# A Note on Additional Materials for Macro-Finance Decoupling: Robust Evaluations of Macro Asset Pricing Models

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**Abstract**

This note contains additional model derivation and numerical details of the main text Cheng, Dou, and Liao (2021). Section A derives the Euler equations that serve as the asset pricing moment conditions in the disaster risk model and the long-run risk model. Section B considers the long-run risk model and shows that the Gaussian limit is an innocuous assumption. Section C provides derivations for the time-varying disaster risk model in the empirical application of the main text and some additional discussions on the literature. Section D contains an additional robustness check for the simulation results in the main text.

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# A Derivations of Asset Pricing Moments in Examples

## A.1 Solution of the Disaster Risk Model

The dividend-price ratio is constant in the equilibrium, denoted by $C/P$. The stock return is

$$R_t = (C_t + P_t)/P_{t-1} = (C/P + 1)C_t/C_{t-1} = (C/P + 1)e^{\sigma\varepsilon_t - \zeta_t}. \tag{A.1}$$

Thus, the expected log stock return is

$$\mathbb{E}_{t-1}[\ln R_t] = \ln(C/P + 1) - p\left(\underline{v} + 1/\alpha\right). \tag{A.2}$$

The equilibrium dividend-price ratio is characterized by

$$\begin{aligned} P/C &= \mathbb{E}_{t-1}\left[e^{-\delta}\left(C_t/C_{t-1}\right)^{-\gamma}(P_t/C_t + 1)C_t/C_{t-1}\right] \\ &= (P/C + 1)e^{-\delta}\mathbb{E}_{t-1}\left[\left(C_t/C_{t-1}\right)^{1-\gamma}\right] \\ &= (P/C + 1)e^{-\delta + \frac{1}{2}(1-\gamma)^2\sigma^2}\left[1 - p + pe^{-(1-\gamma)\underline{v}}\frac{\alpha}{\alpha - \gamma + 1}\right]. \end{aligned} \tag{A.3}$$

As a result, the term $\ln(C/P + 1)$ in (A.1) equals to

$$\begin{aligned} \ln(C/P + 1) &= \delta - \frac{1}{2}(1-\gamma)^2\sigma^2 - \ln\left[1 - p + pe^{-(1-\gamma)\underline{v}}\frac{\alpha}{\alpha - \gamma + 1}\right] \\ &\approx \delta - \frac{1}{2}(1-\gamma)^2\sigma^2 + p - pe^{-(1-\gamma)\underline{v}}\frac{\alpha}{\alpha - \gamma + 1}. \end{aligned} \tag{A.4}$$

Define the log return as $r_t \equiv \ln R_t$. Equations (A.4) and (A.2) imply the following relation:

$$\mathbb{E}_{t-1}[r_t] \approx \delta - \frac{1}{2}(1-\gamma)^2\sigma^2 - p\left(\underline{v} + 1/\alpha\right) + p - pe^{(\gamma-1)\underline{v}}\frac{\alpha}{\alpha - \gamma + 1}. \tag{A.5}$$

The equilibrium risk-free rate satisfies that

$$R_f = \mathbb{E}_{t-1}\left[e^{-\delta}(C_t/C_{t-1})^{-\gamma}\right]^{-1} = e^{\delta - \frac{1}{2}\gamma^2\sigma^2}\left[1 - p + pe^{\gamma\underline{v}}\frac{\alpha}{\alpha - \gamma}\right]^{-1}. \tag{A.6}$$

Define the log risk-free rate as $r_f \equiv \ln R_f$. Thus, the log risk-free rate satisfies

$$\begin{aligned} r_f &= \delta - \frac{1}{2}\gamma^2\sigma^2 - \ln\left[1 - p + pe^{\gamma\underline{v}}\frac{\alpha}{\alpha - \gamma}\right] \\ &\approx \delta - \frac{1}{2}\gamma^2\sigma^2 + p - pe^{\gamma\underline{v}}\frac{\alpha}{\alpha - \gamma}. \end{aligned} \tag{A.7}$$

The excess log return, defined as $r_t^e \equiv r_t - r_f$, is

$$\mathbb{E}_{t-1}\left[r_t^e\right] = \gamma\sigma^2 - \frac{1}{2}\sigma^2 - p(\underline{v} + 1/\alpha) + p\alpha\left[\frac{e^{\gamma\underline{v}}}{\alpha-\gamma} - \frac{e^{(\gamma-1)\underline{v}}}{\alpha-\gamma+1}\right]. \tag{A.8}$$

The excess log return $r_t^e$ has the same conditional exposure to the shocks as the log return $r_t$ since the log risk-free rate $r_f$ has zero exposure to the shocks. Therefore, the excess log return in the equilibrium can be represented as follows:

$$r_t^e = \gamma\sigma^2 - \frac{1}{2}\sigma^2 - p(\underline{v} + 1/\alpha) + p\alpha\left[\frac{e^{\gamma\underline{v}}}{\alpha-\gamma} - \frac{e^{(\gamma-1)\underline{v}}}{\alpha-\gamma+1}\right] + \varepsilon_{\ell,t}^e, \tag{A.9}$$

where $\varepsilon_{\ell,t}^e \equiv \sigma\varepsilon_t - [x_t(\underline{v} + J_t) - p(\underline{v} + 1/\alpha)] + \sigma_r\varepsilon_{\ell,t}$. The shock of the consumption growth is $\varepsilon_t$, the jump shock of the consumption growth is $-[x_t(\underline{v} + J_t) - p(\underline{v} + 1/\alpha)]$, and the shock of the measurement error is $\varepsilon_{\ell,t}$. The measurement error $\varepsilon_{\ell,t}$ is i.i.d. standard normal and is independent of other shocks.

## A.2 Solution of the Long-Run Risk Model

The stochastic discount factor (SDF) can be expressed as follows:

$$M_t = \delta^{\vartheta}\left(\frac{C_t}{C_{t-1}}\right)^{-\vartheta/\psi} R_{c,t}^{\vartheta-1}, \quad \text{with } \vartheta \equiv \frac{1-\gamma}{1-1/\psi}, \tag{A.10}$$

where $R_{c,t}$ is the return on the consumption claim. The log SDF can be written as

$$m_t = \vartheta\log\delta - \frac{\vartheta}{\psi}\Delta c_t + (\vartheta-1)r_t. \tag{A.11}$$

The state variable in the simplest long-run risk model is $x_t$. To turn the system into an affine model, we first exploit the Campbell-Shiller log-linearization approximation:

$$r_t = \kappa_0 + \kappa_1 z_t + \Delta c_t - z_{t-1}, \tag{A.12}$$

where $z_{t-1} = \ln(W_{t-1}/C_{t-1})$ is the log wealth-consumption ratio and wealth is the "price" of consumption claims. The log-linearization constants are determined by long-run steady state:

$$\kappa_0 = \ln(1+e^{\overline{z}}) - \kappa_1\overline{z} \quad \text{and} \quad \kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}, \tag{A.13}$$

where $\overline{z}$ is the mean of the log price-consumption ratio. Given the log-linearization approximation

(A.12) – (A.13), we can search the equilibrium characterized by

$$z_t = A_0 + A_1 x_t, \tag{A.14}$$

where the constants $A_0$ and $A_1$ are to be determined by the equilibrium conditions.

Thus, the log return on consumption claim can be written as

$$r_t = \kappa_0 + \kappa_1 (A_0 + A_1 x_t) + \Delta c_t - (A_0 + A_1 x_{t-1}). \tag{A.15}$$

Therefore, the log SDF can be re-written in terms of state variables and exogenous shocks

$$m_t = \Gamma_0 + \Gamma_1 x_{t-1} - \lambda_c \sigma_c \epsilon_{c,t} - \lambda_x \phi \epsilon_{x,t}, \tag{A.16}$$

where predictive coefficients are

$$\Gamma_0 = \ln \delta \quad \text{and} \quad \Gamma_1 = -\psi^{-1}, \tag{A.17}$$

and the market price of risk coefficients are

$$\lambda_c = \gamma \quad \text{and} \quad \lambda_x = \left(\gamma - \psi^{-1}\right) \frac{\kappa_1 \phi}{1 - \kappa_1 \rho}. \tag{A.18}$$

The coefficients $A_j$'s are determined by the equilibrium condition (i.e., the Euler equation for the price of consumption claim) as follows:

$$1 = \mathbb{E}_{t-1}[M_t R_{c,t}] = \mathbb{E}_{t-1}\left[e^{m_t + r_t}\right], \tag{A.19}$$

where $\mathbb{E}_{t-1}[\cdot]$ denote the expectation given the information at time $t-1$. It leads to the equilibrium conditions:

$$A_0 = \frac{1}{1-\kappa_1}(\ln \delta + \kappa_0) \quad \text{and} \quad A_1 = \frac{1-\psi^{-1}}{1-\kappa_1 \rho}\phi. \tag{A.20}$$

The long-run mean $\overline{z}$ is also determined endogenously in the equilibrium. In the long-run steady state, we have

$$\overline{z} = A_0. \tag{A.21}$$

We first derive $\kappa_1$ in equilibrium. After taking log on the both sides of (A.13) and plugging (A.20) and (A.21) into the equation, we can obtain the following relation:

$$\begin{aligned} \ln \kappa_1 &= \overline{z} - \ln(1 + e^{\overline{z}}) = \overline{z} - \kappa_0 + \kappa_1 \overline{z} \\ &= (1-\kappa_1)\overline{z} - \kappa_0 = \ln \delta. \end{aligned} \tag{A.22}$$

Thus, the equilibrium log-linearization coefficient is equal to the representative agent's time preference parameter; that is, $\kappa_1 = \delta$, in equilibrium.

From (A.15) and (A.20), it follows that

$$r_t - \mathbb{E}_{t-1}[r_t] = \beta_c \sigma_c \epsilon_{c,t} + \beta_x \epsilon_{x,t}, \tag{A.23}$$

where the betas are

$$\beta_c = 1 \quad \text{and} \quad \beta_x = \kappa_1 A_1. \tag{A.24}$$

The Euler equation for the log market return, denoted by $r_t$, and the risk free rate, denoted by $r_{f,t-1}$, can be written in one equation

$$\mathbb{E}_{t-1}\left[e^{m_t}\right] = \mathbb{E}_{t-1}\left[e^{m_t + r_t^e}\right], \tag{A.25}$$

where $r_t^e \equiv r_t - r_{f,t-1}$ is the excess log return.

The Euler equation (A.25) leads to

$$\begin{aligned}
\mathbb{E}_{t-1}\left[r_t^e\right] &= \lambda_c \beta_c \sigma_c^2 + \lambda_x \beta_x - \frac{1}{2}\left(\beta_c^2 \sigma_c^2 + \beta_x^2 \phi^2\right) \\
&= \gamma \sigma_c^2 - \frac{1}{2}\sigma_c^2 + \frac{1}{2}(2\gamma - \psi^{-1} - 1)(1 - \psi^{-1}) \frac{\kappa_1^2 \phi^2}{(1 - \kappa_1 \rho)^2} \\
&= \gamma \sigma_c^2 - \frac{1}{2}\sigma_c^2 + \frac{1}{2}(2\gamma - \psi^{-1} - 1)(1 - \psi^{-1}) \frac{\phi^2}{(\delta^{-1} - \rho)^2}.
\end{aligned} \tag{A.26}$$

The excess log return $r_t^e$ has the same conditional exposure to the shocks as the log return $r_t$ since the log risk-free rate $r_{f,t-1}$ has zero conditional exposure to the shocks $\varepsilon_{c,t}$ and $\varepsilon_{x,t}$. Therefore, combining (A.20), (A.24), and (A.26), we can obtain that the excess log return in the equilibrium can be represented as follows:

$$r_t^e = \gamma \sigma_c^2 - \frac{1}{2}\sigma_c^2 + \frac{1}{2}(2\gamma - \psi^{-1} - 1)(1 - \psi^{-1}) \frac{\phi^2}{(\delta^{-1} - \rho)^2} + \varepsilon_{d,t}^e, \tag{A.27}$$

where $\varepsilon_{d,t}^e \equiv \sigma_c \varepsilon_{c,t} + (1-\psi^{-1})(1-\rho)^{-1} \phi \varepsilon_{x,t} + \sigma_r \varepsilon_{d,t}$. The shock of the contemporaneous consumption growth is $\varepsilon_{c,t}$, the shock of the low-frequency component is $\varepsilon_{x,t}$, and the shock of the measurement error is $\varepsilon_{d,t}$. The measurement error $\varepsilon_{d,t}$ is i.i.d. standard normal and is independent of other shocks.

# B Gaussian Limit in the Long-Run Risk Model

For the long-run risk model, we show that the Gaussian limit for the moment conditions is an innocuous assumption even if $\rho_n \to 1$ and $\phi_n \to 0$ at any rate, as long as $\theta_n = (1 - \rho_n)/\phi_n$ is bounded from above and below by some finite positive constants, where the subscript $n$ denotes the sample size $n$.

We start with the first moment condition. The observed process $\Delta c_t$ satisfies

$$\begin{aligned}\Delta c_{t+1} - \rho \Delta c_t &= \phi_n \left(\rho_n x_{t-1} + \epsilon_{x,t}\right) + \sigma_c \epsilon_{c,t+1} - \rho \left(\phi_n x_{t-1} + \sigma_c \epsilon_{c,t}\right) \\ &= \phi_n(\rho_n - \rho) x_{t-1} + \phi_n \epsilon_{x,t} + \sigma_c \epsilon_{c,t+1} - \rho \sigma_c \epsilon_{c,t}. \end{aligned} \tag{B.1}$$

Multiply the first difference $\Delta c_{t+1} - \rho \Delta c_t$ in (B.1) by $\Delta c_{t-1} = \phi_n x_{t-2} + \sigma_c \epsilon_{c,t-1}$. We obtain

$$M_{1t} \equiv \Delta c_{t-1} \left(\Delta c_{t+1} - \rho \Delta c_t\right) = M_{1a,t} + M_{1b,t} + M_{1c,t} + M_{1d,t} + \mu_1, \tag{B.2}$$

where

$$\begin{aligned} \mu_1 &\equiv \mathbb{E}\left[\Delta c_{t-1} \left(\Delta c_{t+1} - \rho \Delta c_t\right)\right] = \phi_n^2 (\rho_n - \rho) \rho_n \mathbb{E}[x_{t-2}^2], \\ M_{1a,t} &\equiv \phi_n^2 (\rho_n - \rho) \rho_n \left(x_{t-2}^2 - \mathbb{E}[x_{t-2}^2]\right), \\ M_{1b,t} &\equiv \phi_n (\rho_n - \rho) \left(\sigma_c x_{t-1} \epsilon_{c,t-1} + \phi_n x_{t-2} \varepsilon_{x,t-1}\right), \\ M_{1c,t} &\equiv \phi_n x_{t-2} \left(\phi_n \epsilon_{x,t} + \sigma_c \epsilon_{c,t+1} - \rho \sigma_c \epsilon_{c,t}\right), \\ M_{1d,t} &\equiv \sigma_c \epsilon_{c,t-1} \left(\phi_n \epsilon_{x,t} + \sigma_c \epsilon_{c,t+1} - \rho \sigma_c \epsilon_{c,t}\right). \end{aligned} \tag{B.3}$$

The moment condition satisfies

$$M_1 \equiv n^{-1/2} \sum_{t=1}^{n} \left(M_{1t} - \mathbb{E}\left[M_{1t}\right]\right) = M_{1a} + M_{1b} + M_{1c} + M_{1d}, \tag{B.4}$$

where $M_{1j} \equiv n^{-1/2} \sum_{t=1}^{n} M_{1j,t}$ for $j = a, b, c, d$.

Below we consider three separate cases: (i) $\rho_n$ is bounded away from 1; (ii) $\rho_n$ converges to 1 at the rate slower than $n^{-1}$, i.e., $(1 - \rho_n) n \to \infty$; (iii) $\rho_n$ converges to 1 at the rate $n^{-1}$ or faster, i.e., $(1 - \rho_n) n \to c \in [0, \infty)$. In all three cases, $\theta_n = (1 - \rho_n)/\phi_n$ is bounded below from 0 and above from infinity. Therefore, $\phi_n$ always converges to 0 at the same rate at which $\rho_n$ converges to 1.

In case (i), we can apply the central limit theorem (CLT) for weakly dependent triangular arrays and $M_1$ has a Gaussian limit.

Now we consider case (ii). We assume the initial condition satisfies $\mathbb{E}[x_0^2] = o(n^{1/2})$ as in GP. Following Lemma 1 of Giraitis and Phillips (2006) (hereafter GP),

$$\frac{(1 - \rho_n^2)^{1/2}}{n^{1/2}} \sum_{t=1}^{n} x_{t-1} \epsilon_{x,t} \to_d N(0, 1) \tag{B.5}$$

using $Var(\epsilon_{x,t}) = 1$. We assume the initial condition satisfies $\mathbb{E}[x_0^2] = o(n^{1/2})$ as in GP. Following the proof for Lemma 1 of GP we also have

$$\frac{(1-\rho_n^2)^{1/2}}{n^{1/2}} \sum_{t=1}^{n} x_{t-1}\epsilon_{x,t+1} \to_d N(0,1),$$
$$\frac{(1-\rho_n^2)^{1/2}}{n^{1/2}} \sum_{t=1}^{n} x_{t-1}\epsilon_{c,t+j} \to_d N(0,\sigma_c^2), \tag{B.6}$$

for any $j$ following the independence of $\epsilon_{x,t}$ and $\epsilon_{c,t'}$ for any $t \neq t'$. Following equation (20) in the proof of Lemma 2 of GP,

$$n^{-1/2} \sum_{t=1}^{n} \left(x_{t-1}^2 - \mathbb{E}[x_{t-1}^2]\right) = \left(1-\rho_n^2\right)^{-3/2} 2\rho_n Z_1 + \left(1-\rho_n^2\right)^{-1} Z_2 + Z_3, \tag{B.7}$$

where

$$Z_1 \equiv \frac{(1-\rho_n^2)^{1/2}}{n^{1/2}} \sum_{t=1}^{n} x_{t-1}\epsilon_{x,t} \to_d N(0,1),$$
$$Z_2 \equiv n^{-1/2} \sum_{t=1}^{n} \left(\epsilon_{x,t}^2 - 1\right) \to_d N(0, V_{x^2}),$$
$$Z_3 \equiv \frac{x_0^2 - \mathbb{E}[x_0^2]}{n^{1/2}\left(1-\rho_n^2\right)} - \frac{x_n^2 - \mathbb{E}[x_n^2]}{n^{1/2}\left(1-\rho_n^2\right)}, \tag{B.8}$$

where the convergence for $Z_1$ holds by (B.5), the convergence for $Z_2$ follows from the CLT with $V_{x^2}$ being the variance of $\epsilon_{x,t}^2$. Then, we have $M_{1a} = o_p(1)$ because

$$\phi_n^2 \left(1-\rho_n^2\right)^{-3/2} 2\rho_n Z_1 = o_p(1),$$
$$\phi_n^2 \left(1-\rho_n^2\right)^{-1} Z_2 = o_p(1),$$
$$\phi_n^2 Z_3 = o_p(1), \tag{B.9}$$

which in turn holds because $\theta_n = (1-\rho_n)/\phi_n$ is bounded, $x_0^2 = o_p(n^{1/2})$ by the initial condition, and $\phi_n^2 \left(1-\rho_n^2\right)^{-1} n^{-1/2} x_n^2 = o_p(1)$ by Lemma 3 of GP and the Markov inequality.

The other terms satisfy

$$M_{1b} = \phi_n O_p((1-\rho_n^2)^{-1/2}) = o_p(1),$$
$$M_{1c} = \phi_n O_p((1-\rho_n^2)^{-1/2}) = o_p(1), \tag{B.10}$$

by $\phi_n = O((1-\rho_n))$, (B.5) and (B.6). Then we can apply the CLT for triangular array of i.i.d. random variables to $M_{1d}$ and the Gaussian limit holds for $M_1$.

Finally, we consider the case (iii), where $n(1-\rho_n) \to c$ for $c \in [0, \infty)$. Following Phillips (1987),

$$n^{-2} \sum_{t=1}^{n} x_{t-1}^2 = O_p(1), \;\; n^{-1} \sum_{t=1}^{n} x_{t-1}\epsilon_{x,t} = O_p(1). \tag{B.11}$$

We also have

$$n^{-1} \sum_{t=1}^{n} x_{t-1}\epsilon_{x,t+1} = O_p(1) \quad \text{and} \quad n^{-1} \sum_{t=1}^{n} x_{t-1}\epsilon_{c,t+j} = O_p(1) \tag{B.12}$$

for any $j$, because $\epsilon_{x,t}$ is i.i.d. with mean zero and it is independent of $\epsilon_{c,t+j}$, following results for a vector process, see Park and Phillips (1988). Therefore,

$$\begin{aligned} M_{1a} &= \phi_n^2 \left[ O_p(n^{3/2}) - O(n^{1/2})(1-\rho_n^2)^{-1} \right] = o_p(1), \\ M_{1b} &= \phi_n O_p(n^{1/2}) = o_p(1) \quad \text{and} \quad M_{1c} = \phi_n O_p(n^{1/2}) = o_p(1), \end{aligned} \tag{B.13}$$

because $\phi_n = O(n^{-1})$. As in case (ii), we can apply the CLT to $M_{1d}$ to obtain the Gaussian limit for $M_1$.

Next, we consider multiplying the first difference $\Delta c_{t+1} - \rho \Delta c_t$ in (B.1) by $\Delta c_t = \phi_n x_{t-1} + \sigma_c \epsilon_{c,t}$. We obtain

$$M_{2,t} \equiv \Delta c_t \left( \Delta c_{t+1} - \rho \Delta c_t \right) = M_{2a,t} + M_{2b,t} + M_{2c,t} + M_{2d,t} + \mu_2, \tag{B.14}$$

where

$$\begin{aligned} \mu_2 &\equiv \mathbb{E}\left[\Delta c_t \left(\Delta c_{t+1} - \rho \Delta c_t\right)\right] = \phi_n^2(\rho_n - \rho)\mathbb{E}[x_{t-1}^2] - \rho\sigma_c^2, \\ M_{2a,t} &\equiv \phi_n^2(\rho_n - \rho)(x_{t-1}^2 - \mathbb{E}[x_{t-1}^2]), \\ M_{2b,t} &\equiv \phi_n(\rho_n - \rho)\sigma_c x_{t-1}\epsilon_{c,t}, \\ M_{2c,t} &\equiv \phi_n x_{t-1}\left(\phi_n \epsilon_{x,t} + \sigma_c \epsilon_{c,t+1} - \rho\sigma_c\epsilon_{c,t}\right), \\ M_{2d,t} &\equiv \sigma_c\epsilon_{c,t}\left(\phi_n \epsilon_{x,t} + \sigma_c \epsilon_{c,t+1} - \rho\sigma_c\epsilon_{c,t}\right). \end{aligned} \tag{B.15}$$

As above, we consider three cases. In case (i) where $\rho$ is bounded away from 1, we can apply the CLT directly. In case (ii),

$$M_{2a} = n^{-1/2} \sum_{t=1}^{n} M_{2a,t} = \phi_n^2(\rho_n - \rho) n^{-1/2} \sum_{t=1}^{n} \left( x_{t-1}^2 - \mathbb{E}[x_{t-1}^2] \right) = o_p(1) \tag{B.16}$$

following the same arguments for $M_{1a}$. Similarly, we can show that $M_{2b}$ and $M_{2c}$, defined similarly to $M_{1b}$ and $M_{1c}$ respectively, are both $o_p(1)$ following the arguments for $M_{1b}$ and $M_{1c}$. Finally, the CLT always applies to $M_{2d}$, the counterpart of $M_{1d}$. In case (iii), same arguments for $M_{1a}$ give

$$M_{2a} = \phi_n^2 \left[ O_p(n^{3/2}) - O(n^{1/2})(1-\rho_n^2)^{-1} \right] = o_p(1). \tag{B.17}$$

Arguments for $M_{1b}$, $M_{1c}$ and $M_{1d}$ can be used to show the same results hold for their counterparts $M_{2b}$, $M_{2c}$ and $M_{2d}$ respectively.

Finally, the asset pricing moment condition always has a Gaussian limit because it is a location model with an i.i.d. error.

## C Additional Materials for the Empirical Application

First, we start with a discussion on the importance of studying the robust evaluation of time-varying disaster risk models. After that, we verify the baseline moment conditions since they do not depend on the model solution. Then, we solve the model and derive the equilibrium relations. Lastly, we verify the asset pricing moment conditions.

**Motivation.** The time-varying disaster risk model has been one of the most influential frameworks in the literature. For instance, the time-varying disaster risk mechanism has been used to explain important empirical patterns in macroeconomic quantities (e.g., Gourio, 2012, 2013), exchange rates and international capital flows (e.g., Gourio, Siemer, and Verdelhan, 2013; Martin, 2013; Farhi and Gabaix, 2015; Dou and Verdelhan, 2017; Lewis and Liu, 2017), global imbalances (e.g., Gourinchas, Rey, and Govillot, 2017), volatile unemployment flows (e.g., Kilic and Wachter, 2018; Petrosky-Nadeau, Zhang, and Kuehn, 2018), prices of derivatives (e.g., Gabaix, 2012; Farhi and Gabaix, 2015; Seo and Wachter, 2018, 2019), credit spreads (e.g., Gourio, 2013), and term structure of return volatility and risk premia (e.g., Hasler and Marfè, 2016). Tsai and Wachter (2015) and Welch (2016) provide lucid summaries of (time-varying) disaster risk mechanisms.

The feature of information imbalance is ubiquitous in macro asset pricing structural models, especially in time-varying rare-disaster risk models (e.g., Chen, Dou, and Kogan, 2021). However, the literature lacks a robust and efficient way to statistically evaluate these influential models or reliably quantify model uncertainty embedded in these potentially fragile structural models. Using this full-fledged example of the time-varying disaster risk mechanism, we show how to use the real data, the structural asset pricing model, and our robust specification test procedure to construct joint uncertainty sets of average return and volatility for the optimal robust portfolio problem. Besides robust portfolio problems, data-driven model uncertainty sets are crucial for robustness analysis of structural economic models in other dimensions (e.g., Hansen and Sargent, 2001, 2008, 2020; Cagetti, Hansen, Sargent, and Williams, 2002; Hansen, 2014; Bidder and Dew-Becker, 2016). In particular, model uncertainty is intrinsic to climate change and thus is an essential element in climate economics (e.g., Brock and Hansen, 2019; Barnett, Brock, and Hansen, 2020; Diebold and Rudebusch, 2021). In practice, constructing statistically valid uncertainty sets of structural models is also useful for formal econometric analysis accounting for model misspecifications (e.g., Andrews, Gentzkow, and Shapiro, 2017; Cheng, Liao, and Shi, 2019; Bonhomme and Weidner, 2020; Armstrong and Kolesar, 2021). Robust inference methods with various identification problems recently also are considered by Chen, Christensen, and Tamer (2018), Chen and Santos (2018), Andrews and Guggenberger (2019), Andrews, Marmer, and Yu (2019), Han and McCloskey (2019), Moreira and Moreira (2019), Cox (2020), Evdokimov and Zeleneev (2020), Kaji (2021), Montiel Olea (2020), among others. Hansen, Lunde, and Nason (2011) provide a model confidence set for the selection of forecasting models.

Although we focus on a real-data application on structural asset pricing models in the paper, we emphasize that the proposed conditional specification test can also be applied to the setting of linear asset pricing models studied by Kan and Zhang (1999), Kleibergen (2009), Beaulieu, Dufour, and Khalaf (2013), Beaulieu, Dufour, and Khalaf (2020), Gospodinov, Kan, and Robotti (2014), Burnside (2015); Kleibergen and Zhan (2015), Kleibergen and Zhan (2020), Anatolyev and Mikusheva (2020), Antoine, Proulx, and Renault (2020), Laurinaityte, Meinerding, Schlag, and Thimme (2020), and Manresa, Penaranda, and Sentana (2020).[1] Recently, as a complementary contribution, a growing body of literature has started to apply machine learning techniques to evaluate linear asset pricing models (e.g., Kelly, Pruitt, and Su, 2019; Feng, Giglio, and Xiu, 2020; Giglio and Xiu, 2020).

**Verifying the Baseline Moment Conditions.** We now verify the baseline moment conditions. The first moment condition is:

$$\begin{aligned}\mathbb{E}\left[(\Delta c_t - g_c) + p\mu_1(\alpha)\right] &= \mathbb{E}\left[\sigma_c\varepsilon_{c,t+1} - \zeta_{t+1}\right] + p\mu_1(\alpha) \\ &= -\mathbb{E}\left[x_{t+1}(\underline{v} + J_{t+1})\right] + p\mu_1(\alpha) \\ &= -\mathbb{E}\left[p_t\right]\mu_1(\alpha) + p\mu_1(\alpha) = 0. \end{aligned} \tag{C.1}$$

The second moment condition can be derived similarly. The third moment condition can also be easily verified as follows:

$$\begin{aligned}\mathbb{E}\left[(\Delta c_t - g_c)^2 - \sigma_c^2 - p\mu_2(\alpha)\right] &= \mathbb{E}\left[(\sigma_c\varepsilon_{c,t+1} - \zeta_{t+1})^2 - \sigma_c^2 - p\mu_2(\alpha)\right] \\ &= \mathbb{E}\left[\sigma_c^2\varepsilon_{c,t+1}^2 + x_{t+1}(\underline{v} + J_{t+1})^2\right] - \sigma_c^2 - p\mu_2(\alpha) \\ &= \mathbb{E}\left[p_t\right]\mu_2(\alpha) - p\mu_2(\alpha) = 0. \end{aligned} \tag{C.2}$$

[1] Instead of focusing on the test statistic itself, Lewellen, Nagel, and Shanken (2010), Daniel and Titman (2012), Ahn, Conrad, and Dittmar (2009), and Nagel and Singleton (2011) emphasize and propose new methods for constructing informative test assets to increase the power of testing linear factor models.

The fourth moment condition can be verified in the same way. The fifth moment condition is verified below.

$$\begin{aligned} &\mathbb{E}\left[\Delta c_{t-1}\left[\Delta c_{t+1} - \rho\Delta c_t + (1-\rho)(p\mu_1(\alpha) - g_c)\right]\right] \\ &= \mathbb{E}\left[\Delta c_{t-1}\left[(\sigma_c\varepsilon_{c,t+1} - \zeta_{t+1}) - \rho(\sigma_c\varepsilon_{c,t} - \zeta_t) + (1-\rho)p\mu_1(\alpha)\right]\right]. \end{aligned} \tag{C.3}$$

The law of iterated expectation further leads to

$$\begin{aligned} &\mathbb{E}\left[\Delta c_{t-1}\left[(\sigma_c\varepsilon_{c,t+1} - \zeta_{t+1}) - \rho(\sigma_c\varepsilon_{c,t} - \zeta_t) + (1-\rho)p\mu_1(\alpha)\right]\right] \\ &= -\mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[\zeta_{t+1} - \rho\zeta_t - (1-\rho)p\mu_1(\alpha)\right]\right] \\ &= -\mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[x_{t+1}(\underline{v} + J_{t+1}) - \rho x_t(\underline{v} + J_t) - (1-\rho)p\mu_1(\alpha)\right]\right]. \end{aligned} \tag{C.4}$$

Applying again the law of iterated expectation leads to

$$\begin{aligned} &\mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[x_{t+1}(\underline{v} + J_{t+1}) - \rho x_t(\underline{v} + J_t) - (1-\rho)p\mu_1(\alpha)\right]\right] \\ &= \mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[\mathbb{E}_t\left[x_{t+1}(\underline{v} + J_{t+1})\right] - \rho p_{t-1}\mu_1(\alpha) - (1-\rho)p\mu_1(\alpha)\right]\right] \\ &= \mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[p_t - \rho p_{t-1} - (1-\rho)p\right]\right]\mu_1(\alpha) \\ &= \mathbb{E}\left[\Delta c_{t-1}\mathbb{E}_{t-1}\left[\sigma_p p\varepsilon_{p,t}\right]\right]\mu_1(\alpha) = 0. \end{aligned} \tag{C.5}$$

The sixth moment condition can be verified in the same way.

**Model Solution.** We next derive the equilibrium of the model. Because the EIS coefficient is one, the first-order condition of optimal consumption leads to $C_t = (1-\delta)W_t$. Because of the homotheticity of the preference, it is natural to conjecture that

$$V_t = \mathcal{I}(p_t)C_t, \tag{C.6}$$

where $\mathcal{I}(p_t)$ is a deterministic function of $p_t$, capturing the marginal value of net worth. The specification of the dynamics is consistent with the exponential-affine models, and thus, we further conjecture that

$$\mathcal{I}(p_t) = e^{I_0 + I_1 p_t}, \tag{C.7}$$

with constants $I_0$ and $I_1$ to be determined by the equilibrium conditions.

The constants $I_0$ and $I_1$ can be solved by plugging (C.6) and (C.7) into the recursive value function relation. Specifically, it holds that

$$I_0 + I_1 p_t + \ln C_t = (1-\delta)\ln C_t + (1-\gamma)^{-1}\delta \ln \mathbb{E}_t\left[e^{(1-\gamma)(I_0 + I_1 p_{t+1})}C_{t+1}^{1-\gamma}\right]. \tag{C.8}$$

The relation above can be rewritten as

$$I_0 + I_1 p_t = \delta \left[ I_0 + I_1(1-\rho)p + I_1 \rho p_t + g_c \right] + \frac{1}{2}\delta(1-\gamma)\left( I_1^2 \sigma_p^2 p^2 + \sigma_c^2 \right) + (1-\gamma)^{-1}\delta \left[ \mathbf{1}_{\{p_t \geq 0\}} p_t \ell(\alpha, \gamma - 1) - \mathbf{1}_{\{p_t < 0\}} p_t \ell(\alpha, 1-\gamma) \right], \tag{C.9}$$

with $\ell(\alpha, x) \equiv e^{x\underline{v}} \frac{\alpha}{\alpha - x} - 1$.

Because $\mathbf{1}_{\{p_t<0\}} p_t \approx 0$ under the relevant calibrations, we can rewrite (C.9) as

$$I_0 + I_1 p_t \approx \delta \left[ I_0 + I_1(1-\rho)p + I_1 \rho p_t + g_c \right] + \frac{1}{2}\delta(1-\gamma)\left( I_1^2 \sigma_p^2 p^2 + \sigma_c^2 \right) + (1-\gamma)^{-1}\delta \mathbf{1}_{\{p_t \geq 0\}} p_t \ell(\alpha, \gamma - 1). \tag{C.10}$$

By matching the constant term and $p_t$ term, we obtain that

$$I_1 \approx \delta I_1 \rho + (1-\gamma)^{-1} \delta \ell(\alpha, \gamma - 1) \tag{C.11}$$

$$I_0 \approx \delta I_0 + \delta I_1 (1-\rho) p + \delta g_c + \frac{1}{2}\delta(1-\gamma)\left( I_1^2 \sigma_p^2 p^2 + \sigma_c^2 \right). \tag{C.12}$$

Equation (C.11) has one root, which is the solution for $I_1$ in equilibrium:

$$I_1 = \frac{\ell(\alpha, \gamma - 1)}{(1-\gamma)(\delta^{-1} - \rho)}. \tag{C.13}$$

Thus, after plugging (C.13) into (C.12), we can obtain the solution for $I_0$ in equilibrium:

$$I_0 = \frac{\delta}{1-\delta} \left[ I_1(1-\rho)p + g_c + \frac{1}{2}(1-\gamma)(I_1^2 \sigma_p^2 p^2 + \sigma_c^2) \right]. \tag{C.14}$$

The equilibrium stochastic discount factor (SDF) is

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]}. \tag{C.15}$$

Thus, by combining (C.6), (C.7), (C.13), and (C.14), the SDF expression in (C.15) can be rewritten as follows:

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{e^{(1-\gamma)(I_0 + I_1 p_{t+1})}}{\mathbb{E}_t \left[ e^{(1-\gamma)(I_0 + I_1 p_{t+1})} (C_{t+1}/C_t)^{1-\gamma} \right]} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{e^{(1-\gamma)[I_0 + I_1(1-\rho)p + I_1 \rho p_t + I_1 \sigma_p p \varepsilon_{p,t+1}]}}{\mathbb{E}_t \left[ e^{(1-\gamma)(I_0 + I_1 p_{t+1})} (C_{t+1}/C_t)^{1-\gamma} \right]}. \tag{C.16}$$

Here, the expected value function with power $1-\gamma$ can be computed as follows:

$$
\begin{aligned}
&\mathbb{E}_t\left[e^{(1-\gamma)(I_0+I_1p_{t+1})}(C_{t+1}/C_t)^{1-\gamma}\right] \\
&\quad= \mathbb{E}_t\left[e^{(1-\gamma)(I_0+I_1(1-\rho)p+I_1\rho p_t+I_1\sigma_p p\varepsilon_{p,t+1}+g_c+\sigma_c\varepsilon_{c,t+1}-\zeta_{t+1})}\right] \\
&\quad= e^{(1-\gamma)(I_0+I_1(1-\rho)p+I_1\rho p_t+g_c)+\frac{1}{2}(1-\gamma)^2(I_1^2\sigma_p^2p^2+\sigma_c^2)}\mathbb{E}_t\left[e^{-(1-\gamma)\zeta_{t+1}}\right] \\
&\quad= e^{(1-\gamma)(I_0+I_1(1-\rho)p+I_1\rho p_t+g_c)+\frac{1}{2}(1-\gamma)^2(I_1^2\sigma_p^2p^2+\sigma_c^2)}\left[1+p_t\ell(\alpha,\gamma-1)\right].
\end{aligned}
\tag{C.17}
$$

After plugging in the equilibrium value function and rearranging the terms, we get the log SDF, denoted by $m_{t+1}\equiv\ln M_{t+1}$ as follows:

$$
\begin{aligned}
m_{t+1} &= \ln\delta-\gamma(g_c+\sigma_c\varepsilon_{c,t+1}-\zeta_{t+1})+(1-\gamma)I_1\sigma_p p\varepsilon_{p,t+1} \\
&\quad-(1-\gamma)g_c-\frac{1}{2}(1-\gamma)^2(I_1^2\sigma_p^2p^2+\sigma_c^2)-p_t\ell(\alpha,\gamma-1).
\end{aligned}
\tag{C.18}
$$

Re-arranging terms leads to

$$
m_{t+1}=\Gamma_0+\Gamma_1p_t-\lambda_c\sigma_c\varepsilon_{c,t+1}-\lambda_p\sigma_p p\varepsilon_{p,t+1}+\lambda_\zeta\zeta_{t+1},
\tag{C.19}
$$

where the predictive coefficients are

$$
\begin{aligned}
\Gamma_0 &= \ln\delta-g_c-\frac{1}{2}(1-\gamma)^2\left(I_1^2\sigma_p^2p^2+\sigma_c^2\right) \\
\Gamma_1 &= -\ell(\alpha,\gamma-1),
\end{aligned}
\tag{C.20}
$$

and the loading coefficients are

$$
\lambda_c=\gamma,\quad \lambda_p=(\gamma-1)I_1,\quad \text{and } \lambda_\zeta=\gamma.
\tag{C.21}
$$

The log risk-free rate, denoted by $r_{f,t}=-\ln\mathbb{E}_t\left[M_{t+1}\right]$, is

$$
\begin{aligned}
r_{f,t} &= -\ln\mathbb{E}_t\left[e^{m_{t+1}}\right] \\
&= -\Gamma_0-\frac{1}{2}\left(\lambda_c^2\sigma_c^2+\lambda_p^2\sigma_p^2p^2\right)-\left[\Gamma_1+\ell(\alpha,\lambda_\zeta)\right]p_t \\
&= -\ln\delta+g_c-\frac{1}{2}(2\gamma-1)\sigma_c^2-\left[\ell(\alpha,\gamma)-\ell(\alpha,\gamma-1)\right]p_t.
\end{aligned}
\tag{C.22}
$$

Using the Campbell-Shiller decomposition and log-linearization approximation, we can represent the log return in terms of log price-dividend ratio and log dividend growth:

$$
r_{m,t+1}=\kappa_{m,0}+\kappa_{m,1}z_{m,t+1}+\Delta d_{t+1}-z_{m,t},
\tag{C.23}
$$

where

$$\kappa_{m,0} = \ln(1 + e^{\overline{z}_m}) - \kappa_{m,1}\overline{z}_m \tag{C.24}$$

and

$$\kappa_{m,1} = \frac{e^{\overline{z}_m}}{1 + e^{\overline{z}_m}} \tag{C.25}$$

and $\overline{z}_m$ is long-run mean of market log price-dividend ratio.

Using the log-linearization approximation, we search the equilibrium characterized by

$$z_{m,t} = A_{m,0} + A_{m,1}p_t, \tag{C.26}$$

where the constants $A_{m,0}$ and $A_{m,1}$ can be computed recursively as follows.

Define the period-$t$ price of the dividend strip paid at the period $t+n$ as $H(D_t, p_t, n) = \mathbb{E}_t\left[M_{t,t+n}D_{t+n}\right]$ where $M_{t,t+n} \equiv e^{\sum_{i=1}^{n} m_{t+i}}$. The price function $H(D_t, p_t, n)$ satisfies the following recursive relations:

$$H(D_t, p_t, n) = \mathbb{E}_t\left[e^{m_{t+1}}H(D_{t+1}, p_{t+1}, n-1)\right] \tag{C.27}$$

$$H(D_t, p_t, 0) = D_t, \tag{C.28}$$

for arbitrary $t$ and $n \geq 1$.

We conjecture that $H(D_t, p_t, n) = D_t e^{A_n + B_n p_t}$. Then, the recursive relations in (C.27) and (C.28) can be rewritten as follows:

$$\begin{aligned} e^{A_n + B_n p_t} &= \mathbb{E}_t\left[e^{\Delta d_{t+1} + m_{t+1} + A_{n-1} + B_{n-1}p_{t+1}}\right] \\ &= \mathbb{E}_t\left[e^{(g_d + \phi\sigma_c\varepsilon_{c,t+1} - \phi\zeta_{t+1}) + (\Gamma_0 + \Gamma_1 p_t - \lambda_c\sigma_c\varepsilon_{c,t+1} - \lambda_p\sigma_p p\varepsilon_{p,t+1} + \lambda_\zeta\zeta_{t+1}) + (A_{n-1} + B_{n-1}p_{t+1})}\right] \\ &= e^{\tilde{A}_n + \tilde{B}_n p_t}\mathbb{E}_t\left[e^{(\phi - \lambda_c)\sigma_c\varepsilon_{c,t+1} + (B_{n-1} - \lambda_p)\sigma_p p\varepsilon_{p,t+1} + (\lambda_\zeta - \phi)\zeta_{t+1}}\right] \\ &= e^{\tilde{A}_n + \tilde{B}_n p_t + \frac{1}{2}(\phi - \gamma)^2\sigma_c^2 + \frac{1}{2}[B_{n-1} - (\gamma - 1)I_1]^2\sigma_p^2 p^2}\mathbb{E}_t\left[e^{(\gamma - \phi)\zeta_{t+1}}\right], \end{aligned} \tag{C.29}$$

where $\tilde{A}_n = g_d + \Gamma_0 + A_{n-1} + B_{n-1}(1-\rho)p$, and $\tilde{B}_n = \Gamma_1 + B_{n-1}\rho$.

The moment generating function of $\zeta_{t+1}$ gives

$$\ln \mathbb{E}_t\left[e^{(\gamma - \phi)\zeta_{t+1}}\right] \approx p_t\ell(\alpha, \gamma - \phi). \tag{C.30}$$

Thus, $A_n$ has the following recursive relation:

$$\begin{aligned}
A_n &= \tilde{A}_n + \frac{1}{2}(\phi-\gamma)^2\sigma_c^2 + \frac{1}{2}\left[B_{n-1} - (\gamma-1)I_1\right]^2\sigma_p^2p^2 \\
&= g_d + \Gamma_0 + A_{n-1} + B_{n-1}(1-\rho)p + \frac{1}{2}(\phi-\gamma)^2\sigma_c^2 + \frac{1}{2}\left[B_{n-1} - (\gamma-1)I_1\right]^2\sigma_p^2p^2 \\
&= \ln\delta + (g_d - g_c) - \frac{1}{2}(1-\gamma)^2\left(I_1^2\sigma_p^2p^2 + \sigma_c^2\right) + \frac{1}{2}(\phi-\gamma)^2\sigma_c^2 \\
&\quad + A_{n-1} + B_{n-1}(1-\rho)p + \frac{1}{2}\left[B_{n-1} - (\gamma-1)I_1\right]^2\sigma_p^2p^2,
\end{aligned} \tag{C.31}$$

and $B_n$ has the following recursive relation:

$$\begin{aligned}
B_n &= \tilde{B}_n + \ell(\alpha, \gamma-\phi) \\
&= \rho B_{n-1} + \ell(\alpha, \gamma-\phi) - \ell(\alpha, \gamma-1),
\end{aligned} \tag{C.32}$$

with initial values $A_0 = B_0 = 0$.

Therefore, it holds that

$$B_n = \frac{1-\rho^n}{1-\rho}\left[\ell(\alpha, \gamma-\phi) - \ell(\alpha, \gamma-1)\right]. \tag{C.33}$$

Because $\sigma_p^2 p \approx 0$, equation (C.35) can be rewritten as

$$\begin{aligned}
A_n - A_{n-1} &\approx \ln\delta + (g_d - g_c) + \frac{1}{2}(\phi-1)(\phi+1-2\gamma)\sigma_c^2 \\
&\quad + (1-\rho^n)\left[\ell(\alpha, \gamma-\phi) - \ell(\alpha, \gamma-1)\right]p.
\end{aligned} \tag{C.34}$$

Thus, it holds that

$$A_n = n\ln\bar{\delta} - \frac{1-\rho^n}{1-\rho}\left[\ell(\alpha, \gamma-\phi) - \ell(\alpha, \gamma-1)\right]p, \tag{C.35}$$

where $\bar{\delta}$ is referred to as the effective time preference coefficient (Barro, 2009), and the log of the effective time preference coefficient is equal to

$$\ln\bar{\delta} \equiv \ln\delta + (g_d - g_c) + \frac{1}{2}(\phi-1)(\phi+1-2\gamma)\sigma_c^2 + \left[\ell(\alpha, \gamma-\phi) - \ell(\alpha, \gamma-1)\right]p. \tag{C.36}$$

Therefore, the log price-dividend ratio is

$$z_{m,t} = \ln\left[\sum_{n=1}^{+\infty} e^{A_n + B_n p_t}\right]. \tag{C.37}$$

According to Taylor expansion in terms of $p_t$ around $p$, it follows that

$$A_{m,0} = \ln\left[\sum_{n=1}^{+\infty} e^{A_n+B_n p}\right] - A_{m,1}p \quad \text{and} \quad A_{m,1} = \frac{\sum_{n=1}^{+\infty} B_n e^{A_n+B_n p}}{\sum_{n=1}^{+\infty} e^{A_n+B_n p}}. \tag{C.38}$$

And thus, it holds that

$$\begin{aligned}
A_{m,1} &= \frac{1}{1-\rho}\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right] - \frac{\sum_{n=1}^{+\infty} e^{n(\ln\bar{\delta}+\ln\rho)}}{\sum_{n=1}^{+\infty} e^{n\ln\bar{\delta}}}\frac{1}{1-\rho}\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right] \\
&= \left[\frac{1}{1-\rho} - \frac{\rho}{1-\rho}\frac{1-\bar{\delta}}{1-\rho\bar{\delta}}\right]\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right] \\
&= \frac{1-\rho\bar{\delta}-\rho(1-\bar{\delta})}{(1-\rho)(1-\rho\bar{\delta})}\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right] \\
&= \frac{1-\rho}{(1-\rho)(1-\rho\bar{\delta})}\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right] \\
&= \frac{1}{1-\rho\bar{\delta}}\left[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)\right].
\end{aligned} \tag{C.39}$$

In principle, the persistence parameter $\rho$ can be infinitely close to 1, and thus, we require the effective time preference coefficient to be less than one (i.e., $\bar{\delta} < 1$).

According to (C.33) and (C.35), it follows that

$$\begin{aligned}
A_{m,0} &= \ln\left[\sum_{n=1}^{\infty} e^{n\ln\bar{\delta}}\right] - A_{m,1}p \\
&= \ln\left[\frac{\bar{\delta}}{1-\bar{\delta}}\right] - A_{m,1}p,
\end{aligned} \tag{C.40}$$

and the long-run average log price-dividend ratio is

$$\bar{z}_m = A_{m,0} + A_{m,1}p = \ln\left[\frac{\bar{\delta}}{1-\bar{\delta}}\right]. \tag{C.41}$$

Plugging back into (C.25), we obtain that

$$\kappa_{m,1} = \bar{\delta} = \delta e^{(g_d-g_c)+\frac{1}{2}(\phi-1)(\phi+1-2\gamma)\sigma_c^2+[\ell(\alpha,\gamma-\phi)-\ell(\alpha,\gamma-1)]p}. \tag{C.42}$$

Because $\sigma_c^2 \approx 0$ and $p \approx 0$, the following approximation works well for relevant parameter regions:

$$\kappa_{m,1} = \bar{\delta} \approx \delta e^{(g_d-g_c)}. \tag{C.43}$$

According to (C.23) and (C.26), the log market return can be rewritten as

$$r_{m,t+1} - \mathbb{E}_t\left[r_{m,t+1}\right] = \beta_c\sigma_c\varepsilon_{c,t+1} + \beta_p\sigma_p p\varepsilon_{p,t+1} - \beta_\zeta\left[\zeta_{t+1} - p_t\mu_1(\alpha)\right], \tag{C.44}$$

where $\beta_c = \phi$, $\beta_p = \kappa_{m,1}A_{m,1}$, and $\beta_\zeta = \phi$. The Euler equation for the log market return is

$$1 = \mathbb{E}_t\left[e^{r_{m,t+1}+m_{t+1}}\right], \tag{C.45}$$

which leads to

$$\mathbb{E}_t\left[e^{m_{t+1}}\right] = \mathbb{E}_t\left[e^{r_{m,t+1}-r_{f,t}+m_{t+1}}\right]. \tag{C.46}$$

It further leads to

$$\begin{aligned} & e^{\mathbb{E}_t[m_{t+1}]+\frac{1}{2}\left[\gamma^2\sigma_c^2+\lambda_p^2\sigma_p^2p^2\right]-\gamma p_t\mu_1(\alpha)}\mathbb{E}_t\left[e^{\gamma\zeta_{t+1}}\right] \\ &\quad = e^{\mathbb{E}_t[m_{t+1}]+\mathbb{E}_t[r_{m,t+1}]-r_{f,t}+\frac{1}{2}\left[(\phi-\gamma)^2\sigma_c^2+(\beta_p-\lambda_p)^2\sigma_p^2p^2\right]-(\gamma-\phi)p_t\mu_1(\alpha)}\mathbb{E}_t\left[e^{(\gamma-\phi)\zeta_{t+1}}\right]. \end{aligned} \tag{C.47}$$

Finally, rearranging terms leads to

$$\mathbb{E}_t\left[r_{m,t+1}\right] - r_{f,t} = \phi\gamma\sigma_c^2 + \beta_p\lambda_p\sigma_p^2p^2 + \left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-\phi) - \phi\mu_1(\alpha)\right]p_t - \frac{1}{2}\left(\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2p^2\right).$$

Now, we derive the yield of the defaultable government bond, denoted by $y_{b,t}$, and we express the log government bond return as follows:

$$r_{b,t+1} = y_{b,t} - x_{b,t+1}(\underline{v} + J_{t+1}). \tag{C.48}$$

A default on the government bond occurs with probability $q$ conditional on the occurrence of a disaster. Thus, by definition, it holds that

$$\mathbb{E}_t\left[r_{b,t+1}\right] = y_{b,t} - p_tq\mu_1(\alpha). \tag{C.49}$$

According to the Euler equation of the defaultable government bond and the risk-free bond, it holds that

$$\mathbb{E}_t\left[e^{m_{t+1}+r_{b,t+1}-r_{f,t}}\right] = \mathbb{E}_t\left[e^{m_{t+1}}\right]. \tag{C.50}$$

Some calculations show that the following relation approximately holds:

$$\begin{aligned}
&\ln \mathbb{E}_t\left[e^{m_{t+1}+r_{b,t+1}-r_{f,t}}\right] \\
&\quad = \Gamma_0 + \Gamma_1 p_t + y_{b,t} - r_{f,t} + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}\lambda_p^2\sigma_p^2 p^2 + \ln \mathbb{E}_t\left[e^{\gamma x_{t+1}v_{t+1} - x_{b,t+1}v_{t+1}}\right] \\
&\quad = \Gamma_0 + \Gamma_1 p_t + y_{b,t} - r_{f,t} + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}\lambda_p^2\sigma_p^2 p^2 + \left[(1-q)\ell(\alpha,\gamma) + q\ell(\alpha,\gamma-1)\right] p_t. \qquad \text{(C.51)}
\end{aligned}$$

Combining (C.22), (C.50), and (C.51), it follows that

$$y_{b,t} - r_{f,t} = q\left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-1)\right] p_t. \qquad \text{(C.52)}$$

Further, putting (C.49) and (C.52) together, we obtain the following relation:

$$\mathbb{E}_t\left[r_{b,t+1}\right] - r_{f,t} = q\left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-1) - \mu_1(\alpha)\right] p_t. \qquad \text{(C.53)}$$

Therefore, the conditional mean of excess log returns of the market portfolio relative to the defaultable government bill is

$$\begin{aligned}
\mathbb{E}_t\left[r_{m,t+1} - r_{b,t+1}\right] = {} & \phi\gamma\sigma_c^2 + \beta_p\lambda_p\sigma_p^2 p^2 + \left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-\phi) - \phi\mu_1(\alpha)\right] p_t \qquad \text{(C.54)} \\
& - \frac{1}{2}\left(\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2\right) - q\left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-1) - \mu_1(\alpha)\right] p_t,
\end{aligned}$$

and the conditional variance of excess returns of the market portfolio relative to the defaultable government bill is

$$\begin{aligned}
\mathbb{V}ar_t\left[r_{m,t+1} - r_{b,t+1}\right] &= \mathbb{V}ar_t\left[\phi\sigma_c\varepsilon_{c,t+1} + \beta_p\sigma_p p\varepsilon_{p,t+1} - (\phi x_{t+1} - x_{b,t+1})(\underline{v} + J_{t+1})\right] \\
&= \phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2 + \left(\phi^2 - 2\phi q + q\right) p_t\mu_2(\alpha) - (\phi - q)^2 p_t^2\mu_1(\alpha)^2. \qquad \text{(C.55)}
\end{aligned}$$

**Verifying the Asset Pricing Moment Conditions.** Lastly, we verify the asset pricing moment conditions. The first moment condition is the expected log return of defaultable government bills:

$$\mathbb{E}\left[r_{b,t}\right] - \omega_1(\vartheta) = \left(\mathbb{E}\left[r_{b,t}\right] - r_{f,t-1}\right) + r_{f,t-1} - \omega_1(\vartheta). \qquad \text{(C.56)}$$

Plugging (C.22) and (C.53) into the equation above, it follows that

$$\begin{aligned}
\mathbb{E}\left[r_{b,t}\right]-\omega_1(\vartheta) &= q\left[\ell(\alpha,\gamma)-\ell(\alpha,\gamma-1)-\mu_1(\alpha)\right]\mathbb{E}\left[p_{t-1}\right] \\
&\quad -\ln\delta+g_c-\frac{1}{2}(2\gamma-1)\sigma_c^2-\left[\ell(\alpha,\gamma)-\ell(\alpha,\gamma-1)\right]\mathbb{E}\left[p_{t-1}\right]-\omega_1(\vartheta) \\
&= -\ln\delta+g_c-\frac{1}{2}(2\gamma-1)\sigma_c^2-(1-q)p\left[\ell(\alpha,\gamma)-\ell(\alpha,\gamma-1)\right]-qp\mu_1(\alpha)-\omega_1(\vartheta) \\
&= -\ln\delta+g_c-\frac{1}{2}(2\gamma-1)\sigma_c^2-qp\mu_1(\alpha)-(1-q)h_1(\vartheta)\frac{p}{\alpha-\gamma}-\omega_1(\vartheta) \\
&= 0, \qquad \text{(C.57)}
\end{aligned}$$

where $h_1(\vartheta)\equiv\alpha\left[e^{\underline{v}\gamma}-e^{\underline{v}(\gamma-1)}\frac{\alpha-\gamma}{\alpha-\gamma+1}\right]$.

The second asset pricing moment condition is the unconditional variance of the log government bill return:

$$\mathbb{E}\left[r_{b,t}-\omega_1(\vartheta)\right]^2-\omega_2(\vartheta)=\mathbb{V}ar\left[\mathbb{E}_{t-1}(r_{b,t})\right]+\mathbb{E}\left[\mathbb{V}ar_{t-1}(r_{b,t})\right]-\omega_2(\vartheta) \qquad \text{(C.58)}$$

Plugging in the equilibrium expressions for $\mathbb{V}ar\left[\mathbb{E}_{t-1}(r_{b,t})\right]$ and $\mathbb{E}\left[\mathbb{V}ar_{t-1}(r_{b,t})\right]$, the equation above can further lead to

$$\begin{aligned}
&\mathbb{E}\left[r_{b,t}-\omega_1(\vartheta)\right]^2-\omega_2(\vartheta) \\
&= \left[(1-q)(\ell(\alpha,\gamma)-\ell(\alpha,\gamma-1))+q\mu_1(\alpha)\right]^2\frac{\sigma_p^2p^2}{1-\rho^2} \\
&\quad +qp\mu_2(\alpha)-q^2p^2\mu_1(\alpha)^2\left(1+\frac{\sigma_p^2}{1-\rho^2}\right)-\omega_2(\vartheta). \qquad \text{(C.59)}
\end{aligned}$$

According to the definition of the function $\omega_2(\vartheta)$ and the equation above, it follows that

$$\mathbb{E}\left[r_{b,t}-\omega_1(\vartheta)\right]^2-\omega_2(\vartheta)=0. \qquad \text{(C.60)}$$

The third asset pricing moment condition is the unconditional mean of the excess log market return relative to the defaultable government bills. The excess log market return is

$$\begin{aligned}
r_{m,t}^e &= \mathbb{E}_{t-1}\left[r_{m,t}^e\right]+\phi\sigma_c\varepsilon_{c,t}+\beta_p\sigma_p\sqrt{p}\varepsilon_{p,t} \qquad \text{(C.61)} \\
&\quad -\phi\left[x_t(\underline{v}+J_t)-p_{t-1}\mu_1(\alpha)\right]+\left[x_{b,t}(\underline{v}+J_t)-qp_{t-1}\mu_1(\alpha)\right],
\end{aligned}$$

with the conditional expected excess log market return to be

$$\mathbb{E}_{t-1}\left[r^e_{m,t}\right] = \phi\gamma\sigma_c^2 + \beta_p\lambda_p\sigma_p^2 p^2 + \left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-\phi) - \phi\mu_1(\alpha)\right] p_{t-1}$$
$$- \frac{1}{2}\left(\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2\right) - q\left[\ell(\alpha,\gamma) - \ell(\alpha,\gamma-1) - \mu_1(\alpha)\right] p_{t-1}. \tag{C.62}$$

Therefore, the unconditional mean of excess log market returns is

$$\mathbb{E}\left[r^e_{m,t}\right] = h_0(\vartheta) + h_3(\vartheta)\frac{p}{\alpha-\gamma}, \tag{C.63}$$

$$\text{where } \; h_0(\vartheta) = \phi\gamma\sigma_c^2 + \beta_p\lambda_p\sigma_p^2 p^2 - \frac{1}{2}\left(\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2\right),$$
$$h_3(\vartheta) = \alpha\left[(1-q)e^{\underline{v}\gamma} - e^{\underline{v}(\gamma-\phi)}\frac{\alpha-\gamma}{\alpha-\gamma+\phi} + qe^{\underline{v}(\gamma-1)}\frac{\alpha-\gamma}{\alpha-\gamma+1}\right] - (\alpha-\gamma)(\phi-q)\mu_1(\alpha).$$

The fourth asset pricing moment condition is the unconditional variance of the excess log market return relative to the defaultable government bills. The unconditional variance has the following decomposition:

$$\mathbb{V}ar\left[r^e_{m,t+1}\right] = \mathbb{E}\left[\mathbb{V}ar_t(r^e_{m,t+1})\right] + \mathbb{V}ar\left[\mathbb{E}_t\left(r^e_{m,t+1}\right)\right]. \tag{C.64}$$

The unconditional mean of the conditional variance is

$$\mathbb{E}\left[\mathbb{V}ar_t\left(r^e_{m,t+1}\right)\right] = \mathbb{E}\left[\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2 + p_t(q - 2\phi q + \phi^2)\mu_2(\alpha) - p_t^2(q-\phi)^2\mu_1(\alpha)^2\right] \tag{C.65}$$
$$= \phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 p^2 + (q - 2\phi q + \phi^2)\mu_2(\alpha)p - (q-\phi)^2\mu_1(\alpha)^2\left(\frac{\sigma_p^2 p^2}{1-\rho^2} + p^2\right).$$

The unconditional variance of the conditional mean is

$$\mathbb{V}ar\left[\mathbb{E}_t\left(r^e_{m,t+1}\right)\right] = \mathbb{V}ar\left[\left((1-q)\ell(\alpha,\gamma) - \ell(\alpha,\gamma-\phi) + q\ell(\alpha,\gamma-1) - (\phi-q)\mu_1(\alpha)\right)p_t\right]$$
$$= \left[(1-q)\ell(\alpha,\gamma) - \ell(\alpha,\gamma-\phi) + q\ell(\alpha,\gamma-1) - (\phi-q)\mu_1(\alpha)\right]^2\frac{\sigma_p^2 p^2}{1-\rho^2}$$
$$= h_3(\vartheta)^2\frac{1}{(\alpha-\gamma)^2}\frac{\sigma_p^2 p^2}{1-\rho^2}. \tag{C.66}$$

The sixth asset pricing moment condition can be derived as follows, and the fifth asset pricing moment can be derived similarly. The excess log market return in period $t+1$ is

$$r^e_{m,t+1} = h_0(\vartheta) + h_3(\vartheta)\frac{p_t}{\alpha-\gamma} + e_{t+1}, \tag{C.67}$$

where $e_{t+1}$ is a random variable such that $\mathbb{E}_t\left[e_{t+1}\right] = 0$. Thus, the excess log market return in period $t + 1$ can further expressed in terms of $p_{t-1}$:

$$
\begin{aligned}
r^e_{m,t+1} &= h_0(\vartheta) + h_3(\vartheta)(\alpha - \gamma)^{-1}\left[(1-\rho)p + \rho p_{t-1} + \sigma_p p \varepsilon_{p,t}\right] + e_{t+1} \\
&= h_0(\vartheta) + h_3(\vartheta)\frac{p}{\alpha - \gamma} + h_3(\vartheta)(\alpha - \gamma)^{-1}\rho(p_{t-1} - p) + \tilde{e}_t, \qquad \text{(C.68)}
\end{aligned}
$$

where $\mathbb{E}_{t-1}\left[\tilde{e}_t\right] = 0$ with $\tilde{e}_t \equiv h_3(\vartheta)(\alpha - \gamma)\sigma_p p \varepsilon_{p,t} + e_{t+1}$.

The log price-dividend ratio is

$$
z_{m,t} - \overline{z}_m = \frac{1}{1 - \rho\overline{\delta}} h_2(\vartheta)(p_t - p). \qquad \text{(C.69)}
$$

Plugging (C.69) into (C.68), it follows that

$$
r^e_{m,t+1} = \omega_3(\vartheta) + \frac{\rho(1 - \rho\overline{\delta})}{\alpha - \gamma} h_2(\vartheta)^{-1} h_3(\vartheta)(z_{m,t-1} - \overline{z}_m) + \tilde{e}_t. \qquad \text{(C.70)}
$$

Therefore, according to the definition of $\omega_6(\vartheta)$ and the equation above, the asset pricing moment condition follows,

$$
\mathbb{E}_{t-1}\left[r^e_{m,t+1} - \omega_6(\vartheta)(z_{m,t-1} - \overline{z}_m) - \omega_3(\vartheta)\right] = 0. \qquad \text{(C.71)}
$$

# D Robustness Check for the Simulation Result

In Figure 1 of the main text, for each model, the true value of $\theta$ and its parameter space are both calibrated based on the asset pricing moment condition, which links the equity premium to the structural parameter $\theta$. Specifically, the true value of $\theta$ is calibrated to match an annual equity premium of 6% and the bounds of the parameter spaces are calibrated to match 3% and 9%. Here we report a figure similar to Figure 1 of the main text, but the parameter spaces for both models are calibrate to match equity premium between 2% and 10%, as robustness checks. Compared to Figure 1 of the main text, this additional figure demonstrates similar patterns. For each test, the level of the local power does slightly vary with the parameter space because the baseline moments are nearly flat.

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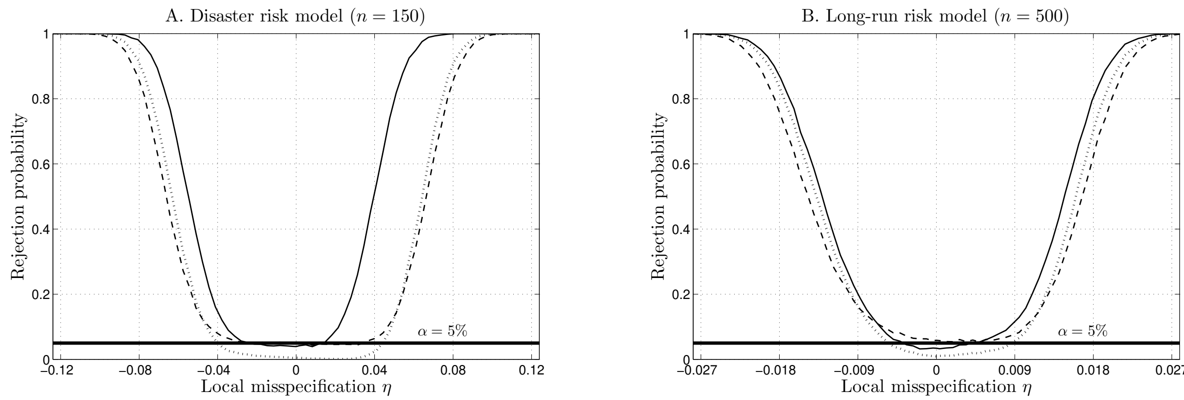
Figure 1: Parameter space calibrated with equity premium between 2% and 10%




Note: Panels A and B plot the rejection probabilities of three different specification tests for the disaster risk model and the long-run risk model, respectively, based on the simulated data. In both panels, the solid curve represents the rejection probability of the conditional specification test; the dashed curve represents the rejection probability of the $J$ test; the dotted curve represents the rejection probability of the $C$ test; and the bold solid horizontal line represents the 5% nominal size for all three specification tests.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