Contrarian Opinion and Its Predictability: Application to Exchange Rates*

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This Version: August 2018

Abstract

Although exchange rates exhibit trends, they are difficult to predict out-of-sample. Puzzlingly when trends end, speculators behave overly-optimistically (overly-pessimistically). We formalize a novel mechanism for generating trends, where this contrarian-pattern arises in equilibrium. News are impounded into exchange rates via trending bubbly-paths, which informed-speculators ride until trend-reversals approach. When informed-speculators exit, trends continue because uninformed-speculators trade more optimistically (pessimistically) with hedgers. We estimate these switching-times and forecast trend-reversals using COT-speculator-position data. Our directional-forecasts achieve 53%-72% success-ratios over 1-to-12-month horizons, outperforming the random-walk for most currency-horizon pairs using the Binomial test and a new test weighting directional-forecasts by subsequent exchange rate changes.

Keywords: Bubbly Equilibrium, Contrarian Opinion, Exchange Rate Trends, Markov-switching Model, Trend Reversal

JEL Classification: F31; F37; G12

*We thank Saki Bigio, Graham Elliott, Roger Farmer, Olivier Gossner, Pierre Olivier Gourinchas, Jin Hahn, Jim Hamilton, Rosa Matzkin, Romain Ranciere, Allan Timmermann, Pierre Olivier Weil, Frank Westermann as well as seminar participants at the 2018 IEFS-EAER Conference and UCLA, for helpful comments.

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1 Introduction

Exchange rates tend to follow trends that often last several quarters.\footnote{Engel and Hamilton (1990) show the existence of long swings in the Dollar-German Mark, French Franc, and British Pound.} Although these trends can be identified in-sample, they are difficult to predict out-of-sample. It is not only difficult for econometricians to assess whether a trend may end soon, but for market participants as well. In fact, speculator position data reveals that typically the average speculator is overly optimistic at the end of an uptrend, while at the end of a downtrend she is overly pessimistic.

Figure 1 exhibits this contrarian pattern, depicting the Yen-US Dollar exchange rate and the speculators’ net positions in the Yen from the Commitments-of-traders (COT) report. As we can see, speculators’ net positions in the Yen typically exhibit a maximum around the time when a Yen’s uptrend ends and exhibit a minimum near the end of a Yen’s downtrend.

This contrarian pattern can also be observed in other currencies and may be associated with the "principle of contrarian opinion" (PCO): At extremes, what the majority believes is wrong. When a majority believes the market is going up—by the PCO—it is destined to plunge, and fast. Similarly, when a majority believes the downtrend will continue, an upward reversal is likely to come soon.

Contrarian opinion has a long tradition among practitioners, going back to Lefevre (1923) and Soros (1987). In this paper, we formalize the PCO within an exchange rate determination framework and show that it is part of an internally consistent mechanism—i.e., a PCO equilibrium. We then use the empirical implications of the PCO equilibrium to construct out-of-sample directional forecasts over 1 to 12 months horizons. Our model helps to address three questions: Why do trends arise across several currencies? Why would the average speculator systematically be wrong at extremes? And could one design a forecasting strategy based on the contrarian pattern of market-participants’ position data?
Our setup combines elements of exchange rate determination models with positive-feedback bubble models. There are three types of traders who may take positions on domestic and foreign bonds: informed speculators, uninformed speculators, and hedgers. News about future interest rate differentials are observed only by informed speculators and hedgers. Big positive/negative news lead informed speculators to take long/short position in the foreign bond, triggering a trend, i.e., a bubbly path along which a gap grows between the exchange rate and its fundamental value. Informed speculators have no incentive to arbitrage the increasing mispricing, instead finding it optimal to ride the trend for a while, exiting only when the gap becomes so large that uninformed speculators may no longer be able to finance their trades, thus making a trend reversal very likely. At this switching time, informed speculators close their long/short positions in the foreign bond and open short/long positions.

In order for this contrarian pattern to arise in equilibrium, informed speculators must expect that the exchange rate will move in their favor at the time they will exit their positions. Otherwise, by Tirole’s (1982) backwards induction argument, the bubble would unravel. We show that in a positive/negative bubbly equilibrium, along which the foreign currency appreciates/depreciates, the foreign currency necessarily appreciates/depreciates at switching time.
This is because a necessary condition for the existence of bubbly equilibria is that uninformed speculators’ demand for the foreign bond be more price elastic than the hedgers’ supply. Thus, when the informed speculators’ demand falls, market clearing can be attained only if the foreign currency appreciates. Of course, the uptrend is very likely to reverse soon after switching time, proving right the contrarian informed speculators and proving wrong the uninformed momentum-driven speculators.\(^2\)

The model is simple enough so as to allow for closed-form solutions and guide the design of forecasts of incoming trend reversals. In the PCO equilibrium, the aggregate speculators’ net positions can be expressed as the sum of a Markov switching term and an exchange rate autoregressive term, which correspond to informed and to uninformed speculators, respectively. That is, the speculators’ position data follows the process

\[ Y_t = c(\kappa_t) + \theta \Delta S_t + \varepsilon_t, \]

where the hidden state \( \kappa_t \) indicates whether the informed speculators’ net position is long \((\kappa_t = +1)\), short \((\kappa_t = -1)\) or zero \((\kappa_t = 0)\). Given the observations on \( Y_t \) and \( \Delta S_t \), we estimate \( \hat{\kappa}_t \) and \( \hat{\theta} \). We then forecast the imminent end of an uptrend if the estimated state is \( \hat{\kappa}_t = -1 \) and there is evidence of an uptrend, i.e., the estimate \( \hat{\theta} > 0 \) is significantly different from zero and \( \Delta S_t > 0 \). Analogously, we forecast an imminent end of a downtrend if \( \hat{\kappa}_t = +1 \) and there is evidence of a downtrend. Lastly, we forecast end of a trendless path if we detect a shift in \( \hat{\kappa}_t \) to either \(+1\) or \(-1\), along a trendless path.

We take these indicators as leading indicators of trend reversals and use them to construct \textit{out-of-sample directional forecasts} of exchange rates over horizons ranging from 1 to 12 months. We use the net speculators positions data of the COT report, and consider five currencies vis-a-vis the US dollar over the 1992-2017 period.\(^3\) Over horizons ranging from 1 to 12 months, our out-of-sample directional forecasts have a 61\% average success ratio across the five currencies. The forecast success ratios are particularly accurate at the 6-

\(^2\)Similarly, a negative shock to expectations may trigger a downtrend, along which informed speculators are short the foreign bond until a switching time when a melt-down occurs. Shortly thereafter the downtrend is very likely to end.

\(^3\)We consider the Euro, Japanese Yen, British Pound, Australian Dollar, and Canadian Dollar at 1m, 3m, 6m, 9m, and 12m forecasting horizons.
to-12 months horizons, reaching 66.4% for the Yen, 68.6% for the Canadian Dollar, and 67.1% for the Euro.

Interestingly, our model’s forecasts are not of mere statistical importance. Analyzing the Yen and the Euro we find that the signals of our model often foresee major monetary shocks, and in several occasions signal when the associated exchange rate move may end. For instance, the launch of the quantitative easing programs in Japan (2001), in the USA (2009) and in the Eurozone (2015). Similar signals are exhibited by our model preceding the Lehman (2008) crisis and the implementation of Abenomics in 2013.

To evaluate whether our directional forecasts succeed in predicting big swings in exchange rates, we propose a new directional forecast test. Unlike the traditional directional test, our test weights each directional forecast by the realized exchange rate change, and evaluates whether the weighted directional forecasts outperform the driftless random walk forecasts. At the 9- and 12-month horizons, the weighted directional test rejects the random-walk null in favour of our model across all currencies, controlling for auto-correlation using the long-run variance estimators proposed by Newey and West (1987) and Andrews (1991). The same holds true at the 6-month horizon across all currencies, except the Australian Dollar. At the 1-month and 3-month horizons, the null is rejected in 6 out of the 10 currency-horizon pairs. The standard binomial test generates similar results.\(^4\)

The high success ratio of our directional forecasts is reflected in Figure 2, which depicts the densities of one-year ahead changes in the Yen-US Dollar exchange rate. As we can see, the mean of the unconditional density is -0.8%. Meanwhile, the mean of the density conditional on a Yen appreciation (resp. depreciation) forecast is 2.4% (resp. -5.7%).

Engel and West (2005) have shown that the poor performance of funda-

\(^4\)Throughout this paper, when citing the Newey-West long-run variance (LRV) estimator, we refer to the kernel smoothed LRV estimator proposed in Newey and West (1987) and bandwidth selection rule suggested in Newey and West (1994). When citing the Andrews LRV estimator, we refer to the kernel smoothed LRV estimator using Bartlett kernel and bandwidth selection rule suggested in Andrews (1991). See Section E of the Extended Appendix for the construction of the above LRV estimators.
Figure 2: Kernel Density Function of 1-year Exchange Rate Changes

![Kernel Density Function of 1-year Exchange Rate Changes](image)

Note: The solid blue line depicts unconditional kernel density function of 1-year exchange rate changes. The orange (yellow) line with circle (star) markers is the kernel density function of 1-year exchange rate changes conditional on our appreciation (depreciation) forecasts.

mentals’ based models arises because exchange rates may be driven by news about future events. In our model, the current exchange rate also reacts to news about future events. The novel element is that such news may trigger a bubbly path, which rational-informed speculators have no incentives to arbitrage away until the end of the bubble is sufficiently likely.

By investigating the information content of market participant’s positions, our paper is related to Evans and Lyons (2002, 2005, 2012), who argue that order flow contains relevant information about the determinants of exchange rates. Using linear prediction models, they find that order flow forecasts exchange rates successfully over up to one-month horizons. This finding reflects the refined information contained in their order flow data.\(^5\) In contrast, the COT report is noisy since it contains a mixture of informed and uninformed traders’ positions. Therefore, higher/lower positions in the COT report are not necessarily associated with higher/lower future values of foreign currencies and thereby linear models fail to accurately forecast future exchange rate

\(^5\)In our setup there is no order flow among informed speculators, as they all have the same information.
changes over 1 to 12 months horizons. Our non-linear forecasts are designed for extracting hidden information from this type of noisy data. Another difference is that Evans and Lyons’ order flow data is proprietary, while COT data is public and available on a weekly basis.

Our bubbly paths have the same spirit as those in positive-feedback models. In Abreu and Brunnermeier (2003), momentum traders interact with rational but not perfectly-informed traders, each of whom only becomes aware of the bubble at a random time and does not know when other informed traders become aware of the bubble. Because of this asymmetry of information across rational traders, they may continue to ride the bubble even after all of them are aware of it. The price falls when they exit the market. In our setup, by contrast, there is no asymmetric information across informed traders and the foreign currency’s price necessarily increases when they exit. This is because of the PCO mechanism. Barberis et al. (2018) consider fundamental traders and extrapolator traders, who put weight both on price momentum and on fundamentals. They show that a bubble arises if there is a sequence of positive shocks to fundamentals. Their focus is on generating increasing volume, rather than forecasting trend reversals. In their model, fundamental traders exit early along a bubble, while in our PCO equilibrium informed speculators ride the bubble until the probability of its end is high. This property allows us to derive an estimation strategy that forecasts incoming trend reversals.

In contrast to much of the literature, our forecasts are non-linear, which naturally follows from the implications of our theoretical model. Our estimation method relies on the maximum likelihood estimation of the Markov switching model (hereafter MSM) proposed by Hamilton (1989, 1990) and applied by Engel and Hamilton (1990) to exchange rate data. In this paper, we apply a MSM to the speculators’ aggregate net position data, rather than to the exchange rate itself. Moreover, rather than a two-state MSM, our model implies three regimes corresponding to the sign of the informed speculators’

\[6\]Granger causality tests reveal that across the five currencies, net speculator positions do not Granger-cause exchange rates over 1 to 12 months horizons. See Table F.1 in the Extended Appendix for details.
Lastly, our weighted directional test is in the spirit of Elliott and Ito (1999) and may be more relevant than the standard binomial test to situations faced by investors, who look at the profitability associated with their forecasts rather than the share of correct forecasts. As Elliott and Timmermann (2016) point out, the standard binomial test does not capture any notion of profitability.

The structure of the paper is the following. Section 2 presents the model. In Section 3, we apply the forecasting strategy implied by the equilibrium to COT data and present our directional forecasts, and the accuracy tests. In Section 4, we present the literature review. Section 5 contains the conclusions. The main proofs are contained in the Appendix. Lastly, the Extended Appendix contains auxiliary material and robustness checks of the empirical estimations.

2 Model

We combine a minimal model of exchange rate fluctuations with a positive-feedback bubble model, where forward-looking informed speculators trade with uniformly trend-chasing speculators and with hedgers. We characterize equilibria where the response to transitory expectational shocks exhibits short-run momentum and reversals. In equilibrium, switches in the sign of the net positions of informed speculators have predictive power over exchange rates. Estimating such switching times is the basis of our forecasting strategy.

There is an infinite horizon, one good, which is the numeraire, and two one-period bonds. The domestic bond pays next period a continuous compounding interest rate $i_t$, and the foreign bond pays next period a continuous compounding interest rate $i^f_t$. The domestic bond has a perfectly elastic supply and its price is one. The foreign bond is in zero net supply and we will refer to its price, $S_t$, as the exchange rate.

We take the interest rates as primitives and assume that for $0 \leq t \leq T - 1$ the interest rate differential is zero, while for $t \geq T$ it follows an AR(1) process.
with a transient shock at time $T$

$$i_t - i^f_t = \begin{cases} 
0 & \text{if } t < T \\
 a(i_{t-1} - i^f_{t-1}) + \omega_t & \text{if } t \geq T 
\end{cases}, \quad \text{with } \omega_t = \begin{cases} 
\omega & \text{if } t = T \\
0 & \text{otherwise} 
\end{cases}, \quad (1)$$

where $a \in (0, 1)$ and $\omega$ is a random variable.

There are three types of overlapping generations of traders who live 2 periods and can take short and long positions in the two bonds: informed speculators, fundamental traders, and momentum speculators. The population of each type of trader has measure one. Informed speculators are active over the infinite horizon, while fundamental and momentum traders are active only up to $t = T - 1$. We consider each in turn.

*Informed Speculators.* The young representative informed speculator is a risk-neutral rational agent that engages in the carry-trade. At any time $t$, she takes a position $q^I_t$ in foreign bonds and a position $-S_t q^I_t$ in domestic bonds to maximize her next period’s expected profits:

$$E_t^I [W_{t+1}^I] = q^I_t \cdot \left( E_t^I[S_{t+1}] \exp(i^f_t) - S_t \exp(i_t) \right), \quad (2)$$

where $E_t^I[\cdot]$ denotes the conditional expectation operator given her information set $I_t$ at time $t$. Each informed speculator can go long, short or stay off the market. Her choice set is $\{-c, 0, c\}$, where $c$ is a real positive number. Since $i_t = i^f_t$ for any $t \leq T - 1$, by (1), the representative informed speculator’s payoff is $\pi^I_t = q^I_t \exp(i_t) \cdot (E_t^I[S_{t+1}] - S_t)$. Thus, on $t \leq T - 1$ her demand for the foreign bond is:

$$q^I_t(S_t, E_t^I[S_{t+1}]) = C(E_t^I[\Delta S_{t+1}]) = \begin{cases} 
c & \text{if } E_t^I[\Delta S_{t+1}] > 0 \\
0 & \text{if } E_t^I[\Delta S_{t+1}] = 0 \\
-c & \text{if } E_t^I[\Delta S_{t+1}] < 0 
\end{cases}, \quad (3)$$

where $E_t^I[\Delta S_{t+1}] = E_t^I[S_{t+1}] - S_t$. Informed speculators know the interest rate differential process (1), but do not observe the future shock $\omega_T$. At time $t < T$, they only observe a noisy signal $\eta^A_t$ about the shock $\omega_T$. Using the observation
on $\eta_t^A$, the time-$t$ ($t < T$) representative informed speculator makes a forecast of the time-$T$ exchange rate, which we denote by $Z_t$

$$Z_t \equiv E[S^*_T | \eta_t^A],$$

where $S_t^*$ denotes the exchange rate at time $T$, which is a known function of $\omega_T$ as we show in Appendix B.1. We will refer to the random variable $Z_t$ as the fundamental.

**Fundamental Traders.** These agents are hedgers that short the foreign bond when $S_t$ is above the fundamental $Z_t$ and go long when $S_t$ is below $Z_t$. Their demand function is:

$$q^F_t(S_t, Z_t) = \phi (Z_t - S_t), \quad \phi > 0$$

We show in Appendix B.2 that (5) arises in a setup where the representative fundamental trader chooses his demand for the foreign bond $q^F_t \in R$ in order to maximize $E^F_t \left[ - \exp(-\gamma^FW^F_t) \right]$, where $W^F_t = q^F_t(S^*_t - S_t)$ and $E^F_t \left[ \cdot \right]$ is the conditional expectation operator taken with respect to his information set.

The time-$t$ representative fundamental trader observes the signal $\eta_t^A$ and his forecast of $S^*_t$ is $Z_t = E[S^*_T | \eta_t^A]$ in (4). He assumes that, for any $t$, the forecast error $S^*_t - Z_t \sim u_F$ is a normal random variable with mean zero and variance $\sigma^2_u$. Therefore, he believes that, at any time $t$, the conditional distribution of $S^*_t$ given $\eta_t^A$ is $N(Z_t, \sigma_u^2)$. Since the population of fundamental traders has measure one, the resulting demand function is given by (5) with $\phi = \left(\gamma^F \sigma_u^2\right)^{-1}$, as we show in Appendix B.2.

**Momentum Speculators.** The representative momentum speculator does not observe the fundamental $Z_t$ and has behavioral biases. She believes that all other market participants are fundamental traders and that the fundamental follows the process $Z_{t+1} = Z_t + u_{t+1}$, with $u_{t+1} = au_t + v_{t+1}$ and $v_{t+1} \sim N(0, \sigma_v^2)$.\(^7\) She faces no short-sales constraints and chooses her demand for

\(^7\)A similar assumption is made by Banerjee and Green (2015) to generate momentum strategies. Having agents with distorted beliefs is typically done in the behavioral finance literature. For instance, Barberis et al. (2018), and Gourinchas and Tornell (2004).
the foreign bond $q^m_t \in R$ to maximize $E^m_t \left[ - \exp \left( -\gamma^m W^m_{t+1} \right) \right]$, with $W^m_{t+1} = q^m_t \cdot (S_{t+1} - S_t)$, where we have used $i_t = i^f_t$ for any $t \leq T - 1$. Young momentum speculators do not have any funds. In order to meet margin requirements, they borrow from financiers, which are outside of the model. There may be situations in which they are not able to find financing and hence cannot trade. When such event occurs we say that there is a liquidity crunch.⁸ We define a binary state variable $L_t$ which takes the value $1$ if a liquidity crunch is triggered at time $t$ and the value $0$ otherwise. When $L_t = 1$, momentum speculators cannot borrow at both time $t$ and time $t + 1$. We show in Appendix B.3 that for $t \leq T - 1$, the momentum speculators’ demand function for the foreign bond is

$$q^m_t(S_t, S_{t-1}) = \begin{cases} \theta (S_t - S_{t-1}), & \text{if } L_{t-1} = 0 \text{ and } L_t = 0 \\ 0, & \text{if } L_{t-1} = 1 \text{ or } L_t = 1 \end{cases}, \quad \theta = (\gamma^m \sigma^2_{\nu})^{-1}.$$ 

Equation (6) captures the essence of trend-following observed in several asset markets. It implies that if momentum speculators can find financing and $S_t$ has been increasing/decreasing, then their demand for the foreign bond increases/decreases over time. However, if there is a liquidity crunch, they cannot find financing and their demand becomes zero.

As the exchange rate diverges from the fundamental $Z_t$, it becomes less likely that momentum speculators will be able to find financing. Concretely, we assume that the probability of a liquidity crunch next period $\sigma_{t+1}$ is increasing in the distance between the mean of $Z_t$ and the exchange rate:

$$\sigma_{t+1} \equiv \Pr(L_{t+1} = 1|S_t) = \begin{cases} 0, & \text{if } |S_t - \mu_z| \leq \vartheta \\ \bar{\sigma}, & \text{if } \vartheta < |S_t - \mu_z| \leq \overline{\vartheta} \\ 1 & \text{if } \overline{\vartheta} < S_t - \mu_z \end{cases}, \quad \mu_z \equiv E[Z_t], \quad \overline{\vartheta} > \vartheta > 0 \text{ and } \bar{\sigma} \in (0, 1) \text{ are some fixed constants.}$$

Even though the carry trade is a zero-cost portfolio, traders are typically required to put down a deposit to meet potential margin calls. Traders typically need to borrow to put down the deposit, and such liquidity may dry from time to time, e.g. Brunnermeier and Pedersen (2008).
the model we assume that the economy starts at time $0$ with $L_0 = 1$.

**Equilibrium Concept.** We consider competitive equilibria in which each trader submits her demand schedule for the foreign bond, taking exchange rates as given, and the market of the foreign bond clears every period. In order to derive our forecasting rules in the empirical section, we characterize bubbly equilibrium paths, along which the exchange rate follows either a strictly increasing or a strictly decreasing path during a period over which $Z_t$ experiences no shocks.

**Definition 1 (Bubbly Path).** Suppose that in the time interval $\{t, \ldots, t'\}$ with $t' > t + 3$, there are no shocks to the fundamental and no liquidity crunch, i.e., $Z_j = \mu z$ and $L_j = 0$ at any $j \in \{t, \ldots, t'\}$. Then there is a positive bubbly path on $\{t, \ldots, t'\}$ if $S_j > S_{j'}$ for any $j, j' \in \{t, \ldots, t'\}$ with $j > j'$. Similarly, there is a negative bubbly path on $\{t, \ldots, t'\}$ if $S_j < S_{j'}$ for any $j, j' \in \{t, \ldots, t'\}$ with $j > j'$. We say that there exists a bubbly path on $\{t, \ldots, t'\}$ if there is either a positive bubbly path or a negative bubbly path.

Equipped with the definition of a bubbly path we now define a bubbly equilibrium.

**Definition 2 (Bubbly Equilibrium).** A bubbly equilibrium is an exchange rate and liquidity crunch process $\{(S^*_j, L^*_j)\}_{j=1}^\infty$, such that: (i) taking the exchange rates and liquidity crunch as given, the demand schedules $q^I_t(S^*_t, E^I_t[S^*_{t+1}])$, $q^F_t(S^*_t, Z_t)$ and $q^m_t(S^*_t, S^*_{t-1})$ solve the optimization problems of informed, fundamental, and momentum traders, respectively; (ii) the foreign bond’s market clears every period:

\[
q^F_t(S^*_t, Z_t) + q^m_t(S^*_t, S^*_{t-1}) + q^I_t(S^*_t, E^I_t[S^*_{t+1}]) = 0 \quad \text{if } t \leq T - 1
\]

\[
q^I_t(S^*_t, E^I_t[S^*_{t+1}]) = 0 \quad \text{if } t \geq T .
\]

(iii) there is a time interval $\{t, \ldots, t'\}$, with $T - 1 > t' > t + 1$, on which a bubbly path exists; and (iv) in the absence of shocks, the exchange rate reverts back to the fundamental $Z_t$ in finite time; (v) given exchange rates, the liquidity crunch is determined by (7).
2.1 Discussion of the Setup

The three types of traders in the model are typically present in several asset markets. Lefevre (1923) refers to the informed speculators as manipulators, to the uninformed speculators simply as speculators, and to the fundamental traders as investors. These three types of traders are necessary for our argument. Informed speculators can correctly forecast the distribution of future equilibrium exchange rates and initiate bubbly paths when they observe shocks to the fundamental. The role of momentum speculators is to drive the bubble larger and larger. The fundamental traders’ role is to disconnect the equilibrium trade volume of informed and momentum speculators.

The fundamental $Z_t$, defined in (4), will play an important role because the equilibrium boom & bust paths will be characterized in terms of deviations of $S_t$ from $Z_t$. To interpret $Z_t$, notice that it would equal the time-$t$ exchange rate in an hypothetical economy with only rational informed speculators who know the interest rate differential process (1), and at any time $t < T$ observe a noisy signal $\eta^I_t$ about the future shock to fundamentals $\omega_T$, with which they forecast $S^*_T$. In such economy $E^I_t[\cdot] = E_t[\cdot | \eta^I_t]$ and the market-clearing condition would be $C(E^I_t[\Delta S_{t+1}^*]) = 0$ for all $t < T$. It follows from (3) that market clearing requires that $E^I_t[\Delta S_{t+1}^*] = 0$ for all $t < T$. Thus, the time-$t$ equilibrium exchange rate in this hypothetical economy is the one expected to prevail at time $T$: $S^*_t = E^I_t[S^*_T] = Z_t$ for all $t < T$. In other words, because $i_t = i^I_t$ on $t \leq T - 1$, any time-$t$ news of a future time-$T$ shock, makes the time-$t$’s exchange rate jump to the level it is expected to have at time $T$.

Two key properties of the bubbly equilibria we characterize will be quite useful in designing our empirical forecasting strategy. First, a single transitory shock to the fundamental $Z_t$ can trigger a bubbly path. Second, if a bubbly equilibrium exists, then along a positive bubbly path, $S^*_t$ must go up at the time informed speculators switch their long positions for short positions. This result implies that backwards induction does not unravel to initial time, and so informed speculators find it profitable to ride the bubble initially, and exit only when it is very likely that momentum traders will not be able to obtain financing anymore. At this switching time $\tau^*$, informed speculators know that
the exchange rate is very likely to change direction and revert back to $Z_t$.\(^9\) Therefore, their entry and exit decisions may act as leading indicators of trend reversals. Hence, the empirical implication of our bubbly equilibria is that if one has time series data which contains a mixture of informed and momentum speculators, then one can estimate the switching times when the trend is very likely to change direction by identifying times when such speculator position data has a regime switch.\(^10\)

Lastly, in the empirical section we will consider the US Dollar versus other currencies, and so we will refer to an increase/decrease in the price of the foreign bond, $S_t$, as a Dollar depreciation/appreciation.

### 2.2 Bubbly Equilibria

On $t \geq T$, since only informed speculators are active, $S^*_t$ is determined by iterating forward the uncovered interest parity condition $E_t^I[S^*_{t+1}] \exp(i'_t) - S^*_t \exp(i_t) = 0$, as in standard exchange rate determination models. We show in Appendix B.1 that if there is no long-run bubble, the time-$T$ exchange rate is

$$S^*_T = \alpha_T \exp \left( -\frac{\omega}{1 - a} \right) \text{ almost surely,} \quad \text{(9)}$$

where $\omega$ is the shock to the interest rate differential that occurs at time $T$ and $\alpha_T = E_T^I [S^*_{\infty}]$. The term $\omega/(1 - a)$ is the present value of future interest rate differentials and $\alpha_T$ may be interpreted as the conditional time-$T$ expectation of the long-run exchange rate. Our model has nothing to say about the determinants of $\alpha_T$. Our analysis is about the short-run boom & bust equilibrium paths around this long-run level.

We next consider the time interval $t \leq T - 1$. Substituting (9) in (4), we have that along the equilibrium path, $Z_t$ follows the process:

$$Z_t = \mu_z + \zeta_t, \quad \text{with} \quad \begin{cases} 
\mu_z = E \left[ \alpha_T \exp \left( \frac{\omega}{a-1} \right) \right] \\
\zeta_t = E \left[ \alpha_T \exp \left( \frac{\omega}{a-1} \right) \eta^A_t \right] - \mu_z.
\end{cases} \quad \text{(10)}$$

\(^9\)The analogous pattern arises along a negative bubbly path.
\(^10\)As we shall see in the next section, the conditions we impose on the model parameters ensure that in all cases the equilibrium switching time $\tau^*$ is always smaller than $T$.\[13\]
That is, the informed speculators’s time-$t$ forecast of $S_t^*$ is the sum of a constant $\mu_z$—the unconditional mean of $S_t^*$—and a time-varying term $\zeta_t$ that reflects shocks to expectations about $\omega_T$. We assume that the informed speculators take $\{\zeta_t\}_t$ as a martingale difference sequence. In subsection 2.3, we characterize bubbly equilibria for an economy that is hit by a positive transitory shock to the fundamental $Z_t$, i.e., a shock to expectations of the time-$T$ interest rate differential. Concretely, we consider the following trajectory of $\zeta_t$:

$$\zeta_t = \begin{cases} z & \text{if } t = 1 \\
0 & \text{if } t > 1 \end{cases},$$

(11)

which is induced by a trajectory of the signal $\eta_t^A$ through (10). In subsection 2.4, we consider a negative transitory shock.

To derive the equilibrium exchange rate for $t \leq T - 1$ notice that the uncertainty comes from the random shocks $\zeta_t$ to the fundamental $Z_t$, and the possibility of a liquidity crunch. The endogenous variables are the demands for the foreign bond, the equilibrium exchange rate $S_t^*$, and the binary variable $L_t^*$ indicating whether a liquidity crunch is triggered at time $t$. The time-$t$ demands for the foreign bond are calculated using $\{S_{t-1}, S_t, E_t^I[S_{t+1}], L_{t-1}, L_t\}$ together with equations (3), (5), and (6). If no liquidity crunch is triggered at $t - 1$ and $t$, the market-clearing condition at time $t$ is

$$0 = \theta(S_t^* - S_{t-1}^*) + \phi(Z_t - S_t^*) + C(E_t^I[\Delta S_{t+1}^*]).$$

(12)

Otherwise, if a liquidity crunch is triggered at $t - 1$ or $t$, market-clearing implies

$$0 = \phi(Z_t - S_t^*) + C(E_t^I[\Delta S_{t+1}^*]).$$

(13)

It follows that for $t \leq T - 1$, the equilibrium exchange rate $S_t^*$ satisfies

$$S_t^* = \begin{cases} \frac{\theta S_{t-1}^* - \phi Z_t - C_t}{\theta - \phi}, & \text{if } L_{t-1}^* = 0 \text{ and } L_t^* = 0 \\
Z_t + \frac{C_t}{\phi}, & \text{if } L_{t-1}^* = 1 \text{ or } L_t^* = 1 \end{cases},$$

(14)

where $C_t^* = C(E_t^I[\Delta S_{t+1}^*])$. The equilibrium exchange rate $\{S_t^*\}_t$ and the law
of motion of the liquidity crunch (6) determine the state variable \( \{ L_t^* \} \).

Before proceeding it will prove useful to determine the values of \( \theta \) and \( \phi \) over which bubbly equilibria exist.

**Lemma 3.** An equilibrium bubbly path exists only if the momentum speculator’s demand is more sensitive to current exchange rates than the fundamental traders’ demand. That is:

\[
\theta > \phi.
\]  

(15)

To grasp the intuition of Lemma 3, suppose on the contrary that \( \theta \leq \phi \) and that a bubbly path exists. We show in the Appendix B.4 that along this hypothetical bubbly path, price changes would satisfy

\[
(\theta - \phi) \Delta S_j^* = \theta \Delta S_{j-1}^* - \Delta C_j^* \text{ for any } j \in \{ t + 2, \ldots, t' \}.
\]

Since \( \theta > 0 \), \( \Delta S_j^* > 0 \) and \( \Delta S_{j-1}^* > 0 \) for any \( j \in \{ t + 2, \ldots, t' \} \), the above equation holds under \( \theta \leq \phi \) only when \( \Delta C_j^* > 0 \) for any \( j \in \{ t + 2, \ldots, t' \} \) which is impossible since \( t' - t > 3 \) and \( C_j^* \) can only take three possible values for any \( j \).

Restriction (15) will be key to characterizing the equilibrium paths, and so we will impose this condition throughout the rest of the paper. Both \( \theta \) and \( \phi \) are latent parameters, so the empirical validity of condition (15) cannot be directly determined. However, given the existence of an equilibrium bubbly path, condition (15) implies that price changes whose magnitude may increase over time, a condition which is observable.

**Lemma 4.** Suppose that an equilibrium bubbly path exists. Then restriction (15) implies that for at least one \( j \) such that \( \Delta S_j^* > \Delta S_{j-1}^* \) along the positive bubbly path, and \( \Delta S_j^* < \Delta S_{j-1}^* \) along the negative bubbly path.

We next characterize bubbly equilibria by first showing, in Proposition 5, that if a liquidity crunch were to occur, so that fundamental traders and informed speculators were the only market participants, the equilibrium exchange rate would revert to the fundamental \( Z_t \).
Proposition 5. Consider a time \( t_0 \) when a liquidity crunch is triggered and there is no shock to the fundamental (i.e., \( L^{*}_{t_0} = 1 \) and \( Z^{*}_{t_0} = \mu_z \)). Then at time \( t_0 \), the equilibrium exchange rate is equal to \( \mu_z \) and informed speculators do not participate in the market for the foreign bond, i.e., \( S^{*}_{t_0} = \mu_z \), \( C^{*}_{t_0} = 0 \) and \( L^{*}_{t_0+1} = 0 \).

To grasp the intuition of Proposition 5, note that when a liquidity crunch is triggered at time \( t \), the momentum traders’ demand is zero and so market clearing condition (13) implies that \( S^{*}_{t_0} = \mu_z + \phi^{-1}C^{*}_{t_0} \). We show in the Appendix that only a zero demand by informed speculators \( (C^{*}_{t_0} = 0) \) can be part of an equilibrium and so we must have \( S^{*}_{t_0} = \mu_z \). Suppose instead that the representative informed speculator expected a Dollar depreciation and chose a long position in the foreign bond \( (C^{*}_{t_0} = c) \). Then, regardless of the sign of future informed speculators’s positions, the expected value of the Dollar would not go down (i.e., \( E^{I}_{t_0}[\Delta S^{*}_{t_0+1}] \leq 0 \)). This clearly contradicts the portfolio rule of informed speculators in (3). An analogous argument holds for \( C^{*}_{t_0} = -c \).

2.3 Positive Bubbly Equilibria

In Theorem 6 below, we characterize positive bubbly equilibria, in which informed speculators take long positions in the foreign bond in response to a positive shock to \( Z_t \). The resulting Dollar depreciation leads momentum speculators to take long positions, initiating a bubbly path. Informed speculators find it optimal to ride this bubble for a while: at \( t \) they choose \( C^{*}_t = c \) expecting \( \Delta S^{*}_{t+1} > 0 \), at \( t+1 \) they set \( C^{*}_{t+1} = c \) expecting \( \Delta S^{*}_{t+2} > 0 \), and so on until \( \tau^* \), which we call switching time. At \( \tau^* \), a liquidity crunch becomes a possibility and so \( E^{I}_{\tau^*}[\Delta S^{*}_{\tau^*+1}] < 0 \). Thus, informed speculators close their long positions and start to short the foreign bond, i.e., they set \( C^{*}_t = -c \) between time \( \tau^* \) and time \( l^* \), when a liquidity crunch is triggered.

Theorem 6 (Positive Bubbly Equilibrium). Consider an economy that is in a liquidity crunch at time \( t = 0 \) i.e., \( L^{*}_0 = 1 \), and is hit by a transitory positive shock at time \( t = 1 \) (i.e., \( \zeta_0 = 0, \zeta_1 = z \) and \( \zeta_t = 0 \) for all \( t \geq 2 \)). If demand
parameters $\phi$ and $\theta$ satisfy (15), the magnitude of the transitory shock satisfies
\begin{equation}
    z > 2c\phi^{-1},
\end{equation}
and the parameters that govern the probability of liquidity crunch satisfy
\begin{equation}
    \sigma > \phi\theta^{-1} \text{ and } \vartheta \geq (1 - \phi/\theta)^{-1}
    \left[ z + (1 + \phi/\theta)c\phi^{-1} \right],
\end{equation}
then positive bubbly equilibria exist and have the following properties.

(i) At time 0, before the shock occurs, the equilibrium exchange rate satisfies $S^*_0 = \mu z$, and the informed speculators’ demand is $C^*_0 = 0$.

(ii) There exists a switching time $\tau^* > 2$ with $1 \leq \tau^* - \tau_* \leq 2$, where $\tau_*$ is defined in (21) below, such that
\begin{equation}
    S_t^* = \begin{cases}
        \mu z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{t-1}}, & 1 \leq t < \tau^* \\text{or} \\
        \mu z + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau_*+1}} - 1 \right], & \tau^* \leq t < l^*.
    \end{cases}
\end{equation}

(iii) The liquidity crunch will be triggered in a finite time, i.e., $\tau^* \leq l^* \leq l^\text{max}$ where $l^\text{max}$ is a finite integer defined in (22) below.

(iv) The informed speculators’ demand satisfies
\begin{equation}
    C_t^* = \begin{cases}
        c, & 1 \leq t < \tau^* \\
        -c, & \tau^* \leq t < l^*.
    \end{cases}
\end{equation}

(v) The momentum speculators’s demand satisfies
\begin{equation}
    q_t^{m*} = \begin{cases}
        0, & t \in \{0, 1, l^*\} \\
        \frac{z^\phi}{(1 - \phi/\theta)^{t-1}}, & 2 \leq t < \tau^* \\
        \frac{z^\phi}{(1 - \phi/\theta)^{t-1}} + \frac{2c}{(1 - \phi/\theta)^{t-\tau_*+1}}, & \tau^* \leq t < l^*.
    \end{cases}
\end{equation}

(vi) At time $l^*$, $S_l^* = \mu z$ and $C_l^* = 0$. 

17
Remark 2.1. This theorem makes two key points that will be exploited by our forecasting strategy. First, initially, informed speculators find it optimal to choose long positions in the foreign bond and ride the positive bubble. Second, the Dollar necessarily depreciates (i.e., $S_t^*$ increases) at the time when a liquidity crunch becomes imminent and informed speculators close their long positions in the foreign bond. Thus, backwards induction about the eventual fall in $S_t^*$, does not unravel all the way to initial time. The intuition is the following. Even though $S_t^*$ is above its fundamental value, initially informed speculators find it optimal to ride the positive bubble because they know momentum speculators will be increasing their long positions. However, when $S_t^*$ gets sufficiently far away from the mean of the fundamental $\mu_z$, a liquidity crunch becomes a possibility. Because informed speculators are forward-looking, at this "switching time" $\tau^*$ they rationally close their long positions and establish short positions in the foreign bond. At $\tau^*$, $S_t^*$ necessarily increases and momentum speculators increase their long positions (with fundamental traders taking the corresponding larger short positions). Thereafter, momentum speculators continue choosing long positions up to liquidity crunch time, when $S_t^*$ falls back to $\mu_z$.

$S_t^*$ necessarily increases when informed speculators’ demand falls because the demand of momentum speculators is more sensitive to the current exchange rate than that of fundamental traders ($\theta > \phi$), which is a necessary condition for the existence of bubbly equilibria (Lemma 3). To see this consider Figure 3, which depicts market clearing condition (8). The supply schedule $q_t^F = \phi(S_t - \mu_2)$ is the fundamental traders’ net supply function (5), while the demand schedule $Q_t = \theta(S_t - S_{t-1}) + C_t^I$ is the sum of momentum and informed speculators’ net demands (3) and (6). Notice that the demand schedule is upward sloping because $\theta > 0$, and it is less steep than the supply schedule because $\theta > \phi$. Now, a fall in the informed speculators’ demand from $C_t^I = +c$ to $C_t^I = -c$ shifts the demand schedule to the right. Because the supply is steeper than the demand, the equilibrium $S_t$ must increase in response to the fall in $C_t^I$. If instead $S_t^*$ went down, the momentum speculators’ demand for the foreign bond would fall more than the fundamental traders’ supply, and
Figure 3: Principle of Contrarian Opinion

![Diagram showing the principle of contrarian opinion]

so $S_t^*$ would fall even more. □

**Remark 2.2.** The pattern characterized in Theorem 6 is the principle of contrarian opinion. Near an extreme overvaluation, when the reversal of a Dollar depreciation trend becomes nigh, a majority of speculators (which are momentum driven) continue accumulating long positions in the foreign asset as the Dollar keeps depreciating, while a few speculators (the informed) close their long positions. Shortly thereafter the Dollar appreciates back to its fundamental value, proving the majority to be wrong.\(^{11}\) □

**Remark 2.3.** The proof of Theorem 6 in the Appendix has six steps. First, claims (i) and (vi) follow directly from Proposition 5. Second, Lemma 10 shows that the time-1 shock induces informed speculators to set $C_t^* = c$, expecting $\Delta S_t^* > 0$ and knowing $L_t^* = 0$ with probability 1. The resulting $\Delta S_t^* > 0$ activates the time-2 demand from momentum speculators, which in turn triggers a positive bubbly path. Third, Lemma 12, Lemma 13 and Lemma 14 prove that informed speculators find it optimal to ride this bubble

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\(^{11}\)Referring to Jess Livermore’s sale of Consolidated Steel, *Lefevre* (1923), Ch. XXII, explains how through cleverly timed purchases, the informed speculator can create the aura of such interest in an asset that she might move the price higher even while, on a net basis, dispensing of a position in it: "I didn’t even do my principal selling on the way down, but on the way up. It was like a dream of Paradise to find an adequate buying power created for you without stirring a finger to bring it about, particularly when you were in a hurry."
until a switching time $\tau^*$, which must equal either $\tau_* + 1$ or $\tau_* + 2$, with
\[
\tau_* = \max \left\{ t \in \mathbb{N} : \frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta} \phi \leq \vartheta \right\}, \tag{21}
\]
where $\mathbb{N}$ denotes the set of natural numbers and $\tau_*$ must be greater than 2 by (17). Fourth, Lemma 16 shows that for any $t \geq \tau^*$, informed speculators set $C^*_t = -c$, expecting $\Delta S^*_{t+1} < 0$. Therefore, the equilibrium $S^*_t$ satisfies (18) and the demand of the informed speculators satisfies (19), which proves claims (ii) and (iv). Fifth, Lemma 15 shows that the time $l^*$ when the liquidity crunch is triggered is larger than or equal to the switching time $\tau^*$. Moreover, Lemma 16 shows that $l^*$ is smaller than a fixed and finite integer $l_{\text{max}}$ defined as
\[
l_{\text{max}} = \min \left\{ t \in \mathbb{N} : t \geq \tau_* + 3 \text{ and } \frac{z}{(1-\phi/\theta)^{t-2}} + \frac{2c}{\phi (1-\phi/\theta)^{t-\tau_*-2}} - \frac{c}{\phi} > \bar{\vartheta} \right\}. \tag{22}
\]
This proves claim (iii). Lastly, Lemma 17 derives the demand of momentum speculators in claim (v). □

**Remark 2.4.** Because $\theta > \phi$, from (18) we see that the equilibrium $S^*_t$ is strictly increasing for $t < \tau^*$ because
\[
\Delta S^*_t = \frac{z}{(1-\phi/\theta)^{t-1}} \frac{\phi}{\theta} > 0. \tag{23}
\]
Moreover, the magnitude of the exchange rate changes increases strictly over time
\[
\Delta S^*_t / \Delta S^*_t = (1-\phi/\theta)^{-1} > 1, \tag{24}
\]
which implies that the increment of $S^*_t$ along the bubbly path is explosive. □

**Remark 2.5.** At switching time $\tau^*$, when informed speculators close their long positions and start to short the foreign bond, $S^*_t$ increases even faster than before
\[
\Delta S^*_{\tau^*} = \frac{z}{(1-\phi/\theta)^{\tau^*-1}} \frac{\phi}{\theta} + \frac{2c}{\phi} \frac{\phi/\theta}{1-\phi/\theta}. \tag{25}
\]
Comparing equations (23) with (25), we can see that there is a jump in the equilibrium exchange rate change at switching time $\tau^*$. Furthermore, at time $\tau^*$, there is a jump in the ratio of the changes of the equilibrium exchange rate:

$$\frac{\Delta S_{\tau^*}^*}{\Delta S_{\tau^*-1}^*} = \frac{1}{1 - \phi/\theta} + \frac{c\theta}{\phi^2 \phi} \left( \frac{\phi/\theta}{1 - \phi/\theta} \right)^{\tau^*-1}. \quad (26)$$

Despite the larger exchange rate depreciation at time $\tau^*$, informed speculators expect the Dollar to appreciate, i.e., $E_{\tau^*}^I[\Delta S_{\tau^*+1}^*] < 0$, because the probability of a liquidity crunch increases from 0 to at least $\sigma$.

The increase in volatility at time $\tau^*$ resembles the explosive behavior of asset prices typically observed near the so-called top formations that tend to mark the end of price uptrends and the beginning of price downtrends. Investment practitioners refer to these switching times as "distribution times" because positions are distributed from smart-money to dumb-money.\(^\text{12}\)

**Remark 2.6.** After switching time $\tau^*$, we have

$$\Delta S_t^* = \frac{2c}{\phi} \frac{\phi/\theta}{(1 - \phi/\theta)^{t-\tau^*+1}} + \frac{\phi^2 z}{(1 - \phi/\theta)^{t-1}} \text{ and } \frac{\Delta S_t^*}{\Delta S_{t-1}^*} = \frac{1}{1 - \phi/\theta}. \quad (27)$$

As we can see, the jump in the ratio of changes of the equilibrium exchange rate is temporary. It comes back to the level it had before the switching time $\tau^*$. □

**Remark 2.7.** If there is no further restriction on the latent parameters of the model, the equilibrium switching time $\tau^*$ may occur either at $\tau^* + 1$ or $\tau^* + 2$ because during these two periods there are multiple equilibrium exchange rates. For any $t$ with $1 \leq t - \tau^* \leq 2$, a long position by informed speculators ($C_t^* = c$) induces positive expected returns ($E_t^I[\Delta S_{t+1}^*] > 0$), while a short position induces negative expected returns. However, for any $t \geq \tau^* + 3$, expected returns are negative regardless of the informed speculators positions (i.e., $E_t^I[\Delta S_{t+1}^*] < 0$ for all $C_t^* \in \{-c, 0, c\}$). From Lemmas 14 and 16 in the Appendix, we see that the switching time $\tau^*$ is unique, and equal to $\tau^* + 1$, if

---

\(^{12}\)See for instance the accounts of Lefèvre (1923) and Soros (1987).
and only if
\[
\frac{z}{(1 - \phi/\theta)\tau^*} + \frac{c}{\phi} > \vartheta. \tag{28}
\]

The above condition ensures that \( S_{\tau^* + 1} - \mu_z > \vartheta \) regardless of the informed speculators’ demand at time \( \tau^* + 1 \), which implies that the probability of a liquidity crunch next period becomes positive and at least \( \bar{\sigma} \). In contrast, if (28) does not hold, then \( \tau^* \) may be either \( \tau^* + 1 \) or \( \tau^* + 2 \). □

**Remark 2.8.** Condition (16) requires that the magnitude of the transitory shock to the expected fundamental be large enough. This condition is necessary for a transitory shock to induce informed speculators to take a non-zero position, and hence initiate a bubbly equilibrium path. The second restriction in (17) sets the threshold \( \vartheta \) large enough so that the liquidity crunch is triggered neither at \( t = 1 \) nor at \( t = 2 \). This restriction is necessary for there to be a non-trivial time interval during which there is zero probability of a liquidity crunch, so there is enough time for a bubble to develop. □

### 2.4 Negative Bubbly Path

An analogous result to that in the Theorem 6 applies to a negative bubbly equilibrium. In particular, we characterize the conditions under which starting with \( S^*_0 = \mu_z \), a transitory shock to the expected fundamental can induce a negative bubbly path. Concretely, we consider the following trajectory for the shock in (10)
\[
\zeta_t = \begin{cases} 
-z & \text{if } t = 1 \\
0 & \text{if } t > 1
\end{cases}. \tag{29}
\]

Along the negative bubbly path, informed speculators anticipate a continuation of the negative bubble and find it optimal to ride the bubble for a while: they choose \( C^*_t = -c \) expecting \( \Delta S^*_{t+1} < 0 \) at time \( t \), and at \( t + 1 \) they set \( C^*_{t+1} = -c \) expecting \( \Delta S^*_{t+2} < 0 \), and so on until a critical time \( \tau^* \)—which we call switching time.

Since \( S_t \) is decreasing along the negative bubbly path, there should be some device in the model to prevent the exchange rate from becoming negative. For
this purpose, we revise the law of motion of the liquidity crunch process:

\[ \sigma_{t+1} \equiv \Pr(L_{t+1} = 1 | S_t) = \begin{cases} 0, & |S_t - \mu| \leq \theta \text{ and } S_t > S \\ \sigma, & |S_t - \mu| > \theta \text{ and } S_t > S \\ 1, & S_t \leq S \text{ or } S_t - \mu > \theta \end{cases} \tag{30} \]

where \( S \) is a small non-random positive lower bound. By (30), the probability of a liquidity crunch becomes \( \sigma \) when the exchange rate \( S_t \) deviates from \( Z_t \) by more than \( \theta \), and it becomes 1 when \( S_t \) is smaller than \( S \). To ensure that a liquidity crunch is not triggered immediately following a negative shock at time 1, we impose the following condition on \( S \).

**Condition 7.** The lower exchange rate bound \( S \) satisfies

\[ S < \mu - z - \frac{1}{ \phi / \theta} \frac{c}{ \phi} \]

To ensure that equilibrium exchange rates are positive along the equilibrium negative bubbly path, we impose the following parametric conditions.

**Condition 8.** The model parameters satisfy

\[ \begin{cases} \mu - \frac{z}{(1 - \phi / \theta)^{\tau + 1}} - \frac{c}{ \phi} > 0, & \text{if } \tau \leq \tau_* \\ \mu - \frac{z}{(1 - \phi / \theta)^{\tau_e - 1}} - \frac{c}{ \phi} \left[ \frac{2}{(1 - \phi / \theta)^{\tau_e - \tau_*}} - 1 \right] > 0, & \text{if } \tau_* < \tau \end{cases} \tag{32} \]

where \( \tau_* \) is defined in (21),

\[ \tau_e = \min \left\{ t > \tau_* : \mu - \frac{z}{(1 - \phi / \theta)^{t-1}} - \frac{c}{ \phi} \left[ \frac{2}{(1 - \phi / \theta)^{t-\tau_*}} - 1 \right] \leq S \right\} \tag{33} \]

and

\[ \tau = \max \left\{ t \in \mathbb{N} : \mu - \frac{z}{(1 - \phi / \theta)^{t-1}} - \frac{1 + \phi / \theta}{1 - \phi / \theta} \frac{c}{ \phi} > S \right\} \tag{34} \]

Conditions (17) and (31) imply that \( \tau \geq 1 \), which ensures that \( \tau \) is well defined. To grasp the intuition for Condition 8, notice that on one hand, when \( \tau \leq \tau_* \), a liquidity crunch will be triggered no later than \( \tau + 3 \) (by Lemmas
22, 23 and 26 in Appendix). When a liquidity crunch happens at $\tau + 3$, the equilibrium exchange rate at time $\tau + 2$ is

$$S^*_{\tau+2} = \mu_z - \frac{z}{(1 - \phi/\theta)\tau+1} - \frac{c}{\phi} \theta + \phi,$$

which is the first line in (32). On the other hand, when $\tau < \tau$ a liquidity crunch will be triggered no later than $\tau_e + 1$ (as we show in the proof of Corollary 9). When a liquidity crunch happens at $\tau_e + 1$, the equilibrium exchange rate at time $\tau_e$ is

$$S^*_{\tau_e} = \mu_z - \frac{z}{(1 - \phi/\theta)\tau_e-1} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)\tau_e-\tau_e-1} - 1 \right],$$

which is the second line in (32). Therefore, Condition 8 ensures that exchange rates are positive along the negative bubbly path. We are now equipped to establish the existence of negative bubbly equilibria.

Corollary 9 (Negative Bubbly Equilibrium). Let $\tau_* = \min\{\tau, \tau_e\}$. Consider an economy that has a liquidity crunch at time $t = 0$, i.e., $L^*_0 = 1$, and is hit by a transitory negative shock at time $t = 1$ (i.e., $\zeta_0 = 0$, $\zeta_1 = -z$ and $\zeta_t = 0$ for all $t \geq 2$). Under (15), (16), (17) and Conditions 7 and 8, the equilibrium bubbly path has the following properties.

(i) At time 0, the equilibrium exchange rate is $S^*_0 = \mu_z$ and the informed speculators’ demand is $C^*_0 = 0$.

(ii) There exists a switching time $\tau^*$ with $1 \leq \tau^* - \tau_* \leq 2$ such that

$$S^*_t = \begin{cases} 
\mu_z - \frac{\xi}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}}, & 1 \leq t < \tau^* \\
\mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1 \right], & \tau^* \leq t < \tau_* 
\end{cases},$$

where $S^*_t > 0$ for any $t$ with $1 \leq t < \tau^*$.

(iii) The liquidity crunch is triggered in finite time, i.e., $\tau^* \leq \tau^* \leq \tau^*_{\text{max}}$, where $\tau^*_{\text{max}} = \max\{\tau + 3, \tau_e + 2\}$.
(iv) The informed speculators’ demand satisfies

\[ C_t^* = \begin{cases} 
-c, & 1 \leq t < \tau^* \\
\phantom{+}c, & \tau^* \leq t < l^* \end{cases} \]  

(36)

(v) The momentum speculators’ demand satisfies

\[ q_t^{m^*} = \begin{cases} 
0, & t \in \{0, 1, l^*\} \\
-\frac{-z\phi}{(1-\phi/\theta)^{t-\tau}}, & 2 \leq t < \tau^* \\
-\frac{z\phi}{(1-\phi/\theta)^{t-\tau}} - \frac{2c}{(1-\phi/\theta)^{1-l^*+1}}, & \tau^* \leq t < l^* \end{cases} \]  

(37)

(vi) At time \(l^*\), \(S_{l^*} = \mu_z\) and \(C_{l^*}^* = 0\).

**Remark 2.9.** The equilibrium along the negative bubbly path has similar properties to the positive bubbly path. The informed speculators find it optimal to short the foreign bond and ride the negative bubble until a liquidity crunch becomes possible. The principle of contrarian opinion is also present: At switching time \(\tau^*\), informed speculators switch their foreign bond’s short positions to long positions and the Dollar necessarily appreciates (i.e., \(S_t\) decreases) even faster. After \(\tau^*\), momentum speculators continue increasing their short positions at a faster pace, with fundamental traders taking the corresponding increasing long positions. This equilibrium process continues until a liquidity crunch is triggered and the exchange rate jumps up to the fundamental value \(Z_{l^*}\). □

3 Empirical Implications of the Model

In the PCO equilibrium characterized in Lemma 4, Theorem 6 and Corollary 9, informed speculators observe news about future interest rate differentials and about the probability of a liquidity crunch \(\sigma_{t+1}\), which is an increasing function of the gap between the exchange rate \(S_t\) and the mean of the fundamental \(\mu_z\). They then calculate the conditional expectation of future exchange rate changes \(E_t^f[\Delta S_{t+1}]\) and choose their demand for the foreign bond \(q_t^f\). Along
a trendless path, informed speculators switch their zero position in the foreign bond from to $+c/−c$ if they observe a large enough positive/negative shock. This switching time marks the beginning of an uptrend/downtrend, along which informed speculators find it optimal to choose a long/short position until the probability of a liquidity crunch $\sigma_{t+1}$ becomes large enough. At this switching time, informed speculators close their long/short positions and open the reverse short/long positions. This switch forecasts that the uptrend/downtrend in $S_t$ will reverse soon.

If one had data that included exclusively informed speculators’ positions, then one could forecast the incoming trend reversals directly. Unfortunately, publicly available speculator-position data is scarce and noisy, as it is typically a mixture of informed and uninformed market participants’ positions. So higher/lower speculators’ net positions in the foreign currency are not always associated with a Dollar depreciation/appreciation in the near future. Here is where our theoretical model is handy. The equilibrium of the model implies that if speculator-position data contains a mixture of informed and uninformed speculators, the econometrician can backup useful information about an imminent exchange rate trend reversal by estimating dates when informed speculators switch their positions along paths that satisfy the properties of a PCO equilibrium characterized in Theorem 6 and Corollary 9. Concretely, suppose that net speculators’ position data is a mixture of informed speculators $q_t^I$, uninformed momentum-driven speculators $q_t^m$, and noise $\varepsilon_t$:

$$ Y_t = q_t^I + q_t^m + \varepsilon_t, $$  

(38)

where $\varepsilon_t$ is assumed to be i.i.d. normal with mean zero and variance $\sigma^2_\varepsilon$. Then the PCO equilibria imply that (38) can be expressed as:

$$ Y_t = c(\kappa_t) + \theta \Delta S_t^* + \varepsilon_t, $$  

(39)

where $\Delta S_t^* = S_t^* - S_t^{*-1}$ is the demand of momentum speculators, and the discrete component $c(\kappa_t)$ is the demand of informed speculators. Depending on the state $\kappa_t$, the mean $c(\kappa_t)$ of the net demand of informed speculators may
take a positive (net long-position), negative (net short position) or zero value:

\[
c(\kappa_t) = \begin{cases} 
  c(1) = c, & \text{if } \kappa_t = 1 \ (E^I_t [\Delta S^*_t + 1] > 0) \\
  c(0) = 0, & \text{if } \kappa_t = 0 \ (E^I_t [\Delta S^*_t + 1] = 0) \\
  c(-1) = -c, & \text{if } \kappa_t = -1 \ (E^I_t [\Delta S^*_t + 1] < 0)
\end{cases}
\]

(40)

The state variable \( \kappa_t \) is assumed to follow a first-order Markov process with transition matrix \( \Pi = [p_{i,j}]_{i,j=-1,0,1} \), where \( p_{i,j} = \Pr(\kappa_t = j | \kappa_{t-1} = i) \) is the conditional transition probability that the state is \( j \) at time \( t \) given that it was \( i \) at time \( t - 1 \). The empirical model specified in (39) and (40) is a particular example of the MSM. Our forecasting strategy consists of two steps. First, we estimate the MSM in (39) and find the most likely hidden state \( \hat{\kappa}_t \), which allows us to uncover the sign of the informed speculators’s conditional expectation about exchange rate changes \( E^I_t [\Delta S^*_t + 1] \). Second, using \( \hat{\kappa}_t \) and the properties of the PCO equilibria, we generate out-of-sample directional forecasts of exchange rate changes over several horizons.

**Data.** We use speculator position data from the COT report of the Commodity Futures Trading Commission (CFTC). The CFTC requires all large traders to identify themselves as either non-commercial or commercial. Non-commercial and commercial traders may be associated with our model’s speculators and fundamental traders, respectively. We will proxy our model’s speculators’ positions in the foreign bond \( Y_t \) with the foreign currency futures net contracts held by non-commercial traders. Typically, on each Tuesday the CFTC gathers position data and on the Friday of the same week, it publishes the COT report which includes the non-commercial and commercial position data. We consider five major currencies vis-a-vis the US dollar: the Australian dollar (AUD), the Canadian Dollar (CAD), the Euro (EUR), the Japanese yen (JPY), and the British pound (GBP). We use weekly spot exchange rates at the end of trading on Friday, released by the Federal Reserve Board. Our sample begins in September 1992, which is when the CFTC started to release the COT report on a weekly basis, except for the Euro, for which the data begins in January 1999. Our sample ends on the last week of 2017.
For each of the five currencies, we estimate the MSM specified in (39) using the EM algorithm proposed by Hamilton (1990) with a rolling sample \( \{Y_s, \Delta_k S^*_s\}_{s=r-R+1}^{s=r+T} \), where \( r \) is the index of the rolling sample, \( R \) is the window size, and \( T \) is size of the full sample. For each \( r \in \{R, R+1, \ldots, T\} \), we use the sample \( \{Y_s, \Delta_k S^*_s\}_{s=r-R+1}^{s=r+T} \) to estimate the unknown parameters in the model (39), and then compute the estimator of the unconditional probability \( \xi_{j,s} = \Pr(\kappa_s = j) \) for \( j \in \{1, 0, -1\} \) and \( s \in \{r-R+1, \ldots, r\} \). Let \( \hat{\xi}_{j,s} \) denote the estimator of \( \xi_{j,s} \), i.e., the so-called filtered probability. We consider exchange rate changes \( \Delta_k S^*_t \) with a lag \( k = 4 \) weeks and three rolling window sizes: \( R = 70, 90, \) and \( 110 \) weeks respectively.

To ensure that a global maximum is attained, we consider 150 different initial values of parameters for each estimation.

For each time \( r \), we use the filtered probabilities \( (\hat{\xi}_{-1,r}, \hat{\xi}_0,r, \hat{\xi}_{1,r}) \) to determine the most likely state of the informed speculators’ positions at time \( r \) \( (r = R, R+1, \ldots, T) \) as follows:

\[
\hat{\kappa}_r = \begin{cases} 
1, & \text{if } \hat{\xi}_{1,r} > \max\left\{\hat{\xi}_{0,r}, \hat{\xi}_{-1,r}\right\} \\
0, & \text{if } \hat{\xi}_{0,r} > \max\left\{\hat{\xi}_{1,r}, \hat{\xi}_{-1,r}\right\} \\
-1, & \text{if } \hat{\xi}_{-1,r} > \max\left\{\hat{\xi}_{1,r}, \hat{\xi}_{0,r}\right\}
\end{cases}
\]

If the most likely state is \( \hat{\kappa}_r = 1 \) (resp. \(-1\)), our estimate is that informed speculators take net long positions (resp. net short portions) at time \( r \). Meanwhile, if \( \hat{\kappa}_r = 0 \), our estimate is that they take a zero position.\(^{16}\)

\(^{13}\) The main advantage of the EM algorithm over direct numerical optimization methods is its robustness with respect to the multiple local maxima problem.

\(^{14}\) We have used \( \Delta_k S_t \equiv S_t - S_{t-k} \), with \( k = 4 \) rather than \( S_t - S_{t-1} \) in estimation and forecasting to better capture momentum effects.

\(^{15}\) The estimation results are quite robust with respect to the lag \( k \) and the size of the rolling window \( R \). In Tables F.2-F.6 (in the Extended Appendix), we report the correlation of the MSM estimation results with \( k \in \{4, 8\} \) and \( R \in \{70, 80, 90, 100, 110, 120, 130\} \).

\(^{16}\) Figure F.2 in Appendix depicts the series of the estimated intercepts \( (c_{(-1)}, c_{(0)}, c_{(1)}) \) with rolling samples. For all currencies, the estimators of \( c_{(1)} \) are mostly positive; the estimators of \( c_{(-1)} \) are mostly negative; while the estimators of \( c_{(0)} \) fluctuate around zero. Figure F.3 in Appendix shows the evolution of the estimated filtered probabilities \( (\hat{\xi}_{-1,r}, \hat{\xi}_0,r, \hat{\xi}_{1,r}) \). Figure F.4 in Appendix depicts the series of the estimators of \( \theta \) and the lower bounds of the 0.95 confidence intervals in (39) for each currency.
3.1 Directional Forecasts

Given the sequence of most likely states \( \{\hat{\kappa}_r\}_{r=R}^T \), we backup the information of informed speculators as follows. First, by Theorem 6, times when an uptrend is likely to end soon \( \hat{\tau}_{(-)}^* \), correspond to dates \( r \) when: (i) the most likely state is \( \hat{\kappa}_r = -1 \); (ii) \( S_t^* \) has been in an uptrend, i.e., \( \Delta_m S_t^* = S_t^* - S_{t-m}^* > 0 \) over the past \( m = 8 \) weeks; and (iii) there has been positive momentum, i.e., the estimated coefficient of the momentum traders’ demand \( \hat{\theta}_r \) is positive and statistically significant: \( \hat{\theta}_r / \hat{\sigma}_r > z_{0.05} \), where \( \hat{\sigma}_r \) is the estimated standard deviation of \( \hat{\theta}_r \), and \( z_{0.05} \) is the 95% quantile of the standard normal distribution. Second, by Corollary 9, times when a downtrend is likely to end soon \( \hat{\tau}_{(+)}^* \), correspond to dates \( r \) when: (i) the most likely state is \( \hat{\kappa}_r = 1 \); (ii) \( S_t^* \) has been in a downtrend \( \Delta_m S_t^* = S_t^* - S_{t-m}^* < 0 \) over the past \( m = 8 \) weeks; and (iii) there has been negative momentum: \( \hat{\theta}_r / \hat{\sigma}_r > z_{0.05} \).

Third, by Theorem 6 (resp. Corollary 9), times when a trendless path is likely to end soon and give way to an uptrend \( \hat{\tau}_{(0+)}^* \) (resp. downtrend \( \hat{\tau}_{(0-)}^* \), correspond to dates \( r \) when we detect the most likely state shifts to \( \hat{\kappa}_r = 1 \) (resp. \( \hat{\kappa}_r = -1 \)) after having been zero over the past 4 weeks.

As we can see, Theorem 6 and Corollary 9 imply contrarian forecasts. Thus, whenever we detect that \( r = \hat{\tau}_{(+)}^* \) or \( r = \hat{\tau}_{(0+)}^* \), we predict a trend reversal within a short period and forecast that \( S_{r+h}^* - S_r^* > 0 \) over the next \( h \) weeks, i.e., the Dollar will depreciate (appreciate). Analogously, whenever we detect \( r = \hat{\tau}_{(-)}^* \) or \( r = \hat{\tau}_{(0-)}^* \), we predict \( S_{r+h}^* - S_r^* < 0 \) over the next \( h \) weeks. That is, our directional forecasting strategy is

\[
D_{r,h} = \begin{cases} 
-1, & \text{if } r = \hat{\tau}_{(-)}^* \text{ or } \hat{\tau}_{(0-)}^* \\
1, & \text{if } r = \hat{\tau}_{(+)}^* \text{ or } \hat{\tau}_{(0+)}^* \\
0, & \text{otherwise}
\end{cases}
\]

where \( D_{r,h} = 1 \) (resp. \(-1\)) means that our directional forecast over an \( h \)-period horizon is a decline in \( S^* \) (resp. an increase in \( S^* \)), i.e., a Dollar appreciation (resp. depreciation). Meanwhile, if \( D_{r,h} = 0 \), we predict no change.

During every week, we generate out-of-sample exchange rate directional
forecasts for five horizons: \( h \in \{4, 13, 26, 39, 52\} \) weeks which correspond to 1, 3, 6, 9 and 12 months forecasting horizons respectively. For each currency \( i \), our directional forecasts start in the week after the first estimation of the MSM, and end \( h \) weeks prior to the end of our sample. That is, they start in week \( R \) and end in week \( T - h \). Our sample starts on 10/2/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017. Thus, our directional forecasts start on 01/20/1995 for the Pound; 09/01/1994 for the Canadian Dollar; 04/15/1994 for the Yen and the Australian Dollar; and 05/04/2001 for the Euro.\(^{17}\)

Table 1: Success Ratio of the Directional Forecasts

<table>
<thead>
<tr>
<th>Currency</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>0.582</td>
<td>0.577</td>
<td>0.617</td>
<td>0.565</td>
<td>0.603</td>
</tr>
<tr>
<td></td>
<td>(201)</td>
<td>(201)</td>
<td>(201)</td>
<td>(200)</td>
<td>(199)</td>
</tr>
<tr>
<td>CAD</td>
<td>0.577</td>
<td>0.640</td>
<td>0.720</td>
<td>0.690</td>
<td>0.647</td>
</tr>
<tr>
<td></td>
<td>(175)</td>
<td>(175)</td>
<td>(175)</td>
<td>(174)</td>
<td>(173)</td>
</tr>
<tr>
<td>EUR</td>
<td>0.586</td>
<td>0.559</td>
<td>0.699</td>
<td>0.669</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td>(145)</td>
<td>(143)</td>
<td>(143)</td>
<td>(142)</td>
<td>(141)</td>
</tr>
<tr>
<td>JPY</td>
<td>0.553</td>
<td>0.551</td>
<td>0.614</td>
<td>0.689</td>
<td>0.689</td>
</tr>
<tr>
<td></td>
<td>(208)</td>
<td>(205)</td>
<td>(197)</td>
<td>(190)</td>
<td>(180)</td>
</tr>
<tr>
<td>GBP</td>
<td>0.563</td>
<td>0.591</td>
<td>0.534</td>
<td>0.529</td>
<td>0.554</td>
</tr>
<tr>
<td></td>
<td>(208)</td>
<td>(208)</td>
<td>(208)</td>
<td>(208)</td>
<td>(204)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.

In all tables and figures in the main text and Appendix, we set the rolling window size \( R = 70 \) weeks for the Pound; \( R = 90 \) weeks for the Euro and Canadian Dollar; and \( R = 110 \) weeks for the Yen and the Australian Dollar.\(^{18}\)

\(^{17}\)The dates of the last forecasts are 12/08/2017 for \( h = 4 \), 10/06/2017 for \( h = 13 \), 07/06/2017 for \( h = 26 \), 04/07/2017 for \( h = 39 \), and 01/06/2017 for \( h = 52 \).

\(^{18}\)See Tables F.7 - F.13 in the Extended Appendix for the directional forecasting results.
Table 1 exhibits the forecast success ratios at the 1m, 3m, 6m, 9m, and 12m horizons. The forecast success ratio is the number of successful forecasts divided by the total number of forecasts. As we can see, the aggregate forecast success ratio is 61% for all currencies over the period October 1992-December 2016. Interestingly, the forecast accuracy is greater at the 6m to 12m horizons than at the 1m to 3m months horizons. If we confine our attention to 6m, 9m and 12m forecasting horizons, the aggregate success ratio is 63.1%. When taken individually, we can see that the success ratios are greater than 52.9% in all cases and that in several cases the success ratio is larger than 60%.

The high average success ratios observed in Table 1 are not dominated by either appreciation or depreciation forecasts. Table C.2 in Appendix reports the success ratios of the appreciation and depreciation forecasts, separately. Moreover, the high forecast success ratios are not limited to specific periods. In most country-horizon pairs, the performance of the directional forecast is stable over the sample period.\textsuperscript{19}

### 3.2 Model Forecasts and Monetary Shocks

In this subsection, we discuss the link between our directional forecasts and major monetary shocks in Japan and the Eurozone. Figure 4 depicts the resulting 9-month ($h = 39$) ahead forecasts using strategy $D_{r,h}$ in (41) for the Yen and the Euro. The green balls depict Yen (or Euro) appreciation forecasts and the red balls depict Yen (or Euro) depreciation forecasts. The full balls are the forecasts that turned out to be successful, while the empty balls are unsuccessful forecasts.\textsuperscript{20}

As we can see in Figure 4, several of the signals generated by our forecasts with window sizes $R = 70, 80, 90, 100, 110, 120, 130$ for all five countries.\textsuperscript{19}Figures F.5 through F.9 (in the Extended Appendix) exhibit this stability by plotting the evolution of the cumulative forecast success ratios for each forecasting horizon. Approximately after 400 weeks, these ratios converge to a stable level, above 50% in most country-horizon pairs. Initially, however, these ratios fluctuate quite a bit because the sample size is small. For example, in Figure F.8 of the Extended Appendix, the 9m ahead forecasts for the Japanese Yen initially fluctuate between 60% and 90% before converging to nearly 70%.

\textsuperscript{20}The results for the other currencies are plotted in Figure C.1 in the Appendix.
Note: The figure illustrates the performance of 9-month ahead directional forecasts. Green (red) dots depict the time when an appreciation (depreciation) forecast has been made. If a forecast turned out to correctly predict the direction of movements of the currency, then the dot is filled. If the forecast is wrong, the dot is empty.
casting strategy may be associated with major monetary shocks. Our model’s informed speculators seem to have been able to interpret the effects of crisis events and of monetary policies, often foreseeing them and in several occasions signaling when the associated exchange rate moves may end. For instance, the launch of QE by the Bank of Japan in March 2001 was preceded by our strategy’s October 2000 bearish signal in the Yen. Then, in October 2001, while the first QE was still underway, our strategy correctly signaled the end of the Yen’s downward move induced by QE. More recently, in June 2012 and in mid-2014, our strategy forecasted the incoming Yen’s downtrend, prior to the implementation of Abenomics in December 2012 and prior to the acceleration of quantitative and qualitative easing (QQE) in October 2014, respectively. In April 2015, while Abenomics and QQE continued to be active, our strategy forecasted the incoming Yen’s uptrend, which started about two months later, in June 2015.

Our forecasting strategy has tracked the tumultuous period for the Eurozone between 2008-2016 fairly well. First, after exhibiting only bullish signals for three years, in December 2008 our forecasting strategy started to exhibit a series of bearish signals. Tellingly, the Lehman crisis of September 2008 was preceded by these successful bearish signals by two months. Second, following the announcement of quantitative easing by the US Fed, the model issued a bull signal in August 2009. Third, although the Euro appreciated for a few months, the forecast of an appreciation over a 9-month horizon ultimately proved to be wrong. Fourth, the Greek crisis erupted in December 2009, an event around which our model does not exhibit any bearish signal. Fifth, in November 2010, the US Fed announced the second phase of quantitative easing; we observe successful bullish signals in October 2010 before the program’s announcement and slightly after. Sixth, in December 2011, to stem the Eurozone banking crisis, the ECB announced the Long Term Refinancing Operations (LTRO) program; it is similarly preceded and followed by successful bearish signals in November 2011. Seventh, the deepening of the crisis compelled Mario Draghi, president of the ECB, to make his “whatever it takes” statement in the summer of 2012, as well as the ECB’s announcement of the
Outright Monetary Transactions (OMT) program in September of the same year. Our model exhibits a series of wrong bearish signals in September 2012. Perhaps this is because bad news for the Euro were dominated by the QE3 of the US Fed. Shortly thereafter, in January 2013 our model exhibits a series of successful bullish signals. Eighth, in a speech at Jackson Hole in August 2014 Draghi promised to initiate a QE program, a big surprise to onlookers. The ECB formally announced its QE decision in January 2015, beginning buying in March. Our strategy issued a bearish signal two months before Draghi’s speech, in June 2014. Interestingly, most of the Euro’s downward 20% move occurred between the time of our signal in June 2014 and January 2015 ECB’s announcement of QE (15%). Following the launch of QE in March 2015, our model exhibits a bearish signal. Starting in early 2016, while QE continued to be active, our model exhibits a series of bullish signals, which correctly signal the end of the Euro’s downward move.

### 3.3 A New Weighted Directional Test

The high forecasting success ratios in Table 1 illustrate the usefulness of the COT data and our forecasting method in generating directional exchange rate predictions. Two issues remain, however. First, wouldn’t flipping a coin result in similar or even better success ratios? To answer this question, we conduct the standard binomial test to evaluates whether our directional forecasts outperform the random walk forecasts. The second and probably more important issue is whether our directional forecasts succeed in predicting big moves in exchange rates. This question captures the spirit of George Soros’s observation: It’s not whether you’re right or wrong, but how much money you make when you’re right and how much you lose when you’re wrong. To answer this question, we propose a novel directional test that weights each directional forecast by the realized exchange rate change. Our test has the same spirit as the trading rule considered by Elliott and Ito (1999) to evaluate the predictability content of survey forecasts. In order to motivate such

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21Draghi’s speech was made in a context where the 3-year loans made under the LTRO program were coming due, and there were fears of deflation.
Table 2: Weighted Directional Forecasts Test

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon (h)</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>Panel A: Newey-West</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.063*</td>
<td>0.177**</td>
<td>0.199</td>
<td>0.335*</td>
<td>0.470**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.596)</td>
<td>(1.691)</td>
<td>(1.263)</td>
<td>(1.502)</td>
<td>(1.714)</td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.047*</td>
<td>0.161***</td>
<td>0.253***</td>
<td>0.304***</td>
<td>0.258*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.538)</td>
<td>(2.519)</td>
<td>(2.334)</td>
<td>(2.450)</td>
<td>(1.753)</td>
</tr>
<tr>
<td>EUR</td>
<td></td>
<td>0.042</td>
<td>0.159*</td>
<td>0.521***</td>
<td>0.746***</td>
<td>0.629***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.926)</td>
<td>(1.289)</td>
<td>(2.729)</td>
<td>(2.897)</td>
<td>(2.346)</td>
</tr>
<tr>
<td>JPY</td>
<td></td>
<td>0.029</td>
<td>0.055</td>
<td>0.439***</td>
<td>0.648***</td>
<td>0.659***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.581)</td>
<td>(0.602)</td>
<td>(2.749)</td>
<td>(3.681)</td>
<td>(2.958)</td>
</tr>
<tr>
<td>GBP</td>
<td>Panel B: Andrews</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.061*</td>
<td>0.144*</td>
<td>0.221*</td>
<td>0.327*</td>
<td>0.383**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.568)</td>
<td>(1.638)</td>
<td>(1.319)</td>
<td>(1.592)</td>
<td>(1.885)</td>
</tr>
</tbody>
</table>

Note: Panel A reports the test results using the Newey-West LRV estimator to control for auto-correlation. Panel B reports the test results using the Andrews LRV estimator to control for auto-correlation. t-values are in parentheses. We consider an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The information on our directional forecasts is described in the note to Table 1.

test, consider the following investment rule. At time \( r \), if the \( h \)-period-ahead directional forecast is an increase in \( S \), buy 1 unit of the foreign bond at time \( r \) and sell it at time \( r + h \); if the \( h \)-period-ahead directional forecast is a fall in \( S \), sell short 1 unit of the foreign bond at time \( r \) and buy it back at time \( r + h \); if the directional forecast is zero, then take no position at time \( r \). The
resulting profit is given by the following weighted sum of directional forecasts.

\[ T_{a,n} = \frac{1}{T_1} \sum_{r=R}^{T-h} D_{r,h}(S_{r+h}^* - S_r^*), \text{ where } T_1 = T - R - h + 1. \] (42)

Under the random walk assumption, the exchange rate is not predictable since the process \( \{D_{r,h}(S_{r+h}^* - S_r^*)\} \) is a martingale difference array. Therefore, the expected profit of the above investment strategy must be zero. Thus, the null hypothesis is that the weighted sum of our directional forecasts (42) is zero:

\[ H_0 : E \left[ D_{r,h}(S_{r+h}^* - S_r^*) \right] = 0. \] (43)

That is, under the null, our directional forecasts are uncorrelated with future realized exchange rate changes. The alternative hypothesis is that the weighted sum of our directional forecasts is positive. That is, our forecasts generate expected positive profits:

\[ H_1 : E \left[ D_{r,h}(S_{r+h}^* - S_r^*) \right] > 0. \] (44)

Let \( V_{T_{a,n}} \) denote the consistent estimator of the asymptotic variance of \( T_{a,n} \). Then by Slutsky’s theorem and the martingale central limit theorem, we deduce that

\[ \sqrt{T_1} V_{T_{a,n}}^{-1/2} T_{a,n} \rightarrow_d N(0,1) \] (45)

under the null hypothesis (43). For the empirical implementation, we construct \( V_{T_{a,n}} \) using two LRV estimators that control for auto-correlation in \( \{D_{r,h}(S_{r+h}^* - S_r^*)\}_{r=R}^{T-h} \): the Newey-West LRV estimator and the Andrews LRV estimator.

Table 2 presents the values of the \( T_{a,n} \) statistic, and its t-values (test statistics), for the five currencies and the 5 horizons we consider. The null is rejected if the test statistic \( T_{a,n} \) is significantly larger than zero. For the one-sided test, a t-value greater than 1.282 implies a 10% significance level. As we can see in panel A of Table 2, using the Newey-West LRV estimator, 21 out of 25 currency-horizon pairs are statistically significant. At the 9-month and 12-
month horizons, the weighted directional test rejects the random-walk null in favour of our model across all currencies. The same holds true at the 6-month horizon (except for the Australian Dollar) and at the 3-month horizon (except for the Japanese Yen). At the 1-month horizon, the null is rejected for the Australian Dollar and the British Pound.

Comparing Tables 1 and 2 we can see that the high forecast success ratios translate into a rejection of the null of our weighted directional test. However, they are not the same. For example, even though the success ratio of the British Pound at the 9-month horizon is slightly above 50 percent, the null of the weighted directional test is rejected at the 10 percent level after controlling autocorrelation. In contrast, while the success ratio of the Australian Dollar at the 6-month horizon is above 60 percent, the null of the weighted directional test is not rejected.\textsuperscript{22}

The high success ratios of our directional forecasts are reflected in Table C.1, which reports the means of one-year ahead changes in the exchange rates. For all currencies, the mean exchange rate change conditional on an appreciation (resp. depreciation) forecast is greater (resp. smaller) than the unconditional mean.

4 Related Literature

\textit{Exchange rate determination models.} Numerous empirical studies have found that in the short-run (less than a year) there is a disconnect between standard observable fundamentals and exchange rates in terms of both in-sample fit and out-of-sample predictability. Meanwhile, over long horizons (3-5 years) there is some evidence of a connection. \textit{Engel and West (2005)}, hereafter EW, express the exchange rate as the current observed fundamentals plus the present

\textsuperscript{22}Even after controlling for autocorrelation, over the 6m to 12m horizons, there is very strong evidence of exchange rate predictability in all currencies. As we can see in panel A of Table 2, using the Newey-West LRV estimator, the null is rejected in 14 out of 15 country-horizon pairs at the 6m, 9m and 12m horizons. Similarly, Panel B shows that using the Andrews LRV estimator to control for autocorrelation, the null is rejected in 13 out of 15 country-horizon pairs. At the 1m and 3m horizons there is weak evidence of predictability: the random walk null is rejected in only 6 out of 12 currency-horizon pairs.
discounted value of expected future observed fundamentals \( \{ f_t \} \), as well as current and expected future unobserved fundamentals and non-fundamental factors \( \{ U_t \} \): \( s_t = f_t + E_t[\sum_{j=1}^{\infty} \beta^j \cdot \Delta f_{t+j}] + U_t \). The disconnect reflects the fact that the deviation of the exchange rate from observed fundamentals \( (s_t - f_t) \) can be large and persistent. EW show that if fundamentals have a unit root and the discount factor \( \beta \) is close to one, then exchange rates exhibit near-random walk behavior and they help predict future fundamentals. The implication is that fundamentals are not useful for predicting future exchange rate movements out of sample. Engel and West (2005), and Mark (2009) investigate the contribution of expectations of future fundamentals. As in these papers, time-\( t \) news about the future, i.e., shocks to \( Z_t \) in (10), are reflected in the time-\( t \) exchange rate \( S_t \). The new element here is that a shock to \( Z_t \), in addition to affecting \( S_t \), may trigger a bubbly path. Thus, the equilibrium exchange rate may exhibit short-run momentum.

By investigating the information content of market participant’s positions, our paper is linked to the literature on order flow. In Evans and Lyons (2002, 2012), is the difference between buyer- and seller-initiated trades across a set of traders that posses dispersed micro-level news about both \( \{ U_t \} \) and \( \{ f_t \} \). This order flow information is then impounded into exchange rates by dealers. Evans and Lyons (2002, 2012) show that the daily order flow in the interbank foreign exchange market explains a large share of contemporaneous daily exchange rate movements. Evans and Lyons (2005) and Evans and Lyons (2012) show that end-user order flow has significant forecasting power up to a 1-month horizon over exchange rates and fundamentals. Bacchetta and Van Wincoop (2006) consider an infinite horizon setup where order flow, as measured by the market orders’s net volume, aggregates heterogeneous information about future fundamentals and hedging demands. The main difference with Evans and Lyons is that while they focus on order flow, the key forecasting variable in our model is the net position of informed speculators. In our setup, there is no order flow among informed speculators as they all have the same information. Furthermore, Evans and Lyons use order flow data to make up-to-a-month in a linear fashion. We, instead, make forecasts of incoming trend reversals
by estimating switching times when the subset of informed speculators close their long/short positions along an uptrend/downtrend. Lastly, while the order flow data among informed traders used by Evans and Lyons is proprietary, our strategy applies to the less pristine and noisy data in the COT report, which contains a mixture of informed and uninformed traders, but is public and readily available.

**Non-linear forecasts.** In contrast to much of the literature, our forecasts are non-linear, which naturally follows from the implications of our theoretical model. Our forecasts are based on our estimates of the informed speculators’ switching times. Our estimation method relies on the maximum likelihood estimation of the Markov switching model (hereafter MSM) proposed by Hamilton (1989, 1990). Engel and Hamilton (1990) apply the MSM to exchange rate data to capture the long swings exhibited by exchange rates from the mid 1970s to the end of the 1980s. In this paper, we apply an MSM to the speculators’ aggregate net position data, instead of the exchange rate itself. Moreover, rather than a two-state MSM, our model implies three regimes corresponding to the sign of the informed speculators’ positions: long, short, and nil. Jordà and Taylor (2012) construct a carry trade strategy using a threshold error correction model, where the adjustment of exchange rates is more likely to be precipitated when fundamentals exhibit deviations exceeding median in-sample levels.

**Out-of-sample Forecast Accuracy Tests.** Since Meese and Rogoff (1983), it has been difficult for fundamentals-augmented point forecasts to beat the random walk, with few exceptions. The null hypothesis of the CW-test has been rejected by few authors such as Gourinchas and Rey (2007), using net foreign assets, and Molodtsova and Papell (2009) using Taylor-rule fundamentals.24

23Engel (1994), however, finds that this model does not clearly outperform the random walk in out-of-sample exchange rate forecasts.

But according to recent surveys by Rossi (2013) and Cheung et al. (2018), it is not yet clear whether these results are robust with respect to the sample period, the choice of the currency and forecast evaluation methods. Rossi (2013) reports that at both the 1-quarter and the 4-year forecasting horizons, the DMW-test finds no evidence of forecastability for the monetary model, the PPP model, and the Taylor rule based model across all currencies she considers, while the CW-test finds relatively robust success of the Taylor-rule based model. Furthermore, Cheung et al. (2018) find that still, no existing model can consistently outperform the random walk, especially at less than 1 year horizons. This paper focuses on directional forecasts, not point forecasts. We conduct the familiar binomial test based on the forecast success ratio, as well as a new directional test that weights each directional forecast by the subsequent exchange rate change, and so gives more weight to the directional forecasts associated with bigger exchange rate moves. Across 5 currency pairs, over the 1-month to 12-month horizons, both tests show that our out-of-sample directional forecasts outperform the random walk in a majority of cases.

Models of Bubbles. Our model is related to positive-feedback bubble models in the spirit of De Long et al. (1990). Abreu and Brunnermeier (2003) consider a finite-time economy with financially constrained momentum traders and rational-but-not-perfectly-informed traders. Each informed trader only becomes aware of the bubble at a random time and does not know when other informed traders become aware of the bubble. Because there is an upper bound on the momentum traders demand, the bubble may burst if a critical mass of informed traders exit. In equilibrium, even after all informed traders are aware of the bubble, they ride it for a while, and in some cases all the way to terminal time. Tirole (1982)’s backwards induction argument breaks down because, at least in the initial phase, the expected gain of waiting and getting the higher bubbly return dominates the potential loss. Thus, an informed trader has no incentives to exit. This mechanism is not operative in our model can predict the exchange rate better than the driftless random walk at both long and short horizons. Net foreign assets is the deviation from trend of a weighted combination of gross assets, gross liabilities, gross exports and gross imports.

She does not carry out the DMW test for the net foreign assets model.
because (i) there is no asymmetric information across informed traders and (ii) the momentum traders have a large enough demand capacity so that the exit of all informed traders does not necessarily lead to a price fall. In our setup, backwards induction breaks down too because of the PCO mechanism: informed traders ride the bubble until switching time $\tau^*$ because they know that the equilibrium $S_t$ will necessarily increase between $\tau^* - 1$ and $\tau^*$, when all informed traders close their positions, i.e., the asset is ‘distributed’ from informed to uninformed traders. This equilibrium PCO dynamics is the key identification restriction that we use to estimate likely trend reversals. In other words, Abreu and Brunnermeier (2003) focus on the preemption game within the group of rational traders, while we focus on the distribution between two different groups of traders.

Barberis et al. (2018) consider two types of bounded-rational traders: fundamental traders and extrapolator traders. The latter put weight both on price momentum and on fundamentals. The key is that this weight wavers randomly over time and across extrapolators. They show that a bubble arises if there is a sequence of positive shocks to fundamentals that first increase and then decrease. They also show that trading volume may increase sharply along the bubble. This is because when fundamental traders hit their short-sale constraint, all trade is among extrapolators, and their wavering may induce intense trading volume, especially when some extrapolators hit their short-sale constraint. In our model, total trading volume does not play the main role. The key variable in our setup is the net position of rational-informed speculators, not total trading volume. As in Barberis et al. (2018), volume is increasing along a bubble, but it is driven by trade between momentum and fundamental traders. Notice that a key property of our PCO equilibrium, that allows us to derive a simple econometric strategy to recover useful information from net-speculators’ positions is that rational-informed speculators ride the bubble until the probability of its end is high, while in Barberis et al. (2018) fundamental traders exit early.\footnote{Our rational-informed speculators are not present in their model. The momentum traders in our setup may be associated with their extrapolators by setting to zero the}
our econometric strategy—is that in our setup a single transitory shock can generate a bubble, while in Barberis et al. (2018) a bubble is conditioned on a sequence of (increasing and then decreasing) positive shocks. One may say that their model is designed to account for large bubbles that occur with a decennary frequency, such as the dot-com bubble, while our model is designed to detect likely imminent trend reversals in garden variety up- and down-trends with frequencies of a few quarters.

**Momentum.** In analyzing price trends and their reversals, this paper is related to the time-series momentum literature, which documents the short-run positive and long-run negative autocorrelation of returns exhibited by individual securities across many asset classes including stocks, commodities, and currencies (e.g., Brunnermeier et al. (2008), Moskowitz et al. (2012), Menkhoff et al. (2012) and Fama and French (1998)). This literature documents in-sample predictability. We contribute to this literature by proposing an econometric strategy to generate out-of-sample forecasts about incoming trend-reversals. To our knowledge, this is an original contribution. Our method does not only capture the reversals associated with the rare crashes observed at decennary frequencies, but also the reversals of garden variety up- and down-trends observed at frequencies of several quarters.

The time-series momentum literature is related to, but is distinct from the cross-sectional momentum literature, which analyzes the over-performance of past winners relative to past losers (e.g., Jegadeesh and Titman (1993)). This paper is not focused on the construction of momentum portfolios. Such a portfolio may be derived from the informed traders’ equilibrium strategy in our model, which entails a trend-following strategy at the onset of a trend and then switching to a contrarian strategy at a critical switching time.

**Predictability in Futures Markets.** There is a large literature on futures’ predictability on hedging pressure dating back to Keynes (1930) which has been generalized since then (e.g., Hirshleifer (1990)). The idea is that if there is a demand for hedging (by ‘producers’), then there has to be a risk premium weight on fundamentals and eliminating wavering. Their and our fundamental traders are analogous.
to induce speculators to supply such insurance in the futures market. This means that if hedgers are net short/long—and by consequence speculators are net long/short, then ceteris paribus there is a tendency for the futures price to increase/fall towards the expected future spot price. Notice that this predictability is distinct from our forecasts of a trend reversal. This hedging pressure theory has been examined in currency futures using COT data by Bessembinder (1992) and Wang (2004) among others.

There are some other papers that analyze COT data. Moskowitz et al. (2012) document time-series momentum across different asset classes and find that in-sample speculators profit on average from this momentum. Hong and Yogo (2012) find that open-interest in futures markets has in-sample predictive power over the excess returns of several assets classes over an one-month horizon. Brunnermeier et al. (2008) find that, in-sample, currency crash risk increases with speculators’ positions in currency futures markets. Wang (2004) finds in-sample a positive correlation between lagged speculator sentiment and future exchange rate changes over horizons from 2 to 8 weeks. This correlation is stronger if one considers the extreme 20% of speculators sentiment. None of these studies performs out-of-sample forecasts.

Our Previous Work. This paper is a much revised and extended version of Kim et al. (2014), hereafter KLT. In KLT, the exchange rate is exogenous and follows a three-state Markov process, and there is a representative investor that observes a noisy signal of the state. By inverting the demand of the investor, KLT derive a MSM that predicts whether speculators will be increasing or decreasing their positions. Out-of-sample forecasts are then formed using this predicted state. In this paper, the exchange rate is endogenous and may follow trends. We consider a richer heterogenous-agent model, where we can ask why rational informed speculators have incentives to ride bubble-like paths and close their positions only when a trend reversal is nigh. The PCO equilibrium we have characterized implies a different estimation model than the one in KLT. The model estimated by KLT uncovers information about the ongoing trend, while this paper forecasts an incoming trend reversal. That is, in contrast to KLT, this paper makes out-of-sample forecasts about future exchange
rate movements in a contrarian manner. Lastly, Tornell and Yuan (2012) find that the peaks and troughs of speculators net positions are generally useful predictors of future exchange rates.

5 Conclusion

Exchange rates tend to exhibit swings of appreciation and depreciation that often last several quarters and sometimes continue for several years. Although these swings can be identified in-sample, they have proven difficult to predict out-of-sample. Our model provides a novel way of thinking about exchange rate trends, and generates a novel exchange rate forecasting strategy that extracts useful information from speculator position data.

Our starting point is the contrarian pattern observed in speculator position data, whereby at the end of an upswing (downswing), the average currency speculator tends to be overoptimistic (overpessimistic). This contrarian pattern has a long tradition in the practitioners’ literature; Lefevre (1923) eloquently described a mechanism that generates it. Our contribution is to formalize such a mechanism in a simple heterogeneous-agent model.

In our model, news about future interest rate differentials are observed only by a set of informed speculators, and are impounded into exchange rates via trending bubbly paths, along which there is a growing gap between the exchange rate and its fundamental value. Informed speculators find it optimal to ride the trend until the gap becomes so large that uninformed momentum-driven speculators may not be able to finance their trades, thus making a trend reversal very likely. At this time, informed speculators switch the sign of their positions in the foreign currency.

Our forecasting method estimates this switching strategy of informed speculators from noisy market-position data, which contains a mixture of informed and uninformed market participants’ positions.

Our out-of-sample directional forecasts attain success ratios ranging from 52.9% to 72% across five major currencies. Both the standard Binomial test and a new weighted directional test reject the random-walk null in favor of
our model across most currency-horizon pairs.

We have shown that our model’s forecast signals are not simply statistical objects; several signals may be associated with major monetary shocks. Our model’s informed speculators seem to have been able to interpret the effects of crisis events and of monetary policies, often foreseeing them and in several occasions signaling when the associated exchange rate moves may end. For instance, the launch of QE by the Bank of Japan in March 2001 was preceded by our strategy’s bearish signal in the Yen. Then, in October 2001, while QE was still underway, our strategy correctly signaled the end of the Yen’s downward move induced by QE. Similar signals were generated by our strategy around the implementation of Abenomics in 2013. In the Euro, our forecasting strategy exhibits bearish signals a few months before the September 2008 Lehman bankruptcy, and before the announcement of the ECB’s QE program in January 2015.

References


Appendix For Online Publication

This online Appendix provides supporting materials, proofs, further theoretical exercises and extra empirical results to complement Kim, Kim, Liao and Tornell (2018). The Binomial test of the directional forecasts and associated empirical results are in Section A. Section B presents the proof of the results stated in Section 2 of the main text. Empirical results which are discussed in Section 3 of the main text but not presented there are included in Section C.

A Binomial Test

Here, we test the significance of our model in forecasting the sign of \( S_{r+h}^* - S_r^* \), i.e., \( D_{r,h} \) defined in (41). The null hypothesis we test is that our directional forecasts \( D_{r,h} \) are uncorrelated with the sign of the realized exchange rate change \( D_{r,h}^* = \text{sgn}(S_{r+h}^* - S_r^*) \):

\[
H_0: \text{Cov}(D_{r,h}^*, D_{t,h}) = 0. \tag{A.46}
\]

The alternative hypothesis could be one sided or two sided:

\[
H_1^{\text{one-sided}}: \text{Cov}(D_{r,h}^*, D_{t,h}) > 0 \quad \text{or} \quad H_1^{\text{two-sided}}: \text{Cov}(D_{r,h}^*, D_{t,h}) \neq 0.
\]

In order to test the null consider the following test statistic:

\[
T_{b,n} = \frac{1}{T_1} \sum_{r=R}^{T-h} D_{r,h}^* D_{r,h} - \frac{1}{T_{1}^{r_1=R}} \sum_{r_1=R}^{T-h} D_{r_1,h}^* \frac{1}{T_1^{r_2=R}} \sum_{r_2=R}^{T-h} D_{r_2,h},
\]

which is the sample covariance of the two random variables: \( D_{r,h}^* \) and \( D_{r,h} \). Let \( V_{T_{b,n}} \) denote the consistent estimator of the asymptotic variance of \( T_{b,n} \). Then we have

\[
\sqrt{n_1} V_{T_{b,n}}^{-\frac{1}{2}} T_{b,n} \rightarrow_d N(0, 1).
\]

The null is rejected if the t-value of the \( T_{b,n} \) statistic is positive and statisti-
Table A.1: Binomial Directional Forecasts Test

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon (h)</th>
<th>Panel A: Newey-West</th>
<th>Panel B: Andrews</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1m</td>
<td>3m</td>
<td>6m</td>
</tr>
<tr>
<td>AUD</td>
<td>0.025**</td>
<td>0.025*</td>
<td>0.039**</td>
</tr>
<tr>
<td></td>
<td>(1.389)</td>
<td>(2.067)</td>
<td>(1.062)</td>
</tr>
<tr>
<td>CAD</td>
<td>0.022**</td>
<td>0.040***</td>
<td>0.062***</td>
</tr>
<tr>
<td></td>
<td>(2.506)</td>
<td>(3.612)</td>
<td>(3.016)</td>
</tr>
<tr>
<td>EUR</td>
<td>0.027*</td>
<td>0.017</td>
<td>0.063***</td>
</tr>
<tr>
<td></td>
<td>(0.795)</td>
<td>(2.858)</td>
<td>(2.182)</td>
</tr>
<tr>
<td>JPY</td>
<td>0.016</td>
<td>0.015</td>
<td>0.035**</td>
</tr>
<tr>
<td></td>
<td>(0.902)</td>
<td>(1.871)</td>
<td>(3.176)</td>
</tr>
<tr>
<td>GBP</td>
<td>0.022*</td>
<td>0.032**</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(1.868)</td>
<td>(0.812)</td>
<td>(0.561)</td>
</tr>
</tbody>
</table>

Note: Panel A reports the test results using the Newey-West LRV estimator to control for auto-correlation. Panel B reports the test results using the Andrews LRV estimator to control for auto-correlation. t-values are in parentheses. We consider an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The information on our directional forecasts is described in the note to Table 1.
than 1.282 implies a 10\% significance level.

As Table A.1 show, overall the binomial test indicates that our directional forecasts have strong predictability. Using the Newey-West LRV estimator, the null is rejected 18 out of 25 cases. The results are exceptionally good for the Canadian Dollar, with the null rejected in all 5 forecasting horizons. They are also good for the Euro and the Australian Dollar, with the null rejected in 4 out of 5 forecasting horizons.

B Proof of the Main Results

B.1 Derivation of (9)

The market clearing condition for \( t \geq T \) (i.e., \( q_{T+j}^I = 0 \)) implies that

\[ E_t^I [S_{t+1}^*] \exp(i_t^f) - S_t^* \exp(i_t) = 0, \]  

(B.47)

which can be rewritten as

\[ S_t^* = \exp(i_t^f - i_t) E_t^I [S_{t+1}^*]. \]  

(B.48)

Iterating forward this condition and using the law of iterated expectations, we have

\[ S_t^* = E_t^I \left[ \exp \left( \sum_{j=0}^{k} (i_{t+j}^f - i_{t+j}) \right) S_{t+k+1}^* \right] \text{ for any } k \geq 0. \]  

(B.49)

By (1),

\[ i_{t+j}^f - i_{t+j}^I = a^{j+1} (i_{t-1} - i_{t-1}^I) + \sum_{s=0}^{j} a^{j-s} \omega_{t+s} \text{ for any } j \geq 0. \]  

(B.50)
Using (B.49) with \( t = T \) and then applying (B.50), we get

\[
S_T^* = E_T^I \left[ \exp \left( (i_{T-1}^f - i_{T-1}) \sum_{j=0}^{k} a^{j+1} - \sum_{j=0}^{k} \sum_{s=0}^{j} a^{j-s} \omega_{T+s} \right) S_{T+k+1}^* \right]
\]

\[
= E_T^I \left[ \exp \left( -\omega_T \sum_{j=0}^{k} a^j \right) S_{T+k+1}^* \right]
\]

\[
= \exp \left( \frac{\omega(1 - a^{k+1})}{a - 1} \right) E_T^I [S_{T+k+1}^*],
\]

(B.51)

where the second equality is by \( i_{T-1}^f = i_{T-1} \) and \( \omega_{T+s} = 0 \) for any \( s > 0 \), the third equality is by the assumption that \( \omega_T = \omega \), which is observable at time \( T \) (and hence is included in \( \eta_T^A \)). Since \( a \in (0, 1) \),

\[
\exp \left( \frac{\omega(1 - a^{k+1})}{a - 1} \right) \to \exp \left( \frac{\omega}{a - 1} \right)
\]

(B.52)

as \( k \to \infty \) almost surely. Assuming that the long-run exchange rate \( S_{\infty}^* \) satisfies \( E_T^I [S_{T+k}^*] \to E_T^I [S_{\infty}^*] \) as \( k \to \infty \) almost surely and that \( E_T^I [S_{\infty}^*] \) is a bounded random variable, it follows from (B.49) and (B.52) that

\[
\exp \left( \frac{\omega(1 - a^{k+1})}{a - 1} \right) E_T^I [S_{T+k+1}^*] \to \exp \left( \frac{\omega}{a - 1} \right) E_T^I [S_{\infty}^*]
\]

(B.53)

as \( k \to \infty \) almost surely. Combining the results in (B.51) and (B.53), we get

\[
S_T^* = \exp \left( \frac{\omega}{a - 1} \right) E_T^I [S_{\infty}^*]
\]

(B.54)

almost surely, which is (9) in the main text.

**B.2 Demand of Fundamental Traders (5)**

The period \( t \) fundamental traders hold the belief that \( S_T^* \) has the same distribution as \( Z_t + u_F \), where \( u_F \sim N(0, \sigma_u^2) \) and \( u_F \) is independent with
respect to $Z_t$ and $S_t$, and $Z_t$ is the fundamental process defined in (4). The period $t$ fundamental traders observe $Z_t$, know the distribution of $u_F$, and solve the following problem

$$\max_{q_t^F \in \mathbb{R}} E_t^F \left[ - \exp(-\gamma^F W_t^F) \right], \text{ subject to } W_t^F = q_t^F (S_t^* - S_t), \quad (B.55)$$

where $\gamma^F > 0$ is a finite constant and $E_t^F [\cdot]$ denotes the conditional expectation operator taken with respect to the fundamental traders’ belief of $S_T^*$ given their knowledge of $Z_t$ and $S_t$. Since the conditional distribution of $S_T^*$ given $Z_t$ and $S_t$ is Gaussian, we can express problem (B.55) as follows

$$\max_{q_t^F \in \mathbb{R}} \left\{ E_t^F [W_t^F] - \gamma^F \frac{1}{2} \text{Var}_t^F [W_t^F] \right\}, \quad (B.56)$$

where $\text{Var}_t^F [\cdot]$ denotes the conditional variance operator taking with respect to the fundamental traders’ belief of $S_T^*$ given their knowledge of $Z_t$ and $S_t$. By definition,

$$E_t^F [W_t^F] = E_t^F [S_T^* - S_t] q_t^F = (Z_t - S_t)q_t^F,$$

$$\text{Var}_t^F [W_t^F] = \text{Var}_t^F [S_T^*] (q_t^F)^2 = \sigma_u^2 (q_t^F)^2,$$

which implies that

$$E_t^F [W_t^F] - \gamma^F \text{Var}_t^F [W_t^F] = (Z_t - S_t)q_t^F - \gamma^F \frac{1}{2} \sigma_u^2 (q_t^F)^2.$$

Therefore, the representative fundamental trader chooses $q_t^F$ through the following maximization problem

$$\max_{q_t^F \in \mathbb{R}} \left\{ (Z_t - S_t)q_t^F - \gamma^F \frac{1}{2} \sigma_u^2 (q_t^F)^2 \right\}.$$

The solution is given by (5) in the main text.
B.3 Demand of Momentum Speculators (6)

Because each momentum trader believes that all other traders are fundamental traders, the representative momentum trader at time $t$ believes that the market prices equal fundamentals

$$S_{t+h} = Z_{t+h} \text{ for } h \in \{-1, 0, 1\},$$  \hspace{1cm} (B.57)

where $S_{t-1}$ and $S_t$ are observed by the momentum trader at time $t$. Second, she believes that the fundamental $Z_t$ follows the process

$$Z_{t+1} = Z_t + u_{t+1},$$  \hspace{1cm} (B.58)

where the shock $u_{t+1}$ follows an autoregressive process

$$u_{t+1} = au_t + v_{t+1}, \text{ with } v_{t+1} \sim N(0, \sigma^2_v),$$  \hspace{1cm} (B.59)

where $a \in (0, 1)$ and $\sigma^2_v > 0$ are finite constants, and $\{v_t\}$ is an i.i.d. process. The representative young momentum speculator maximizes the expected value of an exponential expected utility function defined over her wealth $W_{t+1}^m$ next period

$$\max_{q_t^m} E_{t}^m \left[ - \exp(-\gamma^m W_{t+1}^m) \right],$$  \hspace{1cm} (B.60)

subject to the following borrowing constraint

$$b_t = S_t |q_t^m| \leq \overline{B}_t,$$  \hspace{1cm} (B.61)

where the operator $E_t^m [\cdot]$ is the conditional expectation taken with respect to the momentum trader’s beliefs specified in (B.57), (B.58) and (B.59) and her knowledge on $S_{t-1}$ and $S_t$. The upper bound on debt $\overline{B}_t$ is a random variable which may take two possible values: $\infty$ and 0. Specifically, $\overline{B}_t$ satisfies the following properties: (a) $\overline{B}_t = \infty$ if $(\overline{B}_{t-2}, \overline{B}_{t-1}) = (0, 0)$; (b) $\overline{B}_t = 0$ if $(\overline{B}_{t-2}, \overline{B}_{t-1}) = (\infty, 0)$; (c) $\overline{B}_t = \infty$ if $|S_{t-1} - \mu_z| \leq \vartheta$ and $\overline{B}_{t-1} = \infty$; (d) $\overline{B}_t =$
0 with probability $\bar{\sigma}$ and $\overline{B}_t = \infty$ with probability $1 - \bar{\sigma}$ if $\vartheta < |S_t - \mu_z| \leq \overline{\vartheta}$ and $\overline{B}_{t-1} = \infty$; (e) $B_t = 0$ if $\mu_z + \overline{\vartheta} < S_t$ and $\overline{B}_{t-1} = \infty$. \footnote{Behind the law of motion of $\overline{B}_t$ one may think of a "credit-market game" (as in Schneider and Tornell (2004) and Ranci`ere and Tornell (2016)) with risk neutral financiers that may lend to the momentum speculators at time $t$ if they expect all other financiers will lend at time $t + 1$.}

A liquidity crunch is triggered at time $t$, i.e., $L_t = 1$, if $(\overline{B}_{t-1}, \overline{B}_t) = (\infty, 0)$. By definition, when the liquidity crunch is triggered at time $t$, we must have $B_{t+1} = 0$. Moreover, when $|S_{t-1} - \mu_z| \leq \vartheta$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity crunch is triggered at time $t$ is 0. When $\vartheta < |S_t - \mu_z| \leq \overline{\vartheta}$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity crunch is triggered at time $t$ is $\bar{\sigma}$. Finally, when $\mu_z + \overline{\vartheta} < S_t$ and $\overline{B}_{t-1} = \infty$, the probability that the liquidity crunch is triggered at time $t$ is 1. Therefore, the definition of liquidity crunch is consistent to the one defined in the main text.

By the momentum trader’s beliefs specified in (B.57), (B.58) and (B.59),

$$W_{t+1}^m = (S_{t+1} - S_t)q_t^m = (Z_{t+1} - Z_t)q_t^m = (au_t + v_{t+1})q_t^m.$$  

Since $u_t$ and $q_t^m$ are non-random given $S_{t-1}$ and $S_t$, and $v_{t+1}$ is independent with respect to $S_{t-1}$ and $S_t$, the conditional distribution of $W_{t+1}^m$ given $S_{t-1}$ and $S_t$ is Gaussian with conditional mean and conditional variance

$$E_t^m[W_{t+1}^m] = E_t^m[au_t]q_t^m = a(S_t - S_{t-1})q_t^m \quad \text{and} \quad (B.62)$$

$$\text{Var}_t^m[W_{t+1}^m] = \text{Var}_t^m[v_{t+1}](q_t^m)^2 = \sigma_v^2(q_t^m)^2 \quad (B.63)$$

respectively, where $\text{Var}_t^m[\cdot]$ denotes the conditional variance operator taking with respect to the momentum trader’s belief specified in (B.57), (B.58) and (B.59) and her knowledge on $S_{t-1}$ and $S_t$. Using the properties of the Gaussian
distribution we can express the objective function (B.60) as follows

$$
\max_{q^m_t} \left\{ E^m_t [W^m_t] - \frac{\gamma^m}{2} \text{Var}^m_t [W^m_{t+1}] \right\}
= \max_{q^m_t} \left\{ a(S_t - S_{t-1}) q^m_t - \frac{\gamma^m \sigma^2_v}{2} (q^m_t)^2 \right\},
$$

subject to borrowing constraint (B.61). Therefore, the demand of the representative momentum trader $q^{m*}_t$ satisfies

$$
q^{m*}_t = \begin{cases} 
\theta (S_t - S_{t-1}) & \text{if } |\theta (S_t - S_{t-1})| S_t \leq \overline{B}_t \\
\frac{\overline{B}_t}{S_t} & \text{if } \theta (S_t - S_{t-1}) S_t \geq \overline{B}_t \\
-\frac{\overline{B}_t}{S_t} & \text{if } \theta (S_t - S_{t-1}) S_t \leq -\overline{B}_t 
\end{cases},
$$

where $\theta = a (\gamma^m \sigma^2_v)^{-1} > 0$. We next link $q^{m*}_t$ with the indicator variable of liquidity crunch $L_t$. First, at any period $t$, we have and only have one of the following three cases: (i) $\theta (S_t - S_{t-1}) S_t = 0$; (ii) $\theta (S_t - S_{t-1}) S_t > 0$; (iii) $\theta (S_t - S_{t-1}) S_t < 0$. When $L_{t-1} = 1$ or $L_t = 1$, we have $\overline{B}_t = 0$. Therefore the demand of the momentum traders $q^{m*}_t = 0$ by (B.65) in all three cases (i), (ii) and (iii). Second, if $L_{t-1} = 0$ and $L_t = 0$, we have $\overline{B}_t = \infty$ which together with (B.65) implies that $q^{m*}_t = \theta (S_t - S_{t-1})$. In sum, we have

$$
q^{m*}_t = \begin{cases} 
\theta (S_t - S_{t-1}) , & \text{if } L_{t-1} = 0 \text{ and } L_t = 0 \\
0 , & \text{if } L_{t-1} = 1 \text{ or } L_t = 1 
\end{cases}, \quad \theta = \frac{a}{\gamma^m \sigma^2_v} > 0, \quad (B.66)
$$

which is the demand of momentum traders (6) in the main text.

**B.4 Proof of the results in subsection 2.2**

In the rest of the proof, we write $E^t_t[\cdot] = E^t_t[\cdot]$ for any $t$.

*Proof of Lemma 3.* Suppose that $\theta \leq \phi$ and that a positive bubbly path exists. Since for any $j \in \{t + 1, \ldots, t'\}$, we have $Z_j = \mu_z$, $L^*_j - 1 = 0$ and $L^*_j = 0$, the market clearing condition (12) implies that the equilibrium prices along this
positive bubbly path would be

\[(\theta - \phi)S_j^* = \theta S_{j-1}^* - (\phi \mu_z + C_j^*)\]

for any \(j \in \{t + 1, \ldots, t'\}\), \(B.67\)

which implies that

\[(\theta - \phi)\Delta S_j^* = \theta \Delta S_{j-1}^* - \Delta C_j^*\]

for any \(j \in \{t + 2, \ldots, t'\}\). \(B.68\)

Since \(\theta > 0\), \(\Delta S_j^* > 0\) and \(\Delta S_{j-1}^* > 0\) along the positive bubbly path, \(B.68\) holds under \(\theta \leq \phi\) only when \(\Delta C_j^* > 0\) for any \(j \in \{t + 2, \ldots, t'\}\). This is impossible since \(C_j^* \in \{-c, 0, c\}\) for any \(j \in \{t, \ldots, t'\}\) and \(t' > t + 3\). Hence, a positive bubbly path does not exist if \(\theta \leq \phi\).

The same argument shows that a negative bubbly path does not exist if \(\theta \leq \phi\).

Proof of Lemma 4. Consider the positive bubbly path first. Suppose that \(15\) holds. Since \(C_j^* \in \{-c, 0, c\}\) for any \(j \in \{t, \ldots, t'\}\) and \(t' > t + 3\), there exists at least one \(j^* \in \{t + 2, \ldots, t'\}\) such that \(\Delta C_j^* \leq 0\). Under \(15\) and the conditions of the bubbly path, we can invoke \(B.68\) and \(\Delta C_j^* \leq 0\) to deduce that

\[\Delta S_j^{*}\cdot = \frac{\theta \Delta S_{j-1}^* - \Delta C_j^*}{\theta - \phi} \geq \frac{\theta}{\theta - \phi} \Delta S_{j-1}^*\cdot]. \(B.69\)

By \(15\)

\[\frac{\theta}{\theta - \phi} > 1, \quad (B.70)\]

which together with \(B.69\) implies that

\[\Delta S_j^{*}\cdot > \Delta S_{j-1}^*\cdot]. \(B.71\)
The same argument applies to the negative bubbly path. ■

**Proof of Proposition 5.** Since the liquidity crunch is triggered at time \( t_0 \), (6) implies that the momentum speculator’s demand is zero at time \( t_0 \) and at time \( t_0 + 1 \). Moreover, since at time \( t_0 \) there is no shock to fundamentals \( \zeta_{t_0} = 0 \), market clearing condition (13) implies that

\[
S^*_t = \mu_z + \frac{C^*_t}{\phi} \quad \text{and} \quad S^*_{t_0+1} = \mu_z + \zeta_{t_0+1} + \frac{C^*_{t_0+1}}{\phi}.
\]

(B.72)

Therefore, the equilibrium price change is

\[
\Delta S^*_{t_0+1} = \zeta_{t_0+1} + \frac{C^*_{t_0+1} - C^*_t}{\phi}.
\]

(B.73)

Suppose that \( C^*_t = c \). In this case

\[
\zeta_{t_0+1} + \frac{C^*_{t_0+1} - c}{\phi} \leq \zeta_{t_0+1} \quad \text{for any} \quad C^*_{t_0+1} \in \{-c, 0, c\}.
\]

It then follows from (B.73) that \( E_{t_0} [\Delta S^*_{t_0+1}] \leq 0 \). This non-positive expected price change is inconsistent with \( C^*_t = c \), as it contradicts the demand policy of informed speculators (3). Next, suppose that \( C^*_t = -c \). In this case

\[
\zeta_{t_0+1} + \frac{C^*_{t_0+1} + c}{\phi} \geq \zeta_{t_0+1} \quad \text{for any} \quad C^*_{t_0+1} \in \{-c, 0, c\},
\]

which implies that \( E_{t_0} [\Delta S^*_{t_0+1}] \geq 0 \). This also contradicts the demand rule of the informed speculators. Hence, in an equilibrium, we must have \( C^*_t = 0 \), which together with the first equation in (B.72) implies that

\[
S^*_t = \mu_z.
\]
Because \( S^*_{t_0} - \mu_z = 0 \), the law of motion of the probability of liquidity crunch (7) implies that \( L^*_{t_0+1} = 0 \) with probability 1. This proves the claim of the lemma.

B.5 Proof of the results for the positive bubbly path

Consider any period \( t > 1 \). If liquidity crunch is not triggered in both \( t - 1 \) and \( t \), the market clearing condition is

\[
\phi (Z_t - S^*_t) + \theta (S^*_t - S^*_{t-1}) + C_t [E_t[S^*_{t+1}]] = 0,
\]

and hence the equilibrium price is

\[
S^*_t = \frac{\theta S^*_{t-1} - \phi Z_t - C_t [E_t[S^*_{t+1}]]}{\theta - \phi}.
\]

Therefore, the price change is

\[
\Delta S^*_t = \frac{\phi(S^*_{t-1} - Z_t) - C^*_t}{\theta - \phi} = \frac{\phi(S^*_{t-1} - \mu_z) - C^*_t}{\theta - \phi} = \frac{\phi \zeta_t}{\theta - \phi},
\]

where \( C^*_t = C_t [E_t[S^*_{t+1}]] \). Alternatively, if liquidity crunch is triggered in period \( t - 1 \) and/or period \( t \), momentum speculators do not participate in the market. Thus, the market clearing condition is

\[
\phi (Z_t - S^*_t) + C^*_t = 0,
\]

which implies that

\[
S^*_t = Z_t + \phi^{-1} C^*_t.
\]

The above equations will be used extensively in the proof.

Proof of Theorem 6. The claim in part (i) of the theorem follows from Proposition 5. The results in parts (ii) and (iii) are implied by Lemma 10, Lemma

11
Lemma 10. Under the conditions of Theorem 6, we have

\[ S_1^* = \mu_z + z + \phi^{-1}c, \quad C_1^* = c, \quad L_1^* = 0 \quad \text{and} \quad L_2^* = 0. \]  

(B.79)

Proof of Lemma 10. By Proposition 5, at time 0 we have

\[ S_0^* = \mu_z, \quad C_0^* = 0, \quad L_0^* = 1 \quad \text{and} \quad L_1^* = 0. \]  

(B.80)

At time \( t = 1 \), the informed speculators observe \( Z_1 = \mu_z + z \). Moreover since \( L_0^* = 1 \), by (B.78) the market clearing condition in this period is

\[ S_1^* = \mu_z + z + \phi^{-1}C_1^*. \]  

(B.81)

Because (i) \( z + \phi^{-1}c < (1 - \phi/\theta)^{-1} [z + (1 + \phi/\theta)c\phi^{-1}] \) by (15), (ii) \( |z + \phi^{-1}C_1^*| \leq z + \phi^{-1}c \) for any \( C_1^* \in \{-c, 0, c\} \), and (iii) \( \mu_z + z - \phi^{-1}c > \mu_z \), it follows from (17) that

\[ |S_1^* - \mu_z| < \vartheta, \]  

(B.82)

regardless of the demand of the informed speculators at time 1. Hence the law of motion of the probability of liquidity crunch (7) implies that at time 1, informed speculators know that \( L_2^* = 0 \) with probability 1. Since \( L_1^* = 0 \) and
\( L_2^* = 0 \), we can use (B.76) with \( t = 2 \) and (B.81) to get

\[
\Delta S_2^* = \frac{\phi(S_2^* - \mu_z) - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} = \frac{\phi z + C_2^* - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} > -\frac{\phi \zeta_2}{\theta - \phi} \tag{B.83}
\]

for any \( C_1^*, C_2^* \in \{-c, 0, c\} \), where the last inequality follows from \( \theta > \phi \) and \( z > 2c/\phi \). Since \( E_1[\zeta_2] = 0 \), the inequality in (B.83) implies that \( E_1[\Delta S_2^*] > 0 \), which together with the demand function of the informed speculators (3) and the price equation (B.81), implies that \( C_1^* = c \) and \( S_1^* = \mu_z + z + c/\phi \). This completes the proof of (B.79).

**Lemma 11.** Suppose (15), (16) and (17) hold. Then \( \tau_* \), defined in (21), satisfies: (i) \( \tau_* \geq 2 \); and (ii)

\[
\frac{z}{(1 - \phi/\theta)^{\tau_* - 1}} + \frac{c}{1 - \phi/\theta} > \vartheta. \tag{B.84}
\]

**Proof of Lemma 11.** (i) By (15), \( \phi/\theta \in (0, 1) \) which together with \( z > 0 \) implies that \( (1 - \phi/\theta)^{-t+1} \) is an increasing sequence of \( t \). When \( t = 2 \), we have

\[
\left[ \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{1 - \phi/\theta} \right]_{t=2} = \frac{z}{1 - \phi/\theta} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{1 - \phi/\theta} = \frac{1}{1 - \phi/\theta} \left[ z + (1 + \phi/\theta) \frac{c}{1 - \phi/\theta} \right] \leq \vartheta \tag{B.85}
\]

where the inequality is by condition (17). Therefore the claim in part (i) of the lemma follows by the definition of \( \tau_* \). To prove (ii) note that by the definition of \( \tau_* \) in (21),

\[
\vartheta < \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{1 - \phi/\theta} = \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{c}{1 - \phi/\theta} + \frac{\phi/\theta}{1 - \phi/\theta} \frac{2c}{1 - \phi/\theta}. \tag{B.86}
\]
Since $\phi/\theta \in (0, 1)$, $(1 - \phi/\theta)^{-1} = \sum_{k=0}^{\infty} (\phi/\theta)^k$. Therefore

$$\frac{z}{(1 - \phi/\theta)^{\tau_* + 1}} + \frac{c}{\phi} = \frac{z}{(1 - \phi/\theta)^{\tau_*}} \frac{1}{1 - \phi/\theta} + \frac{c}{\phi}$$

$$= \frac{z}{(1 - \phi/\theta)^{\tau_*}} \sum_{k=0}^{\infty} (\phi/\theta)^k + \frac{c}{\phi}$$

$$> \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{c}{\phi} + \frac{\phi/\theta}{1 - \phi/\theta} z,$$

(B.87)

where the inequality is by (i) $\sum_{k=0}^{\infty} (\phi/\theta)^k > \sum_{k=0}^{1} (\phi/\theta)^k$, (ii) $\phi/\theta \in (0, 1)$, and (iii) $\tau_* \geq 2$ (which has been proved above). Since $z > 2c\phi^{-1}$ (by (16)) and $\phi/\theta \in (0, 1)$, by (B.86) and (B.87) we immediately prove (B.84).

Lemma 12. Under the conditions of Theorem 6, we have

$$S_t^* = \mu z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{t-1}}, C_t^* = c, L_t^* = 0 \text{ and } L_{t+1}^* = 0$$

(B.88)

for any $t$ with $1 < t \leq \tau_*$, where $\tau_*$ is defined in (21).

Proof of Lemma 12. By (B.79), we know that (B.88) holds for $t = 1$. We prove that (B.88) holds for $t$ with $2 \leq t \leq \tau_*$ by mathematical induction. Suppose that (B.88) holds for $t - 1$, i.e.,

$$S_{t-1}^* = \mu z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{t-2}}, C_{t-1}^* = c, L_{t-1}^* = 0 \text{ and } L_t^* = 0$$

(B.89)

Since $L_{t-1}^* = 0$ and $L_t^* = 0$, at time $t$ we can use $\zeta_t = 0$, (B.75) and (B.89) to get

$$S_t^* = \frac{\theta S_{t-1}^* - \phi Z_t - C_t^*}{\theta - \phi} = \mu z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi}.$$  

(B.90)
For any \( t \leq \tau^* \), by the definition of \( \tau^* \) we know that

\[
|S^*_t - \mu_z| = \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi - C^*_t}{\theta - \phi} \right| 
\leq \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta c}{1 - \phi/\theta \phi} \leq \vartheta,
\]

(B.91)

where the first inequality is by noting that \((\theta - \phi)^{-1}(\theta c/\phi - C^*_t)\) is non-negative and attains it maximum at \( C^*_t = -c \), and the second inequality is by \( t \leq \tau^* \), the definition of \( \tau^* \) and the fact that \( z(1 - \phi/\theta)^{-t+1} \) is a strictly increasing sequence of \( t \). Therefore by (7) we have \( L^*_{t+1} = 0 \). Since \( L^*_t = 0 \) and \( L^*_{t+1} = 0 \), we can apply (B.76) and (B.90) to get

\[
\Delta S^*_{t+1} = \frac{\phi(S^*_t - \mu_z) - C^*_{t+1}}{\theta - \phi} = \frac{\phi z - (1 - \phi/\theta)^{t-1}C^*_{t+1}}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} + \frac{\theta c - \phi C^*_t}{(\theta - \phi)^2} - \frac{\phi \zeta_{t+1}}{\theta - \phi}.
\]

(B.92)

Under the restrictions \( \theta > \phi > 0 \) and \( z > 2c/\phi \), we have

\[
\frac{\phi z - (1 - \phi/\theta)^{t-1}C^*_{t+1}}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} > 0 \quad \text{and} \quad \frac{\theta c - \phi C^*_t}{(\theta - \phi)^2} > 0
\]

(B.93)

for any \( C^*_t, C^*_{t+1} \in \{-c, 0, c\} \), which together with (B.92) and \( E_t[\zeta_{t+1}] = 0 \) implies that

\[
E_t[\Delta S^*_{t+1}] = \frac{\phi z - (1 - \phi/\theta)^{t-1}E_t[C^*_{t+1}]}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} + \frac{\theta c - \phi C^*_t}{(\theta - \phi)^2} > 0.
\]

(B.94)

Combing (B.94) with the demand function of the informed speculators we get \( C^*_t = c \), which together with (B.90) shows that

\[
S^*_t = \mu_z + \phi^{-1}c + \frac{z}{(1 - \phi/\theta)^{t-1}}.
\]

(B.95)
Therefore (B.88) also holds for $t$. The claim of the lemma follows by mathematical induction. 

**Lemma 13.** Under the conditions of Theorem 6 and condition

$$\frac{z}{(1 - \phi/\theta)^\tau} + \frac{c}{\phi} > \vartheta,$$  \hspace{1cm} (B.96)

there exists a unique equilibrium at time $\tau^* + 1$:

$$C^*_{\tau^*+1} = -c \text{ and } S^*_{\tau^*+1} = \mu_z + \frac{z}{(1 - \phi/\theta)^\tau} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \hspace{1cm} (B.97)$$

**Proof of Lemma 13.** By (B.88), we have $L^*_{\tau^*} = 0$ and $L^*_{\tau^*+1} = 0$. Since $\zeta_{\tau^*+1} = 0$ at time $\tau^* + 1$, by (B.75) and (B.88) with $t = \tau^* + 1$, we have

$$S^*_{\tau^*+1} = \mu_z + \frac{z}{(1 - \phi/\theta)^\tau} + \frac{\theta c/\phi - C^*_{\tau^*+1}}{\theta - \phi}.$$  \hspace{1cm} (B.98)

where $(\theta - \phi)^{-1}(\theta c/\phi - C^*_{\tau^*+1})$ is non-negative for any $C^*_{\tau^*+1} \in \{-c, 0, c\}$ and it achieves its minimum $c/\phi$ at $C^*_{\tau^*+1} = c$. Therefore,

$$S^*_{\tau^*+1} = \mu_z > \frac{z}{(1 - \phi/\theta)^\tau} + \frac{c}{\phi} > \vartheta,$$  \hspace{1cm} (B.99)

where the second inequality is by (B.96). Under (B.99), the law of motion of $L^*_{\tau^*+2}$ in (7) implies that, regardless of the informed speculators’ demand at $\tau^* + 1$, the probability of liquidity crunch in period $\tau^* + 2$ becomes at least $\sigma$. Let $\sigma_{\tau^*+2}$ denote this probability. On the one hand if $L^*_{\tau^*+2} = 0$, (B.76) and (B.98) imply that

$$\Delta S^*_{\tau^*+2} = \frac{\phi z/\theta}{(1 - \phi/\theta)^\tau} - \frac{C^*_{\tau^*+2}}{\theta - \phi} - \frac{\theta c - \phi C^*_{\tau^*+1}}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau^*+2}}{\theta - \phi}.$$  \hspace{1cm} (B.100)
On the other hand if $L^s_{\tau+2} = 1$, equilibrium price equation (B.78) and (B.98) imply that

$$\Delta S^*_{\tau+2} = \zeta_{\tau+2} + \frac{C^*_{\tau+2}}{\phi} \frac{z}{(1 - \phi/\theta)^{\tau^*}} - \frac{\theta c}{\phi} - \frac{C^*_{\tau+1}}{\theta - \phi}.$$  \hspace{1cm} (B.101)

Combining the results in (B.100) and (B.101), and applying $E_{\tau+1}[\zeta_{\tau+2}] = 0$, we get

$$E_{\tau+1}[\Delta S^*_{\tau+2}] = (1 - \sigma_{\tau+2}) \left[ \frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau^*}} - \frac{E_{\tau+1}[C^*_{\tau+2}]}{\theta - \phi} + \frac{\theta c}{\phi} - \frac{C^*_{\tau+1}}{(\theta - \phi)^2} \right]$$

$$+ \sigma_{\tau+2} \left[ \frac{E_{\tau+1}[C^*_{\tau+2}]}{\phi} \frac{z}{(1 - \phi/\theta)^{\tau^*}} - \frac{\theta c}{\phi} - \frac{C^*_{\tau+1}}{(\theta - \phi)^2} \right]$$

$$= \frac{z}{(1 - \phi/\theta)^{\tau^*}} \left( \frac{\phi}{\theta} - \sigma_{\tau+2} \right) E_{\tau+1}[C^*_{\tau+2}] + \frac{\theta (\sigma_{\tau+2} - \phi/\theta)}{\phi (\theta - \phi)} C^*_{\tau+1}.$$  \hspace{1cm} (B.102)

Define

$$Q(c_1, c_2) = \frac{\theta^2 (\phi/\theta - \sigma_{\tau+2}) c_1}{\phi (\theta - \phi)^2} + \theta (\sigma_{\tau+2} - \phi/\theta) c + \frac{\theta (\sigma_{\tau+2} - \phi/\theta)}{\phi (\theta - \phi)} c_2.$$  \hspace{1cm} (B.103)

Note that

$$Q(c, c) = \frac{\theta^2 (\phi/\theta - \sigma_{\tau+2}) c}{\phi (\theta - \phi)^2} + \frac{\theta (\sigma_{\tau+2} - \phi/\theta)}{(\theta - \phi)^2} c + \frac{\theta (\sigma_{\tau+2} - \phi/\theta)}{\phi (\theta - \phi)} c$$

$$= \frac{(\phi/\theta - \sigma_{\tau+2}) \theta c}{(\theta - \phi)} \left[ \frac{\phi}{\theta} - \frac{1}{\theta - \phi} - \frac{1}{\phi} \right] = 0.$$  \hspace{1cm} (B.104)

As $\sigma_{\tau+2} \geq \overline{\sigma} > \phi/\theta$ and $\theta > \phi$, we know that $Q(c_1, c_2) \leq Q(c, c)$ for any
$c_1, c_2 \in \{-c, 0, c\}$, which together with (B.104) implies that

$$Q(c_1, c_2) \leq 0 \text{ for any } c_1, c_2 \in \{-c, 0, c\}. \quad (B.105)$$

From (B.102) we have that

$$E_{\tau_0+1}[\Delta S_{\tau_0+2}^*] = \frac{z}{(1 - \phi/\theta)\tau_0} \frac{\phi/\theta - \sigma_{\tau_0+2}}{1 - \phi/\theta} + E_{\tau_0+1}[Q(C_{\tau_0+2}^*, C_{\tau_0+1}^*)]. \quad (B.106)$$

Using (B.105), (B.106) and restrictions $\theta > \phi$ and $\sigma_{\tau_0+2} > \phi/\theta$, we deduce that $E_{\tau_0+1}[\Delta S_{\tau_0+2}^*] < 0$. From the demand function of the informed speculators (3), we then get $C_{\tau_0+1}^* = -c$, which together with (B.98) implies that

$$S_{\tau_0+1}^* = \mu z + \frac{z}{(1 - \phi/\theta)\tau_0} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \quad (B.107)$$

This completes the proof.  

**Lemma 14.** Under the conditions of Theorem 6 and condition

$$\frac{z}{(1 - \phi/\theta)\tau_0} + \frac{c}{\phi} \leq \vartheta, \quad (B.108)$$

there exist and only exist two possible equilibria at time $\tau_0 + 1$. The first equilibrium is the same as the one derived in Lemma 13 under restriction (B.96). In the second equilibrium, $C_{\tau_0+1}^* = c$, $C_{\tau_0+2}^* = -c$,

$$S_{\tau_0+1}^* = \mu z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)\tau_0}. \quad (B.109)$$

and

$$S_{\tau_0+2}^* = \mu z + \frac{c}{\phi} + \frac{1 + \phi/\theta}{\phi} + \frac{z}{(1 - \phi/\theta)(\tau_0+1)}. \quad (B.110)$$

**Proof of Lemma 14.** By (B.88), we have $L_{\tau_0}^* = 0$ and $L_{\tau_0+1}^* = 0$. Since $\zeta_{\tau_0+1} = \ldots$
at time $\tau^* + 1$, by (B.75) and (B.88) with $t = \tau^* + 1$, we have

$$S^*_{\tau^* + 1} = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{\theta c/\phi - C^*_{\tau^* + 1}}{\theta - \phi}, \quad (B.111)$$

Therefore there are three possible conjectured equilibria at time $\tau^* + 1$:

$$S^*_{\tau^* + 1} = \begin{cases} 
\mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}, & \text{if } C^*_{\tau^* + 1} = -c \\
\mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{\theta}{\phi} \frac{1}{1 - \phi/\theta}, & \text{if } C^*_{\tau^* + 1} = 0 \\
\mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi}, & \text{if } C^*_{\tau^* + 1} = c 
\end{cases} \quad (B.112)$$

To show that

$$C^*_{\tau^* + 1} = -c \text{ and } S^*_{\tau^* + 1} = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} \quad (B.113)$$

is an equilibrium, it is sufficient to show that $E_{\tau^* + 1}[\Delta P^*_{\tau^* + 2}] < 0$. Given $C^*_{\tau^* + 1} = -c$,

$$S^*_{\tau^* + 1} - \mu_z = \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} > 0 \quad (B.114)$$

by the definition of $\tau^*$ in (21). Therefore, by (7) the informed speculators know that the probability of liquidity crunch in period $\tau^* + 2$ becomes at least $\bar{\sigma}$. Let $\sigma_{\tau^* + 2}$ denote this probability. On the one hand if $L^*_{\tau^* + 2} = 0$, (B.76) and (B.113) imply that

$$\Delta P^*_{\tau^* + 2} = \frac{\phi z / \theta}{(1 - \phi/\theta)^{\tau^* + 1}} - \frac{C^*_{\tau^* + 2}}{\theta - \phi} + \frac{c}{\theta - \phi} - \frac{(\theta + \phi)c}{\theta - \phi} \quad (B.115)$$

On the other hand if $L^*_{\tau^* + 2} = 1$, equilibrium price equation (B.78) and (B.98) imply that

$$\Delta S^*_{\tau^* + 2} = \zeta_{\tau^* + 2} + \frac{C^*_{\tau^* + 2}}{\phi} - \frac{z}{(1 - \phi/\theta)^{\tau^*}} - \frac{c}{\phi} \frac{\theta + \phi}{\theta - \phi} \quad (B.116)$$
Combining the results in (B.115) and (B.116), and applying \( E_{\tau+1}[\xi_{\tau+2}] = 0 \), we get

\[
E_{\tau+1}[\Delta S_{\tau+2}^*] = (1 - \sigma_{\tau+2}) \left[ \frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau+1}} - \frac{E_{\tau+1}[C_{\tau+2}^*]}{\theta - \phi} + \frac{(\theta + \phi)c}{(\theta - \phi)^2} \right] + \sigma_{\tau+2} \left[ \frac{E_{\tau+1}[C_{\tau+2}^*]}{\phi} - \frac{z}{(1 - \phi/\theta)^{\tau+1}} \right] - \frac{c \theta + \phi}{\theta - \phi} \\
= \frac{z}{(1 - \phi/\theta)^{\tau+1}} \left[ \phi/\theta - \sigma_{\tau+2} \right] + \frac{\theta(\phi/\theta - \sigma_{\tau+2})c}{(\theta - \phi)^2} (1 + \theta/\phi) + \frac{\theta(\sigma_{\tau+2} - \phi/\theta)C_{\tau+2}^*}{\phi(\theta - \phi)}.
\]  

(B.117)

Since \( \sigma_{\tau+2} \geq \sigma > \phi/\theta \) and \( \theta > \phi \),

\[
\frac{z}{(1 - \phi/\theta)^{\tau+1}} \left[ \phi/\theta - \sigma_{\tau+2} \right] + \frac{\theta(\phi/\theta - \sigma_{\tau+2})c}{(\theta - \phi)^2} (1 + \theta/\phi) + \frac{\theta(\sigma_{\tau+2} - \phi/\theta)C_{\tau+2}^*}{\phi(\theta - \phi)} \\
\leq \frac{z}{(1 - \phi/\theta)^{\tau+1}} \left[ \phi/\theta - \sigma_{\tau+2} \right] + \frac{\theta(\phi/\theta - \sigma_{\tau+2})c}{(\theta - \phi)^2} (1 + \theta/\phi) + \frac{\theta(\sigma_{\tau+2} - \phi/\theta)c}{\phi(\theta - \phi)} \\
= \frac{z}{(1 - \phi/\theta)^{\tau+1}} \left[ \phi/\theta - \sigma_{\tau+2} \right] + \frac{2\theta(\phi/\theta - \sigma_{\tau+2})c}{(\theta - \phi)^2} < 0
\]

(B.118)

where the second inequality is by \( z > 0, c > 0, \sigma_{\tau+2} \geq \sigma > \phi/\theta \) and \( \theta > \phi > 0 \).

By (B.117) and (B.118),

\[
E_{\tau+1}[\Delta S_{\tau+2}^*] < 0
\]

(B.119)

which shows that (B.113) is indeed an equilibrium.

We next show that

\[
C_{\tau+1}^* = c \quad \text{and} \quad S_{\tau+1}^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{\tau+1}},
\]

\[
C_{\tau+2}^* = -c \quad \text{and} \quad S_{\tau+2}^* = \mu_z + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} + \frac{z}{(1 - \phi/\theta)^{\tau+1}}
\]

(B.120)

is also an equilibrium under (B.108). Given the demand of the informed specu-
lators $C_{\tau+1}^* = c$ and $C_{\tau+2}^* = -c$, it is sufficient to show that $E_{\tau+1}[\Delta S_{\tau+2}^*] > 0$ and $E_{\tau+2}[\Delta S_{\tau+3}^*] < 0$. By (7) and (B.108), at time $\tau+1$ informed speculators know that $L_{\tau+2}^* = 0$ with probability 1 if they choose $C_{\tau+1}^* = c$. Since $L_{\tau+1}^* = 0$ and $L_{\tau+2}^* = 0$, by (B.76) and (B.120), the informed speculator knows at time $\tau+1$ that the market clearing condition at time $\tau+2$ implies

$$\Delta S_{\tau+2}^* = \frac{z\phi/\theta}{(1 - \phi/\theta)^{\tau+1}} + \frac{c - C_{\tau+2}^*}{\theta - \phi} - \frac{\phi\zeta_{\tau+2}}{\theta - \phi}, \quad (B.121)$$

which together with $C_{\tau+2}^* \leq c$, $E_{\tau+1}[\xi_{\tau+2}] = 0$ and $\theta > \phi$ implies that

$$E_{\tau+1}[\Delta S_{\tau+2}^*] \geq \frac{z\phi/\theta}{(1 - \phi/\theta)^{\tau+1}} > 0. \quad (B.122)$$

This rationalizes the informed speculator’s choice $C_{\tau+1}^* = c$ and the equilibrium price

$$S_{\tau+1}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau+1}} + \frac{c}{\phi}, \quad (B.123)$$

at time $\tau+1$. We have shown above that $L_{\tau+1}^* = 0$ and $L_{\tau+2}^* = 0$. Hence at time $\tau+2$, by $\zeta_{\tau+2} = 0$, (B.75) and (B.123), the market clearing condition is

$$S_{\tau+2}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau+1}} + \frac{\theta c - \phi C_{\tau+2}^*}{\phi(\theta - \phi)}, \quad (B.124)$$

Since $\phi > 0$, by (B.124) and (B.84)

$$S_{\tau+2}^* - \mu_z \geq \frac{z}{(1 - \phi/\theta)^{\tau+1}} + \frac{c}{\phi(\theta - \phi)} > \theta. \quad (B.125)$$

Hence regardless of the informed speculators’ demand at $\tau+2$, the probability of liquidity crunch in period $\tau+3$ becomes at least $\sigma$. Let $\sigma_{\tau+3}$ denote this
probability. On the one hand if $L^*_{τ+3} = 0$, (B.76) and (B.124) imply that

$$\Delta S^*_{τ+3} = \frac{φz - (1 - φ/θ)^{τ+1}C^*_{τ+3}}{(θ - φ)(1 - φ/θ)^{τ+1}} + \frac{θc - φC^*_{τ+2}}{(θ - φ)^2} - \frac{φζ_{τ+3}}{θ - φ}. \quad (B.126)$$

On the other hand if $L^*_{τ+3} = 1$, equilibrium price equation (B.78) and (B.124) imply that

$$\Delta S^*_{τ+3} = ζ_{τ+3} + \frac{C^*_{τ+3}}{φ} - \frac{z}{(1 - φ/θ)^{τ+1}} - \frac{θc/φ - C^*_{τ+2}}{θ − φ}. \quad (B.127)$$

Combining the results in (B.126) and (B.127), and applying $E^{τ+2}_{τ+2}[ζ_{τ+3}] = 0$, we get

$$E^{τ+2}_{τ+2}[ΔS^*_{τ+3}] = (1 - σ_{τ+3}) \left[ \frac{φz/θ}{(1 - φ/θ)^{τ+2}} - \frac{E^{τ+2}_{τ+2}[C^*_{τ+3}]}{θ - φ} + \frac{θc/φ - C^*_{τ+2}}{(θ - φ)^2} \right]$$

$$+ σ_{τ+3} \left[ \frac{E^{τ+2}_{τ+2}[C^*_{τ+3}]}{φ} - \frac{z}{(1 - φ/θ)^{τ+1}} - \frac{θc/φ - C^*_{τ+2}}{θ - φ} \right]$$

$$= \frac{z}{(1 - φ/θ)^{τ+1}} \frac{φ/θ - σ_{τ+3}}{1 - φ/θ} + \frac{θ(φ/θ - σ_{τ+3})c}{φ(θ - φ)}$$

$$+ \frac{θ(σ_{τ+3} - φ/θ)}{φ(θ - φ)} E^{τ+2}_{τ+2}[C^*_{τ+3}] + \frac{θ(σ_{τ+3} - φ/θ)}{(θ - φ)^2} C^*_{τ+2}. \quad (B.128)$$

Using the same arguments of showing $E^{τ+1}_{τ+1}[ΔS^*_{τ+2}] < 0$ in Lemma 13, we can show that $E^{τ+2}_{τ+2}[ΔS^*_{τ+3}] < 0$ regardless of the informed speculators’ demand at $τ_s + 2$ and $τ_s + 3$. Therefore at time $τ_s + 2$,

$$C^*_{τ+2} = -c \text{ and } S^*_{τ+2} = μz + \frac{z}{(1 - φ/θ)^{τ+1}} + \frac{c θ + φ}{φθ - φ}. \quad (B.129)$$

which shows that (B.120) is indeed an equilibrium.
Finally, we show that

\[ C^*_{\tau^* + 1} = 0 \text{ and } S^*_{\tau^* + 1} = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1}{1 - \phi/\theta} \]  

(B.130)

cannot be an equilibrium. There are two cases to consider:

\begin{align*}
\text{case 1:} & \quad \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1}{1 - \phi/\theta} \leq \vartheta, \quad (B.131) \\
\text{case 2:} & \quad \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \frac{1}{1 - \phi/\theta} > \vartheta. \quad (B.132)
\end{align*}

We investigate the conjectured equilibrium (B.130) in case 1 first. Under (B.131), at time \( \tau^* + 1 \) the informed speculators’ know that \( L^*_{\tau^* + 2} = 0 \) with probability 1 given the demand \( C^*_{\tau^* + 1} = 0 \). Therefore by (B.76) and (B.130), the informed speculator knows that market clearing condition at time \( \tau^* + 2 \) implies

\[ \Delta S^*_{\tau^* + 2} = \frac{\phi z}{(1 - \phi/\theta)^{\tau^* + 1}} + \frac{c(1 - \phi/\theta)^{-1} - C^*_{\tau^* + 2}}{\theta - \phi} - \frac{\phi \zeta_{\tau^* + 2}}{\theta - \phi}, \quad (B.133) \]

which together with \( E_{\tau^* + 1}[\zeta_{\tau^* + 2}] = 0 \) and \( \theta > \phi \) implies that

\[ E_{\tau^* + 1}[\Delta S^*_{\tau^* + 2}] \geq \frac{\phi z}{(1 - \phi/\theta)^{\tau^* + 1}} > 0. \quad (B.134) \]

This contradicts the informed speculators’ demand \( C^*_{\tau^* + 1} = 0 \). Next we investigate the conjectured equilibrium (B.130) in case 2. Under (B.132), the informed speculators’ know that \( L^*_{\tau^* + 2} = 1 \) with probability at least \( \sigma \) given the demand \( C^*_{\tau^* + 1} = 0 \). Let \( \sigma_{\tau^* + 2} \) denote this probability. On the one hand if
\( L_{\tau_{r+2}}^* = 0 \), (B.76) and (B.130) imply that
\[
\Delta S_{\tau_{r+2}}^* = \frac{\phi z - (1 - \phi / \theta)^{\tau_{r+2}} C_{\tau_{r+2}}^*}{(\theta - \phi)(1 - \phi / \theta)^{\tau_{r+2}}} \frac{\theta c}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau_{r+2}}}{\theta - \phi}. \tag{B.135}
\]

On the other hand if \( L_{\tau_{r+2}}^* = 1 \), equilibrium price equation (B.78) and (B.130) imply that
\[
\Delta S_{\tau_{r+2}}^* = \xi_{\tau_{r+2}} + \frac{C_{\tau_{r+2}}^*}{\phi} - \frac{z}{(1 - \phi / \theta)^{\tau_{r+2}}} - \frac{c}{\phi} \frac{1}{1 - \phi / \theta}. \tag{B.136}
\]

Combining the results in (B.135) and (B.136), and applying \( E_{\tau_{r+2}}[\xi_{\tau_{r+2}}] = 0 \), we get
\[
E_{\tau_{r+2}}[\Delta S_{\tau_{r+3}}^*] = (1 - \sigma_{\tau_{r+2}}) \left[ \frac{\phi z}{\theta} - \frac{E_{\tau_{r+1}}[C_{\tau_{r+2}}^*]}{\theta - \phi} + \frac{\theta c}{(\theta - \phi)^2} \right]
+ \sigma_{\tau_{r+2}} \left[ \frac{E_{\tau_{r+1}}[C_{\tau_{r+2}}^*]}{\phi} - \frac{z}{(1 - \phi / \theta)^{\tau_{r+1}}} - \frac{\theta c}{\phi} \frac{1}{1 - \phi / \theta} \right]
= \frac{z}{(1 - \phi / \theta)^{\tau_{r+1}}} \frac{\phi / \theta - \sigma_{\tau_{r+2}}}{1 - \phi / \theta} + \frac{\theta^2(\phi / \theta - \sigma_{\tau_{r+2}})c}{\phi(\theta - \phi)}
+ \frac{\theta(\sigma_{\tau_{r+2}} - \phi / \theta)}{\phi(\theta - \phi)} E_{\tau_{r+1}}[C_{\tau_{r+2}}^*]. \tag{B.137}
\]

Since \( \sigma_{\tau_{r+2}} \geq \sigma > \phi / \theta \), we have
\[
E_{\tau_{r+2}}[\Delta S_{\tau_{r+3}}^*] \leq \frac{z}{(1 - \phi / \theta)^{\tau_{r+1}}} \frac{\phi / \theta - \sigma_{\tau_{r+2}}}{1 - \phi / \theta} + \frac{\theta^2(\phi / \theta - \sigma_{\tau_{r+2}})c}{\phi(\theta - \phi)} + \frac{\theta(\sigma_{\tau_{r+2}} - \phi / \theta)c}{\phi(\theta - \phi)} \frac{\phi}{\theta - \phi} \tag{B.138}
< 0
\]
where the last inequality is by \( \sigma_{\tau_{r+2}} \geq \sigma > \phi / \theta \) and \( \theta > \phi > 0 \). This contradicts \( C_{\tau_{r+1}}^* = 0 \) in the conjectured equilibrium (B.130). □

**Lemma 15.** Under the conditions of Theorem 6, we have \( l^* \geq r^* \).
Proof of Lemma 15. By Lemma 13 and Lemma 14, we have $\tau^* = \tau_s + 1$ and $L_{\tau_s+1}^* = 0$ when (B.96) holds, and $\tau^* = \tau_s + 1$ and $L_{\tau_s+1}^* = 0$ or $\tau^* = \tau_s + 2$ and $L_{\tau_s+2}^* = 0$ when (B.108) holds. This implies that $L_{\tau_s}^* = 0$ and hence $l^* \geq \tau^*$. \hfill \qed

Lemma 16. Under the conditions of Theorem 6 and $l^* > \tau^*$, we have

\[ S_t^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{-1}} + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*} - 1} - \left( \frac{1 + \phi/\theta}{\phi (1 - \phi/\theta)} \right) \frac{1}{\theta - \phi} \right] \quad \text{and} \quad C_t^* = -c \]  

(B.139)

for any $t$ with $\tau^* \leq t < l^*$ where $l^*$ satisfies $l^* \leq \bar{l}_{\text{max}}$ with probability 1.

Proof of Lemma 16. By Lemma 13 and Lemma 14, at time $\tau^*$, we have

\[ C_{\tau^*}^* = -c \quad \text{and} \quad S_{\tau^*}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{c}{\phi} \left[ \frac{1}{(1 - \phi/\theta)^{t-\tau^*} - 1} \right] \quad \text{where} \quad \tau^* = \tau_s + 1 \]  

when (B.96) holds, and $\tau^* = \tau_s + 1$ or $\tau^* = \tau_s + 2$ when (B.108) holds. Therefore (B.139) holds for $t = \tau^*$. If $l^* = \tau^* + 1$, then $l^* \leq \bar{l}_{\text{max}}$ by the definition of $\bar{l}_{\text{max}}$ which finishes the proof.

We next consider the case that $l^* > \tau^* + 1$. Suppose that (B.139) holds for $t - 1$ with $\tau^* \leq t - 1 < l^*$, we will show that it also holds for $t$ with $t < l^*$. At period $t$, liquidity crunch is not triggered because $t < l^*$, which together with $Z_t = \mu_z$ and (B.75) implies that

\[ S_t^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*} - 1} - \left( \frac{1}{1 - \phi/\theta} \right) \right] \frac{c}{\phi} - \frac{C_t^*}{\theta - \phi}. \]  

(B.141)

Since $\theta > \phi$ and $t - \tau^* \geq 1$, by $(1 - \phi/\theta)^{-1} = \sum_{k=0}^{\infty} (\phi/\theta)^k > \sum_{k=0}^{1} (\phi/\theta)^k$, the
definition of \( \tau \) and (B.141)

\[
S^*_t > \mu + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} - \frac{c}{\theta - \phi}
\]

\[
> \mu + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^2} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} - \frac{c}{\theta - \phi}
\]

\[
= \mu + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} - \frac{c}{1 - \phi/\theta \theta}
\]

\[
> \mu + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} \sum_{k=0}^{1} (\phi/\theta)^k - \frac{1}{1 - \phi/\theta \theta}
\]

\[
> \mu + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi}
\]  \hspace{1cm} (B.142)

where the last inequality is by

\[
\frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} - \frac{1}{1 - \phi/\theta \theta} \frac{c}{\theta - \phi} = \frac{\phi/\theta}{1 - \phi/\theta \theta} > 0. \quad (B.143)
\]

Hence the informed speculators’ know that \( L^*_t = 1 \) with probability at least \( \sigma \) regardless of the demand \( C_t^* \) at time \( t \). Let \( \sigma_{t+1} \) denote this probability. On the one hand, when there is no liquidity crunch in period \( t+1 \), (B.76) implies that

\[
\Delta S^*_{t+1} = \frac{\phi S_t^* - \phi \mu}{\theta - \phi} - \frac{C_{t+1}^* + \phi \zeta_{t+1}^*}{\theta - \phi}
\]

\[
= \frac{z\phi/\theta}{(1 - \phi/\theta)^t} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+2}} - \frac{1}{(1 - \phi/\theta)^2} \right] \frac{c}{\theta}
\]

\[
- \frac{\phi C_{t+1}^*}{(\theta - \phi)^2} - \frac{C_{t+1}^* + \phi \zeta_{t+1}^*}{\theta - \phi}. \quad (B.144)
\]

On the other hand, when there is liquidity crunch in period \( t+1 \), (B.78) implies that

\[
S^*_{t+1} = \mu + \zeta_{t+1} + \phi^{-1} C_{t+1}^*. \quad (B.145)
\]
Combining this expression with (B.141), we have

\[
\begin{align*}
\Delta S_{t+1}^{*} & = \zeta_{t+1} + \frac{C_{t+1}^*}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}} \\
& \quad - \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} + \frac{C_t^*}{\theta - \phi} \\
& = - \frac{z}{(1 - \phi/\theta)^{t-1}} - \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} \\
& \quad + \frac{C_{t+1}^*}{\phi} + \frac{C_t^*}{\theta - \phi} + \zeta_{t+1}. \tag{B.146}
\end{align*}
\]

Next, we compute \( E_t [\Delta S_{t+1}^*] = (1 - \sigma_{t+1}) E_t [\Delta S_{t+1}^{*,nl}] + \sigma_{t+1} E_t [\Delta S_{t+1}^{*,l}] \), where \( \sigma_{t+1} \) is the probability of liquidity crunch in period \( t + 1 \). To do so note that considering the first two terms in (B.144) and in (B.146), we get

\[
\begin{align*}
\frac{(1 - \sigma_{t+1})z\phi/\theta}{(1 - \phi/\theta)^t} & + \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1} - \frac{1}{1 - \phi/\theta} \left[ (1 - \sigma_{t+1}) - \frac{\sigma_{t+1} c}{\phi} \right] \\
= \frac{\phi/\theta - \sigma_{t+1} z}{1 - \phi/\theta (1 - \phi/\theta)^{t-1}} & + \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1} - \frac{1}{1 - \phi/\theta} \left[ (1 - \sigma_{t+1}) - \frac{\sigma_{t+1} c}{\phi} \right] \\
= \frac{\phi/\theta - \sigma_{t+1} z}{1 - \phi/\theta (1 - \phi/\theta)^{t-1}} & + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1} - \frac{1}{1 - \phi/\theta} \left[ (1 - \sigma_{t+1}) - \frac{\sigma_{t+1} c}{\phi} \right] \right] \phi/\theta - \sigma_{t+1} \frac{c}{1 - \phi/\theta}. \tag{B.147}
\end{align*}
\]

Considering the third and fourth terms in (B.144) and in (B.146) and setting \( E_t [\zeta_{t+1}] = 0 \) we get

\[
\begin{align*}
\frac{C_{t+1}^*}{\phi} \sigma_{t+1} + \frac{C_t^*}{\theta - \phi} \sigma_{t+1} & - \frac{(1 - \overline{\sigma}) C_{t+1}^*}{\theta - \phi} - \frac{(1 - \overline{\sigma}) \phi C_t^*}{(\theta - \phi)^2} \\
= \frac{\sigma_{t+1} - \phi/\theta C_{t+1}^*}{\phi} + \frac{\sigma_{t+1} - \phi/\theta C_t^*}{(1 - \phi/\theta)^2 \theta} \\
& \leq \frac{\sigma_{t+1} - \phi/\theta c}{1 - \phi/\theta} + \frac{\sigma_{t+1} - \phi/\theta c}{(1 - \phi/\theta)^2 c} = \frac{\sigma_{t+1} - \phi/\theta c}{(1 - \phi/\theta)^2 \phi}. \tag{B.148}
\end{align*}
\]

\[27\]
where the last inequality follows from parameter restrictions $\theta > \phi$ and $\sigma_{t+1} \geq \bar{\sigma} > \phi/\theta$. Collecting the results in (B.144)-(B.148), we have that

$$E_t [\Delta S_{t+1}^*] \leq \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{\phi/\theta - \sigma_{t+1}}{1 - \phi/\theta} + \frac{\sigma_{t+1} - \phi/\theta c}{(1 - \phi/\theta)^2} \frac{\phi/\theta - \sigma_{t+1}}{1 - \phi/\theta} \frac{z}{(1 - \phi/\theta)^{t-1}}$$

$$= \frac{2c}{\phi} \frac{\phi/\theta - \sigma_{t+1}}{(1 - \phi/\theta)^2} \left[ \frac{1}{(1 - \phi/\theta)^{t-\tau^*}} - 1 \right] + \frac{\phi/\theta - \sigma_{t+1}}{1 - \phi/\theta} \frac{z}{(1 - \phi/\theta)^{t-1}} < 0$$

(B.149)

where the inequality follows from parameter restrictions $\theta > \phi$ and $\sigma_{t+1} \geq \bar{\sigma} > \phi/\theta$. It follows from (B.149) that $E_t [\Delta S_{t+1}^*] < 0$, which combined with the demand function of the informed speculators imply that $C_t^* = -c$. Plugging $C_t^* = -c$ into equation (B.141), we get

$$S_t^* = \mu z + \frac{\phi}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} + \frac{c}{\theta - \phi}$$

$$= \mu z + \frac{\phi}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1 \right] \frac{c}{\phi}.$$ (B.150)

This completes the proof of (B.139).

Finally we show that $l^* \leq \bar{l}_{\text{max}}$ when $l^* > \tau^* + 1$. In this case, we have show that

$$S_t^* = \mu z + \frac{\phi}{(1 - \phi/\theta)^{t-1}} + \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1 \right]$$ (B.151)

for any $\tau^* + 1 \leq t < l^*$. Since $\theta > \phi > 0$ and $c > 0$, we see that $S_t^*$ diverges
with $t$ at a geometric rate. Suppose that $l^* > \bar{l}_{\text{max}}$. Then by (B.151)

$$S^*_{l_{\text{max}}-1} - \mu_z = \frac{z}{(1 - \phi/\theta)^{\bar{l}_{\text{max}}-2}} + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{\bar{l}_{\text{max}}-\tau^*}} - 1 \right]$$

$$\geq \frac{z}{(1 - \phi/\theta)^{\bar{l}_{\text{max}}-2}} + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{\bar{l}_{\text{max}}-\tau^*}} - 1 \right] > \bar{\nu}$$

where the first inequality is by $\tau^* \leq \tau_\nu + 2$ which is proved in Lemma 13 and Lemma 14, the second inequality is by the definition of $\bar{l}_{\text{max}}$. Since $S^*_{l_{\text{max}}-1} - \mu_z > \bar{\nu}$, by (7) we deduce that $L^*_{l_{\text{max}}} = 1$ with probability 1. Hence $l^* = \bar{l}_{\text{max}}$ which contradicts $l^* > \bar{l}_{\text{max}}$. 

**Lemma 17.** Under the conditions of Theorem 6, the demand of the momentum speculators satisfy

$$q^m_t = \begin{cases} 
0, & t = 0, 1, l^* \\
\frac{z\phi}{(1 - \phi/\theta)^{\tau^* - t}}, & 2 \leq t < \tau^* \\
\frac{z\phi}{(1 - \phi/\theta)^{\tau^* - t}} + \frac{c}{\phi} \frac{2\phi}{(1 - \phi/\theta)^{\tau^* - t + 1}} & \tau^* \leq t < l^* 
\end{cases} \quad (B.152)$$

**Proof of Lemma 17.** The demand of the momentum speculators $q^m_t$ at time $t = 0, 1, l^*$ is zero because $L^*_0 = 1$ and $L^*_1 = 1$. For any $t$ with $2 \leq t < l^*$, we have $L^*_t = 0$. Therefore, the demand of the momentum speculators is

$$q^m_t = \theta \Delta S^*_t \quad (B.153)$$

By Lemma 12 and Lemma 14, we know that for any $t$ with $2 \leq t < \tau^*$,

$$S^*_t = \mu_z + \phi^{-1} c + \frac{z}{(1 - \phi/\theta)^{t-1}} \quad (B.154)$$
which together with (B.153) implies that

$$q_{m^*}^t = \theta z \frac{(1 - \phi/\theta)^{t-1}}{(1 - \phi/\theta)^{t-2}} = \frac{z\phi}{(1 - \phi/\theta)^{t-1}}$$

(B.155)

for any $t$ with $2 \leq t < \tau^*$. By Lemma 13 and Lemma 14,

$$S_{\tau^*}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}$$

(B.156)

which together with (B.154) implies that

$$\Delta S_{\tau^*}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} - \left[ \mu_z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{\tau^* - 2}} \right]$$

(B.157)

Therefore

$$q_{m^*}^{\tau^*} = \frac{z\phi}{(1 - \phi/\theta)^{\tau^* - 1}} + \frac{c}{\phi} \frac{2\phi}{1 - \phi/\theta}$$

(B.158)

For any $t$ with $\tau^* \leq t < l^*$, by Lemma 16

$$\Delta S_t^* = \frac{z\phi/\theta}{(1 - \phi/\theta)^{t-1}} + \frac{c}{\phi} \frac{2\phi/\theta}{(1 - \phi/\theta)^{t-\tau^*+1}}$$

(B.159)

which together with (B.153) implies that

$$q_{m^*}^t = \frac{z\phi}{(1 - \phi/\theta)^{t-1}} + \frac{c}{\phi} \frac{2\phi}{(1 - \phi/\theta)^{t-\tau^*+1}}$$

for any $t$ with $\tau^* \leq t < l^*$. This finishes the proof.
C Extra Empirical Results

Figure C.1: Success and Failures of 9-month Ahead Directional Forecasts

Note: The figure illustrates the performance of 9-month ahead directional forecasts. Green (red) dots depict the time when an appreciation (depreciation) forecast has been made. If a forecast turned out to correctly predict the direction of movements of the currency, then the dot is filled. If the forecast is wrong, the dot is empty.
Table C.1: Mean of 1-year Exchange Rate Changes

<table>
<thead>
<tr>
<th>Currency</th>
<th>Unconditional</th>
<th>Conditional on appreciation forecasts</th>
<th>Conditional on depreciation forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>0.14</td>
<td>1.41</td>
<td>−4.28</td>
</tr>
<tr>
<td>CAD</td>
<td>0.27</td>
<td>2.02</td>
<td>−1.48</td>
</tr>
<tr>
<td>EUR</td>
<td>1.46</td>
<td>3.07</td>
<td>−4.60</td>
</tr>
<tr>
<td>JPY</td>
<td>−0.81</td>
<td>2.35</td>
<td>−5.69</td>
</tr>
<tr>
<td>GBP</td>
<td>−0.78</td>
<td>2.06</td>
<td>−2.37</td>
</tr>
</tbody>
</table>

Note: The table reports the unconditional mean of 1-year exchange rate changes and the mean of 1-year exchange rate changes conditional on appreciation/depreciation forecasts based on our directional forecasts. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table C.2: Success Ratio of the Directional Forecasts

<table>
<thead>
<tr>
<th>Currency</th>
<th>Panel A: Appreciation Forecasts</th>
<th>Panel B: Depreciation Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1m</td>
<td>3m</td>
</tr>
<tr>
<td>AUD</td>
<td>0.600</td>
<td>0.618</td>
</tr>
<tr>
<td>CAD</td>
<td>0.616</td>
<td>0.651</td>
</tr>
<tr>
<td>EUR</td>
<td>0.675</td>
<td>0.590</td>
</tr>
<tr>
<td>JPY</td>
<td>0.610</td>
<td>0.570</td>
</tr>
<tr>
<td>GBP</td>
<td>0.529</td>
<td>0.624</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Extended Appendix

This online Appendix provides supporting materials complement Kim, Kim, Liao and Tornell (2018). Section D includes the proof of the negative bubbly path. Section E describes the construction of the long-run variance estimators used in the main text. Section F presents empirical results which serve as robustness check of Section 3 in the main text.

D Proof of Negative Bubbly Equilibrium

Lemma 18. Under conditions (15), (16) and (31), we have

\[ \mu_z - \frac{z}{(1 - \phi/\theta)z + 1} - \frac{c}{\phi} \leq S. \]  \hfill (D.160)

Proof of Lemma 18. By the definition of \( \tau \),

\[ S \geq \mu_z - \frac{z}{(1 - \phi/\theta)z} - \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} = \mu_z - \frac{z}{(1 - \phi/\theta)z} - \frac{c}{\phi} - \frac{2c}{1 - \phi/\theta}. \]  \hfill (D.161)

By \( z > 2c/\phi \) and \( \tau \geq 1 \),

\[ \frac{z\phi/\theta}{(1 - \phi/\theta)z} \geq \frac{z\phi/\theta}{1 - \phi/\theta} > \frac{2c/\theta}{1 - \phi/\theta}. \]  \hfill (D.162)

Since \( \phi/\theta \in (0, 1) \), \( (1 - \phi/\theta)^{-1} = \sum_{k=0}^{\infty} (\phi/\theta)^k \) which together with (D.162) implies that

\[ \mu_z - \frac{z}{(1 - \phi/\theta)z + 1} - \frac{c}{\phi} = \mu_z - \frac{z}{(1 - \phi/\theta)z} \sum_{k=0}^{\infty} (\phi/\theta)^k - \frac{c}{\phi} \]

\[ < \mu_z - \frac{z}{(1 - \phi/\theta)z} (1 + \phi/\theta) - \frac{c}{\phi} \]

\[ < \mu_z - \frac{z}{(1 - \phi/\theta)z} - \frac{c}{\phi} - \frac{2c}{1 - \phi/\theta}. \]  \hfill (D.163)

Combining the results in (D.161) and (D.163), we immediately get (D.160).

Lemma 19. Under the conditions of Corollary 9, we have

\[ S^*_1 = \mu_z - z - \phi^{-1}c, \quad C^*_1 = -c, \quad L^*_1 = 0 \quad \text{and} \quad L^*_2 = 0. \]  \hfill (D.164)
Proof of Lemma 19. By Proposition 5, at time 0 we have
\[ S^*_0 = \mu z, \quad C^*_0 = 0, \quad L^*_0 = 1 \quad \text{and} \quad L^*_1 = 0. \] (D.165)

At time \( t = 1 \), the informed speculators observe \( Z_1 = \mu z - z \). Moreover since \( L^*_0 = 1 \), by (B.78) the market clearing condition in this period is
\[ S^*_1 = Z_1 + \frac{C^*_1}{\phi}. \] (D.166)

Because (i) \(|z - \phi^{-1}C^*_1| \leq z + \phi^{-1}c\) for any \( C^*_1 \in \{-c, 0, c\} \), and (ii) \( z + \phi^{-1}c < (1 - \phi/\theta)^{-1}[z + (1 + \phi/\theta)c\phi^{-1}] \) by (15), it follows from (17) that
\[ |S^*_1 - \mu z| < \vartheta. \] (D.167)

By Condition (31) and (D.166),
\[ S^*_1 \geq \mu z - z - \phi^{-1}c > S \] (D.168)

regardless of the demand of the informed speculators at time 1. Hence the law of motion of the probability of liquidity crunch (30) implies that \( L^*_2 = 0 \) with probability 1. Since \( L^*_1 = 0 \) and \( L^*_2 = 0 \), we can use (B.76) with \( t = 2 \) and (D.166) to get
\[
\Delta S^*_2 = \frac{\phi(P^*_1 - \mu z) - C^*_2}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} = \frac{-\phi z + C^*_1 - C^*_2}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} < -\frac{\phi \zeta_2}{\theta - \phi} \] (D.169)

for any \( C^*_1, C^*_2 \in \{-c, 0, c\} \), where the last inequality follows from \( \theta > \phi > 0 \) and \( z > 2c/\phi \). Since \( E_1[\zeta_2] = 0 \), the inequality in (D.169) implies that \( E_1[\Delta S^*_2] < 0 \), which together with the demand function of the informed speculators (3) and the price equation (D.166), implies that \( C^*_1 = -c \) and \( S^*_1 = \mu z - z - c/\phi \). This finishes the proof of (D.164). \( \blacksquare \)

Lemma 20. Under the conditions of Corollary 9, we have
\[ S^*_t = \mu z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}} , \quad C^*_t = -c, \quad L^*_t = 0 \quad \text{and} \quad L^*_{t+1} = 0 \] (D.170)

for any \( t \) with \( 1 \leq t \leq \tau^*_s \).

Proof of Lemma 20. By Lemma 19, (D.170) holds for \( t = 1 \). Since \( \tau_s \geq 2 \), the
proof is finished if $\tau = 1$. In the rest of the proof, we assume that $\tau \geq 2$. We shall prove that (D.170) holds for $t$ with $2 \leq t < \tau_*$ by mathematical induction.

Suppose that (D.170) holds for $t - 1$. By Lemma 19, (D.170) holds for $t = 1$. We shall prove that (D.170) holds for $t$ with $2 \leq t < \tau_*$ by mathematical induction. Suppose that (D.170) holds for $t - 1$. Since $L_{t-1}^* = 0$, $L_t^* = 0$ and $\xi_t = 0$, at period $t$ we have $Z_t = \mu_z$ and

$$S_t^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{\theta c/\phi + C_t^*}{\theta - \phi}. \quad (D.171)$$

Because $t \leq \tau_*$, we know that

$$|S_t^* - \mu_z| = \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi + C_t^*}{\theta - \phi} \right| \leq \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} c < \vartheta. \quad (D.172)$$

Moreover since $t \leq \tau$, by (D.171) and the definition of $\tau$,

$$S_t^* \geq \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{1 + \phi/\theta}{1 - \phi/\theta} c > S. \quad (D.173)$$

From (D.172), (D.173) and the law of motion of the probability of liquidity crunch (30), the informed speculators know that $L_{t+1}^* = 0$ with probability 1. Hence, by (B.76) and (D.171), we have

$$\Delta S_{t+1}^* = \frac{\phi(S_t^* - \mu_z) - C_{t+1}^*}{\theta - \phi} - \frac{\phi \zeta_{t+1}}{\theta - \phi} = \frac{\phi z + (1 - \phi/\theta)^{t-1}C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{t+1}}{\theta - \phi}. \quad (D.174)$$

Under the restrictions $\theta > \phi$ and $z > 2c/\phi$, we have

$$-\frac{\phi z + (1 - \phi/\theta)^{t-1}C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} < 0 \quad \text{and} \quad -\frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} < 0 \quad (D.175)$$

for any $C_t^*, C_{t+1}^* \in \{-c, 0, c\}$, which together with (D.174) and $E_t[\zeta_{t+1}] = 0$ implies that

$$E_t[\Delta S_{t+1}^*] = -\frac{\phi z + (1 - \phi/\theta)^{t-1}E_t[C_{t+1}^*]}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} < 0. \quad (D.176)$$

Combing (D.176) with the demand function of the informed speculators we
get \( C^*_t = -c \), which together with (D.171) shows that (D.170) holds at \( t \). This finishes the proof. 

Lemma 21. Suppose that \( L^*_t = L^*_{t+1} = 0 \), \( \xi_{t+1} = 0 \) and

\[
S^*_t = \mu_z - \frac{c}{\frac{1}{\phi} - \frac{1}{\theta}}\frac{1}{(1 - \phi/\theta)^{t-1}}. \tag{D.177}
\]

If the probability of liquidity crunch at \( t + 2 \) is larger than \( \phi/\theta \), then

\[
E_{t+1}[\Delta S^*_{t+2}] > 0, \quad C^*_{t+1} = c \quad \text{and} \quad S^*_{t+1} = \mu_z - \frac{c(1 + \phi/\theta)}{\phi(1 - \phi/\theta)} - \frac{z}{(1 - \phi/\theta)^t}. \tag{D.178}
\]

Proof of Lemma 21. Since \( L^*_t = L^*_{t+1} = 0 \) and \( \zeta_{t+1} = 0 \), by (B.75) and (D.177), we have

\[
S^*_{t+1} = \mu_z - \frac{z}{(1 - \phi/\theta)^t} - \frac{\theta c/\phi + C^*_{t+1}}{\theta - \phi}. \tag{D.179}
\]

Let \( \sigma_s \) denote the probability of \( L^*_{t+2} = 1 \). On the one hand if there is \( L^*_{t+2} = 0 \), (B.76) and (D.177) imply that

\[
\Delta S^*_{t+2} = \frac{-\phi z/\theta}{(1 - \phi/\theta)^{t+1}} - \frac{\theta c + \phi C^*_{t+2}}{(\theta - \phi)^2} - \frac{C^*_{t+2} + \phi \zeta_{t+2}}{\theta - \phi}. \tag{D.180}
\]

On the other hand if \( L^*_{t+2} = 1 \), equilibrium price equation (B.78) and (D.177) imply that

\[
\Delta S^*_{t+2} = \frac{\zeta_{t+2}}{\phi} + \frac{C^*_{t+2}}{(1 - \phi/\theta)^t} + \frac{\theta c/\phi + C^*_{t+1}}{\theta - \phi}. \tag{D.181}
\]

Combining the results in (D.180) and (D.181), and applying \( E_{t+1}[\zeta_{t+2}] = 0 \), we get

\[
E_{t+1}[\Delta S^*_{t+2}] = (1 - \sigma_s) \left[ \frac{-\phi z/\theta}{(1 - \phi/\theta)^{t+1}} - \frac{E_{t+1}[C^*_{t+2}]}{\theta - \phi} - \frac{\theta c + \phi C^*_{t+1}}{(\theta - \phi)^2} \right] + \sigma_s \left[ \frac{E_{t+1}[C^*_{t+2}]}{\phi} + \frac{z}{(1 - \phi/\theta)^t} + \frac{\theta c/\phi + C^*_{t+1}}{\theta - \phi} \right] = \frac{z}{(1 - \phi/\theta)^t} (1 - \phi/\theta) + \frac{\theta^2(\sigma_s - \phi/\theta)c}{\phi(\theta - \phi)^2} + \frac{\theta(\sigma_s - \phi/\theta)}{\phi(\theta - \phi)} E_{t+1}[C^*_{t+2}] + \frac{\theta(\sigma_s - \phi/\theta)}{(\theta - \phi)^2} C^*_{t+1}. \tag{D.182}
\]
Define

\[ Q(c_1, c_2) = \frac{\theta^2(\sigma_s - \phi/\theta)c}{\phi(\theta - \phi)^2} + \frac{\theta(\sigma_s - \phi/\theta)}{\phi(\theta - \phi)} c_2 + \frac{\theta(\sigma_s - \phi/\theta)}{(\theta - \phi)^2} c_1. \quad (D.183) \]

Then

\[ Q(-c, -c) = \frac{\theta^2(\sigma_s - \phi/\theta)c}{\phi(\theta - \phi)^2} - \frac{\theta(\sigma_s - \phi/\theta)}{\phi(\theta - \phi)} c - \frac{\theta(\sigma_s - \phi/\theta)}{(\theta - \phi)^2} c \]

\[ = \frac{\theta(\sigma_s - \phi/\theta)c}{\theta - \phi} \left( \frac{\theta}{\phi(\theta - \phi)} - \frac{1}{\phi} - \frac{1}{\theta - \phi} \right) = 0, \quad (D.184) \]

which together with \( \sigma_s > \phi/\theta \) implies that

\[ E_{t+1}[\Delta S_{t+2}^*] \geq \frac{z}{(1 - \phi/\theta)^t} \frac{\sigma_s - \phi/\theta}{1 - \phi/\theta} > 0 \quad (D.185) \]

regardless the demand of the informed speculators. By (D.185) and demand law of the informed speculators, we have \( C_{t+1}^* = c \) which together with (D.177) proves (D.178).

**Lemma 22.** Suppose that the conditions of Corollary 9 hold. If

\[ \tau \leq \tau_s \text{ and } \mu_z - \frac{z}{(1 - \phi/\theta)^t} \frac{c}{\phi} \leq S, \quad (D.186) \]

then

\[ C_{\tau_s+1}^* = c, \quad S_{\tau_s+1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^t} - \frac{c}{\phi} 1 + \frac{\phi/\theta}{1 - \phi/\theta} \text{ and } l^* = \tau_s + 2. \quad (D.187) \]

**Proof of Lemma 22.** Since \( \tau \leq \tau_s \), we have

\[ \tau_s = \tau. \quad (D.188) \]

By (D.170), we have \( L_{\tau_s}^* = 0 \) and \( L_{\tau_s+1}^* = 0 \). Since \( \zeta_{\tau_s+1} = 0 \) at time \( \tau_s + 1 \), by (B.75) and (D.170) with \( t = \tau_s + 1 \), we have

\[ S_{\tau_s+1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^t} - \frac{\theta c/\phi + C_{\tau_s+1}^*}{\theta - \phi}. \quad (D.189) \]

Since \( \theta > \phi \), \( (\theta - \phi)^{-1}(\theta c/\phi + C_{\tau_s+1}^*) \) is an increasing function of \( C_{\tau_s+1}^* \). There-
fore,
\[ S^*_\tau_{\tau+1} \leq \mu_z - \frac{z}{(1 - \phi/\theta)L^*} - \frac{c}{\phi} \] (D.190)
regardless of the demand of the informed speculators at time \( \tau_* + 1 \). By (D.188), (D.189) and the second inequality in (D.186), the law of motion of \( L^*_\tau \) in (30) implies that \( L^*_{\tau_*+2} = 1 \) with probability 1, regardless of the informed speculators’ demand at \( \tau_* + 1 \). By Lemma 20,

\[ S^*_\tau = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)L^* - 1}. \] (D.191)

Hence by Lemma 21,

\[ C^*_\tau_{\tau+1} = c \text{ and } S^*_\tau_{\tau+1} = \mu_z - \frac{z}{(1 - \phi/\theta)L^*} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \] (D.192)

Since \( L^*_{\tau_*+2} = 1 \) with probability 1 and \( \tau_* = \tau \), we have \( t^* = \tau + 2 \). This finishes the proof. \( \blacksquare \)

**Lemma 23.** Suppose that the conditions of Corollary 9 hold. If

\[ \tau < \tau_* \text{ and } \mu_z - \frac{z}{(1 - \phi/\theta)L^*} - \frac{c}{\phi} > S^*, \] (D.193)

then there exist two possible equilibriums at time \( \tau_* + 1 \). The first equilibrium is the same as the one derived in Lemma 22. In the second equilibrium, \( C^*_\tau_{\tau_*+1} = -c \), \( C^*_\tau_{\tau_*+2} = c \), \( l^* = \tau + 3 \),

\[ S^*_\tau_{\tau+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)L^*}. \] (D.194)

and

\[ S^*_\tau_{\tau+2} = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)L^*_{\tau_*+1}}. \] (D.195)

**Proof of Lemma 23.** In the proof of Lemma 22, we have show that

\[ \tau_* = \tau \text{ and } S^*_\tau_{\tau_*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)L^*} - \frac{\theta c/\phi + C^*_\tau_{\tau_*+1}}{\theta - \phi}. \] (D.196)

To prove that (D.187) is an equilibrium, it is sufficient to show that

\[ E_{\tau+1} [\Delta S^*_\tau_{\tau_*+2}] > 0. \] (D.197)
Given $C_{\tau^*+1} = c$, we have

$$S^\ast_{\tau^*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)\bar{\zeta}} - \frac{c + \phi/\theta}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}.$$  \hspace{1cm} (D.198)

By the definition of $\tau$, $S^\ast_{\tau^*+1} \leq \bar{S}$ which together with the law of motion of $L^\ast_t$ in (30) implies that

$$L^\ast_{\tau^*+2} = 1 \text{ with probability } 1. \hspace{1cm} (D.199)$$

Hence by Lemma 21,

$$C^\ast_{\tau^*+1} = c \text{ and } S^\ast_{\tau^*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)\bar{\zeta}} - \frac{c + \phi/\theta}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \hspace{1cm}$$

Since $L^\ast_{\tau^*+2} = 1$ with probability 1 and $\tau^* = \tau$, we have $l^* = \tau + 2$.

To derive the second equilibrium, it is sufficient to show that

$$E_{\tau^*+1}[\Delta S^\ast_{\tau^*+2}] < 0. \hspace{1cm} (D.200)$$

Given $C^\ast_{\tau^*+1} = -c$, by (D.196) we have

$$S^\ast_{\tau^*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)\bar{\zeta}} - \frac{c}{\phi}, \hspace{1cm} (D.201)$$

which together with the second inequality in (D.193) implies that

$$S^\ast_{\tau^*+1} > \bar{S}. \hspace{1cm} (D.202)$$

Since $\bar{\tau} < \tau^*$, by the definition of $\tau^*$

$$\left| S^\ast_{\tau^*+1} - \mu_z \right| = \frac{z}{(1 - \phi/\theta)\bar{\zeta}} + \frac{c}{\phi} \hspace{1cm}$$

$$< \frac{z}{(1 - \phi/\theta)\bar{\tau}} + \frac{c + \phi/\theta}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} \leq \theta. \hspace{1cm} (D.203)$$

By (D.202), (D.203) and the law of motion of $L^\ast_t$ in (7),

$$L^\ast_{\tau^*+2} = 0 \text{ with probability } 1. \hspace{1cm} (D.204)$$
Under (D.204), we can invoke (B.75) to get

$$\Delta S^*_{\tau^*+2} = \frac{\phi}{\theta - \phi} S^*_{\tau^*+1} - \frac{\phi}{\theta - \phi} Z^*_{\tau^*+2} - \frac{C^*_{\tau^*+2}}{\theta - \phi} \tag{D.205}$$

which together with $E_{\tau^*+1}[\xi^*_{\tau^*+2}] = 0$ and (D.201) implies that

$$E_{\tau^*+1}[\Delta S^*_{\tau^*+2}] = \frac{\phi}{\theta - \phi} S^*_{\tau^*+1} - \frac{\phi}{\theta - \phi} \mu_z - \frac{E_{\tau^*+1}[C^*_{\tau^*+2}]}{\theta - \phi} \leq -\frac{\phi}{\theta - \phi} \frac{z}{(1 - \phi/\theta)\xi} < 0 \tag{D.206}$$

where the first inequality is by $E_{\tau^*+1}[C^*_{\tau^*+2}] \geq -c$ and $\theta > \phi$. This proves (D.200) and hence the equilibrium in time $\tau^* + 1$. Since $L^*_{\tau^*+1} = 0$, $L^*_{\tau^*+2} = 0$ and $\xi^*_{\tau^*+2} = 0$, at time $\tau^* + 2$, the equilibrium price satisfies

$$S^*_{\tau^*+2} = \frac{\theta}{\theta - \phi} S^*_{\tau^*+1} - \frac{\phi}{\theta - \phi} Z^*_{\tau^*+2} - \frac{C^*_{\tau^*+2}}{\theta - \phi} = \mu_z - \frac{z}{(1 - \phi/\theta)\xi} \frac{c\theta/\phi + C^*_{\tau^*+2}}{\theta - \phi}, \tag{D.207}$$

which implies that

$$S^*_{\tau^*+2} \leq \mu_z - \frac{z}{(1 - \phi/\theta)\xi} - \frac{c}{\phi} \leq S \tag{D.208}$$

regardless the demand of the informed speculators in time $\tau^* + 2$. By the law of motion of $L^*_t$ in (30),

$$L^*_{\tau^*+3} = 1 \text{ with probability 1.} \tag{D.209}$$

Therefore, by (D.201) and (D.209), we can invoke Lemma 21 to show $C^*_{\tau^*+2} = c$ which finishes the proof of the second equilibrium. 

**Lemma 24.** Suppose that the conditions of Corollary 9 hold. If

$$\tau_* \leq \tau \text{ and } \frac{z}{(1 - \phi/\theta)\tau^*} + \frac{c}{\phi} > \vartheta, \tag{D.210}$$

there is no equilibrium in time $\tau^* + 2$. By the law of motion of $L^*_t$ in (30),

$$L^*_{\tau^*+3} = 1 \text{ with probability 1.} \tag{D.209}$$

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Therefore, by (D.201) and (D.209), we can invoke Lemma 21 to show $C^*_{\tau^*+2} = c$ which finishes the proof of the second equilibrium.
then
\[ C^*_{\tau_*} = c \text{ and } S^*_{\tau_*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)^\tau_*} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} \] (D.211)

Proof of Lemma 24. Since \( \tau_* \leq \tau \), by the definition of \( \tau_* \), we have
\[ \tau_* = \tau_* \] (D.212)

By (D.170), we have \( L^*_{\tau_*} = 0 \) and \( L^*_{\tau_*+1} = 0 \). Since \( \zeta_{\tau_*+1} = 0 \) at time \( \tau_* + 1 \), by (B.75) and (D.170) with \( t = \tau_* + 1 \), we have
\[ S^*_{\tau_*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)^\tau_*} - \frac{\theta c/\phi + C^*_{\tau_*+1}}{\theta - \phi}. \] (D.213)

Since \( C^*_{\tau_*+1} \in \{-c, 0, c\} \),
\[ \frac{\theta c/\phi + C^*_{\tau_*+1}}{\theta - \phi} \leq \frac{\theta c/\phi - c}{\theta - \phi} = \frac{c}{\phi}, \] (D.214)

which implies that
\[ S^*_{\tau_*+1} > \vartheta. \] (D.215)

Hence by the law of motion of \( L^*_t \) in (7),
\[ L^*_{\tau_*+2} = 1 \] with probability \( \sigma \). (D.216)

The equilibrium in (D.211) follows by Lemma 21.

Lemma 25. Suppose that the conditions of Corollary 9 hold. If
\[ \tau_* < \tau \text{ and } \frac{z}{(1 - \phi/\theta)^\tau_*} + \frac{c}{\phi} \leq \vartheta, \] (D.217)

then there exist two possible equilibriums at time \( \tau_*+1 \). The first equilibrium is the same as the one derived in Lemma 24. In the second equilibrium, \( C^*_{\tau_*+1} = -c \), \( C^*_{\tau_*+2} = c \),
\[ S^*_{\tau_*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^\tau_*}, \] (D.218)

and
\[ S^*_{\tau_*+2} = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau_*+1}}. \] (D.219)
Proof of Lemma 25. In the proof of Lemma 24, we have show that

\[ \tau_* = \tau_* \quad \text{and} \quad S_{\tau_{*,+1}} = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_*}} - \frac{\theta c/\phi + C_{\tau_{*,+1}}^*}{\theta - \phi}. \]  

(D.220)

To prove that (D.211) is an equilibrium, it is sufficient to show that

\[ E_{\tau_{*,+1}}[\Delta S_{\tau_{*,+2}}^*] > 0. \]  

(D.221)

Given \( C_{\tau_{*,+1}}^* = c \), we have

\[ S_{\tau_{*,+1}}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_*}} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \]  

(D.222)

By the definition of \( \tau_* \),

\[ \left| S_{\tau_{*,+1}}^* - \mu_z \right| = \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} > \vartheta, \]  

(D.223)

which together with the law of motion of \( L_t^* \) in (7) implies that

\[ L_{\tau_{*,+2}}^* = 1 \]  

with probability at least \( \sigma \).  

(D.224)

Therefore, (D.221) can be proved with the same arguments in the proof of Lemma 24.

To derive the second equilibrium, it is sufficient to show that

\[ E_{\tau_{*,+1}}[\Delta S_{\tau_{*,+2}}^*] < 0. \]  

(D.225)

Given \( C_{\tau_{*,+1}}^* = -c \), we have

\[ S_{\tau_{*,+1}}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_*}} - \frac{c}{\phi}, \]  

(D.226)

which together with the second inequality in (D.217) implies that

\[ \left| S_{\tau_{*,+1}}^* - \mu_z \right| = \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{c}{\phi} \leq \vartheta. \]  

(D.227)
Moreover, since $\tau_* < \tau$,
\[
\mu_z - \frac{z}{(1 - \phi/\theta)\tau_*} - \frac{c}{\phi} \geq \mu_z - \frac{z}{(1 - \phi/\theta)\tau_*} - \frac{c}{\phi} > \mu_z - \frac{z}{(1 - \phi/\theta)\tau_*} - \frac{1 + \phi/\theta}{1 - \phi/\theta} c > S(D.228)
\]
where the last inequality is by the definition of $\tau$. Hence by the law of motion of $L^*_t$ in (7),
\[
L^*_t = 0 \text{ with probability } 1. \quad \text{(D.229)}
\]
Under (D.229), we can invoke (B.75) to get
\[
\Delta S^*_{\tau_* + 2} = \frac{\phi}{\theta - \phi} S^*_{\tau_* + 1} - \frac{\phi}{\theta - \phi} Z^*_{\tau_* + 2} - \frac{C^*_{\tau_* + 2}}{\theta - \phi} \quad \text{(D.230)}
\]
which together with (D.226) implies that
\[
E_{\tau_* + 1}[\Delta S^*_{\tau_* + 2}] = \frac{\phi}{\theta - \phi} P^*_{\tau_* + 1} - \frac{\phi}{\theta - \phi} \mu_z - \frac{E_{\tau_* + 1}[C^*_{\tau_* + 2}]}{\theta - \phi} = - \frac{\phi}{\theta - \phi} \frac{z}{(1 - \phi/\theta)\tau_*} - \frac{c + E_{\tau_* + 1}[C^*_{\tau_* + 2}]}{\theta - \phi} \leq - \frac{\phi}{\theta - \phi} \frac{z}{(1 - \phi/\theta)\tau_*} < 0. \quad \text{(D.231)}
\]
This proves (D.225). At time $\tau_* + 2$, the equilibrium price satisfies
\[
S^*_{\tau_* + 2} = \frac{\theta}{\theta - \phi} S^*_{\tau_* + 1} - \frac{\phi}{\theta - \phi} Z^*_{\tau_* + 2} - \frac{C^*_{\tau_* + 2}}{\theta - \phi} = \mu_z - \frac{z}{(1 - \phi/\theta)\tau_* + 1} - \frac{c\theta/\phi + C^*_{\tau_* + 2}}{\theta - \phi}, \quad \text{(D.232)}
\]
which implies that
\[
S^*_{\tau_* + 2} \leq \mu_z - \frac{z}{(1 - \phi/\theta)\tau_* + 1} - \frac{c}{\phi} \quad \text{(D.233)}
\]
regardless the demand of the informed speculators in time $\tau_* + 2$. Therefore,
\[
\left| S^*_{\tau_* + 2} - \mu_z \right| = \frac{z}{(1 - \phi/\theta)\tau_* + 1} + \frac{c}{\phi} > 0, \quad \text{(D.234)}
\]
which implies that
\[ L^\ast_{\tau+3} = 1 \text{ with probability at least } \sigma. \] (D.235)

Hence by Lemma 21,
\[ E_{\tau+2}[S^\ast_{\tau+3}] > 0, \quad C^\ast_{\tau+2} = c \quad \text{and} \quad S^\ast_{\tau+2} = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)\tau_1} \] (D.236)
which finishes the proof. \( \Box \)

**Lemma 26.** Suppose that the conditions of Corollary 9 hold. If
\[ \tau^* = \tau, \quad \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \leq \vartheta \quad \text{and} \quad \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^*}} - \frac{c}{\phi} > S \] (D.237)
then there exist two possible equilibriums at time \( \tau^* + 1 \). The first equilibrium is the same as the one derived in Lemma 22. In the second equilibrium, \( C^\ast_{\tau^*+1} = -c \), \( C^\ast_{\tau^*+2} = c \), \( L^\ast_{\tau^*+3} = 1 \) with probability 1,
\[ S^\ast_{\tau^*+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^{\tau^*}} \] (D.238)
and
\[ S^\ast_{\tau^*+2} = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau^*+1}}. \] (D.239)

**Proof of Lemma 26.** The proof of the first equilibrium follows the same arguments in the proof of Lemma 23, and hence is omitted. To derive the second equilibrium, we first show that
\[ E_{\tau^*+1}[S^\ast_{\tau^*+2}] < 0. \] (D.240)
Given \( C^\ast_{\tau^*+1} = -c \), we have
\[ S^\ast_{\tau^*+1} = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^*}} - \frac{c}{\phi}, \] (D.241)
which together with \( \tau^* = \tau^* \) and the first inequality in (D.237) implies that
\[ \left| S^\ast_{\tau^*+1} - \mu_z \right| = \frac{z}{(1 - \phi/\theta)^{\tau^*}} + \frac{c}{\phi} \leq \vartheta. \] (D.242)
Moreover by $\tau_* = \tau$ and the second inequality in (D.237),
\[ S_{\tau_* + 1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_*}} - \frac{c}{\phi} > S. \]  
(D.243)

Hence by (D.242), (D.243) and the law of motion of $L_t^*$ in (7),
\[ L_{\tau_* + 2}^* = 0 \text{ with probability } 1. \]  
(D.244)

Using the same arguments in the proof of Lemma 24 and Lemma 26, we can show that
\[ E_{\tau_* + 1}[\Delta S_{\tau_* + 2}^*] < 0, \]  
(D.245)

which verifies
\[ C_{\tau_* + 1}^* = -c \text{ and } S_{\tau_* + 1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_*}} - \frac{c}{\phi}. \]  
(D.246)

At time $\tau_* + 2$, the equilibrium price satisfies
\[ S_{\tau_* + 2}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_* + 1}} - \frac{c}{\phi} \]  
(D.247)

which implies that
\[ S_{\tau_* + 2}^* \leq \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_* + 1}} - \frac{c}{\phi} \]  
(D.248)

regardless the demand of the informed speculators in time $\tau_* + 2$. Therefore,
\[ S_{\tau_* + 2}^* \leq \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau_* + 1}} - \frac{c}{\phi} \leq S. \]  
(D.249)

which implies that
\[ L_{\tau_* + 3}^* = 1 \text{ with probability } 1. \]  
(D.250)

The rest of the proof follows the same arguments of the proof of Lemma 23 and hence is omitted.

Lemma 27. Suppose that $L_t^* = L_{t+1}^* = 0$, $\zeta_{t+1} = 0$ and
\[ S_t^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t-\tau^* + 1}} - 1 \right], \]  
(D.251)

where $\tau^* < t$. If the probability of liquidity crunch at $t + 2$ is larger than $\phi/\theta$, ...
then $E_{t+1}[\Delta S_{t+2}^*] > 0$, $C_{t+1}^* = c$ and
\[
S_{t+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)^{t+1}} - \frac{c}{\phi} \left[ \frac{2}{(1-\phi/\theta)^{t-\tau_*+2}} - 1 \right]. \tag{D.252}
\]

**Proof of Lemma 27.** Since $L_t^* = L_{t+1}^* = 0$ and $\zeta_{t+1} = 0$, by (B.75) and (D.177) with $t = \tau_* + 1$, we have
\[
S_{t+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)^{t+1}} - \frac{c}{\phi} \left[ \frac{2}{(1-\phi/\theta)^{t-\tau_*+2}} - 1 \right] + \frac{c - C_{t+1}^*}{\theta - \phi}. \tag{D.253}
\]

Let $\sigma_s$ denote the probability of $L_{t+2}^* = 1$. On the one hand if there is $L_{t+2}^* = 0$, (B.76) and (D.253) imply that
\[
\Delta S_{t+2}^* = \frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \frac{2}{(1-\phi/\theta)^{t-\tau_*+3}} - \frac{1}{(1-\phi/\theta)^2} \frac{c}{\theta}
- \phi C_{t+1}^* \frac{C_{t+2}^*}{(\theta - \phi)^2} - \frac{C_{t+2}^* + \phi \zeta_{t+2}^*}{\theta - \phi}. \tag{D.254}
\]

On the other hand if $L_{t+2}^* = 1$, equilibrium price equation (B.78) and (D.253) imply that
\[
\Delta S_{t+2}^* = \zeta_{t+2}^* + \frac{C_{t+2}^*}{\phi} + \frac{C_{t+1}^*}{\theta - \phi} + \frac{z}{(1-\phi/\theta)^t}
+ \frac{c}{\phi} \left[ \frac{2}{(1-\phi/\theta)^{t-\tau_*+2}} - 1 \right]. \tag{D.255}
\]

Combining the results in (D.254) and (D.255), and applying $E_{t+1}[\zeta_{t+2}] = 0$, we get
\[
E_{t+1}[\Delta S_{t+2}^*] = (1 - \sigma_s) \left[ \frac{-\phi z/\theta}{(1-\phi/\theta)^{t+1}} - \frac{E_{t+1}[C_{t+2}^*]}{\theta - \phi} - \frac{\phi C_{t+1}^*}{(\theta - \phi)^2} \right]
+ \sigma_s \left[ \frac{E_{t+1}[C_{t+2}^*]}{\phi} + \frac{C_{t+1}^*}{\theta - \phi} + \frac{z}{(1-\phi/\theta)^t}
+ \frac{c}{\phi} \left[ \frac{2}{(1-\phi/\theta)^{t-\tau_*+3}} - 1 \right] \right]
= A_{1,t}(\sigma_s) + A_{2,t}(\sigma_s, C_{t+1}^*, C_{t+2}^*) \tag{D.256}
\]
where

\[ A_{1,t}(\sigma_s) = \frac{z(\sigma_s - \phi/\theta)}{(1 - \phi/\theta)^{t+1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t+2}} - \frac{1}{1 - \phi/\theta} \right] \phi/\theta - \sigma_s \]

and

\[ A_{2,t}(\sigma_s, C_{t+1}^*, C_{t+2}^*) = \frac{\sigma_s - \phi/\theta}{1 - \phi/\theta} \frac{E_{t+1}[C_{t+2}^*]}{\phi} + \frac{\sigma_s - \phi/\theta}{(1 - \phi/\theta)^2} C_{t+1}^*. \]

Since \( \sigma_s > \phi/\theta \) and \( \theta > \phi > 0 \),

\[ A_{2,t}(\sigma_s, C_{t+1}^*, C_{t+2}^*) \geq -\frac{\sigma_s - \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} - \frac{\sigma_s - \phi/\theta}{1 - \phi/\theta} \frac{c}{(1 - \phi/\theta)^2} \frac{\phi/\theta - \sigma_s}{(1 - \phi/\theta)^2} \frac{c}{\phi} \]

regardless of the demand of the informed speculators. Moreover

\[ -\frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t+2}} - \frac{1}{1 - \phi/\theta} \right] \phi/\theta - \sigma_s \]

\[ = \frac{2c}{\phi} \frac{\sigma_s - \phi/\theta}{1 - \phi/\theta} \left[ \frac{1}{(1 - \phi/\theta)^{t+1}} - 1 \right]. \] (D.258)

Collecting the results in (D.256), (D.257) and (D.258), we have

\[ E_{t+1}[\Delta S_{t+2}^*] \geq \frac{z(\sigma_s - \phi/\theta)}{(1 - \phi/\theta)^{t+1}} + \frac{2c}{\phi} \frac{\sigma_s - \phi/\theta}{(1 - \phi/\theta)^2} \left[ \frac{1}{(1 - \phi/\theta)^{t+1}} - 1 \right] > 0 \]

where the second inequality is by \( \sigma_s > \phi/\theta \) and \( \theta > \phi > 0 \). By (D.259) and the demand law of the informed speculators, \( C_{t+1}^* = c \) which together with (D.253) implies that

\[ S_{t+1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^t} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t+2}} - 1 \right]. \] (D.260)

This finishes the proof. \( \blacksquare \)

**Lemma 28.** For any \( t > \tau_* \) and for any \( C^* \in \{-c, 0, c\} \),

\[ \frac{z}{(1 - \phi/\theta)^t} + \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{t+2}} - 1 \right] + \frac{C^*}{\theta - \phi} > \theta. \] (D.261)
Proof of Lemma 28. Since $\frac{\phi}{\theta} \in (0, 1)$,

$$\frac{1}{1 - \frac{\phi}{\theta}} = \sum_{k=0}^{\infty} \left(\frac{\phi}{\theta}\right)^k > 1 + \frac{\phi}{\theta} \tag{D.262}$$

which implies that

$$\frac{2}{(1 - \frac{\phi}{\theta})^{t - \tau_\ast} + 1} \geq \frac{2}{1 - \frac{\phi}{\theta}} \frac{1}{\frac{\phi}{\theta}} > \frac{2(1 + \frac{\phi}{\theta})}{1 - \frac{\phi}{\theta}}. \tag{D.263}$$

Therefore,

$$\frac{c}{\phi} \left[ \frac{2}{(1 - \frac{\phi}{\theta})^{t - \tau_\ast} + 1} - \frac{1}{1 - \frac{\phi}{\theta}} \right] + \frac{C^*}{\theta - \phi} > \frac{c}{\phi} \frac{1 + \frac{\phi}{\theta}}{1 - \frac{\phi}{\theta}} + \frac{c + C^*}{\theta - \phi} \geq \frac{c}{\phi} \frac{1 + \frac{\phi}{\theta}}{1 - \frac{\phi}{\theta}} \tag{D.264}$$

for any $C^* \in \{-c, 0, c\}$. By (D.255),

$$\frac{z}{(1 - \frac{\phi}{\theta})^t} + \frac{c}{\phi} \left[ \frac{2}{(1 - \frac{\phi}{\theta})^{t - \tau_\ast} + 1} - \frac{1}{1 - \frac{\phi}{\theta}} \right] + \frac{C^*}{\theta - \phi} > \frac{z}{(1 - \frac{\phi}{\theta})^t} + \frac{c}{\phi} \frac{1 + \frac{\phi}{\theta}}{1 - \frac{\phi}{\theta}}$$

for any $t > \tau_\ast$ and any $C^* \in \{-c, 0, c\}$, which together with the definition of $\tau_\ast$ proves (D.261). \qed

Proof of Corollary 9. The claim in part (i) and part (vi) of the theorem follow by Proposition 5.

We next prove the results in parts (ii)-(v). In Lemma 19 and Lemma 20, we have show that

$$S_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \frac{\phi}{\theta})^{t-1}}, \quad C_t^* = -c, \quad L_t^* = 0 \text{ and } L_{t+1}^* = 0 \tag{D.265}$$

for any $t$ with $1 \leq t \leq \tau_\ast$. The momentum speculators’ demand is 0 at time 0, 1 and $t^*$ due to liquidity crunch. For any $t$ with $2 \leq t \leq \tau_\ast$, $L_{t-1}^* = 0$ and $L_t^* = 0$ by (D.265). Hence the demand of the momentum speculators’ demand
is not zero. By the first equality in (D.265),

\[ \triangle S_t^* = -\frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{z}{(1 - \phi/\theta)^{t-2}} = -\frac{z\phi/\theta}{(1 - \phi/\theta)^{t-1}} \]  
(D.266)

which together with the demand law of the momentum speculators implies that

\[ q_t^* = -\frac{z\phi}{(1 - \phi/\theta)^{t-1}} \text{ for any } t \text{ with } 2 \leq t \leq \tau^*. \]  
(D.267)

To prove the rest of the results, we have to consider 3 different cases.

**Case 1.** \( \tau_* = \tau \).

**Case 1.1.** \( \mu_z - \frac{z}{(1 - \phi/\theta)\xi} - \frac{c}{\phi} \leq S \). By Lemma 22, \( \tau^* = \tau + 1 \) and \( l^* = \tau + 2 \).

At time \( \tau^* \), \( C_{\tau_*}^* = c \) and

\[ S_{\tau_*}^* = \mu_z - \frac{z}{(1 - \phi/\theta)\xi} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}. \]  
(D.268)

By Lemma 20,

\[ S_{\xi}^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)\xi - 1}, \]  
(D.269)

which together with \( \tau^* = \tau + 1 \) and (D.268) implies that

\[ \triangle S_{\tau_*}^* = -\frac{2c/\theta}{1 - \phi/\theta} - \frac{z\phi/\theta}{1 - \phi/\theta} \]  
(D.270)

By the demand law of the momentum speculators,

\[ q_{m\tau_*}^* = \frac{-2c}{1 - \phi/\theta} - \frac{z\phi}{(1 - \phi/\theta)\xi} = \frac{-2c/\theta}{1 - \phi/\theta} - \frac{z\phi}{(1 - \phi/\theta)^{\tau_*-1}} - \frac{2c}{1 - \phi/\theta}. \]  
(D.271)

By Condition 8, \( S_{\tau_*}^* > 0 \). Since the equilibrium prices are decreasing and \( l^* = \tau^* + 1 \), we have \( S_t^* > 0 \) for any \( t \) with \( 1 \leq t < l^* \). This proves the results when \( \mu_z - \frac{z}{(1 - \phi/\theta)\xi} - \frac{c}{\phi} \leq S \).

**Case 1.2.** \( \frac{z}{(1 - \phi/\theta)\tau} + \frac{c}{\phi} > \theta \). By Lemma 24,

\[ C_{\xi+1}^* = c \text{ and } S_{\xi+1}^* = \mu_z - \frac{z}{(1 - \phi/\theta)\tau} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}, \]  
(D.272)

which together with Lemma 20 implies that \( \tau^* = \tau_* + 1 \). By (D.269) and (D.272),

\[ \triangle S_{\tau_*}^* = -\frac{2c/\theta}{1 - \phi/\theta} - \frac{z\phi/\theta}{(1 - \phi/\theta)^{\tau}}. \]  
(D.273)
which together with the demand law of the momentum speculators implies

$$q_m^* = -\frac{z\phi}{(1 - \phi/\theta)^{\tau^*+1}} - \frac{2c}{1 - \phi/\theta}$$

(D.274)

Since $\tau^* = \tau$, by (D.272) and the definition of $\tau$,

$$S_{\tau^*+1}^* = \mu z - \frac{z}{(1 - \phi/\theta)\tau} - \frac{c}{\phi} \frac{1 + \phi/\theta}{(1 - \phi/\theta)} \leq S$$

(D.275)

which combined with (30) implies that

$$L_{\tau^*+1}^* = 1 \text{ with probability 1.}$$

(D.276)

Hence $l^* = \tau + 2 = \tau^* + 1$. By Condition 8, $S_{\tau^*}^* > 0$. Since the equilibrium prices are decreasing and $l^* = \tau^* + 1$, we have $S_t^* > 0$ for any $t$ with $1 \leq t < l^*$. This proves the results when $\tau^* = \tau$.}

**Case 1.3.** $\mu z - \frac{z}{(1 - \phi/\theta)\tau} > S$ and $\frac{c}{\phi} \frac{1 + \phi/\theta}{(1 - \phi/\theta)} \leq \vartheta$. By Lemma 26, there are two equilibriums at time $\tau^* + 1$. The first equilibrium is the same as the one derived in Lemma 22. Hence the proof of parts (ii)-(v) follows the same arguments in case 1.1. In the second equilibrium, $C_{\tau^*+1}^* = -c$, $C_{\tau^*+2}^* = c$, $L_{\tau^*+3}^* = 1 \text{ with probability 1,}$

$$S_{\tau^*+1}^* = \mu z - \frac{c}{\phi} \frac{z}{(1 - \phi/\theta)\tau^*}$$

(D.277)

and

$$S_{\tau^*+2}^* = \mu z - \frac{c}{\phi} \frac{1 + \phi/\theta}{(1 - \phi/\theta)} \frac{z}{(1 - \phi/\theta)^{\tau^*+1}}.$$  

(D.278)

Hence we have $\tau^* = \tau + 2$ and $l^* = \tau + 3$. Note that

$$\triangle S_{\tau^*+1}^* = -\frac{z\phi}{(1 - \phi/\theta)^{\tau}} \text{ and } \triangle S_{\tau^*+2}^* = \frac{2c/\theta}{1 - \phi/\theta} - \frac{z\phi/\theta}{(1 - \phi/\theta)^{\tau+1}}.$$  

(D.279)

which together with the demand law of the momentum speculators implies

$$q_{\tau^*+1}^* = -\frac{z\phi}{(1 - \phi/\theta)^{\tau}} \text{ and } q_{\tau^*+2}^* = \frac{2c}{1 - \phi/\theta} - \frac{z\phi}{(1 - \phi/\theta)^{\tau+1}}.$$  

(D.280)

By Condition 8, $S_{\tau^*}^* > 0$. Since the equilibrium prices are decreasing and $l^* = \tau + 3$, we have $S_t^* > 0$ for any $t$ with $1 \leq t < l^*$. This finishes the proof in the case that $\tau^* = \tau$.}

18
Case 2. $\tau < \tau_*$.  

Case 2.1. $\mu_z - \frac{z}{(1-\phi/\theta)\tau} - \frac{c}{\phi} \leq S$. By Lemma 22,

$$C_{\tau_*+1}^* = c, \ S_{\tau_*+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)\tau} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}$$

and $l^* = \tau + 2$.  \hspace{1cm} (D.281)

The proof follows the same arguments above in case 1.1.

Case 2.2. $\mu_z - \frac{z}{(1-\phi/\theta)\tau} - \frac{c}{\phi} > S$. By Lemma 23, there exist and only exist two possible equilibriums at time $\tau_* + 1$. The first equilibrium is the same as the one derived in Lemma 22 and hence the proof of parts (ii)-(v) follows the same arguments above in case 1.1. when $\tau = \tau_*$. In the second equilibrium, $C_{\tau_*+1}^* = -c, \ C_{\tau_*+2}^* = c, \ l^* = \tau + 3$,

$$S_{\tau_*+1}^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1-\phi/\theta)\tau}$$ \hspace{1cm} (D.282)

and

$$S_{\tau_*+2}^* = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1-\phi/\theta)\tau_*+1}.$$ \hspace{1cm} (D.283)

Since $\tau_* = \tau$, the proof follows the same arguments above in case 1.3.

Case 3. $\tau_* < \tau$.  

Case 3.1. $\frac{z}{(1-\phi/\theta)\tau} + \frac{c}{\phi} > \vartheta$. By Lemma 24,

$$C_{\tau_*+1}^* = c \text{ and } S_{\tau_*+1}^* = \mu_z - \frac{z}{(1-\phi/\theta)\tau_*} - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta}.$$ \hspace{1cm} (D.284)

which together with Lemma 20 implies that $\tau^* = \tau_* + 1$. Moreover,

$$\triangle S_{\tau_*+1}^* = \frac{-2c/\theta}{1 - \phi/\theta} - \frac{z\phi/\theta}{(1-\phi/\theta)\tau_*}.$$ \hspace{1cm} (D.285)

which together with the demand law of the momentum speculators implies that

$$q_{\lambda, \tau_*+1}^{m*} = \frac{-2c}{1 - \phi/\theta} - \frac{z\phi}{(1-\phi/\theta)\tau_*}.$$ \hspace{1cm} (D.286)

This proves (35) (36) and (37) for $t \leq \tau^*$. Suppose that the results in (35) (36) and (37) hold for $t = \tau^* + k - 1$, $L_{\tau^*+k-1}^* = 0$ and $L_{\tau^*+k}^* = 0$ for some $k \geq 1$. 


We next show that (35) (36) and (37) also hold for \( t = \tau^* + k \). By (B.75),
\[
S^*_{\tau^*+k} = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^*+k-1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{\tau^*+k-\tau^*}} - \frac{1}{1 - \phi/\theta} \right] - \frac{C^*_{\tau^*+k}}{\theta - \phi},
\]  
(D.287)
Since \( \tau^* + k - 1 > \tau^* \), by Lemma 28,
\[
|S^*_{\tau^*+k} - \mu_z| > \vartheta
\]  
(D.288)
which together with (30) implies that
\[
L^*_{\tau^*+k+1} = 1 \text{ with probability at least } \sigma.
\]  
(D.289)
By Lemma 27, \( C^*_{t+1} = c \) and
\[
S^*_{\tau^*+k} = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^*+k-1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{\tau^*+k-\tau^*+1}} - 1 \right].
\]  
(D.290)
Moreover,
\[
\Delta S^*_{\tau^*+k} = \frac{-\phi z/\theta}{(1 - \phi/\theta)^{\tau^*+k-1}} - \frac{2c}{\theta} \frac{1}{(1 - \phi/\theta)^{\tau^*+k-\tau^*+1}}
\]  
(D.291)
which together with the demand law of the momentum speculators implies that
\[
q^m_{\tau^*+k} = \frac{-z \phi}{(1 - \phi/\theta)^{\tau^*+k-1}} - \frac{2c}{(1 - \phi/\theta)^{\tau^*+k-\tau^*+1}}.
\]  
(D.292)
This shows that (35) (36) and (37) hold for \( t = \tau^* + k \). Hence they hold for any \( t \) with \( t < l^* \). By (35), the equilibrium prices are strictly decreasing along the negative bubbly path. By (D.288) and (30),
\[
L^*_t = 1 \text{ with probability at least } \sigma.
\]  
(D.293)
for any \( t \) with \( \tau^* \leq t < l^* \). Hence with nonzero probability, the liquidity crunch may be triggered at a time before \( \tau_e \). In such case, by (35),
\[
S^*_t > \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^*-1}} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)^{\tau^*-\tau^*}} - 1 \right]
\]  
(D.294)
for any \( t \) with \( 1 \leq t < l^* \), which together with Condition 8 implies that \( S^*_t > 0 \)
for any $t$ with $1 \leq t < l^*$. On the other hand, if the liquidity crunch is not triggered before $\tau_e$ then at time $\tau_e$ by (35)

$$S^*_\tau_e = \mu_z - \frac{z}{(1 - \phi/\theta)\tau_e - 1} - \frac{c}{\phi} \left[ \frac{2}{(1 - \phi/\theta)\tau_e - \tau^* + 1} - 1 \right] \quad \text{(D.295)}$$

which together with the definition of $\tau_e$ implies that

$$S^*_\tau_e \leq S.$$ \quad \text{(D.296)}

Hence by (30)

$$L^*_{\tau_e+1} = 1 \text{ with probability } 1 \quad \text{(D.297)}$$

which implies that $l^* = \tau_e + 1$. By (D.295) and $\tau^* = \tau_e + 1$, we can use Condition 8 to deduce that $S^*_t > 0$. Since $l^* = \tau_e + 1$ and $S^*_t > S^*_\tau_e$ for any $t$ with $1 \leq t < \tau_e$, we have $S^*_t > 0$ for any $t$ with $1 \leq t < l^*$.

**Case 3.2.** $\frac{\tilde{z}}{(1 - \phi/\theta)^\tau_e} + \frac{c}{\phi} \leq \vartheta$. By Lemma 25, there are two equilibria. The first equilibrium is the same as the one derived in Lemma 24 and hence the proof of parts (ii)-(iv) follows the same arguments in case 3.1. In the second equilibrium, $C^*_{\tau_e+1} = -c$, $C^*_{\tau_e+2} = c$,

$$S^*_{\tau_e+1} = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)\tau_e},$$

and

$$S^*_{\tau_e+2} = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)\tau_e + 1},$$

which together with Lemma 20 implies that $\tau^* = \tau_e + 2$. Moreover,

$$\delta^*\tau_e = -\frac{2c}{1 - \phi/\theta} - \frac{z\phi}{(1 - \phi/\theta)\tau_e + 1},$$

which together with the demand law of the momentum speculators implies that

$$\theta^*_{\tau_e+1} = -\frac{2c}{1 - \phi/\theta} - \frac{z\phi}{(1 - \phi/\theta)\tau_e + 1}.$$  

This proves the results in (35) (36) and (37) for $t \leq \tau^*$. Using similar arguments in case 3.1, we can show that if parts (ii)-(v) hold for $t = \tau^* + k - 1$ for some $k \geq 1$, then they also hold for $t = \tau^* + k$. Hence results in (35) (36) and (37) of the corollary hold for any $t < l^*$. Using the same arguments in case 3.1, we can show that $S^*_t > 0$ for any $t$ with $1 \leq t < l^*$. This finishes the proof. □
Constructing the LRV Estimators

For any weakly dependent process \( \{ W_{t,n} \}_{t=1}^{n} \) with

\[ E[W_{t,n}] = 0 \text{ for all } t \text{ and } n, \quad (E.298) \]

and finite positive LRV \( V_{W} \), its sample autocovariance can be defined as

\[ \Gamma_{W,n}(j) = \frac{1}{n-j} \sum_{t=1}^{n-j} (W_{t,n} - \overline{W}_{n})(W_{t+j,n} - \overline{W}_{n}) \quad (E.299) \]

for \( j = 0, \ldots, n-1 \). It is clear that the sample autocovariance satisfies \( \Gamma_{W,n}(-j) = \Gamma_{W,n}(j) \) for \( j = 0, \ldots, n-1 \). Note that the sample autocovariance is sample mean centered, which improves the power of the test of the hypothesis in (E.298).

The kernel based LRV estimator for \( \{ W_{t,n} \}_{t=1}^{n} \) is then defined as

\[ V_{W,n} = \sum_{j=-n+1}^{n+1} K(j/M) \Gamma_{W,n}(j) \quad (E.300) \]

where \( K(\cdot) \) is some kernel smoothing function with bandwidth \( M \). Under some regularity conditions (see, e.g., Newey and West (1987) and Andrews (1991)), there is

\[ V_{W,n} \to_p V_{W}. \quad (E.301) \]

One key condition for the above consistency result is that \( M \) goes to infinity at certain rate. In finite samples, there are two different rules of selecting \( M \): One is the rule proposed in Newey and West (1994) and the other is the parametric (AR(1)) approximation rule in Andrews (1991).

Table E.1: Construction of LRV Estimators

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{a,n} )</td>
<td>( D_{t,h}(S_{t+h}^{<em>} - S_{t}^{</em>}) )</td>
</tr>
<tr>
<td>( T_{b,n} )</td>
<td>( (D_{t,h} - \overline{D}<em>{n,h})(D</em>{t,h}^{<em>} - \overline{D}_{n,h}^{</em>}) )</td>
</tr>
</tbody>
</table>
In the rest of this appendix, we briefly describe how to construct the LRV estimators for the test statistics presented in the main text. The Newey-West and Andrews LRV estimators can be constructed using the formula in (E.299). Hence for each test statistic, we only need to define its corresponding $W_{t,n}$ for the construction of LRV estimators, which are summarized in Table E.1. For the ease of notation, we ignore the index $i$ in each test statistic.
F Extra Results

Figure F.1: Net Speculators’ Position

Note: This Figure presents the net position of speculators normalized by open interest. The data is from the Commitment-of-Traders Report of the CFTC.
Figure F.2: Evolution of Estimated $c_t$

Note: This figure presents the estimated means of each state in the Markov-switching model. The blue and yellow lines depict the estimated mean of the appreciation and depreciation state, respectively. Meanwhile the red line represents the estimated mean of the range state.
Figure F.3: Evolution of Estimated Filtered Probabilities

Note: This figure presents the estimated filtered probabilities of the states in the Markov-switching model.
Figure F.4: Evolution of Estimated $\theta_t$

Note: This figure presents the estimated coefficient corresponding to the exchange rate term in the Markov-switching model. The blue solid line depicts the estimated coefficient and the orange dashed line depicts the lower 5% confidence interval of the estimated coefficient.
Figure F.5: Cumulative Forecast Success Ratio ($h = 1m$)

Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.
Figure F.6: Cumulative Forecast Success Ratio \((h = 3m)\)

Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.
Figure F.7: Cumulative Forecast Success Ratio ($h = 6m$)

Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.
Figure F.8: Cumulative Forecast Success Ratio \((h = 9m)\)

Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.
Figure F.9: Cumulative Forecast Success Ratio ($h = 12m$)

Note: This figure plots the cumulative forecasting success ratio, which is defined as the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.
Table F.1: Linear Granger-causality Test Results

<table>
<thead>
<tr>
<th>Currency</th>
<th>$H_0$: FX Do Not Cause COT</th>
<th>Probability</th>
<th>$H_0$: COT Do Not Cause FX</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>4.703</td>
<td>0.000</td>
<td>0.846</td>
<td>0.774</td>
</tr>
<tr>
<td>CAD</td>
<td>3.906</td>
<td>0.000</td>
<td>0.673</td>
<td>0.964</td>
</tr>
<tr>
<td>EUR</td>
<td>4.306</td>
<td>0.000</td>
<td>1.458</td>
<td>0.021</td>
</tr>
<tr>
<td>JPY</td>
<td>6.970</td>
<td>0.000</td>
<td>0.748</td>
<td>0.908</td>
</tr>
<tr>
<td>GBP</td>
<td>5.895</td>
<td>0.000</td>
<td>0.791</td>
<td>0.857</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Currency</th>
<th>$H_0$: ΔFX Do Not Cause ΔCOT</th>
<th>Probability</th>
<th>$H_0$: ΔCOT Do Not Cause ΔFX</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>4.656</td>
<td>0.000</td>
<td>0.792</td>
<td>0.855</td>
</tr>
<tr>
<td>CAD</td>
<td>3.710</td>
<td>0.000</td>
<td>0.705</td>
<td>0.944</td>
</tr>
<tr>
<td>EUR</td>
<td>4.375</td>
<td>0.000</td>
<td>1.370</td>
<td>0.046</td>
</tr>
<tr>
<td>JPY</td>
<td>6.787</td>
<td>0.000</td>
<td>0.796</td>
<td>0.850</td>
</tr>
<tr>
<td>GBP</td>
<td>5.690</td>
<td>0.000</td>
<td>0.808</td>
<td>0.833</td>
</tr>
</tbody>
</table>

Note: This table reports the results of the linear Granger-causality test. The number of lags included is 52 weeks. However, the results are robust with respect to the number of lags. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.2: Robustness of the MSM Estimation Results (Australian Dollar)

<table>
<thead>
<tr>
<th></th>
<th>$R = 70$</th>
<th>$R = 80$</th>
<th>$R = 90$</th>
<th>$R = 100$</th>
<th>$R = 110$</th>
<th>$R = 120$</th>
<th>$R = 130$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(R = 70, k = 4)$</td>
<td>1.00</td>
<td>0.95</td>
<td>0.92</td>
<td>0.89</td>
<td>0.90</td>
<td>0.88</td>
<td>0.87</td>
</tr>
<tr>
<td>$(R = 80, k = 4)$</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
<td>0.93</td>
<td>0.92</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>$(R = 90, k = 4)$</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
<td>0.96</td>
<td>0.95</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>$(R = 100, k = 4)$</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>$(R = 110, k = 4)$</td>
<td>0.90</td>
<td>0.92</td>
<td>0.95</td>
<td>0.97</td>
<td>1.00</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>$(R = 120, k = 4)$</td>
<td>0.88</td>
<td>0.91</td>
<td>0.93</td>
<td>0.95</td>
<td>0.96</td>
<td>1.00</td>
<td>0.97</td>
</tr>
<tr>
<td>$(R = 130, k = 4)$</td>
<td>0.87</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
<td>0.95</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>$(R = 70, k = 8)$</td>
<td>0.72</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$(R = 80, k = 8)$</td>
<td>0.89</td>
<td>0.92</td>
<td>0.91</td>
<td>0.90</td>
<td>0.90</td>
<td>0.88</td>
<td>0.87</td>
</tr>
<tr>
<td>$(R = 90, k = 8)$</td>
<td>0.87</td>
<td>0.90</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>$(R = 100, k = 8)$</td>
<td>0.86</td>
<td>0.88</td>
<td>0.91</td>
<td>0.92</td>
<td>0.91</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>$(R = 110, k = 8)$</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.91</td>
<td>0.92</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>$(R = 120, k = 8)$</td>
<td>0.83</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>$(R = 130, k = 8)$</td>
<td>0.83</td>
<td>0.85</td>
<td>0.87</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Panel A: $k = 4$

Panel B: $k = 8$

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$
Table F.3: Robustness of the MSM Estimation Results (Canadian Dollar)

<table>
<thead>
<tr>
<th></th>
<th>R = 70</th>
<th>R = 80</th>
<th>R = 90</th>
<th>R = 100</th>
<th>R = 110</th>
<th>R = 120</th>
<th>R = 130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: k = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R = 70, k = 4)</td>
<td>1.00</td>
<td>0.94</td>
<td>0.93</td>
<td>0.91</td>
<td>0.89</td>
<td>0.87</td>
<td>0.84</td>
</tr>
<tr>
<td>(R = 80, k = 4)</td>
<td>0.94</td>
<td>1.00</td>
<td>0.96</td>
<td>0.94</td>
<td>0.92</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>(R = 90, k = 4)</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>0.94</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>(R = 100, k = 4)</td>
<td>0.91</td>
<td>0.94</td>
<td>0.96</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>(R = 110, k = 4)</td>
<td>0.89</td>
<td>0.92</td>
<td>0.94</td>
<td>0.97</td>
<td>1.00</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>(R = 120, k = 4)</td>
<td>0.87</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td>0.95</td>
<td>1.00</td>
<td>0.96</td>
</tr>
<tr>
<td>(R = 130, k = 4)</td>
<td>0.84</td>
<td>0.87</td>
<td>0.88</td>
<td>0.90</td>
<td>0.92</td>
<td>0.96</td>
<td>1.00</td>
</tr>
</tbody>
</table>

|                  |        |        |        |         |         |         |         |
| (R = 70, k = 8)  | 0.67   | 0.69   | 0.66   | 0.67    | 0.68    | 0.66    | 0.65    |
| (R = 80, k = 8)  | 0.87   | 0.89   | 0.88   | 0.87    | 0.86    | 0.83    | 0.81    |
| (R = 90, k = 8)  | 0.84   | 0.87   | 0.89   | 0.88    | 0.87    | 0.84    | 0.82    |
| (R = 100, k = 8) | 0.82   | 0.85   | 0.88   | 0.89    | 0.88    | 0.85    | 0.83    |
| (R = 110, k = 8) | 0.82   | 0.84   | 0.86   | 0.88    | 0.89    | 0.87    | 0.85    |
| (R = 120, k = 8) | 0.80   | 0.83   | 0.84   | 0.85    | 0.87    | 0.88    | 0.86    |
| (R = 130, k = 8) | 0.77   | 0.81   | 0.83   | 0.84    | 0.85    | 0.87    | 0.88    |

|                  |        |        |        |         |         |         |         |
| (R = 70, k = 4)  | 0.67   | 0.69   | 0.68   | 0.67    | 0.68    | 0.66    | 0.66    |
| (R = 80, k = 4)  | 0.66   | 0.68   | 0.69   | 0.68    | 0.68    | 0.68    | 0.66    |
| (R = 90, k = 4)  | 0.65   | 0.64   | 0.65   | 0.64    | 0.64    | 0.64    | 0.64    |
| (R = 100, k = 4) | 0.68   | 0.68   | 0.68   | 0.68    | 0.68    | 0.68    | 0.68    |
| (R = 110, k = 4) | 0.68   | 0.67   | 0.67   | 0.67    | 0.67    | 0.67    | 0.67    |
| (R = 120, k = 4) | 0.68   | 0.67   | 0.67   | 0.67    | 0.67    | 0.67    | 0.67    |
| (R = 130, k = 4) | 0.68   | 0.67   | 0.67   | 0.67    | 0.67    | 0.67    | 0.67    |

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$.
Table F.4: Robustness of the MSM Estimation Results (Euro)

<table>
<thead>
<tr>
<th></th>
<th>( R = 70 )</th>
<th>( R = 80 )</th>
<th>( R = 90 )</th>
<th>( R = 100 )</th>
<th>( R = 110 )</th>
<th>( R = 120 )</th>
<th>( R = 130 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (R = 70, k = 4) )</td>
<td>1.00</td>
<td>0.94</td>
<td>0.91</td>
<td>0.86</td>
<td>0.82</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>( (R = 80, k = 4) )</td>
<td>0.94</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
<td>0.87</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>( (R = 90, k = 4) )</td>
<td>0.91</td>
<td>0.95</td>
<td>1.00</td>
<td>0.94</td>
<td>0.91</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>( (R = 100, k = 4) )</td>
<td>0.86</td>
<td>0.90</td>
<td>0.94</td>
<td>1.00</td>
<td>0.96</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>( (R = 110, k = 4) )</td>
<td>0.82</td>
<td>0.87</td>
<td>0.91</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>( (R = 120, k = 4) )</td>
<td>0.80</td>
<td>0.85</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.97</td>
</tr>
<tr>
<td>( (R = 130, k = 4) )</td>
<td>0.80</td>
<td>0.86</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>( (R = 70, k = 8) )</td>
<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
<td>0.83</td>
<td>0.81</td>
<td>0.80</td>
<td>0.79</td>
</tr>
<tr>
<td>( (R = 80, k = 8) )</td>
<td>0.88</td>
<td>0.91</td>
<td>0.90</td>
<td>0.87</td>
<td>0.85</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>( (R = 90, k = 8) )</td>
<td>0.84</td>
<td>0.88</td>
<td>0.90</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>( (R = 100, k = 8) )</td>
<td>0.80</td>
<td>0.83</td>
<td>0.86</td>
<td>0.90</td>
<td>0.90</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>( (R = 110, k = 8) )</td>
<td>0.79</td>
<td>0.83</td>
<td>0.85</td>
<td>0.90</td>
<td>0.92</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>( (R = 120, k = 8) )</td>
<td>0.75</td>
<td>0.82</td>
<td>0.85</td>
<td>0.89</td>
<td>0.90</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>( (R = 130, k = 8) )</td>
<td>0.73</td>
<td>0.80</td>
<td>0.84</td>
<td>0.89</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Panel B: \( k = 8 \)

<table>
<thead>
<tr>
<th></th>
<th>( R = 70 )</th>
<th>( R = 80 )</th>
<th>( R = 90 )</th>
<th>( R = 100 )</th>
<th>( R = 110 )</th>
<th>( R = 120 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (R = 70, k = 4) )</td>
<td>0.89</td>
<td>0.88</td>
<td>0.84</td>
<td>0.80</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>( (R = 80, k = 4) )</td>
<td>0.89</td>
<td>0.91</td>
<td>0.88</td>
<td>0.83</td>
<td>0.83</td>
<td>0.82</td>
</tr>
<tr>
<td>( (R = 90, k = 4) )</td>
<td>0.88</td>
<td>0.90</td>
<td>0.90</td>
<td>0.86</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>( (R = 100, k = 4) )</td>
<td>0.83</td>
<td>0.87</td>
<td>0.89</td>
<td>0.90</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>( (R = 110, k = 4) )</td>
<td>0.81</td>
<td>0.85</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>0.90</td>
</tr>
<tr>
<td>( (R = 120, k = 4) )</td>
<td>0.80</td>
<td>0.84</td>
<td>0.87</td>
<td>0.88</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>( (R = 130, k = 4) )</td>
<td>0.79</td>
<td>0.84</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>( (R = 70, k = 8) )</td>
<td>1.00</td>
<td>0.94</td>
<td>0.89</td>
<td>0.84</td>
<td>0.83</td>
<td>0.80</td>
</tr>
<tr>
<td>( (R = 80, k = 8) )</td>
<td>0.94</td>
<td>1.00</td>
<td>0.93</td>
<td>0.88</td>
<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td>( (R = 90, k = 8) )</td>
<td>0.89</td>
<td>0.93</td>
<td>1.00</td>
<td>0.93</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>( (R = 100, k = 8) )</td>
<td>0.84</td>
<td>0.88</td>
<td>0.93</td>
<td>1.00</td>
<td>0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>( (R = 110, k = 8) )</td>
<td>0.83</td>
<td>0.87</td>
<td>0.91</td>
<td>0.96</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>( (R = 120, k = 8) )</td>
<td>0.80</td>
<td>0.85</td>
<td>0.89</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>( (R = 130, k = 8) )</td>
<td>0.76</td>
<td>0.82</td>
<td>0.87</td>
<td>0.90</td>
<td>0.92</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Note: This table reports the pairwise correlation of the MSM estimation results with \( k \in \{4, 8\} \) and \( R \in \{70, 80, 90, 100, 110, 120, 130\} \)
Table F.5: Robustness of the MSM Estimation Results (Japanese Yen)

<table>
<thead>
<tr>
<th></th>
<th>$R = 70$</th>
<th>$R = 80$</th>
<th>$R = 90$</th>
<th>$R = 100$</th>
<th>$R = 110$</th>
<th>$R = 120$</th>
<th>$R = 130$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(R = 70, k = 4)$</td>
<td>1.00</td>
<td>0.95</td>
<td>0.92</td>
<td>0.89</td>
<td>0.87</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>$(R = 80, k = 4)$</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
<td>0.92</td>
<td>0.90</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>$(R = 90, k = 4)$</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
<td>0.93</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>$(R = 100, k = 4)$</td>
<td>0.89</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
<td>0.96</td>
<td>0.94</td>
<td>0.93</td>
</tr>
<tr>
<td>$(R = 110, k = 4)$</td>
<td>0.87</td>
<td>0.90</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>$(R = 120, k = 4)$</td>
<td>0.85</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
<td>0.98</td>
<td>1.00</td>
<td>0.97</td>
</tr>
<tr>
<td>$(R = 130, k = 4)$</td>
<td>0.83</td>
<td>0.86</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>$(R = 70, k = 8)$</td>
<td>0.65</td>
<td>0.66</td>
<td>0.68</td>
<td>0.67</td>
<td>0.65</td>
<td>0.64</td>
<td>0.65</td>
</tr>
<tr>
<td>$(R = 80, k = 8)$</td>
<td>0.88</td>
<td>0.90</td>
<td>0.89</td>
<td>0.87</td>
<td>0.86</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td>$(R = 90, k = 8)$</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.89</td>
<td>0.88</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>$(R = 100, k = 8)$</td>
<td>0.85</td>
<td>0.87</td>
<td>0.89</td>
<td>0.91</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>$(R = 110, k = 8)$</td>
<td>0.85</td>
<td>0.87</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
<td>0.92</td>
<td>0.91</td>
</tr>
<tr>
<td>$(R = 120, k = 8)$</td>
<td>0.84</td>
<td>0.85</td>
<td>0.88</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>$(R = 130, k = 8)$</td>
<td>0.81</td>
<td>0.84</td>
<td>0.86</td>
<td>0.89</td>
<td>0.91</td>
<td>0.92</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Panel B: $k = 8$

<table>
<thead>
<tr>
<th></th>
<th>$R = 70$</th>
<th>$R = 80$</th>
<th>$R = 90$</th>
<th>$R = 100$</th>
<th>$R = 110$</th>
<th>$R = 120$</th>
<th>$R = 130$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(R = 70, k = 4)$</td>
<td>0.65</td>
<td>0.88</td>
<td>0.86</td>
<td>0.85</td>
<td>0.85</td>
<td>0.84</td>
<td>0.81</td>
</tr>
<tr>
<td>$(R = 80, k = 4)$</td>
<td>0.66</td>
<td>0.90</td>
<td>0.88</td>
<td>0.87</td>
<td>0.87</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>$(R = 90, k = 4)$</td>
<td>0.68</td>
<td>0.89</td>
<td>0.90</td>
<td>0.89</td>
<td>0.90</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>$(R = 100, k = 4)$</td>
<td>0.67</td>
<td>0.87</td>
<td>0.89</td>
<td>0.91</td>
<td>0.92</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>$(R = 110, k = 4)$</td>
<td>0.65</td>
<td>0.86</td>
<td>0.88</td>
<td>0.89</td>
<td>0.93</td>
<td>0.92</td>
<td>0.91</td>
</tr>
<tr>
<td>$(R = 120, k = 4)$</td>
<td>0.64</td>
<td>0.84</td>
<td>0.86</td>
<td>0.89</td>
<td>0.92</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>$(R = 130, k = 4)$</td>
<td>0.65</td>
<td>0.83</td>
<td>0.85</td>
<td>0.87</td>
<td>0.91</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>$(R = 70, k = 8)$</td>
<td>1.00</td>
<td>0.67</td>
<td>0.68</td>
<td>0.67</td>
<td>0.66</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$(R = 80, k = 8)$</td>
<td>0.67</td>
<td>1.00</td>
<td>0.94</td>
<td>0.92</td>
<td>0.90</td>
<td>0.87</td>
<td>0.84</td>
</tr>
<tr>
<td>$(R = 90, k = 8)$</td>
<td>0.68</td>
<td>0.94</td>
<td>1.00</td>
<td>0.95</td>
<td>0.93</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>$(R = 100, k = 8)$</td>
<td>0.67</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>$(R = 110, k = 8)$</td>
<td>0.66</td>
<td>0.90</td>
<td>0.93</td>
<td>0.95</td>
<td>1.00</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>$(R = 120, k = 8)$</td>
<td>0.65</td>
<td>0.87</td>
<td>0.90</td>
<td>0.93</td>
<td>0.97</td>
<td>1.00</td>
<td>0.97</td>
</tr>
<tr>
<td>$(R = 130, k = 8)$</td>
<td>0.66</td>
<td>0.84</td>
<td>0.88</td>
<td>0.90</td>
<td>0.95</td>
<td>0.97</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$
Table F.6: Robustness of the MSM Estimation Results (British Pound)

<table>
<thead>
<tr>
<th></th>
<th>R = 70</th>
<th>R = 80</th>
<th>R = 90</th>
<th>R = 100</th>
<th>R = 110</th>
<th>R = 120</th>
<th>R = 130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: k = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R = 70, k = 4</td>
<td>1.00</td>
<td>0.92</td>
<td>0.89</td>
<td>0.87</td>
<td>0.84</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>R = 80, k = 4</td>
<td>0.92</td>
<td>1.00</td>
<td>0.94</td>
<td>0.91</td>
<td>0.88</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>R = 90, k = 4</td>
<td>0.89</td>
<td>0.94</td>
<td>1.00</td>
<td>0.94</td>
<td>0.91</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>R = 100, k = 4</td>
<td>0.87</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
<td>0.95</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>R = 110, k = 4</td>
<td>0.84</td>
<td>0.88</td>
<td>0.91</td>
<td>0.95</td>
<td>1.00</td>
<td>0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>R = 120, k = 4</td>
<td>0.85</td>
<td>0.88</td>
<td>0.91</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.94</td>
</tr>
<tr>
<td>R = 130, k = 4</td>
<td>0.83</td>
<td>0.86</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>R = 70, k = 8</td>
<td>0.64</td>
<td>0.65</td>
<td>0.65</td>
<td>0.63</td>
<td>0.61</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>R = 80, k = 8</td>
<td>0.86</td>
<td>0.90</td>
<td>0.88</td>
<td>0.86</td>
<td>0.85</td>
<td>0.84</td>
<td>0.82</td>
</tr>
<tr>
<td>R = 90, k = 8</td>
<td>0.83</td>
<td>0.87</td>
<td>0.90</td>
<td>0.88</td>
<td>0.87</td>
<td>0.86</td>
<td>0.84</td>
</tr>
<tr>
<td>R = 100, k = 8</td>
<td>0.80</td>
<td>0.84</td>
<td>0.87</td>
<td>0.89</td>
<td>0.88</td>
<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td>R = 110, k = 8</td>
<td>0.79</td>
<td>0.83</td>
<td>0.85</td>
<td>0.88</td>
<td>0.89</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>R = 120, k = 8</td>
<td>0.81</td>
<td>0.85</td>
<td>0.86</td>
<td>0.88</td>
<td>0.89</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>R = 130, k = 8</td>
<td>0.80</td>
<td>0.83</td>
<td>0.84</td>
<td>0.86</td>
<td>0.87</td>
<td>0.89</td>
<td>0.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R = 70</th>
<th>R = 80</th>
<th>R = 90</th>
<th>R = 100</th>
<th>R = 110</th>
<th>R = 120</th>
<th>R = 130</th>
</tr>
</thead>
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<tr>
<td>Panel B: k = 8</td>
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<td></td>
</tr>
<tr>
<td>R = 70, k = 4</td>
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<td>0.86</td>
<td>0.83</td>
<td>0.80</td>
<td>0.79</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
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<td>0.83</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>R = 90, k = 4</td>
<td>0.65</td>
<td>0.88</td>
<td>0.90</td>
<td>0.87</td>
<td>0.85</td>
<td>0.86</td>
<td>0.84</td>
</tr>
<tr>
<td>R = 100, k = 4</td>
<td>0.63</td>
<td>0.86</td>
<td>0.88</td>
<td>0.89</td>
<td>0.88</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>R = 110, k = 4</td>
<td>0.61</td>
<td>0.85</td>
<td>0.87</td>
<td>0.88</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>R = 120, k = 4</td>
<td>0.62</td>
<td>0.84</td>
<td>0.86</td>
<td>0.87</td>
<td>0.88</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>R = 130, k = 4</td>
<td>0.61</td>
<td>0.82</td>
<td>0.84</td>
<td>0.85</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>R = 70, k = 8</td>
<td>1.00</td>
<td>0.65</td>
<td>0.65</td>
<td>0.62</td>
<td>0.61</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>R = 80, k = 8</td>
<td>0.65</td>
<td>1.00</td>
<td>0.94</td>
<td>0.91</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>R = 90, k = 8</td>
<td>0.65</td>
<td>0.94</td>
<td>1.00</td>
<td>0.94</td>
<td>0.92</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>R = 100, k = 8</td>
<td>0.62</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
<td>0.95</td>
<td>0.94</td>
<td>0.91</td>
</tr>
<tr>
<td>R = 110, k = 8</td>
<td>0.61</td>
<td>0.89</td>
<td>0.92</td>
<td>0.95</td>
<td>1.00</td>
<td>0.96</td>
<td>0.93</td>
</tr>
<tr>
<td>R = 120, k = 8</td>
<td>0.62</td>
<td>0.89</td>
<td>0.91</td>
<td>0.94</td>
<td>0.96</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>R = 130, k = 8</td>
<td>0.61</td>
<td>0.87</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td>0.95</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: This table reports the pairwise correlation of the MSM estimation results with $k \in \{4, 8\}$ and $R \in \{70, 80, 90, 100, 110, 120, 130\}$
Table F.7: Success Ratio of the Directional Forecasts ($R = 70$)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon ($h$)</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td></td>
<td>0.599</td>
<td>0.565</td>
<td>0.571</td>
<td>0.520</td>
<td>0.565</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(177)</td>
<td>(177)</td>
<td>(177)</td>
<td>(171)</td>
<td>(170)</td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.544</td>
<td>0.569</td>
<td>0.631</td>
<td>0.623</td>
<td>0.589</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(160)</td>
<td>(160)</td>
<td>(160)</td>
<td>(159)</td>
<td>(158)</td>
</tr>
<tr>
<td>EUR</td>
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<td>0.570</td>
<td>0.553</td>
<td>0.702</td>
<td>0.657</td>
<td>0.630</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(142)</td>
<td>(141)</td>
<td>(141)</td>
<td>(140)</td>
<td>(138)</td>
</tr>
<tr>
<td>JPY</td>
<td></td>
<td>0.549</td>
<td>0.471</td>
<td>0.476</td>
<td>0.510</td>
<td>0.510</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(175)</td>
<td>(174)</td>
<td>(166)</td>
<td>(157)</td>
<td>(151)</td>
</tr>
<tr>
<td>GBP</td>
<td></td>
<td>0.563</td>
<td>0.591</td>
<td>0.534</td>
<td>0.529</td>
<td>0.554</td>
</tr>
<tr>
<td></td>
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<td>(208)</td>
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<td>(208)</td>
<td>(208)</td>
<td>(204)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.8: Success Ratio of the Directional Forecasts ($R = 80$)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon ($h$)</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td></td>
<td>0.579</td>
<td>0.515</td>
<td>0.520</td>
<td>0.453</td>
<td>0.521</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(171)</td>
<td>(171)</td>
<td>(171)</td>
<td>(170)</td>
<td>(169)</td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.567</td>
<td>0.632</td>
<td>0.690</td>
<td>0.655</td>
<td>0.606</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(171)</td>
<td>(171)</td>
<td>(171)</td>
<td>(171)</td>
<td>(170)</td>
</tr>
<tr>
<td>EUR</td>
<td></td>
<td>0.567</td>
<td>0.567</td>
<td>0.660</td>
<td>0.627</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(150)</td>
<td>(150)</td>
<td>(150)</td>
<td>(150)</td>
<td>(149)</td>
</tr>
<tr>
<td>JPY</td>
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<td>0.527</td>
<td>0.503</td>
<td>0.526</td>
<td>0.590</td>
<td>0.590</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(186)</td>
<td>(183)</td>
<td>(175)</td>
<td>(166)</td>
<td>(156)</td>
</tr>
<tr>
<td>GBP</td>
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<td>0.553</td>
<td>0.558</td>
<td>0.512</td>
<td>0.522</td>
<td>0.556</td>
</tr>
<tr>
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<td></td>
<td>(208)</td>
<td>(208)</td>
<td>(207)</td>
<td>(203)</td>
<td>(198)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.9: Success Ratio of the Directional Forecasts ($R = 90$)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon ($h$)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>1m</td>
</tr>
<tr>
<td>AUD</td>
<td>0.566</td>
</tr>
<tr>
<td></td>
<td>(189)</td>
</tr>
<tr>
<td>CAD</td>
<td>0.577</td>
</tr>
<tr>
<td></td>
<td>(175)</td>
</tr>
<tr>
<td>EUR</td>
<td>0.586</td>
</tr>
<tr>
<td></td>
<td>(145)</td>
</tr>
<tr>
<td>JPY</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td>(186)</td>
</tr>
<tr>
<td>GBP</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td>(206)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.10: Success Ratio of the Directional Forecasts ($R = 100$)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Forecasting Horizon ($h$)</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td></td>
<td>0.605</td>
<td>0.592</td>
<td>0.612</td>
<td>0.533</td>
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</tr>
<tr>
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<td>(196)</td>
<td>(195)</td>
<td>(194)</td>
<td></td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.577</td>
<td>0.587</td>
<td>0.661</td>
<td>0.640</td>
<td>0.612</td>
</tr>
<tr>
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<td>(189)</td>
<td>(189)</td>
<td>(189)</td>
<td>(189)</td>
<td>(188)</td>
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<tr>
<td>EUR</td>
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<td>0.593</td>
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<td>0.615</td>
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<td>0.575</td>
</tr>
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<td>(135)</td>
<td>(135)</td>
<td>(134)</td>
<td>(134)</td>
<td></td>
</tr>
<tr>
<td>JPY</td>
<td></td>
<td>0.525</td>
<td>0.532</td>
<td>0.617</td>
<td>0.688</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td>(204)</td>
<td>(201)</td>
<td>(193)</td>
<td>(186)</td>
<td>(176)</td>
<td></td>
</tr>
<tr>
<td>GBP</td>
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<td>0.500</td>
<td>0.483</td>
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<td>0.500</td>
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<tr>
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<td>(209)</td>
<td>(205)</td>
<td>(200)</td>
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</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.11: Success Ratio of the Directional Forecasts \((R = 110)\)

<table>
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<th>3m</th>
<th>6m</th>
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<th>12m</th>
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</thead>
<tbody>
<tr>
<td>AUD</td>
<td></td>
<td>0.582</td>
<td>0.577</td>
<td>0.617</td>
<td>0.565</td>
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</tr>
<tr>
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<td>(201)</td>
<td>(201)</td>
<td>(201)</td>
<td>(200)</td>
<td>(199)</td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.525</td>
<td>0.559</td>
<td>0.647</td>
<td>0.627</td>
<td>0.601</td>
</tr>
<tr>
<td></td>
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<td>(204)</td>
<td>(204)</td>
<td>(204)</td>
<td>(204)</td>
<td>(203)</td>
</tr>
<tr>
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<td>0.521</td>
<td>0.629</td>
<td>0.607</td>
<td>0.600</td>
</tr>
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<td>(140)</td>
<td>(140)</td>
<td>(140)</td>
</tr>
<tr>
<td>JPY</td>
<td></td>
<td>0.553</td>
<td>0.551</td>
<td>0.614</td>
<td>0.689</td>
<td>0.689</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(208)</td>
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<td>(197)</td>
<td>(190)</td>
<td>(180)</td>
</tr>
<tr>
<td>GBP</td>
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<td>0.548</td>
<td>0.524</td>
<td>0.483</td>
<td>0.507</td>
<td>0.510</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(210)</td>
<td>(206)</td>
<td>(205)</td>
<td>(201)</td>
<td>(196)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.12: Success Ratio of the Directional Forecasts \((R = 120)\)

<table>
<thead>
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<th>Currency</th>
<th>Forecasting Horizon ((h))</th>
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<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td></td>
<td>0.603</td>
<td>0.582</td>
<td>0.622</td>
<td>0.557</td>
<td>0.581</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(194)</td>
<td>(194)</td>
<td>(193)</td>
<td>(192)</td>
<td>(191)</td>
</tr>
<tr>
<td>CAD</td>
<td></td>
<td>0.563</td>
<td>0.568</td>
<td>0.623</td>
<td>0.618</td>
<td>0.586</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(199)</td>
<td>(199)</td>
<td>(199)</td>
<td>(199)</td>
<td>(198)</td>
</tr>
<tr>
<td>EUR</td>
<td></td>
<td>0.553</td>
<td>0.538</td>
<td>0.659</td>
<td>0.614</td>
<td>0.591</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(132)</td>
<td>(132)</td>
<td>(132)</td>
<td>(132)</td>
<td>(132)</td>
</tr>
<tr>
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<td>0.519</td>
<td>0.612</td>
<td>0.663</td>
<td>0.677</td>
</tr>
<tr>
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<td></td>
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<td>(214)</td>
<td>(206)</td>
<td>(199)</td>
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<td>(201)</td>
<td>(200)</td>
<td>(196)</td>
<td>(191)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.
Table F.13: Success Ratio of the Directional Forecasts ($R = 130$)

<table>
<thead>
<tr>
<th>Currency</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>0.586</td>
<td>0.545</td>
<td>0.602</td>
<td>0.538</td>
<td>0.569</td>
</tr>
<tr>
<td></td>
<td>(215)</td>
<td>(213)</td>
<td>(211)</td>
<td>(210)</td>
<td>(209)</td>
</tr>
<tr>
<td>CAD</td>
<td>0.526</td>
<td>0.541</td>
<td>0.592</td>
<td>0.587</td>
<td>0.564</td>
</tr>
<tr>
<td></td>
<td>(196)</td>
<td>(196)</td>
<td>(196)</td>
<td>(196)</td>
<td>(195)</td>
</tr>
<tr>
<td>EUR</td>
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<td>0.550</td>
<td>0.674</td>
<td>0.643</td>
<td>0.605</td>
</tr>
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<td>(129)</td>
<td>(129)</td>
<td>(129)</td>
<td>(129)</td>
</tr>
<tr>
<td>JPY</td>
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<td>0.544</td>
<td>0.612</td>
<td>0.683</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td>(220)</td>
<td>(217)</td>
<td>(209)</td>
<td>(202)</td>
<td>(192)</td>
</tr>
<tr>
<td>GBP</td>
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<td>0.491</td>
<td>0.465</td>
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<td>0.485</td>
</tr>
<tr>
<td></td>
<td>(220)</td>
<td>(216)</td>
<td>(215)</td>
<td>(211)</td>
<td>(206)</td>
</tr>
</tbody>
</table>

Note: The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. The total number of appreciation and depreciation forecasts is in parentheses. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 12/31/2017 for all currencies.