Sovereign Risk Premia *

Nicola Borri           Adrien Verdelhan
Boston University       MIT Sloan

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Abstract

Emerging countries tend to default when their economic conditions worsen. If bad times in an emerging country correspond to bad times for the US investor, then these foreign sovereign bonds are particularly risky and should offer high returns. We explore how this mechanism plays out in the data and in a general equilibrium model of optimal borrowing and default. Empirically, we obtain a cross-section of sovereign bond returns: the higher the correlation between past sovereign bond returns and US corporate bond returns, the higher the average sovereign excess returns. A model of risk-averse lenders with external habit preferences qualitatively replicates this feature.

* Borri: Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215; borri@bu.edu; Tel: (617) 353-5690; http://people.bu.edu/borri. Verdelhan: Department of Finance, MIT Sloan School of Management, E52-436, 50 Memorial Drive, Cambridge, MA 02142; adrienv@mit.edu; Tel: (617) 253-5123; http://web.mit.edu/adrienv/www/. The authors thank seminar participants at Boston University, Boston College GLMM Macro Meeting, IHS Vienna, École Polytechnique, LUISS, Warwick Business School, Bank of Italy, EPFL, CEPR and the North American Econometric Society Meeting for helpful comments. All errors are our own.
In this paper, we study sovereign bonds issued by emerging countries in US dollars. The Euler equation for an American investor implies that sovereign bond prices depend on their default probabilities and on covariances between bond payoffs and the investor’s marginal utility of wealth. Default probabilities are a well-known driver of emerging bond yields. The worse the Standard & Poor’s credit rating, for example, the higher the yield on average. In this paper, we show both theoretically and empirically that covariances between bond returns and risk factors are key determinants of bond prices and debt quantities.

To illustrate the intuition behind this result, assume that an American investor has constant relative risk-aversion and invests in one-period foreign government bonds. Emerging countries tend to default in “bad times”, when foreign consumption is low. If bad times in the foreign economy correspond to bad times in the domestic economy, then foreign countries tend to default in bad times for the US investor. In this case, sovereign bonds are particularly risky, and the US investor expects to be compensated for that risk through a high return. Alternatively, if bad times in the foreign economy correspond to good times for the US investor, then sovereign bonds are less risky and may even hedge domestic consumption. As a result, sovereign bond prices depend on both expected probabilities of repayment and the timing of the bond payoffs.

With this price mechanism in mind, we turn to the data on sovereign debt. We look at bonds issued by emerging market countries that are included in JP Morgan’s EMBI Global index. Yields on EMBI bonds increase with the probability of default as measured by Standard and Poor’s credit ratings. However, for a given default probability, there is significant cross-sectional variation in yields; at the end of August 2008, for example, spreads were up to 300 basis points. To disentangle the two price mechanisms, we build portfolios of sovereign bonds by sorting countries along two dimensions: their default probabilities and their covariance with US economic conditions. For the first dimension, we use Standard and Poor’s credit ratings to measure the probability of sovereign default. Credit ratings are not investor-specific and do not account for the timing of a potential default. For the second dimension, we compute bond betas, which are defined as the slope coefficients in regressions of one-month sovereign bond returns on one-month US corporate bond returns at daily frequency. In our framework, US corporate bond returns proxy for domestic economic conditions. Our intuition starts off the correlation between macroeconomic conditions in emerging countries and in the US, but most emerging countries lack high frequency macroeconomic data. Bond returns offer a high frequency measurement of investor marginal utility of wealth. This is consistent with the literature on corporate bond indices: Krainer (2004) shows that US corporate credit spreads are counter-cyclical; Elton, Gruber, Agrawal, and Mann (2001) find that only a quarter of the spread on US corporate bonds is due to expected default probabilities, and that the remaining portion represents compensation for co-movement with Fama and French (1993) risk factors. After sorting countries
along these two dimensions, we obtain six portfolios and a large cross-section of holding period excess returns. The average spread between countries with low and high default probabilities is about 500 basis points. The average spread between countries with low and high bond betas is also about 500 basis points.

We study this cross-section of excess returns from the perspective of a US investor. We find that a large fraction of the cross-section of average EMBI excess returns can be explained by their covariances with just one risk factor: the return on a US BBB corporate bond. Portfolios with higher exposure to this risk factor are riskier and have higher average excess returns because they offer lower returns when US corporate default risks are higher. The market price of risk is in line with the mean of the risk factor, as implied by a no-arbitrage condition. Pricing errors are not statistically significant. Looking at the time-variation in the market price of risk, we find that it increases in bad times, as measured by a high value of the equity option-implied VIX index.

To interpret our findings and uncover their implications in terms of optimal borrowing, we build on the seminal work by Eaton and Gersovitz (1981) and use a dynamic general equilibrium model of sovereign lending with endogenous default choice. In the model, a set of small open economies borrow from a large developed country (the US). We consider endowment economies. The only source of heterogeneity across small open economies is their correlation with the US business cycle. We introduce a key modification to the literature: we assume that investors are risk-averse and have external habit preferences as in Campbell and Cochrane (1999). This feature helps our understanding of the data: without it, e.g. when investors are risk-neutral, there is no role for covariances in sovereign bond prices. A model with risk neutral investors cannot account for the results of our empirical analysis on EMBI bonds. Moreover, Campbell and Cochrane (1999) preferences entail time-varying risk aversion, and thus a time-variation in the market price of risk.

The rest of the model is similar to Arellano (2008) and Aguiar and Gopinath (2006). As in the former paper, we assume a nonlinear cost of default. As in the latter, we assume that foreign endowments present a time-varying long-run mean. In the model, countries default after receiving a series of negative shocks. When business cycles in emerging countries and in the US are positively correlated, defaults tend to occur when US consumption is low relative to the habit level. Bonds issued by these countries are riskier and have lower prices because they have low payoffs when the lender's marginal utility of consumption is high. The model matches important features of the emerging markets business cycle. Consumption is more volatile than output; interest rate spreads and trade balances are strongly counter-cyclical. We thus focus on the model's implications for bond pricing. In the model, we can precisely measure expected default probabilities and consumption correlation, so we do not need to rely on proxies like Standard and Poor's ratings or corporate spreads. The model offers a general equilibrium view of debt quantities and prices.
and defaults are endogenous choices: countries facing high borrowing costs choose to borrow less, thereby lowering their default risk. In the simulations, high beta countries pay higher interest rates even if they borrow less in equilibrium.

Two discrepancies between the model and the data are worth mentioning. First, the spread in returns between high and low default probability countries and the spread in returns between high and low beta countries is smaller than in the data. Second, the model only considers one-period bonds, whereas actual bonds have longer maturities. As a result, the model does not take into account interest rate risk. We leave this interesting case out for future research.

This paper is related to two strands of existing literature on sovereign debt. First, this paper contributes to the large body of empirical literature on emerging market bond spreads. The paper closest to ours is Longstaff, Pan, Pedersen, and Singleton (2007). They study changes in emerging market credit default swaps spreads and find that global factors, like the return on the U.S. stock market and changes in the VIX index, explain a large fraction of the common variation in swap spreads. They argue convincingly that excess returns are mostly compensation for bearing global risk, with little or no country specific risk premia. Second, our paper contributes to the theoretical literature on sovereign lending with defaults. Here, the papers closest to ours are Arellano (2008) and Aguiar and Gopinath (2006).

The paper is organized as follows. Section 1 describes the data, how we build EMBI portfolios, and the main characteristics of the EMBI portfolio excess returns. Section 2 shows that two global risk factors explain most of the time series variation in portfolio excess returns. In section 3, we interpret these findings by describing a general equilibrium model of sovereign borrowing. Section 4 considers a calibrated version of the model that qualitatively replicates our empirical findings. Section 5 concludes. All the tables and figures are in the appendix.

1 The Cross-Section of EMBI Returns

We focus on sovereign bonds issued in US dollars by emerging countries. To study these bonds, we take the perspective of a US investor who borrows in dollars to invest in this bond market. We check that these bond returns increase with the probability of default, as measured by Standard

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and Poor’s country ratings. We uncover a second mechanism: the higher the sovereign bond’s covariance with US corporate bond returns, the higher the average sovereign excess returns. Using these two results, we build portfolios along two dimensions and obtain a cross-section of EMBI excess returns.

We start by describing the raw data and setting up some notations. Then we turn to our portfolio-building methodology, and report the main characteristics of our cross-section of EMBI excess returns.

1.1 Data and notations

Data on Emerging Markets We focus on the set of countries included in JP Morgan’s EMBI Global index. JP Morgan publishes country-specific and aggregate indices that market participants consider as benchmarks. The EMBI Global index covers low or middle income per capita countries (according to the World Bank’s classification). It also includes countries that are currently - or have been in the past ten years - restructuring their external or local debts. Our main dataset thus contains 36 countries: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote D’Ivoire, Dominican Republic, Ecuador, Egypt, El Salvador, Hungary, Indonesia, Iraq, Kazakhstan, Lebanon, Malaysia, Mexico, Morocco, Pakistan, Panama, Peru, Philippine, Poland, Russia, Serbia, South Africa, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam. The sample period runs from January 1995 to May 2009.

The JP Morgan EMBI Global total return price index includes accrued dividends and cash payments. In each country, the index is a market capitalization-weighted aggregate of US dollar-denominated Brady Bonds, Eurobonds, traded loans and local market debt instruments issued by sovereign and quasi-sovereign entities. The weight of each instrument in each country-specific index is determined by dividing the issue’s market capitalization by the total market capitalization for all instruments in the index. The market capitalization of each issue corresponds to its face value outstanding multiplied by its bid-side settlement price. Weights are updated at the end of each month (see Cavanagh and Long (1999)). These bonds are liquid debt instruments that are actively traded. Their notional sizes are at least equal to $500 million. Each issue included in the EMBI Global index must have at least 2.5 years until maturity when it enters the index and at least 1 year until maturity to remain in the index. Moreover, JP Morgan sets liquidity criteria such as easily accessible and verifiable daily prices either from an inter-dealer broker or a certified JP Morgan source.

To assess the default probability of each country, we rely on Standard and Poor’s ratings. Standard and Poor’s credit ratings take the form of letter grades ranging from AAA (highest credit worthiness) to SD (selective default). They are available for a large set of countries over a long time
period. We collect Standard and Poor's ratings for all the 36 countries in the EMBI index, except Côte d'Ivoire and Iraq. We focus on ratings for long-term debt denominated in foreign currencies and convert ratings into numbers ranging from 1 (highest credit worthiness) to 23 (lowest credit worthiness). Our sample contains several default episodes. Argentina, the Dominican Republic, Ecuador, Russia and Uruguay defaulted on their external debt during our sample period. Argentina was in default status from November, 2001 to May, 2005, the Dominican Republic from February, 2005 to May, 2005, Ecuador in July, 2000 for only one month, Russia from January, 1999 to November, 2000 and Uruguay in May, 2003 for only one month.

Ratings are not traded prices. This obvious fact has two consequences. First, ratings are not tailored to a particular investor. For example, they are the same for a US and a Japanese investor. As a result, ratings do not take into account the timing of a potential sovereign default: a country that might default in good times for the US has the same rating as a country that might default in bad times. Second, for most countries, credit ratings do not encompass all the information on expected defaults. They are not updated on a regular basis, but rather when new information or events suggest the need for additional Standard and Poor's studies and grade revisions.

To complement the Standard and Poor's ratings, it is now common to rely on credit default swaps (CDS) and debt to GNP ratios. These two measures do not seem appropriate for our study. CDS are insurance contracts against the event that a sovereign defaults on its debt over a given horizon (see Pan and Singleton (2008)). These contracts are traded in US dollars. As a result, their prices should reflect both the magnitude and the timing of expected defaults. This conflicts with our goal to disentangle these two effects. Moreover, CDS data are only available from December 2002 on, and for a subset of the EMBI Global countries. Debt to GNP ratios are available for many countries, but at annual frequency. These ratios do not predict default probabilities and returns as well than Standard and Poor's ratings. To check, however, that high debt levels do not drive our results, we report debt to GNP ratios. Our series come from the World Bank Global Development Finance annual data set. We linearly interpolate the annual debt to GNP ratios to obtain monthly series.

As a snapshot of our data set, Figure 1 reports, for each country in JP Morgan's EMBI Global Index, the annual stripped spread plotted against the Standard and Poor's credit rating at the end of August 2008. The stripped spread is equal to the difference between the average yield to maturity in the emerging country and the corresponding yield to maturity on the US Treasury spot curve, after 'stripping' out the value of any collateralized cash flows. These spreads correspond to the usual representation of sovereign risk premia. Throughout the rest of the paper though, we use index prices to compute returns.
**Notation**  Before turning to our portfolio-building strategy, we introduce here some useful notations. Let $r_{t+1}^{*i}$ denote the log excess return, including any accrued dividends, of an American investor who borrows funds in US dollars at the log risk free rate $r_t^f$ in order to buy country $i$'s EMBI bond, and then sells this bond after one month and pays back his debt. His log excess return is equal to:

$$r_{t+1}^{*i} = p_{t+1}^i - p_t^i - r_t^f,$$

where $p_t^i$ denotes the log market price of an EMBI bond in country $i$ at date $t$.

We define the bond beta ($\beta_{EMBI}^i$) of each country $i$'s as the slope coefficient in a regression of EMBI bond returns on US BBB-rated corporate bond returns:

$$\Delta p_t^i = \alpha^i + \beta_{EMBI}^i r_t^{BBB} + \epsilon_t,$$

where $r_t^{BBB}$ denotes the log total return on the Merrill Lynch US BBB corporate bond index.

We compute betas on 100-day rolling windows to obtain time-series of $\beta_{EMBI,t}$. As a timing convention, we date $t$ the beta estimated with returns up to date $t$. For each regression, we estimate betas only if at least 50 observations for both the left- and right-hand side variables are available over the previous 100-day rolling window period.

### 1.2 Portfolios of Excess Returns

**EMBI portfolios** We build portfolios of EMBI excess returns by sorting countries along two dimensions: their probabilities of defaults and their bond betas. First, at the end of each period $t$, we sort all countries in the sample in two groups on the basis of their bond betas $\beta_{EMBI,t}$. The first group contains the countries with the lowest $\beta_{EMBI,t}$, the second group contains the countries with the highest $\beta_{EMBI,t}$. Second, we sort all countries within each of the two groups in three portfolios ranked from low to high probabilities of default. We measure default probabilities with Standard and Poor’s credit ratings. As a result, we obtain six portfolios. Portfolios 1, 2 and 3 contain countries with the lowest betas, portfolios 4, 5 and 6 contain countries with the highest betas. Portfolios 1 and 4 contain countries with the lowest default probabilities, portfolios 3 and 6 contain countries with the highest default probabilities. Portfolios are re-balanced at the end of every month, using information available at that point. We compute the EMBI excess returns $r_{t+1}^{*ij}$ for portfolios $j$ by taking the average of the EMBI excess returns in each portfolios $j$ over the subsequent period (e.g between $t$ and $t+1$). The total number of countries in our portfolios varies over time. We have 6 countries at the beginning of the sample in January, 1995 and 32 at the end in August, 2008.\(^3\)

\(^3\)Daily historical levels of the EMBI indices are available from December 31, 1993 onwards for a limited set of countries. We need at least six countries in the sample to start building our six portfolios and thus start in January.
The maximum number of countries attained during the sample is 32.

Table II provides an overview of our six EMBI portfolios. For each portfolio $j$, we report the average foreign bond beta $\hat{\beta}_{EMBI}^j$, the average total excess return $r^{e,j}$, the average Standard and Poor’s credit rating and the average external debt to GNP ratio. All returns are reported in US dollars and the moments are annualized: we multiply the mean of the monthly return by 12 and the standard deviation by $\sqrt{12}$. The Sharpe ratio is the ratio of the annualized mean to the annualized standard deviation.

Our portfolios highlight two simple empirical facts. First, excess returns increase from low to high betas: portfolio 1, 2 and 3 (low betas) offer lower excess returns than portfolios 4, 5 and 6 (high betas). The average excess return on all the low beta portfolios is 505 basis points per annum. For the high beta portfolios, it is 1020 basis points. As a result, there is on average a 500 basis points difference between high and low beta portfolios. Bilateral comparisons (portfolio 1 versus portfolio 4, 2 versus 5, and 3 versus 6) all show that, for similar credit ratings, high beta bonds always offer higher returns. Second, excess returns also increase with default probabilities: portfolios 1 and 4 (low default probabilities) offer lower excess returns than portfolios 3 and 6 (high default probabilities). For low beta countries, the spread between low and high default probabilities entails a 350 basis point difference in returns. For high beta countries, this difference jumps to 650 basis points. These two empirical facts square well with intuition. An investor receives higher returns to compensate for higher default probabilities. If the investor is risk-averse, then he expects higher returns for assets that co-vary with his return on wealth.

These spreads are economically and statistically significant. As a back-of-the-envelope check to this point, note that the standard error on the mean estimate is approximately equal to the standard deviation of the excess returns divided by the square root of the number of observations (assuming $i.i.d$ returns). The average standard deviation is approximately equal to 13 percent. The sample size is 164 quarters ($12.8^2$). The standard error on the mean is thus around 1 percent, or 100 basis points. A spread of 500 basis points corresponds to five times the standard deviation of the mean.

Patton and Timmermann (2008) propose a more precise test of these cross-sectional properties. We use their non-parametric test to examine whether there exists a monotonic mapping between the observable variables used to sort EMBI countries into portfolios and expected returns. The test rejects at standard significance levels the null of the absence of a monotonic relationship between portfolio ranks and returns against the alternative of an increasing pattern (the $p$-value is 1.5%).

We conduct two robustness checks: value-weighted portfolios (instead of equal weights in our main sample) and stock market betas (instead of bond betas). We find a similar cross-section of

\footnote{Table II in the appendix reports the frequency of reallocation across portfolios. Figure II in the appendix focuses on the examples of Argentina and Mexico.}
excess returns as before when we build value-weighted portfolios using again bond betas and credit ratings. We also find a similar cross-section when we use stock market betas and credit ratings. The stock market betas correspond to slope coefficients in regressions of sovereign bond returns on the US stock market return. We report summary statistics on these additional portfolios in Tables 8 and 9 in a separate appendix. High beta sovereign bonds tend to offer higher returns.

To sum-up, by sorting countries along their Standard and Poor’s ratings and bond betas, we have obtained a rich cross-section of average excess returns. We now turn to the dynamic properties of these portfolios.

2 Common Risk Factors in EMBI Excess Returns

In this section, we show that covariances with US corporate bond returns account for a large share of our cross-section of average excess returns.

2.1 Asset Pricing Methodology

Linear factor models of asset pricing predict that average returns on a cross-section of assets can be attributed to risk premia associated with their exposure to a small number of risk factors. In the arbitrage pricing theory of Ross (1976) these factors capture common variation in individual asset returns.

Cross-Sectional Asset Pricing We use \( R_{t+1}^{e,j} \) to denote the average excess return on portfolio \( j \) in period \( t + 1 \). In the absence of arbitrage opportunities, this excess return has a zero price and satisfies the following Euler equation:

\[
E_t[M_{t+1} R_{t+1}^{e,j}] = 0,
\]

where \( M \) denotes the stochastic discount factor of the US investor. We assume that the log stochastic discount factor \( m \) is linear in the pricing factors \( f \):

\[
m_{t+1} = 1 - b(f_{t+1} - \mu),
\]

where \( b \) is the vector of factor loadings and \( \mu \) denotes the factor means. This linear factor model implies a beta pricing model: the log expected excess return is equal to the factor price \( \lambda \) times the beta of each portfolio \( \beta^i \):

\[
E[r_{t}^{e,j}] = \lambda^i \beta^i
\]
where \( r^\sim_j \) denotes the log excess return on portfolio \( j \) corrected for its Jensen term, \( \lambda = \Sigma_{ff} b, \Sigma_{ff} = E(f_t - \mu_f)' \) is the variance-covariance matrix of the factor, and \( \beta^j \) denotes the regression coefficients of the return \( R^e_j \) on the factors. To estimate the factor prices \( \lambda \) and the portfolio betas \( \beta \), we use two different procedures: a Generalized Method of Moments (GMM) applied to linear factor models, following Hansen (1982), and a two-stage OLS estimation following Fama and MacBeth (1973), henceforth FMB.

We briefly describe these two techniques.

**GMM** The moment conditions are the sample analog of the populations pricing errors:

\[ g_t(b) = E_T(m_t \widetilde{r}_t) = E_T(\widetilde{r}_t) - E_T(\widetilde{r}_t f'_t) b, \]

where \( \widetilde{r}_t = [\widetilde{r}_t^1, \widetilde{r}_t^2, \ldots, \widetilde{r}_t^N]' \) groups all the \( N \) EMBI portfolios. In the first stage of the GMM estimation, we use the identity matrix as the weighting matrix, while in the second stage we use the inverse of the spectral density \( S \) matrix of the pricing errors in the first stage: \( S = \sum E[(m_t \widetilde{r}_t')(m_t - \mu_f)' \widetilde{r}_t' f_t] \). We use demeaned factors in both stages. Since we focus on linear factors models, the first stage is equivalent to an OLS cross-sectional regression of average returns on the second moment of returns and factors. The second stage is a GLS cross-sectional regression of average excess returns on the second moment of returns and factors.

**FMB** In the first stage of the FMB procedure, for each portfolio \( j \), we run a time-series regression of the EMBI excess returns \( \widetilde{r}_t^j \) on a constant and the factors \( f_t \), in order to estimate \( \beta^j \). The only difference with the first stage of the GMM procedure stems from the presence of a constant in the regressions. In the second stage, we run a cross-sectional regression of the average excess returns \( E_T(m_t \widetilde{r}_t^j) \) on the betas that were estimated in the first stage, to estimate the factor prices \( \lambda \). The first stage GMM estimates and the FMB point estimates are identical, because we do not include a constant in the second step of the FMB procedure. Finally, we can back out the factor loadings \( b \) from the factor prices and covariance matrix of the factors.

### 2.2 Results

We use a single risk factor to account for the returns on our EMBI portfolios. This risk factor is log total return on the Merrill Lynch US BBB corporate bond index that we used to form portfolios. Table 2 reports our asset pricing results. We focus first on market prices of risk and then turn to the quantities of risk in our portfolios.

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5We use a Newey and West (1987) approximation of the spectral density matrix. The optimal number of lags is determined using Andrews (1991)'s criterion with a maximum of 6 lags.
**Market Prices of Risk**  The top panel of the table reports estimates of the market price of risk $\lambda$ and the SDF factor loadings $b$, the adjusted $R^2$, the square-root of mean-squared errors $RMSE$ and the p-values of $\chi^2$ tests (in percentage points). The market price of risk is equal to 693 basis points per annum. The FMB standard error is 271 basis points. The risk price is more than two standard errors from zero, and thus highly statistically significant. Overall, asset pricing errors are small. The RMSE is 158 basis points and the $R^2$ is 73 percent. The null that the pricing errors are zero cannot be rejected, regardless of the estimation procedure. Figure 2 plots predicted against realized excess returns for the six EMBI portfolios. Clearly, the model’s predicted excess returns are consistent with the average excess returns. Note that predicted excess returns correspond here simply the OLS estimates of the betas times the sample mean of the factors, not the estimated prices of risk.

Since the factors are returns, the no arbitrage condition implies that risk prices should be equal to the factors’ average excess returns. This condition stems from the fact that the Euler equation applies to the risk factor too, which clearly has a regression coefficient $\beta$ of one on itself. In our estimation, this no-arbitrage condition is satisfied. The average of the risk factor is 652 basis points. So the estimated price of risk is 41 basis points removed from the point estimate. The standard error on the mean estimate is equal to 49 basis points. As a consequence, the mean is not statistically different from the market price of risk, and the no-arbitrage condition is satisfied.

**Alphas and betas in EMBI returns**  The bottom panel of Table 2 reports the constants (denoted $\alpha$) and the slope coefficients (denoted $\beta_{US_{BBB}}$) obtained by running time-series regressions of each portfolio’s excess returns $r_{x,e,j}$ on a constant and the $US_{BBB}$ risk factor.

The first column reports $\alpha$’s estimates. The $\alpha$s for each portfolio are generally small and not significantly different from zero. The null that the $\alpha$s are jointly zero is clearly rejected. The second column reports the $\beta$s for our risk factor. These $\beta$s increase from 0.91 to 1.05 for the low $\beta_{EMBI}$ group, while for the second $\beta_{EMBI}$ group they increase from 0.92 for portfolio 4 to 1.78 for portfolio 6. Betas line up with average excess returns for two reasons: pre-formation betas predict post-formation betas, and bonds with higher default probabilities tend to load more on the risk factor. Comparing portfolios 1 and 4, 2 and 5, and 3 and 6, we note that asset pricing (eg post-formation) betas are always higher in the second group, as they should.

As a robustness check, we run the same asset pricing tests on a different set of returns. We use the EMBI returns sorted on the US stock market betas and credit ratings. We use the same risk factor as before, the US BBB corporate bond return. Table 3 reports the results. Figure 3 plots predicted against realized excess returns for these EMBI portfolios. Results are very similar to the previous ones. The market price of risk is positive and significantly different from zero. It is not
statistically different from the mean of the factor. Pricing errors are small and not significant.

Finally, we study the time-variation in the market price of risk, starting from the conditional Euler equation. Hansen and Richard (1987) shows that a simple conditional factor model can be turned into an unconditional factor model using all the variables $z_t$ in the information set of the investor. The conditional Euler equation for portfolio $j$, $E_t[M_{t+1}R^j_{t+1}] = 1$, is then equivalent to the following unconditional condition:

$$E_t[M_{t+1}z_t R^j_{t+1}] = 1$$

Following Cochrane (2001), we can also interpret this condition as an Euler equation applied to a managed portfolio $z_t R^j_{t+1}$. This managed portfolio corresponds to an investment strategy that goes long portfolio $j$ when $z_t$ is positive and short otherwise. We assume that one scaling variable $z_t$ summarizes all the information set of the investor. Our conditioning variable $z$ is the CBOE volatility index VIX, which is lagged, demeaned and scaled by its standard deviation. We multiply both returns and risk factors by $z_t$. As a result, we obtain twelve test assets: the original six EMBI portfolios, and the same portfolios multiplied by the scaling variable. For the risk factors, we use the US high yield return $US - BBB$ and the same return multiplied by our conditioning variable $US - BBB R_{t+1}$. Table 2 in the separate appendix reports the results. We find that the implied market prices of risk associated with the bond risk factor vary significantly through time. They tend to increase in bad times, when the implied US stock market volatility is high. Time-varying risk-aversion is one way to interpret this finding.

Let us summarize this section. By sorting countries along their Standard and Poor's ratings and bond betas, we have obtained a cross-section of average excess returns which reflects different risk exposures. To study the implications of such risk exposure on debt quantity and prices, we now specify a general equilibrium model of sovereign borrowing that can potentially replicate our previous findings. The main intuition is as follows. When investors are risk averse and the endowment process in the borrowing country is correlated with lenders’ marginal utilities of consumption, the pricing of a sovereign bond depends not only on the probability of default but also on its correlation with the investor stochastic discount factor.

3  A General Equilibrium Model of Sovereign Borrowing

In this section, we build a $N$-country model of sovereign borrowing to interpret the empirical properties of the EMBI portfolios documented in the previous section. We start off the seminal two-country model of Eaton and Gersovitz (1981) and its recent version in Aguiar and Gopinath (2006). But we depart from the previous literature and assume that lenders are risk averse, instead of being risk-
neutral, and that emerging countries’ business cycles differ in their correlations to the US business cycle.

This simple departure has key implications on sovereign bond prices. We know that emerging countries tend to default when they experience difficult economic conditions. Again, if bad times in emerging countries correspond to bad times for the investor, then sovereign bonds appear risky: they pay badly in bad times. A risk-averse investor will expect to be compensated for that risk: he will earn on average a premium on these bonds, or equivalently, these bonds will trade at a lower value than their simple, discounted expected payoffs. If bad times in emerging countries correspond to good times for the investor, then sovereign bonds appear less risky: they pay badly in good times, and well in bad times. If the investor is risk-averse, these bonds trade at a higher value than their simple, discounted expected payoffs.

3.1 Setup

We explore this mechanism and its general equilibrium implications. In the model, there are N-1 small, emerging open economies, and one large developed economy. In each small open economy, there is a representative agent who receives a stochastic endowment stream. In what follows, the superscript B (for 'borrowers') denotes variables corresponding to the N-1 small open economies, the superscript L (for 'lenders') the large developed economy. Upper case variables denote levels, lower case variables denote logs.

Endowments Endowments are composed of a transitory component $z_t$ and a trend $\Gamma_t$ as in Aguiar and Gopinath (2006). The countries’ log endowments evolve as:

$$Y_t^{B,i} = e^{z_t^i} \Gamma_t^i.$$  \hspace{1cm} (3.1)

The transitory components, $z_t^i$ follows an AR(1) around a long run mean $\mu_z$:

$$z_t^i = \mu_z (1 - \alpha_z) + \alpha_z z_{t-1}^i + e_{t}^{z,i}.$$  

The trend is described by:

$$\Gamma_t^i = G_t^i \Gamma_{t-1}^i$$  \hspace{1cm} (3.2)

where:

$$g_t^i = \log(G_t^i) = \mu_g (1 - \alpha_g) + \alpha_g g_{t-1}^i + e_{t}^{g,i}.$$  

Note that a positive shock $e^{g,i}$ implies a permanent higher level of output. We assume that $e^{g,i}, e^{z,i}$ are i.i.d normal and that shocks to the transitory and trend components are orthogonal.
\( E(e^{g,i}^t e^z,i^t) = 0 \). All emerging countries have the same endowment persistence and volatility: 
\( E([e^z,i^t]^2) = \sigma^2_{e^z} \) and \( E([e^{g,i}]^2) = \sigma^2_{e^g} \).

In the large developed economy, there is a representative agent that receives every period an exogenous consumption endowment. We assume that idiosyncratic shocks to consumption growth are \( i.i.d. \) log-normally distributed:

\[ \Delta c^t_i = \tau^i_t + \epsilon^i_t. \]

The emerging countries only differ according to their conditional correlation to the developed economy: \( E(e^{d,i}^t e^L_1) = \rho^{d,i} \) and \( E(e^{z,i}^t e^L_1) = \rho^{z,i} \). This is the key source of heterogeneity across countries in our model. In a separate appendix, we report some evidence that such heterogeneity exists in the data. Correlation coefficients between foreign and US HP-filtered GDP range in our sample from -0.3 to 0.6 on annual data, and from -0.3 to 0.5 on quarterly data. Our model predicts that, everything else equal, countries that are positively correlated with the US, should pay higher spreads to US investors. In our empirical section, we show that countries with high EMBI market betas exhibit higher spreads, once we control for S&P ratings. The intuition for this finding is that market betas offer high frequency measures of the links between emerging countries and the US.

**Debt contracts** All variables in the model are real, and we abstract from monetary policies. In each emerging economy, a benevolent government maximizes the welfare of its representative citizen. To do so, the government can borrow resources from the developed country. The government, however, can only trade non-contingent one-period zero-coupon bonds. These debt contracts are not enforceable: the government can choose to default on its debts at any point in time. In this set-up, if investors are risk neutral, the price of a sovereign bond depends exclusively on the endogenous probability of default, which varies with the amount of funds borrowed and the expected next-period endowment. But if investors are risk-averse, then sovereign bond prices reflect the correlation between the emerging economy’s business cycle and the US economy.

### 3.2 Borrowers

We start with the description of the borrowers. The representative agent in each small open economy maximizes the stream of discounted utilities \( U^B_t \):

\[ U^B = E_0 \sum_{t=0}^{\infty} (\beta^B)^t U^B_t = E_0 \sum_{t=0}^{\infty} (\beta^B)^t \frac{(C^B_t)^{1-\gamma}}{1-\gamma}, \]
where $\beta^B$ denotes the time discount factor, and $C_t^B$ denotes consumption at time $t$. We let the lenders’ and borrowers’ discount factors ($\beta^B$ and $\beta^L$) differ because developing countries tend to have higher real risk free rates than emerging countries.

The representative household receives a stochastic stream of the tradable good $Y_t^B$ every period. We assume that $y_t^B$, the log of the borrower’s endowment, follows a Markov process. The representative agent also receives a goods transfer from the government in a lump-sum fashion: i.e., any proceeds from international operations are rebated lump-sum from the government to its citizens. The government has access to international capital markets: at the beginning of period $t$, it can purchase $B_t^t+1$ one-period zero-coupon bonds at price $Q_t$. $B_t^t+1$ denotes the quantity of one-period zero-coupon bonds purchased at date $t$ and coming to maturity at date $t+1$. A positive value for $B_t^t+1$ represents a saving for the borrowing country, which supplies $Q_tB_t^t+1$ units of period $t$ goods in order to receive $B_t^t+1 > 0$ units of goods in the following period. On the contrary, a negative value $B_t^t+1 < 0$ implies borrowing $Q_tB_t^t+1$ units of goods at $t$ and promising to repay, conditional on not defaulting, $B_t^t+1$ units of $t+1$ good. The representative household’s budget constraint conditional on not defaulting at time $t$ is then:

$$C_t^B = Y_t^B - Q_tB_t^t+1 + B_t^t-1.$$  \hspace{1cm} (3.3)

In case of default, all current debt disappears. This simplifying assumption implies that the sovereign cannot selectively default on parts of its debt.\footnote{Political economists argue that politicians tend to have shorter time horizons in small developing countries. In Amador (2003) for example, a low value for the discount factor $\beta^B$ corresponds to the high short-term discount rate of an incumbent party with low probability of remaining in power in a model where different parties alternate.} A sovereign that defaults at date $t$ is excluded from international capital markets for a stochastic number of periods and suffers a direct output loss. In this case, consumption is constrained by the value of output during autarky, which is denoted $Y_t^{B,\text{def}}$, and the budget constraint is simply:

$$C_t^B = Y_t^{B,\text{def}}.$$  \hspace{1cm} (3.4)

Following Arellano (2008), we assume an asymmetric direct output cost of default. More precisely, we assume that $Y_t^{B,\text{def}} = \min\{Y_t^B, \bar{Y}^B\}$, where $\bar{Y}^B$ is the output upper bound in case of a default and it is defined as $(1 - \theta) \text{mean}(Y^B)$. This form of direct output cost implies that defaults are more costly in good times. A country that receives a high value of $Y^B$ expects high values of the endowment also in the near future, given the high persistence of the endowment process. If the country defaults when $Y^B$ is high, its consumption is set to be low for the entire time of exclusion.

\footnote{Bolton and Jeanne (2008) and Duffie, Pedersen, and Singleton (2003) propose models where the sovereign can selectively default on part of the outstanding debt.}
from capital markets according to the budget constraint (3.4). When the endowment is high, the utility cost of default (which lasts several periods) is likely to outweigh the utility benefit from not repaying the outstanding debt (which lasts one period). As a result, the country has less incentives to default. In general equilibrium, lenders take that into account, and sovereign countries can borrow more in good times. Take now the opposite case. Consider a country that receives a particularly low value of the endowment. This country would like to borrow to smooth out consumption. Given the high persistence of the endowment process, this country also expects low values of the endowment in the near future. If the endowment is low enough and the country defaults, the direct output cost is likely to be low for the entire exclusion period (because $Y^B < \hat{Y}^B$). At the same time, when the endowment is low, the marginal utility cost of a net capital outflow is very high for a risk averse borrower. Investors anticipate that the borrower is likely to default in this case and they require a high premium to supply any funds. In equilibrium, when $Y^B$ is low enough, there is no borrowing and the sovereign is credit-rationed.

Therefore, this assumption on output cost affects both the size and the timing of debt in equilibrium. It is a convenient way to ensure that countries borrow more when output is above trend, a robust feature of emerging economies’ business cycles (see for example Neumeyer and Perri (2005), Aguiar and Gopinath (2007) or Uribe and Yue (2006)). It also implies that countries tend to default when output is below trend, as they do (Tomz (2007)). Note, however, that it is empirically difficult to determine whether the fall in output is the reason for defaulting, or rather the consequence of the default.

A second consequence of a country’s default is exclusion from international capital markets. In Eaton and Gersovitz (1981) exclusion is permanent, and default is not an equilibrium outcome. We follow Arellano (2008) and Aguiar and Gopinath (2006) and assume that exclusion lasts a stochastic number of periods. Although this assumption implies a degree of coordination by foreign investors that is partially at odds with the assumption that investors behave competitively, it captures the fact that countries in default do not access international capital markets for some time. As Hatchondo, Martinez, and Sapriza (2007) note, in this framework, the equilibrium size of debt is smaller when the exclusion from capital markets is shorter. This is because exclusion works an incentive to repay, thus reassuring lenders, decreasing the risk premium and allowing more borrowing.

### 3.3 Lenders

We now turn to the description of the lenders. The representative agent receives an exogenous stochastic consumption endowment every period denoted $C^L_t$. Lenders are risk-averse and behave competitively. In order to reproduce the large spread between low and high beta countries, we rely on habit preferences similar to Campbell and Cochrane (1999). We assume that lenders maximize
the stream of discounted utilities $U^L_t$:

$$U^L = E_t \sum_{t=0}^{\infty} (\beta^L)^t U^L_t = E_t \sum_{t=0}^{\infty} (\beta^L)^t \frac{(C^L_t - H_t^L)^{1-\gamma} - 1}{1-\gamma},$$

where $\beta^L$ denotes the lenders' discount factor and $H_t$ the external habit level. The external habit level corresponds to a time-varying subsistence level or social externality.

**Why not power utility?** We show in the appendix that a model where borrowers and lenders have the same power utility preferences does not produce a large spread in excess returns. The maximum spread between high and low correlation groups in the latter case is only 55 basis points, an order of magnitude smaller than in the data. This result parallels the equity premium puzzle in Mehra and Prescott (1985). To illustrate this point, assume that two countries have the same default probability and the same yield volatility. Then the spread between their bond returns depend on the covariance between the US marginal utility of consumption and the return differences. As a result, the maximum spread between these two countries is twice the product of the risk-aversion coefficient times the standard deviation of consumption growth (around 1.5 percent) and the standard deviation of the returns (around 13 percent). A risk-aversion coefficient of 2 would imply a maximum spread of around 80 basis points. A risk-aversion coefficient close to 13 would then lead to a spread of 5 percent as in the data, but it would also imply a high and volatile risk free rate. On the contrary, the introduction of habit preferences implies that lenders' risk aversion is time-varying, and higher in 'bad times'. As consumption declines toward the habit in 'bad times', the curvature of the utility function rises, so risky assets prices fall and expected returns rise. Local risk-aversion is sometimes very high, even if the risk-aversion coefficient remains low and the real interest rate in line with the data.

Following Campbell and Cochrane (1999), we assume that the external habit level depends on the consumption endowment through the following autoregressive process for the surplus consumption ratio, defined as the percentage gap between the endowment and habit ($S^{L}_t \equiv [C^L_t - H^L_t] / C^L_t$):

$$s^{L}_{t+1} = (1- \phi) s^L_t + \phi s^L_t + \lambda(s^L_t)(\Delta c^L_{t+1} - g^L),$$

where $g^L$ is the average consumption growth. The sensitivity function $\lambda(s^L_t)$ describes how habits are formed from past aggregate endowments. In this framework, 'bad times' refers to times
of low surplus consumption ratios (when consumption is close to the habit level), and 'negative shocks' refers to negative consumption growth shocks $\varepsilon^t$. The sensitivity function $\lambda(s^L_t)$ governs the dynamic of the surplus consumption ratio:

$$\lambda(s^L_t) = \begin{cases} \frac{1}{\bar{s}} \sqrt{1 - 2(s^L_t - \bar{s})^2} - 1 & \text{if } s^L_t \leq s^L_{\text{max}} \\ 0 & \text{elsewhere,} \end{cases}$$

where $\bar{s}$ and $s^L_{\text{max}}$ are respectively the steady-state and upper bound of the surplus-consumption ratio. $\bar{s}$ measures the steady-state gap (in percentage) between consumption and habit levels. Note that the non-linearity of the surplus consumption ratio keeps habits always below consumption and marginal utilities always positive and finite. Assuming that $\bar{s} = \sigma e^{\gamma} \sqrt{1 - \phi}$ and $s^L_{\text{max}} = \bar{s} + (1 - \bar{s})/2$, the sensitivity function leads to a constant risk free rate: $r_t = \bar{r} = -\ln(\beta^t) + \gamma g^t - \frac{\gamma^2 \sigma^2}{2 \bar{s}^2}$. This model delivers time-varying risk aversion for the lenders. Since the habit level depends on aggregate consumption, the local curvature of the lenders’ utility function is $\gamma_t = \gamma/S^L_t$. When the endowment is close to the habit level, the surplus consumption ratio is low and the lender very risk averse.

Lenders supply any quantity of funds demanded by the small open economy, but they require compensation for the risk they bear. Lenders cannot default. When lenders are risk-neutral, they charge the borrower the interest rate that makes them break-even in expected value. In our model, lenders are risk-averse, and require not only a default premium, but also a default risk premium. They expect a higher return on average if defaults are more likely in bad times for them, i.e when their endowment is close to the habit level.

### 3.4 Recursive equilibrium

In order to describe the economy at time $t$, we need to keep track of the borrower’s endowment stream, his outstanding debt, and the lender’s past surplus consumption ratios. Let $y^B$ and $s^L$ denote the history of events up to $t$: $y^B = (y^B_0, ..., y^B_t)$ and $s^L = (s^L_0, ..., s^L_t)$. We denote $x$ a column vector that summarizes this information: $x = [y^B, s^L]'$. Given that the two stochastic endowment processes are Markovian, we denote $f(x', x)$ the conditional density of $x'$, e.g. the value of $x$ at time $t+1$ given the initial value of $x$ at time $t$. In what follows, the value of a variable in period $t+1$ is denoted with a prime superscript.

Given the initial state of the economy, the value of the default option is:

$$v^o(B, x) = \max\{v^e(B, B', x), v^d(x)\},$$
where $v^c(B, B', x)$ denotes the contract continuation value, $v^d$ the value of defaulting and $v^o$ the value of function of being in good credit standing at the start of the period. If the government chooses to repay the debt coming to maturity, it can purchase some new debt. As a result, the value of staying in the contract is a function of the exogenous states $y^B$ and $s^L$, the quantity of debt coming to maturity at time $B$ and future debt $B'$. In case of default, all outstanding debt is erased, and the small economy is forced into autarky for a stochastic number of periods. Hence, the only state variables that influence the value $v^d$ of defaulting are $y^B$ and $s^L$. We now define more precisely $v^c$ and $v^d$.

The value of default depends on the probability of re-accessing financial markets in the future and on the current output loss:

$$v^d(x) = u_B(y^{def}) + \beta \int_{x'} \left[ \lambda v^o(0, x') + (1 - \lambda) v^d(x') \right] f(x', x) dx',$$

where $\lambda$ is the exogenous probability of re-entering international capital markets after a default. As we have seen, when a borrower defaults, consumption is equal to the autarky value of output. In the following period, the borrower regains access to international capital markets with no outstanding debt with probability $\lambda$, or remains in autarky with probability $1 - \lambda$.

The value of staying in the contract and repaying debt coming to maturity is:

$$v^c(B, x) = \max_{B'} \{ u(c) + \beta \int_x v^o(B', x') f(x', x) dx' \},$$

subject to the budget constraint (5.3). The borrower chooses $B'$ to maximize utility and anticipates that the equilibrium bond price depends on the exogenous states variable and on the new debt $B'$. Figure 3 plots the difference between the value of staying in the contract $v^c(B, x)$ and the value of defaulting $v^d(x)$ as a function of the log trend growth $g$, for an initial debt level equal to 15 percent of the current endowment and when the investors’ surplus consumption ratio is equal to $s^L$. When $g$ is low, the borrower prefers to default. The intersection between the red line $(v^c(B, x) - v^d(x))$ and the blue line (0) determines a threshold such that the borrower will default (repay) whenever $g$ is smaller (bigger) than it.

Let $\mathcal{T}$ denotes the set of possible values for the exogenous states $x$. For each value of $B$, the small open economy default policy is the set $D(B)$ of exogenous states such that the value of default is larger than the value of staying in the contract:

$$D(B) = \{ x \in \mathcal{T} : v^d(x) > v^c(B, x) \}.$$

---

---

\footnote{Kovrijnykh and Szentes (2007) explore the possibility of endogenizing $\lambda$.}
Similarly, \( R(B) \) is the set of exogenous states such that the value of default is smaller than the value of staying in the contract. The repayment set \( R(B) \) is the complement to \( D(B) \):

\[
R(B) = \{ x \in \mathcal{T} : v^d(x) \leq v^c(B, x) \}.
\]

The default probability \( dp \) is endogenous and depends on the amount of outstanding debt and on the endowment realization. In particular, the default probability is related to the default set through:

\[
 dp(B', x) = \int_{D(B')} f(x', x) dx',
\]

where \( dp(B', x) \) denotes the expectation at time \( t \) of a default at time \( t + 1 \) for a given level \( B' \) of outstanding debt due at time \( t + 1 \). Figure 4 plots the default policy set \( D(B) \) as a function of the beginning of period asset position \( B \) and the trend growth \( g \), when the investors’ surplus consumption ratio is equal to \( \mathcal{Z}^{-} \). The grey shaded area denotes combinations of \( B \) and \( g \) such that the borrower optimally choose to default. Countries tend to default for larger debt levels and when the endowment is low.

### 3.5 Bond Prices

Bond prices \( Q(B', x) \) are a function of the current state vector \( x \) and the desired level of borrowing \( B' \). If borrowers do not default at date \( t + 1 \), lenders receive payoffs equal to the face value of the bonds, which is normalized to 1. In case of default at date \( t + 1 \), payoffs are zero. Starting from the investor’s Euler equation, the bond price function is:

\[
Q(B', x) = E[M'1_{1-dp(B', x)}] = E[M']E[1_{1-dp(B', x)}] + \text{cov}[M', 1_{1-dp(B', x)}],
\]

where \( M' \) is the investors’ stochastic discount factor and is equal to:

\[
M' = \beta^L \frac{U_{ct}(C', H')}{U_{ct}(C, H)} = \beta^L \left( \frac{SI'}{S^I} \right)^{-\gamma} = \beta^L e^{-\gamma[\phi+(\phi-1)(\xi'-\overline{\xi}^f)+(1+\lambda)(\xi')\lambda]}.
\]

A risk free asset pays one unit of consumption good in any state of the world and has a price equal to \( Q^{rf} = E[M'] \). If investors are risk-neutral, sovereign bond prices depend only on expected default probabilities: \( Q(B', x) = E[1_{1-dp(B', x)}] \times Q^{rf} \). Investors’ risk aversion introduce a new component to sovereign bond pricing. For a given default probability, bond prices depend on the covariance between investors’ stochastic discount factors and default events. If defaults tend to occur in bad times for investors (e.g. when their marginal utility of consumption is high), the covariance term in (3.5) is negative, bond prices are low and yields are high. Likewise, if defaults tend to occur in good times for investors, yields are low. Figure 5 plots the bond price (solid line) in (3.5) as a function of
the borrowing choice \( B' \) for the lowest (red) and highest (blue) values of the log trend growth \( g \) in our grid. The dashed line in the figure represents the bond price function for a model with risk neutral investors. The figure shows two important implications of our model. First, for a given borrowing choice, bond prices are lower in bad times for the borrower. The blue line is always above the red line, but when the two lines are both at 0 or at the risk-free level. Second, bonds issued by countries that tend to default more frequently when the investors’ marginal utility is high are riskier and have lower prices. This effect is captured by a model with risk averse investors. In the figure, we plot the bond price functions of a country with a business cycle that is positively related to the investors’ consumption growth (in the figure the correlation coefficient \( \rho_g \) is equal to 0.5). Bond prices of the model with risk averse investors (solid line) are always below or equal bond prices of the model with risk neutral investors (dashed line) and never above it.

4 Simulation

We simulate the model at quarterly frequency. We start by rapidly reviewing its parameters.

4.1 Calibration

We calibrate the borrower’s endowment process described in (3.1) using the parameters in Aguiar and Gopinath (2006). These parameters describe Argentina. In order to focus on permanent shocks to trend growth, we shut down the transitory components of the endowment by setting \( \sigma_{e,\iota} = 0 \). We calibrate lenders’ consumption growth using the post-war U.S. economy as a reference. Habit preference parameters are from Campbell and Cochrane (1999). Table 3 reports all the parameters used in the simulation.

The direct output cost of default \( \theta \) is equal to 2 percent per period in line with the evidence of a significant output drop in the aftermath of a default (see, for example, Rose (2005)). The probability of re-entering capital markets after a default \( \lambda \) is equal to 15 percent per period, implying an average exclusion of 6.6 quarters. The empirical evidence on the time-length of exclusion is mixed. For example, Gelos, Sahay, and Sandleris (2004) find that in the 1980s the average time of exclusion is 4.7 years, while only 0.3 years in the 1990s.\(^{10}\)

The risk aversion parameter \( \gamma \) in the borrowers’ and lenders’ utility function is set equal to 2. Lenders discount future at the annualized rate \( \beta^L = 0.89 \), while the borrower has a lower time

\(^{10}\) Argentina defaulted in 2001 and then restructured three quarters of the $95 billion defaulted debt in a 2005 swap. But in September 2008, Argentina still faced legal actions by investors holding out for full repayment. Argentina could not issue new debt on international capital markets for fear it would be embargoed (Financial Times, 25 September, 2008).
discount factor $\beta^B = 0.40$. The value of $\gamma$ and $\beta^L$ are calibrated in order to match an average US real log risk-free rate of 0.94 percent per annum. Models of this class require low values for $\beta^B$ in order to generate larger values for the debt to GDP ratio. We use the same number as in Aguiar and Gopinath (2006). A low value for $\beta^B$ matches the usually high real interest rates in emerging markets. The computational algorithm is described in the appendix.

4.2 Building Portfolios of Simulated Data

In equilibrium, investors know expected default probabilities and require higher risk premia from borrowers that are more likely to default when investors’ consumption is close to their habit levels. We solve our model for a set of 15 uniformly spaced different values of $\rho^i$, which is the correlation between investors’ consumption growth and borrower’s endowments. These correlation coefficients vary from $-0.5$ to $0.5$. Each $\rho^i$ corresponds to a different sovereign borrower. We simulate time series data for countries that differ only with respect to $\rho^i$ and face the same time series for investors’ consumption growth. The values for all the other parameters are those in Table 3.

We use the simulated data to build portfolios that mimic the EMBI portfolios described in section 1. What are the equivalents to the Standard and Poor’s ratings and EMBI bond betas that we used in section 1 on actual data? In the model, expected default probabilities exist in closed form. We do not need to rely on ratings to proxy them. We denote $E[dp^i]$ the investors’ expectation that country $i$ will default next period. In the model, we also have a more direct measure of the business cycle’s correlation with the US economy than the bond betas we previously computed. Here, we obtain $\beta_{SIM}^i$ as the slope coefficient from a regression of the borrower $i$’s past output growth up to time $t$ on a constant and the investor’s past endowment growth up to time $t$. We use a rolling window of 250 periods.

The building portfolio strategy runs again in two steps. First, at the end of each period $t$, we sort all countries in the sample into 2 groups on the basis of the observed $\beta_{SIM}^i$ at that time. The first group contains countries with the lowest $\beta_{SIM}^i$, the second group contains countries with the highest $\beta_{SIM}^i$. Second, at the end of each period $t$, we sort all countries within each of the previous 2 groups into 3 portfolios on the basis of the expected default probability $E[dp^i]$ at that time. Within each group, the first portfolio contains countries with the smallest expected default probabilities and the last portfolio contains countries with the highest default probabilities. The 6 portfolios are re-balanced at the end of every period. For each portfolio $j$, we compute the excess returns $r_{t+1}^{ej}$ by taking the average of the excess returns in the portfolio. Excess returns correspond to the returns in emerging countries minus the risk-free rate in the large, developed economy. We have a total of 34 simulated countries, for 5,000 quarters. We compute $\beta_{SIM}^i$ starting in quarter 500 and use the last 600 quarters for our analysis (150 years). Countries in default in a given
quarter are excluded from the sample, given that they do not have access to international capital markets. As a result, the total number of countries in our portfolios varies slightly over time. Table 4 provides an overview of the 6 portfolios.

For each portfolio \( j \), we report the average value for \( \beta_{SIM} \), the excess return \( r_e^j \), the expected default probability \( E[d Pf] \) and the debt to output ratio. All the moments are annualized: we multiply the mean of the quarterly data by 4 and the standard deviation by 2. The Sharpe ratio is the ratio of the annualized mean to the annualized standard deviation.

The first panel reports the average \( \beta_{SIM} \) for countries in portfolio \( j \). There is a stark contrast between the first three and the last three portfolios. The business cycle of countries with a low \( \beta_{SIM} \) is negatively correlated with the investors’ endowment growth. These countries on average default more frequently when investors’ consumption is high and above their habit levels. On the contrary, countries with a high \( \beta_{SIM} \) default more frequently when investors’ consumption is low and close to their habit levels. The second panel reports average expected default probabilities. Within each \( \beta_{SIM}^{low,high} \)-group, there is a cross-section of average default probabilities, with a spread up to 0.5 percent. These first two panels correspond to the sorting variables.

Let us turn now to average excess returns. Countries with higher default probabilities offer higher returns. This is the first order effect, with a difference of around 25 basis points between portfolios with low default probabilities (1 and 4) and portfolios with high default probabilities (3 and 6). Countries with larger values of \( \beta_{SIM} \) pay higher returns. This is true at all levels of default probabilities. This is the second order effect. The difference in excess returns between low and high beta countries is particularly striking for countries with high default probabilities. It amounts to 15 basis points annually. This spread is significant. It is not due to higher levels of debt, as the last panel shows. It is actually the opposite: high beta countries pay higher interest rates even if they borrow less in equilibrium. These features echo the characteristics of our EMBI bond portfolios. Comparing these spreads to their actual counterparts, we note, however, that both default probability and beta spreads are much larger in the data than in the model.

5 Conclusion

In this paper, we show that sovereign bond betas govern sovereign bond spreads. In the data, countries with higher bond betas pay higher borrowing rates. The difference in spreads between countries with high and low betas is large. Models of optimal borrowing and endogenous defaults with risk neutral investors cannot account for our empirical findings. We study one example of

\[111\] Here again, we use Patton and Timmermann (2008)’s MR non parametric test. It rejects at any conventional significance levels the null of the absence of a monotonic relationship between portfolio ranks and expected returns against the alternative of an increasing pattern.
a general equilibrium model of sovereign borrowing and defaults with risk-averse investors. In the model, borrowing countries only differ along one dimension: their endowments are more or less correlated to the lenders’ consumption. Lenders’ habit preferences lead to spreads in returns between low and high default probability countries, and between high and low beta countries.
References


Table 1: EMBI Portfolios Sorted on Credit Ratings and Bond Market Betas (Equal Weights)

<table>
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<th>3</th>
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</tr>
<tr>
<td>Mean</td>
<td>3.01</td>
<td>5.62</td>
<td>6.54</td>
<td>7.03</td>
<td>10.15</td>
<td>13.50</td>
</tr>
<tr>
<td>Std</td>
<td>10.39</td>
<td>11.54</td>
<td>16.63</td>
<td>9.10</td>
<td>12.84</td>
<td>19.26</td>
</tr>
<tr>
<td>Excess Return: $r_{ej}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.26</td>
<td>0.32</td>
<td>0.50</td>
<td>0.21</td>
<td>0.39</td>
<td>0.66</td>
</tr>
<tr>
<td>Std</td>
<td>0.10</td>
<td>0.09</td>
<td>0.15</td>
<td>0.07</td>
<td>0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>EMBI Stock Market Beta: $\beta_{EMBI}'$ (Post-Formation)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports, for each portfolio $j$, the average beta $\beta_{EMBI}$ from a regression of EMBI returns on the total returns on the Merrill Lynch US BBB corporate bond index, the average EMBI log total excess return, the average Standard and Poor’s credit rating, post-formation betas and the average external debt to GNP ratio. Post-formation betas correspond to slope coefficients in regressions of monthly EMBI returns on monthly US MSCI stock market returns. Excess returns are annualized and reported in percentage points. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. Panel I reports equally-weighted statistics. Panel II reports value-weighted statistics. The portfolios are constructed by sorting EMBI countries on two dimensions: every month countries are sorted on their probability of default, measured by the S&P credit rating, and on $\beta_{EMBI}$. Note that Standard and Poor’s uses letter grades to describe a country’s credit worthiness. We index Standard and Poor’s letter grade classification with numbers going from 1 to 23. Data are monthly, from JP Morgan and Standard and Poor’s (Datastream). The sample period is 1/1995 - 5/2009.
Table 2: Asset Pricing: Portfolios Sorted on Credit Ratings and Bond Market Betas

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{US-BBB}$</th>
<th>$b_{US-BBB}$</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GMM_1$</td>
<td>6.93</td>
<td>1.54</td>
<td>77.83</td>
<td>1.58</td>
<td>22.02</td>
</tr>
<tr>
<td></td>
<td>[4.63]</td>
<td>[1.03]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$GMM_2$</td>
<td>6.49</td>
<td>1.45</td>
<td>75.50</td>
<td>1.66</td>
<td>22.19</td>
</tr>
<tr>
<td></td>
<td>[2.92]</td>
<td>[0.65]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$FMB$</td>
<td>6.93</td>
<td>1.53</td>
<td>73.00</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.63]</td>
<td>[0.58]</td>
<td></td>
<td></td>
<td>43.79</td>
</tr>
<tr>
<td></td>
<td>(2.71)</td>
<td>(0.60)</td>
<td></td>
<td></td>
<td>49.89</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><strong>6.52</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>[0.49]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\alpha_0$ (%)</th>
<th>$\beta_{US-BBB}$</th>
<th>$R^2$ (%)</th>
<th>$\chi^2(\alpha)$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.93</td>
<td>0.91</td>
<td>28.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.42]</td>
<td>[0.12]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.16</td>
<td>0.88</td>
<td>22.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.89]</td>
<td>[0.12]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.33</td>
<td>1.05</td>
<td>15.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.04]</td>
<td>[0.24]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>0.92</td>
<td>38.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.21]</td>
<td>[0.11]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.81</td>
<td>1.28</td>
<td>37.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.63]</td>
<td>[0.16]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.86</td>
<td>1.78</td>
<td>32.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.06]</td>
<td>[0.35]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| All       | 6.27            | 0.39             |           |                  |           |

Notes: Panel I reports results from GMM and Fama-McBeth asset pricing procedures. Market prices of risk $\lambda$, the adjusted $R^2$, the square-root of mean-squared errors RMSE and the p-values of $\chi^2$ tests on pricing errors are reported in percentage points. $b$ denotes the vector of factor loadings. All excess EMBI returns are multiplied by 12 (annualized). The standard errors in brackets are Newey and West (1987) standard errors with the optimal number of lags according to Andrews (1991). Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. Panel II reports OLS estimates of the factor betas. $R^2$s and p-values are reported in percentage points. The $\chi^2$ test statistic $\alpha_0V_\alpha^{-1}\alpha$ tests the null that all intercepts are jointly zero. This statistic is constructed from the Newey-West variance-covariance matrix (1 lag) for the system of equations (see Cochrane (2001), page 234). Data are monthly, from JP Morgan in Datastream. The sample period is 1/1995-5/2009. The alphas are annualized and in percentage points.
Table 3: Parameters Choices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean lenders’ consumption growth (%)</td>
<td>$g^l$</td>
<td>1.89</td>
</tr>
<tr>
<td>Standard deviation of lenders’ consumption growth (%)</td>
<td>$\sigma_{\epsilon_l}$</td>
<td>1.50</td>
</tr>
<tr>
<td>Persistence of the lenders’ surplus consumption ratio</td>
<td>$\phi$</td>
<td>.87</td>
</tr>
<tr>
<td>Persistence of borrowers’ endowment</td>
<td>$\alpha_g$</td>
<td>.17</td>
</tr>
<tr>
<td>Standard deviation of borrowers’ endowments (%)</td>
<td>$\sigma_{\phi}$</td>
<td>3</td>
</tr>
<tr>
<td>Mean trend growth rate (%)</td>
<td>$\mu_g$</td>
<td>2.5</td>
</tr>
<tr>
<td>Direct default cost (%)</td>
<td>$\theta$</td>
<td>2</td>
</tr>
<tr>
<td>Probability of re-entry(%)</td>
<td>$\lambda$</td>
<td>15</td>
</tr>
<tr>
<td>Risk-aversion parameter</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Lenders’ time discount</td>
<td>$\beta^l$</td>
<td>.89</td>
</tr>
<tr>
<td>Borrowers’ time discount</td>
<td>$\beta^B$</td>
<td>.40</td>
</tr>
</tbody>
</table>

The table reports benchmark values for the parameters used in the simulation. These parameters imply an annualized risk-free rate $r^*$ in the large developed country equal to .94 percent per annum, a steady-state endowment ratio $S$ equal to 5.7 percent and a maximum surplus endowment ratio $S_{\text{max}}$ of 9.4 percent. The values for the direct output cost and the probability of re-entering financial markets after a default are per quarter. All the other parameters are annualized, e.g., they are reported as $4g^l$, $2\sigma_{\epsilon_l}$, $2\sigma_{\phi}$, $\phi^l$, $\beta^l$, $\beta^B$ and $4r^*$ since the model is simulated at quarterly frequency. Values describing lenders’ consumption growth are from Campbell and Cochrane (1999) and correspond to post-war US consumption data. Values describing the borrowers’ endowments are from Aguiar and Gopinath (2006).
Table 4: Portfolios of Simulated Data

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{SIM}^j$</td>
<td>Low</td>
<td></td>
<td></td>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[dp^j]$</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumption beta: $\beta_{SIM}^j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std 0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Default probability: $E[dp^j]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Excess Return: $r^{e,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std</td>
</tr>
<tr>
<td>SR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Debt/GNP: $d^j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std</td>
</tr>
</tbody>
</table>

Notes: This table reports, for each portfolio $j$, the slope coefficient $\beta_{SIM}^j$ from a regression of borrowers’ output growth on the investors’ consumption growth, the average excess return, the average expected probability of default and the debt to output ratio. Excess returns are annualized and reported in percentage points. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. Data comes from simulating our model under the assumption of habit preferences for foreign lenders. The portfolios are constructed by sorting data for different countries obtained by simulating our model in two dimensions: every month, countries are sorted on expected default probabilities and on $\beta_{SIM}^j$. The sample has 600 quarters.
Figure 1: EMBI Global Annual Spreads and Standard and Poor’s Ratings

The figure plots, for each country in the EMBI Global Index, the annual stripped spread against the Standard and Poor’s credit rating at the end of May 2008. Spreads are in basis points. Standard and Poor’s credit ratings are indexed from 1 (AAA) to 23 (SD). A higher number implies a lower credit worthiness. Data are from Datastream.
Figure 2: Predicted versus Realized Average Excess Returns

The figure plots realized average EMBI excess returns on the vertical axis against predicted average excess returns on the horizontal axis. We regress actual excess returns on a constant and the return on the US - BBB bond index to obtain slope coefficient $\beta_i$. Each predicted excess return is obtained using the OLS estimate $\beta_i$ times the sample mean of the factor. All returns are annualized. Data is monthly. The sample period is 1/1995-5/2009.
Figure 3: The Value of Repaying vs the Value of Defaulting

This figure plots the difference between the value of staying in the contract \( v^c(B, x) \) and the value of defaulting \( v^d(x) \) as a function of the trend growth \( g \), when the investors’ surplus consumption ratio is equal to \( \beta \) and the initial asset position is equal to \( B = -0.15 \). In order to focus on trend shocks, we set \( \sigma_x = 0 \) in the endowment process. The correlation coefficient between shocks to trend growth and shocks to investors’ consumption growth \( \rho_g \) is equal to zero.
Figure 4: Default Policy Set

This figure plots the default policy set $D(B)$ as a function of the beginning of period asset position and trend shock $g$ when the investors’ surplus consumption ratio is equal to $\mathbb{E}$. In order to focus on trend shocks, we set $\sigma_z = 0$ in the endowment process. The correlation coefficient between shocks to trend growth and shocks to investors’ consumption growth $\rho_g$ is equal to zero.
Figure 5: Bond Price Function

This figure plots the bond price $Q(B', x)$ of a model with risk averse investors (solid line) and the bond price $Q_n(B', x)$ of a model with risk neutral investors (dashed line) as a function of the borrowing choice ($B'$) for the lowest and highest values of the log trend growth in our grid. In the figure, the investors' surplus consumption ratio is equal to $\frac{e}{e}$ and we set $\sigma_z = 0$ in the endowment process. The correlation coefficient between shocks to trend growth and shocks to investors' consumption growth $\rho_g$ is equal to 0.5.