Costly Contracts and Consumer Credit*

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Abstract

This paper explores the implications of technological progress in consumer lending. The model features households whose endowment risk is private information, and intermediaries which observe a noisy signal of each borrower's default risks. To offer a lending contract, an intermediary incurs a fixed cost. Each lending contract is comprised of an interest rate, a borrowing limit and a set of eligible borrowers. Technological improvements which lead to more accurate signals of a borrowers type or lower the cost of offering a contract increase the number of contracts offered and lead to the extension of credit to riskier households. This results in higher aggregate levels of defaults and borrowing. To corroborate the predictions of the model, we examine data on credit card borrowing reported by households in the Survey of Consumer Finance. We find that the number of different credit card interest rates reported (one measure of the "number" of contracts) has increased, that the empirical density of credit card interest rates has become much more disperse and lower income households' share of outstanding credit card debt has increased since 1983.

Keywords: Consumer Credit, Endogenous Financial Contracts, Bankruptcy.

JEL Classifications: E21, E49, G18, K35

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1 Introduction

Banks used to give credit cards only to the best consumers and charge them a flat interest rate of about 20 percent and an annual fee. But with the relaxing of usury laws in some states, and the ready availability of credit scores in the late 1980s, banks began offering cards with a variety of different interest rates and fees, tying the pricing to the credit risk of the cardholder. (New York Times, May 19, 2009)

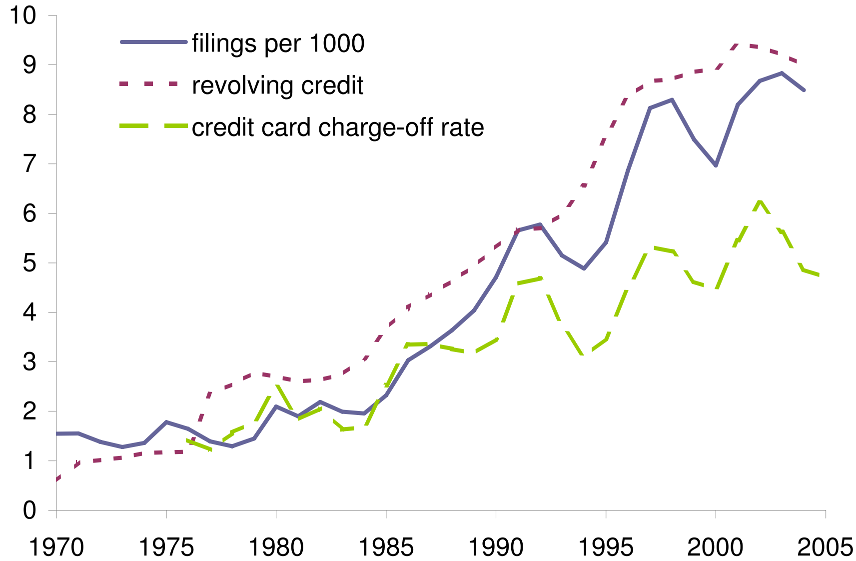
The market for unsecured consumer borrowing has undergone dramatic changes over the past thirty years. As Figure 1 illustrates, the surge in consumer bankruptcies was accompanied by a significant increase in both the outstanding volume of unsecured credit (especially credit card debt) and charge-offs rates. These aggregate changes have been accompanied by the diffusion and increased use of credit cards, which some have argued play a central role in the rise in bankruptcy and unsecured consumer borrowing (White (2007), Ellis (1998)).[1] In turn, it has been argued that the increase in credit card usage is driven by technological change in credit markets, which has changed the ability of lenders to accurately price risk and to offer contracts more closely tailored to the risk characteristics of different groups (Mann (2006)).

While the role of improved credit technology is frequently asserted, little work been done on whether the implications of improved credit evaluation technology for the degree of segmentation are consistent with the empirical evidence. To address this gap, we develop a simple incomplete markets model of bankruptcy to analyze the qualitative implications of improved credit technology on the equilibrium set of lending contracts. Following the consumer credit industry definition, we define a credit contract as an interest rate, a credit limit and a set of risk types which the lender will accept.[2] The model incorporates two elements which play a key role in determining

[1]This argument has been buttressed by recent quantitative incomplete market models of bankruptcy which have argued that changes in the supply of credit appear to have played a significant role in the rise of bankruptcies and unsecured borrowing over the past 30 years (Athreya (2004), Livshits, MacGee, and Tertilt (forthcoming)).

[2]The standard definition of a product in the consumer loan industry is "a collection of loans or

Figure 1: Aggregate Facts



Source: Livshits, MacGee, and Tertilt (forthcoming)

the conditions of credit contracts: asymmetric information about borrowers risk of default and a fixed cost to create each contract offered by lenders. This fixed cost of creating contracts means that some pooling of different observable risk types is optimal. Asymmetric information – which we introduce by assuming that lenders observe a noisy signal of a borrowers true default risk while the borrower is perfectly informed about their type – allows us to explore the effects of variations in the extent of adverse selection on consumer credit markets. We use this framework to analyze the effect of technological change which improves the accuracy of signals (thus mitigating adverse selection) as well as reductions in the fixed cost of creating contracts. Comparing the model predictions to the data allows us to evaluate the plausibility of stories which assign a key role to credit market innovations in the rise of unsecured borrowing and bankruptcies, as well as to asses the welfare consequences of these innovations.

While asymmetric information is a common element of credit market models, fixed costs of contract design are relatively unexplored. This is somewhat surprising since

lines of credit governed by *standard terms and conditions*." (Lawrence and Solomon (2002, p. 23)) texts targeted at credit market practitioners discuss significant fixed costs associated with developing consumer credit contracts. According to Lawrence and Solomon (2002), a prominent consumer credit industry handbook, the development of a consumer lending product development involves selecting the target market, researching the competition in the target market, designing the terms and conditions of the product, (potentially) testing the product[3], brand creation through advertisements, point-of-sale promotions and mass mailings, forecasting profitability, preparing a formal documentation of the product, an annual formal review of the product, and providing well-trained customer service tailored specifically to the needs of the product.[4] To a large extent, these costs are fixed for each product, rather than a function of the number of loans. Even after the initial product launch, account maintenance requires additional fixed costs, such as customer data base maintenance, costs involved in changing in the terms of the product, etc. Finally, it is worth noting that fixed costs are consistent with the fact that credit contracts are a differentiated product, with each product tailored to a specific segment of the market.

The model environment builds on the classic contribution of Jaffee and Russell (1976). Borrowers live for two periods, and have a stochastic endowment in the second period. Borrowers are hetereogeneous with respect to their probabilities of different endowment realizations. To offer a lending contract (comprised of a bond price, a borrowing limit, and a set of eligible consumer types), an intermediary incurs a fixed

[3]This involves the actual testing costs plus the delay induced by testing. The typical testing period in this industry is eighteen month (Lawrence and Solomon 2002).

[4]A similar process is described in other industry guidebooks. For example, Siddiqi (2006), outlines the development process of credit risk *scorecards.* A scorecard is a mapping from individual characteristics to a risk score for a particular subset of the population. Large issuers develop their own "custom scorecards" based on data from their own customers, while some firms use purchased data. Because of industry change as well as changes to the overall economic environment, scorecards are constantly updated (a scorecard is usually developed on data that's up to two years old), i.e. there is not one "true" mapping that once developed becomes a public good. Siddiqi (2006) gives an of a financial company that outsourced scorecard development and purchased ten different cards at an average cost of \$27,000 a card.

cost. There is free entry into the credit market, so that in equilibrium each contract earns zero profit. We assume that with probability α, this signal is accurate and with complementary probability $1-\alpha$ the signal is a random draw from the distribution of households types. Our equilibrium concept (which we largely formalize in the timing of the lending game) builds on work by Hellwig (1987), who discusses under what condition (pooling) equilibria exist in environments similar to ours.

We show that this environment generates a finite set of contracts. This is driven by the assumption that there is a fixed cost of contracts which implies that *some* "pooling" is optimal. A pooling contract offers cost savings per borrower, since the fixed cost can be spread across more consumers. The cost to the lowest risk types within a pool of a larger pool however, is that expanding the pool to include higher risk types increases the interest rate by increasing the average default premium. With free entry of intermediaries, these two forces lead to a finite set of contracts for any (strictly positive) fixed cost. Since a disproportionate share of the fixed cost is paid by lower risk types, this equilibrium features a form of cross-subsidization of borrowing by higher risk types within each contract segment.

We use the model to analyze the qualitative implications of two mechanisms via which improved information technology may have impacted credit markets. The first mechanism we explore is that improved information technology reduced adverse selection problems by improving the ability of lenders to predict prospective borrowers default risk, which facilitated the expansion of credit. The second possibility we consider is that improved information technology reduced the cost of designing and marketing financial contracts, which led to more contracts being offered, each targeted at smaller subsets of the population.

We find that technological improvements which make signals about a borrowers type more accurate lead to an increase in the number of contracts offered. The increase in the number of contracts leads to the extension of credit to riskier households. This generates more unsecured borrowing and an increase in defaults, since the "new" borrowers are more likely to default. Risk based pricing is increased both due to a reduction in the misclassification of high risk borrowers as low risk types as well a reduction in the measure of households served by each contract shrinks, which reduces the extent of cross-subsidization. We further find that technological improvements which lower the cost of offering a contract also generates an increase in the number of contracts, more risk based pricing and an extension of credit to riskier households.

The model also generates interesting insights into the possible relationship between the risk free interest rate and the average borrowing interest rate. In an influential paper, Ausubel (1991) documented that the decline in the risk-free rate in the U.S. in the 1980s was not accompanied by a decline in the average credit card rate. This led to a debate over whether or not the credit card industry was competitive. We show in our model that a decline in the risk free rate can sometimes lead to higher average borrowing interest rate. The mechanism is that a decline in the risk free rate makes borrowing more attractive, and can thus lead to an increase in the number of contracts offered in equilibrium. Since new contracts are offered to riskier borrowers, the average borrowing interest can increase if the average risk premium on borrowing increases by more than the fall in the risk-free rate.

The second task we tackle in this paper is to assemble extensive data on the number of unsecured consumer credit contracts targeted at specific types (groups) of borrowers. To measure the number and distribution of credit contracts across consumers we look at two specific features of credit contracts: the interest rate and credit limit. We pay particular attention to the distribution of credit card interest rates. Using data from the Survey of Consumer Finance, we document a large increase in the number of different credit card interest rates reported by households since 1983. More strikingly, the empirical density of credit card interest rates has become much "flatter" since 1983. While in 1983 nearly 55% of households reported they faced the same credit rate (18%), by the late 1990s no single credit card interest rate was reported by more than 10% of households. We also document a similar pattern in data on interest rates for 24-month consumer loans and credit cards from surveys of banks conducted by the Board of Governors. These shifts in the distribution of interest rates have also been accompanied by increased lending to lower income households. These features – an increase in contract variety, an increased spread between the lowest and highest interest rates offered, and an expansion of credit to lower income consumers – are all consistent with the predictions of our model.

The equilibrium model of bankruptcy that we use is related to recent work on equilibrium models of consumer bankruptcy (See Athreya (2005) for a survey). Both Livshits, MacGee, and Tertilt (2007) and Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) outline dynamic equilibrium models where interest rates vary with borrowers' characteristics, and show that for reasonable parameter values, these models can match the level of U.S. bankruptcy filings and debt-income ratios. Livshits, MacGee, and Tertilt (forthcoming) argue that a rise in income and expense (such as uninsured medical expenses) plays a small role in accounting for the rise in filings and unsecured credit. Instead, they (and Athreya (2004)) model financial innovation as impacting consumer lending via two adhoc channels: a fall in the cost ("stigma") of bankruptcy and reduced transaction cost of lending. These papers conclude that the changes in credit markets appear to be largely driven by these "supply-side" factors. One limitation of this literature is that it relies on reduced form ways of modeling financial innovation. As a result, this literature has relatively little to say about how improved information technology may have changed the set of consumer credit contracts offered or facilitated the extension of credit to riskier borrowers by more accurate pricing of borrowers default risk (Barron and Staten (2003)). In recent work, Chatterjee, Corbae, and Rios-Rull (2007) and Chatterjee, Corbae, and Rios-Rull (2006) present the first formal model of the role of credit histories and credit scoring in supporting the repayment of unsecured credit.

Closely related to the story we explore is recent work by Narajabad (2008), Drozd and Nosal (2008), Sanchez (2008), and Athreya, Tam, and Young (2008) who also explore whether improved information technologies led to an extension of credit to riskier borrowers. Narajabad (2008) formalizes this mechanism in a model without adverse selection, since he assumes that consumers do not know their own riskiness, while lenders see a noisy signal on a borrowers type. The key mechanism in his framework is a shift in the intensive margin, as relatively low risk borrowers are able to borrow higher amounts, which in turn increases their probability of defaulting. In a relevant empirical contribution, Edelberg (2006) examines PSID and SCF data and finds that the risk-based pricing of consumer loans has increased over the past twenty years.

Our work is also closely related to Adams, Einav, and Levin (2009) and Einav, Jenkins, and Levin (2008) who analyze contract pricing in the context of subprime auto loans. Adams, Einav, and Levin (2009) find evidence that subprime lenders face both moral hazard and adverse election problems in this market. Building on these findings, Einav, Jenkins, and Levin (2008) use a monopolistic lending model to study pricing and contract design in this market. They conclude that the return to investing in technology to evaluate loan applicants (i.e. credit scoring) is significant.

The remainder of the paper is organized as follows. Section 1.1 documents technological progress in the financial sector over the last couple decades, Section 2 sets up the general model. In Section 3 we characterize the set of equilibrium contracts, while in Section 4 we explore the implications of improved signal accuracy and a decline in the fixed cost. Section 6 examines data on the terms of consumer unsecured borrowing (especially interest rates). Section 7 concludes.

1.1 Financial Innovation

It is frequently asserted that the past thirty years have witnessed the diffusion and introduction of numerous innovations in consumer credit markets (Mann (2006)). Many of these changes are related to the rapid improvements in information technology, which has significantly reduced the cost of processing information and led to large increases in information sharing on borrowers between financial intermediaries (Barron and Staten (2003), Berger (2003), Evans and Schmalnsee (1999)). It has been argued that this has increased the analysis of the relationship between borrower characteristics and loan performance by lenders to better price loans (Barron and Staten 2003). Despite the existence of a broad descriptive literature on financial innovation, there are very few empirical studies documenting the extent of it quantitatively.[5] Here we briefly outline some suggestive evidence of significant technological innovations, especially in the credit card market where most of the expansion of unsecured consumer credit has taken place. One very broad measure of technological progress is reported in Berger (2003) who compares labor productivity in the commercial banking sector with labor productivity in the rest of the economy. Figure 2 shows that between 1980 and 2000, productivity growth in the banking sector was about twice as high as in the non-farm business sector. More specific evidence on crucial innovations in the credit card industry is as follows:

- The development of improved credit-scoring techniques to identify and then monitor creditworthy customers, during the 1970s. These systems became increasingly widely used during the 1980s and 1990s.[6]

- Increased used of computers to process information to facilitate customer acquisition, designing credit cards, marketing, as well as monitoring repayment, and debt collection.

- Increased securitization of credit card debt (starting in 1987).

There are two direct pieces of evidence that suggest that technological innovations related to shifts in the cost of processing information have diffused and become widely used. First, there was a substantial spread of credit scoring throughout the consumer credit industry during the 1980s and 1990s (McCorkell (2002), Engen (2001), Asher

[5] Frame and White (2004) in a recent survey of the literature on financial innovation noted that: "A striking feature of this literature [...] is the *relative dearth of empirical studies* that [...] provide a quantitative analysis of financial innovation."

[6] The most prominent player is Fair Isaac Cooperation, the developer of the FICO score. Fair Isaac started building credit scoring systems in late 1950s, although the first credit card scoring system was not delivered until 1970. In 1975 Fair Isaac introduced the first behavior scoring system to predict credit risk related to existing customers. In 1981 the Fair Isaac credit bureau scores were introduced. For details see: http://en.wikipedia.org/wiki/Fair_Isaac

(1994)).[7] Several authors have argued that the development and spread of credit scoring was necessary for the growth of the credit card industry (Evans and Schmalnsee (1999), Johnson (1992)). The diffusion of credit scoring is reflected in usage figures reported by the American Banking Association (ABA). The fraction of large banks using credit scoring as a loan approval criteria increased from half in 1988 to nearly seven-eights in 2000, and the fraction of large banks using fully automated loan processing (for direct loans) increased from 12 percent in 1988 to nearly 29 percent in 2000 (Installment Lending Report 2000). While larger banks are more likely to adopt credit scoring than smaller banks (Berger (2003)), banks of any size can access this technology by purchasing scores from other providers.[8] Barron and Staten (2003) argue that credit cards companies during the early 1990s rapidly expanded their use of risk based pricing, which led to substantial declines in interest rates for low risk customers and increased interest rates for higher risk consumers. There is also evidence that the adoption of automated credit scoring systems decreased the time and cost required to evaluate loan applications (Mester (1997)).

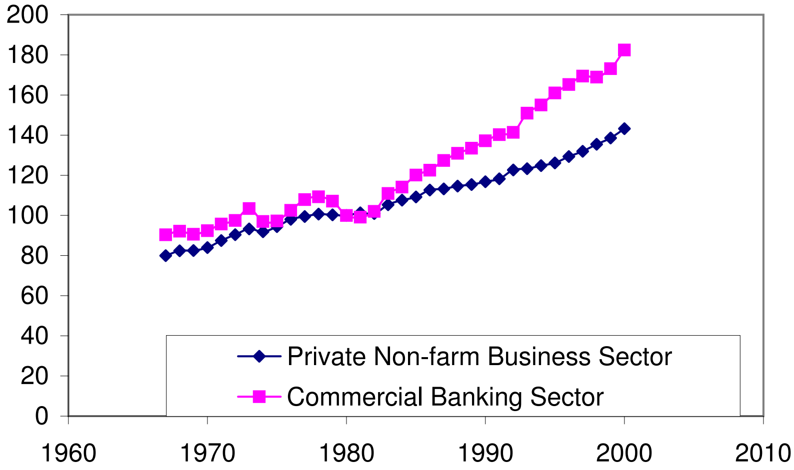
The second (related) piece of direct evidence is the rapid increase in information on borrowers collected by credit bureaus and purchased by lenders. For every credit-using person in the United States, there is at least one (more likely three) credit bureau files (Hunt 2002). The number of credit reports issued has increased dramatically from 100 million in 1970 to 400 million in 1989, to more than 700 million today. This reflects the widespread adoption of credit scoring to evaluate loan applicants. The information in these files is widely used by lenders, as more than 2 million credit

[7] Credit scoring is the evaluation of the credit risk of loan applicants using historical data and statistical techniques ((Mester 1997)). Credit scores are used both to evaluate initial loan applications, and to adjust the interest rates and credit limits of revolving (credit card) debts (up and down).

[8] Further support for the significant impact of credit scoring on lending comes from studies of small business lending. Frame et al (2001) find that the adoption of credit scoring by banks to evaluate small business loans led to lending, while Berger, Frame and Miller (2002) found that credit scoring led to the extension of credit to "marginal applicants" at higher interest rates.

reports are sold by credit bureaus in the U.S. daily (Riestra (2002)).[9]

Figure 2: Productivity: Commercial Banking vs. Private Business Sector



Source: Berger (2003), Table 5.

There has also been significant innovations in how credit card companies finance their operations. Beginning in 1987, credit card companies began to securitize their portfolio of credit card receivables. As can be seen form Table 1, by 2005 nearly half of all balances are now securitized. This has led to a reduction in the costs of financing credit card operations.

However, not all technological progress in the financial sector was directly related to a better assessment of credit risk. Other innovations took place that simply increased the efficiency of designing credit cards, marketing credit cards, and processing accounts. Some of this progress can be thought of a reduction in the cost of credit per account, while other parts may be interpreted as a reduction in the fixed cost of entering this market with a differentiated product.

[9]In Canada and the U.S., credit bureaus report data on borrowers payment history, the stock of current debt and any public judgments (such as bankruptcy).

Table 1: **Measures of Technological Progress in the Financial Sector**

Measure of Innovation	then	and now	Source
Credit scoring as loan approval tool*	50% (1988)	85% (2000)	ILR 2000
Credit Reports issues	400 mln (1989)	700 mln (2001)	Riestra (2002)
Credit Card Payments	10.4 bln (1995)	15 bln (2000)	Berger (2003)
Fully automated loan processing*	12% (1988)	29% (2000)	ILR 2000
Mail solicitations	1.1 bln (1990)	5.23 bln (2004)	Synovate**
Securitization as a share of all credit card balances held by banks	26.7% (1991)	48.3% (2005)	Fed§

* for large banks

** *Mail Monitor,* Synovate, as cited in Federal Reserve Board (2006).

§ Federal Reserve Board (2006)

2 Model Environment

We analyze a two period small open economy with incomplete markets. The economy is populated by a continuum of borrowers each of whom faces stochastic income in period 2. Markets are incomplete in that only non-contingent contracts can be issued. Borrowers can default on contracts and incur exogenous costs associated with bankruptcy. Financial intermediaries are competitive and have access to funds at an exogenously given (risk-free) interest rate. The creation of each financial contract (characterized by a lending rate, a borrowing limit and eligibility requirement for borrowers) requires the payment of a fixed cost χ.

2.1 People

The economy is populated by a continuum of 2-period lived households. For simplicity, we assume that borrowers are risk-neutral, with preferences represented by:

$$c_1 + \beta E c_2$$

Each household receives the same deterministic endowment of y_1 units of the consumption good in period 1. The second period endowment, y_2^i, is stochastic. The endowment can take one of two possible values: $y_2^i \in \{y_h, y_l\}$, where $y_h > y_l$. Households differ in their probability ρ_i of receiving the high endowment y_h. The expected value of income of household i is

$$E_i y_2 = (1 - \rho_i) y_l + \rho_i y_h$$

We identify households with their type ρ_i. ρ is distributed uniformly on $[a, 1]$, where $a \geq 0$. Households know their own type.

2.2 Signals

While each household knows their own type, other agents can only observe a public signal, σ_i, regarding household i's type. With probability α, this signal is accurate: $\sigma_i = \rho_i$. With complementary probability $(1 - \alpha)$, the signal is an independent draw from the ρ distribution ($U[a, 1]$). Thus, α is the precision of the public signal.

2.3 Bankruptcy

There is limited commitment by borrowers. We model this as a bankruptcy system, whereby borrowers can declare bankruptcy in period 2. The cost of bankruptcy to a borrower is the loss of fraction γ of the second-period endowment. Lenders do not recover any funds from bankrupt borrowers.

2.4 Financial Market

Financial markets for borrowing and lending are competitive. Financial intermediaries can borrow (or save) from the (foreign) market at the exogenously given interest rate r. Financial intermediaries accept deposits from savers and make loans to borrowers. Loans take the form of one period non-contingent bond contracts. However, the bankruptcy option introduces a partial contingency by allowing bankrupts to discharge their debts.

Throughout the paper, we assume that $\beta < \frac{1}{1+r} = \bar{q}$, so that households want to borrow as much as possible (at actuarially fair prices), and never want to save. What limits the households' ability to borrow is their inability to commit to repaying loans.

Financial intermediaries must incur a fixed cost χ in order to offer a non-contingent lending contract to (an unlimited number of) households. Endowment-contingent contracts are ruled out (due to non-verifiability of the endowment realization). A contract is characterized by $(L, q, \underline{\sigma})$, where L is the face value of the loan, q is the per-unit price of the loan (so that qL is the amount advanced in period 1 in exchange for a promise to pay L in period 2), and $\underline{\sigma}$ is the minimal value of public signal that makes a household eligible for the contract.[10]

Intermediaries observe the public signal about a household's type, but not the actual type. Households are allowed to accept only one contract, so the intermediaries know the total amount being borrowed. Intermediaries forecast the default probability of loan applicants, and decide to whom to grant loans.

Profit maximization implies that intermediaries never offer loans to types on which they would make negative expected profits, which implies that the expected value of repayments cannot be lower than the cost of the loan to the intermediary.

In equilibrium, free entry implies that intermediaries earn zero expected profits on their *loan portfolio.* The bond price incorporates the fixed cost of offering the contract, so that in equilibrium the operating profit of each contract equals the fixed cost.

2.5 Timing

The timing of events in the financial markets is as follows:[11]

[10] Alternatively, we can specify the contract as just (L, q) and have the eligibility set (characterized by $\underline{\sigma}$) be an equilibrium outcome.

[11] This timing is necessary for the existence of (partially) pooling equilibria in the environment with imperfect public signals. (Hellwig 1987) discusses the key role of timing in guaranteeing existence of equilibrium.

1.a. Intermediaries pay fixed costs χ of entry and announce their contracts. While this stage can be modeled as simultaneous move game, we prefer to think of it as sequential – the stage does not end until no new intermediary wants to enter (having observed the contracts already being offered).

1.b Households observe all offered contracts and choose which one to apply for (realizing that some intermediaries may choose to exit the market).[12]

1.c Intermediaries, who paid the entry cost, decide whether to stay in the market and advance loans to qualified applicants or to exit the market.[13]

1.d Loans are advanced to qualified applicants by lenders who remain in the market. We can further split this stage into two sub-stages: Successful applicants are notified, and then they make their choice of lenders.

2.a Households realize their endowments in period 2, and make their default decisions.

2.b Non-defaulting households repay their loans.

2.6 Equilibrium

We defer the complete definition of equilibrium — which involves specifying agents' beliefs on and off the equilibrium path — to Section 3. Abstracting from the question of beliefs, a standard (non-game-theoretic) equilibrium satisfies the following conditions.

An *equilibrium*[14] is a set of active contracts $\mathcal{K}^* = \{(q_k, L_k, \underline{\sigma}_k)_{k=1,\ldots,N}\}$ and con-

[12]To simplify the analysis, we could introduce ϵ cost of sending an application, so that each household applies only for a single contract which will be offered in equilibrium.

[13]This stage is not necessary in the environment with perfect signals (Section 3.2) but is essential to ensure existence of equilibria under asymmetric information.

[14]This is a description of a competitive equilibrium that comes out of the (sequential) game specified in section 2.5. For a full description of (sequential) equilibrium, which also includes the set of beliefs of all players (entrants and households) on and off the equilibrium path, see Section **??**.

sumer contract decision rules $\kappa(\rho, \mathcal{K}) \in \mathcal{K} \cup \{(0,0,0)\}$ for each type ρ such that

1. Given $\{(q_k, L_k, \underline{\sigma}_k)_{k \neq j}\}$ and consumer contract decision rules, each (potential) bank j maximizes profits by making the following choice: to enter or not, and if it enters, it chooses contract $(q_j, L_j, \underline{\sigma}_j)$ and incurs fixed cost χ.

2. Given any $\mathcal{K}$, a consumer of type ρ chooses which contract (if any) to accept so as to maximize expected utility. Note that a consumer of type ρ can choose a contract k only if $\rho \geqslant \underline{\sigma}_k$.

3 Characterizing Equilibrium

We focus on a pure strategy equilibrium with pooling within the public types. A contract is characterized by $(L, q, \underline{\sigma})$, where L is the maximum face value of the loan, q is the per-unit price of the loan (so that qL is the amount advanced in period 1 in exchange for the promise to pay L in period 2), and $\underline{\sigma}$ is the minimal value of public signal that makes a household eligible for the contract.

3.1 Characterizing Equilibrium Contracts

We begin by characterizing the face value of possible equilibrium contracts. Contracts in the model can vary along two key dimensions: the face value L to be repaid in period 2, and bond price q. The key result is that all possible lending contracts are characterized by one of two face values. The risk-free borrowing contract has a face value equal to the cost of bankruptcy in the low income state, so households are always willing to repay this contract in equilibrium. Risky-lending contracts have the maximum face value such that in the high income state borrowers are always willing to repay. Contracts with lower face value are not offered in equilibrium since, if (risk-neutral) households are willing to borrow at a given price, they want to borrow as much as possible at that price.

Formally, the first result is that free entry leads to zero profits net of the cost of offering contracts.

Proposition 3.1. *All contracts offered earn exactly χ profits (valued as of period 1).*

Proof. Profits of less than χ preclude entry in the pure strategy equilibrium. Profits of more than χ would generate entry of a competing contract with better terms. □

The second result is that the face value of all risky contracts is the same. This result is tied to the fact that separating contracts are not an equilibrium outcome of the game.

Proposition 3.2. *There are at most two types of contracts offered in equilibrium: risk-free contract with $L = \gamma y_l$ and $\underline{\sigma} = a$, and N risky contracts with $L = \gamma y_h$. Risky contracts are repaid by all households who realize high endowment y_h in period 2.*

Proof. The risk-free contract with $L = \gamma y_l$ dominates all other risk-free contracts (and thus generates the highest possible profit). Risky contracts with $L > \gamma y_h$ cannot be offered as they would never be repaid.

The more interesting result is that no risky contracts with $L' < \gamma y_h$ are offered. It is clear that, keeping the price q constant, all households who would apply for (L', q) would prefer $(\gamma y_h, q)$, and it would generate greater profits for the lender. However, there is potential for cream-skimming by offering a smaller loan L' with a slightly better price $q' > q$ (such that $q'L' < qL$) so that (L', q') is preferred to (L, q) by households with high unobservable type ρ. Households with a low ρ would prefer (L, q), since they are less likely to repay and hence are willing to promise the larger repayment L in exchange for the larger advance qL. Such cream-skimming is never an equilibrium outcome, due to the specific timing (see Section 2.5). If an entrant attempts to cream-skim from an existing contract by offering a contract (L', q') that is preferred to the existing contract only by the better types, the equilibrium of that subgame has the incumbent contract exiting the market (having realized that the "good" customers have applied for the new contract). As a result, the "bad" customers (with low unobservable ρ) also apply for the new contract, even though they prefer the terms of the incumbent contract. Thus, "cream-skimming" fails, and the entrant makes lower profit than would the incumbent who was "cream-skimmed" (since both $q' > q$ and $L' < L$). Since the incumbent contract was exactly recovering the fixed cost χ, such an entry is unprofitable. □

Proposition 3.3. *Every lender offering a risky contract at price q rejects an applicant* **iff** *the expected profit from that applicant is negative.*

Corollary 3.4. *When $\alpha = 1$ (and hence, $\sigma_i = \rho_i$), every lender offering a risky contract at price q rejects an applicant* **iff** $\rho < \underline{\rho}(q) = \frac{q}{\bar{q}}$.

This implies that the "riskiest" household accepted by a risky contract makes no contribution to overhead cost χ. If a risk-free contract is offered in equilibrium, the eligibility set for that contract is unrestricted.

Proposition 3.5. *The interval of public types served by each risky contract is of size* $\theta = \sqrt{\frac{2\chi(1-a)}{\alpha\bar{q}\gamma y_h}}$.

Proof. This result follows from Propositions 3.1 and 3.3 and the assumption of uniform distribution of types. From Proposition 3.3, the lowest public type accepted, $\underline{\sigma}$, generates zero expected profits. Any public type greater than that, $\sigma' = \underline{\sigma} + \delta$, generates positive profits of $\alpha\delta\bar{q}\gamma y_h$ per customer. It is worth noting that the measure of households with public signal σ' who actually borrow is not the same across contracts — this measure is greater for the "top" contracts (lower k's). This follows from the fact that some households with inaccurately low signals ($\rho > \sigma$) will opt out of the risky borrowing contract — we denote by $\overline{\rho}(q)$ the highest underlying type willing to accept a risky contract with price q. Yet, the *difference* in profitability between σ' and $\underline{\sigma}$ is the same across the contracts. To see that, consider first the expected profits *per customer* of public type σ':

$$\begin{aligned}
\mathrm{E}\pi(\underline{\sigma}+\delta) &= \overline{q}\mathrm{E}(\rho|\sigma=\underline{\sigma}+\delta,\rho>\overline{\rho}(q_k))\gamma y_h - q_k\gamma y_h \\
&= \overline{q}\left(\mathrm{E}(\rho|\sigma=\underline{\sigma},\rho>\overline{\rho}(q_k)) + \frac{\alpha\delta+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}0}{\alpha+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}}\right)\gamma y_h - q_k\gamma y_h \\
&= \mathrm{E}\pi(\underline{\sigma}) + \frac{\alpha\delta\overline{q}\gamma y_h}{\alpha+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}} \quad = \quad \frac{\alpha}{\alpha+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}}\delta\overline{q}\gamma y_h
\end{aligned}$$

The expression $\frac{\alpha}{\alpha+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}}$ is the fraction of *participating* customers for whom the signal is accurate. It is critical to note that the participation cutoff $\overline{\rho}(q_k)$ is the same for all public types within the k-contract. While a lower $\overline{\rho}$ effectively makes signal (locally) more precise by lowering the fraction of customers with inaccurate signals (thus increasing the difference in profitability between σ' and $\underline{\sigma}$), it also lowers the number of customers of a given observed type. These two forces exactly offset each other — the expected profits from the *whole* public type σ' are:

$$\begin{aligned}
\mathrm{E}\Pi(\underline{\sigma}+\delta) &= \mathrm{E}\pi(\underline{\sigma}+\delta)\cdot\frac{\alpha+(1-\alpha)\frac{\overline{\rho}(q_k)-a}{1-a}}{1-a} \\
&= \frac{\alpha\delta\overline{q}\gamma y_h}{1-a}
\end{aligned}$$

From Proposition 3.1, we know that

$$\int_0^\theta \mathrm{E}\Pi(\underline{\sigma}+\delta)d\delta = \int_0^\theta \frac{\alpha\delta\overline{q}\gamma y_h}{1-a}d\delta = \frac{\alpha\overline{q}\gamma y_h}{1-a}\cdot\frac{\theta^2}{2} = \chi$$

□

Proposition 3.6. *With α sufficiently high, we can support the equilibria with partial pooling within public types.*

Proof. What we have to show is twofold: 1) that there is no profitable deviation (by other intermediaries) which would unravel the pooling equilibrium, and 2) that the participation cutoff is above the target group: $\overline{\rho}(q_k) \geqslant \sigma_{k-1}$.

(1) A profitable deviation which could unravel the pooling equilibrium would offer an alternative contract that is attractive only to "good" (private) types. In our environment, such deviation would include slightly lower face value of the debt L'

with slightly (but sufficiently) better price q'. What rules out such deviation is our timing which includes application and exit stages (see 1.b and 1.c in Section 2.5). If such a deviation were introduced, the households would recognize that the original pooling contract is no longer viable and would not be offered in equilibrium. Thus, both "good" and "bad" private types would apply for the new (deviation) contract, thus making it unprofitable ex-ante.

(2) In the case $\alpha = 1$, we determine the number of contracts by effectively comparing $\overline{\rho}(q_k)$ with $\underline{\rho}_{k-1}$. Since everything is continuous in α, the inequalities $\overline{\rho}(q_k) > \sigma_{k-1}$ should still hold when α is close enough to 1. We may lose the last risky contract, especially if $\overline{\rho}(q_K) = \underline{\rho}_{K-1}$ held with equality for the last contract under $\alpha = 1$. □

3.1.1 Household Problem and Participation Constraints

Given a choice between multiple risky contracts, households always prefer the risky contract with the highest q that they are eligible for. Thus, the households' decision problem can be characterized as choosing between the best risky contract (if one exists) offered that will accept them, the risk-free contract and autarky (conditional on the risk-free contract and a risky contract being offered to them in equilibrium). We formalize this choice problem in three household participation constraints.

We begin by considering the participation constraints of households with accurate signals ($\sigma_i = \rho_i$). The problem of a consumer of type ρ with public signal $\sigma = \rho$ is:

$$\max \left\{ v_\rho(q, L), v_\rho(q_{rf}, L_{rf}), v_\rho(0, 0) \right\},$$

where the value of an arbitrary risky contract (q, L) is

$$v_\rho(q, L) = qL + \beta \left(\rho(y_h - L) + (1 - \rho)(1 - \gamma)y_l \right), \tag{3.1}$$

the value of a risk-free contract (q_{rf}, L_{rf}) is

$$v_\rho(q_{rf}, L_{rf}) = q_{rf} L_{rf} + \beta \left(\rho y_h + (1 - \rho)y_l - L_{rf} \right), \tag{3.2}$$

and the value of autarky is

$$v_\rho(0, 0) = \beta \left(\rho y_h + (1 - \rho)y_l \right). \tag{3.3}$$

The easiest participation constraint to analyze is that comparing autarky (equation (3.3)) and the risk-free contact (equation (3.2)). The risk free contract dominates autarky whenever

$$q_{rf} \geqslant \beta \tag{3.4}$$

This has a straightforward interpretation. The risk-free contract will be accepted whenever the number of people willing to accept the risk-free contract is large enough relative to the fixed cost that the bond price exceeds the discount factor.

We now turn to the two participation constraints involving the risky contract. A household will prefer the risky contract (q, L) to autarky whenever $v_\rho(q, L) \geqslant v_\rho(0, 0)$. This reduces to

$$q \geqslant \beta \left(\rho + (1-\rho) \frac{\gamma y_l}{L} \right). \tag{3.5}$$

for all ρ in the eligibility set who are not eligible for a better risky contract. For the risk-free household ($\rho = 1$), this collapses to $q \geqslant \beta$.

Finally, if the risk-free contract is offered in equilibrium (and thus preferred to autarky), households face a choice between a risky and the risk-free contract. The risky contract then has to satisfy $v_\rho(q, L) \geqslant v_\rho(q_{rf}, L_{rf})$, which reduces to

$$q \geqslant (q_{rf} - \beta) \frac{\overline{L}_{rf}}{L} + \beta \left(\rho + (1-\rho) \frac{\gamma y_l}{L} \right) \tag{3.6}$$

for all ρ in the eligibility set.

These constraints have important implications for the set of equilibrium contracts. First, consider a risky contract (q, L) offered to households in an interval $[\underline{\rho}, \overline{\rho}]$. Recall that the risky contracts have $L > \gamma y_l$ (otherwise the contract would be risk-free), which implies that $\frac{\gamma y_l}{L} < 1$, and the right-hand side of both equation (3.5) and (3.6) are increasing in ρ. Hence, we only need to check the participation constraint for the least risky type in an interval covered by a risky contract since if a participation constraint does not bind for the highest type in the interval, $\overline{\rho}$, it will not bind for any household $\rho < \overline{\rho}$. Second, note that as one moves from one risky interval to a riskier contract, q decreases. This makes the left-hand side of equations (3.5) and

(3.6) smaller, which makes it less likely that the risky contract for that risk bin will be preferred to the risk-free contract or autarky.

Imperfect signals complicate households' participation decisions. Since the right-hand side of the participation constraints above is increasing in ρ, households with signals greater than their true type ($\sigma_i > \rho_i$) will always accept the risky contracts they are eligible for if households with $\sigma_i = \rho_i$ find it optimal to accept them. On the other hand, households with public signals below their true type ($\sigma_i < \rho_i$) may choose to opt out of the risky contracts offered to them and choose autarky or the risk-free contract (if it is offered in equilibrium).

Exploiting the monotonicity of the incentive constraint (3.5) in ρ, we can specify the highest true type that would choose a risky contract (L, q) over autarky:

$$\overline{\rho}_{aut}(q, L) = \frac{qL - \beta\gamma y_l}{\beta(L - \gamma y_l)}. \tag{3.7}$$

Analogously, the highest type that would prefer a risky contract (L, q) to the risk-free contract (L_{rf}, q_{rf}) (where in equilibrium $L_{rf} = \gamma y_l$) is

$$\overline{\rho}_{rf}(q, L) = \frac{qL - (q_{rf} - \beta)L_{rf} - \beta\gamma y_l}{\beta(L - \gamma y_l)} = \frac{qL - q_{rf}\gamma y_l}{\beta(L - \gamma y_l)}. \tag{3.8}$$

3.1.2 The Set of Equilibrium Contracts

As can be seen from Proposition 3.1, all risky contracts have the same face value, $L = \gamma y_h$, and differ only in the price (and eligibility set). We order these contracts by the riskiness of the clientele served by the contract, from the least to the most risky. We assume throughout our characterization that there is a strictly positive cost $\chi > 0$ of creating a contract.

The equilibrium is characterized by a finite number N of risky contracts, which serve consecutive intervals $[\underline{\sigma}_n, \underline{\sigma}_{n-1})$ of public types (which we continue to order from the best to the worst types targeted), and a possible risk-free contract, all of which generate operating profit of exactly χ. Each of these contracts has a bond price q_n and an acceptance set of public types. In addition, one has to specify the set of true types $([a, \overline{\rho}(q, L)])$ that accept a risky contract when their public type falls into the eligibility set for that risky contract. Furthermore, the set of potential customers for a risk free contract includes not only all the households who are not eligible for risk-free contracts, but also households with public signals less than their true type who choose not to accept risky contracts they are eligible for.

Proposition 3.7. *The risky contracts are of the form $(q_n, L, \underline{\sigma}_n)$, where $L = \gamma y_h$,*

$$\underline{\sigma}_n = 1 - n\sqrt{\frac{2\chi(1-a)}{\alpha\bar{q}L}}. \tag{3.9}$$

The first contract serves the interval $[\underline{\sigma}_1, 1]$, and each subsequent contract serves the interval $[\underline{\sigma}_n, \underline{\sigma}_{n-1})$. If the participation constraints of borrowers with $\sigma < \rho$ does not bind ($\overline{\rho}(q_n, L) \geqslant 1$), then $q_n = \bar{q}\left(\alpha\underline{\sigma}_n + (1-\alpha)\frac{1+a}{2}\right)$. If the participation constraints of borrowers with $\sigma < \rho$ does bind, then

$$\bar{q}\underline{\sigma}_n\alpha = q_n\left(\alpha + (1-\alpha)\frac{\overline{\rho}_n - a}{1-a}\right) - \bar{q}(1-\alpha)\frac{(\overline{\rho}_n)^2 - a^2}{2(1-a)},$$

where $\overline{\rho}_n = \overline{\rho}(q_n, L)$ is given by equation (3.7) or (3.8).

Proof. (1) We start with the case where the participation constraints of borrowers with $\sigma < \rho$ do not bind. Clearly, intermediaries only accept applicants who deliver non-negative expected profit. That implies that for any contract with price q, the marginal type $\underline{\sigma}$ satisfies

$$\left(\alpha\underline{\sigma} + (1-\alpha)\frac{1+a}{2}\right)\bar{q} = q. \tag{3.10}$$

We now use the free entry condition to pin down the equilibrium values:

$$\left(\alpha\int_{\underline{\sigma}_n}^{\underline{\sigma}_{n-1}} \frac{\rho}{1-a}d\rho + (1-\alpha)\frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{1-a}\frac{1+a}{2}\right)\bar{q} - \frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{1-a}q_n = \frac{\chi}{L} \tag{3.11}$$

Using equation (3.10) to eliminate q_n, and rearranging:

$$\underline{\sigma}_{n-1} - \underline{\sigma}_n = \sqrt{\frac{2\chi(1-a)}{\alpha\bar{q}L}} \tag{3.12}$$

(2) Noisy signals create the possibility of adverse selection. Households with signals $\sigma > \rho$ will always accept a contract that a $\sigma = \rho$ type would. However, high types

with low signals ($\sigma < \rho$) may choose not to borrow via the risky contract they are eligible for and opt for the risk-free contract or autarky.

Note that this is not an issue for the first risky contract $(q_1, \bar{L})$. For each risky contract $(q_n, \overline{L})$ with $n \geqslant 2$, let $\overline{\rho}_n$ denote the cutoff such that all households with $\rho \geqslant \overline{\rho}_n$ (and $\sigma \in (\underline{\sigma}_n, \underline{\sigma}_{n-1}])$ would not purchase contract (q_n, L). Suppose that the participation constraint of high types with low signals does not bind for the first $n-1$ contracts, but binds for contract n. The n^{th} contract covers households with public types in the interval $(\underline{\sigma}_n, \underline{\sigma}_{n-1}]$, where the cut-offs are given by:

$$\underline{\sigma}_{n-1} = \frac{1}{\alpha}\left(\frac{q_{n-1}}{\bar{q}} - (1-\alpha)\frac{1+a}{2}\right) \tag{3.13}$$

$$\underline{\sigma}_n = \frac{1}{\alpha}\left[\frac{q_n}{\bar{q}}\left(\alpha + (1-\alpha)\frac{\overline{\rho}_n - a}{1-a}\right) - (1-\alpha)\frac{(\overline{\rho}_n)^2 - a^2}{2(1-a)}\right] \tag{3.14}$$

The operating profit can be decomposed into the contributions from households with accurate and inaccurate signals. The zero profit condition $\Pi_n = \chi$ is given by:

$$\begin{aligned} 0 = \Pi_n - \chi \quad &= \quad \frac{\bar{q}L}{1-a}\left(\alpha\frac{(\underline{\sigma}_{n-1})^2 - (\underline{\sigma}_n)^2}{2} + (\underline{\sigma}_{n-1} - \underline{\sigma}_n)(1-\alpha)\frac{(\overline{\rho}_n)^2 - a^2}{2(1-a)}\right) \\ &- \quad \frac{q_n L}{1-a}(\underline{\sigma}_{n-1} - \underline{\sigma}_n)\left(\alpha + (1-\alpha)\frac{\overline{\rho}_n - a}{1-a}\right) \end{aligned}$$

Using equation (3.14) to eliminate q_n yields:

$$\underline{\sigma}_{n-1} - \underline{\sigma}_n = \sqrt{\frac{2\chi(1-a)}{\alpha\bar{q}L}} \tag{3.15}$$

Note that this is the same as when the participation constraints of high types with low signals does not bind. Hence, the only effect of binding incentive constraint of high types with low σ binds is to reduce the bond price of a risky contract. This follows from equation (3.15), and from comparing (3.13) and (3.14). □

The equilibrium set of contracts can be solved explicitly by considering several cases. Since the possibility of adverse selection complicate the bond prices (equation (3.14)), one has to simultaneously solve a system of non-linear equations for the cut-offs for the risk-free participation and the bond prices for each risky contract as well as for the risk-free bond price.

3.2 Perfectly Informative Signals ($\alpha = 1$)

We now briefly discuss the (simpler) special case of complete information regarding households' risk types ($\alpha = 1$). This is an interesting special case to explicitly consider for several reasons. First, this environment corresponds to a static version of papers such as Livshits, MacGee, and Tertilt (2007) and Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) which abstract from adverse selection. Second, abstracting from adverse selection helps to illustrate the workings of the model. To save on notation in this section, we will set a, the lower bound on the probability of high income realization, to 0. That is, $\rho \sim U[0, 1]$.

3.2.1 Characterizing Equilibrium with $\alpha = 1$

Here we briefly discuss the key characterization equations for the special case $\alpha = 1$. In the absence of private information, the equations are considerably simpler and thus easier to interpret.

Theorem 3.8. *Finitely many (N) risky contracts are offered. Each contract $(q_n, L = \gamma y_h)$ serves borrowers in the interval $\rho \in (\rho_n, \rho_{n-1}]$, where*

$$\begin{aligned} \rho_n = &\quad 1 - n\sqrt{\tfrac{2\chi}{\gamma y_h \overline{q}}} \\ q_n = &\quad \overline{q}\rho_n \end{aligned}$$

If risk-free contract $(q_{rf}, L_{rf} = \gamma y_l)$ is offered, it serves borrowers with $\rho \in [0, \rho_N]$, and

$$q_{rf} = \overline{q} - \frac{\chi}{\gamma y_l \rho_N}. \tag{3.16}$$

To construct an equilibrium, one needs to check the household participation decisions to solve for N and for whether the risk-free contract if offered in equilibrium. This process is more straightforward when $\alpha = 1$, since for any N it is easy to check if the risk-free contract will be offered since one no longer needs to track the measure of types with lower public signals than their true types who decline the best risky contract they are offered. Combining the household participation constraint (3.4) with the expression for the risk-free bond price (3.16), one can solve for the minimum length of the interval (not served by risky contracts) that makes the risk-free contract viable. Letting the upper-cutoff for this interval be denoted by $\bar{\rho}_{rf}$

$$\begin{aligned} \beta = q_{rf} &= \bar{q} - \tfrac{\chi}{\bar{\rho}_{rf} L_{rf}} \\ \rho_{rf} &= \tfrac{\chi}{(\bar{q}-\beta) L_{rf}} \end{aligned} \tag{3.17}$$

To solve for N, one has to find the (first) risky contract for which the household participation constraint with respect to either autarky or the risk-free contract is violated. Recall that for any risky contract serving the interval $(\underline{\rho}, \overline{\rho}]$ we only need to check the participation constraint of the individual with the highest type, $\overline{\rho}$. Further, if $\underline{\rho} \geqslant \bar{\rho}_{rf}$, then we have to check the household $\overline{\rho}$'s participation constraint with respect to the risk-free contract (serving $[0, \underline{\rho}]$). Conversely, if $\underline{\rho} < \bar{\rho}_{rf}$, we check the value of the risky contract against the autarky participation constraint (3.5).

This observation allows us to put an upper bound on the number of risky contracts offered in equilibrium by considering only the autarky participation constraint.

Theorem 3.9. *Assume that* $\beta \frac{y_L}{y_H} + \bar{q} > 1$. *Taking participation constraints into account, the number of risky contracts offered in equilibrium,* N, *cannot exceed*

$$\overline{N} = \boldsymbol{int}\left(\frac{(\bar{q}-\beta)\sqrt{\frac{\gamma y_h \bar{q}}{2\chi}} - \beta(1 - \frac{y_l}{y_h})}{\beta \frac{y_l}{y_h} + \bar{q} - 1} \right)$$

Proof. We want to find that largest N that is consistent with firm maximization and household participation. In other words, what is the largest n that satisfies

$$\rho_n = 1 - n\sqrt{\frac{2\chi}{y_h \gamma \bar{q}}} \text{ and } q_n = \bar{q}\rho_n$$

and does not violate the household participation constraint for all types in $(\rho_n, \rho_{n-1}]$? Since the autarky participation constraint (equation 3.5) must always hold (and since when the risk-free contract is offered, the autarky constraint is slacker), checking the autarky constraint will give the upper bound on the number of possible contracts. As discussed above, it suffices to check the participation constraint only for type ρ_{n-1}.

Recalling that the autarky-risky participation constraint is $\beta\left(\frac{\gamma y_l(1-\rho)}{L}+\rho\right)\leqslant q$, we have that contract n will satisfy the participation constraint for ρ_{n-1} if and only if:

$$\beta\left(\frac{\gamma y_l(1-\rho_{n-1})}{L}+\rho_{n-1}\right)\leqslant \bar{q}\rho_n$$

Substituting $\rho_n = 1-n\sqrt{\frac{2\chi}{y_h\gamma\bar{q}}}$ from Theorem 3.8, simplifying and collecting terms:

$$n\sqrt{\frac{2\chi}{\gamma y_h\bar{q}}}\left(\beta\frac{y_l}{y_h}-1+\bar{q}\right)\leqslant \bar{q}-\beta-\beta(1-\frac{y_l}{y_h})\sqrt{\frac{2\chi}{\gamma y_h\bar{q}}}$$

Under the condition that $\beta\frac{y_L}{y_H}+\bar{q}>1$, this inequality can be rewritten as:

$$n\leqslant\frac{(\bar{q}-\beta)\sqrt{\frac{\gamma y_h\bar{q}}{2\chi}}-\beta(1-\frac{y_l}{y_h})}{\beta\frac{y_l}{y_h}+\bar{q}-1}$$

The equilibrium number of contracts cannot exceed the largest n that satisfies the inequality above. □

Clearly, this bound $\overline{N}$ is weakly decreasing in χ.

4 Implications of Technological Progress

We now turn to the question of what is the impact of financial market innovations on the set of contracts offered in equilibrium and the cross-sectional pattern of borrowing and defaults. We focus on the comparative static implications of four different channels. First, what is the effect of increased signal accuracy on the number and price of contracts offered in equilibrium? Second, what is the effect of changes in the fixed cost of creating contracts? Finally, we explore the implications of reductions in the cost of funds.

Given the highly stylized nature of the model, we focus on the qualitative predictions of the model. We provide most comparative statics results in the form of theorems. To better illustrate the results, we also present a numerical example. Since the example is intended to help illustrate how the qualitative features of the model (i.e. direction of change) matches up with the data, the parametrization is chosen for simplicity rather than to match any quantitative facts. The parameters used in the example are given in Table 2.

Table 2: Parameters used in the Numerical Example

β	γ	y_h	y_l	$\bar{r}$	χ	a
0.75	0.25	3	0.4	4%	0.0001	0

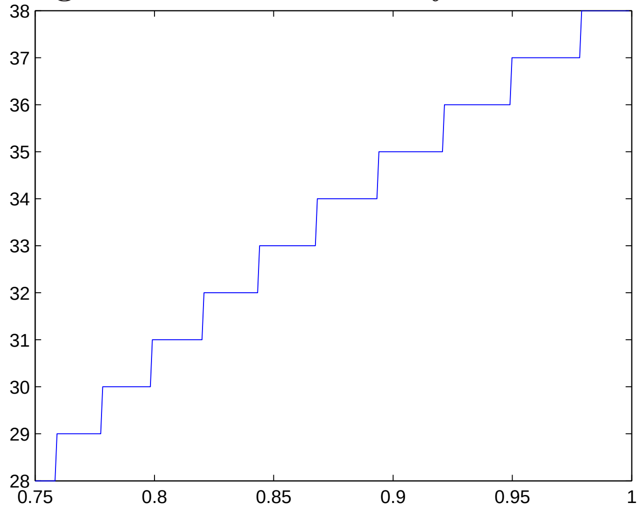
4.1 Improvements in Signal Accuracy

We begin by exploring the implications of improved signal accuracy. This comparative static exercise is motivated by improved credit evaluation technologies such as credit scoring. In addition to presenting some general results, we also illustrate the working by varying $\alpha \in [0.75, 0.9999]$ while holding fixed the other parameters.

Total borrowing and defaults depend upon the fraction of population who borrow via a risky contract. This depends upon three factors: the measure eligible for each contract, the measure of people who accept each contract they are offered and the number of contracts. The qualitative relationship between α and each of these factors is easy to characterize. First, the measure of household eligible for *each individual* contract is decreasing in α. This follows directly from equation (3.15), as the length of the interval eligible for each risky contract is strictly decreasing in α. The intuition is simply that increased signal accuracy reduces the incentive for agents to pool by increasing the return on reducing cross-subsidization via a smaller pool of public types.

The fraction of eligible borrowers who accept the risky contracts they are offered is increasing in α. An increase in signal accuracy reduces the adverse selection problem by shrinking the fraction of misclassified high risk types eligible for the contract. This lowers the default premium (thus increasing the bond price), which reduces the fraction of low risk types with (incorrect) high risk public signals who decline the offered contract. The final channel via which signal accuracy impacts the equilibrium is the number of risky contracts offered. The number of risky contracts is non-decreasing in α. For a *sufficiently large* change in α, the number of risky contracts offered in equilibrium increases. As Figure 3 illustrates, this implies that the number of contracts is an increasing step function in α.

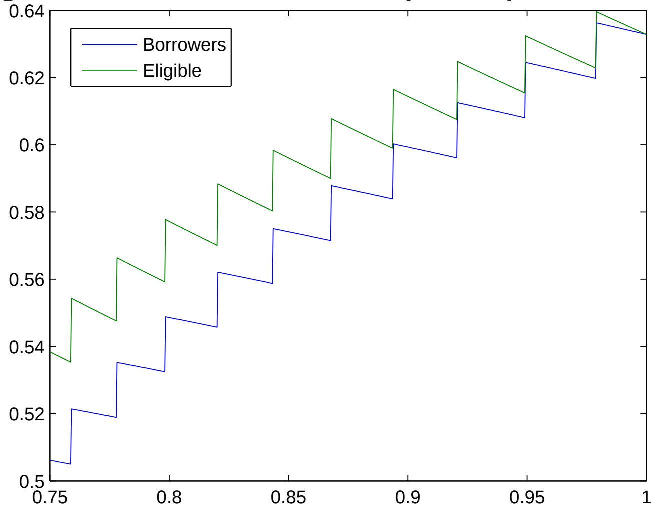
Figure 3: Number of Risky Contracts



The interaction between shrinking interval length and the increased number of contracts determines the total fraction of the population with *access* to risky borrowing. As Figure 4 illustrates, whenever α increases enough to generate an additional risky contract in equilibrium, the total measure of households *eligible* to borrow via risky contracts increases. However, when α increases by less than this amount, the length of each contract interval shrinks, which reduces the fraction of the population with access to risky borrowing.

An increase in the measure of households eligible to borrow goes hand in hand with an increase in the fraction of households borrowing. The increase in the measure of households borrowing strictly exceeds the increase in the fraction of the population eligible to borrow. This reflects the fact that more accurate signals reduce the degree of adverse selection in the economy caused by low risk types with high(er) risk public

Figure 4: Measure Served by Risky Contracts



signals declining to borrow via the risky contract they are eligible for. This can be seen from Figure 4 where for a low signal accuracy roughly 54% of the population has access to borrowing but only 51% borrows, while for perfect signals over 63% of the population has access to risky credit and all eligible households borrow.[15]

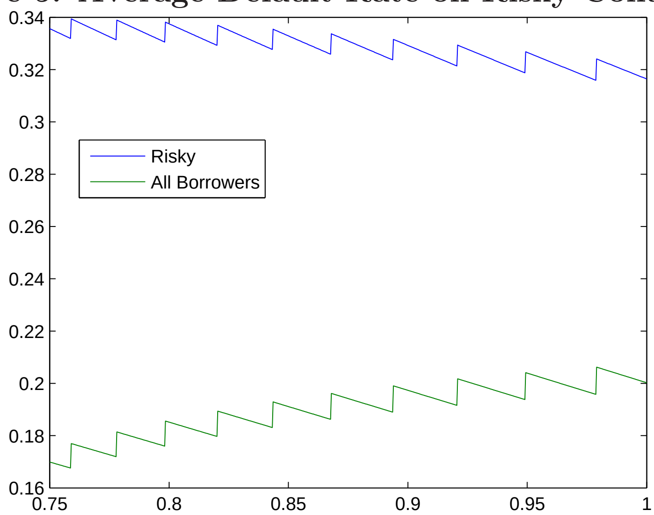
The increase in the number of households eligible to borrow and in the fraction of eligible households who borrow leads to an increase in borrowing and defaults. The expansion of credit to more borrowers in this environment involves the extension of credit to public types with higher default risk than existing borrowers. This tends to increase the average default rate of all borrowers. However, the rise in defaults is partially muted by the fact that improvements in signal accuracy lead to a reduction in the number of high risk people who borrow on terms offered to low risk borrowers. As a result, the default rate (and the default premium) of contracts offered to existing borrowers decreases with improvements in signal accuracy.

The overall effect of these forces is reflected in the average default rate of borrowers. As Figure 5 illustrates, the average default rate on risky contracts spikes up whenever

[15]Recall that the model also allow for a small risk-free loan which is repaid with certainty. These loans are not part of the picture.

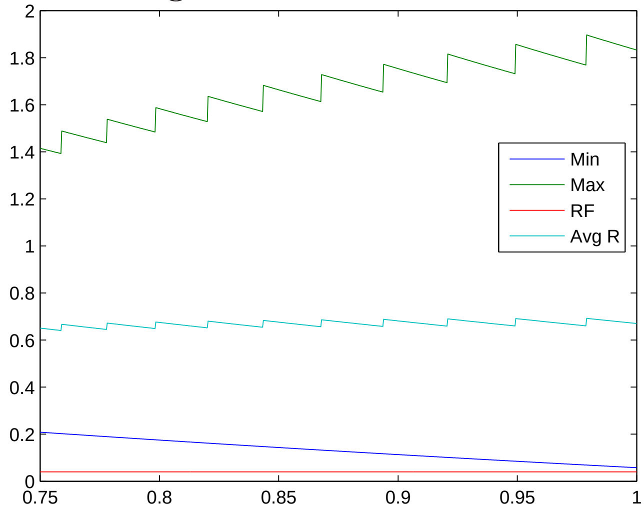
a new contract is introduced. However, improvements in signal quality also tend to lower the average default rate by reducing the number of high risk households whose public type classifies them as low risk. In this example, this effect dominates so that improvements in signal quality leads to a slightly lower average default rate on the risky contract. However, total defaults and the average default rate for all borrowers (which in this example is all households) is increasing in α. This is driven by the fact that the increase in the measure of households borrowing via risky (instead of risk-free) contracts dominates the decline in the default rate on risky contracts.

Figure 5: Average Default Rate on Risky Contracts



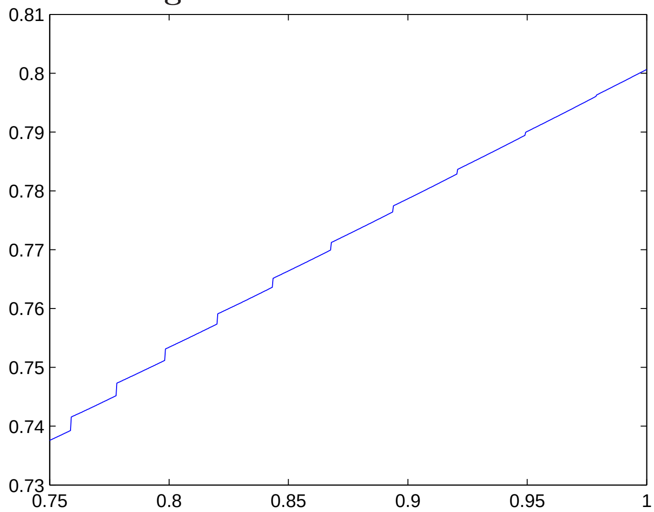
The shifts in default rates shows up in the distribution of interest rates. As α increases, the measure of consumers eligible for each risky interval decreases. The increased accuracy of the signal also implies that fewer high risk borrowers (low ρ types) are incorrectly classified as low risk public types. Both of these two forces combine to drive down the interest rate ($r_1 = \frac{1}{q_1} - 1$) on the lowest risk risky-contract. Since both of these forces are independent of the number of contracts, this implies that the interest rate on the lowest risk types is strictly decreasing in α, as illustrated in Figure 6. As a result, the interest rate on the lowest risk risky contract tends towards the risk-free interest rate in the economy. The maximum interest rate in the economy depends upon these forces as well as on the number of contracts offered. As can be seen from Figure 6, so long as an increase in α is not large enough to generate the entry of an additional contract, the highest borrowing interest rate in the economy decreases. This reflects the fact that the riskiest borrowers are pushed out of the eligibility set for the riskiest contract, which lowers the default premium. However, since the entry of a new contract involves the extension of credit to high risk households previously excluded from risky borrowing, the interest rate on the (new) riskiest contract is increasing in α. The average interest rate (weighted by the face value of borrowing) in the economy reflects these forces, and varies relatively little with α. One additional effect that matters here is that increases in α lead to an increase in the acceptance rate of risky contracts, which increases the share of these contracts in the calculation of the average interest rate in the economy. Finally, it is worth noting that the risk free interest rate in this economy increases slightly with α. This is driven by a reduction in the fraction of the population borrowing via the risk-free contract, which means that the fixed cost of offering this contract is spread over a smaller base.

Figure 6: Interest Rates



The ratio of overhead costs to total borrowing is increasing in α, as can be seen from Figure 7. Since an increase in the accuracy of the signal generates an increase in the number of contracts, the total spent on overhead costs increases. What drives the increase in overhead costs as a share of the face value of lending is that on average the decrease in the measure of each household served by a contract exceeds the reduction in the measure of eligible households who opt not to borrow due to adverse selection. As a result, in this example, improvements in credit technology lead to an increase in average overhead costs. This suggests that cost of operations of banks (or credit card issuers) might not be a good measure of technological progress in the banking sector. Figure 7 also illustrates that overhead costs are relatively small in this example, and are equal to less than 1% of the face value of outstanding debt.

Figure 7: Overhead Costs



4.2 Decline in Cost of Contract χ

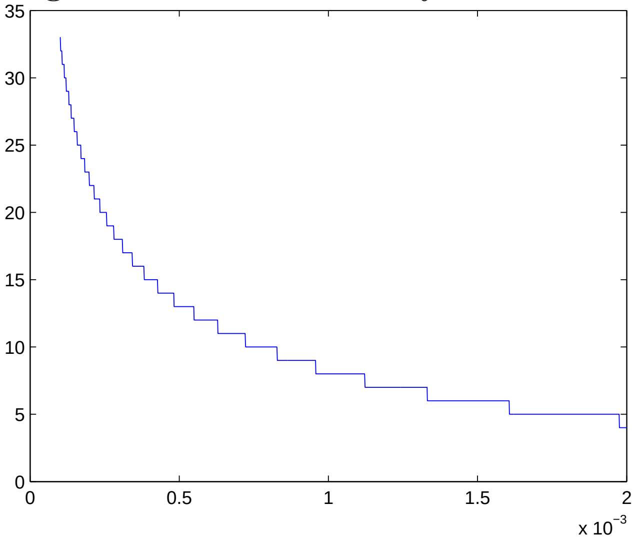
We now consider another possible channel via which financial innovation may have impacted the set of equilibrium contracts: a decline in the fixed cost of offering a contract. As previously discussed, credit contracts are a differentiated product, where each product is tailored to a specific segment of the market. Credit contracts have become more and more differentiated over time (i.e. tailored to finer and finer segments of the population) — as we discuss in Section 6.1. One mechanism which could generate increased market segmentation would be a decline in the fixed cost of designing a particular credit product (contract).[16] Unfortunately, direct evidence on the decline in such a fixed cost is hard to obtain – other than by pointing to general evidence on productivity increases in the credit sector. To the extent that productivity increased in those industries that deliver these services, it seems plausible to believe that these fixed costs have been falling over time.

A change in the fixed cost χ of creating a contract affects both the measure of households served by each contract and the number of contracts offered. The length of the interval served by *each* contract increases in χ. This follows directly from equation (3.15). Intuitively, higher (lower) values of the fixed cost increases (decreases) the benefits of spreading the fixed cost over a larger pool of agents relative to the cost of cross-subsidization of the lowest risk borrowers in a contract pool. The number of contracts is weakly decreasing in χ. For a *sufficiently large* increase (decrease) in χ, the number of risky contracts offered in equilibrium decreases (increases). This follows from Theorem 3.9, which states that the maximum number of contracts is weakly decreasing in χ. The effect of varying χ in our numerical example is illustrated in Figure 8, which shows that the number of contracts is a non-decreasing step function in χ. The intuition for the step function is that for small enough changes in χ, adding a new contract is not profitable.

Since total borrowing and defaults depend upon the fraction of population covered by a risky contract, $(1-\rho_N)$, the interaction between shrinking interval length and the increased number of contracts is key. The size of the group served by each contract is locally increasing in χ almost everywhere. This corresponds to the number of risky contracts remaining the same and the number of households served by each contract

[16] The rise of information technology has also made it easier for companies to offer contracts to a wider geographical area. For example, large credit card providers such as Citi and MBNA offer cards nationally, whereas early credit cards were offered by regional banks. In this model, this would act as an increase in the market size which has a similar effect as a fall in χ.

Figure 8: Number of Risky Contracts



increasing. However, globally, the fraction served by risky contracts is decreasing in χ as can be seen from the fact that both the upper and the lower bounds of the interval are decreasing in χ. As Figure 9 illustrates, whenever χ declines enough to generate an additional risky contract in equilibrium, the total measure of households borrowing via risky contracts increases. However, when χ declines by less than this amount, the only effect is to shrink the length of each interval – which reduces the fraction of the population with risky borrowing. The next theorem provides theoretical bounds for the contract coverage.

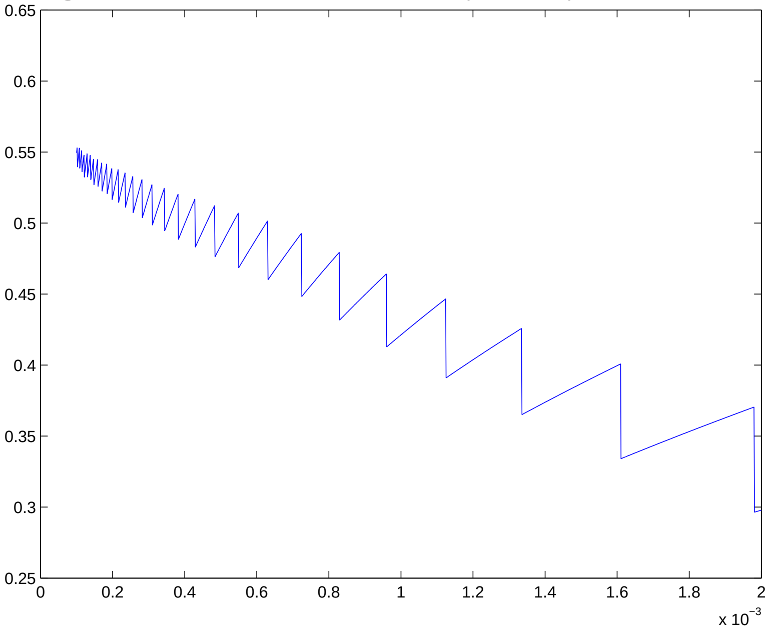
Theorem 4.1. *Assume that $\beta \frac{y_L}{y_H} + \bar{q} > 1$. Then the fraction of households served by risky contracts in equilibrium*

$$(1-\rho_N) \in \left(\frac{\bar{q} - \beta - (\beta + \bar{q} - 1)\sqrt{\frac{2\chi}{\gamma y_h \bar{q}}}}{\beta \frac{y_L}{y_H} - 1 + \bar{q}}, \quad \frac{\bar{q} - \beta - \beta\left(1 - \frac{y_L}{y_H}\right)\sqrt{\frac{2\chi}{\gamma y_h \bar{q}}}}{\beta \frac{y_L}{y_H} - 1 + \bar{q}} \right].$$

Proof. This follows directly from Theorem 3.9. $\square$

The number of risky contracts is thus key to how aggregate borrowing and defaults vary with χ. The reason is that the fraction of the population with access to risky

Figure 9: Measure Served by Risky Contracts



contracts is increasing in the number of risky contracts offered. From the definitions of aggregates in Section C.2, it follows that total defaults and total risky borrowing are increasing in the fraction of the population served by risky contracts. Basically, the previous theorem establishes a weak monotonicity of ρ_N in χ: as contract costs fall enough, the fraction of people covered by risky contracts increases. Building on this, we can now write several additional results on how aggregates behave as a function of contract coverage, $1 - \rho_N$.

Theorem 4.2. *The number of total defaults strictly decreases in ρ_N.*

Proof. Total defaults are given by

$$\int_{\rho_N}^{1} (1-\rho)d\rho = 1/2 - \rho_N + \frac{\rho_N^2}{2}$$

This is strictly decreasing in ρ_N as long as $\rho_N < 1$. □

Theorem 4.3. *Aggregate risky borrowing strictly decreases in ρ_N.*

Proof. Aggregate risky borrowing (in period 1 dollars) is:

$$\int_{\rho_N}^{1} \bar{q} L d\rho = (1 - \rho_N)\bar{q}L$$

This clearly decreases in ρ_N. $\square$

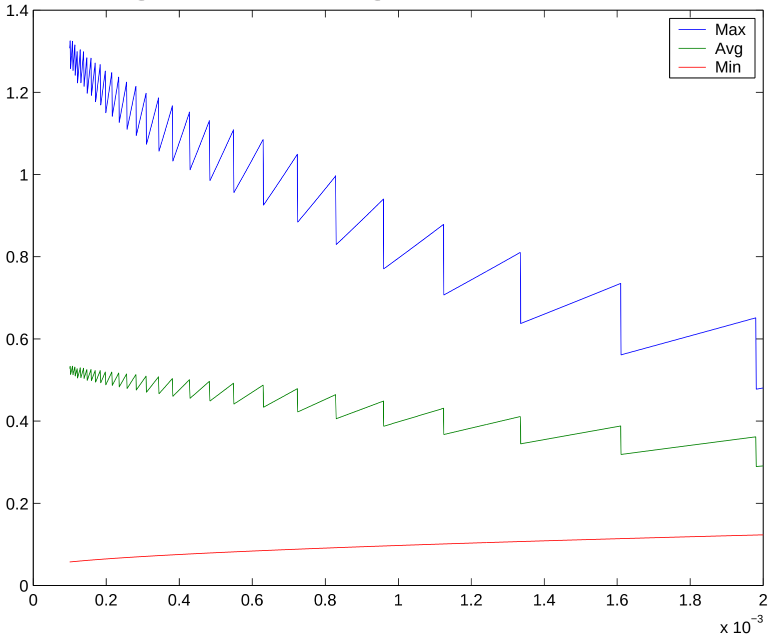
An increase in the number of contracts goes hand in hand with an expansion of credit to more (and riskier) people. This can be seen from Figure 9 where for a high fixed cost only 30% of the population is able to borrow, while for a low fixed cost, about 55% of the population has access to credit.[17] The overall effect on borrowing and defaults is straightforward. Whenever χ decreases enough to increase N, total borrowing and defaults increase. In the model, this also implies that borrowing by lower income (riskier in model) households increases. As a result, defaults increase at a faster rate than debt and the share of risky debt held by lower income households increases.

A fall in χ also reduces the extent of cross-subsidization. The reason is that the lowest risk household in each contract interval "subsidizes" the highest risk households in that pool $\rho \in (\rho_n, \rho_{n-1}]$. The shrinking of each contract interval shrinks the amount of cross-subsidization and hence leads to more accurate risk-based pricing. This pattern can be seen by looking at the bond prices in the example. As χ declines and the number of contracts increases, the model generates both more disperse interest rates and higher average borrowing interest rate. The expansion of credit to higher risk borrowers is accompanied by an increase in the bond price offered to existing borrowers. This can be seen in Figure 10, which plots the average, largest and smallest risky interest rates as a function of χ.

Total overhead costs as a percentage of borrowing are shown in Figure 11. In the example, even though χ falls by a factor of 1:20, total overhead costs (as % of debt) fall only by a factor of 1:4. The reason that a fall in χ lowers overhead costs by less than proportional is that even though fixed costs per contract are falling, fewer

[17] Recall that the model also allow for a small loan which is repaid with certainty. These loans are not part of the picture.

Figure 10: Average, Max and Min r

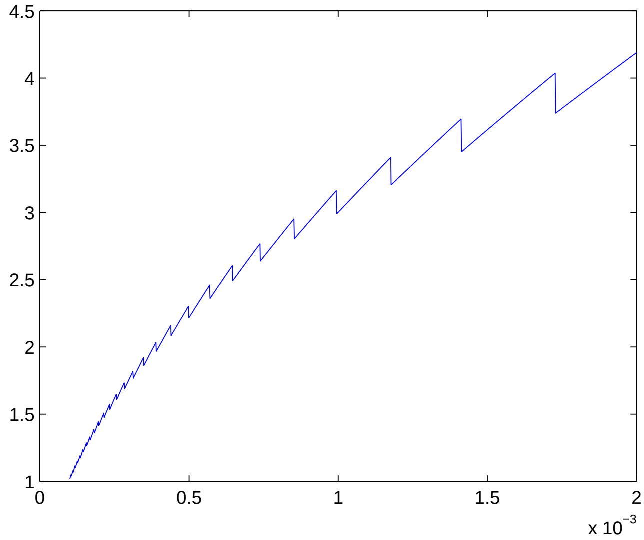


borrowers are now "sharing" a contract, so that each borrower has to pay a larger share of the overhead. This suggests that cost of operations of banks (or credit card issuers) might not be a good measure of technological progress in the banking sector.

4.3 Decline in Risk Free Rate – Increase in $\overline{q}$

In an influential paper, Ausubel (1991) documented that the decline in risk-free interest rates in the U.S. in the 1980s were not accompanied by a decline in the average credit card rates reported by the Board of Governors. This led some to claim that the credit card industry was characterized by imperfect competition. In contrast, others such as Evans and Schmalnsee (1999) argued that significant measurement issues associated with fixed costs of lending and the expansion of credit to riskier households during the late 1980s implied that Ausubel's observation could be consistent with a

Figure 11: Overhead Costs as Percent of Borrowing



competitive credit card industry.[18]

To explore the implications of our model for this debate, we consider the effect of a decline in the risk free interest rate on the number of contracts and average borrowing interest rates. For simplicity, we focus our attention on the perfect signal case ($\alpha = 1$).

We begin by considering the effect of a rise in $\overline{q}$ on the existing contracts. To see this, recall that from Theorem 3.8 that each of N risky contracts offered in equilibrium $(q_n, \gamma y_h)$ serves borrowers in the interval $\rho \in (\rho_n, \rho_{n-1}]$, where

$$\rho_n = \quad 1 - n\sqrt{\tfrac{2\chi}{y_h \gamma \overline{q}}} \quad \text{and} \quad q_n = \quad \overline{q}\rho_n$$

It follows directly that ρ_n is decreasing in $\overline{q}$ and χ, so that an increase in the risk-free bond price reduces the length of each risky contract interval. In addition, the risky bond price is increased.

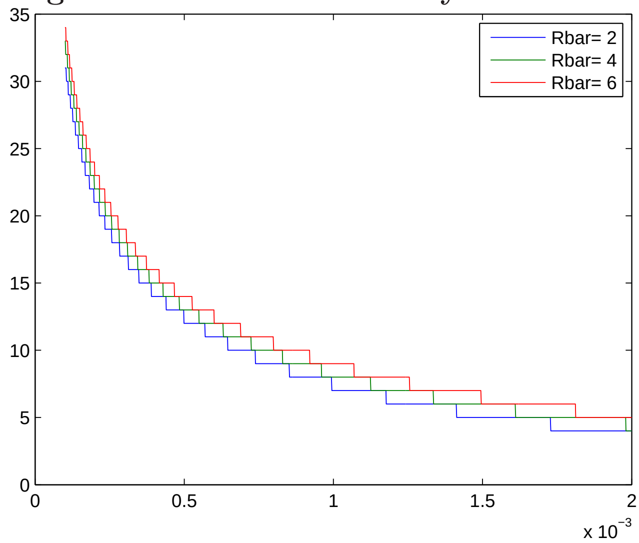
The main determinant of the average borrowing interest rate is the change in

[18] Brito and Hartley (1995) propose a closely related explanation, where they show that a reduction in the cost of fund lends to an extension of credit to riskier borrowers. The key difference between their model and ours is that in our framework the number of contracts (risk categories) is endogenous.

the total number of contracts. As with a change in the fixed cost χ, a sufficiently large rise in the risk free bond price generates an expansion of the number of risky contracts. This leads to an increase in the total measure of risky borrowers, and pulls in borrowers who are riskier than existing borrowers. This generates a rise in defaults. As a result, average borrowing interest rates decline less than proportionally.

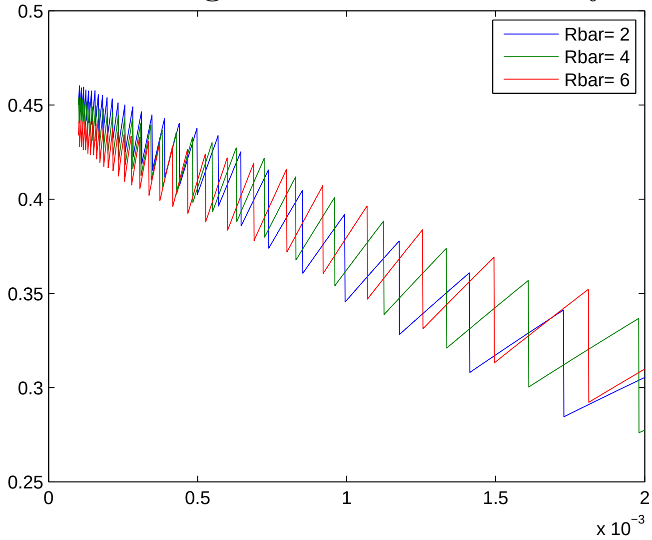
To illustrate this, we extend our numerical example and compare the equilibrium associated with three different risk free interest rates $\bar{r} = 2\%, 4\%, 6\%$ (and corresponding bond prices $0.943, 0.962, 0.980$). Figure 13 plots the number of risky contracts for value of χ for each $\bar{q}$. The figure shows that the number of risky contracts is weakly increasing in the risk free bond price.

Figure 12: Number of Risky Contracts



To illustrate the effect of variations in the risk-free rate on the average borrowing interest rate we plot the average risky bond prices for each value of χ. The figure shows that the effect of a shift in the risk free rate on the average borrowing rate depends upon the level of the fixed cost. When the cost of creating contracts is high and there are very few risky contracts offered, the average borrowing bond price tends to be positively related to the risk free rate. In contrast, for lower values of χ, lower risk free bond prices can have higher average borrowing interest rates. The reason is that the extensive margin effect of extending credit to riskier households can dominate the reduction in interest rates for existing borrowers.

Figure 13: Average Interest Rate on Risky Bonds



5 Pooling Equilibria with No Fixed Costs. A Problem.

The point of this section is to establish that the equilibrium we study in the paper may fail to exist in the limit case of $\chi = 0$.

The candidate equilibrium under no fixed cost is characterized by a risk-free contract with loan size γy_l and price $\overline{q}$ that serves public types $\sigma < \underline{\sigma}$ and a continuum of risky contract targeted to individual observable types that fall into two distinct intervals:

1. For the best public types, the equilibrium contract offers loan size $L = \gamma y_h$, and price

   $q_\sigma = \overline{q}\left(\alpha\sigma + (1-\alpha)\frac{a+b}{2}\right)$, where $[a, b]$ is the support of the type distribution. Everyone with public signal σ is willing to accept this contract.

Such contracts are offered to every public type $\sigma > \overline{\sigma}$. The cut-off $\overline{\sigma}$ is determined by the participation constraint of the best (mislabeled) type: $U^b(L, q_{\overline{\sigma}}) = U^b(\gamma y_l, \overline{q})$. My algebra gave me

$$\overline{\sigma} = \frac{1}{\alpha}\left[\frac{\beta}{\overline{q}}b + \left(1 - \frac{\beta}{\overline{q}}b\right)\frac{y_l}{y_h}\right] - \frac{1-\alpha}{\alpha}\cdot\frac{a+b}{2} \tag{5.1}$$

2. For the public types $\sigma \in [\underline{\sigma}, \overline{\sigma}$, the equilibrium contract offers loan size $L = \gamma y_h$, and price

$$q_\sigma = \overline{q}\left(\frac{\alpha\sigma + (1-\alpha)\frac{\overline{\rho}-a}{b-a}\cdot\frac{a+\overline{\rho}}{2}}{\alpha + (1-\alpha)\frac{\overline{\rho}-a}{b-a}}\right), \tag{5.2}$$

where $\overline{\rho}$ is the highest (mislabeled) private type willing to accept this contract. Clearly, we need to solve jointly for $\overline{\rho}$ and q_σ. I get a quadratic equation.

The last public type served by a risky contract is the one where the participation constraint binds for the accurately labeled borrowers: $\overline{\rho}(q_{\underline{\sigma}}) = \underline{\sigma}$. The equation for $\underline{\sigma}$ is then very similar to that for $\overline{\rho}$ and is also quadratic.

5.1 Unraveling the Candidate Equilibrium

We will now establish that the candidate equilibrium may not in fact be an equilibrium what α is sufficiently close to 1. We will establish that for the top interval of public types as described in point 1 above.

Consider the following deviation by a potential entrant: Separate the top ϵ of mislabeled true types by offering them a smaller loan L_t at a better price q_t. In order to complete the deviation, the entrant also has to offer a new contract to the rest of the borrowers with public type σ so that they do not join the (L_t, q_t) contract after the incumbent exists. The contract offered to the "non-target" group will be denoted (L_d, q_d). This contract is easy to characterize: $L_d = \gamma y_h$, and

$$q_d = \overline{q}\left(\frac{\alpha\sigma + (1-\alpha)\frac{b-\epsilon-a}{b-a}\cdot\frac{a+b-\epsilon}{2}}{\alpha + (1-\alpha)\frac{b-\epsilon-a}{b-a}}\right). \tag{5.3}$$

We will specify the most generous (to the target borrowers) contract that is tailored to the top ϵ - that is the one making 0 profits. The price is then

$$q_t = \overline{q}\left(b - \frac{\epsilon}{2}\right). \tag{5.4}$$

The loan size has to be such that it only attracts the target group and nobody else. That implies that the borrower with the true type $(b-\epsilon)$ has to be indifferent between (L_t, q_t) and (L_d, q_d). That pins down the loan size L_t:

$$q_d\gamma y_h+\beta\left((b-\epsilon)y_h+(1-b+\epsilon)y_l\right)(1-\gamma)=q_tL_t+\beta\left((b-\epsilon)(y_h-L_t)+(1-b+\epsilon)y_l(1-\gamma)\right), \tag{5.5}$$

which can be rearranged to

$$q_d\gamma y_h - q_tL_t = \beta(b-\epsilon)(\gamma y_h - L_t).$$

Incorporating equations (5.3) and (5.4), we get

$$\bar{q}\left[\gamma y_h\left(\frac{\alpha\sigma+(1-\alpha)\frac{b-\epsilon-a}{b-a}\cdot\frac{a+b-\epsilon}{2}}{\alpha+(1-\alpha)\frac{b-\epsilon-a}{b-a}}\right)-\left(b-\frac{\epsilon}{2}\right)L_t\right]=\beta(b-\epsilon)(\gamma y_h-L_t).$$

Rearranging,

$$\left[b-\frac{\epsilon}{2}-\frac{\beta}{\bar{q}}(b-\epsilon)\right]\frac{L_t}{\gamma y_h}=\frac{\alpha\sigma+(1-\alpha)\frac{b-\epsilon-a}{b-a}\cdot\frac{a+b-\epsilon}{2}}{\alpha+(1-\alpha)\frac{b-\epsilon-a}{b-a}}-\frac{\beta}{\bar{q}}(b-\epsilon),$$

we get

$$\frac{L_t}{\gamma y_h}=\frac{\frac{\alpha\sigma+(1-\alpha)\frac{b-\epsilon-a}{b-a}\cdot\frac{a+b-\epsilon}{2}}{\alpha+(1-\alpha)\frac{b-\epsilon-a}{b-a}}-\frac{\beta}{\bar{q}}(b-\epsilon)}{b-\frac{\epsilon}{2}-\frac{\beta}{\bar{q}}(b-\epsilon)}. \tag{5.6}$$

Such a deviation successfully undermines the equilibrium if the very best borrower prefer (L_t, q_t) to the original (L, q_σ):

$$q_tL_t+\beta\left(b(y_h-L_t)+(1-b)y_l(1-\gamma)\right)>q_\sigma\gamma y_h+\beta\left(by_h+(1-b)y_l\right)(1-\gamma).$$

Canceling out the last terms and applying equation (5.5), this is equivalent to

$$q_d\gamma y_h+\beta(b-\epsilon)y_h(1-\gamma)+\beta\epsilon(y_h-L_t)>q_\sigma\gamma y_h+\beta b(1-\gamma)y_h.$$

Simplifying further, we get

$$q_d\gamma y_h+\beta\epsilon(\gamma y_h-L_t)>q_\sigma\gamma y_h.$$

That is, the deviation is successful if

$$\beta\epsilon\left(1-\frac{L_t}{\gamma y_h}\right)>q_\sigma-q_d.$$

Now simply note that as, $\alpha \to 1$, the right-hand-side converges to 0, while the left hand side remain bounded away from 0 (and positive) for any positive ϵ. Hence, for α sufficiently close to 1, there are profitable deviations that unravel the candidate equilibrium when $\chi = 0$. It is important to note that such deviations are unlikely to work for any positive χ as they critically rely on the infinitesimal size of the target group. The group of deviating borrowers is inherently small, too small to justify paying even a tiny fixed cost.

5.2 Welfare Analysis

What are the welfare implications of improved accuracy of signals or lower fixed costs of creating contracts? While the equilibrium allocation in the model is always constrained efficient, technological improvements in the model are in general not Pareto improving, as they generate both winners and losers.

There are two factors which lead to winners and losers as a result of technological change in the model. The first factor is a direct result of a change in households public type that arises when signal accuracy improves. Some borrowers who used to have better public signals than their true type now find themselves with lower public signals, and as a result face worse borrowing terms than before. Conversely, some households who had inaccurately low signals now benefit from better borrowing terms.

The second force that creates winners and losers is tied to the endogenous change of the number of households served by each contract. Both improved signal quality and reduced costs of creating contracts reduces the number of households served by each contract. Borrowers who remain with the same contract benefit from improved lending terms. However, some households located at the bottom of each contract interval (i.e. the marginal borrowers) now find themselves "pushed" down to the next contract. This contract has worse terms, so that these households now face worse borrowing terms. In effect, these borrowers move from a situation where they are being subsidized by better borrowers in their initial contract, to subsidizing others in their new contract, where they are now among the best types. Finally, an additional potential source of losers is from borrowers accepting the risk-free contract — locally, as the number of risky contract increases, the measure of borrowers served by the risk-free contract shrinks (discontinuously) leading to higher interest rates, as the fixed cost is now shared by fewer borrowers.

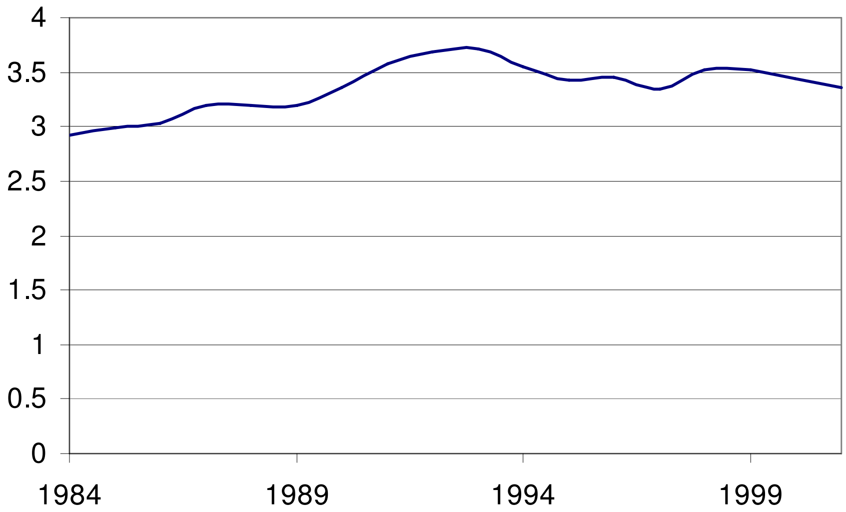
One benchmark to evaluate the welfare effects of the technological improvements is the equally weighted social planner's problem. We assume that this social planner faces the same technological and participation constraints as do intermediaries. This welfare criterion corresponds to the expected utility of an individual about to be born into our economy. From the perspective of this social planner's problem, the market equilibrium features too many contracts. The socially constrained-optimal allocation features fewer contracts each of which serves a larger number of agents. Rather than using the zero expected profit threshold for determining the eligibility set (Proposition 3.3), the social planner extends the eligibility set of the contract to include borrowers who deliver negative expected profits until the participation constraint of the best type (within the contract eligibility set) binds (equation (3.6)). This allocation thus features much more cross-subsidization than the equilibrium allocation. However, since the borrowers are risk-neutral, cross-subsidization is not the direct reason that the market outcome is inefficient here. Instead, the social planner prefers it because it wastes fewer resources on the fixed costs of offering contracts.

Finally, it is worth noting that lowering the fixed costs of offering a contract allows for a Pareto improvement.[19] If the number of contracts and the eligibility set they serve remained unchanged but the prices adjust to keep each contract at zero profits, then every borrower would be better off. That is not an equilibrium allocation however, as the borrowers at the bottom of each (old) bin now make negative contributions towards expected profits.

[19] The same cannot be said about the improvement in information quality. In that case, one borrower's gain implies another's loss.

6 Unsecured Consumer Credit Facts

Figure 14: Non-interest Costs in the Banking Industry



Source: Berger (2003), Table 3, non-interest expenses/gross total assets.

In this section we ask to what extent changes in χ and/or α in the model are consistent with what is observed in the data. We start by revisiting some well-known facts, and then move on to add additional data to better understand changes in the distribution of borrowers.

The first observation is that the model is consistent with the aggregate facts (see Figure 1). As discussed in Section 4, for large enough increases in α or decreases in χ, the model implies an increase in debt, an increase in defaults and a corresponding increase in the discharge rate on debt. Another implication of the model is that the ratio of fixed costs to loans increases in response to an increase in α (see Figure 7), and decreases in response to a decrease in χ (see Figure 11). The closet analog in the data are non-interest costs to total outstanding balances. While we do not have this measure for credit cards, Berger (2003) reports a measure of non-interest costs to total assets of the entire commercial banking sector. Figure 14 shows that this cost measure has been rising steadily from the early 1980s to the mid 1990s, however, it has been constant or declining since. One interpretation of this fact could be that technological progress in the first half of the period was mostly characterized by improvements in the ability of lenders to observe a borrowers type (increases in α), while the second half of the period was marked by banks using more efficient technology to design credit contracts (decrease in χ).

The more interesting aspect of the model predictions have to do with the degree of segmentation of credit market and the relationship between the "extensive margin" (i.e. who has access to borrowing) and improvements in credit technology. Specifically our model has three interesting implications.[20] First, an increase in the "variety" of credit contracts, more specifically in the number of different interest rates offered. Second, an increase in risk-based pricing, i.e. interest rates that are more finely tailored to people's types. Third, increased access to borrowing for more risky people.

To evaluate these predictions, we assemble additional data to gain a more detailed understanding of changes in the distribution of borrowers and terms of credit. We focus our attention on the credit card market. Credit cards are a relatively recent innovation which have become widely used over the past thirty years. While the first bank credit cards were issued during the mid 1960s, by the early 1990s more than 6,000 US institutions issued general purpose credit cards (Canner and Luckett 1992). Credit card borrowing currently accounts for the majority of unsecured borrowing in the United States. Another reason to focus on credit cards is that the cost structure of credit card issuers differs substantially from that of other lenders.[21] This suggests that information technology lowering the cost of originating loans may have a much larger impact on credit card operations than on other consumer lending. Surprisingly, although it is commonly asserted that the past 30 years have witnessed increased segmentation of the consumer credit market and increased expansion of credit to

[20] The model-implied changes refer to those that result from an increase in α or a decrease in χ. We focus on changes that are large enough to increase the number of contracts offered, i.e. we ignore small changes that sometimes (locally) have the opposite implications.

[21] Canner and Luckett (1992) report that operating costs accounted for nearly 60 percent of the costs of credit card operations, compared to less than 20 percent of mortgage lending.

*** still need to update this table***

Table 3: Survey of Consumer Finances

	1983	1989	1995	1998	2001	2004
% Population has card	43%	56%	66%	68%	73%	72%
% Population has balance	51%	52%	56%	55%	54%	56%

Source: Authors' calculations, based on SCF

lower income households, relatively little work has been undertaken to document these changes.

Most of the analysis is based on the Survey of Consumer Finances. As a first check one would like to see that credit card borrowing indeed increased in this survey. In fact, the percent of all households who own a bank credit card has increased in the SCF from 43% in 1983 to 72% in 2004 (see Table 3). Not everyone who owns a card uses the loan function of the card. However, the fraction of the population who carries a positive balance on their credit card has significantly increased as well, almost doubling from about 22% in 1983 to 40% in 2004. (*** check these numbers***)

The comparison between model and data is based on two auxiliary interpretations. First we equate high risk people in the model with low income people in the data. Secondly, we interpret the risky contracts in the model as credit card borrowing in the data. We summarize the model data comparison in Table 6 in Appendix B.

6.1 Increased Variety in Consumer Credit Contracts

If technological progress has been the main driving force for the increase in bankruptcies and consumer debt, and if the details of the mechanism are captured well by our model, then one would expect to observe an increase in contract variety over time. Specifically, the model implies an increase in the number of different interest rates offered. The Survey of Consumer Finance asked questions about the interest rate paid on credit card accounts, which we use to count the number of *different* interest rates. The data reported in Table 4 shows a substantial increase in variety, with the number of different rates roughly tripling between 1983 and 2004.[22]

Table 4: Credit Card Interest Rates, SCF

Year	# of Rates All Households	# of Rates (HH with $B > 0$)	CV All HH	CV (HH with $B > 0$)
1983	78	47	0.22	0.21
1995	142	118	0.30	0.32
1998	136	115	0.32	0.35
2001	222	155	0.37	0.40
2004	211	145	0.56	0.56

Source: Authors' calculations based on Survey of Consumer Finance.

A more nuanced view of variety comes from examining the variance of interest rates across households. Since we are comparing trends in dispersion of a variable with a changing mean, we compute the coefficient of variation (CV).[23] Table 4 reports the CV of the interest rate for six different waves of the SCF. We find a substantial increase in the variability of credit card interest rates across households over time: the CV in interest rates almost triples during the 1983-2004 time period.

The increased dispersion of borrowing interest rates can also be seen using data collected by the Board of Governors directly from banks. We have interest rate data on 24-month consumer loans from a survey of banks (starting in 1971) and credit card interest rates from a survey of credit card issuers (starting in 1990).[24] We find

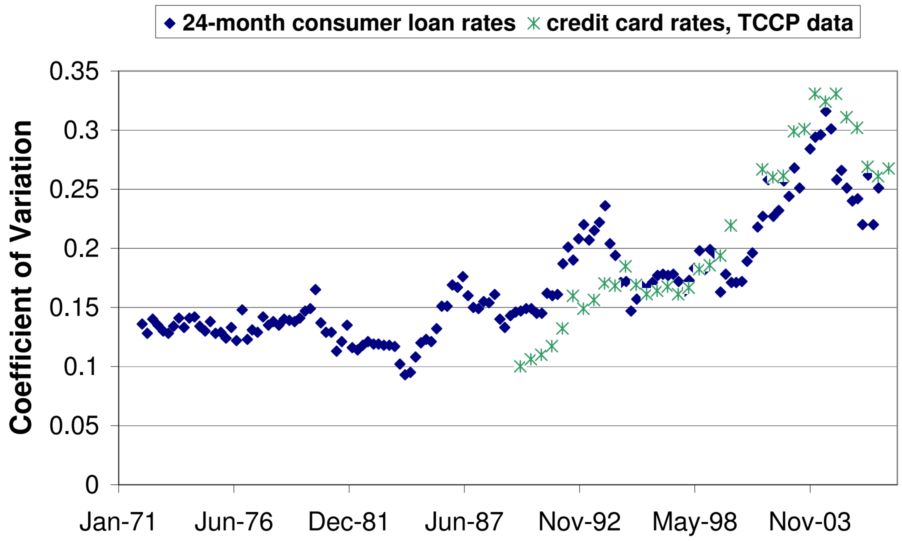
[22]It is worth emphasizing that this measure likely significantly understates the increased variety of credit card contracts, as both Furletti (2003) and Furletti and Ody (2006) argue that credit card providers have made increased use of features such as annual fees, different penalty fees for late payments and other features such as purchase insurance to provide differentiated products.

[23]This is important because the decline in nominal interest rates has shifted down mean borrowing interest rates, which will show up as a decline in the variance of interest rates.

[24]We use data from the Quarterly Report of Interest Rates on Selected Direct Consumer Installment Loans (LIRS) and the Terms of Credit Card Plans (TCCP). The data has to be interpreted with caution, since every bank is asked to report only *one* interest rate (the most commonly used one) and hence likely understates the number of loan options faced by consumers. See Appendix C for a more detailed discussion of these data.

a large increase in the dispersion of interest rates in both series. As can be seen from Figure 15, the CV for 24-month consumer loans increases from roughly 1.5 in the early 1970s to about 3.0 by the late 1990s. A similar increase over time also occurs in credit cards. This finding is consistent with increased banks specialization in different segments of the market.

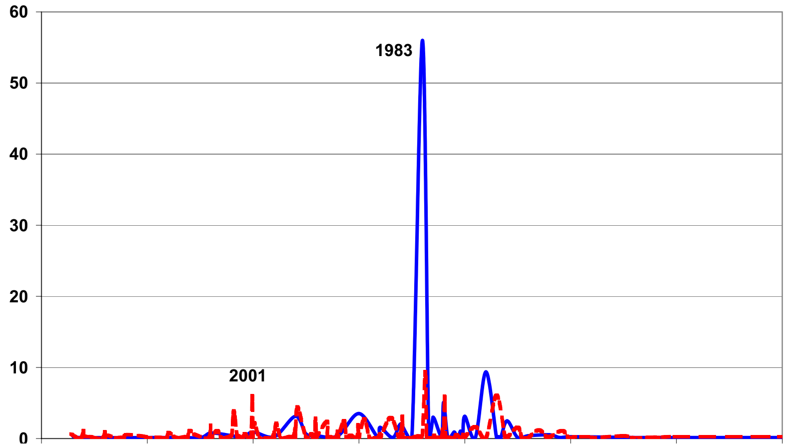
Figure 15: CV Consumer Interest Rates



Source: Authors' calculations based on TCCP and LIRS, see appendix for details.

Even more details about shifts in the terms of borrowing across households over time can be gleaned from changes in the empirical distribution of interest rates across households. Figure 16 displays the fraction of households reporting different interest rates in the SCF (essentially, a normalized histogram) for two different years: 1983 and 2001. This figure clearly shows the increase in interest rate dispersion between these two cross-sections. It is striking that in 1983 more than 50% of households faced a rate of *exactly* 18%. The distribution in 2001 is strikingly "flatter" than the 1983 distribution (the comparison with other years is similar). A very similar figure also emerges for the distribution of interest rates on 24-months consumer loans across banks (not reported here).

Figure 16: Credit Card Interest Rate Distribution



Source: Authors' calculations based on SCF

6.2 Risk Based Pricing

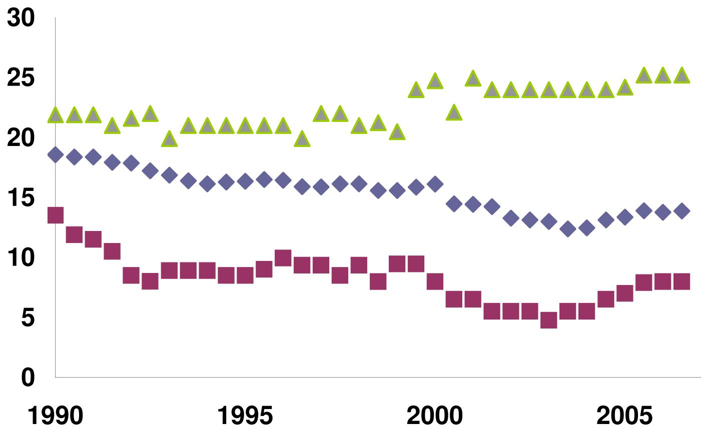
In the model, the increase in interest rate variety goes hand in hand with better risk-based pricing, i.e. people with different public signals are more likely to be offered different contracts.

One interesting fact from the data is that although the average (nominal) interest rate has declined over time, the maximum rate charged by banks has actually increased (see Figure 17). This increased gap between the average and the maximum rate points is consistent with more accurate pricing at the risky end. Of course it could also simply be an immediate consequence of an expansion of credit to riskier households.

More direct evidence of better risk-based pricing has been documented in several papers. For example, Edelberg (2006) combines data from the PSID and the SCF, and finds that lenders have become better at identifying higher risk borrowers and made increased use of risk-based pricing.[25] The timing of the change also coincides with the

[25] Today, credit card companies even make use of consumer purchase information to predict riskiness. For example, consumers who buy premium birdseeds, carbon-monoxide monitors, a device called "snow roof rakes" and those little felt pelts to avoid chairs from scratching the floor almost

Figure 17: Average, Min and Max Interest Rate Across Banks



Source: Authors' calculations, based on TCCP data, national plans only.

observation that in the late 1980s some credit card banks began to offer more different credit card plans "targeted at selected subsets of consumers, and many charge[d] lower interest rates" (Canner and Luckett 1992). The rise in risk-based pricing is also consistent with the entry and expansion of monoline lenders such as Capital One which target specific sub-groups of borrowers with credit card plans priced on their risk characteristics (Mann 2006).[26]

Another (coarse) way of seeing whether the dispersion of interest rates is related to increased risk based pricing is to compare the distribution of interest rates of delinquent and non-delinquents. The SCF asks households if they have been delinquent on a debt payment in the past year. Delinquency on debt is positively correlated with the probability of future default, so delinquent households should be riskier on average than non-delinquents. We find that the distributions for delinquents and

never miss payments. While people who buy cheap generic automotive oil or a chrome-skull car accessory are pretty likely to miss paying a bill eventually (as reported in New York Times, May 17, 2009.

[26]Furletti and Ody (2006) report that credit card issuers also have made increased use of fees as ways to impose a higher price on riskier borrowers.

non-delinquents was nearly identical in 1983 (See Figure 19 in the appendix). However, by 2001, the delinquent distribution has considerable mass to the right of the non-delinquent interest distribution (see Figure 20 in the Appendix). This supports the view that the increase in credit card contracts has led to more accurate pricing of borrowers default risk.

In sum then, it seems that the probability that two people who are observably different face the same interest rate has declined both in the model and the data, another dimension of similarity we add to Table 6.

6.3 Expansion of Credit to Lower Income Households

In the model both an increase in α and a decrease in χ imply an extension of credit to riskier borrowers. It has been well-documented that unemployment risk is higher for less educated (and hence lower income) people (*** add some references here ***). Thus, we now examine the relationship between income and borrowing, and how it has changed over time.

We start by documenting credit card ownership and borrowing across quintiles for six different waves of the SCF (see Table 5). As expected, at any point in time, there is a positive relationship between income and borrowing. For example, in 1983 only 11% of people in the lowest income quintile owned a credit card compared to 79% of people in the highest quintile. Of course not everyone borrows on their credit card, especially many high income people use their card mostly for transactions purposes. However, there is also a positive relationship between income and actual borrowing. In 1983, only 4% of people in the lowest income quintile actually carried a positive balance compared to 37% of households in the highest income quintile. What is interesting for our story is that credit card penetration increased most rapidly for lower income households over this time period. For example, for the lowest quintile, card ownership more than tripled from 11% in 1983 to 38% in 2004, while the fraction of people who carry a balance more than quadrupled from 4% in 1983 to ***% in 2004 (*** check these numbers***).

*** numbers for 2001-04 still need to be updated ***

Table 5: Percent HH with Bank Credit Card, U.S.

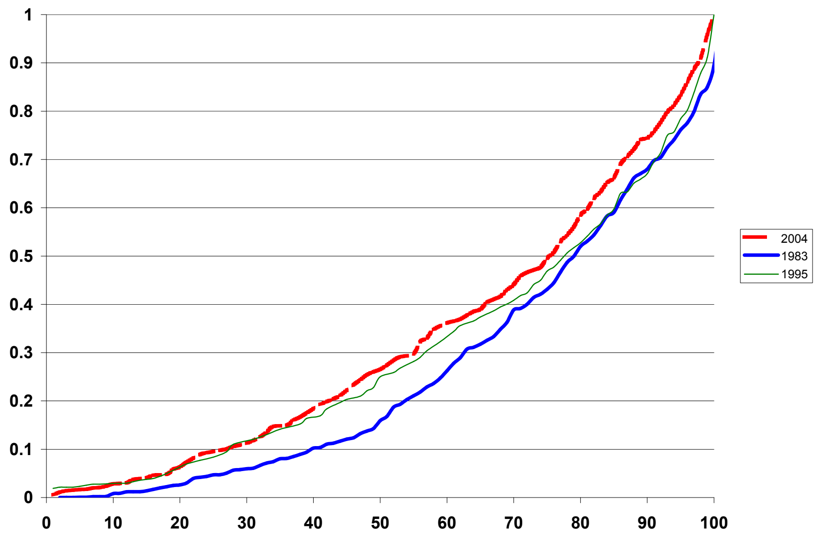
Income Quintile	1983	1989	1995	1998	2001	2004
have card	43%	56%	66%	68%	73%	72%
have balance	22%	29%	37%	37%	54%	56%
Lowest	11%	17%	28%	28%	38%	38%
have balance	4%	7%	16%	17%	60%	61%
2nd Lowest	27%	36%	54%	58%	65%	61%
have balance	13%	17%	30%	34%	59%	60%
Middle	41%	62%	71%	72%	79%	77%
have balance	24%	35%	41%	42%	61%	64%
2nd Highest	57%	76%	83%	86%	88%	87%
have balance	32%	47%	50%	52%	59%	57%
Highest	79%	89%	95%	95%	95%	96%
have balance	37%	41%	48%	43%	38%	44%

Source: 1983-1998 data is based on Durkin (2000), 2001-2004 is from ***, the underlying survey for all waves is the SCF. All CV numbers are the authors' own calculations based on SCF.

The increase in the number of lower income borrowers has been accompanied with a significant increase in their share of total credit card debt outstanding. Figure 18 graphs the cdf for the share of total credit card balances held by various percentiles of the earned income distribution for three different cross-sections: 1983, 1995, and 2004. As can be seen from the graph, the fraction of credit debt held by lower income households has increased significantly over the past twenty years. For example, the fraction of debt held by the bottom 30% (50%) of the earnings distribution nearly doubled from 6.1% to 11.2% (16.8 % to 26.6%). Given that the value of total credit card debt also increased, this figure implies that lower income households' access (and use) of credit card debt has increased significantly.

Figure 18 is consistent with the conclusions of numerous papers (for example, see Black and Morgan (1999), Kennickell, Starr-McCluer, and Surette (2000), Durkin (2000)) that the most rapid increase in credit card usage and debt has been among the poorest households. To the extent that lower income groups are riskier, this

Figure 18: CDF Credit Card Borrowing vs Earned Income



Source: Authors' calculations based on SCF

evidence suggests that borrowing by riskier households has increased over the 1983 - 2004 period.

7 Conclusion

This paper examines whether improved information technology has played a key role in the rapid changes in unsecured credit markets over the past thirty years. To do so, we develop a simple incomplete markets model with bankruptcy to analyze the qualitative implications of two mechanisms for the set of credit contracts offered in equilibrium. We also assemble data on how the number of credit card contracts and the distribution of credit card borrowing has changed over time so as to evaluate the model predictions.

We find that improvements in information technology which facilitate improved accuracy of lenders forecasts of borrowers default risk or the overhead costs of creating and offering contracts have significant implications for the set of contracts offered in equilibrium. For sufficiently large changes, these channels imply that technological change leads to an increased variety of credit contracts, with each contract targeted to a smaller subsets of the population. This increase in the number of contracts leads to an expansion of credit to more households, and involves the extension of credit to riskier households. As a result, these technological innovations can lead to an increase in aggregate borrowing and defaults. We also find that the predictions of both these channels are qualitatively consistent with changes observed in the U.S. in both the aggregate and cross-sectional pattern of borrowing and defaults over the past twenty-five years.

This findings of this paper suggests that interpretations of events in the unsecured credit market using a "standard" competitive framework may be misleading. We find that the introduction of even a small fixed cost of creating a contract leads to significant deviations from the predictions of the standard competitive framework. For example, the predicted relationship between the mean borrowing interest rate and the risk-free costs of funds changes dramatically with fixed costs of contracts, as the extensive margin of changes in the number of contracts leads to an ambiguous relationship between the cost of funds and average borrowing interest rates. This suggests that further explorations of the channels highlighted in our framework in a serious quantitative model could be a promising avenue for future research.

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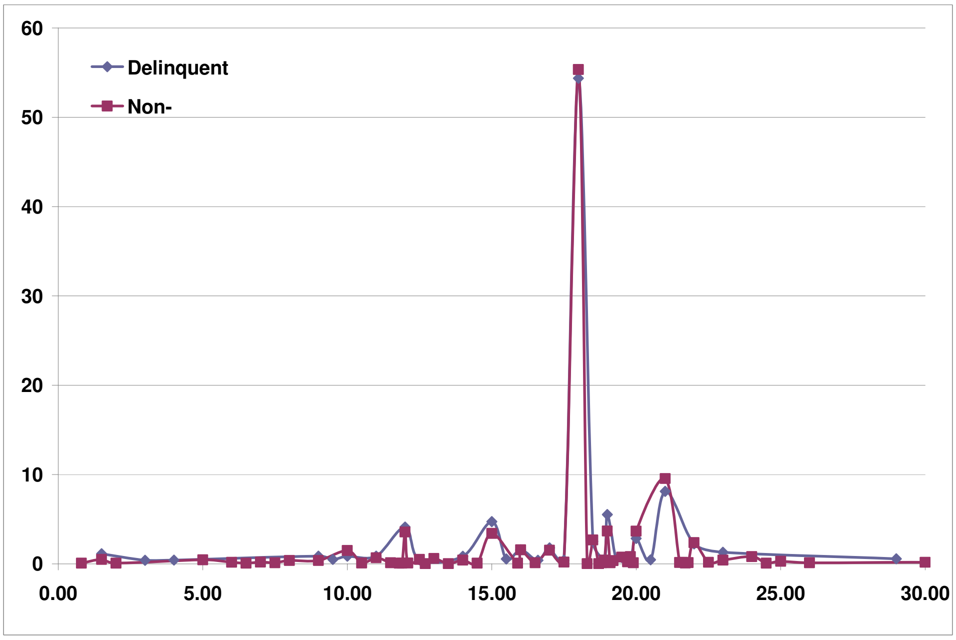
A Additional Figures

Figure 19: Delinquency and Credit Card Interest Rates 1983



B Model vs. Data

In Table 6 we compare the implications of the model with the facts observed in the data.

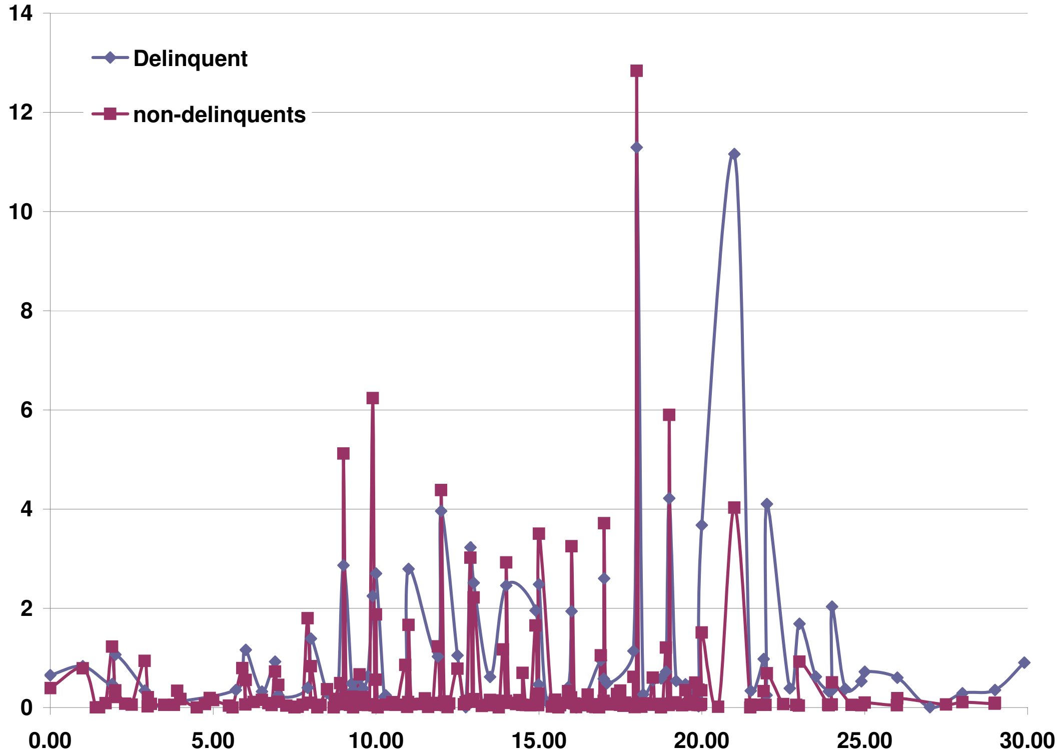
C Data Appendix

The Survey of Consumer Finance asked questions on the credit card interest rate of respondents. The questions asked were for the card with the largest balance (1995 - 2004), while the 1983 survey asked for the best guess of the average annualized interest respondent would pay on the bank or store card he uses most often if the full amount was not paid. One issue that affects the number of different rates reported

Table 6: Comparison Model vs. Data

	Δ in Model		Δ in Data	Data Source
	$\alpha\uparrow$	$\chi\downarrow$	(over time)	
Defaults/Population	$\uparrow$	$\uparrow$	$\uparrow$	Figure 1
Defaults/Borrowers	?	$\uparrow$	?	?
"Risky" Debt/Income	$\uparrow$	$\uparrow$	$\uparrow$	Figure 1
"Risky" Borrowers/Population	$\uparrow$	$\uparrow$	$\uparrow$	Table 3
# interest rates	$\uparrow$	$\uparrow$	$\uparrow$	Table 4
max r	$\uparrow$	$\uparrow$	$\uparrow$	Table 17
min r	$\downarrow$	$\downarrow$	$\downarrow$	Table 17
CV (r)	$\uparrow$	$\uparrow$	$\uparrow$	Table 3, Figure 15
% low income/high risk who borrow	$\uparrow$	$\uparrow$	$\uparrow$	Table 5
debt share low income/high risk	$\uparrow$	$\uparrow$	$\uparrow$	Figure 18

Figure 20: Delinquency and Credit Card Interest Rates 2001



is that in the later years the SCF imputed values for respondents who did not report an interest rate. To count the number of different interest rates, we drop imputed values. The sample size for the various years does increase, but by much less than the reported number of different interest rates.

Year	HH Count	HHs with non-imputed LOC rate	HHs with non-imputed CC rate	HHs with positive CC balance
1983	4103	-	2196	768
1989	3143	263	-	-
1992	3906	282	-	-
1995	4299	251	2458	1072
1998	305	279	2386	1029
2001	4442	265	2523	1085
2004	4519	440	2458	1185

When computing the CDF of credit card debt held by percentiles of the earned income distribution, we define earned income to be the sum of Wages + Salaries + Professional Practice, Business, Limited Partnership, Farm + Unemployment or Worker's Compensation.

C.1 Board of Governor Interest Rate Data

In Figures 15 and **??** we use data collected by the Board of Governors directly from banks on the interest rate for various financial contracts. The Board of Governors conducts several bank surveys asking banks for information on interest rates charged to consumers. The interest rate series for 24-months consumer loans is based on the *Quarterly Report of Interest Rates on Selected Direct Consumer Installment Loans* (LIRS). The data is collected in a quarterly survey (in February, May, August and November) and is available from February 1972 until February 2007.[27] Precisely, the survey asks about the most common rate (annual percentage rate) charged on "other loans for consumer goods and personal expenditures (24-month)." It includes loans for goods other than automobiles or mobile homes whether or not the loan is secured. These loans are typically used for consolidation of debts, medical attention,

[27]The information is collected on form FR 2835 and coded as item LIRS7808 by the Board.

taxes, vacations, and general personal and family expenditures, including student loans currently being repaid. It excludes all home improvement loans, and all loans secured primarily by real estate. This data is used to construct the interest rate series on 24-month consumer loans that the Fed reports in its G.19 series on consumer debt. Unfortunately the data is not a balanced panel. The 1972 data is based on a sample of 296 banks and then drops gradually to 100 in 2007.

The data we use for credit card interest rates is from a second survey, called *Terms of Credit Card Plans* (TCCP) which the Board has conducted bi-annually since 1990. This survey collects data on many dimensions of credit card terms, including finance charges and annual fees. The interest data we use is based on series TCCP6258, including only plans that are nationwide available (i.e. we exclude plans that are offered only in a specific state or region). Response rates vary from year to year between 200 and 400 banks.

Of course both data series have to be interpreted with caution since every bank is asked to report only *one* interest rate (the most commonly used one) and hence does not necessarily represent an accurate picture of *all* loan options faced by consumers.

C.2 Definition of Aggregate Measures

Here we briefly define the main aggregates we are interested in.

Given N risky contracts, the fraction of the population eligible to borrow via risky contracts is $\frac{1-\rho_N}{1-a}$. Since some households with $\sigma < \rho$ may not accept the risky contract they are offered, the fraction of the population with risky borrowing is:

$$\alpha\frac{1-\rho_N}{1-a} + \frac{(1-\alpha)}{1-a}\sum_{j=1}^{N}[(\sigma_{j-1}-\sigma_j)\frac{(\bar{\rho}_j - a)}{1-a}] \tag{C.1}$$

Note that if the participation constraint of types with lower public signals than their true type never binds (so $\bar{\rho}_j = 1$ for all j), this collapses to $\frac{1-\rho_N}{1-a}$.

Total Defaults equals the number of households who borrowed using the risky contract and experienced low income (y_l) in the second period of life:

$$\frac{\alpha}{1-a}\left(1-\rho_N-\frac{1-\rho_N^2}{2}\right)+\frac{(1-\alpha)}{1-a}\sum_{j=1}^{N}[\sigma_{j-1}-\sigma_j)]\frac{1}{1-a}\left(\bar{\rho}_j-a-\frac{(\bar{\rho}_j)^2-a^2}{2}\right)$$

Total borrowing is the sum of risky and risk-free borrowing. *Total Risky Borrowing* in units of the period 1 good is given by[28]

$$\sum_{j=1}^{N}\frac{(\sigma_{j-1}-\sigma_j)}{1-a}q_jL\left(\alpha+(1-\alpha)[\frac{(\bar{\rho}_j-a)}{1-a}]\right)$$

where $\sigma_0=1$. If the risk free contract is offered in equilibrium, then *Total Risk-Free Borrowing* is

$$q_{rf}L_{rf}\left[\frac{1}{1-a}-\sum_{j=1}^{N}\frac{(\sigma_{j-1}-\sigma_j)}{1-a}\left(\alpha+(1-\alpha)[\frac{(\bar{\rho}_j-a)}{1-a}]\right)\right]$$

We define the average *risk premium* using the default rate on a risky contract, expressed in terms of the face value of the debt in period 2. The number of defaults on contract j is

$$D_j=\alpha\frac{\sigma_{j-1}-\sigma_j}{1-a}\left(1-\frac{\sigma_{j-1}+\sigma_j}{2}\right)+(1-\alpha)\frac{\sigma_{j-1}-\sigma_j}{1-a}\frac{1}{1-a}\left(\bar{\rho}_j-a-\frac{(\bar{\rho}_j)^2-a^2}{2}\right)$$

The *Average Overhead on Contract$_j$* is:

$$\text{Average Overhead on Contract}_j=\frac{\chi}{\frac{(\sigma_{j-1}-\sigma_j)}{1-a}q_jL\left(\alpha+(1-\alpha)[\frac{(\bar{\rho}_j-a)}{1-a}]\right)}$$

The *Total Overhead* cost of risky loans is the number of contracts times the cost per contract ($N\chi$). If the risk-free contract is offered, total overhead costs is $(N+1)\chi$.

[28]This is the (present value of) the amount borrowed at date 1, rather than the face value of debt outstanding at $t=2$.