	(0.245)	(0.216)

Note: Panel A reports the results of annualized excess returns and Sharpe ratios of the equally weighted portfolio. Panel B reports the results of annualized excess returns and Sharpe ratio of the volatility-adjusted portfolio. Sharpe ratios are in parentheses.

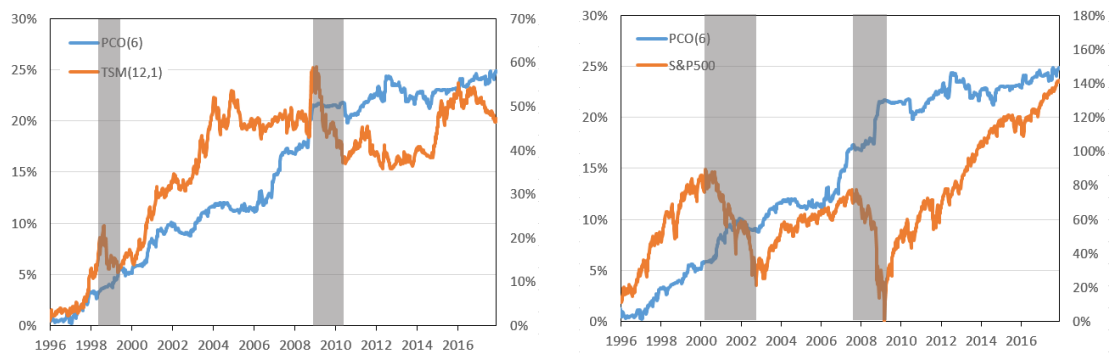
for h months, if the return over the past a months is positive (or negative). Burnside et al. (2011) consider the strategy $TSM(1, 1)$, while Moskowitz et al. (2012) examine various pairs of looking-back and holding periods.

Panel A of Table 4 exhibits the annualized excess returns and the Sharpe ratio of our PCO strategy and the time-series momentum strategy of the equally-weighted portfolio of the five currencies. For the time-series momentum strategy, we report results for the 12-months looking-back period with various holding periods, which provide the highest return and Sharpe ratio, consistent with Moskowitz et al. (2012). The PCO portfolio strategy has an average Sharpe ratio 0.86 across different holding periods, which is four times higher than that (0.22) of the time series momentum strategy.[27]

Table 4 reveals an interesting trade-off. While the PCO portfolio strategy has a lower average annualized excess return (0.82%) than the time-series momentum strategy (1.5%), it has a much higher average Sharpe ratio (0.86 vs. 0.22). The reason for this is that the PCO strategy is designed to perform well during periods when trends switch direction, and thereby its return is crash-free and has low volatility.

[27]The annualized excess returns and the Sharpe ratios of individual currencies are reported in Appendix D.4.

Figure 6: Cumulative Excess Returns



Note: The left figure presents cumulative excess returns of the PCO strategy with 6-months holding periods (blue line, left scale on y-axis), the time-series momentum (TSM) strategy with 12-months lookback periods and 1-month holding periods (orange line, right scale on y-axis). The right figure presents cumulative excess returns of the PCO strategy with 6-months holding periods (blue line, left scale on y-axis) and S&P 500 (orange line, right scale on y-axis). Shaded areas correspond to the periods that TSM (S&P500) loses more than 10% cumulative excess return between the local maximum and local minimum points.

By construction, the PCO strategy is designed not to trade during periods when exchange rates exhibit momentum, and thereby its excess return is relatively low. The crash-free property of the PCO portfolio is illustrated in Figure 6. The left panel compares the cumulative excess returns of the equally-weighted PCO portfolio strategy and the time series momentum strategy $TSM(12, 1)$. The shaded areas correspond to momentum crash episodes: the Russian financial crisis in 1998, the Lehman bankruptcy in 2008. As we can see, the PCO portfolio strategy generates positive returns during these momentum crash episodes. Furthermore, as we can see in the right panel in Figure 6, the PCO portfolio strategy performed well when S&P 500 returns plummeted due to the dot-com bubble crash in 2000 and the Lehman bankruptcy in 2008.

Panel B of Table 4 considers portfolios with the volatility scaling proposed by Moskowitz et al. (2012).[28] The PCO portfolio has an average Sharpe ratio 0.93

[28] Moskowitz et al. (2012) propose a portfolio that is re-scaled so that it has an ex-ante annualized volatility of 40%. The ex-ante annualized volatility of asset i at time t is estimated by its exponentially weighted sample variance of the daily returns:

$$(\sigma_t^i)^2 = 261 \sum_{j=0}^{\infty} (1-\delta)\delta^j (r_{t-1-j} - \overline{r}_t^i)^2,$$

where $\overline{r}_t^i$ is the exponentially weighted average return and δ is chosen so that $\sum_{j=0}^{\infty} (1-\delta)\delta^j = 60$.

across different holding-periods, which is higher than that of the equally-weighted portfolio strategy. As we can see, the performance of the PCO portfolio relative to the momentum portfolio is the same as in Panel A.

5 Related Literature

Exchange rate determination models. Numerous empirical studies have found that in the short-run (less than a year) there is a disconnect between standard observable fundamentals and exchange rates in terms of both in-sample fit and out-of-sample predictability. Meanwhile, over long horizons (3-5 years) there is some evidence of a connection. Engel and West (2005), hereafter EW, express the exchange rate as the current observed fundamentals plus the present discounted value of expected future observed fundamentals $\{f_t\}$, as well as current and expected future unobserved fundamentals and non-fundamental factors $\{U_t\}$: $s_t = f_t + E_t[\sum_{j=1}^{\infty} \beta^j \cdot \Delta f_{t+j}] + U_t$. The disconnect reflects the fact that the deviation of the exchange rate from observed fundamentals $(s_t - f_t)$ can be large and persistent. EW show that if fundamentals have a unit root and the discount factor β is close to one, then exchange rates exhibit near-random walk behavior and they help predict future fundamentals. The implication is that fundamentals are not useful for predicting future exchange rate movements out of sample. Engel and West (2005), and Mark (2009) investigate the contribution of expectations of future fundamentals. As in these papers, time-t news about the future, i.e., shocks to Z_t in (2), are reflected in the time-t exchange rate S_t. The new element here is that a shock to Z_t, in addition to affecting S_t, may trigger a bubbly path. Thus, the equilibrium exchange rate may exhibit momentum.

By investigating the information content of market participant's positions, our paper is linked to the literature on order flow. Evans and Lyons (2002, 2012) consider the difference between buyer- and seller-initiated trades within a set of traders who possess dispersed micro-level news about both $\{U_t\}$ and $\{f_t\}$. This order flow

Therefore, the position size in Moskowitz et al. (2012) is chosen to be $0.4 \times (\sigma_{t-1}^i)^{-1}$ for asset i at time t. Thus, the volatility adjusted excess return of the PCO trade with holding-period h for currency i at time $t+1$ is:

$$r_{t+1,h}^{PCO,i} = \left[h^{-1} \sum_{k=0}^{h-1} D_{t-k,h}^i \right] \times \frac{0.4}{\sigma_t^i} \times er_{t+1}^i$$

information is then impounded into exchange rates by dealers. Evans and Lyons (2002, 2012) show that the daily *order flow* in the interbank foreign exchange market explains a large share of contemporaneous daily exchange rate movements. Evans and Lyons (2005) and Evans and Lyons (2012) show that end-user order flow has significant forecasting power up to a 1-month horizon over exchange rates and fundamentals. Bacchetta and Van Wincoop (2006) consider a setup where order flow, as measured by the market orders's net volume, aggregates heterogeneous information about future fundamentals and hedging demands. While Evans and Lyons focus on order flow, the key forecasting variable in our model is the net position of informed speculators. In our setup, there is no order flow among informed speculators as they all have the same information. Another difference is that Evans and Lyons make up-to-a-month linear forecasts. We, instead, make forecasts of incoming trend reversals by estimating switching times when the subset of informed speculators close their long/short positions along an uptrend/downtrend. Lastly, while the order flow data among informed traders used by Evans and Lyons is proprietary, our strategy applies to the less pristine and noisy data in the COT report, which contains a mixture of informed and uninformed traders, but is public and readily available.

Contrarian forecasts. In contrast to much of the literature, our forecasts are contrarian, which naturally follows from the implications of our theoretical model. Our forecasts are based on our estimates of the informed speculators' switching times. Our estimation method relies on the maximum likelihood estimation of the Markov switching model (hereafter MSM) proposed by Hamilton (1989, 1990). Engel and Hamilton (1990) apply the MSM to exchange rate data to capture the long swings exhibited by exchange rates from the mid 1970s to the end of the 1980s.[29] In this paper, we apply an MSM to the speculators' aggregate net position data, instead of the exchange rate itself. Moreover, rather than a two-state MSM, our model implies three regimes corresponding to the sign of the informed speculators' positions: long, short, and nil. Jordà and Taylor (2012) construct a carry trade strategy using a threshold error correction model, where the adjustment of exchange rates is more likely to be precipitated when fundamentals exhibit deviations exceeding median in-sample levels.

Out-of-sample Forecast Accuracy Tests. Since Meese and Rogoff (1983), it has been

[29] Engel (1994), however, finds that this model does not clearly outperform the random walk in out-of-sample exchange rate forecasts.

difficult for fundamentals-augmented point forecasts to beat the random walk, with few exceptions. The null hypothesis of the CW-test has been rejected by few authors such as Gourinchas and Rey (2007), using net foreign assets, and Molodtsova and Papell (2009) using Taylor-rule fundamentals. But according to recent surveys by Rossi (2013) and Cheung et al. (2018), it is not yet clear whether these results are robust with respect to the sample period, the choice of the currency and forecast evaluation methods. Rossi (2013) reports that at both the 1-quarter and the 4-year forecasting horizons, the DMW-test finds no evidence of forecastability for the monetary model, the PPP model, and the Taylor rule based model across all currencies she considers, while the CW-test finds relatively robust success of the Taylor-rule based model.[30] Furthermore, Cheung et al. (2018) find that still, no existing model can consistently outperform the random walk, especially at less than 1 year horizons. This paper focuses on directional forecasts, not point forecasts. We conduct the familiar binomial test based on the forecast success ratio, as well as a new directional test that weights each directional forecast by the subsequent exchange rate change, and so gives more weight to the directional forecasts associated with bigger exchange rate moves. Across 5 currency pairs, over the 1-month to 12-month horizons, both tests show that our out-of-sample directional forecasts outperform the random walk in a majority of cases.

Models of Bubbles. Our model is related to positive-feedback bubble models in the spirit of De Long et al. (1990). Abreu and Brunnermeier (2003) consider a finite-time economy with financially constrained momentum traders and rational-but-not-perfectly-informed traders. Each informed trader only becomes aware of the bubble at a random time and does not know when other informed traders become aware of the bubble. Because there is an upper bound on the momentum traders demand, the bubble may burst if a critical mass of informed traders exit. In equilibrium, even after all informed traders are aware of the bubble, they ride it for a while, and in some cases all the way to terminal time. Tirole (1982)'s backwards induction argument breaks down because, at least in the initial phase, the expected gain of waiting and getting the higher bubbly return dominates the potential loss. Thus, an informed trader has no incentives to exit. This mechanism is not operative in our model because (i) there is no asymmetric information across informed traders and (ii) the momentum traders

[30] She does not carry out the DMW test for the net foreign assets model.

have a large enough demand capacity so that the exit of all informed traders does not necessarily lead to a price fall. In our setup, backwards induction breaks down too, but for a different reason: informed traders ride the bubble until switching time τ^* because they know that the equilibrium S_t will necessarily increase between $\tau^* - 1$ and τ^*, when all informed traders close their positions and the asset is 'distributed' from informed to uninformed traders. This equilibrium PCO dynamics is the key identification restriction that we use to estimate likely trend reversals. In other words, Abreu and Brunnermeier (2003) focus on the preemption game *within* the group of rational traders, while we focus on the distribution *between* two different groups of traders.

Barberis et al. (2018) consider two types of bounded-rational traders: fundamental traders and extrapolator traders. The latter put weight both on price momentum and on fundamentals. The key is that this weight wavers randomly over time and across extrapolators. They show that a bubble arises if there is a sequence of positive shocks to fundamentals that first increase and then decrease. They also show that trading volume may increase sharply along the bubble. This is because when fundamental traders hit their short-sale constraint, all trade is among extrapolators, and their wavering may induce intense trading volume, especially when some extrapolators hit their short-sale constraint.[31] In contrast, in our model the key variable is the net position of informed speculators, *not* total trading volume. A second difference—which is important for our econometric strategy—is that our informed speculators ride the bubble until its end is nigh, while their fundamental traders exit early. Another difference is that in our setup a single transitory shock can generate a bubble, while in Barberis et al. (2018) a bubble is conditioned on a sequence of increasing and then decreasing positive shocks. One may say that their model is designed to account for large bubbles that occur with a decennary frequency, such as the dot-com bubble, while our model is designed to detect imminent trend reversals in garden variety uptrends and downtrends with frequencies of a few quarters.

Momentum. In analyzing price trends and their reversals, this paper is related to the time-series momentum literature, which documents the short-run positive and long-run negative autocorrelation of returns exhibited by individual securities across

[31] The momentum traders in our setup may be associated with their extrapolators by setting to zero the weight on fundamentals and eliminating wavering. Our hedgers are analogous to their fundamental traders. Our informed speculators are not present in their model.

many asset classes including stocks, commodities, and currencies (e.g., Brunnermeier et al. (2008), Moskowitz et al. (2012), Menkhoff et al. (2012) and Fama and French (1998)). This literature documents *in-sample predictability*. We contribute to this literature by proposing an econometric strategy to generate *out-of-sample contrarian forecasts* about incoming trend-reversals. To our knowledge, this is an original contribution. Our method does not only capture the reversals associated with the rare crashes observed at decennary frequencies, but also the reversals of garden variety uptrends and downtrends observed at frequencies of several quarters. As a result, our contrarian trading strategy based on the PCO attains significantly higher Sharpe ratios than momentum strategies typically used in the literature.

Predictability in Futures Markets. There is a large literature on futures' predictability on hedging pressure dating back to Keynes (1930) which has been generalized since then (e.g., Hirshleifer (1990)). The idea is that if there is a demand for hedging (by 'producers'), then there has to be a risk premium to induce speculators to supply such insurance in the futures market. This means that if hedgers are net short/long—and by consequence speculators are net long/short, then ceteris paribus there is a tendency for the futures price to increase/fall towards the expected future spot price. Notice that this predictability is distinct from our forecasts of a trend reversal. This hedging pressure theory has been examined in currency futures using COT data by Bessembinder (1992) and Wang (2004) among others.

There are some other papers that analyze COT data. Moskowitz et al. (2012) document time-series momentum across different asset classes and find that *in-sample* speculators profit on average from this momentum. Hong and Yogo (2012) find that open-interest in futures markets has *in-sample* predictive power over the excess returns of several assets classes over an one-month horizon. Brunnermeier et al. (2008) find that, *in-sample*, currency crash risk increases with speculators' positions in currency futures markets. Wang (2004) finds *in-sample* a positive correlation between lagged speculator sentiment and future exchange rate changes over horizons from 2 to 8 weeks. This correlation is stronger if one considers the extreme 20% of speculators sentiment. None of these studies performs *out-of-sample* forecasts.

6 Conclusion

Exchange rates tend to exhibit swings of appreciation and depreciation that often last several quarters, sometimes continuing for several years. Although these swings can be identified in-sample, they have proven difficult to predict out-of-sample. Our model provides a novel way of thinking about trends and the contrarian pattern observed across many exchange rates, and generates a novel contrarian forecasting strategy that extracts useful information from noisy speculator position data.

Our starting point is the contrarian pattern observed in speculator position data, whereby at the end of an upswing (downswing), the average currency speculator tends to be overly optimistic (overly pessimistic). This contrarian pattern has a long tradition in the practitioners' literature; one such mechanism is eloquently described by Lefevre (1923). Our contribution is to formalize such a mechanism within a simple heterogeneous-agent model.

In our model, news about future interest rate differentials are observed by informed speculators and by hedgers, becoming impounded into exchange rates via trending bubbly paths, along which a gap grows between the exchange rate and its fundamental value. Informed speculators find it optimal to ride the trend until the gap becomes so large that uninformed momentum-driven speculators may not be able to finance their trades, thus making a trend reversal very likely. At this time, informed speculators switch the sign of their positions in the foreign currency.

Our forecasting method estimates this switching time of informed speculators from noisy market-position data, which contains a mixture of informed and uninformed speculators' positions.

Our out-of-sample contrarian directional forecasts attain success ratios ranging from 53% to 72% across five major currencies. Both the standard Binomial test and a new weighted directional test reject the random-walk null in favor of our model across most currency-horizon pairs.

We have shown that our model's forecast signals are not simply statistical objects; several major events may be associated with our model's signals. Our model's informed speculators seem to have been able to interpret the effects of crisis events and of monetary policies, often foreseeing them and in several occasions even signaling when the associated exchange rate moves may end. For instance, the launch of QE by the Bank of Japan in March 2001 was preceded by our strategy's bearish signal

in the Yen. Then, in October 2001, while QE was still underway, our strategy correctly signaled the end of the Yen's downward move induced by QE. Similar signals were generated by our strategy around the implementation of Abenomics in 2013. In the Euro, our forecasting strategy exhibits bearish signals a few months before the September 2008 Lehman bankruptcy, and before the announcement of the ECB's QE program in January 2015.

Lastly, we find that a contrarian portfolio based on our model's signals is indeed crash-free. During 1995-2017, it generates positive returns during periods when the SP500 or standard momentum strategies suffer severe losses. Furthermore, it attains significantly higher Sharpe ratios than momentum strategies typically used in the literature.

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Appendix For Online Publication

This online Appendix provides supporting materials, proofs, further theoretical exercises and extra empirical results to complement Kim, Kim, Liao and Tornell (2018). The Binomial test of the directional forecasts and associated empirical results are in Section A. Section B presents the proof of the results stated in Section 2 of the main text. Empirical results which are discussed in Section 3 of the main text but not presented there are included in Section D.

A Binomial Test

Here, we test the significance of our model in forecasting the sign of $S^*_{r+h} - S^*_r$, i.e., the forecasting strategy $D_{r,h}$ defined in (28). The null hypothesis we test is that our directional forecasts $D_{r,h}$ are uncorrelated with the sign of the realized exchange rate change $D^*_{r,h} = sgn\left(S^*_{r+h} - S^*_r\right)$:

$$H_0\text{: } Cov\left(D^*_{r,h}, D_{r,h}\right) = 0. \tag{A.39}$$

The alternative hypothesis could be one sided or two sided:

$$H_1^{\text{one-sided}} : Cov\left(D^*_{r,h}, D_{r,h}\right) > 0 \text{ or } H_1^{\text{two-sided}} : Cov\left(D^*_{r,h}, D_{r,h}\right) \neq 0.$$

In order to test the null consider the following test statistic:

$$T_{b,n} = \frac{1}{T_1}\sum_{r=R}^{T-h} D^*_{r,h} D_{r,h} - \frac{1}{T_1}\sum_{r_1=R}^{T-h} D^*_{r_1,h} \frac{1}{T_1}\sum_{r_2=R}^{T-h} D_{r_2,h},$$

which is the sample covariance of the two random variables: $D^*_{r,h}$ and $D_{r,h}$. Let $V_{T_{b,n}}$ denote the consistent estimator of the asymptotic variance of $T_{b,n}$. Then we have

$$\sqrt{n_1} V_{T_{b,n}}^{-\frac{1}{2}} T_{b,n} \to_d N(0,1).$$

The null is rejected if the t-value of the $T_{b,n}$ statistic is positive and statistically significant. For the one-sided test, a t-value greater than 1.282 implies a 10% significance level. Table A.1 reports the test results using the Newey-West LRV estimator

Table A.1: Binomial Directional Forecasts Test

Currency	Forecasting Horizon (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Newey-West				
AUD	0.378*	1.051**	1.169	1.957*	2.723**
	(0.237)	(0.622)	(0.925)	(1.303)	(1.588)
CAD	0.332*	1.116***	1.743***	2.074***	1.748**
	(0.216)	(0.443)	(0.747)	(0.846)	(0.997)
EUR	0.260	0.994*	3.179***	4.516***	3.776***
	(0.280)	(0.771)	(1.165)	(1.559)	(1.610)
JPY	0.168	0.319	2.630***	3.979***	4.224***
	(0.290)	(0.531)	(0.957)	(1.081)	(1.428)
GBP	0.361*	0.853*	1.297*	1.900*	2.242**
	(0.230)	(0.521)	(0.984)	(1.193)	(1.190)
	Panel B: Andrews				
AUD	0.378*	1.051*	1.169	1.957*	2.723**
	(0.235)	(0.648)	(0.961)	(1.358)	(1.649)
CAD	0.332**	1.116***	1.743**	2.074**	1.748*
	(0.216)	(0.465)	(0.814)	(0.950)	(1.145)
EUR	0.260	0.994	3.179**	4.516**	3.776**
	(0.285)	(0.860)	(1.382)	(1.948)	(1.965)
JPY	0.168	0.319	2.630***	3.979***	4.224***
	(0.294)	(0.553)	(1.069)	(1.195)	(1.593)
GBP	0.361*	0.853*	1.297	1.900*	2.242**
	(0.231)	(0.563)	(1.182)	(1.427)	(1.348)

Note: Panel A reports the test results using the Newey-West LRV estimator to control for auto-correlation. Panel B reports the test results using the Andrews LRV estimator to control for auto-correlation. Standard errors are in parentheses. We consider an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The information on our directional forecasts is described in the note to Table 1.

(Panel A) and the Andrews LRV estimator (Panel B) to control for auto-correlation. As Table A.1 shows, overall the binomial test indicates that our directional forecasts have strong predictability. Using the Newey-West LRV estimator, the null is rejected 21 out of 25 cases. The results are exceptionally good for the Canadian Dollar and the British Pound, with the null rejected in all 5 forecasting horizons. They are also good for the Euro and the Australian Dollar, with the null rejected in 4 out of 5 forecasting horizons.

B Proof of the Main Results

B.1 Proof of the Results in Subsection 2.2

In the rest of the proof, we write $E_t^I[\cdot] = E_t[\cdot]$ for any t.

Proof of Lemma 3. Suppose that $\theta \leq \phi$ and that a positive bubbly path exists. Since along a bubbly path, for any $j \in \{t+1, \ldots, t'\}$ we have $Z_j = \mu_z$, $L_{j-1}^* = 0$ and $L_j^* = 0$, the market clearing condition (7) implies that the equilibrium prices along this positive bubbly path would be

$$(\theta - \phi)S_j^* = \theta S_{j-1}^* - (\phi\mu_z + C_j^*) \text{ for any } j \in \{t+1, \ldots, t'\}, \tag{B.40}$$

which implies that

$$(\theta - \phi)\Delta S_j^* = \theta \Delta S_{j-1}^* - \Delta C_j^* \text{ for any } j \in \{t+2, \ldots, t'\}. \tag{B.41}$$

Since $\theta > 0$, $\Delta S_j^* > 0$ and $\Delta S_{j-1}^* > 0$ along the positive bubbly path, (B.41) holds under $\theta \leq \phi$ only when $\Delta C_j^* > 0$ for any $j \in \{t+2, \ldots, t'\}$. This is impossible since $C_j^* \in \{-c, 0, c\}$ for any $j \in \{t, \ldots, t'\}$ and $t' > t+3$. Hence, a positive bubbly path does not exist if $\theta \leq \phi$.

The same argument shows that a negative bubbly path does not exist if $\theta \leq \phi$. ∎

Proof of Lemma 4. Consider the positive bubbly path first. Suppose that (10) holds. Since $C_j^* \in \{-c, 0, c\}$ for any $j \in \{t, \ldots, t'\}$ and $t' > t+3$, there exists at least one $j^* \in \{t+2, \ldots, t'\}$ such that $\Delta C_{j^*}^* \leq 0$. Under (10) and the conditions of the bubbly path, we can invoke (B.41) and $\Delta C_{j^*}^* \leq 0$ to deduce that

$$\Delta S_{j^*}^* = \frac{\theta \Delta S_{j^*-1}^* - \Delta C_{j^*}^*}{\theta - \phi} \geq \frac{\theta}{\theta - \phi} \Delta S_{j^*-1}^*. \tag{B.42}$$

By (10)

$$\frac{\theta}{\theta - \phi} > 1, \tag{B.43}$$

which together with (B.42) implies that

$$\Delta S^*_{j^*} > \Delta S^*_{j^*-1}. \tag{B.44}$$

The same argument applies to the negative bubbly path. ∎

Proof of Proposition 5. Since the liquidity squeeze is triggered at time t_0, (4) implies that the momentum speculator's demand is zero at time t_0 and at time $t_0 + 1$. Moreover, since at time t_0 there is no shock to fundamentals ($\zeta_{t_0} = 0$), market clearing condition (8) implies that

$$S^*_{t_0} = \mu_z + \frac{C^*_{t_0}}{\phi} \text{ and } S^*_{t_0+1} = \mu_z + \zeta_{t_0+1} + \frac{C^*_{t_0+1}}{\phi}. \tag{B.45}$$

Therefore, the equilibrium price change is

$$\Delta S^*_{t_0+1} = \zeta_{t_0+1} + \frac{C^*_{t_0+1} - C^*_{t_0}}{\phi}. \tag{B.46}$$

Suppose that $C^*_{t_0} = c$. In this case

$$\zeta_{t_0+1} + \frac{C^*_{t_0+1} - c}{\phi} \leq \zeta_{t_0+1} \text{ for any } C^*_{t_0+1} \in \{-c, 0, c\}.$$

It then follows from (B.46) that $E_{t_0}[\Delta S^*_{t_0+1}] \leq 0$. This non-positive expected price change is inconsistent with $C^*_{t_0} = c$, as it contradicts the demand policy of informed speculators (1). Next, suppose that $C^*_{t_0} = -c$. In this case

$$\zeta_{t_0+1} + \frac{C^*_{t_0+1} + c}{\phi} \geq \zeta_{t_0+1} \text{ for any } C^*_{t_0+1} \in \{-c, 0, c\},$$

which implies that $E_{t_0}[\Delta S^*_{t_0+1}] \geq 0$. This also contradicts the demand rule of the informed speculators. Hence, in an equilibrium, we must have $C^*_{t_0} = 0$, which together with the first equation in (B.45) implies that

$$S^*_{t_0} = \mu_z.$$

Because $S^*_{t_0} - \mu_z = 0$**,** the law of motion of the probability of liquidity squeeze (5) implies that $L^*_{t_0+1} = 0$ with probability 1. This proves the claim of the Proposition. ∎

B.2 Proof of the Results for the Positive Bubbly Path

In this section, we derive the positive bubbly equilibrium stated in Proposition 7 and Proposition 8. We first provide some useful equations. Consider any period $t > 1$. If liquidity squeeze is not triggered in both $t-1$ and t, the market clearing condition is

$$\phi\left(Z_t - S^*_t\right) + \theta\left(S^*_t - S^*_{t-1}\right) + C\left(E_t[S^*_{t+1}]\right) = 0, \tag{B.47}$$

and hence the equilibrium price is

$$S^*_t = \frac{\theta S^*_{t-1} - \phi Z_t - C\left(E_t[S^*_{t+1}]\right)}{\theta - \phi} \tag{B.48}$$

Therefore, the price change is

$$\Delta S^*_t = \frac{\phi(S^*_{t-1} - Z_t) - C^*_t}{\theta - \phi} = \frac{\phi(S^*_{t-1} - \mu_z) - C^*_t}{\theta - \phi} - \frac{\phi\zeta_t}{\theta - \phi}, \tag{B.49}$$

where $C^*_t = C\left(E_t[S^*_{t+1}]\right)$. Alternatively, if liquidity squeeze is triggered in period $t-1$ and/or period t, momentum speculators do not participate in the market. Thus, the market clearing condition is

$$\phi\left(Z_t - S^*_t\right) + C^*_t = 0, \tag{B.50}$$

which implies that

$$S^*_t = Z_t + \phi^{-1}C^*_t. \tag{B.51}$$

The above equations will be used extensively in the proof.

Proof of Proposition 7 and Proposition 8. The proof has five steps. First, the exchange rates and the demands at time $t = 0$ and l^* claimed in Proposition 8 follow directly from Proposition 5. Second, Lemma 9 shows that the time-1 shock induces informed speculators to set $C_1^* = c$, expecting $\Delta S_2^* > 0$ and knowing $L_2^* = 0$ with probability 1. The resulting $\Delta S_2^* > 0$ activates the time-2 demand from momentum speculators, which in turn triggers a positive bubbly path. Third, Lemma 11, Lemma 12 and Lemma 13 prove that informed speculators find it optimal to ride this bubble until a switching time τ^*, which must equal either $\tau_* + 1$ or $\tau_* + 2$. Fourth, Lemma 15 shows that for any $t \geq \tau^*$, informed speculators set $C_t^* = -c$, expecting $\Delta S_{t+1}^* < 0$. Therefore, the equilibrium S_t^* satisfies (16) and the demand of the informed speculators satisfies (17). Lemma 16 derives the demand of momentum speculators. This proves Proposition 7. Lastly, Lemma 14 shows that the time l^* when the liquidity squeeze is triggered is larger than or equal to the switching time τ^*. Moreover, Lemma 15 shows that l^* is smaller than a fixed and finite integer $\bar{l}_{\max}$. This finishes proving Proposition 8. ∎

Lemma 9. *Under Condition 6, we have*

$$S_1^* = \mu_z + z + \phi^{-1}c,\ C_1^* = c,\ L_1^* = 0 \text{ and } L_2^* = 0. \tag{B.52}$$

Proof of Lemma 9. By Proposition 5, at time 0 we have

$$S_0^* = \mu_z,\ C_0^* = 0,\ L_0^* = 1 \text{ and } L_1^* = 0. \tag{B.53}$$

At time $t = 1$, the informed speculators observe $Z_1 = \mu_z + z$. Moreover since $L_0^* = 1$, by (B.51) the market clearing condition in this period is

$$S_1^* = \mu_z + z + \phi^{-1}C_1^*. \tag{B.54}$$

Because (i) $z + \phi^{-1}c < (1-\phi/\theta)^{-1}\left[z + (1+\phi/\theta)c\phi^{-1}\right]$ by (10), (ii) $|z + \phi^{-1}C_1^*| \leq z + \phi^{-1}c$ for any $C_1^* \in \{-c, 0, c\}$, and (iii) $\mu_z + z - \phi^{-1}c > \mu_z$, it follows from (13) that

$$|S_1^* - \mu_z| < \vartheta, \tag{B.55}$$

regardless of the demand of the informed speculators at time 1. Hence the law of motion of the probability of liquidity squeeze (5) implies that at time 1, informed speculators know that $L_2^* = 0$ with probability 1. Since $L_1^* = 0$ and $L_2^* = 0$, we can use (B.49) with $t = 2$ and (B.54) to get

$$\begin{aligned} \Delta S_2^* &= \frac{\phi(S_1^* - \mu_z) - C_2^*}{\theta - \phi} - \frac{\phi\zeta_2}{\theta - \phi} \\ &= \frac{\phi z + C_1^* - C_2^*}{\theta - \phi} - \frac{\phi\zeta_2}{\theta - \phi} > -\frac{\phi\zeta_2}{\theta - \phi} \end{aligned} \tag{B.56}$$

for any $C_1^*, C_2^* \in \{-c, 0, c\}$, where the last inequality follows from $\theta > \phi$ and $z > 2c/\phi$. Since $E_1[\zeta_2] = 0$, the inequality in (B.56) implies that $E_1[\Delta S_2^*] > 0$, which together with the demand function of the informed speculators (1) and the price equation (B.54), implies that $C_1^* = c$ and $S_1^* = \mu_z + z + c/\phi$. This completes the proof of (B.52). ∎

Lemma 10. *Suppose (10), (12) and (13) hold. Then τ_*, defined in (14), satisfies: (i) $\tau_* \geq 2$; and (ii)*

$$\frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c}{\phi} > \vartheta. \tag{B.57}$$

Proof of Lemma 10. (i) By (10), $\phi/\theta \in (0,1)$ which together with $z > 0$ implies that $(1-\phi/\theta)^{-t+1}$ is an increasing sequence of t. When $t = 2$, we have

$$\begin{aligned} \left[\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\right]_{t=2} &= \frac{z}{1-\phi/\theta} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} \\ &= \frac{1}{1-\phi/\theta}\left[z + (1+\phi/\theta)\frac{c}{\phi}\right] \leq \vartheta, \end{aligned} \tag{B.58}$$

where the inequality is by condition (13). Therefore the claim in part (i) of the lemma

follows by the definition of τ_*. To prove (ii) note that by the definition of τ_* in (14),

$$\vartheta < \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} = \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} + \frac{\phi/\theta}{1-\phi/\theta}\frac{2c}{\phi}. \tag{B.59}$$

Since $\phi/\theta \in (0,1)$, $(1-\phi/\theta)^{-1} = \sum_{k=0}^{\infty}(\phi/\theta)^k$. Therefore

$$\begin{aligned}
\frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c}{\phi} &= \frac{z}{(1-\phi/\theta)^{\tau_*}}\frac{1}{1-\phi/\theta} + \frac{c}{\phi} \\
&= \frac{z}{(1-\phi/\theta)^{\tau_*}}\sum_{k=0}^{\infty}(\phi/\theta)^k + \frac{c}{\phi} \\
&> \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} + \frac{\phi/\theta}{1-\phi/\theta}z,
\end{aligned} \tag{B.60}$$

where the inequality is by (i) $\sum_{k=0}^{\infty}(\phi/\theta)^k > \sum_{k=0}^{1}(\phi/\theta)^k$, (ii) $\phi/\theta \in (0,1)$, and (iii) $\tau_* \geq 2$ (which has been proved above). Since $z > 2c\phi^{-1}$ (by (12)) and $\phi/\theta \in (0,1)$, by (B.59) and (B.60) we immediately prove (B.57). ∎

Lemma 11. *Under Condition 6, we have*

$$S_t^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{t-1}},\ C_t^* = c,\ L_t^* = 0 \text{ and } L_{t+1}^* = 0 \tag{B.61}$$

for any t with $1 < t \leq \tau_*$, where τ_* is defined in (14).

Proof of Lemma 11. By (B.52), we know that (B.61) holds for $t = 1$. We prove that (B.61) holds for t with $2 \leq t \leq \tau_*$ by mathematical induction. Suppose that (B.61) holds for $t-1$, i.e.,

$$S_{t-1}^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{t-2}},\ C_{t-1}^* = c,\ L_{t-1}^* = 0 \text{ and } L_t^* = 0. \tag{B.62}$$

Since $L_{t-1}^* = 0$ and $L_t^* = 0$, at time t we can use $\zeta_t = 0$, (B.48) and (B.62) to get

$$S_t^* = \frac{\theta S_{t-1}^* - \phi Z_t - C_t^*}{\theta - \phi} = \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi}. \tag{B.63}$$

For any $t \leq \tau_*$, by the definition of τ_* we know that

$$\begin{aligned} |S_t^* - \mu_z| &= \left| \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi} \right| \\ &\leq \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} \leq \vartheta, \end{aligned} \tag{B.64}$$

where the first inequality is by noting that $(\theta-\phi)^{-1}(\theta c/\phi - C_t^*)$ is non-negative and attains it maximum at $C_t^* = -c$, and the second inequality is by $t \leq \tau_*$, the definition of τ_* and the fact that $z(1-\phi/\theta)^{-t+1}$ is a strictly increasing sequence of t. Therefore by (5) we have $L_{t+1}^* = 0$. Since $L_t^* = 0$ and $L_{t+1}^* = 0$, we can apply (B.49) and (B.63) to get

$$\begin{aligned} \Delta S_{t+1}^* &= \frac{\phi(S_t^* - \mu_z) - C_{t+1}^*}{\theta - \phi} - \frac{\phi\zeta_{t+1}}{\theta - \phi} \\ &= \frac{\phi z - (1-\phi/\theta)^{t-1} C_{t+1}^*}{(\theta-\phi)(1-\phi/\theta)^{t-1}} + \frac{\theta c - \phi C_t^*}{(\theta-\phi)^2} - \frac{\phi\zeta_{t+1}}{\theta-\phi}. \end{aligned} \tag{B.65}$$

Under the restrictions $\theta > \phi > 0$ and $z > 2c/\phi$, we have

$$\frac{\phi z - (1-\phi/\theta)^{t-1} C_{t+1}^*}{(\theta-\phi)(1-\phi/\theta)^{t-1}} > 0 \text{ and } \frac{\theta c - \phi C_t^*}{(\theta-\phi)^2} > 0 \tag{B.66}$$

for any $C_t^*, C_{t+1}^* \in \{-c, 0, c\}$, which together with (B.65) and $E_t[\zeta_{t+1}] = 0$ implies that

$$E_t[\Delta S_{t+1}^*] = \frac{\phi z - (1-\phi/\theta)^{t-1} E_t[C_{t+1}^*]}{(\theta-\phi)(1-\phi/\theta)^{t-1}} + \frac{\theta c - \phi C_t^*}{(\theta-\phi)^2} > 0. \tag{B.67}$$

Combing (B.67) with the demand function of the informed speculators we get $C_t^* = c$, which together with (B.63) shows that

$$S_t^* = \mu_z + \phi^{-1} c + \frac{z}{(1-\phi/\theta)^{t-1}}. \tag{B.68}$$

Therefore (B.61) also holds for t. The claim of the lemma follows by mathematical induction. ∎

Lemma 12. *Under Condition 6 and condition*

$$\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta, \tag{B.69}$$

there exists a unique equilibrium at time $\tau_* + 1$:

$$C^*_{\tau_*+1} = -c \text{ and } S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{B.70}$$

Proof of Lemma 12. By (B.61), we have $L^*_{\tau_*} = 0$ and $L^*_{\tau_*+1} = 0$. Since $\zeta_{\tau_*+1} = 0$ at time $\tau_* + 1$, by (B.48) and (B.61) with $t = \tau_* + 1$, we have

$$S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{\theta c/\phi - C^*_{\tau_*+1}}{\theta - \phi}, \tag{B.71}$$

where $(\theta-\phi)^{-1}(\theta c/\phi - C^*_{\tau_*+1})$ is non-negative for any $C^*_{\tau_*+1} \in \{-c, 0, c\}$ and it achieves its minimum c/ϕ at $C^*_{\tau_*+1} = c$. Therefore,

$$S^*_{\tau_*+1} - \mu_z > \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta, \tag{B.72}$$

where the second inequality is by (B.69). Under (B.72), the law of motion of L^*_t in (5) implies that, regardless of the informed speculators' demand at $\tau_* + 1$, the probability of liquidity squeeze in period $\tau_* + 2$ becomes at least $\overline{\sigma}$. Let σ_{τ_*+2} denote this probability. On the one hand if $L^*_{\tau_*+2} = 0$, (B.49) and (B.71) imply that

$$\Delta S^*_{\tau_*+2} = \frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+1}} - \frac{C^*_{\tau_*+2}}{\theta-\phi} + \frac{\theta c - \phi C^*_{\tau_*+1}}{(\theta-\phi)^2} - \frac{\phi\zeta_{\tau_*+2}}{\theta-\phi}. \tag{B.73}$$

On the other hand if $L^*_{\tau_*+2} = 1$, equilibrium price equation (B.51) and (B.71) imply that

$$\Delta S^*_{\tau_*+2} = \zeta_{\tau_*+2} + \frac{C^*_{\tau_*+2}}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{\theta c/\phi - C^*_{\tau_*+1}}{\theta-\phi}. \tag{B.74}$$

Combining the results in (B.73) and (B.74), and applying $E_{\tau_*+1}[\zeta_{\tau_*+2}] = 0$, we get

$$\begin{aligned}
E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] &= (1-\sigma_{\tau_*+2})\left[\frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+1}} - \frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\theta-\phi} + \frac{\theta c - \phi C^*_{\tau_*+1}}{(\theta-\phi)^2}\right] \\
&\quad + \sigma_{\tau_*+2}\left[\frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{\theta c/\phi - C^*_{\tau_*+1}}{\theta-\phi}\right] \\
&= \frac{z}{(1-\phi/\theta)^{\tau_*}}\frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta^2(\phi/\theta-\sigma_{\tau_*+2})c}{\phi(\theta-\phi)^2} \\
&\quad + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{\phi(\theta-\phi)}E_{\tau_*+1}[C^*_{\tau_*+2}] + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{(\theta-\phi)^2}C^*_{\tau_*+1}. \qquad \text{(B.75)}
\end{aligned}$$

Define

$$Q(c_1,c_2) = \frac{\theta^2(\phi/\theta-\sigma_{\tau_*+2})c}{\phi(\theta-\phi)^2} + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{(\theta-\phi)^2}c_1 + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{\phi(\theta-\phi)}c_2. \qquad \text{(B.76)}$$

Note that

$$\begin{aligned}
Q(c,c) &= \frac{\theta^2(\phi/\theta-\sigma_{\tau_*+2})c}{\phi(\theta-\phi)^2} + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{(\theta-\phi)^2}c + \frac{\theta(\sigma_{\tau_*+2}-\phi/\theta)}{\phi(\theta-\phi)}c \\
&= \frac{(\phi/\theta-\sigma_{\tau_*+2})\theta c}{(\theta-\phi)}\left[\frac{\theta/\phi}{\theta-\phi} - \frac{1}{\theta-\phi} - \frac{1}{\phi}\right] = 0. \qquad \text{(B.77)}
\end{aligned}$$

As $\sigma_{\tau_*+2} \geq \overline{\sigma} > \phi/\theta$ and $\theta > \phi$, we know that $Q(c_1,c_2) \leq Q(c,c)$ for any $c_1, c_2 \in \{-c, 0, c\}$, which together with (B.77) implies that

$$Q(c_1,c_2) \leq 0 \text{ for any } c_1, c_2 \in \{-c, 0, c\}. \qquad \text{(B.78)}$$

From (B.75) we have that

$$E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] = \frac{z}{(1-\phi/\theta)^{\tau_*}}\frac{\phi/\theta-\sigma_{\tau_*+2}}{1-\phi/\theta} + E_{\tau_*+1}[Q(C^*_{\tau_*+2}, C^*_{\tau_*+1})]. \qquad \text{(B.79)}$$

Using (B.78), (B.79) and restrictions $\theta > \phi$ and $\sigma_{\tau_*+2} > \phi/\theta$, we deduce that $E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] < 0$. From the demand function of the informed speculators (1), we

then get $C^*_{\tau_*+1} = -c$, which together with (B.71) implies that

$$S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{B.80}$$

This completes the proof. ∎

Lemma 13. *Under Condition 6 and condition*

$$\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta, \tag{B.81}$$

there exist and only exist two possible equilibria at time $\tau_* + 1$. The first equilibrium is the same as the one derived in Lemma 12 under restriction (B.69). In the second equilibrium, $C^*_{\tau_*+1} = c$, $C^*_{\tau_*+2} = -c$,

$$S^*_{\tau_*+1} = \mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{\tau_*}} \tag{B.82}$$

and

$$S^*_{\tau_*+2} = \mu_z + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} + \frac{z}{(1-\phi/\theta)^{\tau_*+1}}. \tag{B.83}$$

Proof of Lemma 13. By (B.61), we have $L^*_{\tau_*} = 0$ and $L^*_{\tau_*+1} = 0$. Since $\zeta_{\tau_*+1} = 0$ at time $\tau_* + 1$, by (B.48) and (B.61) with $t = \tau_* + 1$, we have

$$S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{\theta c/\phi - C^*_{\tau_*+1}}{\theta - \phi}, \tag{B.84}$$

Therefore there are three possible conjectured equilibria at time $\tau_* + 1$:

$$S^*_{\tau_*+1} = \begin{cases} \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}, & \text{if } C^*_{\tau_*+1} = -c \\ \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1}{1-\phi/\theta}, & \text{if } C^*_{\tau_*+1} = 0 \\ \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}, & \text{if } C^*_{\tau_*+1} = c \end{cases}. \tag{B.85}$$

To show that

$$C^*_{\tau_*+1} = -c \text{ and } S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \tag{B.86}$$

is an equilibrium, it is sufficient to show that $E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] < 0$. Given $C^*_{\tau_*+1} = -c$,

$$S^*_{\tau_*+1} - \mu_z = \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} > \vartheta \tag{B.87}$$

by the definition of τ_* in (14). Therefore, by (5) the informed speculators know that the probability of liquidity squeeze in period $\tau_* + 2$ becomes at least $\overline{\sigma}$. Let σ_{τ_*+2} denote this probability. On the one hand if $L^*_{\tau_*+2} = 0$, (B.49) and (B.86) imply that

$$\Delta P^*_{\tau_*+2} = \frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+1}} - \frac{C^*_{\tau_*+2}}{\theta-\phi} + \frac{(\theta+\phi)c}{(\theta-\phi)^2} - \frac{\phi\zeta_{\tau_*+2}}{\theta-\phi}. \tag{B.88}$$

On the other hand if $L^*_{\tau_*+2} = 1$, equilibrium price equation (B.51) and (B.71) imply that

$$\Delta S^*_{\tau_*+2} = \zeta_{\tau_*+2} + \frac{C^*_{\tau_*+2}}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{\theta+\phi}{\theta-\phi}. \tag{B.89}$$

Combining the results in (B.88) and (B.89), and applying $E_{\tau_*+1}[\zeta_{\tau_*+2}] = 0$, we get

$$\begin{aligned} E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] &= (1-\sigma_{\tau_*+2})\left[\frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+1}} - \frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\theta-\phi} + \frac{(\theta+\phi)c}{(\theta-\phi)^2}\right] \\ &\quad + \sigma_{\tau_*+2}\left[\frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{\theta+\phi}{\theta-\phi}\right] \\ &= \frac{z}{(1-\phi/\theta)^{\tau_*}}\frac{\phi/\theta-\sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta(\phi/\theta-\sigma_{\tau_*+2})c}{(\theta-\phi)^2}(1+\theta/\phi) \\ &\quad + \frac{\theta(\overline{\sigma}-\phi/\theta)}{\phi(\theta-\phi)}E_{\tau_*+1}[C^*_{\tau_*+2}]. \end{aligned} \tag{B.90}$$

Since $\sigma_{\tau_*+2} \geq \overline{\sigma} > \phi/\theta$ and $\theta > \phi$,

$$\begin{aligned}
& \frac{z}{(1-\phi/\theta)^{\tau_*}} \frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta(\phi/\theta - \sigma_{\tau_*+2})c}{(\theta-\phi)^2}(1+\theta/\phi) + \frac{\theta(\sigma_{\tau_*+2} - \phi/\theta)C^*_{\tau_*+2}}{\phi(\theta-\phi)} \\
\leq \; & \frac{z}{(1-\phi/\theta)^{\tau_*}} \frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta(\phi/\theta - \sigma_{\tau_*+2})c}{(\theta-\phi)^2}(1+\theta/\phi) + \frac{\theta(\sigma_{\tau_*+2} - \phi/\theta)c}{\phi(\theta-\phi)} \\
= \; & \frac{z}{(1-\phi/\theta)^{\tau_*}} \frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{2\theta(\phi/\theta - \sigma_{\tau_*+2})c}{(\theta-\phi)^2} < 0 \qquad \text{(B.91)}
\end{aligned}$$

where the second inequality is by $z > 0$, $c > 0$, $\sigma_{\tau_*+2} \geq \overline{\sigma} > \phi/\theta$ and $\theta > \phi > 0$. By (B.90) and (B.91),

$$E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] < 0, \qquad \text{(B.92)}$$

which shows that (B.86) is indeed an equilibrium.

We next show that

$$\begin{aligned}
C^*_{\tau_*+1} &= c \text{ and } S^*_{\tau_*+1} = \mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{\tau_*}}, \\
C^*_{\tau_*+2} &= -c \text{ and } S^*_{\tau_*+2} = \mu_z + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} + \frac{z}{(1-\phi/\theta)^{\tau_*+1}} \qquad \text{(B.93)}
\end{aligned}$$

is also an equilibrium under (B.81). Given the demand of the informed speculators $C^*_{\tau_*+1} = c$ and $C^*_{\tau_*+2} = -c$, it is sufficient to show that $E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] > 0$ and $E_{\tau_*+2}[\Delta S^*_{\tau_*+3}] < 0$. By (5) and (B.81), at time $\tau_* + 1$ informed speculators know that $L^*_{\tau_*+2} = 0$ with probability 1 if they choose $C^*_{\tau_*+1} = c$. Since $L^*_{\tau_*+1} = 0$ and $L^*_{\tau_*+2} = 0$, by (B.49) and (B.93), the informed speculator knows at time $\tau_* + 1$ that the market clearing condition at time $\tau_* + 2$ implies

$$\Delta S^*_{\tau_*+2} = \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c - C^*_{\tau_*+2}}{\theta-\phi} - \frac{\phi\zeta_{\tau_*+2}}{\theta-\phi}, \qquad \text{(B.94)}$$

which together with $C^*_{\tau_*+2} \leq c$, $E_{\tau_*+1}[\zeta_{\tau_*+2}] = 0$ and $\theta > \phi$ implies that

$$E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] \geq \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau_*+1}} > 0. \qquad \text{(B.95)}$$

This rationalizes the informed speculator's choice $C^*_{\tau_*+1} = c$ and the equilibrium price

$$S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \tag{B.96}$$

at time $\tau_* + 1$. We have shown above that $L^*_{\tau_*+1} = 0$ and $L^*_{\tau_*+2} = 0$. Hence at time $\tau_* + 2$, by $\zeta_{\tau_*+2} = 0$, (B.48) and (B.96), the market clearing condition is

$$S^*_{\tau_*+2} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{\theta c - \phi C^*_{\tau_*+2}}{\phi(\theta-\phi)}. \tag{B.97}$$

Since $\phi > 0$, by (B.97) and (B.57)

$$S^*_{\tau_*+2} - \mu_z \geq \frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c}{\phi(\theta-\phi)} > \vartheta. \tag{B.98}$$

Hence regardless of the informed speculators' demand at $\tau_* + 2$, the probability of liquidity squeeze in period τ_*+3 becomes at least $\overline{\sigma}$. Let σ_{τ_*+3} denote this probability. On the one hand if $L^*_{\tau_*+3} = 0$, (B.49) and (B.97) imply that

$$\Delta S^*_{\tau_*+3} = \frac{\phi z - (1-\phi/\theta)^{\tau_*+1} C^*_{\tau_*+3}}{(\theta-\phi)(1-\phi/\theta)^{\tau_*+1}} + \frac{\theta c - \phi C^*_{\tau_*+2}}{(\theta-\phi)^2} - \frac{\phi \zeta_{\tau_*+3}}{\theta-\phi}. \tag{B.99}$$

On the other hand if $L^*_{\tau_*+3} = 1$, equilibrium price equation (B.51) and (B.97) imply that

$$\Delta S^*_{\tau_*+3} = \zeta_{\tau_*+3} + \frac{C^*_{\tau_*+3}}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*+1}} - \frac{\theta c/\phi - C^*_{\tau_*+2}}{\theta-\phi}. \tag{B.100}$$

Combining the results in (B.99) and (B.100), and applying $E_{\tau_*+2}[\zeta_{\tau_*+3}] = 0$, we get

$$\begin{aligned}
E_{\tau_*+2}[\Delta S^*_{\tau_*+3}] &= (1-\sigma_{\tau_*+3})\left[\frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+2}} - \frac{E_{\tau_*+2}[C^*_{\tau_*+3}]}{\theta-\phi} + \frac{\theta c - \phi C^*_{\tau_*+2}}{(\theta-\phi)^2}\right] \\
&\quad + \sigma_{\tau_*+3}\left[\frac{E_{\tau_*+2}[C^*_{\tau_*+3}]}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*+1}} - \frac{\theta c/\phi - C^*_{\tau_*+2}}{\theta-\phi}\right] \\
&= \frac{z}{(1-\phi/\theta)^{\tau_*+1}}\frac{\phi/\theta - \sigma_{\tau_*+3}}{1-\phi/\theta} + \frac{\theta^2(\phi/\theta-\sigma_{\tau_*+3})c}{\phi(\theta-\phi)^2} \\
&\quad + \frac{\theta(\sigma_{\tau_*+3}-\phi/\theta)}{\phi(\theta-\phi)}E_{\tau_*+2}[C^*_{\tau_*+3}] + \frac{\theta(\sigma_{\tau_*+3}-\phi/\theta)}{(\theta-\phi)^2}C^*_{\tau_*+2}. \qquad \text{(B.101)}
\end{aligned}$$

Using the same arguments of showing $E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] < 0$ in Lemma 12, we can show that $E_{\tau_*+2}[\Delta S^*_{\tau_*+3}] < 0$ regardless of the informed speculators' demand at τ_*+2 and τ_*+3. Therefore at time τ_*+2,

$$C^*_{\tau_*+2} = -c \text{ and } S^*_{\tau_*+2} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c}{\phi}\frac{\theta+\phi}{\theta-\phi}, \qquad \text{(B.102)}$$

which shows that (B.93) is indeed an equilibrium.

Finally, we show that

$$C^*_{\tau_*+1} = 0 \text{ and } S^*_{\tau_*+1} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1}{1-\phi/\theta} \qquad \text{(B.103)}$$

cannot be an equilibrium. There are two cases to consider:

$$\text{case 1:} \qquad \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1}{1-\phi/\theta} \le \vartheta, \qquad \text{(B.104)}$$

$$\text{case 2:} \qquad \mu_z + \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1}{1-\phi/\theta} > \vartheta. \qquad \text{(B.105)}$$

We investigate the conjectured equilibrium (B.103) in case 1 first. Under (B.104), at time τ_*+1 the informed speculators' know that $L^*_{\tau_*+2} = 0$ with probability 1 given the demand $C^*_{\tau_*+1} = 0$. Therefore by (B.49) and (B.103), the informed speculator

knows that market clearing condition at time $\tau_* + 2$ implies

$$\Delta S^*_{\tau_*+2} = \frac{\phi z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c(1-\phi/\theta)^{-1} - C^*_{\tau_*+2}}{\theta - \phi} - \frac{\phi \zeta_{\tau_*+2}}{\theta - \phi}, \tag{B.106}$$

which together with $E_{\tau_*+1}[\zeta_{\tau_*+2}] = 0$ and $\theta > \phi$ implies that

$$E_{\tau_*+1}[\Delta S^*_{\tau_*+2}] \geq \frac{\phi z}{(1-\phi/\theta)^{\tau_*+1}} > 0. \tag{B.107}$$

This contradicts the informed speculators's demand $C^*_{\tau_*+1} = 0$. Next we investigate the conjectured equilibrium (B.103) in case 2. Under (B.105), the informed speculators' know that $L^*_{\tau_*+2} = 1$ with probability at least $\overline{\sigma}$ given the demand $C^*_{\tau_*+1} = 0$. Let σ_{τ_*+2} denote this probability. On the one hand if $L^*_{\tau_*+2} = 0$, (B.49) and (B.103) imply that

$$\Delta S^*_{\tau_*+2} = \frac{\phi z - (1-\phi/\theta)^{\tau_*} C^*_{\tau_*+2}}{(\theta-\phi)(1-\phi/\theta)^{\tau_*}} + \frac{\theta c}{(\theta-\phi)^2} - \frac{\phi \zeta_{\tau_*+2}}{\theta - \phi}. \tag{B.108}$$

On the other hand if $L^*_{\tau_*+2} = 1$, equilibrium price equation (B.51) and (B.103) imply that

$$\Delta S^*_{\tau_*+2} = \zeta_{\tau_*+2} + \frac{C^*_{\tau_*+2}}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{1}{1-\phi/\theta}. \tag{B.109}$$

Combining the results in (B.108) and (B.109), and applying $E_{\tau_*+1}[\zeta_{\tau_*+2}] = 0$, we get

$$\begin{aligned} E_{\tau_*+2}[\Delta S^*_{\tau_*+3}] &= (1-\sigma_{\tau_*+2})\left[\frac{\phi z/\theta}{(1-\phi/\theta)^{\tau_*+1}} - \frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\theta-\phi} + \frac{\theta c}{(\theta-\phi)^2}\right] \\ &\quad + \sigma_{\tau_*+2}\left[\frac{E_{\tau_*+1}[C^*_{\tau_*+2}]}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{\theta c/\phi}{\theta-\phi}\right] \\ &= \frac{z}{(1-\phi/\theta)^{\tau_*}}\frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta^2(\phi/\theta - \sigma_{\tau_*+2})c}{\phi(\theta-\phi)^2} \\ &\quad + \frac{\theta(\sigma_{\tau_*+2} - \phi/\theta)}{\phi(\theta-\phi)} E_{\tau_*+1}[C^*_{\tau_*+2}]. \end{aligned} \tag{B.110}$$

Since $\sigma_{\tau_*+2} \geq \overline{\sigma} > \phi/\theta$, we have

$$\begin{aligned} E_{\tau_*+2}[\Delta S^*_{\tau_*+3}] &\leq \frac{z}{(1-\phi/\theta)^{\tau_*}} \frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta^2(\phi/\theta - \sigma_{\tau_*+2})c}{\phi(\theta-\phi)^2} + \frac{\theta(\sigma_{\tau_*+2} - \phi/\theta)c}{\phi(\theta-\phi)} \\ &= \frac{z}{(1-\phi/\theta)^{\tau_*}} \frac{\phi/\theta - \sigma_{\tau_*+2}}{1-\phi/\theta} + \frac{\theta(\phi/\theta - \sigma_{\tau_*+2})c}{\phi(\theta-\phi)} \frac{\phi}{\theta-\phi} < 0 \end{aligned} \tag{B.111}$$

where the last inequality is by $\sigma_{\tau_*+2} \geq \overline{\sigma} > \phi/\theta$ and $\theta > \phi > 0$. This contradicts $C^*_{\tau_*+1} = 0$ in the conjectured equilibrium (B.103). ∎

Lemma 14. *Under Condition 6, we have $l^* \geq \tau^*$.*

Proof of Lemma 14. By Lemma 12 and Lemma 13, we have $\tau^* = \tau_* + 1$ and $L^*_{\tau_*+1} = 0$ when (B.69) holds, and $\tau^* = \tau_* + 1$ and $L^*_{\tau_*+1} = 0$ or $\tau^* = \tau_* + 2$ and $L^*_{\tau_*+2} = 0$ when (B.81) holds. This implies that $L^*_{\tau^*} = 0$ and hence $l^* \geq \tau^*$. ∎

Lemma 15. *Under Condition 6 and $l^* > \tau^*$, we have*

$$S^*_t = \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right] \text{ and } C^*_t = -c \tag{B.112}$$

for any t with $\tau^* \leq t < l^*$ where l^* satisfies $l^* \leq \bar{l}_{\max}$ with probability 1.

Proof of Lemma 15. By Lemma 12 and Lemma 13, at time τ^*, we have

$$C^*_{\tau^*} = -c \text{ and } S^*_{\tau^*} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}, \tag{B.113}$$

where $\tau^* = \tau_* + 1$ when (B.69) holds, and $\tau^* = \tau_* + 1$ or $\tau_* + 2$ when (B.81) holds. Therefore (B.112) holds for $t = \tau^*$. If $l^* = \tau^* + 1$, then $l^* \leq \bar{l}_{\max}$ by the definition of $\bar{l}_{\max}$ which finishes the proof.

We next consider the case that $l^* > \tau^* + 1$. Suppose that (B.112) holds for $t-1$ with $\tau^* \leq t-1 < l^*$, we will show that it also holds for t with $t < l^*$. At period t, liquidity squeeze is not triggered because $t < l^*$, which together with $Z_t = \mu_z$ and

(B.48) implies that

$$S_t^* = \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} - \frac{C_t^*}{\theta-\phi}. \tag{B.114}$$

Since $\theta > \phi$ and $t - \tau^* \geq 1$, by $(1-\phi/\theta)^{-1} = \sum_{k=0}^{\infty}(\phi/\theta)^k > \sum_{k=0}^{1}(\phi/\theta)^k$, the definition of τ_* and (B.114)

$$\begin{aligned}
S_t^* &> \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} - \frac{c}{\theta-\phi} \\
&> \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{2}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} - \frac{c}{\theta-\phi} \\
&= \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\phi} - \frac{1}{1-\phi/\theta}\frac{c}{\theta} \\
&> \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\sum_{k=0}^{1}(\phi/\theta)^k - \frac{1}{1-\phi/\theta}\frac{c}{\theta} \\
&> \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}
\end{aligned} \tag{B.115}$$

where the last inequality is by

$$\frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\frac{\phi}{\theta} - \frac{1}{1-\phi/\theta}\frac{c}{\theta} = \frac{\phi/\theta}{1-\phi/\theta}\frac{c}{\theta} > 0. \tag{B.116}$$

Hence the informed speculators' know that $L_{t+1}^* = 1$ with probability at least $\overline{\sigma}$ regardless of the demand C_t^* at time t. Let σ_{t+1} denote this probability. On the one hand, when there is no liquidity squeeze in period $t+1$, (B.49) implies that

$$\begin{aligned}
\Delta S_{t+1}^{*,nl} &= \frac{\phi S_t^* - \phi\mu_z}{\theta-\phi} - \frac{C_{t+1}^* + \phi\zeta_{\tau^*+1}}{\theta-\phi} \\
&= \frac{z\phi/\theta}{(1-\phi/\theta)^t} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{(1-\phi/\theta)^2}\right]\frac{c}{\theta} \\
&\quad - \frac{\phi C_t^*}{(\theta-\phi)^2} - \frac{C_{t+1}^* + \phi\zeta_{\tau^*+1}}{\theta-\phi}.
\end{aligned} \tag{B.117}$$

On the other hand, when there is liquidity squeeze in period $t+1$, (B.51) implies that

$$S^*_{t+1} = \mu_z + \zeta_{t+1} + \phi^{-1} C^*_{t+1}. \tag{B.118}$$

Combining this expression with (B.114), we have

$$\begin{aligned}
\Delta S^{*,l}_{t+1} &= \zeta_{t+1} + \frac{C^*_{t+1}}{\phi} - \frac{z}{(1-\phi/\theta)^{t-1}} \\
&\quad - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} + \frac{C^*_t}{\theta-\phi} \\
&= -\frac{z}{(1-\phi/\theta)^{t-1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} \\
&\quad + \frac{C^*_{t+1}}{\phi} + \frac{C^*_t}{\theta-\phi} + \zeta_{t+1}.
\end{aligned} \tag{B.119}$$

Next, we compute $E_t\left[\Delta S^*_{t+1}\right] = (1-\sigma_{t+1})E_t\left[\Delta S^{*,nl}_{t+1}\right] + \sigma_{t+1}E_t\left[\Delta S^{*,l}_{t+1}\right]$, where σ_{t+1} is the probability of liquidity squeeze in period $t+1$. To do so note that considering the first two terms in (B.117) and in (B.119), we get

$$\begin{aligned}
&\frac{(1-\sigma_{t+1})z\phi/\theta}{(1-\phi/\theta)^t} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{(1-\phi/\theta)^2}\right]\frac{(1-\sigma_{t+1})c}{\theta} \\
&\qquad - \frac{\sigma_{t+1}z}{(1-\phi/\theta)^{t-1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\sigma_{t+1}c}{\phi} \\
= &\ \frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\left[\frac{(1-\sigma_{t+1})}{(1-\phi/\theta)\theta} - \frac{\sigma_{t+1}}{\phi}\right]c \\
= &\ \frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta}.
\end{aligned} \tag{B.120}$$

Considering the third and fourth terms in (B.117) and in (B.119) and setting $E_t\left[\zeta_{t+1}\right] =$

0 we get

$$\begin{aligned}
&\frac{C^*_{t+1}}{\phi}\sigma_{t+1} + \frac{C^*_t}{\theta-\phi}\sigma_{t+1} - \frac{(1-\overline{\sigma})C^*_{t+1}}{\theta-\phi} - \frac{(1-\overline{\sigma})\phi C^*_t}{(\theta-\phi)^2} \\
&= \frac{\sigma_{t+1}-\phi/\theta}{1-\phi/\theta}\frac{C^*_{t+1}}{\phi} + \frac{\sigma_{t+1}-\phi/\theta}{(1-\phi/\theta)^2}\frac{C^*_t}{\theta} \\
&\leq \frac{\sigma_{t+1}-\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} + \frac{\sigma_{t+1}-\phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\theta} = \frac{\sigma_{t+1}-\phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\phi},
\end{aligned} \tag{B.121}$$

where the last inequality follows from parameter restrictions $\theta > \phi$ and $\sigma_{t+1} \geq \overline{\sigma} > \phi/\theta$. Collecting the results in (B.117)-(B.121), we have that

$$\begin{aligned}
E_t\left[\Delta S^*_{t+1}\right] &\leq \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta} \\
&\quad + \frac{\sigma_{t+1}-\phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\phi} + \frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} \\
&= \frac{2c}{\phi}\frac{\phi/\theta-\sigma_{t+1}}{(1-\phi/\theta)^2}\left[\frac{1}{(1-\phi/\theta)^{t-\tau^*}} - 1\right] \\
&\quad + \frac{\phi/\theta-\sigma_{t+1}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} < 0
\end{aligned} \tag{B.122}$$

where the inequality follows from parameter restrictions $\theta > \phi$ and $\sigma_{t+1} \geq \overline{\sigma} > \phi/\theta$. It follows from (B.122) that $E_t\left[\Delta S^*_{t+1}\right] < 0$, which combined with the demand function of the informed speculators imply that $C^*_t = -c$. Plugging $C^*_t = -c$ into equation (B.114), we get

$$\begin{aligned}
S^*_t &= \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} + \frac{c}{\theta-\phi} \\
&= \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right]\frac{c}{\phi}.
\end{aligned} \tag{B.123}$$

This completes the proof of (B.112).

Finally we show that $l^* \leq \bar{l}_{\max}$ when $l^* > \tau^* + 1$. In this case, we have show that

$$S^*_t = \mu_z + \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right] \tag{B.124}$$

for any $\tau^* + 1 \leq t < l^*$. Since $\theta > \phi > 0$ and $c > 0$, we see that S_t^* diverges with t at an geometric rate. Suppose that $l^* > \bar{l}_{\max}$. Then by (B.124)

$$\begin{aligned} S^*_{\bar{l}_{\max}-1} - \mu_z &= \frac{z}{(1-\phi/\theta)^{\bar{l}_{\max}-2}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\bar{l}_{\max}-\tau^*}} - 1\right] \\ &\geq \frac{z}{(1-\phi/\theta)^{\bar{l}_{\max}-2}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\bar{l}_{\max}-\tau_*-2}} - 1\right] > \overline{\vartheta} \end{aligned}$$

where the first inequality is by $\tau^* \leq \tau_* + 2$ which is proved in Lemma 12 and Lemma 13, the second inequality is by the definition of $\bar{l}_{\max}$. Since $S^*_{\bar{l}_{\max}-1} - \mu_z > \overline{\vartheta}$, by (5) we deduce that $L^*_{\bar{l}_{\max}} = 1$ with probability 1. Hence $l^* = \bar{l}_{\max}$ which contradicts $l^* > \bar{l}_{\max}$. ∎

Lemma 16. *Under Condition 6, the demand of the momentum speculators satisfies*

$$q_t^{m^*} = \begin{cases} 0, & t = 0, 1, l^* \\ \frac{z\phi}{(1-\phi/\theta)^{t-1}}, & 2 \leq t < \tau^* \\ \frac{z\phi}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\frac{2\phi}{(1-\phi/\theta)^{t-\tau^*+1}} & \tau^* \leq t < l^* \end{cases}. \tag{B.125}$$

Proof of Lemma 16. The demand of the momentum speculators q_t^* at time $t = 0, 1, l^*$ is zero because $L_0^* = 1$ and $L_{l^*}^* = 1$. For any t with $2 \leq t < l_*$, we have $L_t^* = 0$. Therefore, the demand of the momentum speculators is

$$q_t^{m^*} = \theta \Delta S_t^*. \tag{B.126}$$

By Lemma 11 and Lemma 13, we know that for any t with $2 \leq t < \tau^*$,

$$S_t^* = \mu_z + \phi^{-1}c + \frac{z}{(1-\phi/\theta)^{t-1}}, \tag{B.127}$$

which together with (B.126) implies that

$$q_t^{m^*} = \frac{\theta z}{(1-\phi/\theta)^{t-1}} - \frac{\theta z}{(1-\phi/\theta)^{t-2}} = \frac{z\phi}{(1-\phi/\theta)^{t-1}} \tag{B.128}$$

for any t with $2 \leq t < \tau^*$. By Lemma 12 and Lemma 13,

$$S^*_{\tau^*} = \mu_z + \frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}, \tag{B.129}$$

which together with (B.127) implies that

$$\begin{aligned} \Delta S^*_{\tau^*} &= \mu_z + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} + \frac{z}{(1-\phi/\theta)^{\tau^*-1}} - \left[\mu_z + \frac{c}{\phi} + \frac{z}{(1-\phi/\theta)^{\tau^*-2}}\right] \\ &= \frac{c}{\phi}\frac{2\phi/\theta}{1-\phi/\theta} + \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau^*-1}}. \end{aligned} \tag{B.130}$$

Therefore

$$q^{m^*}_{\tau^*} = \frac{z\phi}{(1-\phi/\theta)^{\tau^*-1}} + \frac{c}{\phi}\frac{2\phi}{1-\phi/\theta}. \tag{B.131}$$

For any t with $\tau^* \leq t < l^*$, by Lemma 15

$$\Delta S^*_t = \frac{z\phi/\theta}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\frac{2\phi/\theta}{(1-\phi/\theta)^{t-\tau^*+1}}, \tag{B.132}$$

which together with (B.126) implies that

$$q^{m^*}_t = \frac{z\phi}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\frac{2\phi}{(1-\phi/\theta)^{t-\tau^*+1}}$$

for any t with $\tau^* \leq t < l^*$. This finishes the proof. ∎

C The Time-T Exchange Rate

Here, we discuss the time-T exchange rate S_T and derive the demands of hedgers and momentum speculators in the main text. In the main text we have considered S_T as an exogenous random variable. To show how S_T is related to interest rates consider an infinite horizon economy ($t = T, T+1, ...\infty$) with rational forward-looking speculators and let the interest rate differential follow an AR(1) process with

a transient shock at time T

$$i_t - i_t^f = \begin{cases} 0 & \text{if } t < T \\ a(i_{t-1} - i_{t-1}^f) + \omega_t & \text{if } t \geq T \end{cases}, \text{ with } \omega_t = \begin{cases} \omega & \text{if } t = T \\ 0 & \text{otherwise} \end{cases}, \tag{C.133}$$

where $a \in (0,1)$ and ω_t is a random variable. For $t \geq T$, the market clearing condition (i.e., $q_t^{I*} = 0$) implies that

$$E_t^I[S_{t+1}^*]\exp(i_t^f) - S_t^* \exp(i_t) = 0, \qquad t \geq T, \tag{C.134}$$

which can be rewritten as

$$S_t^* = \exp(i_t^f - i_t) E_t^I[S_{t+1}^*]. \tag{C.135}$$

Iterating forward this condition and using the law of iterated expectations, we have

$$S_t^* = E_t^I\left[\exp\left(\sum_{j=0}^{k}(i_{t+j}^f - i_{t+j})\right) S_{t+k+1}^*\right] \text{ for any } k \geq 0. \tag{C.136}$$

By (C.133),

$$i_{t+j} - i_{t+j}^f = a^{j+1}(i_{t-1} - i_{t-1}^f) + \sum_{s=0}^{j} a^{j-s}\omega_{t+s} \text{ for any } j \geq 0. \tag{C.137}$$

Using (C.136) with $t = T$ and then applying (C.137), we get

$$\begin{aligned} S_T^* &= E_T^I\left[\exp\left((i_{T-1}^f - i_{T-1})\sum_{j=0}^{k} a^{j+1} - \sum_{j=0}^{k}\sum_{s=0}^{j} a^{j-s}\omega_{T+s}\right) S_{T+k+1}^*\right] \\ &= E_T^I\left[\exp\left(-\omega_T \sum_{j=0}^{k} a^j\right) S_{T+k+1}^*\right] \\ &= \exp\left(\frac{\omega(1-a^{k+1})}{a-1}\right) E_T^I\left[S_{T+k+1}^*\right], \end{aligned} \tag{C.138}$$

where the second equality is by $i_{T-1} = i^f_{T-1}$ and $\omega_{T+s} = 0$ for any $s > 0$, the third equality is by the assumption that $\omega_T = \omega$, which is observable at time T. Since $a \in (0, 1)$,

$$\exp\left(\frac{\omega(1-a^{k+1})}{a-1}\right) \to \exp\left(\frac{\omega}{a-1}\right) \tag{C.139}$$

as $k \to \infty$ almost surely. Assuming that the long–run exchange rate S^*_∞ satisfies $E^I_T\left[S^*_{T+k}\right] \to E^I_T\left[S^*_\infty\right]$ as $k \to \infty$ almost surely and that $E^I_T\left[S^*_\infty\right]$ is a bounded random variable, it follows from (C.136) and (C.139) that

$$\exp\left(\frac{\omega(1-a^{k+1})}{a-1}\right) E^I_T\left[S^*_{T+k+1}\right] \to \exp\left(\frac{\omega}{a-1}\right) E^I_T\left[S^*_\infty\right] \tag{C.140}$$

as $k \to \infty$ almost surely. Combining the results in (C.138) and (C.140), we get

$$S^*_T = \exp\left(\frac{\omega}{a-1}\right) E^I_T\left[S^*_\infty\right] \quad \text{almost surely} \tag{C.141}$$

where ω is the shock to the interest rate differential that occurs at time T and $\alpha_T = E^I_T\left[S^*_\infty\right]$. The term $\omega/(1-a)$ is the present value of future interest rate differentials and α_T may be interpreted as the conditional time-T expectation of the long-run exchange rate S^*_∞. Our model has nothing to say about the determinants of α_T. Our analysis is about equilibrium fluctuations around this long-run level.

C.1 Demand of Hedgers (3)

The time-t hedgers hold the belief that S^*_T has the same distribution as $Z_t + u_F$, where $u_F \sim N(0, \sigma^2_u)$ and u_F is independent with respect to Z_t and S_t, and Z_t is the fundamental process defined in (2). The period t hedgers observe Z_t, know the distribution of u_F, and solve the following problem

$$\max_{q^F_t \in R} E^F_t\left[-\exp(-\gamma^F W^F_t)\right], \text{ subject to } W^F_t = q^F_t\left(S^*_T - S_t\right), \tag{C.142}$$

where $\gamma^F > 0$ is a finite constant and $E^F_t[\cdot]$ denotes the conditional expectation operator taken with respect to the hedgers' belief of S^*_T given their knowledge of Z_t and S_t. Since the conditional distribution of S^*_T given Z_t and S_t is Gaussian, we can

express problem (C.142) as follows

$$\max_{q_t^F \in R} \left\{ E_t^F \left[W_t^F \right] - \frac{\gamma^F}{2} \mathrm{Var}_t^F \left[W_t^F \right] \right\}, \tag{C.143}$$

where $\mathrm{Var}_t^F [\cdot]$ denotes the conditional variance operator taking with respect to the hedgers' belief of S_T^* given their knowledge of Z_t and S_t. By definition,

$$E_t^F \left[W_t^F \right] = E_t^F \left[S_T^* - S_t \right] q_t^F = (Z_t - S_t) q_t^F, \text{ and}$$

$$\mathrm{Var}_t^F [W_t^F] = \mathrm{Var}_t^F [S_T^*] (q_t^F)^2 = \sigma_u^2 (q_t^F)^2,$$

which implies that

$$E_t^F \left[W_t^F \right] - \frac{\gamma^F}{2} \mathrm{Var}_t^F \left[W_t^F \right] = (Z_t - S_t) q_t^F - \frac{\gamma^F \sigma_u^2}{2} (q_t^F)^2.$$

Therefore, the representative hedger chooses q_t^F through the following maximization problem

$$\max_{q_t^F \in R} \left\{ (Z_t - S_t) q_t^F - \frac{\gamma^F \sigma_u^2}{2} (q_t^F)^2 \right\}.$$

The solution is $q_t^{F*} = \phi (Z_t - S_t)$, with $\phi = \left(\gamma^F \sigma_u^2 \right)^{-1}$.

C.2 Demand of Momentum Speculators (4)

Each young momentum speculator believes that all other traders are hedgers. Thus, a time-t representative momentum speculator believes that

$$S_{t+h} = Z_{t+h} \text{ for } h \in \{-1, 0, 1\}, \tag{C.144}$$

where S_{t-1} and S_t are observed by the momentum trader at time t. Second, she believes that Z_t follows the process

$$Z_{t+1} = Z_t + u_{t+1}, \tag{C.145}$$

where the shock u_{t+1} follows an autoregressive process

$$u_{t+1} = au_t + v_{t+1}, \text{ with } v_{t+1} \sim N(0, \sigma_v^2), \tag{C.146}$$

where $a \in (0,1)$ and $\sigma_v^2 > 0$ are finite constants, and $\{v_t\}_t$ is an i.i.d. process. The representative young momentum speculator maximizes the expected value of an exponential expected utility function defined over her wealth W_{t+1}^m next period

$$\max_{q_t^m} E_t^m \left[-\exp(-\gamma^m W_{t+1}^m) \right], \tag{C.147}$$

subject to the following borrowing constraint

$$b_t = S_t \left| q_t^m \right| \leq \overline{B}_t, \tag{C.148}$$

where the operator $E_t^m [\cdot]$ is the conditional expectation taken with respect to the momentum trader's beliefs specified in (C.144), (C.145) and (C.146) and her knowledge of S_{t-1} and S_t. The upper bound on debt $\overline{B}_t$ is a random variable which may take two possible values: ∞ and 0. Specifically, $\overline{B}_t$ satisfies the following properties: (a) $\overline{B}_t = \infty$ if $(\overline{B}_{t-2}, \overline{B}_{t-1}) = (0,0)$; (b) $\overline{B}_t = 0$ if $(\overline{B}_{t-2}, \overline{B}_{t-1}) = (\infty, 0)$; (c) $\overline{B}_t = \infty$ if $|S_{t-1} - \mu_z| \leq \vartheta$ and $\overline{B}_{t-1} = \infty$; (d) $\overline{B}_t = 0$ with probability $\overline{\sigma}$ and $\overline{B}_t = \infty$ with probability $1 - \overline{\sigma}$ if $\vartheta < |S_t - \mu_z| \leq \overline{\vartheta}$ and $\overline{B}_{t-1} = \infty$; (e) $\overline{B}_t = 0$ if $\mu_z + \overline{\vartheta} < S_t$ and $\overline{B}_{t-1} = \infty$.[32]

A liquidity squeeze is triggered at time t, i.e., $L_t = 1$, if $(\overline{B}_{t-1}, \overline{B}_t) = (\infty, 0)$. By definition, when the liquidity squeeze is triggered at time t, we must have $\overline{B}_{t+1} = 0$. Moreover, when $|S_{t-1} - \mu_z| \leq \vartheta$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity squeeze is triggered at time t is 0. When $\vartheta < |S_t - \mu_z| \leq \overline{\vartheta}$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity squeeze is triggered at time t is $\overline{\sigma}$. Finally, when $\mu_z + \overline{\vartheta} < S_t$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity squeeze is triggered at time t is 1. Therefore, the definition of liquidity squeeze is consistent with the one defined in the main text.

[32] Behind the law of motion of $\overline{B}_t$ one may think of a credit-market game (as in Schneider and Tornell (2004) and Rancière and Tornell (2016)) with risk neutral financiers that may lend to the momentum speculators at time t if they expect all other financiers will lend at time $t+1$.

By the momentum trader's beliefs specified in (C.144), (C.145) and (C.146),

$$W_{t+1}^m = (S_{t+1} - S_t)q_t^m = (Z_{t+1} - Z_t)q_t^m = (au_t + v_{t+1})q_t^m.$$

Since u_t and q_t^m are non-random given S_{t-1} and S_t, and v_{t+1} is independent with respect to S_{t-1} and S_t, the conditional distribution of W_{t+1}^m given S_{t-1} and S_t is Gaussian with conditional mean and conditional variance

$$E_t^m[W_{t+1}^m] = E_t^m[au_t]q_t^m = a(S_t - S_{t-1})q_t^m \text{ and} \tag{C.149}$$

$$\mathrm{Var}_t^m[W_{t+1}] = \mathrm{Var}_t^m[v_{t+1}](q_t^m)^2 = \sigma_v^2(q_t^m)^2 \tag{C.150}$$

respectively, where $\mathrm{Var}_t^m[\cdot]$ denotes the conditional variance operator taken with respect to the momentum trader's belief specified in (C.144), (C.145) and (C.146) and her knowledge on S_{t-1} and S_t. Using the properties of the Gaussian distribution we can express the objective function (C.147) as follows

$$\begin{aligned} & \max_{q_t^m} \left\{ E_t^m\left[W_{t+1}^m\right] - \frac{\gamma^m}{2}\mathrm{Var}_t^m\left[W_{t+1}^m\right] \right\} \\ = & \max_{q_t^m} \left\{ a(S_t - S_{t-1})q_t^m - \frac{\gamma^m\sigma_v^2}{2}(q_t^m)^2 \right\}, \end{aligned} \tag{C.151}$$

subject to borrowing constraint (C.148). Therefore, the demand of the representative momentum trader q_t^{m*} satisfies

$$q_t^{m*} = \begin{cases} \theta(S_t - S_{t-1}) & \text{if } |\theta(S_t - S_{t-1})|\, S_t \leq \overline{B}_t \\ \frac{\overline{B}_t}{S_t} & \text{if } \theta(S_t - S_{t-1})S_t \geq \overline{B}_t \\ -\frac{\overline{B}_t}{S_t} & \text{if } \theta(S_t - S_{t-1})S_t \leq -\overline{B}_t \end{cases}, \tag{C.152}$$

where $\theta = a(\gamma^m\sigma_v^2)^{-1} > 0$. We next link q_t^{m*} with the indicator variable of liquidity squeeze L_t. First, at any period t, we have and only have one of the following three cases: (i) $\theta(S_t - S_{t-1})S_t = 0$; (ii) $\theta(S_t - S_{t-1})S_t > 0$; (iii) $\theta(S_t - S_{t-1})S_t < 0$. When $L_{t-1} = 1$ or $L_t = 1$, we have $\overline{B}_t = 0$. Therefore the demand of the momentum traders $q_t^{m*} = 0$ by (C.152) in all three cases (i), (ii) and (iii). Second, if $L_{t-1} = 0$ and $L_t = 0$, we have $\overline{B}_t = \infty$ which together with (C.152) implies that $q_t^{m*} = \theta(S_t - S_{t-1})$. In

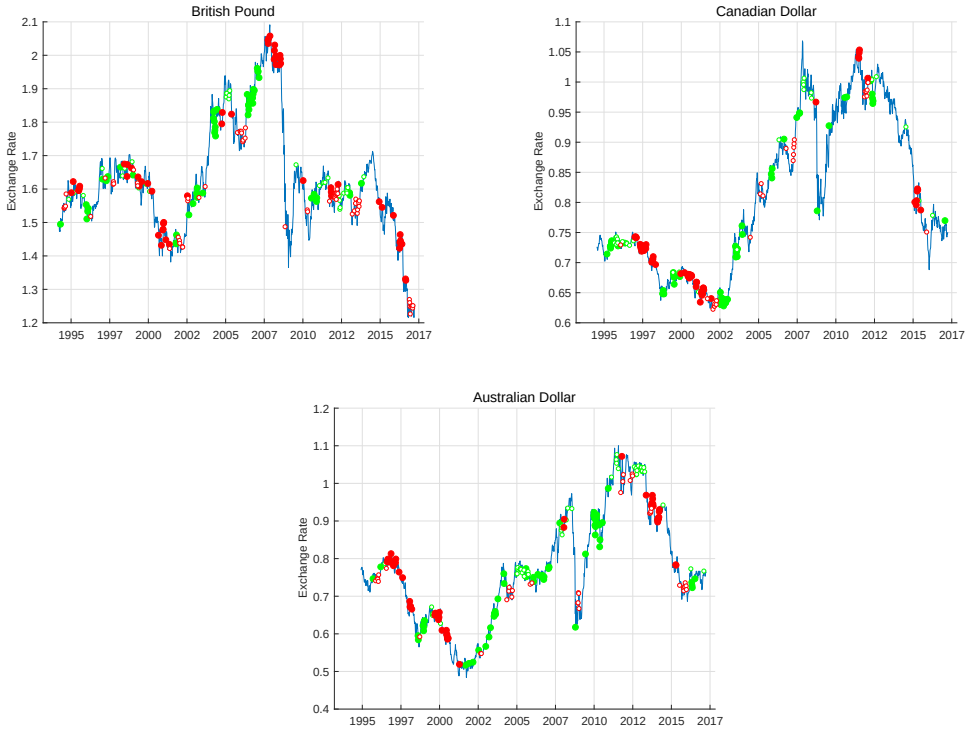
sum, we have

$$q_t^{m*} = \begin{cases} \theta\left(S_t - S_{t-1}\right), & \text{if } L_{t-1} = 0 \text{ and } L_t = 0 \\ 0, & \text{if } L_{t-1} = 1 \text{ or } L_t = 1 \end{cases}, \quad \theta = \frac{a}{\gamma^m \sigma_v^2} > 0, \tag{C.153}$$

which is the demand of momentum traders (4) in the main text.

D Extra Empirical Results

Figure D.1: Success and Failures of 9-month Ahead Directional Forecasts



Note: The figure illustrates the performance of 9-month ahead directional forecasts. Green (red) dots depict the time when an appreciation (depreciation) forecast has been made. If a forecast turned out to correctly predict the direction of movements of the currency, then the dot is filled. If the forecast is wrong, the dot is empty.

Table D.1: Mean of 1-year Exchange Rate Changes

Currency	Unconditional	Conditional on appreciation forecasts	Conditional on depreciation forecasts
AUD	0.14	1.41	−4.28
CAD	0.27	2.02	−1.48
EUR	1.46	3.07	−4.60
JPY	−0.81	2.35	−5.69
GBP	−0.78	2.06	−2.37

Note: The table reports the unconditional mean of 1-year exchange rate changes and the mean of 1-year exchange rate changes conditional on appreciation/depreciation forecasts based on our directional forecasts. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table D.2: Success Ratio of the Directional Forecasts

Currency	Forecasting Horizon (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Appreciation Forecasts				
AUD	0.600	0.618	0.609	0.560	0.574
	(110)	(110)	(110)	(109)	(108)
CAD	0.616	0.651	0.686	0.674	0.588
	(86)	(86)	(86)	(86)	(85)
EUR	0.675	0.590	0.731	0.714	0.658
	(80)	(78)	(78)	(77)	(76)
JPY	0.519	0.468	0.570	0.620	0.646
	(79)	(79)	(79)	(79)	(79)
GBP	0.529	0.624	0.647	0.612	0.576
	(85)	(85)	(85)	(85)	(85)
	Panel B: Depreciation Forecasts				
AUD	0.560	0.527	0.626	0.571	0.637
	(91)	(91)	(91)	(91)	(91)
CAD	0.539	0.629	0.753	0.705	0.705
	(89)	(89)	(89)	(88)	(88)
EUR	0.477	0.523	0.662	0.615	0.631
	(65)	(65)	(65)	(65)	(65)
JPY	0.574	0.603	0.644	0.739	0.723
	(129)	(126)	(118)	(111)	(101)
GBP	0.585	0.569	0.455	0.472	0.538
	(123)	(123)	(123)	(123)	(119)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table D.3: Currency Excess Returns

Currency	Holding Periods (h)				
	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: Equally-weighted Portfolio				
PCO Strategy	0.702	0.629	0.725	0.682	0.531
	(0.525)	(0.652)	(0.813)	(0.811)	(0.750)
$TSM(12, h)$	1.611	1.231	0.951	0.699	0.741
	(0.263)	(0.217)	(0.176)	(0.136)	(0.151)
$Carry(12, h)$	1.679	1.584	1.573	1.769	1.675
	(0.365)	(0.345)	(0.352)	(0.405)	(0.391)
	Panel B: Volatility-adjusted Portfolio				
PCO Strategy	1.368	1.242	1.173	0.929	0.672
	(0.544)	(0.746)	(0.886)	(0.860)	(0.711)
$TSM(12, h)$	3.076	2.436	1.433	0.913	0.583
	(0.447)	(0.370)	(0.226)	(0.148)	(0.101)
$Carry(12, h)$	1.195	1.262	1.294	1.605	1.567
	(0.195)	(0.211)	(0.223)	(0.281)	(0.279)

Note: Panel A reports the results of annualized excess returns and Sharpe ratios of the equally weighted portfolio. Panel B reports the results of annualized excess returns and Sharpe ratios of the volatility-adjusted portfolio. Sharpe ratios are in parentheses.

Table D.4: Annualized Excess Returns and Sharpe Ratios for Individual Currencies

	Holding Periods (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: PCO Strategy				
AUD	1.099	0.973	0.697	0.813	0.666
	(0.324)	(0.404)	(0.343)	(0.473)	(0.395)
CAD	1.017	0.907	0.669	0.563	0.421
	(0.469)	(0.672)	(0.564)	(0.518)	(0.396)
EUR	0.459	0.571	1.005	1.010	0.591
	(0.155)	(0.265)	(0.511)	(0.513)	(0.322)
JPY	1.215	0.678	1.162	0.927	0.778
	(0.453)	(0.351)	(0.522)	(0.520)	(0.482)
GBP	0.867	0.733	0.662	0.700	0.527
	(0.347)	(0.421)	(0.407)	(0.436)	(0.356)
	Panel B: $TSM(12, h)$ Strategy				
AUD	3.408	1.839	1.607	1.282	1.639
	(0.287)	(0.172)	(0.156)	(0.128)	(0.169)
CAD	2.075	1.976	1.188	1.104	1.361
	(0.259)	(0.263)	(0.176)	(0.170)	(0.218)
EUR	−1.072	−3.053	−2.933	−2.336	−1.806
	(−0.118)	(−0.360)	(−0.373)	(−0.327)	(−0.277)
JPY	4.416	4.892	3.697	2.848	1.812
	(0.399)	(0.456)	(0.355)	(0.282)	(0.187)
GBP	0.509	1.136	1.552	1.634	1.559
	(0.058)	(0.136)	(0.193)	(0.213)	(0.217)

Note: Panel A reports the results of annualized excess returns and Sharpe ratios of the PCO strategy for individual currencies. Panel B reports the results of annualized excess return and Sharpe ratio of the $TSM(12, h)$ strategy for individual currencies. Sharpe ratios are in parentheses.

Table D.5: Annualized Excess Returns and Sharpe Ratios for Individual Currencies (Volatility-adjusted Portfolio)

	Holding Periods (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
	Panel A: PCO Strategy				
AUD	0.277	0.670	0.506	0.669	0.534
	(0.058)	(0.193)	(0.175)	(0.280)	(0.255)
CAD	1.801	1.799	1.621	1.186	0.725
	(0.324)	(0.530)	(0.602)	(0.450)	(0.301)
EUR	2.297	1.137	1.195	0.995	0.477
	(0.489)	(0.359)	(0.495)	(0.480)	(0.260)
JPY	2.176	1.784	1.852	1.502	1.347
	(0.452)	(0.475)	(0.682)	(0.689)	(0.671)
GBP	4.747	3.377	2.065	1.763	1.279
	(0.538)	(0.530)	(0.382)	(0.463)	(0.362)
	Panel B: Time-series Momentum Strategy				
AUD	2.848	1.561	1.244	0.460	0.204
	(0.224)	(0.132)	(0.112)	(0.042)	(0.019)
CAD	8.933	8.483	5.366	3.978	3.551
	(0.537)	(0.653)	(0.445)	(0.313)	(0.272)
EUR	0.669	−1.851	−2.169	−1.777	−1.587
	(0.058)	(−0.174)	(−0.244)	(−0.220)	(−0.215)
JPY	5.486	6.494	4.204	3.021	2.043
	(0.437)	(0.540)	(0.366)	(0.266)	(0.195)
GBP	−0.302	0.619	0.924	0.729	0.495
	(−0.022)	(0.049)	(0.079)	(0.069)	(0.052)

Note: Panel A reports the results of annualized excess returns and Sharpe ratios of the PCO strategy for individual currencies with volatility-adjusted. Panel B reports the results of annualized excess returns changes and Sharpe ratios of the $TSM(12, h)$ strategy for individual currencies with volatility-adjusted. Sharpe ratios are in parentheses.

Extended Appendix

This online Appendix provides supporting materials complement Kim, Kim, Liao and Tornell (2019). Sections E and F characterize equilibrium negative bubbly equilibria. Section G describes the construction of the long-run variance estimators used in the main text. Section H presents empirical results which serve as robustness check of Section 3 in the main text.

E Negative Bubbly Path

An analogous result to that in Proposition 7 and Proposition 8 applies to a negative bubbly equilibrium. In particular, we characterize the conditions under which starting with $S_0^* = \mu_z$, a transitory shock to the expected fundamental can induce a negative bubbly path. Concretely, we consider the following trajectory for the shock in (2)

$$\zeta_t = \begin{cases} -z & \text{if } t = 1 \\ 0 & \text{if } t > 1 \end{cases} . \tag{E.154}$$

Along the negative bubbly path, informed speculators anticipate a continuation of the negative bubble and find it optimal to ride the bubble for a while: they choose $C_t^* = -c$ expecting $\Delta S_{t+1}^* < 0$ at time t, and at $t+1$ they set $C_{t+1}^* = -c$ expecting $\Delta S_{t+2}^* < 0$, and so on until a critical time τ^*–which we call switching time.

Since S_t is decreasing along the negative bubbly path, there should be some device in the model to prevent the exchange rate from becoming negative. For this purpose, we revise the law of motion of the liquidity squeeze process:

$$\sigma_{t+1} \equiv \Pr(L_{t+1} = 1 | S_t) = \begin{cases} 0, & |S_t - \mu_z| \leq \vartheta \text{ and } S_t > \underline{S} \\ \overline{\sigma}, & |S_t - \mu_z| > \vartheta \text{ and } S_t > \underline{S} \\ 1, & S_t \leq \underline{S} \text{ or } S_t - \mu_z > \overline{\vartheta} \end{cases} , \tag{E.155}$$

where $\underline{S}$ is a small non-random positive lower bound. By (E.155), the probability of a liquidity squeeze becomes $\overline{\sigma}$ when the exchange rate S_t deviates from Z_t by more than ϑ, and it becomes 1 when S_t is smaller than $\underline{S}$. To ensure that a liquidity squeeze is not triggered immediately following a negative shock at time 1, we impose the following condition on $\underline{S}$.

Condition 17. *The lower exchange rate bound $\underline{S}$ satisfies*

$$\underline{S} < \mu_z - z - \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}. \tag{E.156}$$

To ensure that equilibrium exchange rates are positive along the equilibrium negative bubbly path, we impose the following parametric conditions.

Condition 18. *The model parameters satisfy*

$$\begin{cases} \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi}\frac{\theta+\phi}{\theta-\phi} > 0, & \text{if } \underline{\tau} \leq \tau_* \\ \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_e-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\underline{\tau}_e-\tau_*}} - 1\right] > 0, & \text{if } \tau_* < \underline{\tau} \end{cases}. \tag{E.157}$$

where τ_* is defined in (14),

$$\underline{\tau}_e = \min\left\{t > \tau_* : \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau_*}} - 1\right] \leq \underline{S}\right\}, \tag{E.158}$$

and

$$\underline{\tau} = \max\left\{t \in \mathbb{N} : \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} > \underline{S}\right\}. \tag{E.159}$$

Conditions (13) and (E.156) imply that $\underline{\tau} \geq 1$, which ensures that $\underline{\tau}$ is well defined. To grasp the intuition for Condition 18, notice that on one hand, when $\underline{\tau} \leq \tau_*$, a liquidity squeeze will be triggered no later than $\underline{\tau}+3$ (by Lemmas 24, 25 and 28 in Appendix). When a liquidity squeeze happens at $\underline{\tau}+3$, the equilibrium exchange rate at time $\underline{\tau}+2$ is

$$S^*_{\underline{\tau}+2} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi}\frac{\theta+\phi}{\theta-\phi},$$

which is the first line in (E.157). On the other hand, when $\tau_* < \underline{\tau}$ a liquidity squeeze will be triggered no later than $\underline{\tau}_e+1$ (as we show in the proof of Corollary 19). When a liquidity squeeze happens at $\underline{\tau}_e+1$, the equilibrium exchange rate at time $\underline{\tau}_e$ is

$$S^*_{\underline{\tau}_e} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_e-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\underline{\tau}_e-\tau_*}} - 1\right],$$

which is the second line in (E.157). Therefore, Condition 18 ensures that exchange rates are positive along the negative bubbly path. We are now equipped to establish the existence of negative bubbly equilibria.

Corollary 19 (Negative Bubbly Equilibrium)**.** *Let $\underline{\tau}_* = \min\{\underline{\tau}, \tau_*\}$. Consider an economy that has a liquidity squeeze at time $t = 0$, i.e., $L^*_0 = 1$, and is hit by a*

transitory negative shock at time $t = 1$ (i.e., $\zeta_0 = 0$, $\zeta_1 = -z$ and $\zeta_t = 0$ for all $t \geq 2$). Under (10), (12), (13) and Conditions 17 and 18, the equilibrium bubbly path has the following properties.

(i) At time 0, the equilibrium exchange rate is $S_0^* = \mu_z$ and the informed speculators' demand is $C_0^* = 0$.

(ii) There exists a switching time τ^* with $1 \leq \tau^* - \underline{\tau}_* \leq 2$ such that

$$S_t^* = \begin{cases} \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{t-1}}, & 1 \leq t < \tau^* \\ \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right] & \tau^* \leq t < l^* \end{cases}, \tag{E.160}$$

where $S_t^* > 0$ for any t with $1 \leq t < l^*$.

(iii) The liquidity squeeze is triggered in finite time, i.e., $\tau^* \leq l^* \leq \underline{l}_{\max}$, where $\underline{l}_{\max} = \max\{\underline{\tau} + 3, \underline{\tau}_e + 2\}$.

(iv) The informed speculators' demand satisfies

$$C_t^* = \begin{cases} -c, & 1 \leq t < \tau^* \\ c, & \tau^* \leq t < l^* \end{cases}. \tag{E.161}$$

(v) The momentum speculators' demand satisfies

$$q_t^{m^*} = \begin{cases} 0, & t \in \{0, 1, l^*\} \\ \frac{-z\phi}{(1-\phi/\theta)^{t-1}}, & 2 \leq t < \tau^* \\ -\frac{z\phi}{(1-\phi/\theta)^{t-1}} - \frac{2c}{(1-\phi/\theta)^{t-\tau^*+1}} & \tau^* \leq t < l^* \end{cases}. \tag{E.162}$$

(vi) At time l^*, $S_{l^*}^* = \mu_z$ and $C_{l^*}^* = 0$.

Remark 2.9. The equilibrium along the negative bubbly path has similar properties to the positive bubbly path. The informed speculators find it optimal to short the foreign bond and ride the negative bubble until a liquidity squeeze becomes possible. The principle of contrarian opinion is also present: At switching time τ^*, informed speculators switch their foreign bond's short positions to long positions and the Dollar necessarily appreciates (i.e., S_t decreases) even faster. After τ^*, momentum speculators continue increasing their short positions at a faster pace, with hedgers taking the corresponding increasing long positions. This equilibrium process continues until a liquidity squeeze is triggered and the exchange rate jumps up to the fundamental value Z_{l^*}. □

F Proof of the Negative Bubbly Equilibrium

Lemma 20. *Under conditions (10), (12) and (E.156), we have*

$$\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi} \leq \underline{S}. \tag{F.163}$$

Proof of Lemma 20. By the definition of $\underline{\tau}$,

$$\underline{S} \geq \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} - \frac{2c/\theta}{1-\phi/\theta}. \tag{F.164}$$

By $z > 2c/\phi$ and $\underline{\tau} \geq 1$,

$$\frac{z\phi/\theta}{(1-\phi/\theta)^{\underline{\tau}}} \geq \frac{z\phi/\theta}{1-\phi/\theta} > \frac{2c/\theta}{1-\phi/\theta}. \tag{F.165}$$

Since $\phi/\theta \in (0,1)$, $(1-\phi/\theta)^{-1} = \sum_{k=0}^{\infty}(\phi/\theta)^k$ which together with (F.165) implies that

$$\begin{aligned}
\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi} &= \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}}\sum_{k=0}^{\infty}(\phi/\theta)^k - \frac{c}{\phi} \\
&< \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}}(1+\phi/\theta) - \frac{c}{\phi} \\
&< \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} - \frac{2c/\theta}{1-\phi/\theta}.
\end{aligned} \tag{F.166}$$

Combining the results in (F.164) and (F.166), we immediately get (F.163). ∎

Lemma 21. *Under the conditions of Corollary 19, we have*

$$S_1^* = \mu_z - z - \phi^{-1}c,\ C_1^* = -c,\ L_1^* = 0 \text{ and } L_2^* = 0. \tag{F.167}$$

Proof of Lemma 21. By Proposition 5, at time 0 we have

$$S_0^* = \mu_z,\ C_0^* = 0,\ L_0^* = 1 \text{ and } L_1^* = 0. \tag{F.168}$$

At time $t = 1$, the informed speculators observe $Z_1 = \mu_z - z$. Moreover since $L_0^* = 1$, by (B.51) the market clearing condition in this period is

$$S_1^* = Z_1 + \frac{C_1^*}{\phi}. \tag{F.169}$$

Because (i) $|z - \phi^{-1}C_1^*| \leq z + \phi^{-1}c$ for any $C_1^* \in \{-c, 0, c\}$, and (ii) $z + \phi^{-1}c <$

$(1-\phi/\theta)^{-1}\left[z+(1+\phi/\theta)c\phi^{-1}\right]$ by (10), it follows from (13) that

$$|S_1^* - \mu_z| < \vartheta. \tag{F.170}$$

By Condition (E.156) and (F.169),

$$S_1^* \geq \mu_z - z - \phi^{-1}c > \underline{S} \tag{F.171}$$

regardless of the demand of the informed speculators at time 1. Hence the law of motion of the probability of liquidity squeeze (E.155) implies that $L_2^* = 0$ with probability 1. Since $L_1^* = 0$ and $L_2^* = 0$, we can use (B.49) with $t = 2$ and (F.169) to get

$$\begin{aligned}\Delta S_2^* &= \frac{\phi(P_1^* - \mu_z) - C_2^*}{\theta - \phi} - \frac{\phi\zeta_2}{\theta - \phi} \\ &= \frac{-\phi z + C_1^* - C_2^*}{\theta - \phi} - \frac{\phi\zeta_2}{\theta - \phi} < -\frac{\phi\zeta_2}{\theta - \phi}\end{aligned} \tag{F.172}$$

for any $C_1^*, C_2^* \in \{-c, 0, c\}$, where the last inequality follows from $\theta > \phi > 0$ and $z > 2c/\phi$. Since $E_1[\zeta_2] = 0$, the inequality in (F.172) implies that $E_1[\Delta S_2^*] < 0$, which together with the demand function of the informed speculators (1) and the price equation (F.169), implies that $C_1^* = -c$ and $S_1^* = \mu_z - z - c/\phi$. This finishes the proof of (F.167). ∎

Lemma 22. *Under the conditions of Corollary 19, we have*

$$S_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{t-1}},\ C_t^* = -c,\ L_t^* = 0 \text{ and } L_{t+1}^* = 0 \tag{F.173}$$

for any t with $1 \leq t \leq \underline{\tau}_*$.

Proof of Lemma 22. By Lemma 21, (F.173) holds for $t = 1$. Since $\tau_* \geq 2$, the proof is finished if $\underline{\tau} = 1$. In the rest of the proof, we assume that $\underline{\tau} \geq 2$. We shall prove that (F.173) holds for t with $2 \leq t < \underline{\tau}_*$ by mathematical induction.

Suppose that (F.173) holds for $t-1$. By Lemma 21, (F.173) holds for $t = 1$. We shall prove that (F.173) holds for t with $2 \leq t < \underline{\tau}_*$ by mathematical induction. Suppose that (F.173) holds for $t-1$. Since $L_{t-1}^* = 0$, $L_t^* = 0$ and $\xi_t = 0$, at period t we have $Z_t = \mu_z$ and

$$S_t^* = \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{\theta c/\phi + C_t^*}{\theta - \phi}. \tag{F.174}$$

Because $t \leq \tau_*$, we know that

$$|S_t^* - \mu_z| = \left|\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{\theta c/\phi + C_t^*}{\theta - \phi}\right| \leq \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} < \vartheta. \tag{F.175}$$

Moreover since $t \leq \underline{\tau}$, by (F.174) and the definition of $\underline{\tau}$,

$$S_t^* \geq \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} > \underline{S}. \tag{F.176}$$

From (F.175), (F.176) and the law of motion of the probability of liquidity squeeze (E.155), the informed speculators know that $L_{t+1}^* = 0$ with probability 1. Hence, by (B.49) and (F.174), we have

$$\begin{aligned}\Delta S_{t+1}^* &= \frac{\phi(S_t^* - \mu_z) - C_{t+1}^*}{\theta - \phi} - \frac{\phi\zeta_{t+1}}{\theta-\phi} \\ &= -\frac{\phi z + (1-\phi/\theta)^{t-1}C_{t+1}^*}{(\theta-\phi)(1-\phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta-\phi)^2} - \frac{\phi\zeta_{t+1}}{\theta-\phi}.\end{aligned} \tag{F.177}$$

Under the restrictions $\theta > \phi$ and $z > 2c/\phi$, we have

$$-\frac{\phi z + (1-\phi/\theta)^{t-1}C_{t+1}^*}{(\theta-\phi)(1-\phi/\theta)^{t-1}} < 0 \text{ and } -\frac{\theta c + \phi C_t^*}{(\theta-\phi)^2} < 0 \tag{F.178}$$

for any $C_t^*, C_{t+1}^* \in \{-c, 0, c\}$, which together with (F.177) and $E_t[\zeta_{t+1}] = 0$ implies that

$$E_t[\Delta S_{t+1}^*] = -\frac{\phi z + (1-\phi/\theta)^{t-1}E_t[C_{t+1}^*]}{(\theta-\phi)(1-\phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta-\phi)^2} < 0. \tag{F.179}$$

Combing (F.179) with the demand function of the informed speculators we get $C_t^* = -c$, which together with (F.174) shows that (F.173) holds at t. This finishes the proof. ∎

Lemma 23. *Suppose that $L_t^* = L_{t+1}^* = 0$, $\zeta_{t+1} = 0$ and*

$$S_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{t-1}}. \tag{F.180}$$

If the probability of liquidity squeeze at $t+2$ is larger than ϕ/θ, then

$$E_{t+1}[\Delta S_{t+2}^*] > 0,\ C_{t+1}^* = c \text{ and } S_{t+1}^* = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^t}. \tag{F.181}$$

Proof of Lemma 23. Since $L_t^* = L_{t+1}^* = 0$ and $\zeta_{t+1} = 0$, by (B.48) and (F.180), we have

$$S_{t+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)^t} - \frac{\theta c/\phi + C_{t+1}^*}{\theta - \phi}. \tag{F.182}$$

Let σ_s denote the probability of $L_{t+2}^* = 1$. On the one hand if there is $L_{t+2}^* = 0$,

(B.49) and (F.180) imply that

$$\Delta S^*_{t+2} = \frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \frac{\theta c + \phi C^*_{t+1}}{(\theta-\phi)^2} - \frac{C^*_{t+2} + \phi\zeta_{t+2}}{\theta-\phi}. \tag{F.183}$$

On the other hand if $L^*_{t+2} = 1$, equilibrium price equation (B.51) and (F.180) imply that

$$\Delta S^*_{t+2} = \zeta_{t+2} + \frac{C^*_{t+2}}{\phi} + \frac{z}{(1-\phi/\theta)^t} + \frac{\theta c/\phi + C^*_{t+1}}{\theta-\phi}. \tag{F.184}$$

Combining the results in (F.183) and (F.184), and applying $E_{t+1}[\zeta_{t+2}] = 0$, we get

$$\begin{aligned} E_{t+1}[\Delta S^*_{t+2}] &= (1-\sigma_s)\left[\frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \frac{E_{t+1}[C^*_{t+2}]}{\theta-\phi} - \frac{\theta c + \phi C^*_{t+1}}{(\theta-\phi)^2}\right] \\ &\quad + \sigma_s\left[\frac{E_{t+1}[C^*_{t+2}]}{\phi} + \frac{z}{(1-\phi/\theta)^t} + \frac{\theta c/\phi + C^*_{t+1}}{\theta-\phi}\right] \\ &= \frac{z}{(1-\phi/\theta)^t}\frac{\sigma_s-\phi/\theta}{1-\phi/\theta} + \frac{\theta^2(\sigma_s-\phi/\theta)c}{\phi(\theta-\phi)^2} \\ &\quad + \frac{\theta(\sigma_s-\phi/\theta)}{\phi(\theta-\phi)}E_{t+1}[C^*_{t+2}] + \frac{\theta(\sigma_s-\phi/\theta)}{(\theta-\phi)^2}C^*_{t+1}. \end{aligned} \tag{F.185}$$

Define

$$Q(c_1, c_2) = \frac{\theta^2(\sigma_s-\phi/\theta)c}{\phi(\theta-\phi)^2} + \frac{\theta(\sigma_s-\phi/\theta)}{\phi(\theta-\phi)}c_2 + \frac{\theta(\sigma_s-\phi/\theta)}{(\theta-\phi)^2}c_1. \tag{F.186}$$

Then

$$\begin{aligned} Q(-c,-c) &= \frac{\theta^2(\sigma_s-\phi/\theta)c}{\phi(\theta-\phi)^2} - \frac{\theta(\sigma_s-\phi/\theta)}{\phi(\theta-\phi)}c - \frac{\theta(\sigma_s-\phi/\theta)}{(\theta-\phi)^2}c \\ &= \frac{\theta(\sigma_s-\phi/\theta)c}{\theta-\phi}\left(\frac{\theta}{\phi(\theta-\phi)} - \frac{1}{\phi} - \frac{1}{\theta-\phi}\right) = 0, \end{aligned} \tag{F.187}$$

which together with $\sigma_s > \phi/\theta$ implies that

$$E_{t+1}[\Delta S^*_{t+2}] \geq \frac{z}{(1-\phi/\theta)^t}\frac{\sigma_s-\phi/\theta}{1-\phi/\theta} > 0 \tag{F.188}$$

regardless the demand of the informed speculators. By (F.188) and demand law of the informed speculators, we have $C^*_{t+1} = c$ which together with (F.180) proves (F.181). ∎

Lemma 24. *Suppose that the conditions of Corollary 19 hold. If*

$$\underline{\tau} \leq \tau_* \text{ and } \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} \leq \underline{S}, \tag{F.189}$$

then

$$C^*_{\underline{\tau}_*+1} = c,\ S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \text{ and } l^* = \underline{\tau}+2. \tag{F.190}$$

Proof of Lemma 24. Since $\underline{\tau} \le \tau_*$, we have

$$\underline{\tau}_* = \underline{\tau}. \tag{F.191}$$

By (F.173), we have $L^*_{\underline{\tau}_*} = 0$ and $L^*_{\underline{\tau}_*+1} = 0$. Since $\zeta_{\underline{\tau}_*+1} = 0$ at time $\underline{\tau}_*+1$, by (B.48) and (F.173) with $t = \underline{\tau}_*+1$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} - \frac{\theta c/\phi + C^*_{\underline{\tau}_*+1}}{\theta-\phi}. \tag{F.192}$$

Since $\theta > \phi$, $(\theta-\phi)^{-1}(\theta c/\phi + C^*_{\underline{\tau}_*+1})$ is an increasing function of $C^*_{\underline{\tau}_*+1}$. Therefore,

$$S^*_{\underline{\tau}_*+1} \le \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} - \frac{c}{\phi} \tag{F.193}$$

regardless of the demand of the informed speculators at time $\underline{\tau}_*+1$. By (F.191), (F.192) and the second inequality in (F.189), the law of motion of L^*_t in (E.155) implies that $L^*_{\underline{\tau}_*+2} = 1$ with probability 1, regardless of the informed speculators' demand at $\underline{\tau}_*+1$. By Lemma 22,

$$S^*_{\underline{\tau}_*} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*-1}}. \tag{F.194}$$

Hence by Lemma 23,

$$C^*_{\underline{\tau}_*+1} = c \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{F.195}$$

Since $L^*_{\underline{\tau}_*+2} = 1$ with probability 1 and $\underline{\tau}_* = \underline{\tau}$, we have $l^* = \underline{\tau}+2$. This finishes the proof. ∎

Lemma 25. *Suppose that the conditions of Corollary 19 hold. If*

$$\underline{\tau} < \tau_* \text{ and } \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} > \underline{S}, \tag{F.196}$$

then there exist two possible equilibriums at time $\underline{\tau}_*+1$. The first equilibrium is the same as the one derived in Lemma 24. In the second equilibrium, $C^*_{\underline{\tau}_*+1} = -c$, $C^*_{\underline{\tau}_*+2} = c$, $l^* = \underline{\tau}+3$,

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} \tag{F.197}$$

and

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}}. \tag{F.198}$$

Proof of Lemma 25. In the proof of Lemma 24, we have show that

$$\underline{\tau}_* = \underline{\tau} \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} - \frac{\theta c/\phi + C^*_{\underline{\tau}_*+1}}{\theta - \phi}. \tag{F.199}$$

To prove that (F.190) is an equilibrium, it is sufficient to show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] > 0. \tag{F.200}$$

Given $C^*_{\underline{\tau}_*+1} = c$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{F.201}$$

By the definition of $\underline{\tau}$, $S^*_{\underline{\tau}_*+1} \leq \underline{S}$ which together with the law of motion of L^*_t in (E.155) implies that

$$L^*_{\underline{\tau}_*+2} = 1 \text{ with probability 1.} \tag{F.202}$$

Hence by Lemma 23,

$$C^*_{\underline{\tau}_*+1} = c \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}.$$

Since $L^*_{\underline{\tau}_*+2} = 1$ with probability 1 and $\underline{\tau}_* = \underline{\tau}$, we have $l^* = \underline{\tau} + 2$.

To derive the second equilibrium, it is sufficient to show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] < 0. \tag{F.203}$$

Given $C^*_{\underline{\tau}_*+1} = -c$, by (F.199) we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}, \tag{F.204}$$

which together with the second inequality in (F.196) implies that

$$S^*_{\underline{\tau}_*+1} > \underline{S}. \tag{F.205}$$

Since $\underline{\tau} < \tau_*$, by the definition of τ_*

$$\begin{aligned} \left|S^*_{\underline{\tau}_*+1} - \mu_z\right| &= \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} + \frac{c}{\phi} \\ &< \frac{z}{(1-\phi/\theta)^{\tau_*-1}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \leq \vartheta. \end{aligned} \tag{F.206}$$

By (F.205), (F.206) and the law of motion of L_t^* in (5),

$$L^*_{\underline{\tau}_*+2} = 0 \text{ with probability 1.} \tag{F.207}$$

Under (F.207), we can invoke (B.48) to get

$$\Delta S^*_{\underline{\tau}_*+2} = \frac{\phi}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi} Z_{\underline{\tau}_*+2} - \frac{C^*_{\underline{\tau}_*+2}}{\theta-\phi} \tag{F.208}$$

which together with $E_{\underline{\tau}_*+1}[\zeta^*_{\underline{\tau}_*+2}] = 0$ and (F.204) implies that

$$\begin{aligned} E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] &= \frac{\phi}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi}\mu_z - \frac{E_{\underline{\tau}_*+1}[C^*_{\underline{\tau}_*+2}]}{\theta-\phi} \\ &= -\frac{\phi}{\theta-\phi}\frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c + E_{\underline{\tau}_*+1}[C^*_{\underline{\tau}_*+2}]}{\theta-\phi} \\ &\leq -\frac{\phi}{\theta-\phi}\frac{z}{(1-\phi/\theta)^{\underline{\tau}}} < 0 \end{aligned} \tag{F.209}$$

where the first inequality is by $E_{\underline{\tau}_*+1}[C^*_{\underline{\tau}_*+2}] \geq -c$ and $\theta > \phi$. This proves (F.203) and hence the equilibrium in time $\underline{\tau}_* + 1$. Since $L^*_{\underline{\tau}_*+1} = 0$, $L^*_{\underline{\tau}_*+2} = 0$ and $\zeta^*_{\underline{\tau}_*+2} = 0$, at time $\underline{\tau}_* + 2$, the equilibrium price satisfies

$$\begin{aligned} S^*_{\underline{\tau}_*+2} &= \frac{\theta}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi} Z_{\underline{\tau}_*+2} - \frac{C^*_{\underline{\tau}_*+2}}{\theta-\phi} \\ &= \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c\theta/\phi + C^*_{\underline{\tau}_*+2}}{\theta-\phi}, \end{aligned} \tag{F.210}$$

which implies that

$$S^*_{\underline{\tau}_*+2} \leq \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi} \leq \underline{S} \tag{F.211}$$

regardless the demand of the informed speculators in time $\underline{\tau}_* + 2$. By the law of motion of L_t^* in (E.155),

$$L^*_{\underline{\tau}_*+3} = 1 \text{ with probability 1.} \tag{F.212}$$

Therefore, by (F.204) and (F.212), we can invoke Lemma 23 to show $C^*_{\underline{\tau}_*+2} = c$ which finishes the proof of the second equilibrium. ∎

Lemma 26. *Suppose that the conditions of Corollary 19 hold. If*

$$\tau_* \leq \underline{\tau} \text{ and } \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta, \tag{F.213}$$

then

$$C^*_{\underline{\tau}_*+1} = c \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{F.214}$$

Proof of Lemma 26. Since $\tau_* \leq \underline{\tau}$, by the definition of $\underline{\tau}_*$, we have

$$\underline{\tau}_* = \tau_*. \tag{F.215}$$

By (F.173), we have $L^*_{\underline{\tau}_*} = 0$ and $L^*_{\underline{\tau}_*+1} = 0$. Since $\zeta_{\underline{\tau}_*+1} = 0$ at time $\underline{\tau}_*+1$, by (B.48) and (F.173) with $t = \underline{\tau}_* + 1$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{\theta c/\phi + C^*_{\tau_*+1}}{\theta - \phi}. \tag{F.216}$$

Since $C^*_{\tau_*+1} \in \{-c, 0, c\}$,

$$\frac{\theta c/\phi + C^*_{\tau_*+1}}{\theta - \phi} \leq \frac{\theta c/\phi - c}{\theta - \phi} = \frac{c}{\phi}, \tag{F.217}$$

which implies that

$$S^*_{\underline{\tau}_*+1} > \vartheta. \tag{F.218}$$

Hence by the law of motion of L^*_t in (5),

$$L^*_{\underline{\tau}_*+2} = 1 \text{ with probability } \overline{\sigma}. \tag{F.219}$$

The equilibrium in (F.214) follows by Lemma 23. ∎

Lemma 27. *Suppose that the conditions of Corollary 19 hold. If*

$$\tau_* < \underline{\tau} \text{ and } \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta, \tag{F.220}$$

then there exist two possible equilibriums at time $\underline{\tau}_* + 1$. The first equilibrium is the same as the one derived in Lemma 26. In the second equilibrium, $C^*_{\underline{\tau}_*+1} = -c$, $C^*_{\underline{\tau}_*+2} = c$,

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} \tag{F.221}$$

and

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}}. \tag{F.222}$$

Proof of Lemma 27. In the proof of Lemma 26, we have show that

$$\underline{\tau}_* = \tau_* \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{\theta c/\phi + C^*_{\tau_*+1}}{\theta - \phi}. \tag{F.223}$$

To prove that (F.214) is an equilibrium, it is sufficient to show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] > 0. \tag{F.224}$$

Given $C^*_{\underline{\tau}_*+1} = c$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{F.225}$$

By the definition of τ_*,

$$\left|S^*_{\underline{\tau}_*+1} - \mu_z\right| = \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} > \vartheta, \tag{F.226}$$

which together with the law of motion of L^*_t in (5) implies that

$$L^*_{\underline{\tau}_*+2} = 1 \text{ with probability at least } \overline{\sigma}. \tag{F.227}$$

Therefore, (F.224) can be proved with the same arguments in the proof of Lemma 26.

To derive the second equilibrium, it is sufficient to show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] < 0. \tag{F.228}$$

Given $C^*_{\underline{\tau}_*+1} = -c$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}, \tag{F.229}$$

which together with the second inequality in (F.220) implies that

$$\left|S^*_{\underline{\tau}_*+1} - \mu_z\right| = \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta. \tag{F.230}$$

Moreover, since $\tau_* < \underline{\tau}$,

$$\begin{aligned} \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi} &\geq \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}-1}} - \frac{c}{\phi} \\ &> \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}-1}} - \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi} > \underline{S} \end{aligned} \tag{F.231}$$

where the last inequality is by the definition of $\underline{\tau}$. Hence by the law of motion of L^*_t in (5),

$$L^*_{\underline{\tau}_*+2} = 0 \text{ with probability 1.} \tag{F.232}$$

Under (F.232), we can invoke (B.48) to get

$$\Delta S^*_{\underline{\tau}_*+2} = \frac{\phi}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi} Z_{\underline{\tau}_*+2} - \frac{C^*_{\underline{\tau}_*+2}}{\theta-\phi} \tag{F.233}$$

which together with (F.229) implies that

$$\begin{aligned} E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] &= \frac{\phi}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi}\mu_z - \frac{E_{\underline{\tau}_*+1}[C^*_{\underline{\tau}_*+2}]}{\theta-\phi} \\ &= -\frac{\phi}{\theta-\phi}\frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c+E_{\underline{\tau}_*+1}[C^*_{\underline{\tau}_*+2}]}{\theta-\phi} \\ &\leq -\frac{\phi}{\theta-\phi}\frac{z}{(1-\phi/\theta)^{\tau_*}} < 0. \end{aligned} \tag{F.234}$$

This proves (F.228). At time $\underline{\tau}_* + 2$, the equilibrium price satisfies

$$\begin{aligned} S^*_{\underline{\tau}_*+2} &= \frac{\theta}{\theta-\phi} S^*_{\underline{\tau}_*+1} - \frac{\phi}{\theta-\phi} Z_{\underline{\tau}_*+2} - \frac{C^*_{\underline{\tau}_*+2}}{\theta-\phi} \\ &= \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*+1}} - \frac{c\theta/\phi + C^*_{\underline{\tau}_*+2}}{\theta-\phi}, \end{aligned} \tag{F.235}$$

which implies that

$$S^*_{\underline{\tau}_*+2} \leq \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*+1}} - \frac{c}{\phi} \tag{F.236}$$

regardless the demand of the informed speculators in time $\underline{\tau}_* + 2$. Therefore,

$$\left| S^*_{\underline{\tau}_*+2} - \mu_z \right| = \frac{z}{(1-\phi/\theta)^{\tau_*+1}} + \frac{c}{\phi} > \vartheta, \tag{F.237}$$

which implies that

$$L^*_{\underline{\tau}_*+3} = 1 \text{ with probability at least } \overline{\sigma}. \tag{F.238}$$

Hence by Lemma 23,

$$E_{\underline{\tau}_*+2}[\Delta S^*_{\underline{\tau}_*+3}] > 0,\ C^*_{\underline{\tau}_*+2} = c \text{ and } S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}} \tag{F.239}$$

which finishes the proof. ∎

Lemma 28. *Suppose that the conditions of Corollary 19 hold. If*

$$\tau_* = \underline{\tau},\ \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta \text{ and } \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} > \underline{S} \tag{F.240}$$

then there exist two possible equilibriums at time $\underline{\tau}_* + 1$. *The first equilibrium is the same as the one derived in Lemma 24. In the second equilibrium,* $C^*_{\underline{\tau}_*+1} = -c$, $C^*_{\underline{\tau}_*+2} = c$, $L^*_{\underline{\tau}_*+3} = 1$ *with probability 1,*

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} \tag{F.241}$$

and

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}}. \tag{F.242}$$

Proof of Lemma 28. The proof of the first equilibrium follows the same arguments in the proof of Lemma 25, and hence is omitted. To derive the second equilibrium, we first show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] < 0. \tag{F.243}$$

Given $C^*_{\underline{\tau}_*+1} = -c$, we have

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} - \frac{c}{\phi}, \tag{F.244}$$

which together with $\underline{\tau}_* = \tau_*$ and the first inequality in (F.240) implies that

$$\left|S^*_{\underline{\tau}_*+1} - \mu_z\right| = \frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta. \tag{F.245}$$

Moreover by $\underline{\tau}_* = \underline{\tau}$ and the second inequality in (F.240),

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} > \underline{S}. \tag{F.246}$$

Hence by (F.245), (F.246) and the law of motion of L^*_t in (5),

$$L^*_{\underline{\tau}_*+2} = 0 \text{ with probability 1.} \tag{F.247}$$

Using the same arguments in the proof of Lemma 26 and Lemma 28, we can show that

$$E_{\underline{\tau}_*+1}[\Delta S^*_{\underline{\tau}_*+2}] < 0, \tag{F.248}$$

which verifies

$$C^*_{\underline{\tau}_*+1} = -c \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} - \frac{c}{\phi}. \tag{F.249}$$

At time $\underline{\tau}_* + 2$, the equilibrium price satisfies

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}} - \frac{c\theta/\phi + C^*_{\underline{\tau}_*+2}}{\theta - \phi}, \tag{F.250}$$

which implies that

$$S^*_{\underline{\tau}_*+2} \leq \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}} - \frac{c}{\phi} \tag{F.251}$$

regardless the demand of the informed speculators in time $\underline{\tau}_* + 2$. Therefore,

$$S^*_{\underline{\tau}_*+2} \leq \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}+1}} - \frac{c}{\phi} \leq \underline{S}, \tag{F.252}$$

which implies that

$$L^*_{\underline{\tau}_*+3} = 1 \text{ with probability 1.} \tag{F.253}$$

The rest of the proof follows the same arguments of the proof of Lemma 25 and hence is omitted. ∎

Lemma 29. *Suppose that $L^*_t = L^*_{t+1} = 0$, $\zeta_{t+1} = 0$ and*

$$S^*_t = \mu_z - \frac{z}{(1-\phi/\theta)^{t-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right], \tag{F.254}$$

where $\tau^* < t$. If the probability of liquidity squeeze at $t+2$ is larger than ϕ/θ, then $E_{t+1}[\Delta S^*_{t+2}] > 0$, $C^*_{t+1} = c$ and

$$S^*_{t+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{t}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - 1\right]. \tag{F.255}$$

Proof of Lemma 29. Since $L^*_t = L^*_{t+1} = 0$ and $\zeta_{t+1} = 0$, by (B.48) and (F.180) with $t = \underline{\tau}_* + 1$, we have

$$S^*_{t+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{t}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - 1\right] + \frac{c - C^*_{t+1}}{\theta - \phi}. \tag{F.256}$$

Let σ_s denote the probability of $L^*_{t+2} = 1$. On the one hand if there is $L^*_{t+2} = 0$, (B.49) and (F.256) imply that

$$\begin{aligned}\Delta S^*_{t+2} \;=\; & \frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+3}} - \frac{1}{(1-\phi/\theta)^2}\right]\frac{c}{\theta} \\ & - \frac{\phi C^*_{t+1}}{(\theta-\phi)^2} - \frac{C^*_{t+2} + \phi\zeta_{t+2}}{\theta - \phi}.\end{aligned} \tag{F.257}$$

On the other hand if $L^*_{t+2} = 1$, equilibrium price equation (B.51) and (F.256) imply that

$$\begin{aligned}\Delta S^*_{t+2} \;=\; & \zeta_{t+2} + \frac{C^*_{t+2}}{\phi} + \frac{C^*_{t+1}}{\theta-\phi} + \frac{z}{(1-\phi/\theta)^t} \\ & + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{1-\phi/\theta}\right].\end{aligned} \tag{F.258}$$

Combining the results in (F.257) and (F.258), and applying $E_{t+1}[\zeta_{t+2}] = 0$, we get

$$\begin{aligned} E_{t+1}[\Delta S^*_{t+2}] &= (1-\sigma_s)\left[\begin{array}{c} \frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \frac{E_{t+1}[C^*_{t+2}]}{\theta-\phi} - \frac{\phi C^*_{t+1}}{(\theta-\phi)^2} \\ -\frac{c}{\theta}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+3}} - \frac{1}{(1-\phi/\theta)^2}\right] \end{array}\right] \\ &\quad + \sigma_s \left[\begin{array}{c} \frac{E_{t+1}[C^*_{t+2}]}{\phi} + \frac{C^*_{t+1}}{\theta-\phi} + \frac{z}{(1-\phi/\theta)^t} \\ +\frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{1-\phi/\theta}\right] \end{array}\right] \\ &= A_{1,t}(\sigma_s) + A_{2,t}(\sigma_s, C^*_{t+1}, C^*_{t+2}) \end{aligned} \tag{F.259}$$

where

$$A_{1,t}(\sigma_s) = \frac{z(\sigma_s - \phi/\theta)}{(1-\phi/\theta)^{t+1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta - \sigma_s}{1-\phi/\theta}$$

and

$$A_{2,t}(\sigma_s, C^*_{t+1}, C^*_{t+2}) = \frac{\sigma_s - \phi/\theta}{1-\phi/\theta}\frac{E_{t+1}[C^*_{t+2}]}{\phi} + \frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\frac{C^*_{t+1}}{\theta}.$$

Since $\sigma_s > \phi/\theta$ and $\theta > \phi > 0$,

$$A_{2,t}(\sigma_s, C^*_{t+1}, C^*_{t+2}) \geq -\frac{\sigma_s - \phi/\theta}{1-\phi/\theta}\frac{c}{\phi} - \frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\theta} = -\frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\phi} \tag{F.260}$$

regardless of the demand of the informed speculators. Moreover

$$\begin{aligned} & -\frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta - \sigma_s}{1-\phi/\theta} - \frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\frac{c}{\phi} \\ = \; & \frac{2c}{\phi}\frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\left[\frac{1}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right]. \end{aligned} \tag{F.261}$$

Collecting the results in (F.259), (F.260) and (F.261), we have

$$E_{t+1}[\Delta S^*_{t+2}] \geq \frac{z(\sigma_s - \phi/\theta)}{(1-\phi/\theta)^{t+1}} + \frac{2c}{\phi}\frac{\sigma_s - \phi/\theta}{(1-\phi/\theta)^2}\left[\frac{1}{(1-\phi/\theta)^{t-\tau^*+1}} - 1\right] > 0 \tag{F.262}$$

where the second inequality is by $\sigma_s > \phi/\theta$ and $\theta > \phi > 0$. By (F.262) and the demand law of the informed speculators, $C^*_{t+1} = c$ which together with (F.256) implies that

$$S^*_{t+1} = \mu_z - \frac{z}{(1-\phi/\theta)^t} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - 1\right]. \tag{F.263}$$

This finishes the proof. ∎

Lemma 30. *For any $t > \tau_*$ and for any $C^* \in \{-c, 0, c\}$,*

$$\frac{z}{(1-\phi/\theta)^t} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau_*+1}} - \frac{1}{1-\phi/\theta}\right] + \frac{C^*}{\theta-\phi} > \vartheta. \tag{F.264}$$

Proof of Lemma 30. Since $\phi/\theta \in (0,1)$,

$$\frac{1}{1-\phi/\theta} = \sum_{k=0}^{\infty}(\phi/\theta)^k > 1 + \phi/\theta \tag{F.265}$$

which implies that

$$\frac{2}{(1-\phi/\theta)^{t-\tau_*+1}} \geq \frac{2}{1-\phi/\theta}\frac{1}{1-\phi/\theta} > \frac{2(1+\phi/\theta)}{1-\phi/\theta}. \tag{F.266}$$

Therefore,

$$\begin{aligned} & \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau_*+1}} - \frac{1}{1-\phi/\theta}\right] + \frac{C^*}{\theta-\phi} \\ > \; & \frac{c}{\phi}\left[\frac{1+\phi/\theta}{1-\phi/\theta} + \frac{\phi/\theta}{1-\phi/\theta}\right] + \frac{C^*}{\theta-\phi} \\ = \; & \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} + \frac{c+C^*}{\theta-\phi} \geq \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \end{aligned} \tag{F.267}$$

for any $C^* \in \{-c, 0, c\}$. By (F.258),

$$\begin{aligned} & \frac{z}{(1-\phi/\theta)^t} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau_*+1}} - \frac{1}{1-\phi/\theta}\right] + \frac{C^*}{\theta-\phi} \\ > \; & \frac{z}{(1-\phi/\theta)^t} + \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \end{aligned}$$

for any $t > \tau_*$ and any $C^* \in \{-c, 0, c\}$, which together with the definition of τ_* proves (F.264). ∎

Proof of Corollary 19. The claim in part (i) and part (vi) of the theorem follow by Proposition 5.

We next prove the results in parts (ii)-(v). In Lemma 21 and Lemma 22, we have show that

$$S_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{t-1}},\ C_t^* = -c,\ L_t^* = 0 \text{ and } L_{t+1}^* = 0 \tag{F.268}$$

for any t with $1 \leq t \leq \underline{\tau}_*$. The momentum speculators' demand is 0 at time 0, 1 and l^* due to liquidity squeeze. For any t with $2 \leq t \leq \underline{\tau}_*$, $L_{t-1}^* = 0$ and $L_t^* = 0$ by

(F.268). Hence the demand of the momentum speculators' demand is not zero. By the first equality in (F.268),

$$\triangle S_t^* = -\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{z}{(1-\phi/\theta)^{t-2}} = \frac{-z\phi/\theta}{(1-\phi/\theta)^{t-1}} \tag{F.269}$$

which together with the demand law of the momentum speculators implies that

$$q_t^* = \frac{-z\phi}{(1-\phi/\theta)^{t-1}} \text{ for any } t \text{ with } 2 \le t \le \underline{\tau}_*. \tag{F.270}$$

To prove the rest of the results, we have to consider 3 different cases.

Case 1. $\tau_* = \underline{\tau}$.

Case 1.1. $\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} \le \underline{S}$. By Lemma 24, $\tau^* = \underline{\tau} + 1$ and $l^* = \underline{\tau} + 2$. At time τ^*, $C_{\tau^*}^* = c$ and

$$S_{\tau^*}^* = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}. \tag{F.271}$$

By Lemma 22,

$$S_{\underline{\tau}}^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}-1}}, \tag{F.272}$$

which together with $\tau^* = \underline{\tau} + 1$ and (F.271) implies that

$$\triangle S_{\tau^*}^* = \frac{-2c/\theta}{1-\phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)^{\underline{\tau}}}. \tag{F.273}$$

By the demand law of the momentum speculators,

$$q_{\tau^*}^{m^*} = \frac{-2c}{1-\phi/\theta} - \frac{z\phi}{(1-\phi/\theta)^{\underline{\tau}}} = -\frac{z\phi}{(1-\phi/\theta)^{\tau^*-1}} - \frac{2c}{1-\phi/\theta}. \tag{F.274}$$

By Condition 18, $S_{\tau^*}^* > 0$. Since the equilibrium prices are decreasing and $l^* = \tau^* + 1$, we have $S_t^* > 0$ for any t with $1 \le t < l^*$. This proves the results when $\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} \le \underline{S}$.

Case 1.2. $\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta$. By Lemma 26,

$$C_{\underline{\tau}_*+1}^* = c \text{ and } S_{\underline{\tau}_*+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}, \tag{F.275}$$

which together with Lemma 22 implies that $\tau^* = \tau_* + 1$. By (F.272) and (F.275),

$$\triangle S_{\tau^*}^* = \frac{-2c/\theta}{1-\phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau_*}} \tag{F.276}$$

which together with the demand law of the momentum speculators implies

$$q^{m^*}_{\tau^*} = -\frac{z\phi}{(1-\phi/\theta)^{\tau^*-1}} - \frac{2c}{1-\phi/\theta}. \tag{F.277}$$

Since $\tau_* = \underline{\tau}$, by (F.275) and the definition of $\underline{\tau}$,

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \leq \underline{S} \tag{F.278}$$

which combined with (E.155) implies that

$$L^*_{\tau^*+1} = 1 \text{ with probability 1.} \tag{F.279}$$

Hence $l^* = \underline{\tau} + 2 = \tau^* + 1$. By Condition 18, $S^*_{\tau^*} > 0$. Since the equilibrium prices are decreasing and $l^* = \tau^* + 1$, we have $S^*_t > 0$ for any t with $1 \leq t < l^*$. This proves the results when $\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta$.

Case 1.3. $\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} > \underline{S}$ and $\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta$. By Lemma 28, there are two equilibriums at time $\underline{\tau}_* + 1$. The first equilibrium is the same as the one derived in Lemma 24. Hence the proof of parts (ii)-(v) follows the same arguments in case 1.1. In the second equilibrium, $C^*_{\tau_*+1} = -c$, $C^*_{\tau_*+2} = c$, $L^*_{\tau_*+3} = 1$ with probability 1,

$$S^*_{\tau_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} \tag{F.280}$$

and

$$S^*_{\tau_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}}. \tag{F.281}$$

Hence we have $\tau^* = \underline{\tau} + 2$ and $l^* = \underline{\tau} + 3$. Note that

$$\triangle S^*_{\tau_*+1} = -\frac{z\phi/\theta}{(1-\phi/\theta)^{\underline{\tau}}} \text{ and } \triangle S^*_{\tau_*+2} = \frac{-2c/\theta}{1-\phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)^{\underline{\tau}}} \tag{F.282}$$

which together with the demand law of the momentum speculators implies

$$q^{m^*}_{\tau_*+1} = -\frac{z\phi}{(1-\phi/\theta)^{\underline{\tau}}} \text{ and } q^*_{\tau_*+2} = \frac{-2c}{1-\phi/\theta} - \frac{z\phi}{(1-\phi/\theta)^{\underline{\tau}}}. \tag{F.283}$$

By Condition 18, $S^*_{\tau^*} > 0$. Since the equilibrium prices are decreasing and $l^* = \underline{\tau} + 3$, we have $S^*_t > 0$ for any t with $1 \leq t < l^*$. This finishes the proof in the case that $\tau_* = \underline{\tau}$.

Case 2. $\underline{\tau} < \tau_*$.

Case 2.1. $\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} \leq \underline{S}$. By Lemma 24,

$$C^*_{\underline{\tau}_*+1} = c,\ S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} \text{ and } l^* = \underline{\tau}+2. \tag{F.284}$$

The proof follows the same arguments above in case 1.1.

Case 2.2. $\mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}}} - \frac{c}{\phi} > \underline{S}$. By Lemma 25, there exist and only exist two possible equilibriums at time $\underline{\tau}_*+1$. The first equilibrium is the same as the one derived in Lemma 24 and hence the proof of parts (ii)-(v) follows the same arguments above in case 1.1. when $\underline{\tau} = \tau_*$. In the second equilibrium, $C^*_{\underline{\tau}_*+1} = -c$, $C^*_{\underline{\tau}_*+2} = c$, $l^* = \underline{\tau}+3$,

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}} \tag{F.285}$$

and

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}}. \tag{F.286}$$

Since $\underline{\tau}_* = \underline{\tau}$, the proof follows the same arguments above in case 1.3.

Case 3. $\tau_* < \underline{\tau}$.

Case 3.1. $\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta$. By Lemma 26,

$$C^*_{\underline{\tau}_*+1} = c \text{ and } S^*_{\underline{\tau}_*+1} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau_*}} - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta}, \tag{F.287}$$

which together with Lemma 22 implies that $\tau^* = \tau_*+1$. Moreover,

$$\triangle S^*_{\underline{\tau}_*+1} = \frac{-2c/\theta}{1-\phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau_*}} \tag{F.288}$$

which together with the demand law of the momentum speculators implies that

$$q^{m^*}_{\underline{\tau}_*+1} = \frac{-2c}{1-\phi/\theta} - \frac{z\phi}{(1-\phi/\theta)^{\tau_*}}. \tag{F.289}$$

This proves (E.160) (E.161) and (E.162) for $t \leq \tau^*$. Suppose that the results in (E.160) (E.161) and (E.162) hold for $t = \tau^*+k-1$, $L^*_{\tau^*+k-1} = 0$ and $L^*_{\tau^*+k} = 0$ for some $k \geq 1$. We next show that (E.160) (E.161) and (E.162) also hold for $t = \tau^*+k$. By (B.48),

$$\begin{aligned} S^*_{\tau^*+k} &= \mu_z - \frac{z}{(1-\phi/\theta)^{\tau^*+k-1}} \\ &\quad - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\tau^*+k-\tau_*}} - \frac{1}{1-\phi/\theta}\right] - \frac{C^*_{\tau^*+k}}{\theta-\phi}. \end{aligned} \tag{F.290}$$

Since $\tau^* + k - 1 > \tau_*$, by Lemma 30,

$$\left|S^*_{\tau^*+k} - \mu_z\right| > \vartheta \tag{F.291}$$

which together with (E.155) implies that

$$L^*_{\tau^*+k+1} = 1 \text{ with probability at least } \overline{\sigma}. \tag{F.292}$$

By Lemma 29, $C^*_{t+1} = c$ and

$$S^*_{\tau^*+k} = \mu_z - \frac{z}{(1-\phi/\theta)^{\tau^*+k-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\tau^*+k-\tau^*+1}} - 1\right]. \tag{F.293}$$

Moreover,

$$\triangle S^*_{\tau^*+k} = \frac{-\phi z/\theta}{(1-\phi/\theta)^{\tau^*+k-1}} - \frac{2c}{\theta}\frac{1}{(1-\phi/\theta)^{\tau^*+k-\tau^*+1}} \tag{F.294}$$

which together with the demand law of the momentum speculators implies that

$$q^{m^*}_{\tau^*+k} = \frac{-z\phi}{(1-\phi/\theta)^{\tau^*+k-1}} - \frac{2c}{(1-\phi/\theta)^{\tau^*+k-\tau^*+1}}. \tag{F.295}$$

This shows that (E.160) (E.161) and (E.162) hold for $t = \tau^* + k$. Hence they hold for any t with $t < l^*$. By (E.160), the equilibrium prices are strictly decreasing along the negative bubbly path. By (F.291) and (E.155),

$$L^*_t = 1 \text{ with probability at least } \overline{\sigma}. \tag{F.296}$$

for any t with $\tau^* \leq t < l^*$. Hence with nonzero probability, the liquidity squeeze may be triggered at a time before $\underline{\tau}_e$. In such case, by (E.160),

$$S^*_t > \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_e-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\underline{\tau}_e-\tau_*}} - 1\right] \tag{F.297}$$

for any t with $1 \leq t < l^*$, which together with Condition 18 implies that $S^*_t > 0$ for any t with $1 \leq t < l^*$. On the other hand, if the liquidity squeeze is not triggered before $\underline{\tau}_e$ then at time $\underline{\tau}_e$ by (E.160)

$$S^*_{\underline{\tau}_e} = \mu_z - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_e-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{\underline{\tau}_e-\tau^*+1}} - 1\right] \tag{F.298}$$

which together with the definition of $\underline{\tau}_e$ implies that

$$S^*_{\underline{\tau}_e} \leq \underline{S}. \tag{F.299}$$

Hence by (E.155)

$$L^*_{\underline{\tau}_e+1} = 1 \text{ with probability } 1 \tag{F.300}$$

which implies that $l^* = \underline{\tau}_e + 1$. By (F.298) and $\tau^* = \tau_* + 1$, we can use Condition 18 to deduce that $S^*_{\underline{\tau}_e} > 0$. Since $l^* = \underline{\tau}_e + 1$ and $S^*_t > S^*_{\underline{\tau}_e}$ for any t with $1 \leq t < \underline{\tau}_e$, we have $S^*_t > 0$ for any t with $1 \leq t < l^*$.

Case 3.2. $\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta$. By Lemma 27, there are two equilibria. The first equilibrium is the same as the one derived in Lemma 26 and hence the proof of parts (ii)-(iv) follows the same arguments in case 3.1. In the second equilibrium, $C^*_{\underline{\tau}_*+1} = -c$, $C^*_{\underline{\tau}_*+2} = c$,

$$S^*_{\underline{\tau}_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*}}$$

and

$$S^*_{\underline{\tau}_*+2} = \mu_z - \frac{c}{\phi}\frac{1+\phi/\theta}{1-\phi/\theta} - \frac{z}{(1-\phi/\theta)^{\underline{\tau}_*+1}},$$

which together with Lemma 22 implies that $\tau^* = \tau_* + 2$. Moreover,

$$\triangle S^*_{\underline{\tau}_*+2} = \frac{-2c/\theta}{1-\phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)^{\tau_*+1}}$$

which together with the demand law of the momentum speculators implies that

$$q^{m^*}_{\underline{\tau}_*+1} = \frac{-2c}{1-\phi/\theta} - \frac{z\phi}{(1-\phi/\theta)^{\tau_*+1}}.$$

This proves the results in (E.160) (E.161) and (E.162) for $t \leq \tau^*$. Using similar arguments in case 3.1, we can show that if parts (ii)-(v) hold for $t = \tau^* + k - 1$ for some $k \geq 1$, then they also hold for $t = \tau^* + k$. Hence results in (E.160) (E.161) and (E.162) of the corollary hold for any $t < l^*$. Using the same arguments in case 3.1, we can show that $S^*_t > 0$ for any t with $1 \leq t < l^*$. This finishes the proof. ∎

G Constructing the LRV Estimators

For any weakly dependent process $\{W_{t,n}\}_{t=1}^n$ with

$$E\left[W_{t,n}\right] = 0 \text{ for all } t \text{ and } n, \tag{G.301}$$

and finite positive LRV V_W, its sample autocovariance can be defined as

$$\Gamma_{W,n}(j) = \frac{1}{n-j}\sum_{t=1}^{n-j}\left(W_{t,n} - \overline{W}_n\right)\left(W_{t+j,n} - \overline{W}_n\right) \tag{G.302}$$

for $j = 0, \ldots, n-1$. It is clear that the sample autocovariance satisfies $\Gamma_{W,n}(-j) = \Gamma_{W,n}(j)$ for $j = 0, \ldots, n-1$. Note that the sample autocovariance is sample mean centered, which improves the power of the test of the hypothesis in (G.301).

The kernel based LRV estimator for $\{W_{t,n}\}_{t=1}^{n}$ is then defined as

$$V_{W,n} = \sum_{j=-n+1}^{n+1} K(j/M)\Gamma_{W,n}(j) \tag{G.303}$$

where $K(\cdot)$ is some kernel smoothing function with bandwidth M. Under some regularity conditions (see, e.g., Newey and West (1987) and Andrews (1991)), there is

$$V_{W,n} \to_p V_W. \tag{G.304}$$

One key condition for the above consistency result is that M goes to infinity at certain rate. In finite samples, there are two different rules of selecting M: One is the rule proposed in Newey and West (1994) and the other is the parametric (AR(1)) approximation rule in Andrews (1991).

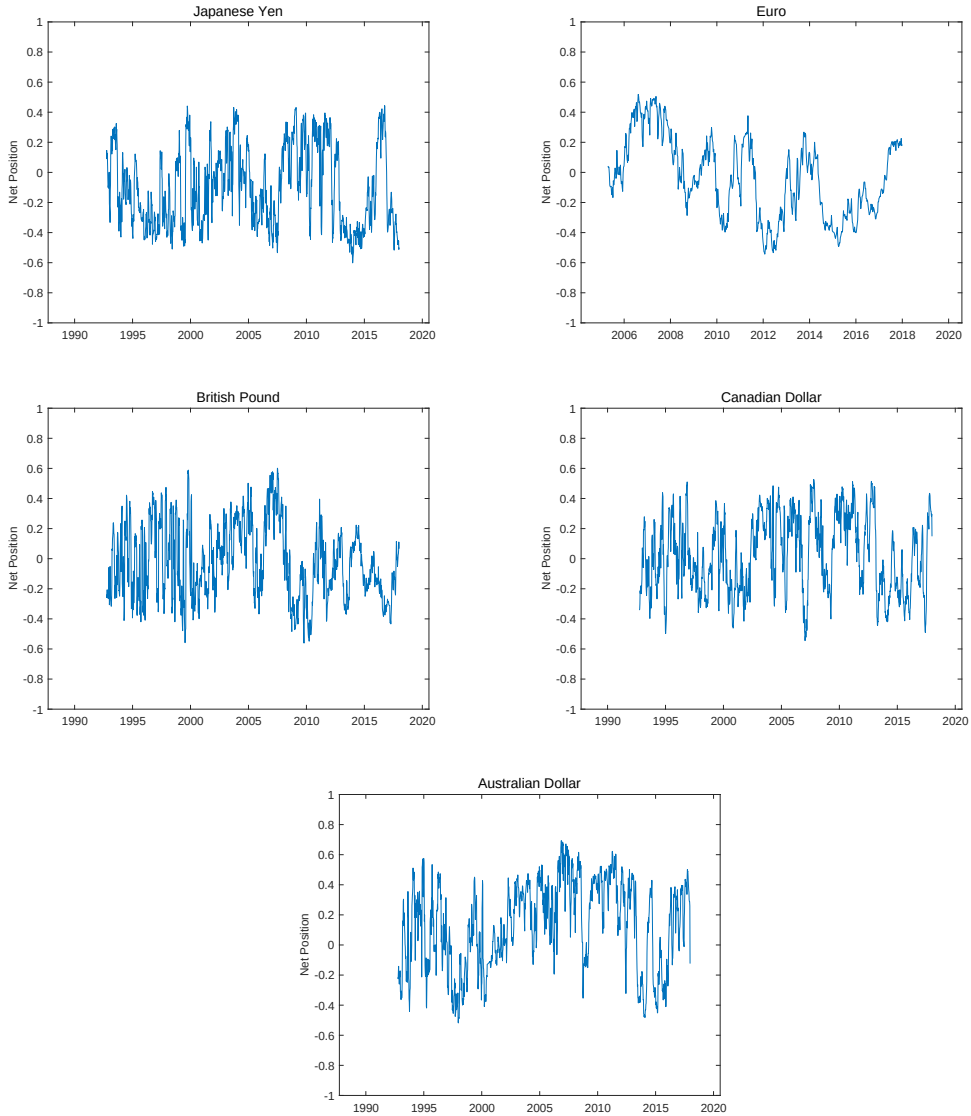
Table G.1: Construction of LRV Estimators

Test Statistic	$W_{t,n}$
$T_{a,n}$	$D_{t,h}(S^*_{t+h} - S^*_t)$
$T_{b,n}$	$(D_{t,h} - \overline{D}_{n,h})(D^*_{t,h} - \overline{D}^*_{n,h})$

In the rest of this appendix, we briefly describe how to construct the LRV estimators for the test statistics presented in the main text. The Newey-West and Andrews LRV estimators can be constructed using the formula in (G.302). Hence for each test statistic, we only need to define its corresponding $W_{t,n}$ for the construction of LRV estimators, which are summarized in Table G.1. For the ease of notation, we ignore the index i in each test statistic.

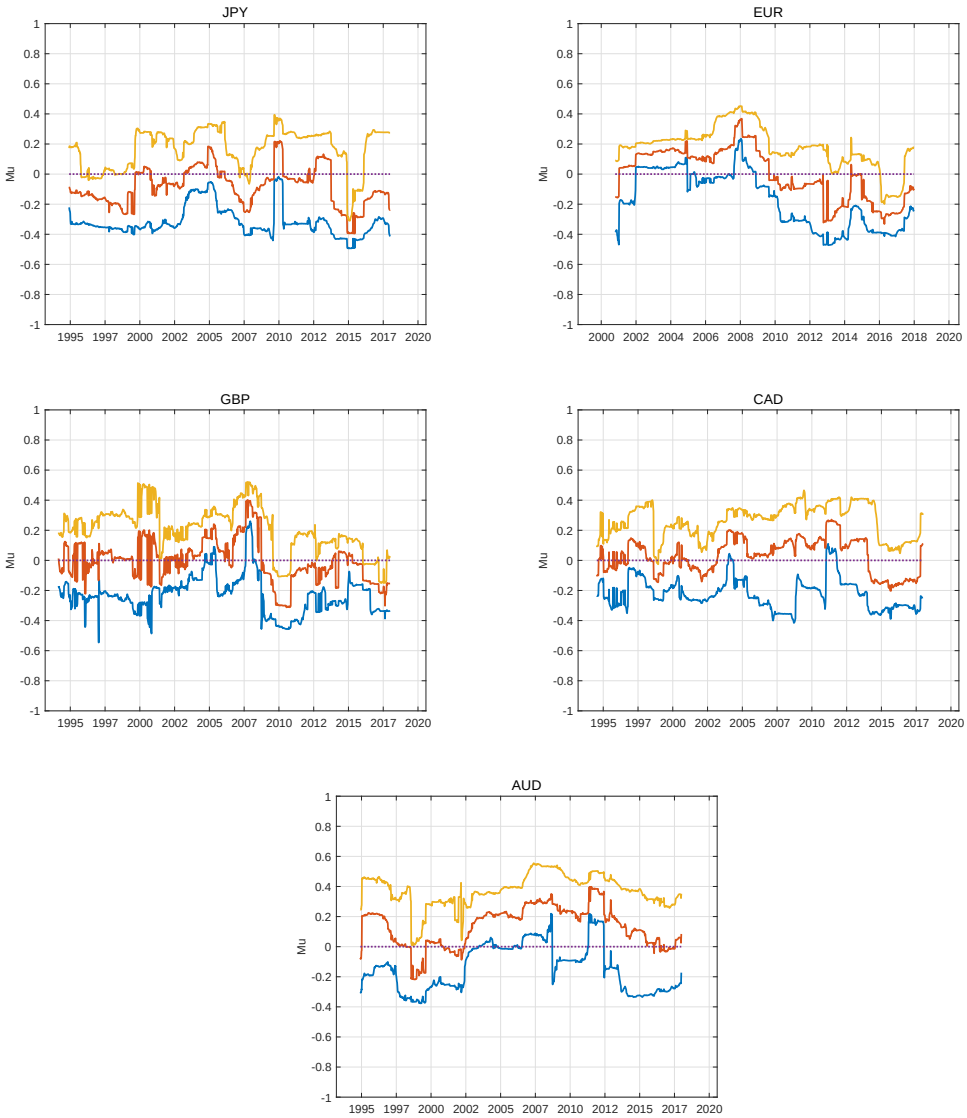
H Extra Results

Figure H.1: Net Speculators' Position



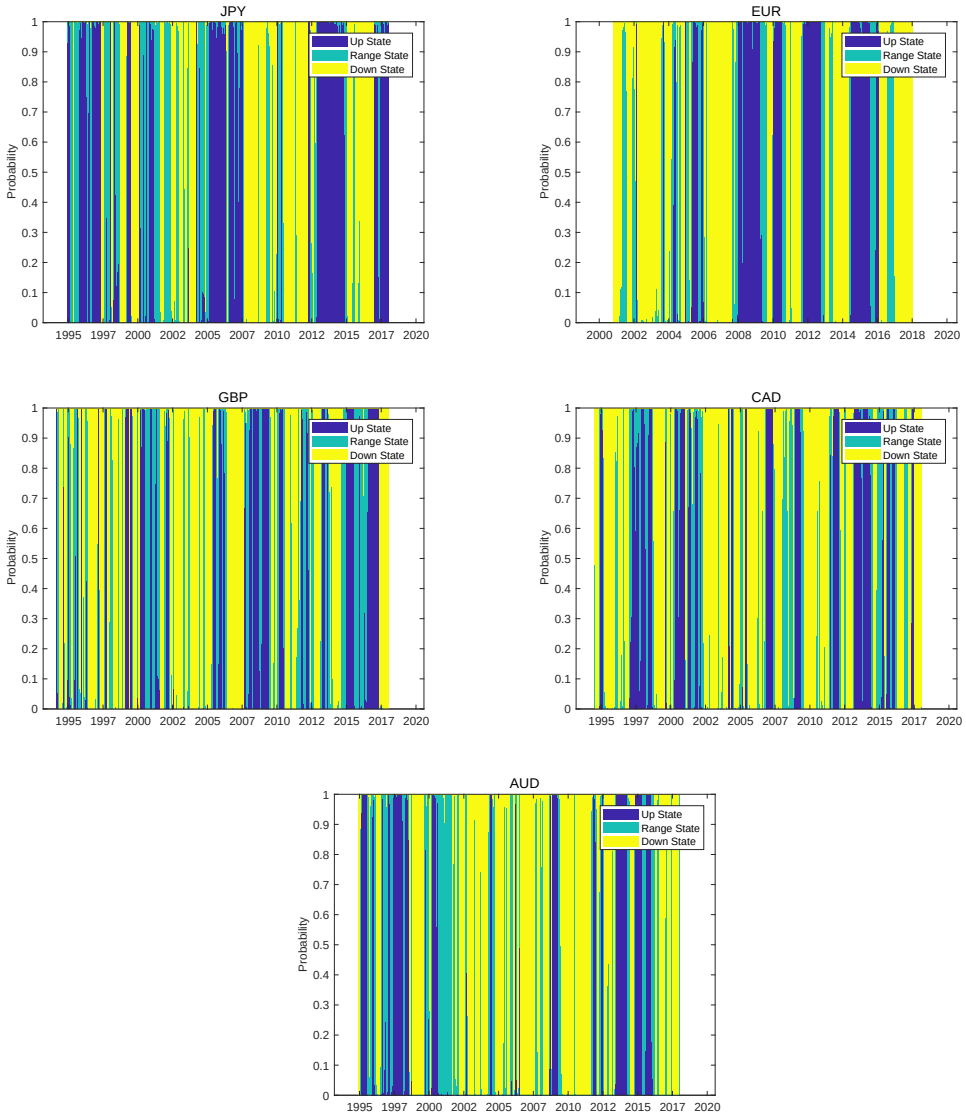
Note: This Figure presents the net position of speculators normalized by open interest. The data is from the Commitment-of-Traders Report of the CFTC.

Figure H.2: Evolution of Estimated c_t



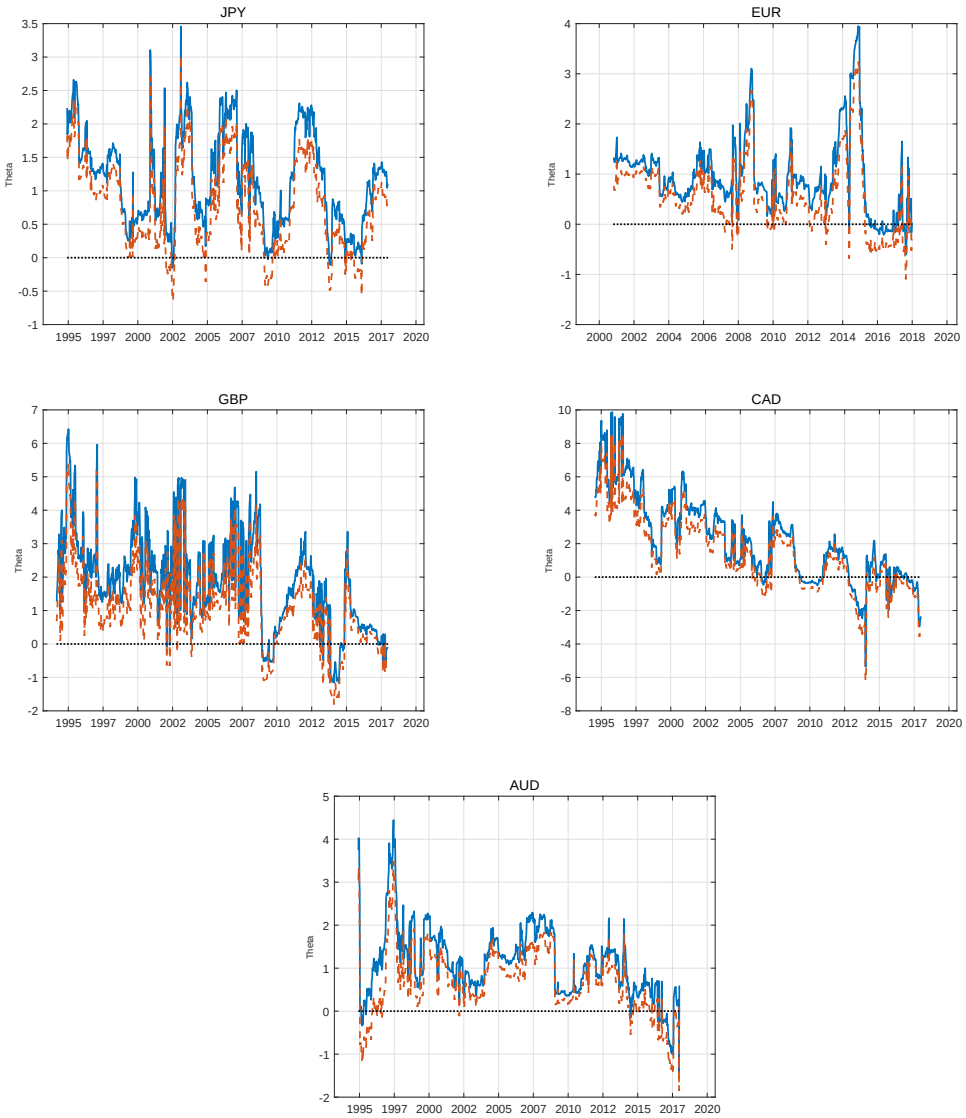
Note: This figure presents the estimated means of each state in the Markov-switching model. The blue and yellow lines depict the estimated mean of the appreciation and depreciation state, respectively. Meanwhile the red line represents the estimated mean of the range state.

Figure H.3: Evolution of Estimated Filtered Probabilities



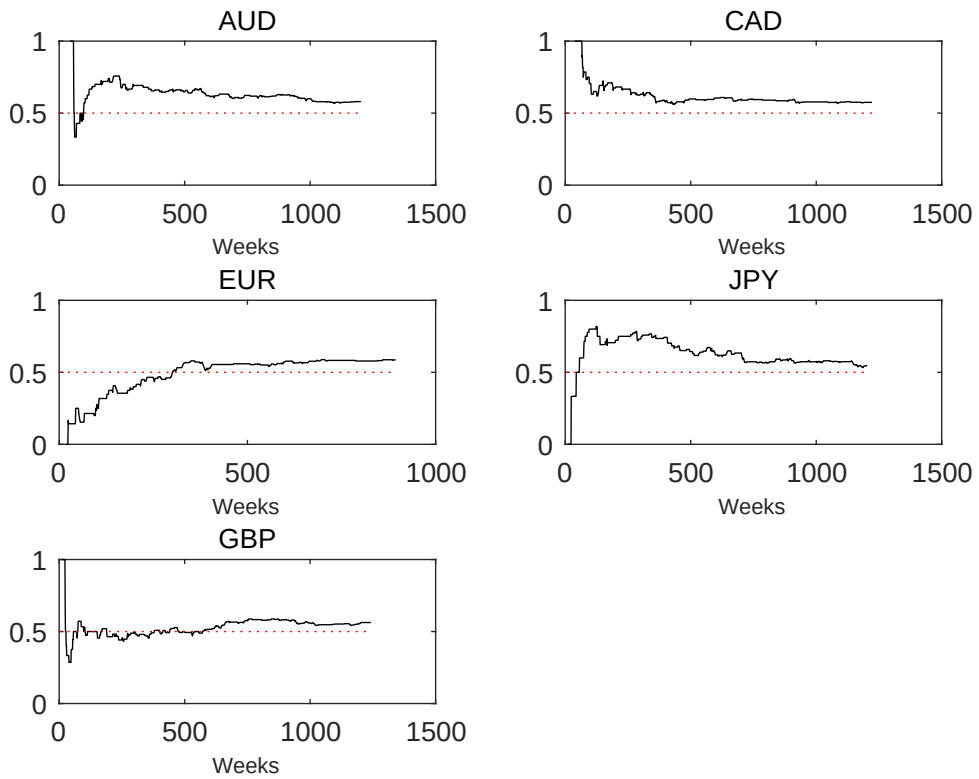
Note: This figure presents the estimated filtered probabilities of the states in the Markov-switching model.

Figure H.4: Evolution of Estimated θ_t



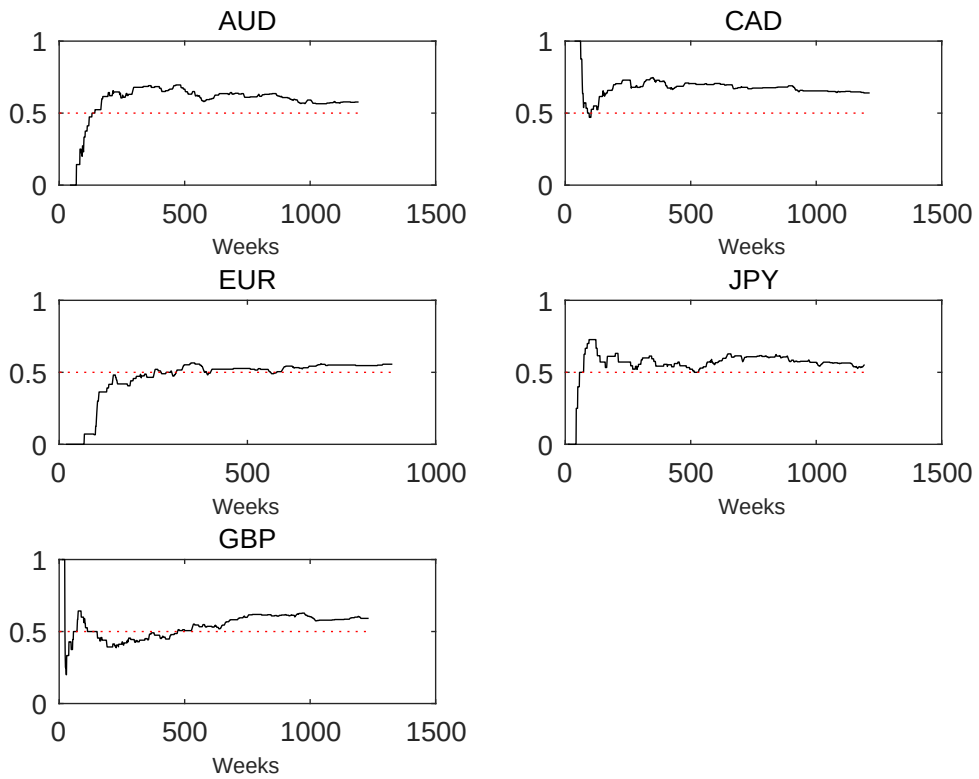
Note: This figure presents the estimated coefficient corresponding to the exchange rate term in the Markov-switching model. The blue solid line depicts the estimated coefficient and the orange dashed line depicts the lower 5% confidence interval of the estimated coefficient.

Figure H.5: Cumulative Forecast Success Ratio ($h = 1m$)



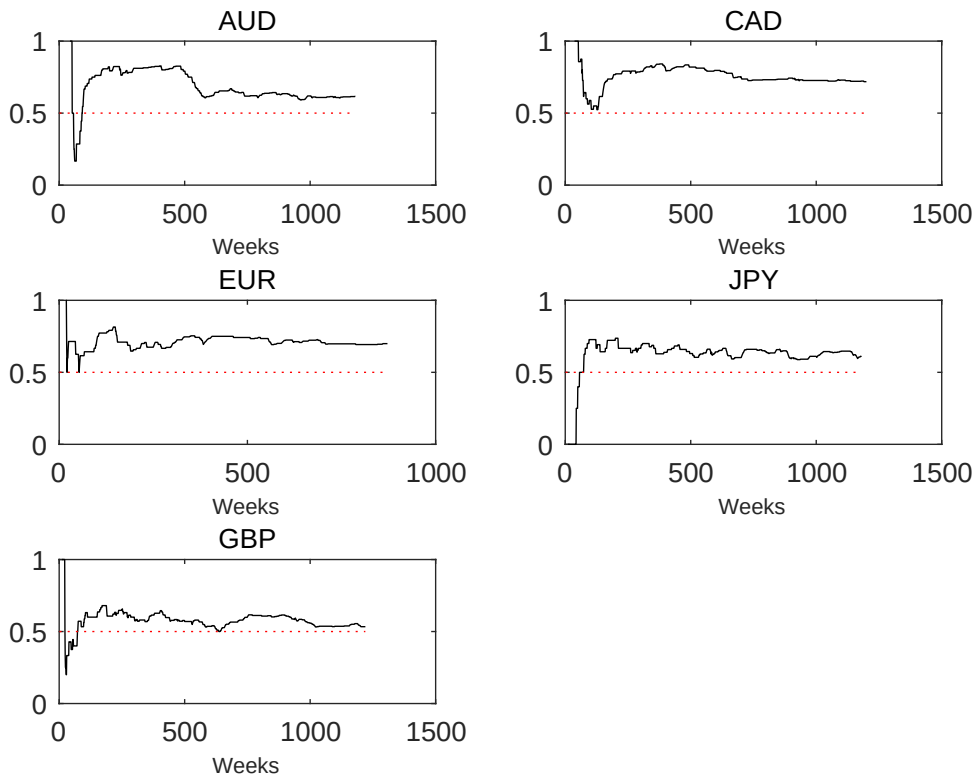
Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

Figure H.6: Cumulative Forecast Success Ratio ($h = 3m$)



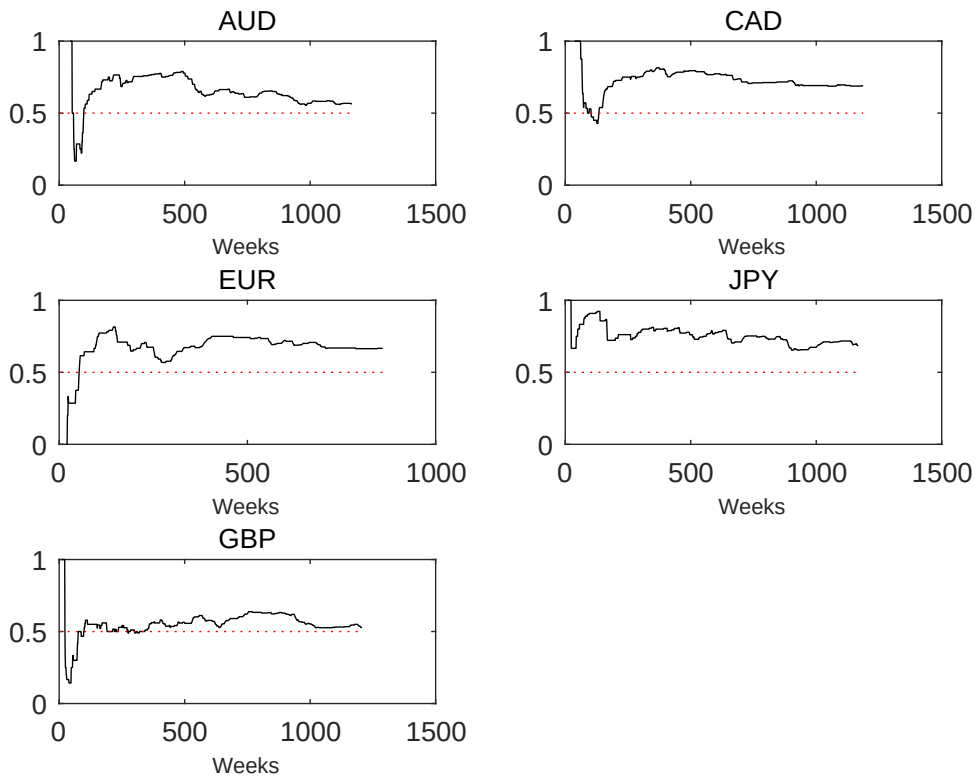
Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

Figure H.7: Cumulative Forecast Success Ratio ($h = 6m$)



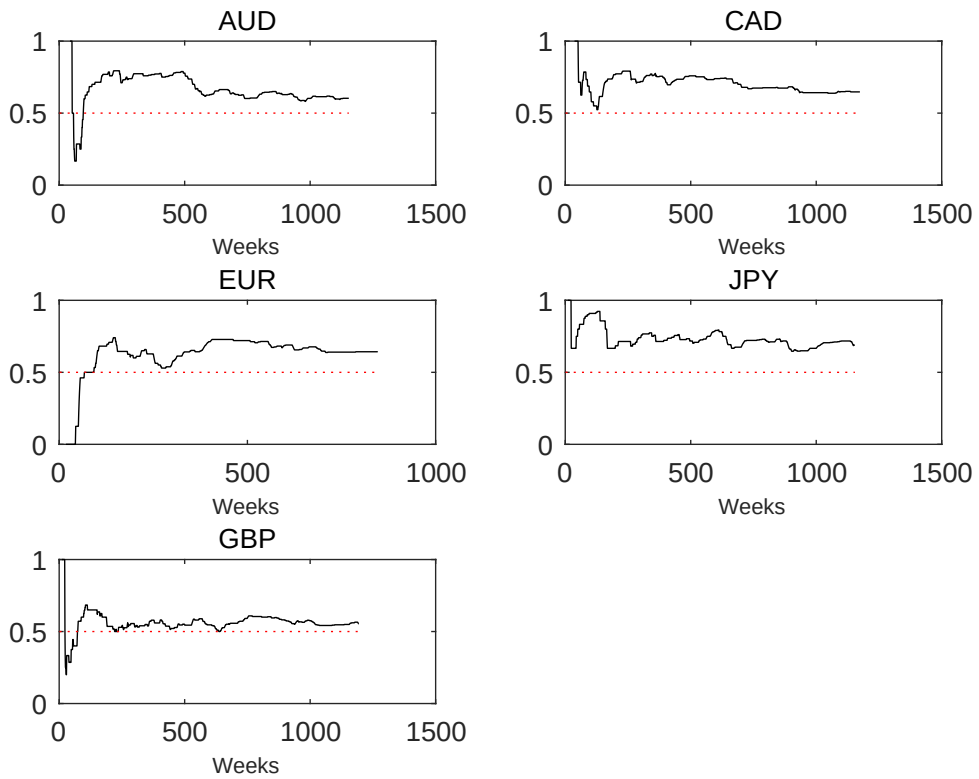
Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

Figure H.8: Cumulative Forecast Success Ratio ($h = 9m$)



Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

Figure H.9: Cumulative Forecast Success Ratio ($h = 12m$)



Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

Table H.1: Linear Granger-causality Test Results

	H_0: FX DO Not Cause COT		H_0: COT DO Not Cause FX	
	F-Statistics	Probability	F-Statistics	Probability
AUD	4.703	0.000	0.846	0.774
CAD	3.906	0.000	0.673	0.964
EUR	4.306	0.000	1.458	0.021
JPY	6.970	0.000	0.748	0.908
GBP	5.895	0.000	0.791	0.857
	H_0: ΔFX DO Not Cause ΔCOT		H_0: ΔCOT DO Not Cause ΔFX	
	F-Statistics	Probability	F-Statistics	Probability
AUD	4.656	0.000	0.792	0.855
CAD	3.710	0.000	0.705	0.944
EUR	4.375	0.000	1.370	0.046
JPY	6.787	0.000	0.796	0.850
GBP	5.690	0.000	0.808	0.833

Note: This table reports the results of the linear Granger-causality test. The number of lags included is 52 weeks. However, the results are robust with respect to the number of lags. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.2: Robustness of the MSM Estimation Results (Australian Dollar)

	$R = 70$	$R = 80$	$R = 90$	$R = 100$	$R = 110$	$R = 120$	$R = 130$
				Panel A: $k = 4$			
$(R = 70, k = 4)$	1.00	0.95	0.92	0.89	0.90	0.88	0.87
$(R = 80, k = 4)$	0.95	1.00	0.95	0.93	0.92	0.91	0.90
$(R = 90, k = 4)$	0.92	0.95	1.00	0.96	0.95	0.93	0.92
$(R = 100, k = 4)$	0.89	0.93	0.96	1.00	0.97	0.95	0.93
$(R = 110, k = 4)$	0.90	0.92	0.95	0.97	1.00	0.96	0.95
$(R = 120, k = 4)$	0.88	0.91	0.93	0.95	0.96	1.00	0.97
$(R = 130, k = 4)$	0.87	0.90	0.92	0.93	0.95	0.97	1.00
$(R = 70, k = 8)$	0.72	0.73	0.73	0.73	0.73	0.72	0.72
$(R = 80, k = 8)$	0.89	0.92	0.91	0.90	0.90	0.88	0.87
$(R = 90, k = 8)$	0.87	0.90	0.92	0.92	0.91	0.90	0.89
$(R = 100, k = 8)$	0.86	0.88	0.91	0.92	0.91	0.90	0.89
$(R = 110, k = 8)$	0.86	0.88	0.90	0.91	0.92	0.91	0.91
$(R = 120, k = 8)$	0.83	0.86	0.88	0.90	0.91	0.91	0.92
$(R = 130, k = 8)$	0.83	0.85	0.87	0.89	0.89	0.89	0.91
				Panel B: $k = 8$			
$(R = 70, k = 4)$	0.72	0.89	0.87	0.86	0.86	0.83	0.83
$(R = 80, k = 4)$	0.73	0.92	0.90	0.88	0.88	0.86	0.85
$(R = 90, k = 4)$	0.73	0.91	0.92	0.91	0.90	0.88	0.87
$(R = 100, k = 4)$	0.73	0.90	0.92	0.92	0.91	0.90	0.89
$(R = 110, k = 4)$	0.73	0.90	0.91	0.91	0.92	0.91	0.89
$(R = 120, k = 4)$	0.72	0.88	0.90	0.90	0.91	0.91	0.89
$(R = 130, k = 4)$	0.72	0.87	0.89	0.89	0.91	0.92	0.91
$(R = 70, k = 8)$	1.00	0.73	0.72	0.72	0.73	0.73	0.71
$(R = 80, k = 8)$	0.73	1.00	0.95	0.93	0.92	0.91	0.90
$(R = 90, k = 8)$	0.72	0.95	1.00	0.96	0.95	0.93	0.92
$(R = 100, k = 8)$	0.72	0.93	0.96	1.00	0.96	0.94	0.93
$(R = 110, k = 8)$	0.73	0.92	0.95	0.96	1.00	0.96	0.95
$(R = 120, k = 8)$	0.73	0.91	0.93	0.94	0.96	1.00	0.96
$(R = 130, k = 8)$	0.71	0.90	0.92	0.93	0.95	0.96	1.00

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$

Table H.3: Robustness of the MSM Estimation Results (Canadian Dollar)

	$R = 70$	$R = 80$	$R = 90$	$R = 100$	$R = 110$	$R = 120$	$R = 130$
				Panel A: $k = 4$			
$(R = 70, k = 4)$	1.00	0.94	0.93	0.91	0.89	0.87	0.84
$(R = 80, k = 4)$	0.94	1.00	0.96	0.94	0.92	0.89	0.87
$(R = 90, k = 4)$	0.93	0.96	1.00	0.96	0.94	0.91	0.88
$(R = 100, k = 4)$	0.91	0.94	0.96	1.00	0.97	0.93	0.90
$(R = 110, k = 4)$	0.89	0.92	0.94	0.97	1.00	0.95	0.92
$(R = 120, k = 4)$	0.87	0.89	0.91	0.93	0.95	1.00	0.96
$(R = 130, k = 4)$	0.84	0.87	0.88	0.90	0.92	0.96	1.00
$(R = 70, k = 8)$	0.67	0.69	0.66	0.67	0.68	0.66	0.65
$(R = 80, k = 8)$	0.87	0.89	0.88	0.87	0.86	0.83	0.81
$(R = 90, k = 8)$	0.84	0.87	0.89	0.88	0.87	0.84	0.82
$(R = 100, k = 8)$	0.82	0.85	0.88	0.89	0.88	0.85	0.83
$(R = 110, k = 8)$	0.82	0.84	0.86	0.88	0.89	0.87	0.85
$(R = 120, k = 8)$	0.80	0.83	0.84	0.85	0.87	0.88	0.86
$(R = 130, k = 8)$	0.77	0.81	0.83	0.84	0.85	0.87	0.88
				Panel B: $k = 8$			
$(R = 70, k = 4)$	0.67	0.87	0.84	0.82	0.82	0.80	0.77
$(R = 80, k = 4)$	0.69	0.89	0.87	0.85	0.84	0.83	0.81
$(R = 90, k = 4)$	0.66	0.88	0.89	0.88	0.86	0.84	0.83
$(R = 100, k = 4)$	0.67	0.87	0.88	0.89	0.88	0.85	0.84
$(R = 110, k = 4)$	0.68	0.86	0.87	0.88	0.89	0.87	0.85
$(R = 120, k = 4)$	0.66	0.83	0.84	0.85	0.87	0.88	0.87
$(R = 130, k = 4)$	0.65	0.81	0.82	0.83	0.85	0.86	0.88
$(R = 70, k = 8)$	1.00	0.69	0.68	0.67	0.68	0.68	0.66
$(R = 80, k = 8)$	0.69	1.00	0.95	0.92	0.90	0.88	0.86
$(R = 90, k = 8)$	0.68	0.95	1.00	0.95	0.91	0.89	0.88
$(R = 100, k = 8)$	0.67	0.92	0.95	1.00	0.95	0.92	0.91
$(R = 110, k = 8)$	0.68	0.90	0.91	0.95	1.00	0.95	0.93
$(R = 120, k = 8)$	0.68	0.88	0.89	0.92	0.95	1.00	0.96
$(R = 130, k = 8)$	0.66	0.86	0.88	0.91	0.93	0.96	1.00

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$

Table H.4: Robustness of the MSM Estimation Results (Euro)

	$R = 70$	$R = 80$	$R = 90$	$R = 100$	$R = 110$	$R = 120$	$R = 130$
				Panel A: $k = 4$			
$(R = 70, k = 4)$	1.00	0.94	0.91	0.86	0.82	0.80	0.80
$(R = 80, k = 4)$	0.94	1.00	0.95	0.90	0.87	0.85	0.86
$(R = 90, k = 4)$	0.91	0.95	1.00	0.94	0.91	0.89	0.89
$(R = 100, k = 4)$	0.86	0.90	0.94	1.00	0.96	0.93	0.93
$(R = 110, k = 4)$	0.82	0.87	0.91	0.96	1.00	0.96	0.96
$(R = 120, k = 4)$	0.80	0.85	0.89	0.93	0.96	1.00	0.97
$(R = 130, k = 4)$	0.80	0.86	0.89	0.93	0.96	0.97	1.00
$(R = 70, k = 8)$	0.89	0.89	0.88	0.83	0.81	0.80	0.79
$(R = 80, k = 8)$	0.88	0.91	0.90	0.87	0.85	0.84	0.84
$(R = 90, k = 8)$	0.84	0.88	0.90	0.89	0.89	0.87	0.86
$(R = 100, k = 8)$	0.80	0.83	0.86	0.90	0.90	0.88	0.88
$(R = 110, k = 8)$	0.79	0.83	0.85	0.90	0.92	0.90	0.90
$(R = 120, k = 8)$	0.75	0.82	0.85	0.89	0.90	0.91	0.90
$(R = 130, k = 8)$	0.73	0.80	0.84	0.89	0.91	0.91	0.92
				Panel B: $k = 8$			
$(R = 70, k = 4)$	0.89	0.88	0.84	0.80	0.79	0.75	0.73
$(R = 80, k = 4)$	0.89	0.91	0.88	0.83	0.83	0.82	0.80
$(R = 90, k = 4)$	0.88	0.90	0.90	0.86	0.85	0.85	0.84
$(R = 100, k = 4)$	0.83	0.87	0.89	0.90	0.90	0.89	0.89
$(R = 110, k = 4)$	0.81	0.85	0.89	0.90	0.92	0.90	0.91
$(R = 120, k = 4)$	0.80	0.84	0.87	0.88	0.90	0.91	0.91
$(R = 130, k = 4)$	0.79	0.84	0.86	0.88	0.90	0.90	0.92
$(R = 70, k = 8)$	1.00	0.94	0.89	0.84	0.83	0.80	0.76
$(R = 80, k = 8)$	0.94	1.00	0.93	0.88	0.87	0.85	0.82
$(R = 90, k = 8)$	0.89	0.93	1.00	0.93	0.91	0.89	0.87
$(R = 100, k = 8)$	0.84	0.88	0.93	1.00	0.96	0.92	0.90
$(R = 110, k = 8)$	0.83	0.87	0.91	0.96	1.00	0.95	0.92
$(R = 120, k = 8)$	0.80	0.85	0.89	0.92	0.95	1.00	0.95
$(R = 130, k = 8)$	0.76	0.82	0.87	0.90	0.92	0.95	1.00

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$

Table H.5: Robustness of the MSM Estimation Results (Japanese Yen)

	$R = 70$	$R = 80$	$R = 90$	$R = 100$	$R = 110$	$R = 120$	$R = 130$
				Panel A: $k = 4$			
$(R = 70, k = 4)$	1.00	0.95	0.92	0.89	0.87	0.85	0.83
$(R = 80, k = 4)$	0.95	1.00	0.95	0.92	0.90	0.88	0.86
$(R = 90, k = 4)$	0.92	0.95	1.00	0.95	0.93	0.91	0.89
$(R = 100, k = 4)$	0.89	0.92	0.95	1.00	0.96	0.94	0.93
$(R = 110, k = 4)$	0.87	0.90	0.93	0.96	1.00	0.98	0.96
$(R = 120, k = 4)$	0.85	0.88	0.91	0.94	0.98	1.00	0.97
$(R = 130, k = 4)$	0.83	0.86	0.89	0.93	0.96	0.97	1.00
$(R = 70, k = 8)$	0.65	0.66	0.68	0.67	0.65	0.64	0.65
$(R = 80, k = 8)$	0.88	0.90	0.89	0.87	0.86	0.84	0.83
$(R = 90, k = 8)$	0.86	0.88	0.90	0.89	0.88	0.86	0.85
$(R = 100, k = 8)$	0.85	0.87	0.89	0.91	0.89	0.89	0.87
$(R = 110, k = 8)$	0.85	0.87	0.90	0.92	0.93	0.92	0.91
$(R = 120, k = 8)$	0.84	0.85	0.88	0.90	0.92	0.93	0.93
$(R = 130, k = 8)$	0.81	0.84	0.86	0.89	0.91	0.92	0.94
				Panel B: $k = 8$			
$(R = 70, k = 4)$	0.65	0.88	0.86	0.85	0.85	0.84	0.81
$(R = 80, k = 4)$	0.66	0.90	0.88	0.87	0.87	0.85	0.84
$(R = 90, k = 4)$	0.68	0.89	0.90	0.89	0.90	0.88	0.86
$(R = 100, k = 4)$	0.67	0.87	0.89	0.91	0.92	0.90	0.89
$(R = 110, k = 4)$	0.65	0.86	0.88	0.89	0.93	0.92	0.91
$(R = 120, k = 4)$	0.64	0.84	0.86	0.89	0.92	0.93	0.92
$(R = 130, k = 4)$	0.65	0.83	0.85	0.87	0.91	0.93	0.94
$(R = 70, k = 8)$	1.00	0.67	0.68	0.67	0.66	0.65	0.66
$(R = 80, k = 8)$	0.67	1.00	0.94	0.92	0.90	0.87	0.84
$(R = 90, k = 8)$	0.68	0.94	1.00	0.95	0.93	0.90	0.88
$(R = 100, k = 8)$	0.67	0.92	0.95	1.00	0.95	0.93	0.90
$(R = 110, k = 8)$	0.66	0.90	0.93	0.95	1.00	0.97	0.95
$(R = 120, k = 8)$	0.65	0.87	0.90	0.93	0.97	1.00	0.97
$(R = 130, k = 8)$	0.66	0.84	0.88	0.90	0.95	0.97	1.00

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$

Table H.6: Robustness of the MSM Estimation Results (British Pound)

	$R = 70$	$R = 80$	$R = 90$	$R = 100$	$R = 110$	$R = 120$	$R = 130$
				Panel A: $k = 4$			
$R = 70, k = 4$	1.00	0.92	0.89	0.87	0.84	0.85	0.83
$R = 80, k = 4$	0.92	1.00	0.94	0.91	0.88	0.88	0.86
$R = 90, k = 4$	0.89	0.94	1.00	0.94	0.91	0.91	0.89
$R = 100, k = 4$	0.87	0.91	0.94	1.00	0.95	0.93	0.90
$R = 110, k = 4$	0.84	0.88	0.91	0.95	1.00	0.96	0.92
$R = 120, k = 4$	0.85	0.88	0.91	0.93	0.96	1.00	0.94
$R = 130, k = 4$	0.83	0.86	0.89	0.90	0.92	0.94	1.00
$R = 70, k = 8$	0.64	0.65	0.65	0.63	0.61	0.62	0.61
$R = 80, k = 8$	0.86	0.90	0.88	0.86	0.85	0.84	0.82
$R = 90, k = 8$	0.83	0.87	0.90	0.88	0.87	0.86	0.84
$R = 100, k = 8$	0.80	0.84	0.87	0.89	0.88	0.87	0.85
$R = 110, k = 8$	0.79	0.83	0.85	0.88	0.89	0.88	0.86
$R = 120, k = 8$	0.81	0.85	0.86	0.88	0.89	0.90	0.88
$R = 130, k = 8$	0.80	0.83	0.84	0.86	0.87	0.89	0.90
				Panel B: $k = 8$			
$R = 70, k = 4$	0.64	0.86	0.83	0.80	0.79	0.81	0.80
$R = 80, k = 4$	0.65	0.90	0.87	0.84	0.83	0.85	0.83
$R = 90, k = 4$	0.65	0.88	0.90	0.87	0.85	0.86	0.84
$R = 100, k = 4$	0.63	0.86	0.88	0.89	0.88	0.88	0.86
$R = 110, k = 4$	0.61	0.85	0.87	0.88	0.89	0.89	0.87
$R = 120, k = 4$	0.62	0.84	0.86	0.87	0.88	0.90	0.89
$R = 130, k = 4$	0.61	0.82	0.84	0.85	0.86	0.88	0.90
$R = 70, k = 8$	1.00	0.65	0.65	0.62	0.61	0.62	0.61
$R = 80, k = 8$	0.65	1.00	0.94	0.91	0.89	0.89	0.87
$R = 90, k = 8$	0.65	0.94	1.00	0.94	0.92	0.91	0.89
$R = 100, k = 8$	0.62	0.91	0.94	1.00	0.95	0.94	0.91
$R = 110, k = 8$	0.61	0.89	0.92	0.95	1.00	0.96	0.93
$R = 120, k = 8$	0.62	0.89	0.91	0.94	0.96	1.00	0.95
$R = 130, k = 8$	0.61	0.87	0.89	0.91	0.93	0.95	1.00

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$

Table H.7: Success Ratio of the Directional Forecasts ($R = 70$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.599	0.565	0.571	0.520	0.565
	(177)	(177)	(177)	(171)	(170)
CAD	0.544	0.569	0.631	0.623	0.589
	(160)	(160)	(160)	(159)	(158)
EUR	0.570	0.553	0.702	0.657	0.630
	(142)	(141)	(141)	(140)	(138)
JPY	0.549	0.471	0.476	0.510	0.510
	(175)	(174)	(166)	(157)	(151)
GBP	0.563	0.591	0.534	0.529	0.554
	(208)	(208)	(208)	(208)	(204)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.8: Success Ratio of the Directional Forecasts ($R = 80$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.579	0.515	0.520	0.453	0.521
	(171)	(171)	(171)	(170)	(169)
CAD	0.567	0.632	0.690	0.655	0.606
	(171)	(171)	(171)	(171)	(170)
EUR	0.567	0.567	0.660	0.627	0.584
	(150)	(150)	(150)	(150)	(149)
JPY	0.527	0.503	0.526	0.590	0.590
	(186)	(183)	(175)	(166)	(156)
GBP	0.553	0.558	0.512	0.522	0.556
	(208)	(208)	(207)	(203)	(198)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.9: Success Ratio of the Directional Forecasts ($R = 90$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.566	0.598	0.594	0.522	0.589
	(189)	(189)	(187)	(186)	(185)
CAD	0.577	0.640	0.720	0.690	0.647
	(175)	(175)	(175)	(174)	(173)
EUR	0.586	0.559	0.699	0.669	0.645
	(145)	(143)	(143)	(142)	(141)
JPY	0.527	0.508	0.585	0.657	0.660
	(186)	(183)	(176)	(169)	(159)
GBP	0.544	0.515	0.478	0.493	0.505
	(206)	(206)	(205)	(201)	(196)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.10: Success Ratio of the Directional Forecasts ($R = 100$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.605	0.592	0.612	0.533	0.582
	(200)	(196)	(196)	(195)	(194)
CAD	0.577	0.587	0.661	0.640	0.612
	(189)	(189)	(189)	(189)	(188)
EUR	0.593	0.481	0.615	0.604	0.575
	(135)	(135)	(135)	(134)	(134)
JPY	0.525	0.532	0.617	0.688	0.682
	(204)	(201)	(193)	(186)	(176)
GBP	0.543	0.500	0.483	0.517	0.500
	(210)	(210)	(209)	(205)	(200)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.11: Success Ratio of the Directional Forecasts ($R = 110$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.582	0.577	0.617	0.565	0.603
	(201)	(201)	(201)	(200)	(199)
CAD	0.525	0.559	0.647	0.627	0.601
	(204)	(204)	(204)	(204)	(203)
EUR	0.550	0.521	0.629	0.607	0.600
	(140)	(140)	(140)	(140)	(140)
JPY	0.553	0.551	0.614	0.689	0.689
	(208)	(205)	(197)	(190)	(180)
GBP	0.548	0.524	0.483	0.507	0.510
	(210)	(206)	(205)	(201)	(196)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.12: Success Ratio of the Directional Forecasts ($R = 120$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.603	0.582	0.622	0.557	0.581
	(194)	(194)	(193)	(192)	(191)
CAD	0.563	0.568	0.623	0.618	0.586
	(199)	(199)	(199)	(199)	(198)
EUR	0.553	0.538	0.659	0.614	0.591
	(132)	(132)	(132)	(132)	(132)
JPY	0.562	0.519	0.612	0.663	0.677
	(217)	(214)	(206)	(199)	(189)
GBP	0.546	0.493	0.445	0.480	0.476
	(205)	(201)	(200)	(196)	(191)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

Table H.13: Success Ratio of the Directional Forecasts ($R = 130$)

	Forecasting Horizon (h)				
Currency	$1m$	$3m$	$6m$	$9m$	$12m$
AUD	0.586	0.545	0.602	0.538	0.569
	(215)	(213)	(211)	(210)	(209)
CAD	0.526	0.541	0.592	0.587	0.564
	(196)	(196)	(196)	(196)	(195)
EUR	0.550	0.550	0.674	0.643	0.605
	(129)	(129)	(129)	(129)	(129)
JPY	0.536	0.544	0.612	0.683	0.682
	(220)	(217)	(209)	(202)	(192)
GBP	0.555	0.491	0.465	0.498	0.485
	(220)	(216)	(215)	(211)	(206)

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.