Growth in the Shadow of Expropriation*

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Abstract

In this paper, we propose a tractable variant of the open economy neoclassical growth model that emphasizes political economy and contracting frictions. The political economy frictions involve disagreement and political turnover, while the contracting friction is a lack of commitment regarding foreign debt and expropriation. We show that the political economy frictions induce growth dynamics in a limited-commitment environment that would otherwise move immediately to the steady state. In particular, greater political disagreement corresponds to a high tax rate on investment, which declines slowly over time, generating slow convergence to the steady state. While in the standard neoclassical growth model capital's share in production plays an important role in determining the speed of convergence, this parameter is replaced by political disagreement in our open economy reformulation. Moreover, while political frictions shorten the horizon of the government, the government may still pursue a path of tax rates in which the first best investment is achieved in the long run, although the transition may be slow. The model rationalizes why openness has different implications for growth depending on the political environment, why institutions such as respect for property rights evolve over time, why governments in open countries that grow rapidly tend to accumulate net foreign assets rather than liabilities, and why foreign aid may not affect growth.

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1 Introduction

In this paper we present a tractable growth model that highlights the interaction of political disagreement, tax policy, and capital flows in a small open economy. We augment the standard neoclassical growth model with two frictions. First, there is limited commitment on the part of the domestic government. Specifically, capital income is subject to ex-post expropriation and the government can default on external debt. Second, political parties with distinct objectives compete for power. We show that the combination of these two frictions generate several prominent features of developing economy growth experiences, including the fact that economies with relatively high growth rates tend to have governments that accumulate large net foreign asset positions.

The model builds on the insight of Alesina and Tabellini (1990) and Persson and Svensson (1989) that political disagreement between potential incumbents makes parties prefer spending to occur while in office. As in Amador (2004), we show that the political environment can be conveniently modeled as a sequence of incumbents that possess quasi-hyperbolic preferences. We embed this political process in a small open economy in which the government can expropriate capital and default on external debt. In this limited commitment environment, we show that the time inconsistent preferences of the government induce interesting dynamics for both investment and external debt that are consistent with prominent features of the data.

Specifically, we consider the path of taxes, consumption, investment, and sovereign debt, that maximizes the population's welfare subject to the constraint that each incumbent has the power to repudiate debt and expropriate capital. Deviation, however, leads to financial autarky and reversion to a high tax-low investment equilibrium. In this sense, we study self-enforcing equilibria in which allocations are constrained by the government's lack of commitment, as in Thomas and Worrall (1994), Alburquerque and Hopenhayn (2004), and Aguiar et al. (2009). These papers discuss how limited commitment can slow capital accumulation. A main result of the current paper is that the political economy frictions generate additional dynamics. In particular, we show that the degree of disagreement among political parties has a first order (negative) effect on the speed of the economy's convergence to the steady state. In the standard closed economy version of the neoclassical growth model, the speed of convergence is governed in large part by the capital share parameter. In our reformulation, capital share is replaced with a parameter reflecting the extent of political disagreement.

The intuition behind the dynamics begins with debt overhang. A country with a large external sovereign debt position has a greater temptation to default, and therefore cannot credibly promise to leave large investment positions un-expropriated. Growth therefore requires the country to pay down its debt, generating a trade off between the incumbent's desire to consume while in office against reducing foreign liabilities and increasing investment. In a political environment with a high degree of disagreement, governments are unwilling to reduce their sovereign debt quickly, as the desire for immediate consumption outweighs the future benefits of less overhanging debt. In this manner, the model is able to reconcile the mixed results that countries have had with financial globalization. Countries with different underlying political environments will have different growth experiences after opening: some economies will borrow and stagnate, while others will experience net capital outflows and grow quickly.

Moreover, we show that introducing foreign aid does not alter the dynamics of the capital stock in our benchmark economy. If foreign aid is not conditioned on ex post capital tax rates or the repayment of external debt, aid does not relax the government's credibility constraint. That is, foreign aid without conditionality does not affect the dynamics of tax rates, capital accumulation, or growth. On the other hand, although debt relief does not have long run effects, it increases investment and output in the short run. This is consistent with the empirical literature that finds mixed or insignificant results regarding the effect of aid on growth, and more robust positive effects of the debt relief programs.[1]

Political disagreement in our model generates short-term impatience, but is distinct from a model of an impatient decision maker with time consistent preferences. For example, in our framework the degree of political disagreement may not affect the long run capital stock. In particular, if the private agents discount at the world interest rate, the economy eventually reaches the first best level of capital for any finite level of disagreement. This reflects the fact that while incumbents disproportionately discount the near future, this relative impatience disappears as the horizon is extended far into the future. However, the level of political disagreement will determine the level of steady state debt that supports the first best capital. Moreover, if private agents are impatient relative to the world interest rate, the private discount factor interacts with the level of political disagreement to determine the steady state level of capital.

The mechanism in our paper is consistent with the empirical fact that fast growth is accompanied by reductions in net foreign liabilities, the so-called "allocation puzzle" of

[1] See Rajan and Subramanian (2008) and Arslanalp and Henry (2006) for recent analysis of aid and debt relief.

Gourinchas and Jeanne (2007) (see also Aizenman et al., 2004 and Prasad et al., 2006). This allocation puzzle represents an important challenge to the standard open economy model which predicts that opening an economy to capital inflows will speed convergence, as the constraint that investment equals domestic savings is relaxed.[2] Our model rationalizes the allocation puzzle as capital will not be invested in an economy with high debt due to the risk of expropriation. This provides an incentive for the government to pay down its external debt along the transition path. The model also addresses the related question of why many countries fail to pursue the high growth-low debt strategy. The incentive to pay down debt is opposed by the desire to spend due to political disagreement. In countries with particularly severe political disagreement, the incentive to pay down external debt takes a back seat and growth suffers.

Our model emphasizes *sovereign* debt overhang. In particular, external debt matters in the model to the extent that the government controls repayment or default. While the allocation puzzle has been framed in the literature in terms of aggregate net foreign assets (both public and private), the appropriate empirical measure for our mechanism is public net foreign assets. Figure 1 documents that the allocation puzzle is driven by the net foreign asset position of the public sector. Specifically, we plot growth in GDP per capita (relative to the U.S.) against the change in the ratio of the government's net foreign assets to GDP, where the net position is defined as international reserves minus public and publicly guaranteed external debt.[3] Figure 1 depicts a clear, and statistically significant, relationship between growth and the change in the government's external net assets.

We should emphasize that this relationship is not driven by fast growing governments borrowing heavily at the beginning of the sample period – the relationship between initial government assets and subsequent growth is weakly positive. Moreover, it is not simply that governments save during transitory booms and borrow during busts. As documented by Kaminsky et al. (2004), fiscal surpluses in developing economies are negatively correlated with income at business cycle frequencies . Figure 1 therefore reflects

[2] Two important papers that study the neoclassical growth model in an open economy setting are Barro et al. (1995), who introduce human capital accumulation and a credit market imperfection to obtain nontrivial dynamics, and Ventura (1997). In the open economy version of the neoclassical model studied by Barro et al. (1995), the debt to output ratio is constant along the transition path if the production function is Cobb-Douglas. More generally, their prediction is for an economy to unambiguously accumulate net liabilities as it accumulates capital. See Castro (2005) for a careful quantitative exploration of whether open economy models with incomplete markets and technology shocks can account for the patterns of development observed in the data.

[3] See the notes to the figures for data sources and sample selection. See also the end of Appendix A for a discussion of an augmented model with exogenous growth.

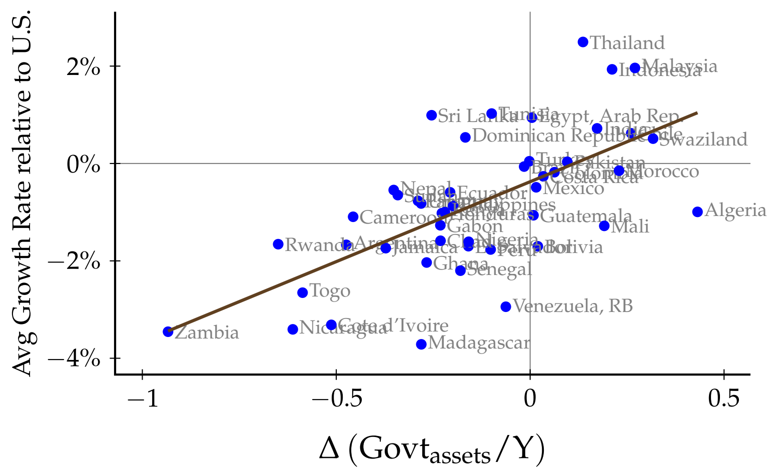

Figure 1: This figure plots average annual growth in real GDP per capita relative to the U.S. against the change in ratio of public net foreign assets to GDP between 1970–2004. Public net foreign assets are international reserves (excluding gold) minus public and publicly guaranteed external debt, both from WDI. Real GDP per capita is constant local currency GDP per capita from World Development Indicators (WDI). The sample includes countries with 1970 GDP per capita less than or equal to USD 10,000 in year 2000 dollars.

long run behavior.

Figure 2 plots growth against the change in private net foreign assets, which is simply total net foreign assets minus public net foreign assets. For the private sector, positive growth is associated with greater net capital inflows on average (albeit weakly), consistent with standard theory. Thus, the puzzle is one regarding government assets, the focus of our model.[4]

Similarly, our paper addresses the issue of "global imbalances" as it relates to the interaction of developing economies with world financial markets. An alternative explanation to ours is that developing economies have incomplete domestic financial markets and therefore higher precautionary savings, which leads to capital outflows (see Willen, 2004 and Mendoza et al., 2008). However, this literature is silent on the heterogeneity across developing economies in terms of capital flows. For example, several Latin American economies have similar or even more volatile business cycle than South Korea (Aguiar and Gopinath, 2007) and less developed financial markets (Rajan and Zingales, 1998),

[4]The public sector asset position is significant when accounting for the total net foreign asset position of countries. The correlation coefficient between the level of total net foreign assets and the level of public net foreign assets, normalized by GDP, is 0.90. The correlation is also high for first differences: The coefficient on an OLS regression of the change in total net foreign assets on the change in the government's foreign asset position from 1970 to 2004 is 0.98.

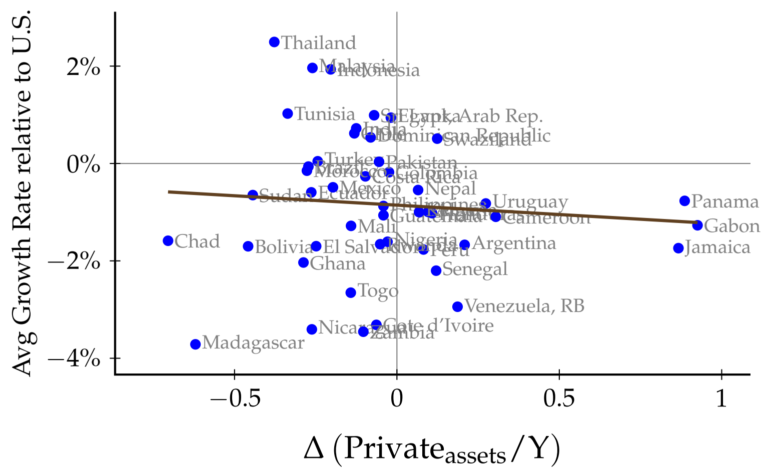

Figure 2: This figure plots average annual growth in real GDP per capita relative to the U.S. against the change in the ration of private net foreign assets to GDP between 1970–2004. Private net foreign assets are total net foreign assets (Net foreign assets are gross foreign assets minus gross liabilities in current US dollars from EWN Mark II) minus public net foreign assets (from Figure 1). Real GDP per capita is constant local currency GDP per capita from World Development Indicators (WDI). The sample includes countries with 1970 GDP per capita less than or equal to USD 10,000 in year 2000 dollars.

yet Latin America is not a strong exporter of capital (Figure 1). Caballero et al. (2008) also emphasize financial market weakness as generating capital outflows. In their model, exogenous growth in developing economies generates wealth but not assets, requiring external savings. Our model shares their focus on contracting frictions in developing economies, but seeks to understand the underlying growth process. As noted above, our paper shares the feature of Thomas and Worrall (1994) that reductions in debt support larger capital stocks. Dooley et al. (2004) view this mechanism through the lens of a financial swap arrangement, and perform a quantitative exercise that rationalizes China's large foreign reserve position. These papers are silent on why some developing countries accumulate collateral and some do not, a primary question of this paper. Our paper also explores how the underlying political environment affects the speed with which countries accumulate collateral or reduce debt.

A predominant explanation of the poor growth performance of developing countries is that weak institutions in general and poor government policies in particular tend to deter investment in capital and/or productivity enhancing technology.[5] A literature has

[5] An important contribution in this regard is Parente and Prescott (2000). Similarly, a large literature links differences in the quality of institutions to differences in income per capita, with a particular emphasis on protections from governmental expropriation (for an influential series of papers along this line, see Acemoglu et al., 2001, 2002 and Acemoglu and Johnson, 2005).

developed that suggests that weak institutions generate capital outflows rather than inflows (see, for example, Tornell and Velasco, 1992 and Alfaro et al., 2008, who address the puzzle raised by Lucas, 1990). While it is no doubt true that world capital avoids countries with weak property rights, our model rationalizes why countries with superior economic performance are net exporters of capital. Our mechanism also endogenizes the evolution of institutions – interpreted as respect for private capital – and the associated growth path.

Our paper also relates to the literature on optimal government taxation with limited commitment. Important papers in this literature are Benhabib and Rustichini (1997) and Phelan and Stacchetti (2001), who share our focus on self-enforcing equilibria supported by trigger strategies (a parallel literature has developed that focuses on Markov perfect equilibria, such as Klein and Rios-Rull, 2003, Klein et al., 2005, and Klein et al., 2008). In this literature, our paper is particularly related to Dominguez (2007), which shows in the environment of Benhabib and Rustichini (1997) that a government will reduce it debt in order to support the first best capital in the long run (see also Reis, 2008). Recently, Azzimonti (2009) has shown how political polarization and government impatience can lead to high levels of investment taxes, slow growth and low levels of output per capita in the context of a closed economy model with capital accumulation, partisan politics and a restriction to Markov strategies. Differently from her work, we are focused on the open economy implications of a political economy model. On the technical side, we analyze trigger strategies and reputational equilibria, as we think these are important elements to consider in any analysis of sovereign debt.

The remainder of the paper is organized as follows. Section 2 presents the environment. Section 3 characterizes the path of equilibrium taxes, investment, and output. Section 4 discusses conditional convergence as it relates to the empirical growth literature, and Section 5 concludes. The appendix features the following extensions of the model: (i) introducing exogenous growth, (ii) allowing capitalists welfare to be in the objective function of the incumbent governments, and (iii) allowing for a more general political process. The appendix also contains all proofs.

2 Environment

In this section we describe the model environment (which is based on Aguiar et al., 2009). Time is discrete and runs from 0 to infinity. There is a small open economy which produces a single good, whose world price is normalized to one. There is also an international financial market that buys and sells risk-free bonds with a return denoted by $R = 1 + r$.

The economy is populated by capitalists, who own and operate capital, workers who provide labor, and a government. In our benchmark analysis, we assume that capitalists do not enter the government's objective function, defined below. This assumption is convenient in that the government has no hard-wired qualms about expropriating capital income and transferring it to its preferred constituency. This assumption is not crucial to the results and we discuss the more general case with "insider" capitalists in Appendix B. The important assumption is that capitalists operate a technology otherwise unavailable to the government and are under the threat of expropriation.

2.1 Firms

Domestic firms use capital together with labor to produce according to a strictly concave, constant returns to scale production function $f(k, l)$.[6] We assume that $f(k, l)$ satisfies the usual Inada conditions. Capital is fully mobile internationally at the beginning of every period,[7] but after invested is sunk for one period. Capital depreciates at a rate d.

Labor is hired by the firms in a competitive domestic labor market which clears at an equilibrium wage w_t. The government taxes the firm profits at a rate τ_t. Let $\pi = f(k, l) - wl$ denote per capita profits before taxes and depreciation, and so $(1 - \tau)\pi$ is after-tax profits. The firm rents capital at the rate $r + d$. Given an equilibrium path of wages and capital taxes, profit maximizing behavior of the representative firm implies:

$$(1 - \tau_t) f_k(k_t, l_t) = r + d \tag{1}$$

$$f_l(k_t, l_t) = w_t. \tag{2}$$

For future reference, we denote $k^\star$ as the first best capital given a mass one of labor: $f_k(k^\star, 1) = r + d$. When convenient in what follows, we will drop the second argument and simply denote production $f(k)$.

[6] The model focuses on transitional dynamics and assumes a constant technology for convenience. The model easily accommodates exogenous technical progress in which the steady state is a balanced growth path. See Appendix A for details.

[7] That is capital will earn the same after tax return in the small open economy as in the international financial markets. See Caselli (2007) on evidence that returns to capital are quantitatively very similar across countries.

It is convenient to limit the government's maximal tax rate to $\bar{\tau} > 0$. We assume that this constraint does not bind along the equilibrium path. Nevertheless, as discussed in Section 2.5, this assumption allows us to characterize possible allocations off the equilibrium path.

2.2 Domestic workers, capitalists and the government

Labor is supplied inelastically each period by a measure-one continuum of domestic workers (there is no international mobility of labor). The representative domestic workers enjoys utility flows over per capita consumption, $u(c)$. Domestic workers discount the future with a discount factor $\beta \in (0, 1/R]$. The representative agent's utility is

$$\sum_{t=0}^{\infty} \beta^t u(c_t). \tag{3}$$

with $u' > 0, u'' \leq 0$, and where we normalize $u(0) \geq 0$.

We assume that domestic workers have no direct access to international capital markets. In particular, we assume that the government can control the consumption/savings decisions of its constituents using lump sum transfers and time varying taxes or subsidies on domestic savings. This is equivalent in our set up to workers consuming their wages plus a transfer: $c_t = w_t + T_t$, where T_t is the transfer from the government.[8]

The government every period receives the income from the tax on profits and transfers resources to the workers subject to its budget constraint:

$$\tau_t \pi_t + b_{t+1} = R b_t + T_t \tag{4}$$

where b_t is debt due in period t. The government and workers combined resource constraint is therefore:

$$c_t + R b_t = b_{t+1} + \tau \pi_t + w_t. \tag{5}$$

Note that output is deterministic, and so a single, risk-free bond traded with the rest of the world is sufficient to insure the economy. However, as described in the next subsection, political incumbents face a risk of losing office, and this risk is not insurable. The fact that sovereign debt is not contingent on individual leader's political fortunes is a realistic assumption. We leave the question of how debt contingent on political outcomes affects

[8]This can be decentralized by consumers having access to a tax distorted bond.

dynamics for future research.

2.3 Political Environment

There is a set $I \equiv \{1, 2, ..., N\}$ of political parties, where N is the number of parties. At any time, the government is under the control of an incumbent party that is chosen at the beginning of every period from set I. As described below, an incumbent party may lose (and regain) power over time. Our fundamental assumption is that the incumbent strictly prefers consumption to occur while in power:

Assumption 1 (Political Economy Friction)**.** *A party enjoys a utility flow $\tilde{\theta}u(c)$ when in power and a utility flow $u(c)$ when not in power, where c is per capita consumption by the domestic workers and where $\tilde{\theta} > 1$.*

The parameter $\tilde{\theta}$ parameterizes the extra marginal utility benefits that a party extracts from expenditures when in power. One motivation for this parameter is political disagreement regarding the type of expenditures, as in, for example, the classic paper of Alesina and Tabellini (1990). Specifically, suppose that the incumbent party selects the attributes of a public good that forms the basis of private consumption. If parties disagree about the desirable attributes of the consumption good, the utility stemming from a given level of spending will be greater for the party in power. We model such disagreement in a simple, reduced form way with the parameter $\tilde{\theta}$.[9] Alternatively, we can think of the incumbent capturing a disproportionate share of per capita consumption.[10]

The transfer of power is modeled as an exogenous Markov process. The fact that the transfer of power is exogenous can be considered a constraint on political contracts between the population (or other parties) and the incumbent. As will be clear, each incumbent will abide by the constrained efficient tax plan along the equilibrium path. However, doing so does not guarantee continued incumbency (although our results easily extend to the case where the incumbent loses office for sure if it deviates). That is, following the prescribed tax and debt plan does not rule out that other factors may lead to a change of government. We capture this with a simple parametrization that nests perpetual office holding, hard term limits, and the probabilistic voting model. Our benchmark case

[9] See Battaglini and Coate (2008) for a recent paper that incorporates pork-barrel spending in a dynamic model of fiscal policy. They obtain a reduced form representation that is similar to ours, except that $\tilde{\theta}$ is also a function of the state of the economy.

[10] Suppose that when in power, a party receives a higher share ϕ of c. Then, the marginal utility when in power is $u'(\phi c)\phi$, and our assumption would be similar to requiring that this marginal utility be increasing in ϕ.

is when the probability of a party being in power at any time is given by $p = 1/N$, and independent of the history. [11]

Given a deterministic path of consumption, the utility of the incumbent in period t can now be expressed as:

$$\tilde{W}_t = \tilde{\theta} u(c_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} (p\tilde{\theta} + 1 - p) u(c_s). \tag{6}$$

Note that the incumbent discounts the future at the private agents' discount factor β, but incumbency implies that it discounts between the current and next period at the rate $\beta(p\tilde{\theta} + 1 - p)/\tilde{\theta} < \beta$. That is, for a deterministic path of consumption, the incumbent's objective is equivalent to a quasi-hyperbolic agent as in Laibson (1997) (the fact that political turnover can induce hyperbolic preferences for political incumbents was explored by Amador, 2004). This framework is rich enough to capture several cases. A situation where the country is ruled by a "dictator for life" who has no altruism for successive generations, can be analyzed by letting $\tilde{\theta} \to \infty$, reflecting the zero weight the dictator puts on aggregate consumption once it is out of power. Letting $\tilde{\theta} \to 1$, the government is benevolent and the political friction disappears.

We simplify the expression for the government's value function by introducing a sequence of fictitious "stand-in" governments, each of which is in power one period and has a value function W_t proportional to $\tilde{W}_t$. Specifically, we can normalize the government's value function (6) by the constant $p\tilde{\theta} + 1 - p$. As will be clear below, the scaling of the incumbent's utility has no effect on the equilibrium allocations, and so we work with W_t. In particular,

$$\begin{aligned} W_t = \frac{\tilde{W}_t}{(p\tilde{\theta} + 1 - p)} &= \theta u(c_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s) \\ &= \theta u(c_t) + \beta V_{t+1}, \end{aligned} \tag{7}$$

where V_t is the value function of private agents and $\theta \equiv \frac{\tilde{\theta}}{p\tilde{\theta}+1-p}$. Note that θ is increasing in $\tilde{\theta}$, but is decreasing in p. That is, a lower probability of retaining office or more political parties increase political disagreement from the perspective of our stand-in government.

[11] The more general case in which an incumbent has a higher probability of retaining office is treated in Appendix C. The main results presented below extend to the more general political process. The appendix also discusses new results that arise when incumbency is persistent.

2.4 Equilibrium Concept

The final key feature of the environment concerns the government's lack of commitment. Specifically, tax policies and debt payments for any period represent promises that can be broken by the government. Given the one-period irreversibility of capital, there exists the possibility that the government can seize capital or capital income. Moreover, the government can decide not to make promised debt payments in any period.

We consider self-enforcing equilibria that are supported by a "punishment" equilibrium. Specifically, let $\underline{W}(k)$ denote the payoff to the incumbent government after a deviation when capital is k, which we characterize in the next subsection. Self-enforcing implies that:

$$W_t \geq \underline{W}(k_t), \forall t, \tag{8}$$

where W_t is given by (7).

Our equilibrium concept assumes that political risk is not insurable. That is, sovereign debt or tax promises cannot be made contingent on the realization of the party in power, which we take as a realistic assumption.[12] We therefore look for equilibria under the following definition:

Definition 1. *A* ***self-enforcing deterministic equilibrium*** *is a deterministic sequence of consumption, capital, debt, tax rates and wages* $\{c_t, k_t, b_t, \tau_t, w_t\}$*, with* $\tau_t \leq \bar{\tau}$ *for all* t *and such that (i) firms maximize profits given taxes and wages; (ii) the labor market clears; (iii) the resource constraint (5) and the associated no Ponzi condition hold given some initial debt* b_0*; and (iv) the participation constraint (8) holds given deviation payoffs* $\underline{W}(k_t)$*.*

2.5 The punishment and the deviation payoff

For simplicity we defined the equilibrium conditional on deviation payoffs $\underline{W}(k)$. Towards obtaining this punishment payoffs, we assume that after any deviation from the equilibrium allocation, the international financial markets shut down access to credit and assets forever.

Given that the country has no access to borrowing nor savings after a deviation, we

[12] That is, foreigners contract with governments, not individual parties. One way to rationalize the absence of political insurance is to assume that foreign creditors cannot distinguish among the various domestic political parties (or factions within a party, and so on). International financial assets therefore cannot make promises contingent on political outcomes.

construct a punishment equilibrium of the game between investors and the government that has the following strategies. For any history following a deviation, the party in power sets the tax rate at its maximum $\bar{\tau}$, and investors invest $\underline{k}$, where $\underline{k}$ solves:

$$(1-\bar{\tau})f'(\underline{k}) = r + d,$$

where $\underline{k} = 0$ if $\bar{\tau} \geq 1$. These strategies form an equilibrium. A party in power today cannot gain by deviating to a different tax rate, given that it is already taxing at the maximum rate and reducing taxes does not increase future investment. On the other hand, investors understand that they will be taxed at the maximum rate, and thus invest up to the point of indifference.

The following lemma establishes that the above equilibrium is the harshest punishment:

Lemma 1. *The continuation equilibrium where $\tau_t = \bar{\tau}$ after any history and the country is in financial autarky generates the lowest utility to the incumbent party of any self-enforcing equilibrium.*

In any self-enforcing equilibrium, once the investors have invested k in the country, the party in power could deviate from the equilibrium path by choosing a different tax rate or by changing the equilibrium path of debt issuance. Such a deviation triggers financial autarky and the lowest possible investment. If the party in power where to deviate, it will find optimal to tax current k at the maximum possible rate, and its deviating payoff will be given by $\underline{W}(k)$:

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \frac{\beta}{1-\beta} u(\bar{c}(\underline{k})), \tag{9}$$

where $\bar{c}(k) = f(k) - (1-\bar{\tau})f'(k)k$.

3 Efficient Allocations

There are in principle multiple equilibria of this economy. In this section we solve for the self-enforcing deterministic equilibrium that maximizes the utility of the population as of time 0 given an initial level of debt. That is, the population chooses its preferred fiscal policy subject to ensuring the cooperation of all future governments, which is a natural benchmark.[13]

We assume for the rest of the paper that the small open economy is at least as impatient as the foreigners:

Assumption 2. *The parameters are such that $\beta R \leq 1$.*

This assumption guarantees that the government of the small open economy does not accumulate assets to infinity. It is also the relevant empirical case. For example, $\beta R = 1$ obtains if the world behaves as a neoclassical growth model, is at a steady state, and domestic agents are as patient as foreigners. The restriction that $\beta R < 1$ for a small open economy is commonly used in quantitative sovereign debt models (see for example, Aguiar and Gopinath, 2006 and Arellano, forthcoming).

As it is standard in the Ramsey taxation literature, we first show that we can restrict attention to allocations, that is, a sequence of consumption and capital: $\{c_t, k_t\}$. To see this note that conditions (i), (ii) and (iii) of Definition 1 can be collapsed to a present value condition:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r+d)k_t - c_t)$$

Importantly, for any allocation $\{c_t, k_t\}$ that satisfies the above present value condition and $k_t \leq \underline{k}$, there exist a tax rate sequence $\{\tau_t \leq \bar{\tau}\}$, a wage sequence $\{w_t\}$, and a debt position $\{b_t\}$ such that $\{c_t, k_t, b_t, \tau_t, w_t\}$ is a competitive equilibrium (satisfies (i), (ii) and (iii)). That is, if an allocation satisfies the present value condition and also satisfies the participation constraint (8), then it is a self enforcing deterministic equilibrium.

We can then obtain the equilibrium allocation that maximizes the utility of the population at time zero, given an initial stock of debt b_0, by solving:

$$V(b_0) = \max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{P}$$

[13] An alternative equilibria is one in which the initial government selects the best self-enforcing fiscal policy from its perspective, where "initial" could be interpreted as the time the economy opens itself to capital flows. This equilibrium has the same dynamics as the one we study in the next subsection, starting from the second period.

subject to:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - (r+d)k_t - c_t \right), \tag{10}$$

$$\underline{W}(k_t) \leq \theta u(c_t) + \beta \sum_{s=t+1}^{\infty} \beta^{s-t-1} u(c_s), \ \forall t \tag{11}$$

$$\underline{k} \leq k_t \tag{12}$$

The first constraint is the present value condition discussed previously; the second constraint is the participation constraint for the sequence of incumbents; and the last constraint, (12), guarantees that $\bar{\tau} \leq \tau_t$. Unless stated otherwise, in what follows we assume that this last constraint does not bind along the equilibrium path.[14]

Let μ_0 be the multiplier on the budget constraint (10) and $R^{-t}\mu_0\eta_t/\theta$ be the multiplier on the sequence of constraints on participation (11). The necessary first order condition for the optimality of consumption is:

$$1 = u'(c_t)\left(\underbrace{\frac{\beta^t R^t}{\mu_0}}_{\text{impatience}} + \underbrace{\sum_{s=0}^{t} \beta^s R^s \frac{\eta_{t-s}}{\theta}}_{\text{limited commitment}} + \underbrace{\left(\frac{\theta-1}{\theta}\right)\eta_t}_{\text{disagreement}} \right), \ \forall t \geq 0. \tag{13}$$

This first order condition for consumption (13) has three terms. The first term, $(\beta R)^t/\mu_0$, is the standard consumption tilting: agents prefer to delay, smooth, or bring forward consumption depending on whether $\beta R \gtreqless 1$. The second term, $\sum_{s=0}^{t}(\beta R)^s \frac{\eta_{t-s}}{\theta}$, reflects the fact that raising consumption in period t relaxes the participation constraints for periods $t - s < t$ as well. This term highlights the efficiency of back-loading payments in one-sided limited commitment models: when $\beta R = 1$, this term is monotone and increasing in t, and thus will lead to an increasing path of consumption. Political disagreement however, introduces one new term: $(\theta - 1)\frac{\eta_t}{\theta}$. This term tells us that the current period is special, as an increase in utility at time t relaxes the current incumbent's participation constraint by an extra $(\theta - 1)$.

The necessary condition for the optimality of the capital stock is:

$$\frac{\eta_t}{\theta} \underline{W}'(k_t) = f'(k_t) - (r+d), \ \forall t \geq 0 \tag{14}$$

The lack of commitment is also evident in this condition, when coupled with the firms'

[14] In the appendix we described the general solution taking into account that this constraint may bind.

first order condition. Note that absent commitment problems ($\eta_t = 0$), capital would be at the first best, as taxing capital in this model is inefficient ex-ante. However, under lack of commitment, a zero tax may not be self-enforcing. When the participation constraint on the incumbent government is binding ($\eta_t > 0$), then $f'(k_t) > r + d$ and so the tax on capital is strictly positive. Nevertheless, the necessary conditions imply that $\tau = 0$ will be sustained in the long run if private agents discount at the world interest rate:

Proposition 1. *If $\beta R = 1$ and $\theta < \infty$, then $k_t \to k^*$.*

The proof of this proposition (see Appendix D) relies on the fact that each time the participation constraint binds, $\eta_t > 0$ and we add a strictly positive term to the sum on the right hand side of (13). This generates a force for increasing consumption over time, which relaxes the government's participation constraint. There is a potentially countervailing force in that the current η_t is weighted by more than the past, as $\theta > 1$. However, eventually the (infinite) sum dominates and consumption levels off at a point such that participation no longer binds at $k^\star$.

As discussed above, a general feature of models with one-sided limited commitment is that the optimal contract "back loads" incentives when agents are patient (see, for example, Ray, 2002). However, if the agent that suffers from lack of commitment is impatient, this is not necessarily the case. For example, in the models of Aguiar et al. (2009) and Acemoglu et al. (2008), governmental impatience prevents the first best level of investment from being achieved in the long run. In our environment, we approach this first best level despite the fact that the incumbent government, which chooses the tax rate at every period, is discounting between today and tomorrow at a *higher rate* than that of the private agents. However, the critical point is that each incumbent discounts the future periods at the same rate $\beta = 1/R$. For this reason, each government is willing to support a path of investment that approaches the first best. This highlights that short term impatience of the incumbents is not sufficient to generate distortions in the long-run.

Another interesting feature of the model is that financial openness always benefits the population in our chosen equilibrium. Financial openness expands the budget set relative to continued financial autarky, starting from zero external debt. Moreover, because deviation leads to financial autarky, no other constraint is affected. It immediately follows that financial openness, all else equal, will (weakly) raise the welfare of the population (or the initial decision maker). Note that in our model, the benefits of financial openness are not just generated from a faster transition, as in the neoclassical growth model, but may also arise from the ability to sustain a different and better steady state due to the accumulation of net foreign assets. The welfare gains from openness may therefore be higher than the transitional gains in the neoclassical growth model, which are quantitatively small as emphasized by Gourinchas and Jeanne (2006).

3.1 Linear Utility

In this subsection, we study the case of linear utility, $u'(c) = 1$, for which we can solve for the equilibrium dynamics in closed form. Although an extreme case, the intuition of the linear case carries over to the concave case studied next. For what follows, we will ignore the non-negativity constraint on consumption (or else, the reader can assume that the analysis is in the neighborhood of the steady state of the economy, which will turn out to feature positive consumption levels).

In the case of linear utility, the first order condition for consumption becomes:

$$1 = \frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^{t} \beta^s R^s \frac{\eta_{t-s}}{\theta} + \left(\frac{\theta - 1}{\theta}\right) \eta_t, \, \forall t \geq 0 \tag{15}$$

The initial period η_0 is therefore $\eta_0 = 1 - \mu_0^{-1}$. As μ_0 is the multiplier on b_0, more debt in period 0 is associated (weakly) with a larger μ_0 and a larger η_0. As can be seen, the multiplier on the resource constraint cannot be smaller than 1, which implies from the associated envelope condition that $V'(b_0) = -\mu_0 \leq -1$. This is intuitive as -1 is the efficient rate of resource transfers between the small open economy and the foreigners in the absence of a binding participation constraint (in an interior solution). The binding participation constraints distort this rate, making it increasingly costly to transfer resources to the foreigners as b_0 increases.

Equation (15) pins down the dynamics of η_t, the multiplier on the government's participation constraint. The dynamics of k_t can be recovered from η_t through the first order condition for capital (14), which states: $\eta_t = \theta(f'(k_t) - r - d)/\underline{W}'(k_t)$. The second order conditions require that, in the neighborhood of the optimum, the latter term is decreasing in k_t. We strengthen this by assuming that this holds globally:

Assumption 3 (Convexity). $\frac{f'(k_t)-r-d}{\underline{W}'(k_t)}$ *is decreasing in* k *for all* $k \in [\underline{k}, k^\star]$.

In the linear case, this holds under mild assumptions.[15] This assumption will also always

[15]For linear utility, sufficient conditions are that $\bar{\tau} = 1$, or that the curvature of the production function, $-\frac{f''(k)k}{f'(k)}$, be non decreasing in k. The latter is satisfied for the usual Cobb-Douglas production function.

be satisfied for concave utility in the neighborhood of $k^\star$. Note that this assumption also ensures that the constraint set in problem (P) is convex, so conditions (13) and (14) are necessary and sufficient.[16] With this assumption in hand, we can explore the dynamics of k by studying η, as k is now monotonically (and inversely) related to η. In the same way, the dynamics of capital taxation are mapped monotonically into the the dynamics of η. A particularly convenient benchmark in the linear utility case is $\bar{\tau} = 1$. In this case, $\tau_t = \eta_t$, so the tax rate is just equal to the multiplier on participation.[17] More generally, monotone convergence of η_t implies monotone convergence of k_t and of the tax rates that decentralize it.

We now characterize the dynamics of η_t:

Proposition 2 (Linear Dynamics)**.** *The multiplier η_t that solves (15) satisfies the following difference equation:*

$$\eta_{t+1} = 1 - \beta R + \beta R \Big(1 - \frac{1}{\theta}\Big)\eta_t \,, \, \forall t \geq 0 \tag{16}$$

with $\eta_0 = 1 - \mu_0^{-1}$ and $\mu_0 \geq 1$. The sequence of η_t converges monotonically towards its steady state value η_∞:

$$\eta_\infty = \frac{\theta(1-\beta R)}{\theta(1-\beta R) + \beta R}.$$

The convergence of η_t implies that the sequence of k_t converges to a steady state. Define $\bar{\theta}$ to be such that $\bar{\theta}(1-\beta R)/(\bar{\theta}(1-\beta R)+\beta R) = (f'(\underline{k}) - (r+d))/\bar{c}'(\underline{k})$. Then,

Corollary 1 (Monotone Convergence)**.** *The sequence of capital, k_t, converges monotonically to its steady state level of capital, k_∞. The value of k_∞ solves*

$$\frac{f'(k_\infty) - (r+d)}{\bar{c}'(k_\infty)} = \frac{\theta(1-\beta R)}{\theta(1-\beta R) + \beta R} \tag{17}$$

[16]To see that Assumption 3 implies convexity of the planning problem, make the following change of variables in problem P: let $h_t = \underline{W}(k_t)$ be our choice variable instead of capital, and define $K(\underline{W}(k)) = k$ to be the inverse of $\underline{W}(k)$. Similarly, we make utility itself the choice variable and let $c(u)$ denote the inverse utility function, that is, the consumption required to deliver the specified utility. In this way, the objective function and constraint (11) are linear in the choice variables. The budget constraint is convex if $f(K(h)) - (r+d)K(h)$ is concave in h, which is the same requirement as Assumption 3.

[17]To see this, note form (9) that if $\bar{\tau} = 1$ we have $\underline{W}'(k) = \theta f'(k)$. The first order condition for capital then implies that $\eta_t = 1 - (r+d)/f'(k)$. The fact that $(1-\tau)f'(k) = r + d$ then gives $\eta = \tau$.

as long as $\theta \leq \bar{\theta}$, and equals $\underline{k}$ otherwise.[18,19] *If country A starts with a higher sovereign debt level than country B, then all else equal, the path of capital for country A will be (weakly) lower than that for country B.*

Note that the value k_∞ is decreasing in θ as long as $\beta R < 1$ and $\theta \leq \bar{\theta}$. That is, more disagreement when coupled with impatience leads to lower steady state levels of investment.

From the fact that $\eta_0 = 1 - \mu_0^{-1}$, whether the government's participation constraint binds in the initial period depends on μ_0, which is the multiplier on initial debt. If the economy starts off with low enough debt (or high enough assets), it can support $k^\star$ in the initial period. If $\beta R = 1$, from (16), it will stay at the first best thereafter. However, if initial debt is such that the first best is not sustainable immediately, then the economy will have non-trivial dynamics. Similarly, if $\beta R < 1$, then (16) implies that $\eta_t > 0$ for $t > 0$ regardless of initial debt, as consumption is front loaded due to impatience. In short, other than the case of patient agents starting off at the first best, the economy will experience non-trivial dynamics as it converges to the steady state. For the remainder of the analysis, we assume this is the case.

Figure 3 shows the transition mapping of η_t. The diagram describes a situation where $\beta R < 1$. Note that the speed of convergence in the neighborhood of the steady state is finite, and given by $-\ln[\beta R(1 - 1/\theta)]$. Note as well that when $\theta = 1$, the speed of convergence is infinite.

Now that we have solved for the dynamics of η and k, we turn to the dynamics of debt. The sequence of binding participation constraints, $W_t = \underline{W}(k_t)$ map the dynamics of capital into that of incumbent utility, given that $\underline{W}(k)$ is strictly increasing in k. Therefore, W_t also monotonically approaches its steady state value. We now show that the information contained in the infinite sequence of incumbent utility values is sufficient to recover the utility of the population at any time:

Lemma 2. *The utility to the population as of time t, $V_t = \sum_{s=0}^{\infty} \beta^s c_{t+s}$, is given by:*

$$V_t = \frac{1}{\theta} \sum_{s=0}^{\infty} \beta^s \left(1 - \frac{1}{\theta}\right)^s W_t.$$

[18] Recall that we have assumed that $k_t > \underline{k}$ along the equilibrium path. That is, the constraint $\tau_t \leq \bar{\tau}$ does not bind. If $\frac{\theta(1-\beta R)}{\theta(1-\beta R)+\beta R} > \frac{f'(\underline{k})-(r+d)}{\bar{c}'(\underline{k})}$, or $\bar{\theta} < \theta$, then the constraint will for sure bind as $t \to \infty$. In this case, k_∞ achieves the lower bound of $\underline{k}$ and further increases in θ do not affect k_∞. See the appendix for a complete treatment.

[19] Note that $\bar{\theta}$ is infinity when $\bar{\tau} = 1$.

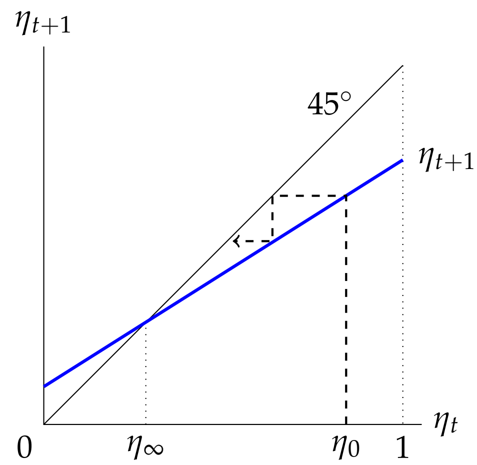

Figure 3: Transition mapping for η_t when $\beta R < 1$. The blue line in the diagram represents the transition mapping as given by equation (16). The dashed line represents a possible equilibrium path for initial condition η_0.

Given that the values k_t are monotonic and that $W_t = \underline{W}(k)$ is an increasing function of k, it follows that:

Proposition 3. *The utility to of the population, V_t, converges monotonically to its steady state value. The sequence of values V_t is increasing (decreasing) if and only if the sequence of k_t is increasing (decreasing).*

We have now shown that the discounted utility of the population and the sequence of incumbent utility move monotonically in the same direction towards their respective steady states. Given that W_t and V_t increase monotonically, it follows that outstanding sovereign debt decreases monotonically:

Corollary 2. *The stock of the economy's outstanding sovereign debt decreases (increases) monotonically to its steady state value if the sequence of k_t is increasing (decreasing).*

The above corollary closes the loop between growth and debt and brings us back to our original motivation. It states, quite generally, that capital accumulation will be accompanied with a reduction in the external debt of the government. Similarly, a country that shrinks, does so while their government accumulates sovereign liabilities.

3.1.1 Interpreting the Results

The political economy frictions are at play when $\theta > 1$. As long as private agents are impatient, (17) implies that increased disagreement (higher θ) leads to a decline in steady state capital. In this situation, political economy considerations exacerbate the relative impatience of private agents. Moreover, from (16) we see that $\theta > 1$ introduces transitional dynamics, and the greater is θ the slower the rate of convergence, which is given by $-\ln[\beta R(1-1/\theta)]$. To the extent that political disagreement varies across countries, we will have heterogeneity in growth rates, and, if $\beta R < 1$, in steady states as well. This addresses the question posed in the introduction regarding cross-country differences in growth rates observed in the data.

We now provide some intuition for why θ governs the rate of convergence and why there are any dynamics at all in an environment where absent political economy frictions, the economy jumps immediately to its steady state.

To simplify the discussion, let's consider the case where $\beta R = 1$. Note first that in an optimal solution, $W_t = \underline{W}(k_t)$. Using $\beta R = 1$, the resource constraint (10) can be rewritten as:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} \left[f(k_t) - (r+d)k_t - \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^t \underline{W}(k_t) \right]$$

The right hand side is the amount of income paid to foreign lenders, which will equal the amount of debt owed in equilibrium. To see the trade-offs inherent in increasing k, consider the maximum amount of debt that can be repaid in equilibrium. That is, the allocation that corresponds to the equilibrium starting from the maximum debt level. That allocation's sequence of k_t solves:

$$\max_{\{k_t\}} \sum_{t=0}^{\infty} R^{-t} \left[f(k_t) - (r+d)k_t - \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^t \underline{W}(k_t) \right]$$

As can be seen from this problem, one can decompose the choice of the optimal k_t in two parts. The first, $f(k_t) - (r+d)k_t$ captures the income generated from investing k_t. The second, $\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^t \underline{W}(k_t)$, behaves as a cost of choosing a given k_t. This cost arises because the binding participation constraints imply that, to increase k_t, resources will have to be transferred to the party in power. Transitional dynamics appear because this cost is strictly decreasing over time, as long as $\theta > 1$, which leads to an optimal path for the capital stock that increases monotonically to its steady state.

This raises the question of why the cost of sustaining k_t is decreasing with time. The answer lies in the provision of incentives to incumbents and their continuing disagreement about the timing of expenditures. To see this, suppose that one would like to increase the capital at some time t, and to achieve this, consumption at time t must be increased by one unit to satisfy the participation constraint. This increase in consumption at time t is costly, but can be compensated with a decrease in consumption at $t-1$, given that the utility of the incumbent at $t-1$ has increased by β. Thus, to keep the incumbent at $t-1$ indifferent, consumption at $t-1$ can be lowered by $-\beta/\theta$. So far, the total cost of increasing k_t is thus $\beta - \beta/\theta$, or:

$$\beta\left(1-\frac{1}{\theta}\right)$$

However, as a consequence of the ongoing disagreement about the timing of expenditures, the changes in the consumption allocation also affect the utility of the incumbent at time $t-2$. Because consumption at time t increases, while consumption at time $t-1$ decreases so as to leave the incumbent at time $t-1$ indifferent, the disagreement implies that the utility of the incumbent at time $t-2$ has *increased*, by an amount equal to $\beta^2\left(1-\frac{1}{\theta}\right) > 0$. Consumption at time $t-2$ can correspondingly be decreased by $\frac{\beta^2}{\theta}\left(1-\frac{1}{\theta}\right)$. So now, the total cost of increasing k_t is $\beta^2\left(1-\frac{1}{\theta}\right) - \frac{\beta^2}{\theta}\left(1-\frac{1}{\theta}\right)$, or:

$$\beta\left(1-\frac{1}{\theta}\right)^2$$

Similarly, because consumption at time $t-2$ decreased so as to leave the incumbent at time $t-2$ indifferent, the disagreement again implies that the incumbent at time $t-3$ now has a higher utility. And thus, consumption at time $t-3$ can be lowered, reducing the total cost of increasing k to $\beta\left(1-\frac{1}{\theta}\right)^3$. This can be repeated in a similar fashion for all previous periods, and *as long as the disagreement is always present across incumbents, the total cost of increasing k_t in period t is decreasing in the number of previous incumbents, generating the dynamics in the path of capital.*

One can also see that when there is no disagreement ($\theta = 1$), convergence is immediate.[20] More generally, note that the higher the disagreement (that is, the higher the θ) the

[20] The linear case in standard models of expropriation has been studied in detail by Thomas and Worrall (1994) and Alburquerque and Hopenhayn (2004) for $\beta R = 1$. In those papers, non-trivial transition dynamics are generated because of the binding requirement that consumption must be positive. The results here

slower the decline in costs and the longer it takes to converge to the steady state.

The above exercise also points to the difference between θ and the discount factor β. A value of $\theta > 1$, makes parties short term impatient, and creates continuous disagreement about the timing of expenditures, making the optimal allocation dynamic. A value of $\beta < 1/R$, also makes the incumbents more impatient, but as long as $\theta = 1$, this impatience does not create disagreement, and no dynamics are generated. Moreover, we see from Proposition 2 that impatience (a low β) speeds convergence to the steady state. If agents are impatient, there is little disagreement across incumbents about spending far in the future, even if delayed spending is compounded at the market interest rate. In this sense, impatience limits the amount of disagreement, speeding convergence to the steady state.

Perhaps the dichotomy between impatience and disagreement is starkest when the economy is shrinking. This will be the case if the economy starts with low enough debt and $\beta R < 1$ (that is, the economy starts to the left of η_∞ in Figure 3). A low β economy (holding θ constant) will collapse relatively quickly to a low steady state. Conversely, a high θ economy (holding $\beta < 1/(1+r)$ constant) will experience a relatively long, slow decline.

The case of a shrinking economy also highlights the distinction between our model and one with a simple borrowing constraint. Borrowing constraints do not induce dynamics if capital starts above its steady state level. However, our model has non-trivial dynamics whether the economy is growing or shrinking.

3.1.2 Steady States of Debt and Consumption

In this section we continue our characterization and solve for the steady state consumption and debt. For that, we use the fact that $W_\infty = \underline{W}(k_\infty)$, and so

$$c_\infty = \frac{\underline{W}(k_\infty)}{\theta + \frac{\beta}{1-\beta}} = \frac{\theta \bar{c}(k_\infty) + \frac{\beta}{1-\beta}\bar{c}(\underline{k})}{\theta + \frac{\beta}{1-\beta}} > 0. \tag{18}$$

make clear that the speed of convergence around the steady state in these models is infinity (independently of whether βR is equal to or less than one), and also that these linear economies will immediately converge if they start with sufficiently low debt. It is possible to generate smoother dynamics in the above models by introducing risk aversion. However in Section 3.2 we argue that numerically, for the neoclassical growth model and standard parameter values, the speed of convergence is determined primarily by θ even in the presence of risk aversion.

This confirms our underlying assumption that in the neighborhood of the steady state consumption is positive, and thus, the non-negativity constraint can be ignored.

The steady state level of debt then follows from the fact that debt equals the present discounted value of net payments to the foreign financial markets:

$$B_\infty = \left(\frac{1+r}{r}\right)\left(f(k_\infty) - (r+d)k_\infty - c_\infty\right). \tag{19}$$

The case of $\beta R = 1$ provides a clear example of the impact of political economy on steady state debt. For this discount factor, the steady state capital is $k^\star$ regardless of $\theta \in [1,\infty)$; that is, $\partial k_\infty/\partial\theta = 0$. Therefore, $\partial B_\infty/\partial\theta = -\left(\frac{1+r}{r}\right)\partial c_\infty/\partial\theta$, and this expression is negative from (18). We thus have the following proposition:

Proposition 4. *Suppose $\beta R = 1$. Then steady state consumption is increasing and steady state debt is decreasing in $\theta \in [1,\infty)$.*

Proposition 4 states that when private agents discount at the world interest rate an increase in political disagreement *reduces* debt levels in the steady state. That is, a country with larger political frictions has a smaller amount of debt in the long run. The intuition for this result follows from the fact that when $\beta R = 1$ the optimal allocation follows a path that leads to first best capital in the long run. However, to sustain this first best capital, a government with a higher θ must have a lower debt, or else it will deviate and expropriate the capital.

While Proposition 4 states that debt will be lower in the steady state for countries with greater political disagreements when $\beta R = 1$, this does not imply that debt will be negatively correlated with disagreement in a cross-section of countries. This does not follow for two reasons. The first is that countries with a large θ converge very slowly to the steady state, with convergence being arbitrarily slow as $\theta \to \infty$. That is, while polarized countries may be heading to a steady state in which debt is low, it will take a very long time to arrive there. Therefore, at any moment in time, countries will be at different points on their transition paths, and the steady state comparative statics are not sufficient to sign the cross-sectional correlation.

Moreover, the result depends on $\beta R = 1$ and does not hold for all β. If agents in developing countries are impatient relative to the world interest rate, k_∞ declines with θ, as well as c_∞, making the net effect of θ on B_∞ ambiguous. In fact, if $\beta R < 1$, then $\partial B_\infty/\partial\theta$ can have either sign, depending on β. For example, suppose that $\bar{\tau} < 1$, so $\underline{k} > 0$. For large enough θ, the upper bound on taxes will bind in the long run, as stated in Proposition 2. Increases in θ above $\bar{\theta}$ do not lead to lower capital in the long run given the bound on tax rates. In this case, the effect of θ on debt once again becomes unambiguous:

Proposition 5. *Let $\beta R < 1$, and $\bar{\tau} < 1$ (so that $\bar{\theta} < \infty$). Then, the following two statements hold:*

$$\lim_{\theta \to \bar{\theta}} B_\infty = 0, \text{ and } \lim_{\theta \to \bar{\theta}^-} \frac{dB_\infty}{d\theta} > 0.$$

Given that $f(k_\infty) > 0$ for all θ, then these statements imply that the debt to output ratio must increase in θ for sufficiently high θ when $\beta R < 1$ and $\bar{\tau} < 1$.

Similarly, if $\beta R < 1$, whether the capital stock converges to its steady state from above or below is ambiguous. That is, if $\beta R = 1$, then the fact that $k_1 < k^\star = k_\infty$ implies that the country unambiguously converges to the steady state from below for all $\theta \in (1, \infty)$. However, if $\beta R < 1$, convergence may be from above, with capital falling over time and debt increasing.

3.1.3 Aid without Conditionality and Debt Forgiveness

Before concluding this section, we will use the model to discuss the role of two policies: foreign aid and debt forgiveness. Our set up delivers a laboratory that allows us to ask whether the introduction of foreign aid and debt relief changes the path of investment and growth. Although, similar in principle (they both represent a transfer from foreigners to the domestic agents), these two policies will end up having different effects on the behavior of the economy.

From our previous analysis, we see that debt forgiveness, as given by a reduction in b_0, will affect the economy in the short run, but will not affect the steady state levels of investment and debt. That is, if b_0 is reduced, then from Corollary 1, we know that the resulting path of capital will be higher, but the long run level of debt, capital, and output will not change as the economy converges to the same steady state.[21] The transitory effect of debt relief is relevant to recent experience in Africa. As Western donor countries consider debt forgiveness, the debtor African countries are simultaneously seeking new

[21] This is consistent with the empirical effect of debt relief on domestic stock market values, investment, and short run GDP growth rates. For a recent survey of the literature see Arslanalp and Henry (2006)

loans from China.[22]

Another common policy aimed at helping developing countries is foreign aid. Among the different emerging market economies, several have received significant amounts of aid from abroad. In the data, however, the relationship between aid and growth seems, if anything, insignificant.[23]

With that goal in mind, let an aid sequence $\{y_t\}_{t=0}^{\infty}$ be a sequence of non-negative values y_t denoting for all times t, the amount of aid provided to the government by unspecified foreign benefactors. The aid sequence is deterministic and non-contingent, that is, there is no conditionality. Importantly, this implies that the autarky values is now given by the following:

$$\underline{W}(k_t) = \theta(\bar{c}(k_t) + y_t) + \sum_{i=1}^{\infty} \beta^i (\bar{c}(\underline{k}) + y_{t+i})$$

for all t. The present value constraint on the resources of the government is:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t}(f(k_t) + y_t - (r+d)k_t - c_t)$$

Then, the following holds:

Proposition 6. *Let $\{k_t\}_{t=0}^{\infty}$ be the optimal sequence of capital that solves the population's problem as of time $t = 0$ without the presence of aid. Then $\{k_t\}_{t=0}^{\infty}$ is also an optimal sequence of capital of the economy with an aid sequence $\{y_t\}_{t=0}^{\infty}$.*

Aid without conditionality improves the utility of the population, as it represents a transfer that will be consumed by them. However, it does not affect the capital accumulation patterns of the country, not even in the short run.[24]

[22] See, for example, the Financial Times article "Donors press Congo over \$9bn China deal" on February 9 of 2009.

[23] See the original article by Burnside and Dollar (2000), Easterly (2003) for a survey, and Rajan and Subramanian (2008) for a more recent analysis.

[24] Note that we consider aid that is not conditional on fiscal policy. Conditional aid may relax the participation constraint and therefore alter investment. See Scholl (2009) for a study of aid and conditionality in an environment with limited commitment.

3.2 Concave Utility

The linear utility case provides simple analytical expressions for the dynamics of the economy that make clear why the transition is slow and how this transition is affected by the parameters of interest. However, linear utility is an extreme assumption and begs the question of whether the insights gleaned from the linear case are robust to more realistic utility functions. To answer this question, we explore the dynamics around the steady state for concave utility.

To simplify expressions, we consider the dynamics of $u_t = u(c_t)$ rather than c_t, and let $c(u)$ denote the inverse utility function. The dynamics of the economy are characterized by the following four equations:

$$c'(u_t) - \beta R c'(u_{t-1}) = \eta_t - \beta R \left(\frac{\theta - 1}{\theta}\right) \eta_{t-1} \tag{20}$$

$$\frac{\eta_t}{\theta} \underline{W}'(k_t) = f'(k_t) - (r + d) \tag{21}$$

$$\theta u_t + \beta V_{t+1} = \underline{W}(k_t) \tag{22}$$

$$V_t = u_t + \beta V_{t+1}, \tag{23}$$

where the first equation is obtained by differencing (13), the second equation is (14), the third equation is the participation constraint with $W_t = \theta u_t + \beta V_{t+1}$, and the last equation is the recursion of V_t.

We first linearize the system around the steady. Letting a hat denote deviations from the steady state values, we obtain:

$$c''(u_\infty)\hat{u}_t - \beta R c''(u_\infty)\hat{u}_{t-1} = \hat{\eta}_t - \beta R \frac{(\theta - 1)}{\theta} \hat{\eta}_{t-1} \tag{20a}$$

$$\eta_\infty \underline{W}''(k_\infty)\hat{k}_t + \underline{W}'(k_\infty)\hat{\eta}_t = \theta f''(k_\infty)\hat{k}_t \tag{21a}$$

$$\theta \hat{u}_t + \beta \hat{V}_{t+1} = \underline{W}'(k_\infty)\hat{k}_t \tag{22a}$$

$$\hat{V}_t = \hat{u}_t + \beta \hat{V}_{t+1}. \tag{23a}$$

where the steady state values are the solution to:

$$\underline{W}(k^\star) = \left(\theta + \frac{\beta}{1-\beta}\right) u^\star$$
$$c'(u^\star) = \eta^\star \left[1 + \left(\frac{\beta R}{1-\beta R}\right)\frac{1}{\theta}\right]$$
$$\eta^\star = \frac{f'(k^\star) - (r+d)}{\underline{W}'(k^\star)/\theta}$$

The system can be simplified down to two variables:

Lemma 3. *Let $x_t = [\hat{u}_t \quad \hat{V}_t]'$. The linearized system (20a)-(23a) simplifies to $x_{t+1} = \mathbf{B}x_t$ where*

$$\mathbf{B} = \begin{bmatrix} \beta R - \frac{1}{1+\kappa}\left(\frac{\beta R}{\theta} - \frac{1}{\beta(\theta-1)}\right) & \frac{1}{1+\kappa}\left(\frac{\beta R}{\theta} - \frac{1}{\beta(\theta-1)}\right) \\ -\frac{1}{\beta} & \frac{1}{\beta} \end{bmatrix},$$

and where $\kappa = \frac{-c''(u_\infty)\underline{W}'(k_\infty)^2}{\theta f''(k_\infty) - \eta_\infty \underline{W}''(k_\infty)}$.

The matrix $\mathbf{B}$ has two eigenvalues, one larger than one in absolute value and the other less than one in absolute value. The convergence of the system is governed by the eigenvalue less than one in magnitude. Setting $c''(u) = 0$ (the linear case), implies that $\kappa = 0$ and we recover an eigenvalue $\beta R\left(1 - \frac{1}{\theta}\right)$, the one for the linear case. For $c''(u) > 0$, convergence is a more complicated expression of underlying parameters. To assess the response of the system to θ and risk aversion, we present some numerical examples in Figure 4. Specifically, we consider power utility: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, for $\sigma = 0$, 1, 1.5, and 2.[25] The figure plots the convergence rate, defined as 1 minus the relevant eigenvalue, for each value σ and for values of $\theta \in [1, 20]$. The first thing to note is how close the speed of convergence are to each other for $\sigma \in [1, 1.5, 2]$. Basically, the lines sit on top of each other. It is also noteworthy how close they are to the one obtained in the linear case. The linear model captures quite accurately the behavior around the steady states. Looking across the plotted lines, we see that all else equal, a higher value of θ is associated with slower convergence, confirming the comparative statics of the linear case.

The results above are about the behavior of the model in a neighborhood of the steady state. Figure 5 presents global results for a set of simulations[26] of the model for different values of the risk aversion parameter σ, where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The figure plots the transition

[25] The other parameters are $f(k) = k^\alpha$, $\alpha = 1/3$, $r = d = 0.05$, $\beta = 1/(1+r)$, and $\bar{\tau} = 0.50$.

[26] The other parameters are $f(k) = k^\alpha$, $\theta = 10$. $\alpha = 1/3$, $r = d = 0.05$, and $\bar{\tau} = 0.5$. The numerical implementation is done through value function iteration. The code is available at authors' website.

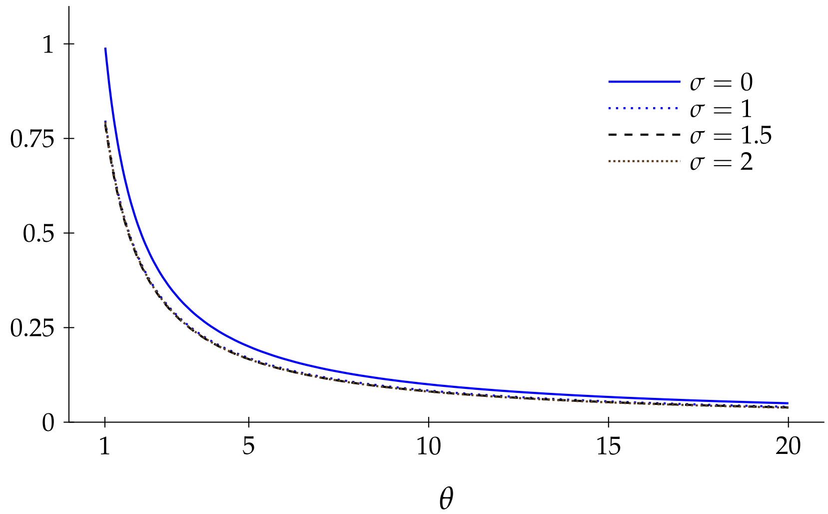

Figure 4: This figure depicts the sensitivity of convergence rates to θ. The vertical axis is the convergence rate, which is calculated as 1 minus the largest eigenvalue less than one of the matrix $\mathbf{B}$ discussed in the text. Each line represents a different coefficient of relative risk aversion (σ), as shown in the legend. Other parameters are: $f(k) = k^{1/3}$, $r = 0.05$, $d = 0.05$, and $\bar{\tau} = 0.5$.

maps of the equilibrium tax rate (written as deviations from its steady state value) for each of the simulations. As can be seen from these plots, the linear utility model accurately describes the dynamic behavior of the concave utility model not only locally, but also globally.

The above two numerical results lead us to conclude that, quite differently from the standard neoclassical model, concave utility does not change the dynamics in a quantitatively significant way with respect to the baseline linear model. This underscores the dominant role political economy considerations play in determining the speed of convergence in our environment.

4 Revisiting Convergence in the Neoclassical Growth Model

A major prediction of the standard neoclassical growth model is conditional convergence; that is, countries which are poor relative to their steady states grow relatively fast. Given that our model is built on a neoclassical foundation, it shares this property as well. However, there are important differences in our model compared to the standard closed economy model, which is the focus of this section.

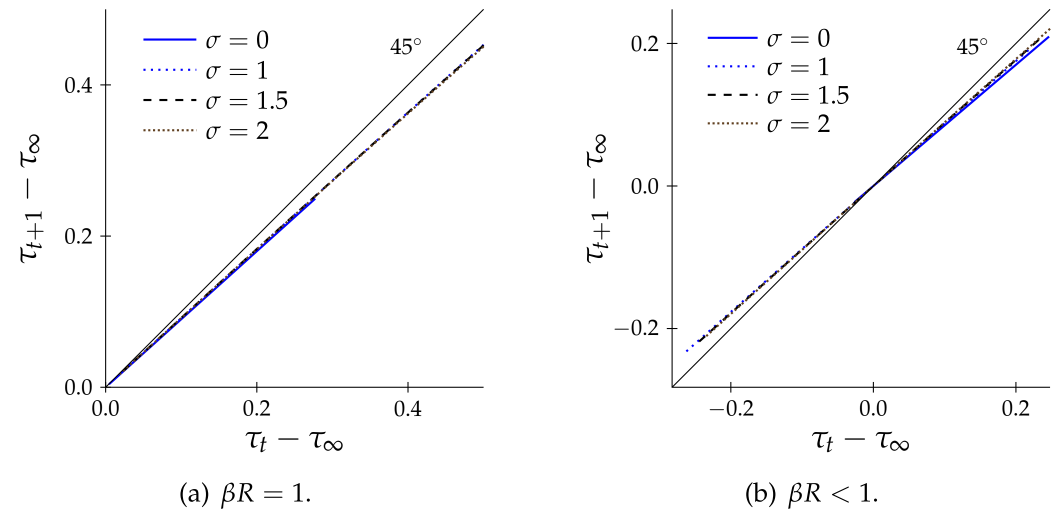

Figure 5: This figure depicts transition mappings for the deviations of the tax rate with respect to its steady state value. Each line represents a different coefficient of relative risk aversion (σ). Other parameters are: $f(k) = k^{1/3}$, $r = 0.05$, $\theta = 10$, $d = 0.05$, and $\bar{\tau} = 0.5$. In panel (b), $\beta = 0.9$.

Consider our benchmark linear model, setting $\beta R = 1$ and $\bar{\tau} = 1$. From equation (16) and the fact that $\eta_t = \tau_t$ when $\bar{\tau} = 1$, we have

$$\tau_t = \left(1 - \frac{1}{\theta}\right)^t \tau_0.$$

The first order condition for capital implies that $\tau_t = 1 - (r+d)/f'(k_t) = 1 - f'(k_\infty)/f'(k_t)$, where the second equality follows from $k_\infty = k^\star$ when $\beta R = 1$. In the case of Cobb-Douglas production with capital share α, we then have

$$\left(\frac{k_t}{k_\infty}\right)^{1-\alpha} = 1 - \left(1 - \frac{1}{\theta}\right)^t \left(\frac{k_0}{k_\infty}\right)^{1-\alpha}.$$

Differentiating with respect to time implies:

$$(1-\alpha)\frac{\dot{k}_t}{k_t} = -\ln\left(1 - \frac{1}{\theta}\right)\left[\left(\frac{k_t}{k_\infty}\right)^{\alpha-1} - 1\right],$$

or, using the fact that $\dot{y}/y = \alpha \dot{k}/k$:

$$\frac{\dot{y}_t}{y_t} = -\ln\left(1 - \frac{1}{\theta}\right)\left(\frac{\alpha}{1-\alpha}\right)\left[\left(\frac{y_t}{y_\infty}\right)^{\frac{\alpha-1}{\alpha}} - 1\right].$$

In the neighborhood of the steady state, we can write this as:

$$\frac{\dot{y}_t}{y_t} \approx -\ln\left(\frac{\theta}{\theta-1}\right)\ln\left(\frac{y_t}{y_\infty}\right). \tag{24}$$

In the terminology of the empirical growth literature, the speed of convergence in the model is given by $\ln\left(\frac{\theta}{\theta-1}\right) \approx 1/(\theta-1)$. For perspective, the comparable speed of convergence in the standard Solow-Swan model is $(1-\alpha)d$.[27] A comparison of this term with that in (24) highlights an important difference. Namely, in the standard model the capital share is an important determinant of convergence, while it plays no role in our version of the neoclassical growth model.[28] In the standard model, the capital share determines how much a unit of savings increases output. This parameter is therefore crucial in the mapping of savings to additional output next period. In our model, external savings by the government does not directly generate capital formation, but does so indirectly through a reduction in the credible tax rate. The tax rate on capital governs the marginal product of capital, and the linear dynamics for the marginal product of capital are, with Cobb-Douglas production, passed through to output independently of the capital share.

The fact that we have replaced capital share with political economy frictions in the speed of convergence is important in interpreting cross-sectional growth regressions. The slow rate of convergence observed empirically, when viewed through the standard model, suggests a large capital share, on the order of 0.75 when using plausible values for other parameters (see Barro and Sala-I-Martin, 2004 p. 59). This has generated a literature on what is the appropriate notion of capital in the neoclassical model, such as Mankiw et al. (1992) which extends the notion of capital to include human capital. In our framework, slow convergence does not require a high capital share, but rather large political economy frictions. For the empirical growth literature, this framework suggests an emphasis

[27]More precisely, the speed of convergence is $(1-\alpha)(g+n+d)$, where g is the rate of exogenous technological progress and n is the population growth rate, both of which we have set to zero in our benchmark model. See Barro and Sala-I-Martin (2004).

[28]In the concave utility version of our model, the curvature of the production function does enter the expression for convergence (See lemma 3). However, we have explored the concave model of Section 3.2 numerically and found that capital share does not have a quantitatively large systematic impact on convergence. This is in line with our findings regarding curvature of the utility function.

on political economy frictions in determining the speed of convergence, in addition to their possible effect on the steady state.[29] However, on the negative side, capital share has a relatively well defined empirical measure and therefore allows quantitative tests of the neoclassical growth model. The empirical measures of political frictions are not as mature as those for capital share, making quantitative empirical tests of equation (24) difficult, but also a fruitful topic for future research.

5 Conclusion

In this paper, we presented a tractable variation on the neoclassical growth model that explains why small open economies have dramatically different growth outcomes, and the ones that grow fast do so while increasing their net foreign asset position. Figures 1 and 2 indicated that this pattern was driven by a net reduction in public debt combined with an inflow of private capital in fast growing economies, and the reverse in shrinking economies, facts consistent with the model developed in this paper. This paper focused on the negative relationship between sovereign debt and growth induced by political economy frictions. In an earlier paper (Aguiar et al., 2009), we explored how debt overhang can exacerbate volatility as well. This raises the intriguing possibility that political economy frictions and the associated debt dynamics may jointly explain the negative relationship between volatility and growth observed in the data, a question we leave for future research.

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[29]Our model, which allows for the possibility of immediate convergence absent political disagreement, is very stylized and not designed for full-fledged calibration. Undoubtedly, a richer model will allow a role for capital share and other elements of the closed economy neoclassical model. Our point is not that these traditional factors are irrelevant, but that all else equal, the extent of political disagreement will have a first order effect on the speed of convergence.

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A Exogenous Growth

In this appendix we extend the model to include exogenous growth and show that the benchmark results are unaffected up to a re-normalization.

Suppose that $y_t = f(k_t, (1+g)^t l_t)$, where g is the rate of exogenous labor-augmenting technical progress. Constant returns to scale in production implies that $y_t = (1+g)^t f((1+g)^{-t} k_t, l_t)$ or $(1+g)^t f(\hat{k}_t, l_t)$, where $\hat{x}_t \equiv \frac{x_t}{(1+g)^t}$, for $x = k, c$. The firm's first order condition can be written:

$$
\begin{aligned}
f_k(k_t, (1+g)^t l_t) &= r + d \\
f_k(\hat{k}_t, l_t) &= r + d,
\end{aligned}
$$

as f_k is homogeneous of degree zero in k and l. We also have $\hat{\underline{k}}_t = (1+g)^{-t} \underline{k}_t$, so that $(1-\bar{\tau}) f_k(\hat{\underline{k}}_t, l_t) = (1-\bar{\tau}) f_k(\underline{k}_t, (1+g)^t l_t) = r + d$. The budget constraint can be re-written:

$$
b_0 \leq \sum_{t=0}^{\infty} R^{-t} (1+g)^t \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right],
$$

where we need $r > g$ to ensure finiteness of the budget set.

Let us assume that $u(c)$ is homogeneous of degree $1-\sigma$, then the objective function can be written:

$$
\sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \beta^t (1+g)^{(1-\sigma)t} u(\hat{c}_t),
$$

where we need $\beta(1+g)^{1-\sigma} < 1$. Turning to the deviation utility:

$$
\begin{aligned}
\bar{c}(k_t) &= f(k_t, (1+g)^t l_t) - (1-\bar{\tau}) f_k(k_t, (1+g)^t l_t) k_t \\
&= (1+g)^t \left[f(\hat{k}_t, l_t) - (1-\bar{\tau}) f_k(\hat{k}_t, l_t) \hat{k}_t \right].
\end{aligned}
$$

and we can define $\hat{\bar{c}}(\hat{k}_t) \equiv (1+g)^{-t} \bar{c}(k_t) = f(\hat{k}_t, l_t) - (1-\bar{\tau}) f_k(\hat{k}_t, l_t) \hat{k}_t$. So, the deviation utility is:

$$
\underline{W}(k_t) = \theta u(\bar{c}(k_t)) + \frac{\beta}{1-\beta} u(\bar{c}(\underline{k})) = (1+g)^{(1-\sigma)t} \left[\theta u(\hat{\bar{c}}(\hat{k}_t)) + + \frac{\beta}{1-\beta} u(\hat{\bar{c}}(\hat{\underline{k}})) \right].
$$

Define $\underline{\hat{W}}(\hat{k}_t) = (1+g)^{(\sigma-1)t} \underline{W}(k_t)$, we have

$$
\underline{\hat{W}}(\hat{k}_t) = \theta u(\hat{\bar{c}}(\hat{k}_t)) + \frac{\beta}{1-\beta} u(\hat{\bar{c}}(\hat{\underline{k}})).
$$

The planning problem can be written:

$$
\max \sum_{t=0}^{\infty} \beta^t (1+g)^{(1-\sigma)t} u(\hat{c}_t)
$$

subject to:

$$
\begin{aligned}
b_0 &\leq \sum_{t=0}^{\infty} R^{-t}(1+g)^t \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right] \\
\underline{\hat{W}}(\hat{k}_t) &\leq \theta u(\hat{c}_t) + \beta(1+g)^{(1-\sigma)} \sum_{s=t+1}^{\infty} \beta^{s-t-1}(1+g)^{(1-\sigma)(s-t-1)} u(\hat{c}_s) \\
\underline{\hat{k}}_t &\leq \hat{k}_t.
\end{aligned}
$$

Now define $\hat{R} \equiv \frac{1+r}{1+g}$ and $\hat{\beta} \equiv \beta(1+g)^{(1-\sigma)}$. Then, the planner's problem above can be re-written:

$$
\max \sum_{t=0}^{\infty} \hat{\beta}^t u(\hat{c}_t)
$$

subject to:

$$
\begin{aligned}
b_0 &\leq \sum_{t=0}^{\infty} \hat{R}^{-t} \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right] \\
\underline{\hat{W}}(\hat{k}_t) &\leq \theta u(\hat{c}_t) + \hat{\beta} \sum_{s=t+1}^{\infty} \hat{\beta}^{s-t-1} u(\hat{c}_s) \\
\underline{\hat{k}}_t &\leq \hat{k}_t.
\end{aligned}
$$

Note that this problem is isomorphic to the original problem, (P).

This discussion is important, not only to show that the results are robust to sustained technological improvements (a fact of the data), but also it highlights the following: a steady state in our model, once augmented with exogenous growth, will feature constant debt to output ratios and an output level that will be growing at the rate of g. In this environment, a country that grows at a slower rate than g will accumulate liabilities as fraction of its output, and the opposite will hold for a country that grows faster than g. If we take g, to a first approximation, to be equal to the growth rate of the U.S., then one should expect that countries that grew faster (slower) than the U.S. should have increased (decreased) external assets relative to GDP. This is exactly what Figure 1 shows.

B Capitalist Insiders

In this section of the appendix we extend the benchmark model to include domestic capitalists that enter the welfare functions of both the private agents setting initial policy and the subsequent governments. Recall that a key distinguishing feature of a capitalist in our environment is the ability to manage firms, a feature which prevented the government from converting savings into productive capital itself. Specifically, suppose that a subset of the domestic population has entrepreneurial ability which enables them to operate the production technology. We assume that all firms are managed by domestic entrepreneurs, but continue to assume the economy is open in that firm financing may originate abroad.

More concretely, consider an entrepreneur who manages a firm with capital stock k. This capital stock is financed through a combination of equity and debt financing, where the entrepreneur may own some of the equity. An entrepreneur hires workers and pays holders of debt and equity using after tax profits. We extend the limited commitment paradigm to encompass domestic entrepreneurs. That is, an entrepreneur can renege on the firm's contracts and divert resources to his or her own private gain. Let $\underline{U}^e(k)$ denote the lifetime utility of a manager who deviates given a firm's capital stock k. We provide a specific formulation of $\underline{U}^e(k)$ below; at this point, there is no need to put additional structure on the deviation utility of the entrepreneurs. Given the lack of commitment, firm financing must be self-enforcing. If c_t^e is the entrepreneur's consumption absent deviation, then the entrepreneur faces a financing constraint of the form $\underline{U}^e(k_t) \leq \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^e)$, for every t. This constraint is the individual firm's counterpart to the government's borrowing constraint, and corresponds to the constraint studied in Alburquerque and Hopenhayn (2004). Note that $\underline{U}^e(k)$ is the utility from deviation for the entrepreneur given the equilibrium actions of all other agents, including the government.

We study the private agents' planning problem.[30] Let the private agents' welfare function be given by $\lambda u(c^w) + (1-\lambda)u(c^e)$, where c^w and c^e are the per capita consumption of workers and entrepreneurs, respectively, and $\lambda \in (0,1]$ is the Pareto weight placed on workers. For ease of exposition, we assume the government places weight λ on workers as well, but this could be relaxed. The planning problem can be written as:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_t^w) + (1-\lambda)u(c_t^e)\right] \tag{P'}$$

subject to

$$\begin{aligned}
b_0 &\leq \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - c_t^w - c_t^e - k_{t+1} + (1-d)k_t\right) - (1+r)k_0 \\
\underline{W}(k_t) &\leq \theta \left[\lambda u(c_t^w) + (1-\lambda)u(c_t^e)\right] + \sum_{s=1}^{\infty} \beta^s \left[(\lambda u(c_{t+s}^w) + (1-\lambda)u(c_{t+s}^e))\right], \ \forall t \\
\underline{U}^e(k_t) &\leq \sum \beta^s u(c_{t+s}^e), \ \forall t.
\end{aligned}$$

The aggregate resource constraint states that the present value of output minus consumption and net investment must equal initial net foreign debt. This constraint is the same as (10), although written in a slightly different way. The second constraint is the government's participation constraint, which is modified to include both types of agents. We assume that the incumbent's preference for current consumption is uniform across agents. The final constraint is the entrepreneur's participation constraint ensuring that firm financing is self enforcing. Note that even though capitalists enter the welfare function of the government there is a temptation for the current incumbent to expropriate capital when $\theta > 1$.

Before solving the planning problem, we discuss how the government's deviation utility $\underline{W}(k)$ is affected by the presence of insider capitalists. We maintain our assumption that if the government deviates, the economy reverts to the Markov Perfect Equilibrium (MPE) under financial autarky. To set notation, let k denote the current capital stock inherited by the current incumbent, and k' the capital stock bequeathed to the next government. Let $V(k')$ denote the continuation value of the current incumbent if it leaves k' to the next government. That is, $V(k_t) =$

[30] The efficient allocation from the planning problem can be decentralized with appropriate taxes and transfers. We omit the details.

$\sum_s \beta^s[\lambda u(c^w_{t+s}) + (1-\lambda)u(c^e_{t+s})]$, where the sequence of consumptions are chosen by future incumbent governments given the inherited state variable k. Similarly, let $U^e(k')$ denote the continuation value of entrepreneurs conditional on k'. The current incumbent's problem is therefore

$$\underline{W}(k) = \max_{c^e, c^w, k'} \theta\left[\lambda u(c^w) + (1-\lambda)u(c^e)\right] + \beta V(k') \tag{25}$$

subject to

$$\begin{aligned} c^w + c^e + k' &\leq f(k) + (1-d)k \\ u(c^e) + \beta U^e(k') &\geq \underline{U}^e(k). \end{aligned}$$

Note that we have set $\bar{\tau} = 1$, so the government has access to total output. We continue to use the notation $\underline{U}^e(k)$ to denote the entrepreneurs' deviation utility, although this is a slight abuse of notation – the value to an entrepreneur diverting with capital stock k will depend on the path of taxation, which in general will be different in the MPE. Other than this last constraint, the MPE is the closed economy neo-classical growth model with a quasi-hyperbolic decision maker.

Returning to the planning problem (P'), we let μ_0, $R^{-t}\frac{\eta_t \mu_0}{\lambda\theta}$, and $R^{-t}\mu_0\gamma_t$ be the multipliers on the three constraints. The first order conditions are:

$$1 = \lambda u'(c^w_t)\left(\frac{\beta^t R^t}{\mu} + \left(\frac{\theta-1}{\theta}\right)\frac{\eta_t}{\lambda} + \sum_{s=0}^{t}\beta^s R^s \frac{\eta_{t-s}}{\lambda\theta}\right) \tag{26}$$

$$\begin{aligned} 1 = (1-\lambda)u'(c^e_t)\Bigg(&\frac{\beta^t R^t}{\mu} + \left(\frac{\theta-1}{\theta}\right)\frac{\eta_t}{\lambda} + \sum_{s=0}^{t}\beta^s R^s \frac{\eta_{t-s}}{\lambda\theta} \\ &+ \frac{1}{1-\lambda}\sum_{s=0}^{t}\beta^s R^s \gamma_{t-s}\Bigg) \end{aligned} \tag{27}$$

$$f'(k_t) = r + d + \frac{\eta_t}{\lambda\theta}\underline{W}'(k_t) + \gamma_t \underline{U}^{e'}(k_t). \tag{28}$$

Before analyzing the problem in detail, a few points are worth mentioning. The benchmark case can be recovered by setting $\lambda = 1$ and relaxing the entrepreneurs borrowing constraint $\gamma_t = 0$. Even if λ is less than one, the first order condition for workers remains essentially the same as before (compare (26) and (13)) – the only difference is a scaling factor. Moreover, conditions (26) and (27) can be combined to yield:

$$\left(\frac{1-\lambda}{\lambda}\right)\frac{u'(c^e_t)}{u'(c^w_t)} + u'(c^e_t)\sum_{s=0}^{t}\beta^s R^s \gamma_{t-s} = 1. \tag{29}$$

This condition says that the plan allocates consumption to workers and entrepreneurs partially according to their Pareto weights, but entrepreneurs may be given additional resources when their borrowing constraint binds.

B.1 The Linear Case Revisited

We now reconsider our benchmark results with linear utility. The case of $\lambda \geq 1/2$ provides a straightforward extension of our basic model as there exists an interior optimum. If $\lambda > 1/2$, then the government strictly prefers workers to entrepreneurs as a group, and transferring resources from the entrepreneurs to the workers relaxes the government's constraint. Similarly, transferring resource from entrepreneurs to workers raises the planner's objective function. However, there is a limit on how many resources can be transferred, as the entrepreneurs always have the option to deviate. This ensures that entrepreneurial consumption is not driven to minus infinity in the linear case. We therefore assume $\lambda \geq 1/2$ in what follows. In the linear case, we also assume that $\underline{U}^e(k) = f(k) + (1-d)k$. That is, an entrepreneur that deviates simply consumes the output and un-depreciated capital. In this formulation, the entrepreneur's deviation utility is independent of government actions.

In the linear case, we can rewrite (26) as:

$$\frac{1}{\lambda} = \frac{\beta^t R^t}{\mu_0} + \left(\frac{\theta - 1}{\theta}\right)\frac{\eta_t}{\lambda} + \sum_{s=0}^{t} \beta^s R^s \frac{\eta_{t-s}}{\lambda\theta}.$$

Solving this equation for the path of η_t, we have

$$\eta_t = 1 - \beta R + \beta R\left(1 - \frac{1}{\theta}\right)\eta_{t-1},$$

for $t \geq 1$, and $\eta_0 = 1 - \frac{\lambda}{\mu_0}$. Thus the dynamics of η_t are the same as before, save for the initial term now has an explicit weight for the workers, λ.

Turning to (29), we have

$$\sum_{s=0}^{t} \beta^s R^s \gamma_{t-s} = 1 - \frac{1-\lambda}{\lambda}.$$

This implies that $\gamma_0 = 1 - \frac{1-\lambda}{\lambda}$, and $\gamma_t = (1-\beta R)\gamma_0$ for $t > 0$. If $\lambda = 1/2$, then $\gamma_t = 0$ for all t. This follows as workers and entrepreneurs receive equal weights and have linear utility, so the optimal plan will transfer resources from workers to entrepreneurs until the entrepreneur's constraint is slack. If $\beta R = 1$, then the entrepreneur's constraint binds only in the initial period for any $\lambda > 1/2$. With linear utility and patience, the entrepreneur is willing to delay consumption into the future (i.e., post a bond), relaxing the borrowing constraint. However, this does not imply that capital is first best – even if the borrowing constraint does not bind, the entrepreneur is subject to government taxation. In all cases, γ_t is constant after the first period and does not depend on the polarization parameter θ: the entrepreneur's lack of commitment does not generate dynamics beyond the first period. Therefore, θ only influences the dynamics of the economy through η, the multiplier on the government's participation constraint.

As in the benchmark case, the dynamics of η_t pin down the dynamics of capital. Specifically, from the first order condition for capital we have $f'(k_t) - r - d = \frac{\eta_t}{\lambda\theta}\underline{W}'(k_t) + \gamma_t \underline{U}^{e'}(k_t)$. After manipulating the envelope and first order conditions from (25), we have $\underline{W}'(k) = \theta(1-\lambda)\left(f'(k) + 1 - d\right)$, where we have used the fact that $\underline{U}^{e'}(k) = f'(k) + 1 - d$ and that $\lambda \geq 1/2$ to guarantee an interior solution. Substituting into the first order condition for capital yields:

$$\eta_t = \frac{\lambda}{1-\lambda}\left(\frac{f'(k_t) - r - d}{f'(k_t) + 1 - d}\right) - (1 - \beta R)\left(\frac{2\lambda - 1}{1-\lambda}\right) \tag{30}$$

for all $t \geq 1$. As in the benchmark model, η_t is inversely related to k_t.

This appendix has shown that the results derived in Section 3.1 carry over directly to an environment in which domestic insiders manage firms.

C Generalized Political Process

In the section of the appendix we extend the political process. The model presented in the text assumed that each party had the same odds of being the next period's incumbent, regardless of which party is currently in power. In this appendix, we extend the model to the case where there may be an advantage to incumbency. Specifically, let $\gamma_{i,o}$ be the probability that the current incumbent loses office, and $\gamma_{o,i}$ be the probability a party will regain office, where all party's out of office are treated symmetrically. We deviate from the text by dropping the assumption $1 - \gamma_{i,o} = \gamma_{o,i}$. If $1 - \gamma_{i,o} > \gamma_{o,i}$, then the incumbent has an advantage in retaining office, and vice versa if $1 - \gamma_{i,o} < \gamma_{o,i}$.

Define $p_{t,s}$ as the probability the incumbent in period t is in power at time $s \geq t$, so $p_{t,s}$ satisfies the difference equation $p_{t,s+1} = (1 - \gamma_{i,o})p_{t,s} + \gamma_{o,i}(1 - p_{t,s})$ with initial condition $p_{t,t} = 1$. Solving for $p_{t,s}$:

$$p_{t,s} = \frac{\gamma_{o,i}}{\gamma_{i,o} + \gamma_{o,i}} + \frac{\gamma_{i,o}(1 - \gamma_{i,o} - \gamma_{o,i})^{s-t}}{\gamma_{i,o} + \gamma_{o,i}}.$$

That is, the probability of being in power at some date s in the future is composed of two terms: a constant term, representing the unconditional probability of being in power, plus an incumbency advantage which vanishes as s goes to infinity.

The counterpart to (6) in the generalized case is:

$$\tilde{W}_t = \sum_{s=t}^{\infty} \beta^{s-t} p_{t,s} \tilde{\theta} u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t}(1 - p_{t,s})u(c_s). \tag{A6}$$

We obtain the generalization of our stand-in government as follows. Define $\gamma = \gamma_{i,o} + \gamma_{o,i}$, and $\theta = 1 + \frac{(\tilde{\theta}-1)\gamma_{i,o}}{\gamma_{o,i}\tilde{\theta}+\gamma_{i,o}} \in (1, \tilde{\theta}]$. Note that this collapses to the case studied in the text by setting $\gamma_{o,1} = 1 - \gamma_{i,o} = p$, so $\gamma = 1$ and $\theta = \tilde{\theta}/(p\tilde{\theta} + 1 - p)$. More generally, $\gamma \in (0, 1]$. Substituting in to (A6), we have the counterpart to (7):

$$\begin{aligned} W_t &= \frac{\gamma_{i,o} + \gamma_{o,i}}{\gamma_{o,i}\tilde{\theta} + \gamma_{i,o}} \tilde{W}_t \\ &= \sum_{s=t}^{\infty} \beta^{s-t}(1-\gamma)^{s-t}\theta u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t}(1 - (1-\gamma)^{s-t})u(c_s). \end{aligned} \tag{A7}$$

Note that we recover the case studied in the text by setting $\gamma = 1$. As in the benchmark case, W_t is proportional to the incumbent's true utility $\tilde{W}_t$, and therefore an allocation satisfies the participation constraints for $\tilde{W}_t$ if and only if it does so for W_t.

All propositions from the benchmark model extend to the general case, as proven in Appendix D, with the appropriate modification of the expressions for general $\gamma \leq 1$.

D Proofs

In this section of the appendix we collect the proofs using the the general political process described in Appendix C. The proofs for the benchmark case follow by setting $\gamma = 1$.

Proof of Lemma 2

We begin with a generalized version of Lemma 2.

Lemma 2 (A). *The utility to the population as of time t, $V_t = \sum_{i=0}^{\infty} \beta^i u_{t+i}$, is given by:*

$$V_t = \left(\frac{1}{\theta}\right) W_t + \frac{\beta\gamma}{\theta}\left(1 - \frac{1}{\theta}\right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta}\right)^i W_{t+1+i}.$$

Proof. Using the definitions, we have

$$\begin{aligned} V_t &= u_t + \beta V_{t+1} \\ W_t &= \theta u_t + \beta(1-\gamma) W_{t+1} + \beta\gamma V_{t+1}. \end{aligned}$$

Eliminating u_t from the above and re-arranging:

$$\begin{aligned} \theta\left(V_t - \beta\left(1 - \frac{\gamma}{\theta}\right) V_{t+1}\right) &= W_t - \beta(1-\gamma) W_{t+1} \\ \theta\left(1 - \beta\left(1 - \frac{\gamma}{\theta}\right) F\right) V_t &= \left(1 - \beta(1-\gamma) F\right) W_t, \end{aligned}$$

where F is the forward operator. Solving for V_t and eliminating explosive solutions:

$$\begin{aligned} \theta V_t &= \left(\frac{1 - \beta(1-\gamma)F}{1 - \beta\left(1 - \frac{\gamma}{\theta}\right) F}\right) W_t \\ &= W_t + \beta\gamma\left(1 - \frac{1}{\theta}\right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta}\right)^i W_{t+1+i}. \end{aligned}$$

Dividing through by θ yields the expression in the lemma. □

Proof of Lemma 1

Define $\underline{W}(k)$ to be the incumbent's value function if it deviates given capital k. We can write this as

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V}, \tag{31}$$

where $\underline{W}$ is the continuation value under the punishment if the incumbent retains power next period, and $\underline{V}$ is the continuation value if it loses power. We normalize $t = 0$ to be the time of the deviation, so we have $\underline{W} = W_1$ and $\underline{V} = V_1$. From Lemma A2, we have:

$$\theta\underline{V} = \theta V_1 = W_1 + \beta\gamma\left(1 - \frac{1}{\theta}\right)\sum_{i=0}^{\infty}\beta^i\left(1 - \frac{\gamma}{\theta}\right)^i W_{1+i}.$$

As the punishment must be self-enforcing, we have $W_t \geq \theta u(\bar{c}(k_t)) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V}$, at each t. Note that a second deviation is punished in the same way as the first. The fact that $\underline{W}(k)$ is the worst possible punishment implies that this maximizes the set of possible self-enforcing allocations, from which we are selecting the one with minimum utility. Substituting in the participation constraint in the above expression yields:

$$\begin{aligned}
\theta\underline{V} &\geq \theta u(\bar{c}(k_1)) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V} \\
&\quad + \beta\gamma\left(1 - \frac{1}{\theta}\right)\sum_{i=0}^{\infty}\beta^i\left(1 - \frac{\gamma}{\theta}\right)^i\left(\theta u(\bar{c}(k_{1+i})) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V}\right) \\
&\geq \left(\frac{1-\beta(1-\gamma)}{1-\beta\left(1-\frac{\gamma}{\theta}\right)}\right)\left(\theta u(\bar{c}(\underline{k})) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V}\right),
\end{aligned}$$

where the last inequality uses the fact that $k_t \geq \underline{k}$, for all t. Rearranging, we have

$$\underline{V} \geq \frac{(1-\beta(1-\gamma))\left(u(\bar{c}(\underline{k})) + \beta(1-\gamma)\underline{W}\right)}{\theta(1-\beta) + \beta^2\gamma(1-\gamma)}. \tag{32}$$

Recall that $W_1 = \underline{W}$. Participation at $t = 1$ requires $W_1 \geq \theta u(\bar{c}(k_1)) + \beta\underline{W} + \beta\gamma\underline{V}$, or using the fact that $k_1 \geq \underline{k}$:

$$\underline{W} \geq \theta u(\bar{c}(\underline{k})) + \beta(1-\gamma)\underline{W} + \beta\gamma\underline{V}.$$

Substituting (32) in for $\underline{V}$ and rearranging yields:

$$\underline{W} \geq \left(\frac{\theta - 1}{1-\beta(1-\gamma)} + \frac{1}{1-\beta}\right)u(\bar{c}(\underline{k})).$$

Substituting back into (32), we have

$$\underline{V} \geq \frac{u(\bar{c}(\underline{k}))}{1-\beta}.$$

The left hand sides of these last two inequalities are the government's and private agent's utility, respectively, from the Nash equilibrium repeated ad infinitum. As repeated Nash is a self enforcing equilibrium and bounds from below the punishment payoff, it is the self-enforcing equilibrium that yields the lowest utility for the deviating government.

The Generalized Problem

One can then write the equivalent of problem (P):

$$V(b_0) = \max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{AP}$$

subject to:

$$b_0 \le \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - (r+d)k_t - c_t \right), \tag{A10}$$

$$\underline{W}(k_t) \le \sum_{s=t}^{\infty} \beta^{s-t}(1-\gamma)^{s-t}\theta u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t}(1-(1-\gamma)^{s-t})u(c_s), \ \forall t \tag{A11}$$

$$\underline{k} \le k_t \tag{A12}$$

Let μ_0 be the multiplier on the budget constraint (A10), $R^{-t}\mu_0\eta_t/\theta$ be the multiplier on the sequence of constraints on participation (A11) and $R^{-t}\phi_t$ be the multiplier on (A12).

The necessary first order conditions are:

$$c'(u_t) = (\beta R)^t \left(\frac{1}{\mu_0} + \sum_{s=0}^{t} (\beta R)^{-s} \left((1-\gamma)^{t-s}(\theta-1) + 1 \right) \frac{\eta_s}{\theta} \right) \tag{A13}$$

$$\frac{\eta_t}{\theta} \underline{W}'(k_t) = f'(k_t) - (r+d) + \phi_t, \ \forall t \ge 0 \tag{A14}$$

where $\phi_t = 0$ if $k_t > \underline{k}$.

Proof of Proposition 1

To prove the proposition, suppose that k_t did not converge to k^*. Define $T_\epsilon = \{t | k_t < k^* - \epsilon\}$. It follows that for some $\epsilon > 0$, T_ϵ has infinite members. Then from (A13):

$$c'(u_t) = \frac{1}{\mu_0} + \sum_{s=0}^{t} \left((1-\gamma)^{t-s}(\theta-1) + 1 \right) \frac{\eta_s}{\theta} \ge \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \le t} \frac{\eta_s}{\theta} \ge \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \le t} C_\epsilon$$

where $C_\epsilon \equiv (f'(k^* - \epsilon) - (r+d))/(\underline{W}'(k^* - \epsilon)/\theta) > 0$, and the inequalities reflect $\eta_s, \phi_s \ge 0$ for all s and $\eta_s \ge C_\epsilon$ for $s \in T_\epsilon$. It follows then that $c'(u_t)$ diverges to infinity, and thus u_t converges to its maximum. But this implies that the participation constraints will stop binding at some finite t_0, which leads to η_s that are zero for all $s > t_0$, a contradiction.

Proof of Proposition 2

The generalized version of Proposition 2 is as follows:

Proposition 2 (A). *The multiplier η_t satisfies the following difference equation:*

$$\eta_{t+1} = (1-(1-\gamma)\beta R)(1-\beta R) + \beta R\Big(1-\frac{\gamma}{\theta}\Big)\eta_t\,,\ \forall t \geq 1 \tag{A16}$$

with $\eta_0 = 1-\mu_0^{-1}$ and $\eta_1 = 1-\beta R + \frac{\beta R(\theta-1)\gamma}{\theta}\eta_0$. The sequence of η_t converges monotonically from $t \geq 1$ towards its steady state value η_∞:

$$\eta_\infty = \frac{\theta(1-(1-\gamma)\beta R)(1-\beta R)}{\theta(1-\beta R)+\beta R\gamma}.$$

Letting $\hat{\eta}_t \equiv \eta_t - \eta_\infty$, convergence to the steady state can be characterized by:

$$\hat{\eta}_{t+1} = \beta R\left(1-\frac{\gamma}{\theta}\right)\hat{\eta}_t.$$

Proof. The generalized counterpart of (15) is:

$$(\beta R)^{-t} - \sum_{s=0}^{t}(\beta R)^{-s}\left((1-\gamma)^{t-s}(\theta-1)+1\right)\frac{\eta_s}{\theta} = \frac{1}{\mu_0},\ \forall t \geq 0 \tag{A15}$$

We have that from (A15) at $t+1$:

$$(\beta R)^{-(t+1)} - \sum_{s=0}^{t}(\beta R)^{-s}\left((1-\gamma)^{t-s}(\theta-1)+1\right)\frac{\eta_s}{\theta} - (\beta R)^{-(t+1)}\eta_{t+1} = \mu_0$$

which can be written as:

$$\begin{aligned}(\beta R)^{-(t+1)} - (1-\gamma)\sum_{s=0}^{t}(\beta R)^{-s}\left((1-\gamma)^{t-s}(\theta-1)+1+\frac{1}{1-\gamma}-1\right)\frac{\eta_s}{\theta}\\ -(\beta R)^{-(t+1)}\eta_{t+1} = \mu_0\end{aligned}$$

where using (A15) at t we get:

$$(\beta R)^{-(t+1)} - (1-\gamma)(\beta R)^{t} - \sum_{s=0}^{t}(\beta R)^{-s}\gamma\frac{\eta_s}{\theta} - (\beta R)^{-(t+1)}\eta_{t+1} = \mu_0 - (1-\gamma)\mu_0 \tag{33}$$

Subtracting (33) at t from (33) at $t+1$:

$$\begin{aligned}&(\beta R)^{-(t+1)} - (\beta R)^{-t} - (1-\gamma)((\beta R)^{t} - (\beta R)^{t-1})\\ &\qquad - \gamma(\beta R)^{-t}\frac{\eta_t}{\theta} + (\beta R)^{-t}\eta_t - (\beta R)^{-(t+1)}\eta_{t+1} = 0\end{aligned}$$

which delivers the result once simplified for $t \geq 1$. Using (A15) at $t=0$ delivers $\eta_0 = (1-\mu_0)/\theta$. And using (A15) at $t=1$ delivers that

$$\eta_1 = 1-\beta R + \frac{\beta R(\theta-1)\gamma}{\theta}\eta_0$$

The steady state value can be computed in the usual way. Given that the slope of (A16) is positive and less than one, convergence and monotonicity follow. □

Note that substituting $\gamma = 1$ delivers the results for Proposition 2.

Dynamics of k_t and the proof of Corollary 1

Proposition 2(A) characterized the dynamics of η_t. One can then, from here, deliver the associated dynamics for k_t. For any given value of η_t, define $K(\eta_t)$ to be the solution to:

$$\eta_t = \frac{f'(K(\eta_t)) - (r+d)}{\underline{W}'(K(\eta_t))/\theta}$$

The convexity assumption guarantees that the above has a unique solution, and that $K(\eta_t)$ is strictly decreasing in η_t. Now, let $\bar{\eta}$ to be such that $K(\bar{\eta}) = \underline{k}$. Then the optimal path for k_t will be:

$$k_t = \begin{cases} K(\eta_t) & ; \text{for } \eta_t < \bar{\eta} \\ \underline{k} & ; \text{otherwise} \end{cases}$$

Given that η_∞ is monotone, this implies that the path for k_t will also be monotone. Figure 6 shows the dynamics of the system, taking the account the possibility that the upper bound constraint on the tax rates might bind.

Define now $\bar{\theta}$ to be the value such that:

$$\frac{f'(\underline{k}) - (r+d)}{\bar{c}'(\underline{k})} = \frac{\bar{\theta}(1-(1-\gamma)\beta R)(1-\beta R)}{\bar{\theta}(1-\beta R) + \beta R \gamma} \equiv \bar{\eta}.$$

Hence, the long run level of capital will be:

$$k_\infty = \begin{cases} K(\eta_\infty) & ; \text{for } \theta < \bar{\theta} \\ \underline{k} & ; \text{otherwise} \end{cases}$$

This proves the first part of the proposition (when plugging in for $\gamma = 1$). For the second part, note that higher debt implies a (weakly) higher multiplier μ_0, and a higher η_0. Given that η_1 and η_t are monotonic in previous values, it follows that the entire path of η increases with μ_0 and debt. That is, a higher level of debt leads to a lower capital path.

Proof of Proposition 3

The proof of this proposition follows directly from Lemma 2(A) , the fact that k_t is monotone, and that $\underline{W}(k)$ is an increasing function of k.

Proof of Corollary 2

Let suppose that k_t is increasing. Let $B_t = \sum_{s=t}^{\infty} R^{s-t}(f(k_s) - (r+d)k_s - c_s)$ denote the stock of debt outstanding in period t. Suppose, to generate a contradiction, that $B_{T+1} > B_T$ for some $T \geq 1$. Let $\{u_t, k_t\}$ denote the equilibrium allocation. Now consider the alternative allocation:

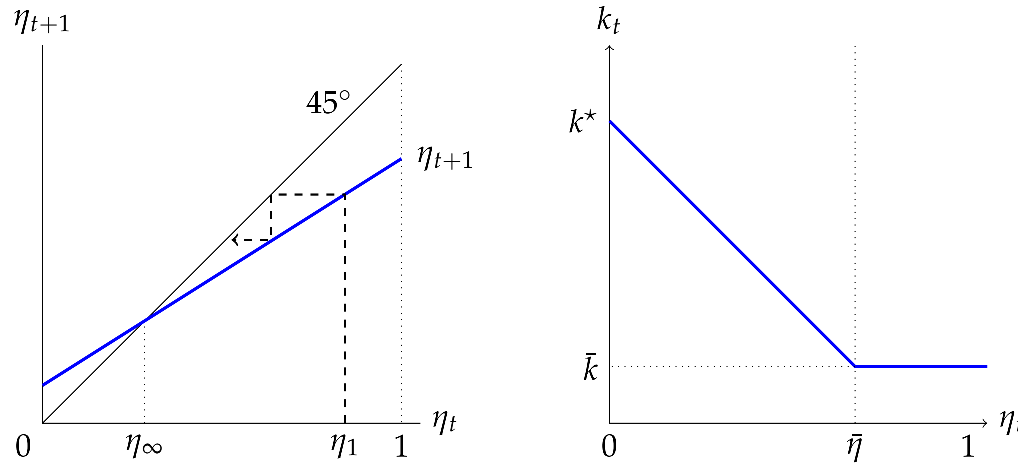

Figure 6: Transition mapping for η_t when $\beta R < 1$. The blue line in the diagram on the left represents the transition mapping as given by equation (A16) for $t \geq 1$. The dashed line represents a possible equilibrium path starting from $\eta_1 = 1 - \beta R + \frac{\beta R(\theta-1)\gamma}{\theta}\frac{(1-\mu)}{\theta}$. The diagram on the right shows the relationship between η_t and the capital k_t.

$\tilde{u}_t = u_t$ and $\tilde{k}_t = k_t$ for $t < T$, and $\tilde{c}_t = c_{t+1}$ and $\tilde{k}_t = k_{t+1}$ for $t \geq T$. That is, starting with period T, we move up the allocation one period. As $\tilde{V}_0 - V_0 = \beta^T(\tilde{V}_T - V_T) = \beta^T(V_{T+1} - V_T) > 0$, the objective function has increased and where the last inequality follows from the monotonicity of V_t. Similarly, $\tilde{B}_0 - B_0 = R^{-T}(\tilde{B}_T - B_T) = R^{-T}(B_{T+1} - B_T) > 0$, the budget constraint is relaxed, where the last inequality follows from the premise $B_{T+1} > B_T$. For $t \geq T$, we have $\tilde{W}_t = W_{t+1} \geq \underline{W}(k_{t+1}) = \underline{W}(\tilde{k}_t)$, so participation holds for period T and after. For $t < T$, note that $W_t = \sum_{s=t}^{T-1} \beta^{s-t}\left[(1-\gamma)^{s-t}(\theta-1)+1\right] u_s + \beta^T(1-\gamma)^T W_T + \beta^T(1-(1-\gamma)^T t)V_{T+1}$. As $\tilde{W}_T > W_T$ and $\tilde{V}_T > V_T$, we have $\tilde{W}_t > W_t$ for all $t < T$. As $\tilde{k}_t = k_t$ for $t < T$, our new allocation satisfies the participation constraints of the governments along the path. Therefore, we have found a feasible allocation that is a strict improvement, a contradiction of optimality. A similar argument works for a decreasing path of k_t.

Proof of Proposition 4

This was proved in the main text.

Proof of Proposition 5

From (19), we have

$$B_\infty = \left(\frac{1+r}{r}\right)\left(f(k_\infty) - (r+d)k_\infty - c_\infty\right).$$

Recall that k_∞ is a continuous function of θ, and by definition, $k_\infty = \underline{k}$ at $\theta = \bar{\theta}$. Therefore, $\lim_{\theta\to\bar{\theta}} k_\infty = \underline{k}$. So

$$\lim_{\theta\to\bar{\theta}} B_\infty = \left(\frac{1+r}{r}\right)\left(f(\underline{k}) - (r+d)\underline{k} - \lim_{\theta\to\bar{\theta}} c_\infty\right). \tag{34}$$

Using the fact that $W_t = \underline{W}(k_t)$ along the path and the definition of $\underline{W}(k)$, we have:

$$\left(\frac{\theta-1}{1-\beta(1-\gamma)} + \frac{1}{1-\beta}\right) c_\infty = \theta\left(\bar{c}(k_\infty) - \bar{c}(\underline{k})\right) + \left(\frac{\theta-1}{1-\beta(1-\gamma)} + \frac{1}{1-\beta}\right)\bar{c}(\underline{k}),$$

or

$$c_\infty = \theta\left(\frac{\theta-1}{1-\beta(1-\gamma)} + \frac{1}{1-\beta}\right)^{-1}\left(\bar{c}(k_\infty) - \bar{c}(\underline{k})\right) + \bar{c}(\underline{k}). \tag{35}$$

This implies, $\lim c_\infty = \bar{c}(\underline{k}) \equiv f(\underline{k}) - (1-\bar{\tau})f'(\underline{k})\underline{k} = f(\underline{k}) - (r+d)\underline{k}$, where the last equality follows from the definition of $\underline{k}$. Plugging into (34) gives $\lim B_\infty = 0$, which is the first part of the proposition.

For the second part, we have

$$\lim_{\theta\to\bar{\theta}^-} \frac{dB_\infty}{d\theta} = \left(\frac{1+r}{r}\right)\left(\left(f'(\underline{k}) - (r+d)\right)\lim_{\theta\to\bar{\theta}^-}\frac{dk_\infty}{d\theta} - \lim_{\theta\to\bar{\theta}^-}\frac{dc_\infty}{d\theta}\right). \tag{36}$$

Equation (35) implies:

$$\begin{aligned}\lim_{\theta\to\bar{\theta}^-}\frac{dc_\infty}{d\theta} &= \bar{\theta}\left(\frac{\bar{\theta}-1}{1-\beta(1-\gamma)} + \frac{1}{1-\beta}\right)^{-1}\bar{c}'(\underline{k})\lim_{\theta\to\bar{\theta}^-}\frac{dk_\infty}{d\theta}\\ &= \frac{\bar{\theta}(1-(1-\gamma)\beta)(1-\beta)}{\bar{\theta}(1-\beta)+\beta\gamma}\bar{c}'(\underline{k})\lim_{\theta\to\bar{\theta}^-}\frac{dk_\infty}{d\theta}\end{aligned}$$

From the definition of $\bar{\theta}$ we have that:

$$f'(\underline{k}) - (r+d) = \frac{\bar{\theta}(1-(1-\gamma)\beta R)(1-\beta R)}{\bar{\theta}(1-\beta R)+\beta R\gamma}\bar{c}'(\underline{k})$$

Plugging these back into (36):

$$\begin{aligned}\lim_{\theta\to\bar{\theta}^-}\frac{dB_\infty}{d\theta} = \left(\frac{1+r}{r}\right)&\left[\frac{\bar{\theta}(1-(1-\gamma)\beta R)(1-\beta R)}{\bar{\theta}(1-\beta R)+\beta R\gamma} - \frac{\bar{\theta}(1-(1-\gamma)\beta)(1-\beta)}{\bar{\theta}(1-\beta)+\beta\gamma}\right]\times\\ &\times \bar{c}'(\underline{k})\lim_{\theta\to\bar{\theta}^-}\frac{dk_\infty}{d\theta}\end{aligned}$$

The term inside the square brackets is strictly negative as long as $R > 1$, and $\bar{c}'(\underline{k}) > 0$ given that $\bar{\tau} > 0$ by assumption. The result follows as k_∞ strictly decreases with θ for $\theta < \bar{\theta}$. This last statement follows from the fact that $\frac{dk_\infty}{d\theta} = K'(\eta_\infty)\frac{d\eta_\infty}{d\theta}$, together with $\frac{d\eta_\infty}{d\theta} > 0$ if $\beta R < 1$ (see Proposition A2), and $K'(\eta) < 0$ (from the convexity assumption).

Proof of Proposition 6

The proof proceeds by noticing that $\hat{c}_t = c_t + y_t$ where c_t is an optimal consumption sequence in the economy without debt is optimal in the economy with debt. To see this, note that the participation constraints are still holding tight, as both W_t and the deviation increase by the same amount. The present value constraint on the resources of the government has not changed. And the first order conditions remain the same as before.

Proof of Lemma 3

From equation (21a) we get:

$$\hat{k}_t = \frac{\underline{W}'(k_\infty)}{\theta f''(k_\infty) - \eta_\infty \underline{W}''(k_\infty)} \hat{\eta}_t$$

Using this and (22a), we get:

$$\hat{\eta}_t = ((\theta - 1)\hat{u}_t + \hat{V}_t) \left(\frac{\underline{W}'(k_\infty)^2}{\theta f''(k_\infty) - \eta_\infty \underline{W}''(k_\infty)} \right)^{-1}$$

where we also used (23a). Plugging back for $\hat{\eta}_t$ and $\hat{\eta}_{t-1}$ into (20a) using the above expression, and simplifying the resulting equation together with (23a), the results follows.