Technological Mismatch: A Model of Internatioal Trade in Goods and Ideas*

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Abstract

I build a model of trade and technology diffusion. The novelty of the model is to separate technology into two different pieces: the technology itself, and the implementation of this technology by a worker. Technological mismatch occurs when an idea is developed in a country but implemented in another country. In contrast with existing models of trade and technology diffusion where diffusion is typically a substitute for trade, I predict that diffusion may enhance trade. Moreover, opening up an international market for ideas increases long term growth, as it reduces the likelihood that ideas are mismatched.

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1 Introduction

> "A month after giving my first conferences on giant magnetoresistance, I received the visit of a delegation from IBM."
>
> Albert Fert, 2007 recipient of the Nobel Prize in Physics.

Many ideas are developed in one country and implemented in another. I develop a simple model to capture this key feature of the development and implementation of ideas: even if ideas can relatively easily travel across countries, the ability to implement these ideas is mostly embedded in actual people. But since the person best able to implement an idea may not live in the country where the idea originated, this does not mean that ideas are locked in separate countries. It means on the contrary that we should expect complex patterns of production and exchange of ideas across countries. I derive two novel predictions from this model of technological mismatch. First, technology transfers and trade flows can be complement. Second, an international market for ideas increases long term growth over and beyond what traditional endogenous growth models would predict.

One prediction of most trade models is that diffusion is a substitute for trade: if one can use the technology of one's trading partners, then there is less need, if any, for trade. The only motive for trade that remains is differences in factor prices, factor endowments or Krugman (1980) type increasing returns to scale. We do however find empirically a strong positive correlation between technology transfers and trade flows between countries. In this paper, I separate out technology and workers' ability to use a given technology. It is possible that one country develops a technology that is better suited to the ability of workers in another country. In such a case, technology transfers may increase trade flows, not lower them. More importantly, in such a world where technologies and abilities are mismatched, trade may enhance the welfare gains from technology diffusion.

Extending the model to a dynamic setting, the presence of technological mismatch reduces the productivity of R&D and lowers long term growth. Allowing technology diffusion in the form of an international market for ideas increases the likelihood that a new idea will be useful, it increases the expected gains from doing R&D, and increases long term growth. I uncover a novel force driving R&D and growth in a world with many countries. Sharing ideas across countries not only increases the total stock of knowledge researchers can build upon, it also allows to waste fewer ideas. In addition, if researchers curtail their creativity to target specific sectors, opening up the flow of ideas across country will allow researchers to unleash their creativity, and enhance growth even further.

The remainder of the paper is organized as follows. In section 2, I develop a simple static model of trade and technology diffusion with technological mismatch. In section 3, I consider the dynamic impact of trade, technology diffusion and technological mismatch on long term growth. In section 4, I allow researcher to target specific sectors, and derive the implications for trade patterns and long term growth of introducing an international market for ideas. Section 5 concludes.

2 A model of trade and technological mismatch

I first develop a model of international trade and technology diffusion in the absence of a market for ideas, neither within countries, or between countries. In order to get the simplest predictions possible, I assume perfect competition in all markets. In the next section only will I depart from perfect competition, and introduce a market for ideas.

2.1 Set-up

There are two countries, home (H) and foreign (F), and one unique factor of production, labor. The cost of one unit of labor in country $i = H, F$ is w_i. The home wage w_H is used as the numeraire, though I will keep w_H for clarity. There is a continuum of workers of mass L_i in country i.

There is continuum of sectors indexed by $\omega \in [0, 1]$. Consumers in each country maximize CES preferences,

$$U = \left(\int_0^1 q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\sigma \geq 1$ is the elasticity of substitution between any two goods.

Mismatch: Each country is able to produce only a fraction $(1-\mu) \in [0, 1]$ of all sectors, where μ stands for mismatch. The distribution of accessible sectors is independent across countries. For each of the $(1-\mu)$ accessible sectors, each country draws its labor productivity $z(\omega)$ from some continuous distribution. In the remaining μ sectors, $z(\omega) = 0$ so that production is not profitable.[1] So μ measures the degree of mismatch between technologies developed locally, and the ability of local workers to use these technologies. When $\mu = 0$, there is a perfect match between technologies and workers' abilities to use those technologies. When $\mu = 1$ on the other hand, there is a perfect mismatch, workers cannot use any technology.

It is costly to ship goods between countries. I assume "iceberg" trade barriers: it takes τ_{ij} units to deliver 1 unit from country i to country j. The marginal cost of good ω in country j is the minimum of the domestic and the foreign cost (inclusive of transportation cost),

$$c_j(\omega) = \min_{i=H,F} \left\{ \frac{\tau_{ij} w_i}{z_i(\omega)} \right\}$$

Assuming perfect competition in each sector, the marginal cost $c_j(\omega)$ will also be the price $p_j(\omega)$ of good ω in country j.

I follow Eaton and Kortum (1999 and 2002) and adopt a probabilistic representation of technologies. For each accessible sector ω, labor productivity $z_i(\omega)$ in country $i = H, F$ is drawn over $\mathbb{R}^+$ from a Fréchet distribution F_i,

$$F_i(z) = \Pr[z_i(\omega) \leq z] = \exp\left(-T_i z^{-\theta}\right) \tag{2}$$

where the scale parameter $T_i > 0$ governs the location of productivities in country i, and the shape parameter $\theta > 0$ governs the dispersion of productivities (same in both countries).

2.2 Trade with no diffusion

In the absence of technology diffusion, firms can only use ideas developed locally. There are four types of sectors. A fraction μ^2 of sectors are inaccessible to either countries ($\omega \in \Omega_N$). A fraction $\mu(1-\mu)$ of sectors are only accessible to country i firms $(\omega \in \Omega_i)_{i=H,F}$. In the remaining $(1-\mu)^2$ sectors ($\omega \in \Omega_{HF}$), both home and foreign firms compete, and in each sector, buyers go to the cheapest supplier.

In country j, prices in the three accessible sectors (Ω_H accessible to home firms only, Ω_F accessible to foreign firms only, and Ω_{HF} accessible to both home and foreign firms) are distributed according to the distributions G's,

$$\begin{cases} G_i^j(p) = \Pr[p_j(\omega) \leq p \mid \omega \in \Omega_i] = 1 - \exp\left(-\left[T_i(\tau_{ij}w_i)^{-\theta}\right]p^{\theta}\right), \; i = H, F \\ G_{HF}^j(p) = \Pr[p_j(\omega) \leq p \mid \omega \in \Omega_{HF}] = 1 - \exp\left(-\left[\sum_{i=H,F} T_i(\tau_{ij}w_i)^{-\theta}\right]p^{\theta}\right) \end{cases} \tag{3}$$

[1] The model extends easily to the case where for each sector, the labor productivity is $a(\omega) z(\omega)$ with a drawn from a uniform distribution.

Unlike in Eaton and Kortum (2002), the distribution of prices is not the same in every sectors, since not every sectors are accessible in each country. The trade volumes will no longer be simply proportional to the fraction of goods exported. To determine the trade volumes, I need to compute the general price index, and for that the price indices for the different subsets of sectors. Given the distribution of prices of different goods, the ideal CES price index in the country j, P_j, is given by,

$$P_j = \left(\lambda \left[\mu (1-\mu) \sum_{i=H,F} \left(T_i (\tau_{ij} w_i)^{-\theta} \right)^{\frac{\sigma-1}{\theta}} + (1-\mu)^2 \left(\sum_{i=H,F} T_i (\tau_{ij} w_i)^{-\theta} \right)^{\frac{\sigma-1}{\theta}} \right] \right)^{1/(1-\sigma)}$$

with λ a constant.[2]

Goods in the $\mu(1-\mu)$ sectors only accessible to home firms ($\omega \in \Omega_H$) are all exported from the home country to the foreign country. The $\mu(1-\mu)$ goods only accessible to foreign firms ($\omega \in \Omega_F$) are all exported from the foreign country to the home country. Of the $(1-\mu)^2$ goods accessible to both home and foreign firms ($\omega \in \Omega_{HF}$), a fraction $\frac{T_i(\tau_{ij} w_i)^{-\theta}}{\sum_{k=H,F} T_k (\tau_{kj} w_k)^{-\theta}}$ are exported from i to j, and the remaining $\frac{T_j(\tau_{jj} w_j)^{-\theta}}{\sum_{k=H,F} T_k (\tau_{kj} w_k)^{-\theta}}$ sold in j are produced locally.

The volume of exports from country i to country j in units of the numeraire is given by $X_{ij} = \sum_{\omega \in \Omega_{ij}} p(\omega) q(\omega)$. From the CES demand structure, $p(\omega) q(\omega) = (p(\omega)/P_j)^{1-\sigma} X_j$, where X_j is the total expenditure by j's consumers. In equilibrium, labor being the only factor of production, and perfect competition driving down profits to zero, we get $X_j = w_j L_j$. So total exports from i to j is given by,

$$X_{ij} = \frac{\int_{\omega \in \Omega_{ij}} p(\omega)^{1-\sigma} d\omega}{\int_{\omega \in \Omega_j} p(\omega)^{1-\sigma} d\omega} w_j L_j$$

Note that $\frac{X_{ij}}{w_j L_j}$ is also a direct measure of trade openness of country j, the ratio of imports to domestic production. In the symmetric and costly trade case ($w_H = w_F = 1$, $T_H = T_F = T$, $L_i = L_j = L$, $\tau_{ij} = \tau_{ji} = \tau$ for $i \neq j$, and $\tau_{ii} = 1$ for $i = H, F$), we get,

$$\frac{X_{ij}}{X_j} = 1 - \frac{\mu + (1-\mu)\left(1+\tau^{-\theta}\right)^{\frac{\sigma-1}{\theta}}}{\mu\left(1+\tau^{-(\sigma-1)}\right) + (1-\mu)\left(1+\tau^{-\theta}\right)^{\frac{\sigma-1}{\theta}}}$$

[2] See Eaton and Kortum (2002), p. 1749, footnote 18, on how to use the moment generating function for a Fréchet distribution to derive this result. The constant λ is given by,

$$\lambda = \Gamma\left(\frac{\theta - (\sigma - 1)}{\theta}\right)$$

where $\Gamma(\cdot)$ is the complete gamma function.

Note that in this symmetric case, $0 < \frac{X_{ij}}{X_j} < \frac{1}{2}$, with $\frac{X_{ij}}{X_j} = \frac{1}{2}$ for $\tau = 1$ and $\frac{X_{ij}}{X_j} = 0$ for $\tau \to +\infty$. Also note that $\frac{X_{ij}}{X_j}$ increases with the level of mismatch μ. If μ is large, not only do countries have access to a very small share of technologies, but the share of sectors accessible to both countries, $(1-\mu)^2$, is also small compared to the set of technologies accessible to only one country, $\mu(1-\mu)$. Countries have more of a comparative advantages, and there is more motive for trade.

In this model, two parameters govern the degree of comparative advantages. As in Eaton and Kortum (2002), θ governs the dispersion of productivity across sectors, and therefore the degree of comparative advantages in the sectors where home and foreign firms compete ($\omega \in \Omega_{HF}$). In addition, μ governs the relative size of sectors where countries have an absolute advantage ($\omega \in \Omega_i$ for country i) versus the size of sectors where they only have a comparative advantage ($\omega \in \Omega_{HF}$), and so the degree of comparative advantages across different types of sectors.

In order to be able to compare trade volumes under different scenarios, nominal variables need to be normalized. I normalize country j's nominal variables by the ideal price index in country j, P_j, so that all quantities are denominated in units of country j's utils. In the symmetric and costly trade case ($w_H = w_F = 1$, $T_H = T_F = T$, $L_i = L_j = L$, $\tau_{ij} = \tau_{ji} = \tau$ for $i \neq j$, and $\tau_{ii} = 1$ for $i = H, F$), aggregate exports from i to j, $\frac{X_{ij}}{P_j}$, are given by,

$$\frac{X_{ij}}{P_j} = \frac{T^{\frac{1}{\theta}} (1-\mu)^{\frac{1}{\sigma-1}} \left[\mu\tau^{-(\sigma-1)} + (1-\mu)\frac{\tau^{-\theta}}{1+\tau^{-\theta}}\left(1+\tau^{-\theta}\right)^{\frac{\sigma-1}{\theta}}\right]}{\left[\Gamma\left(\frac{\theta-(\sigma-1)}{\theta}\right)\right]^{\frac{1}{1-\sigma}} \left[\mu\left(1+\tau^{-(\sigma-1)}\right) + (1-\mu)\left(1+\tau^{-\theta}\right)^{\frac{\sigma-1}{\theta}}\right]^{\frac{\sigma-2}{\sigma-1}}} L$$

Aggregate trade increases with technological progress, T, as it allows to produce more of all goods. Aggregate trade decreases with technological mismatch, μ. This is due to the fact that technological mismatch reduces worldwide production more than it increases comparative advantages and trade openness.

How does technological diffusion affect the patterns of trade flows between countries? In the next section, I allow ideas to freely flow across national borders, and describe the impact on trade patterns, trade flows, and welfare.

2.3 Trade with diffusion

Once diffusion of technology is allowed between the two countries, firms in all sectors can now take the best of the home and the foreign productivity draw. In all sectors, both at home and abroad, technologies are drawn from the same distribution $\tilde{F}$,

$$\tilde{F}(z) = \Pr[z_i(\omega) \leq z] = \exp\left(-T_W z^{-\theta}\right), \text{ with } T_W = T_H + T_F \tag{4}$$

In those sectors accessible to both countries ($\omega \in \Omega_{HF}$), trade disappears entirely. There is no point importing from abroad at a cost what can be done domestically with the same efficiency. This is the traditional sense in which technology diffusion is a substitute for trade in goods in Ricardian models.[3]

However, in the sectors that are only accessible to domestic firms ($\omega \in \Omega_H$), domestic producers now have access to a better technology, since they can use foreign technologies, some of which are better than the domestic ones. The same happens for sectors that are only accessible to foreign firms ($\omega \in \Omega_F$). Trade is going to increase in those sectors. Technology diffusion allows for a better allocation of technologies and abilities worldwide. The patterns of production and trade in those sectors (Ω_H and Ω_F)are the same as before technology diffusion, but with a better technology.

In country j, prices in the three accessible sectors are now drawn from the distributions $\tilde{G}$'s,

$$\begin{cases} \tilde{G}_i^j(p) = \Pr[p_j(\omega) \leq p \mid \omega \in \Omega_i] = 1 - \exp\left(-\left[T_W(\tau_{ij}w_i)^{-\theta}\right]p^\theta\right), \ i = H, F \\ \tilde{G}_{HF}^j(p) = \Pr[p_j(\omega) \leq p \mid \omega \in \Omega_{HF}] = 1 - \exp\left(-T_W(\tau_{jj}w_j)^{-\theta}p^\theta\right) \end{cases} \tag{5}$$

The ideal price index in $j = H, F$ after diffusion becomes,

$$\tilde{P}_j = \left(\lambda\left[\mu(1-\mu)\sum_{i=H,F}\left(T_W(\tau_{ij}w_i)^{-\theta}\right)^{\frac{\sigma-1}{\theta}} + (1-\mu)^2\left(T_W(\tau_{jj}wj)^{-\theta}\right)^{\frac{\sigma-1}{\theta}}\right]\right)^{1/(1-\sigma)}$$

In the symmetric and costly trade case ($w_H = w_F = 1$, $T_H = T_F = T$, $L_i = L_j = L$, $\tau_{ij} = \tau_{ji} = \tau$ for $i \neq j$, and $\tau_{ii} = 1$ for $i = H, F$), aggregate exports from i to j become,

$$\frac{\tilde{X}_{ij}}{\tilde{P}_j} = \frac{\mu(2T)^{\frac{1}{\theta}}(1-\mu)^{\frac{1}{\sigma-1}}\tau^{-(\sigma-1)}}{\left[\Gamma\left(\frac{\theta-(\sigma-1)}{\theta}\right)\right]^{\frac{1}{1-\sigma}}\left[1+\mu\tau^{-(\sigma-1)}\right]^{\frac{\sigma-2}{\sigma-1}}}L$$

How do aggregate trade flows and welfare change when technology diffusion is allowed?

Two forces are present in the model. In the sectors where both home and foreign firms compete ($\omega \in \Omega_{HF}$), the traditional Ricardian force shuts down trade entirely. In a purely Ricardian world,

[3] This is true provided that the difference in wages between the two countries is smaller than trade barriers. Otherwise, one country will be the sole producer of goods in Ω_{HF}. This would occur if one country becomes much smaller than the other, in which case it would eventually specialize entirely in producing goods in its exclusive domain, and its wage would increase above the foreign wage. I assume that the condition, $\frac{1}{\tau_{HF}} < \frac{w_H}{w_F} < \tau_{HF}$, always holds.

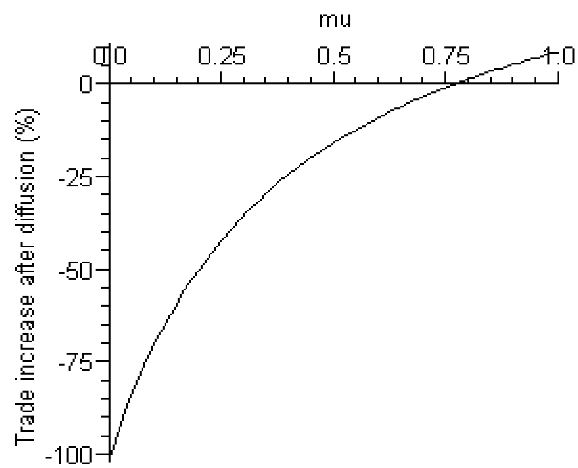

Figure 1: Percentage change in trade volumes after technology diffusion occurs ($100 \times \left[\frac{X_{ij}/P_j\ |_{\text{with diffusion}}}{X_{ij}/P_j\ |_{\text{without diffusion}}} - 1\right]$ as a function of the level of mismatch μ; $(\tau, \sigma, \theta) = (1.5, 6, 8)$).

and in the presence of even arbitrarily small trade barriers, trade in goods disappears when ideas can freely flow across borders. This will be the case here in sectors where home and foreign firms compete ($\omega \in \Omega_{HF}$). In the other sectors though ($\omega \in \Omega_{i=H,F}$), trade increases. Countries now have access to a better technology (the best of the home and the foreign technology), total output increases, and trade increases in those sectors. How much aggregate trade increases depends on the relative size of those two types of sectors (indexed by the measure of mismatch μ), and the elasticity of substitution across goods which governs the relative size of sectors. Figure 1 plots the change in aggregate trade volumes (in percentage points) brought about by technology diffusion as a function of the degree of technological mismatch, μ. In the absence of technological mismatch ($\mu = 0$), I get the traditional Ricardian prediction that diffusion substitutes for trade. As mismatch increases, there is less and less substitution between trade and technology diffusion. For a high enough level of technological mismatch, trade and technology diffusion actually become complements, and trade increases with technology diffusion.

The following proposition states this result formally.

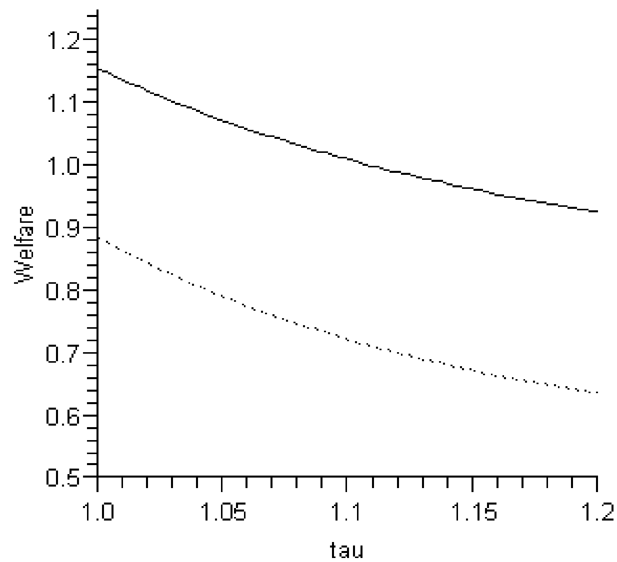

Figure 2: Welfare, diffusion and trade barriers (the dashed line represents welfare without diffusion, the solid line welfare with diffusion; $(\mu, \sigma, \theta) = \left(\frac{1}{2}, 6, 8\right)$).

Proposition 1 *There exists a degree of technological mismatch above which aggregate trade increases when technologies are allowed to diffuse across countries.*

Proof. *See appendix* ■

Both trade and technology diffusion have a positive impact on welfare. This can be seen on figure 2 which plots welfare as a function of the level of trade barriers, both with and without technology diffusion. First note that welfare always decreases with the level of trade barriers, with or without technology diffusion. In the presence of some mismatch, trade does not disappear altogether even after technology diffuses. Impediments to trade always decrease welfare, even in the presence of technology diffusion. Second, technology diffusion always increases welfare, since it improves the technology of production for all firms. For every level of trade barrier (on the x-axis), sharing technologies increases welfare (the solid line is always above the dashed line).

Depending on the degree of mismatch, trade and technology diffusion have a different impact on welfare. Welfare (W) depends on the level of technology (T), on trade barriers (τ) and on the degree of technological mismatch (μ). The following equation (6) describes normalized welfare with and without diffusion, in the case of symmetric and costly trade,

$$\begin{cases} \left(\frac{W_{\text{without diffusion}}}{(1-\mu)^{1/(\sigma-1)}T^{1/\theta}}\right)^{\sigma-1} = (1-\mu)\times\left(1+\tau^{-\theta}\right)^{\frac{\sigma-1}{\theta}} + \mu\times\left(1+\tau^{-(\sigma-1)}\right) \\ \left(\frac{W_{\text{with diffusion}}}{(1-\mu)^{1/(\sigma-1)}T^{1/\theta}}\right)^{\sigma-1} = \underbrace{(1-\mu)\times 2^{\frac{\sigma-1}{\theta}}}_{\Omega_{HF}\text{ sectors}} + \underbrace{\mu\times 2^{\frac{\sigma-1}{\theta}}\left(1+\tau^{-(\sigma-1)}\right)}_{\Omega_H\text{ and }\Omega_F\text{ sectors}} \end{cases} \tag{6}$$

In sectors where both home and foreign firms compete (Ω_{HF}), consumers can buy each good from the cheapest supplier. In the absence of technology diffusion, consumers have to buy foreign goods at a premium to cover the transportation cost. When technologies diffuse between countries, this premium disappears. Welfare increases simply because less resources are wasted as transportation costs ($1+\tau^{-\theta}<2$). In those sectors, the higher transportation costs, the more welfare increases from technology diffusion.

In the sectors only accessible to home or foreign firms (Ω_H and Ω_F), welfare gains only come from the fact that all firms have access to a better technology ($2^{\frac{\sigma-1}{\theta}}>1$). They can use the best of two technology draws instead of a single one. But the patterns of trade are unaffected. This means that goods only accessible to foreign firms still have to be imported from abroad, and transportation costs always have to be paid. In those sectors, the lower transportation costs, the more welfare increases from technology diffusion.

Whether welfare gains from technology diffusion increase or decrease with trade barriers depends on the degree of technological mismatch, which determines the relative size of sectors accessible to home and foreign firms (Ω_{HF}) and sectors accessible only to home or foreign firms (Ω_H and Ω_F). Figure 3 describes the impact of trade and technology diffusion on welfare for different levels of technological mismatch. In the absence of any technological mismatch, the classical Ricardian case, the higher trade barriers, the more welfare increases when technology diffuses. In the absence of trade barriers, trade is a perfect substitute for technology diffusion, so that technology diffusion has no impact at all on welfare. Trade is a perfect substitute for technology diffusion. As trade barriers get larger, the welfare gains from technology get larger, since technology diffusion allows to save trade barriers. The solid line in figure 3 which represents welfare gains from technology diffusion is increasing in the level of trade barriers (τ). In that sense, trade and technology diffusion are substitute.

When technological mismatch is high on the other hand, trade is still needed to reap the benefits of technology diffusion. Foreign firms in those sectors that are only accessible to foreign firms (Ω_F) have access to a better technology. But those goods still have to be imported. So the

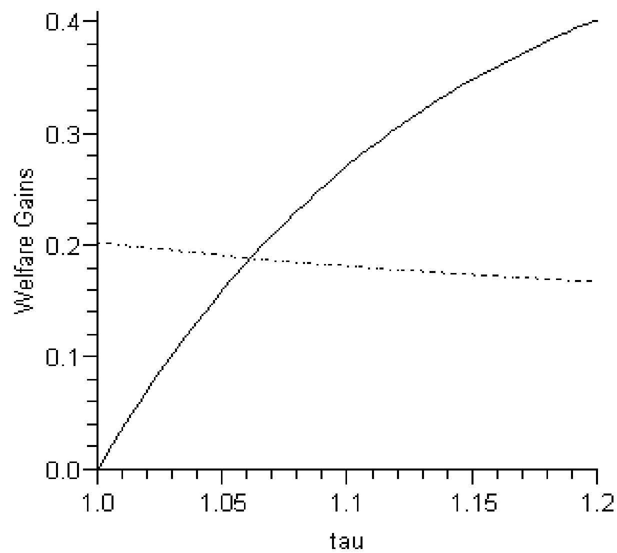

Figure 3: Welfare gains from technology diffusion (the solid line represents welfare gains with no mismatch, $\mu = 0$, the solid line welfare gains when mismatch is high, $\mu = \frac{3}{4}$; $(\sigma, \theta) = (6, 8)$).

lower trade barriers, the more consumers gain from the better technology of foreign firms. The dotted line in figure 3 which represents welfare gains from technology diffusion is decreasing in the level of trade barriers (τ). In that sense, trade and technology diffusion are complement.

The following proposition states this result formally.

Proposition 2 *When the degree of technological mismatch is low, technology diffusion induces a larger welfare gain the larger trade barriers are (technology diffusion and trade are* ***substitute****); when the degree of technological mismatch is large, technology diffusion induces a larger welfare gain the lower trade barriers are (technology diffusion and trade are* ***complement****).*

Proof. *See appendix* ■

In this section, I have established that in the presence of a high enough level of technological mismatch, technology diffusion and trade are complements. This means both that technology diffusion increases trade, and that the welfare gains from technology diffusion are higher the more open to trade countries are. In the next section, I consider the dynamic impact of technological mismatch on innovation and growth. The main finding is that even though trade in goods does not have any long term impact on growth, technology diffusion, or trade in ideas increases long term growth.

3 Technology diffusion as an incentive for research

In this section, I derive a simple dynamic model of endogenous innovation in the presence of technological mismatch based on Eaton and Kortum (1999). The main prediction is that introducing an international market for ideas makes the R&D activity more productive, and increases long term growth. This positive impact on long term growth is larger the larger the degree of technological mismatch. I consider first a case without an international market for ideas, where ideas are used only domestically, and then introduce such a market, where ideas can be sold across national borders. Finally, I develop a simple extension where researchers are allowed to target specific research.

3.1 Endogenous innovation without diffusion

I follow Eaton and Kortum (1999) to derive a simple model of endogenous innovation. In each country, agents can choose between being researchers or being workers. Researchers generate new ideas. Under imperfect (Bertrand) competition, researchers can earn profits if they discover an idea that is better than existing ones: they charge a mark-up over marginal cost as long as their idea is the best. I will first describe the R&D technology, then the expected earnings of a researcher, and finally the equilibrium on the labor market and the steady state equilibrium.

R&D technology: Researchers randomly discover new ideas, and the quality of those new ideas itself is random. The arrival of new ideas follows a Poisson process with parameter normalized to unity. In order to generate endogenous growth, I assume that researchers "build on the shoulders of giants", so that the more ideas have been discovered, the more new ideas are generated. If the total number of ideas discovered until time t in country i is T_{it}, then each researcher discovers T_{it} new ideas. This implies the following law of motion for the stock of ideas (T_{it}) in country i, when a share r_{it} of country i's population do research,[4]

$$\dot{T}_{it} = T_{it} \times r_{it} L_i \tag{7}$$

[4] Note that there is a slight abuse of notations. Formally, $\dot{T}_{it}$ is the Poisson rate at which new ideas are discovered over the time interval $[t; t + dt]$, rather than the exact number of new ideas discovered.

Each new idea corresponds to a technology that may be applied to one single sector $\omega \in [0,1]$, where this sector is randomly selected from a uniform distribution. The quality of a new idea corresponds to the technology for producing good ω. It is drawn from a Pareto distribution F with shape parameter θ,[5]

$$F(q) = \Pr[Q \leq q] = 1 - q^{-\theta}, \text{ for } q \geq 1 \tag{8}$$

The parameter θ is an inverse measure of the dispersion of qualities. When θ is large, most quality draws are close to 1, whereas as θ gets small, quality draws are more dispersed, meaning that it becomes more and more likely to get large quality draws.

Using results from the extreme value distribution theory, Eaton and Kortum (1999) show that if at time t a mass T_{it} of ideas have been discovered, each of them drawn from the Pareto distribution in Eq. (8), then for T_{it} large, the distribution of the best idea is approximately Fréchet,

$$\Pr[Q_{\max}(T_{it}) \leq z] \approx \exp\left(-T_{it} z^{-\theta}\right) \tag{9}$$

where $Q_{\max}(T_{it})$ stands for the highest quality among all the T_{it} draws. This provides a micro-foundation for the distribution of the best productivity assumed in Eq. (2) in the static model of the first section.

The value of an idea: Once a researcher discovers a new idea, she is the sole owner of this idea. She can claim any mark-up that her idea allows her to charge over marginal cost. Imperfect competition allows leaders to extract profits. I assume Bertrand competition. Within any sector ω, all goods are perfect substitutes. Consumers only buy from the cheapest supplier. If a researcher has the best idea so far, she charges a price just above to the second lowest marginal cost. By doing so, she captures all the market in sector ω. In addition, when goods are substitutable across sectors ($\sigma > 1$), the leader will never charge a mark-up above the Dixit-Stiglitz mark-up $\left(\bar{m} = \frac{\sigma}{\sigma-1}\right)$ in order not to lose too much market shares against firms in other sectors.

Eaton and Kortum (1999) show the following important result. If quality is drawn from a Pareto distribution, as in Eq. (8), and if the highest quality is drawn from a Fréchet distribution, as in Eq. (9), then the distribution of mark-ups is invariant.[6] The distribution of mark-ups (M)

[5] I impose the restriction $\theta > \sigma - 1$ to insure that the distribution of firms' sales are not degenerate.

[6] Bernard, Eaton, Jensen and Kortum (2003) extent the result of Eaton and Kortum (1999) from the simple Cobb-Douglas case to the more general case where goods are substitutable (CES preferences with elasticity of susbtitution $\sigma \geq 1$).

at any point in time, in any sector, in any country, is given by,

$$H(m) = \Pr[M \leq m] = \begin{cases} 1 - m^{-\theta} & \text{if } m \leq \bar{m} = \frac{\sigma}{\sigma-1} \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

This implies that, conditional on a new technology being better than existing ones, the expected profits as a share of sales is constant and simply equal to,

$$\pi = \int_1^{\bar{m}} \left(1 - m^{-1}\right) dG(m) = \frac{1 - \bar{m}^{-\theta-1}}{1+\theta} \tag{11}$$

It is invariant through time, across sectors and across countries. This invariance result greatly simplifies the derivation of the expected value of an idea.

A researcher can expect to earn profits from her idea only if her idea is "usable", and only as long as she is the leader in her sector. In the absence of an international market for ideas, a fraction μ of ideas fall into mismatched sectors and cannot be used at all. So any new idea is "usable" with probability $(1-\mu)$ only. At time t, with Pareto distributed quality draws, and Fréchet distributed best technologies, the probability that a researcher's idea is better than existing ideas is arbitrarily close to $1/T_{it}$.[7]

The total expected sales of a firm in country i at time t is equal to the total revenue in country i, X_{it}.[8] A fraction π of those sales goes to the researcher as profits. In the absence of any technology diffusion and any royalties payment across borders, total revenue, X_{it}, is made of the labor bill, plus profits earned by researchers,

$$X_{it} = w_{it}(1 - r_{it}) L_i + \pi X_{it} = \frac{w_{it}(1 - r_{it})}{1 - \pi} L_i \tag{12}$$

where r_{it} is the fraction of i's population working in R&D, and π is the fraction of sales that go to researchers as profit.

The expected value of a new idea in country i at time t, denominated in units of time t's numerarie, is the expected discounted sum of profits from this idea,

$$V_{it} = \int_t^{+\infty} \frac{P_{it}}{P_{is}} e^{-\rho(s-t)} \times (1-\mu)\pi X_{is} \times ds \tag{13}$$

[7] See Eaton and Kortum (1999) for a derivation of this result.

[8] See the appendix to see how this simple expression for total expected profits of i's researchers is derived using trade balance.

where ρ is the discount rate, and the ratio of prices indices, P_{it}/P_{is}, translates the value of future units of the numeraire into time t's value of the numeraire.[9] Since a researcher discovers T_{it} new ideas per unit of time if the stock of knowledge is equal to T_{it}, the expected profits from being a researcher at time t is simply $T_{it}V_{it}$. An interior equilibrium on the labor market requires that agents are indifferent between being researchers and being workers,

$$w_{it} = T_{it}V_{it} \tag{14}$$

If all agents work in the R&D sector, no good is produced and sold (see Eq. (12) with $r_{it} = 0$), and researchers do not earn any profit. As agents switch from being researchers to being workers, total sales increase, so that profits increase, until they reach a point where agents are indifferent between working and doing research.[10]

Proposition 3 (Steady state equilibrium) *There exists a steady state equilibrium in which the stock of knowledge in each country grows at the same constant growth rate g_T, wages and the share of the population in R&D are constant in both countries. In the symmetric and costly trade case, the growth rate, g_T, and the share of the population working in R&D, r, are given by,*[11]

$$\begin{cases} g_T = \frac{L(1-\mu)\frac{\pi}{1-\pi}-\rho}{\frac{\theta-1}{\theta}-(1-\mu)\frac{\pi}{1-\pi}} \\ r = g_T/L \end{cases}$$

Proof. *See appendix.* ■

As in traditional endogenous growth models, the more patient agents are (low ρ), the larger the population (large L), and the higher mark-ups (low σ and/or low θ imply a large π), the higher the share of the population working in R&D and the higher the growth rate. When agents are patient, they value future consumption, and are more willing to work in R&D, where earnings happen over time, than as workers, where earnings are instantaneous, so that the R&D sector is large. As in traditional endogenous growth models, there is a strong size effect so that larger countries generate more ideas and grow more. Finally, when mark-ups are large, R&D is profitable, and the R&D sector is large.

[9] With growth, productivity improves over time, so that prices decrease, and the value of one unit of the numeraire increases over time.

[10] Note that if agents are impatient enough (ρ large), and if the current stock of knowledge is large enough (T_{it} large), it is possible that no one works in the R&D sector, and all agents are workers. I will only consider cases where the solution is interior and some agents are researchers.

[11] As is obvious from this formula, a necessary condition for an interior solution with positive growth rate is $L(1-\mu)\frac{\pi}{1-\pi} > \rho$, which ensures that there are at least some researchers. $r < 1$ is always satisfied.

The existence of technological mismatch introduces a novel force driving long term growth: the lower the level of mismatch, the higher the growth rate. Researchers bear the risk of the R&D activity. When the level of technological mismatch is high, a large fraction of news ideas developed by researchers cannot be implemented. This induces a low return for the R&D activity, and shrinks the size of the R&D sector, ultimately reducing long term growth.

In such a model with no tradable accumulable factor, international trade has no impact on long term growth. International trade does have a direct impact on the level of output and welfare as seen in the previous section, but none on long term growth. This is because international trade benefits workers as much as it benefits researchers, so that it does not affect the trade-off between working and researching.[12]

I show in the next section that opening up the flow of ideas between countries, or opening up an international market for ideas, increases the efficiency of R&D and improves long term growth.

3.2 Endogenous innovation with diffusion

In this section, I introduce an international market for ideas, so that ideas developed in one country can be used in the other country. In order to prevent a mechanical scale effect (building on the shoulders of twice as big a giant), I will arbitrarily assume that researchers still only benefit from the ideas developped in their country, and not on the worldwide stock of knowledge. Allowing researchers to directly benefit from ideas developped abroad would mechanically accelerate the steady state growth, but it would not change qualitatively any of the results I derive. As in the previous section, the stock of knowledge evolves according to Eq. (7).

The solution for the steady state equilibrium is similar to the case without an international market for ideas. I will focus here mostly on the new forces that come with this market for ideas.

A new idea has a probability μ^2 of falling into a mismatched sector, where it is useless both at home and abroad. So only a fraction $\left(1-\mu^2\right)$ of new ideas are potentially useful. Note that because ideas can be used in both countries, and because the mismatched sectors in the two countries are orthogonal, there are fewer mismatched ideas than in the absence of an international market for ideas.

Conditional on being useful, a new idea has a probability $1/\left(T_{Ht}+T_{Ft}\right)$ of being better than existing ideas at time t. The total expected royalties payment from a successful idea in time t

[12] Note that this property is a consequence of the fact that international trade does not affect expected mark-ups through goods market competition.

is $\pi\left(X_{Ht}+X_{Ft}\right)$.[13] In the presence of an international market for ideas, one ought to be careful about royalties payments across national borders. Country i's income comes on the one hand from sales to i's consumers, net of royalties paid to foreigners, and on the other hand from royalties earned from i's technologies that are used abroad. Total income, X_{it}, depends on total output, Y_{it}, which in turn depends on total labor revenue in the following way,

$$\begin{cases} X_{it} = \left(1-\pi\lambda_{jt}\right)Y_{it} + \pi\lambda_{it}Y_{jt} \\ Y_{it} = w_i\left(1-r_{it}\right)L_{it} + \pi Y_{it} = \frac{w_i(1-r_{it})}{1-\pi}L_{it} \end{cases} \tag{15}$$

where π is the fraction of sales that goes to profits, and λ_{it} is the fraction of technologies at time t that has been developed by researchers from country i.[14]

As in the previous section, I can now define the value of an idea at time t,

$$V_{it} = \int_t^{+\infty} \frac{P_{it}}{P_{is}} e^{-\rho(s-t)} \times \frac{1-\mu^2}{T_{Hs}+T_{Fs}} \times \pi\left(X_{Hs}+X_{Fs}\right)ds \tag{16}$$

An interior equilibrium on the labor market is such that in both country, agents are indifferent between being workers or researchers,

$$w_{it} = T_{it}V_{it},\ i = H, F \tag{17}$$

As in the case without a market for ideas, I rule out parameters configurations such that no one does R&D.

Proposition 4 *With an international market for ideas, there exists a steady state equilibrium in which the world stock of knowledge grows at a constant growth rate g_T, wages are constant and equalized across countries, and the share of the population in R&D is constant in both countries. In the symmetric and costly trade case, the growth rate, g_T, and the share of the population working in R&D, r, are given by,*

$$\begin{cases} g_T = \frac{L\left(1-\mu^2\right)\frac{\pi}{1-\pi}-\rho}{\frac{\theta-1}{\theta}-\left(1-\mu^2\right)\frac{\pi}{1-\pi}} \\ r = g_T/L \end{cases}$$

Proof. *See appendix.* ■

[13] See the appendix to see how this simple expression for total expected profits is derived using current account balance.

[14] Because all new ideas are equally good in expectation, this fraction is the share of ideas coming from i among all ideas,

$$\frac{\int_{-\infty}^{t}\left(T_{Hs}+T_{Hs}\right)r_{is}L_{is}ds}{\int_{-\infty}^{t}\left(T_{Hs}+T_{Hs}\right)\left(r_{Hs}L_{Hs}+r_{Fs}L_{Fs}\right)ds}$$

which in the steady state is simply equal to $r_iL_i/\left(r_HL_H+r_FL_F\right)$.

Long term growth increases when an international market for ideas is opened up. This growth effect of globalization is not due to a mechanical scale effect which I rule out by assumption. It is derived from an entirely novel force stemming from the technological mismatch. Once ideas can be sold abroad, the probability that an idea falls. This implies that the R&D activity is more efficient on average. This attracts workers into the R&D sector, and increases long term growth. This can be seen formally from $\left(1-\mu^2\right) > (1-\mu)$, the probability of a new idea being mismatched is lower, and hence the growth rate g_T is higher.

In this section, I have shown that opening up an international market for ideas increases long term growth over and beyond what traditional models of endogenous growth would predict. In the next section, I develop an simple extension of this model with endogenous innovation where I allow researchers to target specific sectors.

Note 1: in a world with constant returns to scale to production, and in a symmetric world, international trade in goods does not have any positive impact on long term growth. It has a positive and possibly large positive impact on the level of production and welfare, but not on technological progress. In this very specific model, opening up an international market for ideas, since it has a positive impact on the productivity in the research sector (through a form of increasing returns to scale in the research sector) does have a positive impact on long term growth.

Note 2: Trade in goods is not even needed at all to reap the benefits from trading ideas. Ideas sold abroad do not have to be paid in goods. It can be paid in exchange for foreign imported ideas that are used domestically. This is potentially an interesting point (maybe the empirical magnitude is small though), especially in an asymmetric case, where the relative price of ideas would have to be pinned down.

4 A model of targeted R&D

I now assume that a researcher can tailor her research effort in order to target ideas that will be useful in some specific sectors. This assumption is meant to capture the trade-off between unleashing the creativity of researchers and targeting useful innovations. In plain words, in the absence of an international market for ideas, one may have wanted to force Albert Fert to come up with inventions that would be useful for the French wine industry, at the cost of him not being a very productive researcher. In a world where ideas cannot cross borders, this may have been a profitable decision. In a world where there is more than the specific strengths of the French industry, this becomes a questionable strategy.

Within each period, a researcher has to decide how to allocate her time between looking for a sector to target, and discovering new ideas for the sector she eventually picks up. There is a trade-off between spending more time looking for an "accessible" sector in order to reduce the likelihood of mismatch, and the efficiency of the research effort. At every point in time, a researcher decides to spend a fraction τ of time targetting sectors, and the remaining $(1-\tau)$ researching, with $\tau \in [0,1]$.

The technology for targetting "accessible" sectors is as follows. When a researcher spends a fraction τ of time targetting sectors, the probability that she finds an "accessible" sector is,

$$\Pr\left[\omega \in \Omega^{H} \mid \tau\right] = (1-\mu^{\tau}) \tag{18}$$

The rationale for this functional form can be described as follows: if a researcher doubles the fraction of time she spends looking for a sector, she gets twice as many chances of finding an "accessible" sector where her innovation will be useful.

The technology for researching is as follows. When a researcher spends a fraction of time $(1-\tau)$ researching, the stochastic rate at which she discovers new ideas is $(1-\tau)$. In other words, the more time she spends researching, the more likely it is that she discovers a new idea. As before, conditional on discovering a new idea, the quality of this idea is drawn from the Pareto distribution $F(\cdot)$.[15]

Definition 5

- *μ is the ex ante level of mismatch.*
- *μ^{τ} is the ex post level of mismatch.*
- *$\varphi(\mu)$ is the overall efficiency of R&D, defined as,*

$$\varphi(\mu) = \max_{\tau} (1-\mu^{\tau})(1-\tau)$$

[15] Note that I assume all along that the information about the highest productivity in a given sector is not known until production starts. Relaxing this assumption would allow researchers to target sectors where the current best technology is low. By losing the random increment over existing ideas, I would lose not only the tractability of the Eaton and Kortum model, but also the dispersion of best ideas that describes the distribution of firm sizes well.

- *$\tau(\mu)$ is the optimal level of targetting, defined as,*

$$\tau(\mu) \equiv \arg\max_{\tau} (1-\mu^{\tau})(1-\tau)$$

Simple algebra allows me to derive the following proposition that describes how the ex ante level of technological mismatch affects targetting and the efficiency of R&D.

Proposition 6 *As mismatch increases, targetting increases $\left(\frac{\partial \tau(\mu)}{\partial \mu} > 0\right)$, and the efficiency of R&D decreases $\left(\frac{\partial \varphi(\mu)}{\partial \mu} < 0\right)$.*

The intuition for this result is straightforward. As the degree of mismatch increases, the likelihood of discovering ideas in mismatched sectors increases. The marginal benefit of targetting accessible sectors increases, whereas the marginal cost of targetting in terms of less effort being put in research remains the same. Targetting accessible sectors becomes more attractive.

The value of being a researcher at time t in country i, v_{it}, is given by,

$$v_{it} = \varphi(\mu) \times T_{it} V_{it} \tag{19}$$

where $\varphi(\mu) = \max_{\tau} (1-\mu^{\tau})(1-\tau)$ is the overall efficiency of R&D when a researcher decides optimally how to allocate her effort between targetting and researching, T_{it} is the stock of ideas in country i at time t, V_{it} is the value of a single idea, defined as in the previous two sections (Eq. (13) without a market for ideas, and Eq. (16) with).

Equilibrium in the labor market requires that in each country, at every point in time, agents are indifferent between working and researching,

$$w_{it} = v_{it} \tag{20}$$

As in the previous sections, I rule out equilibria with no R&D.

Proposition 7 *When researchers are allowed to target their research towards specific sectors, there exist a steady state equilibrium. In the symmetric case and in the absence of an international market for ideas, the steady state growth rate, g_T, is given by,*

$$g_T = \frac{L\varphi(\mu)\frac{\pi}{1-\pi} - \rho}{\frac{\theta-1}{\theta} - \varphi(\mu)\frac{\pi}{1-\pi}}$$

When an international market for ideas exists, the steady state growth rate becomes,

$$g_T = \frac{L\varphi(\mu^2)\frac{\pi}{1-\pi} - \rho}{\frac{\theta-1}{\theta} - \varphi(\mu^2)\frac{\pi}{1-\pi}}$$

Proof. *See appendix.* ■

I can now derive a series of predictions regarding the impact of opening up an international market for ideas on long term growth and on the patterns of international trade in goods.

First, note that opening up an international market for ideas reduces the ex ante level of mismatch worldwide from μ to μ^2. This reduction in the ex ante level of mismatch reduces the incentive for researchers to target specific sectors. As researchers are freed up from targetting sectors and can focus more of their effort doing pure research, this in turn increases the overall efficiency of R&D. This higher efficiency of R&D attracts more researchers, and increases long term growth.

Second, as researchers do less targetting in each country, the ex post level of mismatch in each country increases. We have seen in section 2 that trade openness increases with the degree of technological mismatch, as technological mismatch directly governs the degree of comparative advantage across countries. In other words, opening up an international market for ideas reduces the level of targetting in all countries, and technologies are simply allocated to the workers that can use them most efficiently. Opening up an international market for ideas increases the the degree of trade openness.

The following two propositions present those results formally.

Proposition 8 *Trade in ideas increases growth.*

Proof. *Trade in ideas reduces the ex ante level of worlwide technological mismatch from μ to μ^2. From proposition 6, the lower ex ante level of technological mismatch increases overall efficiency of R&D,*

$$\mu > \mu^2 \Rightarrow \varphi(\mu) < \varphi\left(\mu^2\right)$$

This increase in the overall efficiency of R&D will increases the value of being a researcher, Eq. (19), which in turn attracts more agents into the R&D sector, Eq. (20), and increases long term growth, Eq. (7) and proposition 7. ■

In addition to increasing growth, trade in ideas increases trade in goods by increasing the degree of comparative advantage in each country.

Proposition 9 *Trade in ideas increases trade in goods.*

Proof. *Trade in ideas reduces the incentive to target research, and therefore increases the level of ex post mismatch, which in turn increases trade in goods.* ■

5 Conclusion

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References

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A Appendix

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