Imperfect Information and Optimal Monetary Policy

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Abstract

Should the central bank care whether slow adjustment of the price level is due to adjustment costs as in the standard New Keynesian model or due to imperfect information? Most of the analysis of optimal monetary policy is conducted in the Calvo model. This paper studies optimal monetary policy in a model with exogenous dispersed information and in a rational inattention model.

JEL: E3, E5, D8.

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1 Introduction

To be written.

2 Model

The economy is populated by a representative household, firms and a government.

**Household:** There is a representative household. The household’s preferences in period \( t \) over sequences of composite consumption and labor supply \( \{C_{t+\tau}, L_{t+\tau}\}_{\tau=0}^{\infty} \) are given by

\[
E_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau \left( \frac{C_{t+\tau}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{t+\tau}^{1+\psi}}{1+\psi} \right) \right],
\]

where \( C_{t+\tau} \) is composite consumption and \( L_{t+\tau} \) is labor supply in period \( t+\tau \). The operator \( E_t \) is the expectation operator conditioned on the entire history of the economy up to and including period \( t \). The parameter \( \beta \in (0, 1) \) is the discount factor. The parameter \( \gamma > 0 \) is the inverse of the intertemporal elasticity of substitution and the parameter \( \psi \geq 0 \) is the inverse of the Frisch elasticity of labor supply. Composite consumption in period \( t \) is given by a Dixit-Stiglitz aggregator

\[
C_t = \left( \frac{1}{I} \sum_{i=1}^{I} C_{i,t}^{1+\Lambda_t} \right)^{1+\Lambda_t},
\]

where \( C_{i,t} \) is consumption of good \( i \) in period \( t \). There are \( I \) different consumption goods. The elasticity of substitution between consumption goods equals \( (1 + 1/\Lambda_t) \) in period \( t \). Since \( \Lambda_t \) will equal the desired markup by firms, we call \( \Lambda_t \) the desired markup. We assume that the log of the desired markup follows a Gaussian first-order autoregressive process

\[
\ln(\Lambda_t) = (1 - \rho_\lambda) \ln(\Lambda) + \rho_\lambda \ln(\Lambda_{t-1}) + \nu_t,
\]

where the parameter \( \Lambda > 0 \), the parameter \( \rho_\lambda \in [0, 1) \), and the innovation \( \nu_t \) is i.i.d. \( N(0, \sigma_\nu^2) \).

The flow budget constraint of the representative household in period \( t \) reads

\[
M_t + B_t = R_{t-1} B_{t-1} + W_t L_t + D_t - T_t + \left( M_{t-1} - \sum_{i=1}^{I} P_{i,t-1} C_{i,t-1} \right),
\]

where \( B_{t-1} \) are the household’s holdings of nominal government bonds between period \( t - 1 \) and period \( t \), \( R_{t-1} \) is the nominal gross interest rate on those bond holdings, \( W_t \) is
the nominal wage rate, $D_t$ are nominal aggregate profits, and $T_t$ are nominal lump-sum taxes in period $t$. The term in brackets on the right-hand side of equation (4) are unspent nominal money balances carried over from period $t - 1$ to period $t$. The household can allocate his pre-consumption wealth in period $t$ (i.e. the right-hand side of equation (4)) between nominal money balances, $M_t$, and nominal bond holdings, $B_t$. We assume that the representative household faces the following cash-in-advance constraint in every period

$$\sum_{i=1}^{I} P_{i,t} c_{i,t} = M_t. \quad (5)$$

Furthermore, the representative household faces a no-Ponzi-scheme condition.

In every period, the representative household chooses a consumption vector, labor supply, nominal money balances, and nominal bond holdings. The representative household takes as given the prices of all consumption goods, the nominal interest rate, the nominal wage rate, nominal aggregate profits, and nominal lump-sum taxes.

**Firms:** There are $I$ firms. Firm $i$ supplies good $i$. The technology of firm $i$ is given by

$$Y_{i,t} = A_t L_{i,t}^\alpha, \quad (6)$$

where $Y_{i,t}$ is output and $L_{i,t}$ is labor input of firm $i$ in period $t$. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to labor input. The variable $A_t$ denotes aggregate productivity in period $t$. The log of aggregate productivity follows a Gaussian first-order autoregressive process

$$\ln (A_t) = \rho_a \ln (A_{t-1}) + \varepsilon_t, \quad (7)$$

where the parameter $\rho_a \in [0, 1)$ and the aggregate technology shock $\varepsilon_t$ is i.i.d.$ N \left(0, \sigma^2_\varepsilon\right)$. The process $\{A_t\}$ is independent of the process $\{\Lambda_t\}$.

Nominal profits of firm $i$ in period $t$ equal

$$(1 + \tau_p) P_{i,t} Y_{i,t} - W_t L_{i,t}, \quad (8)$$

where $\tau_p$ is a production subsidy paid by the government.

In every period, each firm sets a price and commits to supply any quantity at that price. The firm takes as given the representative household’s composite consumption, the nominal
wage rate, and the following price index

\[ P_t = I^{1+\Lambda_t} \left( \sum_{i=1}^{I} P_{i,t-i} \right)^{-\Lambda_t}. \]  

**Government:** There is a monetary authority and a fiscal authority. The monetary authority commits to set the money supply according to the following rule

\[ \ln (M_t^s) = F_t(L) \varepsilon_t + G_t(L) \nu_t, \]  

where \( M_t^s \) denotes the money supply in period \( t \) and \( F_t(L) \) and \( G_t(L) \) are infinite-order lag polynomials which can depend on \( t \). The last equation simply says that the log of the money supply in period \( t \) can be any linear function of the sequence of shocks up to and including period \( t \). We will ask the question which linear function is optimal.

How is money injected into the economy and how does the money market clear? The household can transform any fraction of his pre-consumption wealth in period \( t \) into nominal money balances in period \( t \). See equation (4). In equilibrium, the price level and the nominal interest rate will adjust such that the demand for nominal money balances by the representative household will equal the supply of nominal money balances by the monetary authority (i.e. \( M_t = M_t^s \)).

The government budget constraint in period \( t \) reads

\[ T_t + B_t = R_{t-1}B_{t-1} + \tau_p \left( \sum_{i=1}^{I} P_{i,t}Y_{i,t} \right). \]  

The government has to finance interest on nominal government bonds and the production subsidy. The government can collect lump-sum taxes or issue new nominal government bonds. We assume that the government pursues a Ricardian fiscal policy. In particular, for ease of exposition, we assume that the fiscal authority fixes nominal government bonds at some non-negative level

\[ B_t = B \geq 0. \]  

\(^{1}\) Dixit and Stiglitz (1977), in their original article, also assumed that there is a finite number of goods and that firms take as given the price index. Moreover, it seems to be a good description of the U.S. economy that there is a finite number of physical consumption goods and that firms take the consumer price index as given.
We assume that the fiscal authority sets the production subsidy so as to correct, in the non-stochastic steady state, the distortion arising from firms’ market power in the goods market. Formally,
\[ \tau_p = \Lambda. \] (13)

Alternatively, one could assume that the fiscal authority sets the production subsidy so as to correct fully at each point in time the distortion arising from firms’ market power in the goods market. Formally,
\[ \tau_{p,t} = \Lambda_t. \] (14)

However, since in the United States fiscal policy has to be approved by Congress while monetary policy decisions by the Federal Reserve are implemented directly, we find it more realistic to assume that the fiscal authority cannot adjust the production subsidy quickly while the monetary authority can adjust the money supply quickly.

**Information:** The information set of the price setter of firm \( i \) in period \( t \) is given by any initial information that the price setter may have as well as the sequence of all signals that the price setter has received up to and including period \( t \)
\[ \mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \ldots, s_{i,t}\}, \] (15)
where \( \mathcal{I}_{i,-1} \) is the initial information set of the price setter of firm \( i \) in period minus one and \( s_{i,t} \) is the signal that the price setter of firm \( i \) receives in period \( t \). We assume that, in every period \( t \geq 0 \), the price setter of firm \( i \) receives a two-dimensional signal consisting of noisy signals about aggregate productivity and the desired markup:
\[ s_{i,t} = \begin{pmatrix} \ln (A_t) + \eta_{i,t} \\ \ln (\Lambda_t/\Lambda) + \zeta_{i,t} \end{pmatrix}, \] (16)
where the noise in the signal has the following properties: (i) the stochastic processes \( \{\eta_{i,t}\} \) and \( \{\zeta_{i,t}\} \) are independent of the stochastic processes \( \{A_t\} \) and \( \{\Lambda_t\} \), (ii) the stochastic processes \( \{\eta_{i,t}\} \) and \( \{\zeta_{i,t}\} \) are independent across firms and independent of each other, and (iii) the noise \( \eta_{i,t} \) is \( i.i.d. N (0, \sigma^2_\eta) \) and the noise \( \zeta_{i,t} \) is \( i.i.d. N (0, \sigma^2_\zeta) \). We also assume that the number of firms is sufficiently large so that
\[ \frac{1}{T} \sum_{i=1}^{T} \eta_{i,t} = \frac{1}{T} \sum_{i=1}^{T} \zeta_{i,t} = 0. \] (17)
Two remarks are in place before we proceed. First, we think of the noise in the signal as being due to limited attention by decision-makers in firms. Therefore, we find it reasonable to assume that the noise in the signal is idiosyncratic and will wash out in the aggregate. Second, the case of noisy signals about aggregate productivity and the desired markup will turn out to be a useful benchmark. Later we will also consider the case of noisy signals about other variables.

We assume that the monetary authority and the representative household have perfect information (i.e. in every period \( t \geq 0 \), the monetary authority and the representative household know the entire history of the economy up to and including period \( t \)). We assume that the monetary authority has perfect information because we are interested in the optimal conduct of monetary policy. We assume that the representative household has perfect information to isolate the implications of imperfect information by price setters for the optimal conduct of monetary policy. Note that the monetary authority, which has perfect information, could announce in every period the entire history of the economy. It is important to point out that this would make no difference so long as we interpret the noise in the signal as arising from limited attention by decision-makers in firms rather than lack of publicly available information.

### 3 Objective of the central bank

In this section, we state the central bank’s objective and we characterize the feasible allocation that maximizes the central bank’s objective. We also derive a log-quadratic approximation to the central bank’s objective.

We assume that the monetary authority aims to maximize expected utility of the representative household:

\[
E \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right],
\]

where

\[
U(C_t, L_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} - L_t^{1+\psi} \frac{1}{1 + \psi},
\]

\[
C_t = \left( \frac{1}{I} \sum_{i=1}^{I} C_{i,t}^{1+\lambda_i} \right)^{1+\Lambda_t},
\]
and

\[ L_t = \sum_{i=1}^{I} \left( \frac{C_{i,t}}{A_t} \right)^{\frac{1}{\alpha}}. \]  

Equation (19) is the period utility function, equation (20) is the definition of composite consumption, and equation (21) is the feasibility constraint stating that the representative household has to supply the labor that is required to produce the consumption vector.

By substituting equations (20) and (21) into the period utility function (19), one can express period utility as a function only of the consumption vector in period \( t \) and the two exogenous variables \( A_t \) and \( \Lambda_t \). More specifically, it will be convenient to express period utility as a function only of composite consumption in period \( t \), relative consumption of \( I - 1 \) goods in period \( t \), and \( A_t \) and \( \Lambda_t \). In the following, \( \hat{C}_{i,t} = (C_{i,t}/C_t) \) denotes relative consumption of good \( i \) in period \( t \). One can write equation (20) as

\[ 1 = \frac{1}{I} \sum_{i=1}^{I} \hat{C}_{i,t}^{1+\Lambda_t}. \]

Rearranging yields

\[ \hat{C}_{t,t} = \left( I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{1+\Lambda_t} \right)^{1+A_t}. \]  

(22)

Substituting equations (21) and (22) into the period utility function (19) yields the following expression for expected utility of the representative household:

\[ E \left[ \sum_{t=0}^{\infty} \beta^t V \left( C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \ldots, \hat{C}_{I-1,t}, A_t, \Lambda_t \right) \right], \]  

(23)

with

\[ V \left( C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \ldots, \hat{C}_{I-1,t}, A_t, \Lambda_t \right) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} - \frac{1}{1 + \psi} \left( \frac{C_t}{A_t} \right)^{\frac{1}{\alpha}(1+\psi)} \left[ \sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{\alpha}} + \left( I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{1+\Lambda_t} \right)^{\frac{1}{\alpha}(1+\Lambda_t)} \right]^{1+\psi}. \]  

(24)

Equation (24) gives period utility as a function only of composite consumption in period \( t \), the consumption mix in period \( t \), and the two exogenous variables \( A_t \) and \( \Lambda_t \).

**Definition 1** An efficient allocation in period \( t \) is a vector \( \left( C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \ldots, \hat{C}_{I-1,t} \right) \in \mathbb{R}_+^I \) that maximizes expression (24), where \( \hat{C}_{i,t} = (C_{i,t}/C_t) \).
Maximizing expression (24) yields that the unique efficient allocation in period $t$ is

$$C_t^* = \left(\frac{\alpha}{1+\psi}\right)^{1/(1+\psi)} \frac{1}{\gamma-1+\frac{1}{\alpha}(1+\psi)} A_t^{1/\alpha(1+\psi)},$$

and

$$\forall i = 1, 2, \ldots, I - 1: \hat{C}_{i,t}^* = 1.$$  

The efficient composite consumption in period $t$ is increasing in aggregate productivity in period $t$. The efficient consumption mix in period $t$ is to consume an equal amount of each good. The efficient allocation in period $t$ does not depend on $\Lambda_t$.

In Sections 5-7, we work with a log-quadratic approximation to the central bank’s objective (23)-(24). We compute the log-quadratic approximation around the non-stochastic steady state, where a non-stochastic steady state is defined as an equilibrium of the non-stochastic version of the economy with the property that real quantities, relative prices, the nominal interest rate and inflation are constant over time. Let variables without time subscript denote values in the non-stochastic steady state. It is straightforward to show that due to the subsidy (13) we have $C = C^*$ and $\hat{C}_i = \hat{C}_i^*$, that is, in the non-stochastic steady state the equilibrium allocation equals the efficient allocation. Let small variables denote log-deviations from the non-stochastic steady state, that is, $c_t = \ln(C_t/C^*)$, $\hat{c}_{i,t} = \ln(\hat{C}_{i,t}/C^*)$, $a_t = \ln(A_t)$ and $\lambda_t = \ln(\Lambda_t/\Lambda)$. Expressing the function $V$ given by equation (24) in terms of log-deviations from the non-stochastic steady state and using $C = C^*$ and equation (25) yields the following expression for period utility at time $t$

$$v(c_t, \hat{c}_{1,t}, \hat{c}_{2,t}, \ldots, \hat{c}_{I-1,t}, a_t, \lambda_t) = \frac{C^{1-\gamma}e^{(1-\gamma)c_t} - 1}{1-\gamma} - \frac{C^{1-\gamma}e^{\frac{1}{\alpha}(1+\psi)(c_t-a_t)}}{1/\alpha(1+\psi)} \left[ \frac{1}{T} \sum_{i=1}^{I-1} e^{\frac{1}{\alpha} \hat{c}_{i,t}} + \frac{1}{T} \left( I - \sum_{i=1}^{I-1} e^{\frac{1}{\alpha} \hat{c}_{i,t} (1+\Lambda t^\lambda)} \right) \right]^{1+\psi}. \quad (27)$$

Computing a second-order Taylor approximation to the function $v$ around the origin yields the following proposition.

Proposition 1 (Objective of the central bank) Let $v$ denote the period utility function given by equation (27). Let $\tilde{v}$ denote the second-order Taylor approximation to $v$ at the origin.
Let $E$ denote the unconditional expectation operator. Let $x_t$, $z_t$, and $\omega_t$ denote the following vectors

$$x'_t = \begin{pmatrix} c_t & \hat{c}_{1,t} & \cdots & \hat{c}_{I-1,t} \end{pmatrix}, \quad (28)$$

$$z'_t = \begin{pmatrix} a_t & \lambda_t \end{pmatrix}, \quad (29)$$

$$\omega'_t = \begin{pmatrix} x'_t & z'_t & 1 \end{pmatrix}, \quad (30)$$

and let $\omega_{n,t}$ and $\omega_{k,t}$ denote the $n$th and $k$th element of $\omega_t$. Suppose that there exist two constants $\delta < (1/\beta)$ and $\phi \in \mathbb{R}$ such that, for each period $t \geq 0$ and for all $n$ and $k$,

$$E |\omega_{n,t} \omega_{k,t}| < \delta t \phi. \quad (31)$$

Then

$$E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] - E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v}(x^*_t, z_t) \right] = \sum_{t=0}^{\infty} \beta^t E \left[ \frac{1}{2} (x_t - x^*_t)' H (x_t - x^*_t) \right], \quad (32)$$

where the matrix $H$ is given by

$$H = -C^{1-\gamma} \begin{bmatrix} \gamma - 1 + \frac{1}{\alpha} (1 + \psi) & 0 & \cdots & 0 \\ 0 & \frac{2}{\gamma (1+\Lambda)^{\alpha}} & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} & \cdots & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} \\ \vdots & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} \\ 0 & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} & \cdots & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} & \frac{1+\Lambda-\alpha}{\gamma (1+\Lambda)^{\alpha}} \end{bmatrix}, \quad (33)$$

and the vector $x^*_t$ is given by

$$c^*_t = \frac{1}{\gamma - 1 + \frac{1}{\alpha} (1 + \psi)} a_t, \quad (34)$$

and

$$\hat{c}_{i,t}^* = 0. \quad (35)$$

**Proof.** See Appendix A. ■

After the quadratic approximation to the period utility function (27), the efficient composite consumption in period $t$ is given by equation (34) and the efficient consumption mix in period $t$ is given by equation (35). In addition, the loss in utility in period $t$ in the case of a deviation from the efficient allocation is given by the quadratic form in square brackets
on the right-hand side of equation (32). The upper-left element of the matrix $H$ determines the loss in utility in the case of inefficient composite consumption. The lower-right block of the matrix $H$ determines the loss in utility in the case of an inefficient consumption mix. The condition (31) ensures that, in the expression for the expected discounted sum of period utility, after the quadratic approximation to the period utility function (27), one can change the order of integration and summation and the infinite sum converges.

4 Statement of the optimal policy problem

In this section, we state the central bank’s optimal policy problem under commitment. We state the problem for the economy described in Section 2 (“exogenous dispersed information”) and for an economy that is identical apart from the fact that price setters in firms choose the precision of the signal (16) subject to a cost function (“rational inattention”). In the rational inattention model, we start from the assumption that price setters make their information choice after the central bank has committed to a policy and the policy has become common knowledge.\footnote{We think it would be interesting to study the firms’ incentive to learn the central bank’s policy choice.}

4.1 Exogenous dispersed information

When the central bank can commit, the central bank solves

$$\max_{\{F_t(L), G_t(L)\}} \sum_{t=0}^{\infty} \beta^t V \left( C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \ldots, \hat{C}_{I-1,t}, A_t, \Lambda_t \right),$$

subject to (i) the household’s optimality conditions

$$P_tC_t = M_t,$$

$$C_{i,t} = \left( \frac{P_{i,t}}{\bar{P}_t} \right)^{-\left(1 + \frac{1}{\Lambda_t}\right)} C_t,$$

$$P_t = I^{1 + \Lambda_t} \left( \sum_{i=1}^{l} P_{i,t}^{-\frac{1}{\Lambda_t}} \right)^{-\Lambda_t},$$

$$W_t = \frac{L_t^0}{C_t^\gamma},$$

\footnote{We think it would be interesting to study the firms’ incentive to learn the central bank’s policy choice.}
(ii) the firms’ optimality conditions and information sets

\[
E \left[ \left( 1 + \tau_p \right) \frac{1}{A_t} P_{i,t}^{-1/\alpha} \left( \frac{1}{I_P} P_t \right)^{1+\lambda t} C_t + \frac{1 + \lambda t}{\alpha} W_t \frac{1}{A_t} \left[ \frac{P_{i,t}}{P_t} \right]^{1+\lambda t} C_t \right]_{\mathcal{I}_{i,t}} = 0, \quad (41)
\]

\[
\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \ldots, s_{i,t}\},
\]

\[
s_{i,t} = \left( \begin{array}{c}
\ln (A_t) + \eta_{i,t} \\
\ln (\Lambda_t / \Lambda) + \xi_{i,t}
\end{array} \right), \quad (43)
\]

(iii) the labor market clearing condition

\[
L_t = \sum_{i=1}^{I} \left( \frac{C_{i,t}}{A_t} \right)^{\frac{\alpha}{\alpha - 1}}, \quad (44)
\]

(iv) the laws of motion for aggregate productivity and the desired markup

\[
\ln (A_t) = \rho_a \ln (A_{t-1}) + \varepsilon_t, \quad (45)
\]

\[
\ln (\Lambda_t / \Lambda) = \rho_\lambda \ln (\Lambda_{t-1} / \Lambda) + \nu_t, \quad (46)
\]

and (v) the equation for the money supply

\[
\ln (M_t) = F_t (L) \varepsilon_t + G_t (L) \nu_t. \quad (47)
\]

The function \( V \) in objective (36) is given by equation (24), \( F_t (L) \) and \( G_t (L) \) in equation (47) are infinite-order lag polynomials that can depend on \( t \), and the innovations \( \varepsilon_t, \nu_t, \eta_{i,t} \) and \( \xi_{i,t} \) have the properties described in Section 2.

A log-quadratic approximation of objective (36) around the non-stochastic steady state and a log-linear approximation of the equilibrium conditions (37)-(41) and (44) around the non-stochastic steady state yields the following linear-quadratic optimal policy problem

\[
\max_{\{F_t(L), G_t(L)\}} \left[ \sum_{t=0}^{\infty} \beta^t C^{1-\gamma} \left( \gamma - 1 + \frac{1+\psi}{\alpha} (c_{t} - c_{t}^*)^2 \right) + \frac{1+\Lambda_t}{\alpha (1+\Lambda) \alpha} \sum_{i=1}^{I} \left( \xi_{i,t}^2 + \xi_{i,t} \sum_{k=1}^{l-1} \xi_{k,t} \right) \right] , \quad (48)
\]

subject to

\[
p_t + c_t = m_t, \quad (49)
\]
\[ c_{i,t} = - \left( 1 + \frac{1}{\Lambda} \right) (p_{i,t} - p_t) + c_t, \quad (50) \]

\[ p_t = \frac{1}{I} \sum_{i=1}^{I} p_{i,t}, \quad (51) \]

\[ w_t - p_t = \psi l_t + \gamma c_t, \quad (52) \]

\[ p_{i,t} = E \left[ p_{i,t}^{*} | \mathcal{I}_{i,t} \right], \quad (53) \]

\[ p_{i,t}^{*} = p_t + \frac{1}{1 + \frac{1 - \alpha}{\Lambda}} (w_t - p_t) + \frac{\frac{1 - \alpha}{\alpha}}{1 + \frac{1 - \alpha}{\Lambda}} c_t - \frac{\frac{1}{\alpha}}{1 + \frac{1 - \alpha}{\Lambda}} a_t + \frac{\Lambda}{1 + \frac{1 - \alpha}{\Lambda}} \lambda_t, \quad (54) \]

and

\[ l_t = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{\alpha} (c_{i,t} - a_t), \quad (55) \]

where \( c_{i}^{*} \) is given by equation (34), \( \mathcal{I}_{i,t} \) is given by equations (42)-(43), and \( a_t, \lambda_t \) and \( m_t \) are given by equations (45)-(47).

The linear-quadratic optimal policy problem (48)-(55) can be stated more concisely. After substituting equations (50)-(51) into objective (48) and after substituting equations (50)-(52) and equation (55) into equation (54), the linear-quadratic optimal policy problem reads

\[
\max_{\{F_i(L), G_i(L)\}_{t=0}^{\infty}} - \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\gamma}}{2} E \left[ \frac{\left( \gamma - 1 + \frac{1 + \psi}{\alpha} \right) (c_t - c_t^*)^2}{1 + \frac{1 - \alpha}{\Lambda} \lambda_t (1 + \frac{1}{\Lambda})^2 \frac{1}{I} \sum_{i=1}^{I} (p_{i,t} - p_t)^2} \right],
\]

subject to

\[ c_t = m_t - p_t, \quad (57) \]

\[ p_t = \frac{1}{I} \sum_{i=1}^{I} p_{i,t}, \quad (58) \]

\[ p_{i,t} = E \left[ p_{i,t}^{*} | \mathcal{I}_{i,t} \right], \quad (59) \]

\[ p_{i,t}^{*} = p_t + \phi_c c_t - \phi_a a_t + \phi_\lambda \lambda_t, \quad (60) \]

where

\[ c_t^{*} = \frac{\phi_a}{\phi_c} a_t, \quad (61) \]

\[ \mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{ s_{i,0}, s_{i,1}, \ldots, s_{i,t} \}, \quad (62) \]

\[ s_{i,t} = \begin{pmatrix} a_t + \eta_{i,t} \\ \lambda_t + \zeta_{i,t} \end{pmatrix}, \quad (63) \]
\begin{align}
    a_t &= \rho_a a_{t-1} + \varepsilon_t, \quad (64) \\
    \lambda_t &= \rho_\lambda \lambda_{t-1} + \nu_t, \quad (65) \\
    m_t &= F_t (L) \varepsilon_t + G_t (L) \nu_t, \quad (66)
\end{align}

and
\begin{align}
    \phi_c &= \frac{\psi + \gamma + \frac{1-\alpha}{\alpha}}{1 + \frac{1-\alpha}{\alpha} + \frac{\Lambda}{\alpha - \Lambda}}, \quad (67) \\
    \phi_a &= \frac{\psi + \frac{1}{\alpha}}{1 + \frac{1-\alpha}{\alpha} + \frac{\Lambda}{\alpha - \Lambda}}, \quad (68) \\
    \phi_\lambda &= \frac{\Lambda}{1+\Lambda + \frac{1-\alpha}{\alpha}} \quad (69)
\end{align}

In Sections 5-6, we characterize the solution to this problem.

### 4.2 Rational inattention

In the rational inattention model, the precision of the signals (63) is endogenous. We assume that firms choose the precision of the signals after the central bank has committed to a policy. The attention problem of firm $i$ reads

\begin{equation}
    \min_{(\sigma_a^2, \sigma_\lambda^2) \in \mathbb{R}_+^2} \left\{ \frac{\omega}{2} E \left[ \left( p_{i,t} - p_{i,t}^* \right)^2 \right] + \mu \kappa \right\}, \quad (70)
\end{equation}

subject to
\begin{align}
    p_{i,t}^* &= p_t + \phi_c c_t - \phi_a a_t + \phi_\lambda \lambda_t, \quad (71) \\
    p_{i,t} &= E \left[ p_{i,t}^* | s_{\lambda,i,t} \right], \quad (72) \\
    s_{i,t} &= \left( \begin{array}{c} a_t + \eta_{i,t} \\ \lambda_t + \zeta_{i,t} \end{array} \right), \quad (73)
\end{align}

and in each period $t \geq 0$,
\begin{equation}
    \frac{1}{2} \log_2 \left( \frac{\sigma_a^2 | s_{i,t}^{|t-1}}{\sigma_a^2 | s_{i,t}^{|t}} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_\lambda^2 | s_{i,t}^{|t-1}}{\sigma_\lambda^2 | s_{i,t}^{|t}} \right) \leq \kappa. \quad (74)
\end{equation}

Here the coefficient $\omega > 0$ is a non-linear function of the parameters appearing in the profit function, $\mu \geq 0$ is the per-period marginal cost of attention, and $s_{i,t}^{|t}$ is the sequence of signals received by firm $i$ up to and including period $t$. 

13
5 Optimal policy response to aggregate productivity shocks

In this section, we derive the optimal monetary policy response to aggregate productivity shocks.

5.1 Exogenous dispersed information

To derive the optimal monetary policy response to aggregate productivity shocks, note the following. First, solving for the optimal policy response to aggregate productivity shocks and solving for the optimal policy response to markup shocks are two independent problems. Thus, in this section we can assume without loss of generality that \( \lambda_t = 0 \) in every period. Second, suppose that price setters have perfect information. Equations (57)-(60) then imply

\[
\begin{align*}
    c_t & = \frac{\phi_a a_t}{\phi_c} \\
    p_{i,t} - p_t & = 0 \\
    p_t & = m_t - \frac{\phi_a}{\phi_c} a_t.
\end{align*}
\]

Note that when price setters have perfect information, the equilibrium allocation equals the efficient allocation. Equilibrium composite consumption equals efficient composite consumption and the equilibrium consumption mix equals the efficient consumption mix. Furthermore, note that when price setters have perfect information, monetary policy only affects the price level, which has no effect on welfare. Third, suppose that price setters have imperfect information and that the central bank chooses \( m_t = \frac{\phi_a}{\phi_c} a_t \). If \( m_t = \frac{\phi_a}{\phi_c} a_t \), the profit-maximizing price (60) does not depend on aggregate productivity and equations (57)-(60) imply

\[
\begin{align*}
    c_t & = \frac{\phi_a a_t}{\phi_c} \\
    p_{i,t} - p_t & = 0 \\
    p_t & = 0.
\end{align*}
\]

Hence, when price setters have imperfect information, the central bank can replicate one of the equilibria with perfect information: the one with stable prices. Since the allocation associated with this equilibrium is efficient, the central bank can attain the efficient allocation. Fourth, when price setters have imperfect information, no other monetary policy
attains the efficient allocation. For any other monetary policy the profit-maximizing price (60) depends on aggregate productivity, implying that firms put some weight on the signal (63), which creates price dispersion. We arrive at the following proposition.

**Proposition 2** Consider the central bank’s optimal policy problem (56)-(69). If $\sigma^2_\eta > 0$, the unique optimal monetary policy response to aggregate productivity shocks is

$$F_t(L) \varepsilon_t = \frac{\varphi_a}{\varphi_c} a_t.$$  

(75)

*Under this policy, the price level does not respond to an aggregate productivity shock.*

The derivation of this result hopefully also makes clear the limitations and the extensions of this result. If the equilibrium response to aggregate productivity shocks under perfect information was not efficient (e.g., no Dixit-Stiglitz preferences in which case a constant subsidy would no longer be sufficient to correct the distortions due to market power in the goods market), then complete stabilization of the price level in response to an aggregate productivity shock would no longer be optimal. On the other hand, consider any shock with the property that the equilibrium response to this shock under perfect information is efficient. Complete stabilization of the price level in response to this shock is optimal. These examples show that the result stated in Proposition 2 has nothing to do with an aggregate productivity shock being a supply shock rather than a demand shock.

### 5.2 Rational inattention

The same arguments as in Section 5.1 yield the following proposition.

**Proposition 3** Consider the central bank’s optimal policy problem (56)-(69) with (70)-(74). If $\mu > 0$, the unique optimal monetary policy response to aggregate productivity shocks is

$$F_t(L) \varepsilon_t = \frac{\varphi_a}{\varphi_c} a_t.$$  

(76)

*Under this policy, the price level does not respond to an aggregate productivity shock.*
6 Optimal policy response to markup shocks

In this section, we study the optimal monetary policy response to markup shocks. We are interested in markup shocks because a markup shock is a simple example of a shock with the property that the response to this shock under perfect information is not efficient. To see this, suppose that price setters have perfect information. Equations (57)-(60) then imply

\[ c_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t \]

\[ p_{i,t} - p_t = 0 \]

\[ p_t = m_t - \left( \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t \right). \]

If \( \lambda_t \neq 0 \), the equilibrium allocation under perfect information differs from the efficient allocation. In Section 6.3, we show that the main results presented in Sections 6.1-6.2 about the optimal monetary policy response to markup shocks carry over to other shocks which have the property that the response of the economy to the shock under perfect information is not efficient.

In this section, we assume without loss of generality that \( a_t = 0 \) in every period.

6.1 Exogenous dispersed information

In this subsection, we study the optimal monetary policy response to markup shocks in the model with exogenous dispersed information. We first solve for the optimal monetary policy analytically in the case of \( \rho_\lambda = 0 \) and then we solve for the optimal monetary policy numerically in the case of \( \rho_\lambda \neq 0 \).

Proposition 4 Consider the central bank’s optimal monetary policy problem (56)-(69). Suppose \( \sigma^2_\varepsilon = a_{-1} = 0 \) and suppose \( \rho_\lambda = 0 \). Consider policies of the form \( G_t(L) \nu_t = g_0 \nu_t \).
and equilibria of the form $p_t = \theta \lambda_t$. The unique equilibrium for given monetary policy is

$$
\begin{align*}
    p_t &= \frac{\phi_c g_0 + \phi_\lambda \lambda_t}{\phi_c + \frac{\sigma_c^2}{\sigma_\lambda^2}}, \\
    c_t &= \frac{\sigma_c^2 g_0 - \phi_\lambda \lambda_t}{\phi_c + \frac{\sigma_c^2}{\sigma_\lambda^2}}, \\
    p_{i,t} - p_t &= \frac{\phi_c g_0 + \phi_\lambda \zeta_{i,t}}{\phi_c + \frac{\sigma_c^2}{\sigma_\lambda^2}}.
\end{align*}
$$

(77) (78) (79)

If $\sigma_\zeta^2 > 0$, the unique optimal monetary policy is

$$
    g_0 = \frac{\frac{\gamma - 1 + \frac{1 + \psi}{\psi}}{(1 + \Lambda - \alpha)} \phi_\lambda - \phi_c \phi_\lambda}{\frac{\gamma - 1 + \frac{1 + \psi}{\psi}}{(1 + \Lambda - \alpha)} \sigma_\lambda^2 + \phi_c^2}.
$$

(80)

**Proof.** See Appendix B. ■

In the model with exogenous dispersed information, when $\rho_\lambda = 0$, $G_t(L) \nu_t = g_0 \nu_t$ and $p_t = \theta \lambda_t$, complete stabilization of the price level in response to markup shocks is not optimal. We will obtain the opposite result in the rational inattention model.

Next, we solve the central bank’s optimal policy problem (56)-(69) numerically in the case of $\rho_\lambda \neq 0$. When we solve the central bank’s optimal policy problem (56)-(69) numerically, we turn this infinite-dimensional problem into a finite-dimensional problem by parameterizing the lag polynomial $G_t(L)$ as a lag-polynomial of an ARMA(2,2) process and by restricting $G_t(L)$ to be the same in each period. We solve the problem (56)-(69) for the following benchmark parameter values: $\beta = 0.99$, $\gamma = 1$, $\psi = 0$, $\alpha = 2/3$, and $\Lambda = 0.1$.

For comparison, we also solve for the optimal monetary policy response to markup shocks in the Calvo model. In the Calvo model, firms have perfect information, and any given firm can adjust its price in any given period with an exogenous probability equal to $\delta$.

Figure 1 depicts the optimal monetary policy response to a markup shock. The left panels depict the optimal monetary policy when $\rho_\lambda = 0$. The right panels depict the

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3 We also worked with the lag-polynomial of an ARMA(3,3) process. We obtained very similar results.

4 See chapters 6 and 7 in Woodford (2003) for a detailed description of optimal monetary policy in the Calvo model.
optimal monetary policy when $\rho_\lambda = 0.9$. The solid blue lines show the impulse responses of money, output and inflation at the optimal monetary policy in the imperfect information model. For comparison, the dashed red lines show the impulse responses of money, output and inflation at the optimal monetary policy with commitment in the Calvo model.

We obtain the following results. First, full stabilization of the price level after a markup shock is not the optimal monetary policy in the imperfect information model. Second, whether the optimal monetary policy in the imperfect information model is similar to the optimal monetary policy with commitment in the Calvo model depends on the persistence of the markup shock. For $\rho_\lambda = 0.9$, the two policies appear to be identical, while for $\rho_\lambda = 0$ the two policies are quite different. When $\rho_\lambda = 0$, in the imperfect information model it is optimal for the central bank to respond to the shock only in the period of the shock, whereas in the Calvo model it is optimal for the central bank to commit to respond to the shock also in future periods when the shock no longer affects the desired markup.

### 6.2 Rational inattention

**Proposition 5** Consider the central bank’s optimal policy problem (56)-(69) with (70)-(74). Suppose $\sigma_\varepsilon^2 = a_{-1} = 0$ and suppose $\rho_\lambda = 0$. Consider policies of the form $G_t(L) \nu_t = g_0 \nu_t$ and equilibria of the form $p_t = \theta \lambda_t$. Define

$$b = \sqrt{\frac{\mu (\phi_c g_0 + \phi \lambda)^2 \sigma_\lambda^2}{2 \ln(2)}}. \quad (81)$$

If $\mu > 0$, the complete set of equilibria for given monetary policy is: (i) if $b \leq 1$, the following is an equilibrium

$$c_t = g_0 \lambda_t \quad (82)$$

$$p_{i,t} - p_t = 0, \quad (83)$$
(ii) if $b \geq 1$, the following is an equilibrium

\[ c_t = \left[ g_0 - \frac{(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} (\phi_c g_0 + \phi_\lambda) \right] \lambda_t \]  
\[ p_{i,t} - p_t = \frac{(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} (\phi_c g_0 + \phi_\lambda) \zeta_{i,t} \]  
\[ 2\kappa^* = \frac{b + \sqrt{b^2 - 4\phi_c (1 - \phi_c)}}{2\phi_c} \]

and (iii) if $b \leq 1$, $b \geq 2\phi_c$, and $b^2 \geq 4\phi_c (1 - \phi_c)$, the following is an equilibrium

\[ c_t = \left[ g_0 - \frac{(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} (\phi_c g_0 + \phi_\lambda) \right] \lambda_t \]  
\[ p_{i,t} - p_t = \frac{(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} (\phi_c g_0 + \phi_\lambda) \zeta_{i,t} \]  
\[ 2\kappa^* = \frac{b - \sqrt{b^2 - 4\phi_c (1 - \phi_c)}}{2\phi_c} \]

Furthermore, if $\phi_c = 1$, the unique optimal monetary policy is to completely stabilize the price level in response to markup shocks.

**Proof.** See Appendix C.

7  The value of commitment

There is value of commitment in the Calvo model. There is no value of commitment in the model with exogenous dispersed information. Finally, there is value of commitment in the rational inattention model, but it is qualitatively and quantitatively different from the value of commitment in the Calvo model.

8  Conclusion

To be written
A Proof of Proposition 1

First, we introduce notation. Let \( x_t \) denote the vector of all arguments of the function \( v \) given by equation (27) that are endogenous variables:

\[
x_t' = \left( c_t \ c_{1,t} \ \cdots \ c_{I-1,t} \right).
\]

(90)

Let \( z_t \) denote the vector of all arguments of the function \( v \) given by equation (27) that are exogenous variables:

\[
z_t' = \left( a_t \ \lambda_t \right).
\]

(91)

Second, we compute a log-quadratic approximation to the expression for the expected discounted sum of period utility (23). Let \( \tilde{v} \) denote the second-order Taylor approximation to \( v \) at the non-stochastic steady state. We have

\[
E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] = E \left[ \sum_{t=0}^{\infty} \beta^t \left( v(0, 0) + h'_x x_t + h'_z z_t + \frac{1}{2} x'_t H_x x_t + x'_t H_{xz} z_t + \frac{1}{2} z'_t H_z z_t \right) \right],
\]

(92)

where \( h_x \) is the vector of first derivatives of \( v \) with respect to \( x_t \) evaluated at the non-stochastic steady state, \( h_z \) is the vector of first derivatives of \( v \) with respect to \( z_t \) evaluated at the non-stochastic steady state, \( H_x \) is the matrix of second derivatives of \( v \) with respect to \( x_t \) evaluated at the non-stochastic steady state, \( H_z \) is the matrix of second derivatives of \( v \) with respect to \( z_t \) evaluated at the non-stochastic steady state, and \( H_{xz} \) is the matrix of second derivatives of \( v \) with respect to \( x_t \) and \( z_t \) evaluated at the non-stochastic steady state. Third, we rewrite equation (92) using condition (31). Let \( \omega_t \) denote the following vector

\[
\omega_t' = \left( x_t' \ z_t' \ 1 \right).
\]

(93)

Let \( \omega_{n,t} \) and \( \omega_{k,t} \) denote the \( n \)th and \( k \)th element of \( \omega_t \). Condition (31) implies that

\[
\sum_{t=0}^{\infty} \beta^t E \left| v(0, 0) + h'_x x_t + h'_z z_t + \frac{1}{2} x'_t H_x x_t + x'_t H_{xz} z_t + \frac{1}{2} z'_t H_z z_t \right| < \infty.
\]

(94)
It follows that one can change the order of integration and summation on the right-hand side of equation (92):

$$ E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v} (x_t, z_t) \right] = \sum_{t=0}^{\infty} \beta^t E \left[ v(0,0) + h'_tx_t + h'_z z_t + \frac{1}{2} x'_t H_xx_t + x'_t H_xz z_t + \frac{1}{2} z'_t H_z z_t \right]. \quad (95) $$

See Rao (1973), p. 111. Condition (31) also implies that the infinite sum on the right-hand side of equation (95) converges to an element in \( \mathbb{R} \). Fourth, we define the vector \( x^*_t \). In each period \( t \geq 0 \), the vector \( x^*_t \) is defined by

$$ h_x + H_xx^*_t + H_xz z_t = 0. \quad (96) $$

We will show below that \( H_x \) is an invertible matrix. Therefore, one can write the last equation as

$$ x^*_t = -H^{-1}_x h_x - H^{-1}_x H_x z z_t. \quad (97) $$

Hence, \( x^*_t \) is uniquely determined and the vector \( \omega_t \) with \( x_t = x^*_t \) satisfies condition (31). Fifth, equation (95) implies that

$$ E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v} (x_t, z_t) \right] - E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v} (x^*_t, z_t) \right] = \sum_{t=0}^{\infty} \beta^t E \left[ h'_x (x_t - x^*_t) + \frac{1}{2} x'_t H_xx_t - \frac{1}{2} x'_t H_xz z_t + (x_t - x^*_t)' H_x z z_t \right]. \quad (98) $$

Using equation (96) to substitute for \( H_x z z_t \) in the last equation and rearranging yields

$$ E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v} (x_t, z_t) \right] - E \left[ \sum_{t=0}^{\infty} \beta^t \tilde{v} (x^*_t, z_t) \right] = \sum_{t=0}^{\infty} \beta^t E \left[ \frac{1}{2} (x_t - x^*_t)' H_x (x_t - x^*_t) \right]. \quad (99) $$

Sixth, we compute the vector of first derivatives and the matrices of second derivatives appearing in equations (97) and (99). We obtain

$$ h_x = 0, \quad (100) $$

21
and

\[
H_{xz} = C^{1-\gamma} \begin{bmatrix}
1 \alpha (1 + \psi) & 0 \\
0 & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)^{\alpha}} & \frac{1+\Lambda-\alpha}{I(1+\Lambda)^{\alpha}} \\
\vdots & \vdots & \vdots \\
0 & \frac{1+\Lambda-\alpha}{I(1+\Lambda)^{\alpha}} & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)^{\alpha}}
\end{bmatrix},
\]

(102) [72x564]

Seventh, substituting equations (100)-(102) into equation (96) yields the following system of \( I \) equations:

\[
c^*_t = \frac{1}{\gamma - 1 + \frac{1}{\alpha} (1 + \psi)} a_t,
\]

(103) [72x576]

and, for all \( i = 1, \ldots, I - 1 \),

\[
\hat{c}^*_{t,i} + \sum_{k=1}^{I-1} \hat{c}^*_{t,k} = 0.
\]

(104) [72x589]

Finally, we rewrite equation (104). Summing equation (104) over all \( i \neq I \) yields

\[
\sum_{i=1}^{l-1} \hat{c}^*_{t,i} = 0.
\]

(105) [72x599]

Substituting the last equation back into equation (104) yields

\[
\hat{c}^*_t = 0.
\]

(106) [72x611]

Collecting equations (99), (101), (103) and (106), we arrive at Proposition 1.

\section*{B Proof of Proposition 4}

\textbf{Step 1:} Substituting \( a_t = 0 \), the cash-in-advance constraint \( c_t = m_t - p_t \), the monetary policy \( m_t = g_0 \lambda_t \), and \( p_t = \theta \lambda_t \) into the equation for the profit-maximizing price (60) yields

\[
p^*_t = [(1 - \phi_e) \theta + \phi_e g_0 + \phi_\lambda] \lambda_t.
\]
The price set by firm $i$ in period $t$ then equals

$$p_{i,t} = [(1 - \phi_c) \theta + \phi_c g_0 + \phi \lambda] E [\lambda | I_{i,t}]$$

$$= [(1 - \phi_c) \theta + \phi_c g_0 + \phi \lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} (\lambda_t + \zeta_{i,t}).$$

The price level in period $t$ is then given by

$$p_t = [(1 - \phi_c) \theta + \phi_c g_0 + \phi \lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} \lambda_t.$$

Thus, the unique rational expectations equilibrium of the form $p_t = \theta \lambda_t$ is given by the solution to the equation

$$\theta = [(1 - \phi_c) \theta + \phi_c g_0 + \phi \lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2}.$$

Rearranging yields

$$\theta = \frac{(\phi_c g_0 + \phi \lambda) \sigma_\lambda^2}{1 - (1 - \phi_c) \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2}}$$

$$= \frac{\phi_c g_0 + \phi \lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}.$$

Hence,

$$p_t = \frac{\phi_c g_0 + \phi \lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t$$

(107)

$$c_t = \frac{\sigma_\lambda^2}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t$$

(108)

$$p_{i,t} - p_t = \frac{\phi_c g_0 + \phi \lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \zeta_{i,t}.$$ 

(109)

**Step 2:** Substituting equations (108)-(109), equation (61) and $a_t = 0$ into the central bank’s objective (56) yields

$$\frac{1}{1 - \beta} \frac{C^{1-\gamma}}{2} \left[ \left( \gamma - 1 + \frac{1 + \psi}{\alpha} \right) \left( \frac{\sigma_\lambda^2}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \right)^2 \right]$$

$$+ \frac{1 + \Lambda - \alpha}{(1 + \Lambda) \alpha} \left( 1 + \frac{1}{\Lambda} \right)^2 \left( \frac{\phi_c g_0 + \phi \lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \right)^2 \sigma_\zeta^2$$

$$= (107)$$
If $\sigma^2_\zeta > 0$, the $g_0$ that maximizes this expression is

$$g_0 = \frac{\gamma^{-1-\frac{1+\psi}{\gamma}} \sigma_\lambda}{\gamma^{-1-\frac{1+\psi}{\gamma}} \sigma_\lambda + \phi_c^2 \sigma^2_\zeta}.$$  (110)

### C Proof of Proposition 5

**Step 1:** Substituting $a_t = 0$, the cash-in-advance constraint $c_t = m_t - p_t$, the monetary policy $m_t = g_0 \lambda_t$, and the guess $p_t = \theta \lambda_t$ into the equation for the profit-maximizing price yields

$$p^*_{i,t} = [(1-\phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t.$$

The attention problem of firm $i$ reads

$$\min_{\kappa \in \mathbb{R}^+} \left\{ \frac{\omega}{2} E \left[ (p_{i,t} - p^*_{i,t})^2 \right] + \mu \kappa \right\},$$

subject to

$$p^*_{i,t} = [(1-\phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t,$$

$$p_{i,t} = E[p^*_{i,t} | s_{\lambda,i,t}],$$

$$s_{\lambda,i,t} = \lambda_t + \zeta_{i,t},$$

and

$$\frac{1}{2} \log_2 \left( \frac{\sigma^2_\lambda}{\sigma^2_{\lambda|s_{\lambda}}} \right) \leq \kappa,$$

where the coefficient $\omega > 0$ is a non-linear function of the parameters appearing in the profit function and $\mu \geq 0$ is the per-period marginal cost of attention. The solution to this attention problem is

$$\kappa^* = \begin{cases} 
\frac{1}{2} \log_2 \left( \frac{\sigma^2_{\lambda|s_{\lambda}}}{\sigma^2_\lambda} \right) & \text{if } \frac{\sigma^2_{\lambda|s_{\lambda}}}{\sigma^2_\lambda} \geq 1 \\
0 & \text{otherwise} 
\end{cases}$$

The price set by firm $i$ in period $t$ then equals

$$p_{i,t} = [(1-\phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t + \zeta_{i,t},$$

$$p_{i,t} = [(1-\phi_c) \theta + \phi_c g_0 + \phi_\lambda] \frac{\sigma^2_\lambda}{\sigma^2_\lambda + \sigma^2_\zeta} (\lambda_t + \zeta_{i,t})$$

$$p_{i,t} = [(1-\phi_c) \theta + \phi_c g_0 + \phi_\lambda] \left( 1 - 2^{-2\kappa^*} \right) (\lambda_t + \zeta_{i,t}),$$

(111)
where
\[ \frac{\sigma^2}{\sigma^2} = 2^{2\kappa^*} - 1. \]
The price level and composite consumption in period \( t \) are then given by
\[ p_t = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] (1 - 2^{-2\kappa^*}) \lambda_t, \]
and
\[ c_t = \left[ g_0 - [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] (1 - 2^{-2\kappa^*}) \right] \lambda_t. \]

Thus, the set of rational expectations equilibria of the form \( p_t = \theta \lambda_t \) is given by the solutions to the following two equations
\[ \theta = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] (1 - 2^{-2\kappa^*}), \]
and
\[ \kappa^* = \begin{cases} \frac{1}{2} \log_2 \left( \frac{\frac{1}{2}[(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda]^2 \sigma^2_{\lambda}}{2^{\frac{1}{2}\mu(2)}} \right) & \text{if } \frac{\frac{1}{2}[(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda]^2 \sigma^2_{\lambda}}{2^{\frac{1}{2}\mu(2)}} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \]

**Step 2: Corner solution.** We now study under which conditions there exists a solution to equations (114)-(115) with the property \( \kappa^* = 0 \). It follows from equation (114) that \( \kappa^* = 0 \) implies \( \theta = 0 \). It follows from equation (115) that for \( \theta = 0 \) we have \( \kappa^* = 0 \) if and only if
\[ \frac{\frac{1}{2}[(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda]^2 \sigma^2_{\lambda}}{2^{\frac{1}{2}\mu(2)}} \leq 1. \]
Hence, there exists a rational expectations equilibrium of the form \( p_t = \theta \lambda_t \) with \( \kappa^* = 0 \) if and only if the weak inequality (116) is satisfied. Furthermore, equations (111)-(113) imply that in this equilibrium
\[ p_{t,t} = p_t = 0, \]
and
\[ c_t = g_0 \lambda_t. \]

**Step 3: Interior solutions.** Next we study under which conditions there exists a solution to equations (114)-(115) with the property \( \kappa^* > 0 \). Solving equation (114) for \( \theta \) yields
\[ \theta = \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})}. \]
Furthermore, in the case of an interior solution equation (115) reads
\[ \kappa^* = \frac{1}{2} \log_2 \left( \frac{\mu}{2 \ln(2)} \right). \]

Combining the last two equations yields
\[ \kappa^* = \frac{1}{2} \log_2 \left( \frac{\omega^2 \left[ (1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda \right]^2 \sigma^2_\lambda}{2 \ln(2)} \right). \]

Rearranging this equation yields a quadratic equation in \( 2^{\kappa^*} \)
\[ 2^{2\kappa^*} \phi_c - 2^{\kappa^*} \sqrt{\frac{\omega^2 (\phi_c g_0 + \phi_\lambda)^2 \sigma^2_\lambda}{2 \ln(2)}} + (1 - \phi_c) = 0. \]  

An interior solution has to satisfy the last equation as well as: \( 2^{\kappa^*} \in \mathbb{R}, 2^{\kappa^*} \geq 1 \) and
\[ \frac{\omega^2 (\phi_c g_0 + \phi_\lambda)^2 \sigma^2_\lambda}{2 \ln(2)} \geq \left[ 1 - (1 - \phi_c) \left( 1 - 2^{-2\kappa^*} \right) \right]^2. \]  

The two solutions to the quadratic equation (120) are
\[ x = \frac{b + \sqrt{b^2 - 4 \phi_c (1 - \phi_c)}}{2 \phi_c}, \]  
and
\[ x = \frac{b - \sqrt{b^2 - 4 \phi_c (1 - \phi_c)}}{2 \phi_c}, \]

where
\[ x \equiv 2^{\kappa^*}, \]
and
\[ b \equiv \sqrt{\frac{\omega^2 (\phi_c g_0 + \phi_\lambda)^2 \sigma^2_\lambda}{2 \ln(2)}}. \]

These two solutions are real if and only if
\[ b^2 \geq 4 \phi_c (1 - \phi_c). \]

Furthermore, when \( b^2 \geq 4 \phi_c (1 - \phi_c) \), the larger solution (122) satisfies \( x \geq 1 \) if and only if
\[ b \geq 1, \]

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and the smaller solution (123) satisfies $x \geq 1$ if and only if

$$\frac{b}{2\phi_c} \geq 1 \text{ and } b \leq 1.$$ 

Finally, when $x \geq 1$, condition (121) reads

$$0 \geq \phi_c x^2 - bx^2 + 1 - \phi_c.$$ 

Using the fact that

$$\phi_c x^2 - bx + (1 - \phi_c) = 0$$

yields

$$x^2 \geq x.$$ 

Thus, when $x \geq 1$, condition (121) is always satisfied. In summary, when $b \geq 1$, there exists an interior solution which is given by equation (122). When $b \leq 1$, $b \geq 2\phi_c$ and $b^2 \geq 4\phi_c (1 - \phi_c)$, there exists an interior solution which is given by equation (123). Otherwise there exists no interior solution.

**Step 4: Optimal policy.** To be written.
References

