Trade, Multinational Production, and the Gains from Openness*

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Abstract

Much attention has been devoted to the quantification of the gains from trade. In this paper we argue for the need to quantify the gains from *openness*. This notion includes not only trade but all the other ways through which countries interact. We devote special attention to multinational production (MP), which in 2004 was more than twice as large as trade flows. We present and estimate a model where countries interact through trade as well as MP, and then quantify the overall gains from openness and the role of both of these channels in generating those gains. We build a model where trade and MP are alternative ways to serve a foreign market, which makes them substitutes, but MP also relies on imports of intermediate goods from the home country, which make them complements. The model also allows for "bridge" MP, or export platforms, creating an additional channel for complementarities between trade and MP. Our results imply that the gains from openness are much larger than the gains from trade – this is thanks to the large gains from MP, but these gains from trade are larger than the ones calculated using models with only trade. Further, our estimated model suggests complementarity between trade and MP.

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1 Introduction

Much attention has been devoted to the quantification of the gains from trade. In this paper we argue for the need to quantify the gains from *openness*. The notion of openness includes not only trade but all the other ways through which countries interact. Even if a country were to shut down trade, it could still benefit from foreign ideas through the activity of foreign affiliates of multinational firms (what we call multinational production) as well as the flow of ideas through migration, books, journals and the Internet, among others. Multinational production is arguably one of the most important channels through which countries benefit from openness. It is certainly as important as (if not more than) trade: by 2004, total sales of foreign affiliates of multinational firms in the world were more than twice as high as world exports. Furthermore, over the past two decades, while exports increased by a factor of five, sales of foreign affiliates have increased by a factor of seven (UNCTAD's World Investment Report, 2006).

The goal of this paper is to construct and calibrate a general equilibrium model where countries interact through trade and multinational production (MP), and then to quantify the overall gains from openness and the role of both of these channels in generating those gains. Our results imply that the gains from openness are much larger than the gains from trade – this is thanks to the large gains from MP, but these gains from trade are larger than the ones calculated using models with only trade. Further, our estimated model suggests complementarity between trade and MP.

Calculating the gains from trade in a model that allows for trade and MP represents a significant departure from the standard practice in the literature, which is to consider trade as the only channel through which countries interact.[1] Similarly, studies quantifying the gains from MP are based on models that do not allow for trade.[2] Considering each of these channels separately, however, may understate or overstate the associated gains depending on the existence of significant sources of complementarity or substitutability among them. Suppose that MP depends on the ability of foreign affiliates to import inputs from their home country. In this case, shutting down trade would also decrease MP and generate losses beyond those calculated in models with trade but no MP. Alternatively, trade and MP may behave as substitutes because they are competing ways of serving foreign markets. In this case, shutting down trade would

[1] Some recent attempts to quantify gains from trade in Ricardian models are Eaton and Kortum (2002), Alvarez and Lucas (2007), Waugh (2007), and Fieler (2007).

[2] See Burstein and Monge-Naranjo (2007), McGrattan and Precott (2007), and Ramondo (2006).

generate smaller losses than in models with only trade because MP would partially replace the lost trade.

The literature has typically modeled trade and "horizontal" Foreign Direct Investment (FDI) as substitutes in the context of the "proximity-concentration" trade-off: firms choose to either serve a foreign market by exporting or opening an affiliate there (Brainard, 1997; Markusen and Venables, 1997; Helpman, Melitz, and Yeaple, 2004). On the other hand, the literature has modeled trade and "vertical" FDI as complements: foreign affiliates rely on intermediate goods imported from their parent firms to produce goods that are consumed in other markets (Markusen, 1984; Grossman and Helpman, 1985; Antras, 2003). The empirical evidence appears consistent with both of these views. Studies using data at the industry, product, or firm level, have concluded that MP and trade flows in *intermediate inputs*, often conducted within the firm, are complements, while MP and trade flows in *final goods* are substitutes (Belderbos and Sleuwaegen, 1988; Blonigen, 2001; Head and Ries, 2001; Head, Ries, and Spencer, 2004).

This paper presents a general equilibrium, multi-country, Ricardian model of trade and MP. The model has two sectors: tradable intermediate goods, and non-tradable consumption goods. All goods are produced with constant-returns-to-scale technologies which differ across countries, creating incentives for trade and MP. For non-tradable goods, serving a foreign market can only be done through MP, but for tradable goods we have to consider the choice between exports and MP. Trade flows are affected by iceberg-type costs that may vary across country pairs. To avoid these costs or to benefit from lower costs abroad, firms producing tradable goods may prefer to serve a country through MP rather than exports. But MP entails some efficiency losses as well. Moreover, to introduce complementarity between trade and MP, we assume that affiliates rely, at least partially, on imported inputs from their home country; in our empirical approach, we think of this as "intra-firm" trade.[3] Since these imports are affected by trade costs (just as regular "arm-length" trade), this creates an extra cost of MP.

Our set-up allows firms to use a third country as a "bridge" or export platform, to serve a particular market; we refer to this as "bridge MP", or simply BMP.[4] For example, a firm from country i producing a tradable good v can serve country n by doing MP in country l, and

[3]The empirical evidence points to significant intra-firm trade flows related to multinational activities (Hanson, Mataloni, and Slaughter, 2003; Bernard, Jensen, and Schott, 2005; and Alfaro and Charlton, 2007).

[4]We avoid referring to this type of MP as "vertical" MP because the main motivation for BMP in our model is to avoid trade costs rather than allocating different stages of the production process across locations according to their comparative advantage.

ship it to country n. This entails MP costs associated with the pair $\{i, l\}$, and also trade costs associated with the pair $\{l, n\}$.[5]

The multiplicity of choices regarding how to serve a foreign market makes trade and MP substitutes: exports and MP are alternative ways of serving a foreign market. However, the possibility of BMP creates complementarities between trade and MP: the decision by country i of serving market n producing in a third country l generates a trade flow from l to n associated with MP from i to l. Moreover, when country i serves market n through MP, there is an "intra-firm" trade flow in intermediate inputs from country i associated with it. Thus, even in a world without BMP, our model generates complementarities between trade and MP.[6]

We estimate the model using data on bilateral trade and MP flows for a set of OECD countries, as well as data on intra-firm trade flows for U.S. multinationals and foreign multinationals operating in the U.S. However, as it is well known, the trade data alone cannot identify the parameter that determines the strength of comparative advantage – θ in Eaton and Kortum (2002). Thus, we appeal to the model's implications for the long run growth rate to calibrate this parameter. In particular, although the model we present is static, in the Appendix we show that the equilibrium of the static model can be seen as the steady state equilibrium of a dynamic model where productivity evolves according to an exogenous "research" process. This dynamic model exhibits quasi-endogenous growth as in Jones (1995) and Kortum (1997), and is closely related to Eaton and Kortum (2001). Importantly, growth is driven by the same mechanism that generates the gains from openness in the static model, namely the aggregate economies of scale associated with the fact that a larger population is linked to a higher stock of non-rival ideas. This is why calibrating the comparative advantage parameter so that the dynamic model generates a growth rate that matches the one we observe in the data seems appropriate.

We use the estimated model to compute the joint gains from trade and MP; we think of these gains as the overall gains from openness. We also compute the separate gains from these two channels. Our results suggest that the gains from openness are much higher than the gains

[5] Even among rich countries, foreign subsidiaries of multinationals often sell a sizable part of their output outside of the host country. For example, around 30% of total sales of US affiliates in Europe are not done in the host country of production (Blonigen, 2005).

[6] We note that MP can be seen as a channel for international technology diffusion, since a country's technologies end up being used abroad. In this sense, our model captures the idea that trade facilitates technology diffusion, at least diffusion associated with multinational activities. On the other hand, our model does not incorporate any causal link whereby trade or MP enhance international knowledge spillovers. The large literature on this topic is surveyed in Keller (2004).

from trade: the average OECD country's real income is 15% higher thanks to the joint gains from trade and MP compared to isolation (i.e., no trade and no MP). In contrast, shutting down trade would lead to real income losses of only 3% for the average OECD country. The difference between these two numbers arises due to the large gains from MP in our estimated model: shutting down MP would lead to loses of 9% for the average OECD country. Of course, these gains and losses are much higher for smaller countries. For example, gains from openness are 30% for Belgium whereas they are only 5% for the United States. As a small country, Belgium benefits greatly both from trade and MP, although the gains from MP almost double those from trade.

As we mentioned above, the fact that exports and MP are alternative ways to serve foreign consumers makes trade and MP substitutes, but the existence of BMP and the assumption that MP generates a demand for home-country inputs makes trade and MP complements. The estimated model suggests that the second force dominates, so that trade and MP behave as complements.

2 The Model

We extend Eaton and Kortum's (2002) model of trade to incorporate MP. Our model is Ricardian with a continuum of tradable intermediate goods and non-tradable final goods, produced under constant-returns-to-scale. We adopt the probabilistic representation of technologies as first introduced by Eaton and Kortum (2002), but we enrich it to incorporate MP. We embed the model into a general equilibrium framework similar to the one in Alvarez and Lucas (2007).

2.1 The Closed Economy

To introduce the notation and main features of our model, consider first a closed economy with L units of labor. A representative agent consumes a continuum of final goods, indexed by $u \in [0,1]$ in quantities $q_f(u)$, deriving utility

$$U = [\int_0^1 q_f(u)^{\frac{\varepsilon-1}{\varepsilon}} du]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $\varepsilon > 0$. Final goods are produced with labor and a continuum of intermediate goods indexed by $v \in [0,1]$. Formally, intermediate goods are aggregated into a *composite intermediate good* via a CES production function,

$$Q = [\int_0^1 q_g(v)^{\frac{\sigma-1}{\sigma}} dv]^{\frac{\sigma}{\sigma-1}},$$

with $\sigma > 0$. This composite intermediate good and labor are used to produce final goods via Cobb-Douglas technologies with varying productivity levels,

$$q_f(u) = z_f(u) L_f(u)^{\alpha} Q_f(u)^{1-\alpha}. \tag{1}$$

The variables $L_f(u)$ and $Q_f(u)$ denote the quantity of labor and the composite intermediate good used in the production of final good u, respectively, and $z_f(u)$ is a productivity parameter. Similarly, intermediate goods are produced according to

$$q_g(v) = z_g(v) L_g(v)^{\beta} Q_g(v)^{1-\beta}. \tag{2}$$

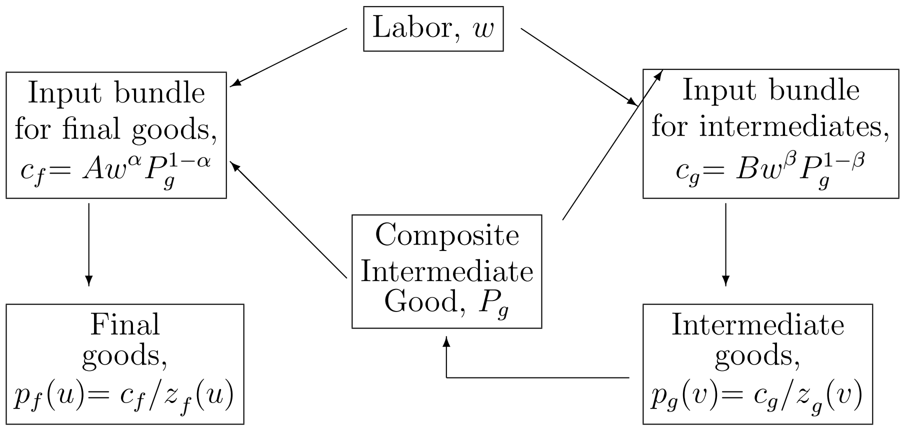
Resource constraints are

$$\begin{aligned} \int_0^1 L_f(u) du + \int_0^1 L_g(v) dv &= L, \\ \int_0^1 Q_f(u) du + \int_0^1 Q_g(v) dv &= Q. \end{aligned}$$

To complete the description of the environment in the closed economy, we assume that the productivity parameters $z_f(u)$ and $z_g(v)$ are random variables drawn independently from a Fréchet distribution with parameters T and $\theta > \max\{1, \sigma - 1\}$, $F(z) = \exp\left(-T z^{-\theta}\right)$, for $z > 0$.

To describe the competitive equilibrium for this economy it is convenient to introduce the notion of an *input bundle for the production of final goods*, and an *input bundle for the production of intermediate goods*, both of which are produced via Cobb-Douglas production functions with labor and the composite intermediate good, and used to produce final and intermediate goods, as specified in (1) and (2), respectively. The unit cost of the input bundle for final goods is $c_f = A w^{\alpha} P_g^{1-\alpha}$, and the unit cost of the input bundle for intermediate goods is $c_g = B w^{\beta} P_g^{1-\beta}$, where w and P_g are the wage and the price of the composite intermediate good, respectively, $A \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$, and $B \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$. In a competitive equilibrium prices of final goods are given by $p_f(u) = c_f / z_f(u)$, and prices of intermediate goods are given by $p_g(v) = c_g / z_g(v)$. In turn, the aggregate price for intermediates, P_g, satisfies $P_g^{1-\sigma} = \int_0^1 p_g(v)^{1-\sigma} dv$. Figure 1 illustrates the cost structure in the closed economy.

Figure 1: Cost Structure in the Closed Economy



The characterization of the equilibrium follows closely the analysis in Eaton and Kortum (2002) and Alvarez and Lucas (2007), so we omit the details. Suffice to say here that the equilibrium real wage is given by

$$\frac{w}{P_f} = \widetilde{\gamma} \cdot T^{\frac{1+\eta}{\theta}}, \tag{3}$$

where $P_f^{1-\varepsilon} = \left(\int_0^1 p_f(u)^{1-\varepsilon} du\right)$ is the ideal price index for final goods, $\eta \equiv (1-\alpha)/\beta$, and $\widetilde{\gamma}$ is a positive constant.[7]

2.2 The World Economy

Now consider a set of countries indexed by $i \in \{1, ..., I\}$ with preferences and technologies described above. Country i has L_i units of labor. Intermediate goods are tradable but final goods are not. Trade is subject to iceberg-type costs: $d_{nl} \geq 1$ units of any good must be shipped from country l for one unit to arrive in country n. We assume that $d_{nn} = 1$, and the triangle inequality holds (i.e., $d_{nl} \leq d_{nj}d_{jl}$ for all n, l, j).

Each country i has a technology to produce each final good and each intermediate good, at home or abroad. These technologies are described by the vectors $\mathbf{z}_{fi}(u) \equiv \{z_{f1i}(u), ..., z_{fIi}(u)\}$ and $\mathbf{z}_{gi}(v) \equiv \{z_{g1i}(v), ..., z_{gIi}(v)\}$. When a country i produces in another country $l \neq i$, we say that there is multinational production (MP) by country i in country l. [Sometimes, we

[7] In Alvarez and Lucas (2007) the real wage is proportional to $T^{\eta/\theta}$ rather than $T^{(1+\eta)/\theta}$. The difference arises because we also have T affecting the productivity for final goods.

just say that MP in country l is carried out by country i "multinationals".] The corresponding productivity parameter in this case is $z_{fli}(u)$, or $z_{gli}(v)$. We adopt the convention that the subscript n denotes the destination country, l the country of production, and i the country where the technology originates. Note that if $z_{fli}(u) = z_{gli}(v) = 0$ whenever $l \neq i$, for all $u, v \in [0, 1]$, our model collapses to the Alvarez and Lucas (2007) version of Eaton and Kortum (2002) model of trade but no MP.

In turns, MP incurs an "iceberg" type efficiency loss of $h_{li} \geq 1$ associated with using an idea from i to produce in l. Thus, whereas national production of final good u in country l entails unit cost $c_{fl}/z_{fll}(u)$, MP of final good u by i in l entails unit cost $c_{fl}h_{li}/z_{fli}(u)$. We assume that $h_{ii} = 1$. Similarly, whereas national production of intermediate good v in l has unit cost $c_{gl}/z_{gll}(v)$, MP of intermediate good v by i in l entails unit cost $c_{gli}/z_{gli}(v)$. The unit cost c_{gli} differs from $c_{fl}h_{li}$ because we assume that MP in intermediate goods requires the use of what we call a *multinational input bundle for the production of intermediate goods*. In particular, we assume that the multinational input bundle combines the national input bundle from the home country (i.e., the country where the technology originates) and the host country (i.e., the country where production takes place). The home country national input bundle must be shipped to the host country of production, and this implies paying the corresponding transportation cost. The cost of the home country national input bundle used in MP by country i in country l is then $c_{gi}d_{li}$. The host country national input bundle has cost c_{gl}, but MP incurs an "iceberg" type efficiency loss of $h_{li} \geq 1$ associated with using an idea from i to produce in l. The cost of the host country national input used in MP by i in l is then $c_{gl}h_{li}$. Combining the costs of home and host country national inputs into a CES aggregator, we get the unit cost of the multinational input bundle for intermediates produced by i in l,

$$c_{gli} = \left[(1-a) \left(c_{gl} h_{li} \right)^{1-\xi} + a \left(c_{gi} d_{li} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}}, \tag{4}$$

where $a \in [0, 1]$ and $\xi > 1$. Note that $c_{gii} = c_{gi}$. Moreover, if $a = 0$, then $c_{gli} = c_{gl}h_{li}$. The parameter ξ indicates the degree of complementarity between the national input bundles from the home and host countries. It is a key parameter for our estimated welfare gains.

Finally, we assume that the productivity vectors $\mathbf{z}_{fi}(u)$ and $\mathbf{z}_{gi}(v)$ for each good are random variables that are drawn independently across goods and countries from a multivariate Fréchet distribution with parameters $(T_{1i}, T_{2i}, ..., T_{Ii})$, $\theta > \max\{1, \sigma - 1\}$, and $\rho \in [0, 1)$,[8]

$$F_i(\mathbf{z}_{si}) = \exp\left[-\left(\sum_l \left(T_{li}\left(z_{sli}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}\right)^{1-\rho}\right]. \tag{5}$$

Note that

$$\lim_{x\to\infty} F_i(x, x, ..., z_{sli}, ..., x) = \exp\left[-T_{li} z_{sli}^{-\theta}\right],$$

so that the marginal distributions are Fréchet. The parameter ρ determines the degree of correlation among the elements of $\mathbf{z}_{si}$: if $\rho = 0$, productivity levels are uncorrelated across production locations, while in the limit as $\rho \to 1$, they are perfectly correlated, so that productivity is independent of the production location (i.e., $z_{sii} = z_{sli}$, for all l).

2.3 Equilibrium Analysis

Since final goods are identical except for their productivity parameters (i.e., they enter preferences symmetrically), we follow Alvarez and Lucas (2007), drop index u, and label final goods by $Z_f \equiv (\mathbf{z}_{f1}, ..., \mathbf{z}_{fI})$. Similarly, we label intermediate goods by $Z_g \equiv (\mathbf{z}_{g1}, ..., \mathbf{z}_{gI})$. The unit cost of a final good Z_f in country n produced with a technology from country i is $c_{fn}h_{ni}/z_{fni}$, while the unit cost of an intermediate good Z_g in country n produced in country l with a technology from country i is $c_{gli}d_{nl}/z_{gli}$.

In a competitive equilibrium the price of final good Z_f in country n is simply the minimum unit cost at which this good can be obtained, which is given by $p_{fn}(Z_f) = \min_i c_{fn}h_{ni}/z_{fni}$. Similarly, the price of intermediate good Z_g in country n is $p_{gn}(Z_g) = \min_{i,l} c_{gli}d_{nl}/z_{gli}$. Note that if $l = i$, then the intermediate good is exported from i to n while if $i \neq l = n$, then there is MP from i to n. Finally, if $i \neq l$ and $l \neq n$, then country l is used as an export platform by country i to serve country n. We say that in this case there is "bridge MP", or simply BMP, by country i in country l.

The following proposition, which is proved in the Appendix, describes a number of convenient results regarding the choice of goods according to the origin of the technology and the location of production, as well as a characterization of prices and expenditure shares.

Proposition 1 *(a) The shares of final and intermediate goods that country n buys produced*

[8] This distribution is discussed in footnote 14, Eaton and Kortum (2002).

with country i technologies are, respectively,

$$\phi_{fni} = \frac{\Phi_{fni}}{\Phi_{fn}} \text{ and } \phi_{gni} = \frac{\Phi_{gni}}{\Phi_{gn}}_{gn},$$

where

$$\Phi_{fni} \equiv T_{ni} \left(c_{fn} h_{ni}\right)^{-\theta}, \Phi_{gni} \equiv \left(\sum_{l} \left(T_{li} \left(c_{gli} d_{nl} \right)^{-\theta} \right)^{\frac{1}{1-\rho}} \right)^{1-\rho}, \text{ and } \Phi_{sn} \equiv \sum_{i} \Phi_{sni}, \text{ for } s = f, g;$$

(b) Of the intermediate goods bought by country n that are produced with country i technologies, the share that is produced in country l is

$$\pi_{gni,l} = \left(\frac{T_{li} \left(c_{gli} d_{nl} \right)^{-\theta}}{\Phi_{gni}} \right)^{\frac{1}{1-\rho}};$$

(c) The average price of goods that are purchased in any market n does not depend on the source of the technology or the location of production; and

(d) The price index in country n, is given by

$$P_{sn} = \gamma \Phi_{sn}^{-1/\theta}, \tag{6}$$

for $s = g, f$, where $\gamma \equiv \Gamma(1 + (1 - \sigma)/\theta)^{1/1-\sigma}$, and $\Gamma(\cdot)$ is the Gamma function.

Since Φ_{sn}, for $s = g, f$ is a function of model's parameters and unit costs, c_{sn}, which in turn is a function of wages and the price index P_{gn}, the set of equations associated with (6), for $s = g$ and $n = \{1, ..., I\}$, implicitly determines P_{gn} as a function of $\mathbf{w} = (w_1, ..., w_I)$. In vector notation, this defines the function $P_g(\mathbf{w})$: $I \to I$ (see Alvarez and Lucas, 2007). Together with $P_g(\mathbf{w})$, equation (6) for $s = f$ also defines a function $P_f(\mathbf{w})$ that determines the price index for final goods as a function of wages.

The total expenditure on final goods by country n is equal to the country's total income, $w_n L_n$. We refer to the total value of final goods produced in n with country i technologies as the value of MP in final goods by i in n, denoted by Y_{fni}. Part (c) of Proposition 1 implies that ϕ_{sni} and $\pi_{sni,l}$ not only represent the share of goods purchased by country n produced with different technologies and in different production locations, but also expenditure shares. Thus,

$$Y_{fni} = \phi_{fni} w_n L_n.$$

Note that $\sum_i Y_{fni} = w_n L_n \sum_i \phi_{fni} = w_n L_n$.

Since total expenditure on intermediates by country n is $P_{gn}Q_n$, the value of MP in intermediates by country i in country l to serve country n is $\phi_{gni}\pi_{gni,l}P_{gn}Q_n$. Thus, total MP by i in l is

$$Y_{gli} = \sum_n \phi_{gni}\pi_{gni,l}P_{gn}Q_n. \tag{7}$$

Total imports by country n from l are given by the sum of intermediate goods produced in country l with technologies from any other country, $\sum_i \phi_{gni}\pi_{gni,l}P_{gn}Q_n$, plus the imports of country l's input bundle for intermediates used by country l's multinationals operating in country n. To compute this second term, let ω_{nl} be the cost share of the home country input bundle for the production of intermediates in country n by multinationals from country l. From (4), $\omega_{nl} = a\left(c_{gl}d_{nl}/c_{gnl}\right)^{1-\xi}$. The value of imports of the input bundle for intermediates by n from l associated with MP by l in n is $\omega_{nl}Y_{gnl}$. Total imports by country n from $l \neq n$ are then given by

$$X_{nl} = \sum_i \phi_{gni}\pi_{gni,l}P_{gn}Q_n + \omega_{nl}Y_{gnl}. \tag{8}$$

For country n , aggregate imports are simply $\sum_{l\neq n} X_{nl}$, while aggregate exports are $\sum_{l\neq n} X_{ln}$. The trade balance condition is then

$$\sum_{l\neq n} X_{nl} = \sum_{l\neq n} X_{ln}. \tag{9}$$

As in Alvarez and Lucas (2007), the total expenditure on the composite intermediate good is proportional to the country's total income.

Proposition 2 *$P_{gn}Q_n = \eta w_n L_n$, for all n, where $\eta \equiv (1-\alpha)/\beta$.*

Since the terms ϕ_{ni}, $\pi_{ni,l}$, and ω_{li} are functions of $\mathbf{w}$, then the trade balance conditions constitute a system of I equations in $\mathbf{w}$. This system of equations together with some normalization of wages yields an equilibrium wage vector $\mathbf{w}$.[9]

2.4 Gravity

In general, it is not possible to express trade flows from l to n as a function of the bilateral trade cost d_{nl} and country specific variables for l and n: the general model does not have a gravity

[9]We use the following normalization: $\sum_{i=1}^{I} w_i L_i = 1$.

equation. However, there is one special case for which a gravity equation can be derived. When $\rho = 0$ and $a = 0$, it can be shown that

$$X_{nl} = \frac{T_l' \left(c_{gl} d_{nl}\right)^{-\theta}}{\sum_k T_k' \left(c_{gk} d_{nk}\right)^{-\theta}} \eta w_n L_n, \tag{10}$$

where $T_l' \equiv \sum_i T_{li} h_{li}^{-\theta}$ is an adjusted technology parameter for l that takes into account the possibility of using technologies from other countries appropriately discounted by the efficiency losses h_{li}. This expression is exactly as the one in Eaton and Kortum (2002), and implies that country $l's$ normalized import share in country n depends only on the trade cost d_{nl}, and the price indices P_{gn} and P_{gl},

$$\frac{X_{nl}/w_n L_n}{X_{ll}/w_l L_l} = \left(\frac{d_{nl} P_{gl}}{P_{gn}}\right)^{-\theta}. \tag{11}$$

In the gravity literature, d_{nl} is called a "bilateral resistance" term while P_{gl} and P_{gn} are called "multilateral resistance" terms.

To understand why this result does not hold in the general case with $\rho > 0$, note that there are I^2 ways to produce any intermediate good, resulting from the combination of I source countries and I production locations. The productivity parameters $z_{g1i}, z_{g2i}, ..., z_{gIi}$ associated with source country i are positively correlated (since $\rho > 0$) whereas the productivity parameters $z_{gl1}, z_{gl2}, ..., z_{glI}$ associated with production location l are uncorrelated (by assumption of independence across the vectors $\mathbf{z}_{gi}$, for $i = 1, 2, ..., I$). The different degrees of correlation among the elements of the columns and rows of the Z_g matrix makes it generally impossible to express all the determinants of bilateral trade flows in a bilateral resistance term together with multilateral resistance terms, as in (11).[10]

Analogously, we can explore whether there is a gravity-like relationship for MP flows. In the special case with $\rho = 0$ and $a > 0$, using (7) and some manipulation, we get

$$Y_{gli} = \eta \gamma^{-\theta} \frac{T_{li} c_{gli}^{-\theta}}{P_{gl}^{-\theta}} \cdot \sum_n \left(\frac{d_{nl} P_{gl}}{P_{gn}}\right)^{-\theta} w_n L_n.$$

The second term on the RHS can be interpreted as the "market potential" of country l, while the term $T_{li} c_{gli}^{-\theta} / P_{gl}^{-\theta}$ captures the "relative competitiveness" of i technologies in country l. If

[10]The only exception is when $a = 0$ and $T_{li} h_{li}^{-\theta}$ is "separable" in the sense that it can be written as the product of a source and a destination-specific terms: $T_{li} h_{li}^{-\theta} = \lambda_l \mu_i$, for all l, i. In this case we obtain an expression similar to (10) but with T_i' substituted by $T_i'' = \lambda_l^{\frac{1}{1-\rho}}$, and θ substituted by $\theta/(1-\rho)$. The reason why this works is that the distribution of $(\tilde{z}_{g1}, \tilde{z}_{g2}, ..., \tilde{z}_{gI})$, for $\tilde{z}_{gl} \equiv \max_i \{z_{gli}/c_{gli}\}$, is multivariate Fréchet with parameters θ and ρ.

we use Ψ_{gl} to denote the market potential of country l in intermediates, then we can write

$$\frac{Y_{gli}/\Psi_l}{Y_{gii}/\Psi_i} = \left(\frac{\widetilde{h}_{gli}P_{gi}}{P_{gl}}\right)^{-\theta}, \tag{12}$$

where $\widetilde{h}_{gli}^{-\theta} = (T_{li}c_{gli}^{-\theta}/T_{ii}c_{gii}^{-\theta})$ is an average relative cost of producing in country l rather than in country i with country i's technologies.

2.5 Symmetry

To gain some additional intuition about the model, we consider the case of symmetric countries. The symmetric case can be solved analytically, yet the basic intuition carries to the general case with asymmetric countries. We calculate the gains from openness, and explore the role of trade and MP in generating those gains. We are particularly interested in understanding the conditions under which trade and MP behave as substitutes or complements. We are also interested in the sources of complementarity – that is, we want to differentiate between the complementarity that arises from the possibility of doing "bridge" MP and the complementarity that arises from the possibility of using the home country's input bundle in MP activities (i.e. the role of the parameters ξ and a).

Symmetry entails $L_i = L$ for all i; $d_{nl} = d$, $h_{nl} = h$, for all $l \neq n$; and $d_{nn} = h_{nn} = 1$ for all n. Moreover, we assume that $T_{li} = T$, for all l, i. In equilibrium wages, costs, and prices are equalized across countries, $w_n = w$, $c_n = c$, and $P_{sn} = P_s$, for $s = g, f$. Thus, the cost of the multinational input bundle collapses to $c_{gli} = m \cdot c_g$ (and $c_{gll} = c_g$), where $m \equiv \left[(1-a)h^{1-\xi} + ad^{1-\xi}\right]^{\frac{1}{1-\xi}}$. Thus, the share of spending in the home input bundle done by MP is simply $\omega = a(d/m)^{1-\xi}$.

The equilibrium is characterized as follows. In the case of final goods the situation is straightforward: a country uses its own technologies for local production to serve domestic consumers, and for MP abroad to serve foreign consumers. For intermediate goods, there is trade, MP and BMP. Countries use some of their own technologies for local production to satisfy domestic and foreign consumers through exports. They also engage in MP abroad, for which the resulting output is sold to local consumers (MP), sent back home or sold to third markets (BMP). There is also trade associated with the import of the home country input bundle for MP.

For intermediate goods, in the special case with $\rho \to 1$, we can show that BMP vanishes, and trade and MP do not overlap: if $d > h$, there is only MP and trade associated with MP (i.e., imports of the home country input bundle), but there is no other trade of individual intermediate goods; while if $h > d$, there is only trade (no MP) (see the Appendix).

In general, we can solve for the real wage in terms of trade and MP costs, d, h and m, and parameters θ, η, and ρ.

Proposition 3 *Under symmetry, the equilibrium real wage is given by*

$$\frac{w}{P_f} = \widetilde{\gamma}^{-1}\left[1+(I-1)h^{-\theta}\right]^{1/\theta}\left[\Delta_0+(I-1)\Delta_1\right]^{\frac{\eta}{\theta}} T^{\frac{1+\eta}{\theta}}, \tag{13}$$

where

$$\Delta_0 \equiv \left(1+(I-1)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho} \tag{14}$$

$$\Delta_1 \equiv \left(d^{-\frac{\theta}{1-\rho}}+m^{-\frac{\theta}{1-\rho}}+(I-2)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}, \tag{15}$$

and $\widetilde{\gamma}$ is a constant.

This result shows how access to foreign ideas through trade and MP increases a country's real wage. We discuss this further in the context of the associated results about the gains from trade and MP.

Gains The gains from openness (GO) are given by the change in the real wage, w/P_f, from isolation ($d, h \to \infty$) to the "benchmark"($d, h < \infty$). The gains from trade (GT) are the gains of moving from isolation to only trade ($d < \infty, h \to \infty$), while the gains from MP (GMP) are the gains of moving from isolation to only MP ($h < \infty, d \to \infty$). Finally, we calculate the gains of moving from a situation with only MP ($h < \infty, d \to \infty$) to a situation with both trade and MP ($h, d < \infty$). This alternative definition of gains from trade is denoted by GT'.

Proposition 4 *Under symmetry, gains from openness are:*

$$GO = \left[1+(I-1)h^{-\theta}\right]^{\frac{1}{\theta}} \cdot \left[\Delta_0+(I-1)\Delta_1\right]^{\frac{\eta}{\theta}}, \tag{16}$$

where Δ_0 and Δ_1 are given by (14) and (15), respectively.

Gains from trade are:

$$GT = \lim_{h\to\infty} GO = \left[1+(I-1)d^{-\theta}\right]^{\frac{\eta}{\theta}}, \tag{17}$$

while gains from MP are:

$$GMP = \lim_{d\to\infty} GO = \left[1 + (I-1)h^{-\theta}\right]^{\frac{1}{\theta}} \cdot \left[1 + (I-1)\widetilde{m}^{-\theta}\right]^{\frac{\eta}{\theta}} \tag{18}$$

where

$$\widetilde{m} \equiv \lim_{d\to\infty} m = (1-a)^{\frac{1}{1-\xi}}\, h. \tag{19}$$

Finally, gains from trade given MP are:

$$GT' = \frac{GO}{GMP} = \left[\frac{\Delta_0 + (I-1)\Delta_1}{1 + (I-1)\widetilde{m}^{-\theta}}\right]^{\frac{\eta}{\theta}}. \tag{20}$$

The expression for GT in equation (17) indicates that a country that opens up to only trade benefits from $I-1$ foreign technologies, but at a "discount" of $d^{-\theta}$. Not surprisingly, GT decreases with d. Similarly, the expression for GMP in equation (18) indicates that a country that opens up to only MP benefits from $I-1$ foreign technologies, but at a discount of $\widetilde{m}^{-\theta}$ (i.e. the cost adjustment of the multinational input bundle when trade is not feasible) in the intermediate goods' sector, and at a discount $h^{-\theta}$ in the final goods' sector. Indeed, GMP decreases with h.

Gains from openness in equation (16) indicate that a country that opens up to both trade and MP in the intermediate goods' sector benefits from using its own technologies, at home and abroad, captured by the term Δ_0, and $I-1$ foreign technologies, captured by the term Δ_1. When domestic technologies are used (the term Δ_0), production can be carried out in $I-1$ foreign locations through MP at the cost m, and then goods shipped back home at the cost d. Hence, technologies are "fully" discounted at $(md)^{-\theta/(1-\rho)}$. In turn, foreign technologies can be accessed by importing goods in which case they are discounted by $d^{-\theta/(1-\rho)}$ (first term in Δ_1), by doing MP in which case they are discounted by $m^{-\theta/(1-\rho)}$ (second term in Δ_1), and by doing BMP in $I-2$ different locations in which case the full discount $(md)^{-\theta/(1-\rho)}$ applies (third term in Δ_1). The term in the first bracket in equation (16) captures the gains from accessing $(I-1)$ foreign technologies through MP in the final goods' sector, at a discount of $h^{-\theta}$.

It is clear that GO decreases with h as well as d: the higher the MP or trade costs, the lower the gains from openness. Additionally, the parameter ρ only appears in GO, in association with intermediates' goods, but not in GT and GMP. Since ρ indicates the correlation between cost draws for a given source country across different production locations, it is only relevant when BMP is feasible, that is, when both trade and MP are allowed. We can see that GO increases with ρ: the lower the correlation between technologies used to produce in different countries by firms of a given origin, the larger the gains from doing BMP (as shown in Lemma 2 in the Appendix).

Finally, gains from trade given MP in equation (20) indicate that, with respect to having only MP, a country that opens up to trade derives extra benefits coming from the possibility of doing BMP, and using the home input bundle for MP.

We talk about complementarity and substitutability between trade and MP in two equivalent ways. Trade and MP are complements when $GO > GT \times GMP$, or $GT' > GT$. Trade and MP are substitutes when $GO < GT \times GMP$ or $GT' < GT$. Finally, trade and MP are independent when $GO = GT \times GMP$ or $GT' = GT$.[11]

The following proposition shows under which parameters' configuration, trade and MP behave as substitutes or complements.

Proposition 5 *(a) For $\rho = 0$ and $a > 0$, trade and MP are complements; (b) for $\rho = 0$ and $a = 0$, trade and MP are independent; (c) for any $0 < \rho < 1$ and $a = 0$, trade and MP are substitutes; and (d) for $a > 0$ and $\xi - 1 > \theta$, there exists $\widehat{\rho}$ such that for $\rho < (>)\widehat{\rho}$, trade and MP are complements (substitutes).*

Moreover, for $a > 0$: (a) $\xi \to 1$ implies $GO > GT \cdot GMP$; and, (b) $\xi \to \infty$ implies $GO < GT \cdot GMP$.

Note that for $a = 0$, trade and MP behave always as substitutes. Thus, BMP is not enough in our model to generate complementarity between trade and MP; at most, when $\rho = 0$, BMP generates independence ($GO = GT \times GMP$, or $GT = GT'$).

Also, the (sufficient) condition in part (d) is $\xi - 1 > \theta$. While $\xi - 1$ governs the effect of trade costs on trade flows in Armington or Krugman models, θ has an analogous role in Ricardian models. Thus, $\xi > 1 + \theta$ means that the effect of trade costs on "intra-firm" trade flows is larger than their effect on "arms-length" trade flows.

3 Model's Calibration

As explained below, we calibrate the model's parameters using data on bilateral trade in manufacturing goods and bilateral gross value of production of foreign affiliates, normalized by

[11] Since $GMP > 0$ then $sign\,(GO - GT \times GMP) = sign\left(\frac{GO - GT \times GMP}{GMP}\right)$, but from (20) we can write $GT' = GO/GMP$, hence $sign\left(\frac{GO - GT \times GMP}{GMP}\right) = sign\,(GT' - GT)$.

expenditure in manufacturing and all sectors, respectively, as well as intra-firm imports of foreign affiliates, a size measure of equipped-labor, and real income per capita. We use standard bilateral variables such as distance, common border, and common language, to calibrate trade and MP costs. We consider nineteen OECD countries.

We use the calibrated version of the model to calculate gains from openness, and in this way, illustrate the rich implications of the model for the interactions among trade and MP. We are particularly interested in understanding the role of intra-firm trade and BMP in generating complementarity, and in affecting the contribution of trade and MP to welfare gains. We show how gains from trade and MP change with some key parameters of the model.

3.1 Data Description

We restrict our analysis to a set of nineteen OECD countries: Australia, Austria, Belgium/Luxemburg, Canada, Denmark, Spain, Finland, France, United Kingdom, Germany, Greece, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, United States.[12] For bilateral variables, we have 342 observations, each corresponding to a country-pair. Depending on availability, our observations are an average over the period 1990-2002, or for the late nineties.

We use data on manufacturing trade flows from country i to country n as the empirical counterpart for trade in intermediates, X_{ni} in the model. These data are from the STAN data set for OECD countries, an average over 1990-2002.

The empirical counterpart for bilateral MP flows is the gross value of production for multinational affiliates from i in n, $Y_{ni} \equiv Y_{fni} + Y_{gni}$ in the model.[13] The available data for this variable include all sectors combined as averages over 1990-2002. The main source of these data is UNCTAD.[14] The number of observations drops to 219 country-pairs for which we have available data for this flow.

We normalize bilateral trade flows by the importer's total expenditure on intermediate goods, and our measure of bilateral MP flows by total expenditure in final goods in the host

[12]These countries are also the ones considered by Eaton and Kortum (2002).

[13]This measure includes both local sales in n and exports to any other country, including the home country i.

[14]See Ramondo (2006) for a detailed description.

country. In the model, trade shares and MP shares are given,respectively, by:

$$\widetilde{X}_{ni} = \frac{X_{ni}}{\eta w_n L_n} \tag{21}$$

$$\widetilde{Y}_{ni} = \frac{Y_{fni} + Y_{gni}}{w_n L_n}, \tag{22}$$

where $\eta w_n L_n$ is total expenditure in intermediate goods, and $w_n L_n$ is total expenditure in final goods (and also total income due to the trade balance condition.)

In the data, we compute total expenditure in intermediate goods as gross production in manufacturing in country n, plus total imports of manufacturing goods into country n from the remaining eighteen OECD countries in the sample, minus total manufacturing exports from country n to the rest of the world. Data on these three variables for each country are from the STAN database. We use an average of these series over the period 1990-2002. Analogously, we compute total expenditure in final goods for country n as gross domestic product for country n, plus total imports into country n from the remaining eighteen OECD countries in the sample, minus total exports from country n to the rest of the world. Data on GDP is from the World Development Indicators, and total exports and imports are from Feenstra and Lipsey (2005), averaged over the period 1990-2002.

We use intra-firm imports from the home country done by foreign affiliates of multinational firms as the empirical counterpart for imports of the national input bundle for intermediates from the home country for MP (i.e., $\omega_{ni} Y_{gni}$ in the model). We only have data on intra-firm trade involving the United States, from the Bureau of Economic Analysis (BEA). We combine data on intra-firm exports from the United States to affiliates of American multinationals in foreign countries with data on imports done by affiliates of foreign multinationals located in the United States from their parent firms, as an average over the period 1999-2003. We normalize intra-firm trade flows from country i to n by gross production of affiliates from i in n,

$$\widetilde{\omega}_{ni} = \frac{\omega_{ni} Y_{gni}}{Y_{gni} + Y_{fni}}, \tag{23}$$

for $n = US$ or $i = US$.

For the empirical counterpart of the bilateral share of MP in intermediate goods,

$$\frac{Y_{gni}}{Y_{gni} + Y_{fni}}, \tag{24}$$

we use data on bilateral gross production of American affiliates abroad and foreign affiliates in the US in the manufacturing sector as share of total gross production in the foreign market and the US, respectively, from BEA (average over the period 1999-2003).

When the US is the source country, we are able to compute the empirical counterpart of the share BMP in the model (i.e. the share of the value of production done by i in l that is sold in a different market n). The BEA divides total sales of American affiliates produced in country l into sales to the local market, to the US, and to third foreign markets. This is the empirical counterpart for $\sum_{n \neq l} \phi_{gni} \pi_{gni,l} X_{gn} / (Y_{gli} + Y_{fli})$, from $i = US$ in a country l belonging to the OECD(19). We average out across l's, and obtain an average bilateral BMP share for the US affiliates in the OECD(19). We consider an average over the period 1999-2003.

We need an empirical counterpart for the variable L_i. We think of this variable as capturing the total number of "equipped" units of labor available for production, so employment must be adjusted to account for human and physical capital available per worker. We use the measure of equipped labor constructed by Klenow and Rodriguez-Clare (2005), for OECD(19) countries, as an average over the nineties. Countries with more equipped labor are considered larger.

We use data on real income per capita (PPP-adjusted) to calibrate the technology parameters T_{li} as explained below. Data are from the Penn World Tables (6.2), as an average over the nineties. In the model, this variable is the ratio of $w_n L_n / P_{fn}$ to population in country n. Both L_n and population are from the data, while the wage w_n and the price index for final goods P_{fn} are a result of computing the model's equilibrium.

Finally, we use bilateral distance, common border, and common language, to calibrate trade and MP cost as explained below. These variables are from the Centre dEtudes Prospectives et Informations internationales (CEPII). Bilateral distance is the distance in kilometers between the largest cities in the two countries. Common language is a dummy equal to one if both countries have the same official language or more than 20% of the population share the same language (even if it is not the official one). Common border is equal to one if two countries share a border.[15]

3.2 Estimation Procedure

We reduce the number of parameters to calibrate by assuming the following functional forms. First, following Fieler (2008), we assume that trade and MP costs in the intermediate goods'

[15]See the Appendix for summary statistics.

sector are approximated by the following functions:

$$
\begin{aligned}
d_{ni} &= 1 + (\delta_0^d + \delta_{dist}^d dist_{ni}) \times (\delta_{border}^d)^{b_{ni}} \times (\delta_{language}^d)^{l_{ni}} &\quad (25)\\
h_{ni} &= 1 + (\delta_0^h + \delta_{dist}^h dist_{ni}) \times (\delta_{border}^h)^{b_{ni}} \times (\delta_{language}^h)^{l_{ni}}, &\quad (26)
\end{aligned}
$$

for all $n \neq i$, with $d_{nn} = 1$ and $h_{nn} = 1$. The variable $dist_{ni}$ is the distance (in thousands of kilometers) between i and n. The term in parenthesis represents the effect of distance on trade and MP costs. The variables b'_{ni}s (l'_{ni}s) equals one if countries share a border (a language), and zero otherwise. Hence, if $\delta_{border} < 1$ sharing a border reduces iceberg costs. Similarly, if $\delta_{language} < 1$, sharing a language reduces iceberg costs. We need to estimate a set of eight parameters, $\Upsilon = \{\delta_0^d, \delta_0^h, \delta_{dist}^d, \delta_{dist}^h, \delta_{border}^d, \delta_{border}^h, \delta_{language}^d, \delta_{language}^h\}$.

The resulting set of parameters to calibrate is

$$
\left\{ \{T_{li}\}_{l,i=1}^{I}, \Upsilon, a, \xi, \rho, \theta, \alpha, \beta \right\}.
$$

We set the labor share in the intermediate goods' sector, β, to 0.5, and the labor share in the final sector, α, to 0.75, as calibrated by Alvarez and Lucas (2007). Then, we have $\eta \equiv (1-\alpha)/\beta = 0.5$.

Turning to θ, it is well known that this parameter cannot be separately identified from the parameters in Υ by running a gravity equation (see Eaton and Kortum, 2002, and Fieler, 2008). Thus, we appeal to the model's implications for the growth rate of real wage. The model in the previous section is static. However, as we show in the Appendix, the equilibrium of the static model can be seen as the steady state equilibrium of a dynamic model where the vectors pf productivity parameters Z_g and Z_f evolve according to an exogenous "research" process. This dynamic model exhibits quasi-endogenous growth as in Jones (1995) and Kortum (1997), and is closely related to Eaton and Kortum (2001). Importantly, growth is driven by the same mechanism that generates the gains from openness in the static model, namely the aggregate economies of scale associated with the fact that a larger population is linked to a higher stock of non-rival ideas. Hence, calibrating θ to generate a growth rate in the dynamic model that matches the one we observe in the data seems reasonable. From Eaton and Kortum (2001), growth rates in the steady state are the same for all countries, and not affected by openness. This implies that the growth rate for the open economy is the same as the one for the closed economy. From equation (3), the growth rate of the real wage in the closed economy is $(1+\eta)/\theta$ times the growth rate of the parameter T. In the growth model for the open economy (in the

Appendix), there is no a single parameter T but rather the whole matrix of T_{li}'s. As shown in the Appendix, our assumptions imply that all these T_{li}'s grow at some common rate g_λ. Hence the steady state growth rate of real wages for all countries is

$$g = \left[\frac{1+\eta}{\theta}\right] g_\lambda. \tag{27}$$

We want the model to be consistent with a growth rate of real output per worker of 1.5%, as calculated by Klenow and Rodríguez-Clare (2005) for the OECD over the last four decades.[16] We set g_λ equal to the growth rate of employment in R&D over the last decades in the five top R&D countries, which according to Jones (2002) has been 4.8%. Using (27) and $\eta = 0.5$, the observed values for g and g_λ imply $\theta = 7.2$.

We assume that the technology parameters T_{li} are given by $T_{li} = \lambda_i \lambda_l^{1-\rho}$, as derived in the Appendix, where the parameters λ's represent the stock of ideas in a country. Thus, we just need to calibrate a vector of nineteen parameters, $\{\lambda_1, ..., \lambda_{19}\}$.

We end up with the following set of parameters to calibrate,

$$\Gamma \equiv \left\{ \{\lambda_i\}_{i=1}^{I}, \Upsilon, a, \xi, \rho \right\}.$$

Our calibration procedure is as follows. Given θ, β, α, a set of parameter values Γ, the matrices for bilateral distance, common border and common language, and the vector of country sizes L_n from the data on R&D employment, we compute the model's equilibrium, and generate a simulated data set with 361 observations (one for each country-pair, including the domestic pairs), for the following variables: trade shares, MP shares, "intra-firm" trade shares ($\widetilde{\omega}_{ni}$ in 23), and real income per capita. Additionally, the model generates MP by i in n in the intermediate goods sector (as share of total MP by i in n), BMP by i in l ((as share of total MP by i in n), trade costs d_{ni}, and MP costs h_{ni}. The algorithm used to compute the model's equilibrium extends the one in Alvarez and Lucas (2007).

The calibration procedure searches for: (i) Υ, a, ξ, and ρ such that the (weighted) sum of the square difference of trade shares, MP shares, and intra-firm trade shares, between the model and the data, respectively, is minimized,

$$\sum_{n,i;n\neq i} \frac{1}{N_X^{obs}} \left(\widetilde{X}_{ni}^{data} - \widetilde{X}_{ni}^{model}\right)^2 + \frac{1}{N_Y^{obs}} \left(\widetilde{Y}_{ni}^{data} - \widetilde{Y}_{ni}^{model}\right)^2 + \sum_{n,i;n\neq i;n,i=USA} \frac{1}{N_\omega^{obs}} \left(\widetilde{\omega}_{ni}^{data} - \widetilde{\omega}_{ni}^{model}\right)^2;$$

[16] To obtain growth rate for TFP g, we need to adjust $g_y = 1.5\%$ by physical capital. Assuming that the share of physical capital in production is $\alpha_k = 1/3$, we get $g = (1-\alpha_k)g_y = 2/3 \times 1.5\% = 1\%$.

and (ii) the technology parameters $\{\lambda_i\}_{i=1}^{I}$ such that the absolute difference of real income per capita (relative to the US) between the model and the data is minimized.

A measure of the explanatory power of the model for trade shares R^2_X, MP shares R^2_Y, and intra-firm trade shares R^2_ω, respectively, is given by:

$$R^2_H \quad = \quad 1 - \frac{\sum_{n,i;n\neq i}\left[\widetilde{H}^{data}_{ni} - \widetilde{H}^{model}_{ni}\right]^2}{\sum_{n,i;n\neq i}(\widetilde{H}^{data}_{ni})^2} \tag{28}$$

where $H = X, Y, \omega$.

Indeed, the chosen moments are informative about the model's parameters. Intuitively, the sources of identification are the following. By decreasing the iceberg trade and MP cost parameters in Υ, trade and MP shares decrease, $\widetilde{X}_{ni}$ and $\widetilde{Y}_{ni}$, respectively. Increasing the parameter a in the cost of the multinational input bundle in intermediates for MP, increases the share of "intra-firm" trade in total MP ($\widetilde{\omega}_{ni}$). The parameter ρ helps matching bilateral trade and MP shares, but mostly through changes in the amount of BMP; the average share of BMP by i in l increases with ρ. Both this moment and the amount of MP in the intermediate relative to the final goods' sector are left out of the calibration procedure but we report them below. Finally, increasing the technology parameter λ_i increases a country's real income per capita.

3.3 Results

Table 1 reports the calibrated parameters while Table 3 reports the targeted moments, both from the data and calibrated model. Table 2 show the implied statistics for trade costs d_{ni}, MP costs, h_{ni}, and the implied correlation between the two. We report the calibrated values for the country-level technology parameters λ_i in the Appendix.

Table 1 shows that the effect of distance on trade costs more than doubles the one on MP costs: the coefficients on distance are $\delta^d_{dist} = 0.13$ for trade, and $\delta^h_{dist} = 0.05$, for MP. A country-pair that is 20% further apart has 21% higher trade cost, and only 2.8% higher MP costs. The effect of border and language is rather similar on both trade and MP costs. Overall, estimates for these "gravity" parameters translate into 25% higher average trade costs (2.1 against 1.7) across country-pairs, as shown in Table 2. This result is driven by the fact that there is more MP than trade in the data. The parameter a together with the elasticity of substitution ξ

Parameter	Value	Definition
δ^d_{dist}	0.13	trade and MP costs parameters
δ^h_{dist}	0.05	in equation (26) and (25)
δ^d_{border}	0.75	
δ^h_{border}	0.79	
$\delta^d_{language}$	0.70	
$\delta^h_{language}$	0.68	
δ^d_0	0.47	
δ^h_0	0.45	
ρ	0.16	correlation between costs draws in 5
a	0.20	weight of Home inputs in 4
ξ	1.9	elasticity of substitution in (4)
α	0.75	labor share in final goods
β	0.5	labor share in intermediate goods
θ	7.2	parameter in (5)

Table 1: Parameters' Estimates.

	(I)	(II)	(III)	(IV)
Avg. trade costs d_{ni}	2.1			
	(0.4)			
Avg. MP costs h_{ni}	1.7			
	(0.2)			
Correlation between d_{ni} and h_{ni}	0.98			

Table 2: Calibrated Trade and MP Costs

in Table 1 regulates the magnitude of intra-firm trade.[17] The parameter ρ, that indicates the degree of correlation between technology draws across different locations of production, regulates the amount of BMP, one of our "out of sample " moments shown in Table 3. Table 3 shows statistics generated by the model's equilibrium at the calibrated parameters. For trade and MP shares, we report mean, standard error and correlation coefficient. The average share of MP in manufacturing as well as the average BMP share (for US affiliates abroad) are not included in the calibration procedure.

[17] Indeed, if we calibrate the model assuming that the bilateral intra-firm trade share is 14% rather than 7.4% as observed in the data, we need to increase a to 0.48 and decrease ξ to 1.3.

Moments	Data	Model
Avg. trade share in manufacturing from i to n	0.019	0.019
	(0.03)	(0.039)
Avg. MP share by i in n	0.022	0.022
	(0.04)	(0.028)
Avg. "intra-firm" trade shares from i to n (for US)	0.074	0.074
(as share of MP by i in n)	(0.07)	(0.02)
Correlation btw. trade and MP shares from i to n	0.73	0.85
Avg. MP in manufacturing by i in n (for US)	0.48	0.49
(as share of total MP by i in n)	(0.12)	(0.03)
Avg. "BMP" by i in n (for US)	0.3	0.09
(as share of MP by i in n)	(0.17)	(0.07)

SE in parenthesis.

Table 3: Moments: Data and Model.

As expected, the model matches the average bilateral trade and MP shares, as well as the average bilateral intra-firm trade share for the US, as observed in the data. However, it delivers a higher correlation between bilateral trade and MP shares than the one observed in the data. Indeed, the model matches fairly well the proportion of MP in manufacturing as observed in the data. One failure of the model is that it generates a low BMP share. While the data for US affiliates in OECD countries shows that 30% of the value of production is sold in countries other than the country of production, the model only generates 9%.[18]

Now, we turn to gravity and the parameter θ. As shown in the previous section, in a model with trade, MP, and "intra-firm"trade in which the costs of doing trade and MP are positively correlated, the elasticity of bilateral trade shares with respect to trade costs is not given in general by the parameter θ –as is standard in Ricardian models of trade. That is, the coefficient b_x in the following equation,

$$\log \widetilde{X}_{ni} = D_n^x + S_i^x + b_x \log d_{ni} + u_{ni}^x, \tag{29}$$

where D_n^x and S_i^x are destination and source country fixed effects, respectively, is not equal to θ, This means that an estimate of θ coming from an OLS estimate of b_x would be bias. We evaluate

[18]Lower ρ implies less correlation across technologies in different production locations, thus, more BMP. Letting $\rho \to 0$ increases the average BMP share to XX% from 9% in Table 3, but it never gets close to the magnitude observed in the data.

this bias by running equation 29 using the calibrated trade costs, d_{ni}, and simulated data for trade shares $\widetilde{X}_{ni}$. We get $\widehat{b}_x = -6.41$.[19] As expected, this coefficient is biased downward (6.41 ¡ θ=7.2). The positive correlation between trade and MP costs, d_{ni} and h_{ni}, creates this bias: the error term u_{ni} is positively correlated with d_{ni}.

We can perform a similar exercise for "intra-firm"trade shares. The coefficient b_ω in the following equation,

$$\log \widetilde{\omega}_{ni} = D_n^\omega + S_i^\omega + b_\omega \log d_{ni} + u_{ni}^\omega, \tag{30}$$

where D_n^ω and S_i^ω are destination and source country fixed effects, respectively, is not equal to $1 - \xi$, as a Krugman-type model of trade would suggest. Running OLS on 30 using simulated data for $\widetilde{\omega}_{ni}$ and the calibrated trade costs for d_{ni} gives $b_\omega = -0.32$. This estimate would implied an elasticity of substitution ξ of 1.3, much lower than the one we calibrate, 1.9. Again, we find a downward bias in the OLS estimate.[20]

The next two tables and figures show how well the calibrated model captures the pattern of trade and MP observed in the data. Table 4 shows the measure of the model's explanatory power in (28), for bilateral trade, MP, and intra-firm trade. Additionally, we present correlations between magnitudes in the model and data for bilateral trade and MP shares across country-pairs, aggregate exports, imports, outward MP and inward MP as share of GDP of the receiving country, for the five model's calibrations. Not surprisingly, the model performs fairly well in terms of matching the observed trade and MP shares across country-pairs, 0.82 and 0.49, respectively. The explanatory power of the model decreases to 0.38 when bilateral intra-firm trade shares for/to the US are considered.

When we aggregate exports and imports as shares of exporter's and importer's GDP, respectively, by country, correlations between model and data are still high, 0.82 and 0.84, respectively. Correlations are slightly lower, 0.45, for aggregate inward MP shares, by country, measured as total sales of foreign affiliates in the country, as share of host's country GDP. However, the model does much more poorly in terms of aggregate outward MP shares, by country, measured as total sales of affiliates abroad (as shares of source country's GDP), dropping to 0.17.

[19] Running OLS on 29 using the observed data for trade shares gives $b_x = -3.8$. Interestingly, for the same set of OECD countries but trade costs estimated in a different way, Eaton and Kortum (2002) get an estimate of b_x between 3.7 and 6.7.

[20] Hanson, Mataloni, and Slaughter (2005) run a similar regression to the one in 30 use data on intra-firm imports in goods for further processing done by foreign affiliates of American multinational firms. Their estimates range from -0.64 to -0.26.

Model's "Explanatory power":	
bilateral trade shares	0.69
bilateral MP shares	0.40
bilateral "intra-firm" trade shares (for US)	0.38
Correlations model data:	
Bilateral Trade Shares	0.82
Bilateral MP Shares	0.49
Total Exports Shares	0.82
Total Imports Shares	0.84
Total Outward MP Shares	0.17
Total Inward MP Shares	0.45

Bilateral MP = gross value of production for affiliates from country i in n; Total Outward MP = total gross value of production for foreign affiliates from country i; Total Inward MP = total gross value of production for foreign affiliates in country n.

Table 4: Goodness of Fit: Calibrated Model.

We further explore the relationship between aggregate exports, imports, inward, and outward MP shares, and country's size in Figures **??** and **??**.

The right panel of Figure **??** shows outward MP as share of recipient country's GDP, for the model and the data, against the model's GDP, $w_i L_i$. The model underestimates large countries as the United States, Japan, and Germany, and overestimates small countries that have very low (e.g. Spain and New Zealand) outward MP relative to size. The left panel shows the analogous scatter for inward MP. The model captures reasonably well large countries, as United States and Japan, and slightly overestimates inward MP into small countries, as share of their size.[21] Figure **??** is analogous to Figure **??** for export and import shares. The model captures fairly accurately the pattern of aggregate export and import shares with respect to country size; it slightly underestimates large countries such as the United States and Japan, and a very small country such as Netherlands with very high export and import shares.[22]

[21]The correlation between outward MP shares and country size in the data is -0.07, while in the model is -0.59; for inward MP, this correlation is -0.32 in the data, and -0.64 in the model.

[22]The correlation between export shares and country size in the data is -0.45, while in the model is -0.63; for import shares, this correlation is -0.54 in the data, and -0.63 in the model.

4 Gains from Openness, Trade, and Multinational Production

We measure gains from openness, trade, and MP by changes in real wages in terms of the final consumption good, w_n/P_{fn}. We calculate real wages under: no trade and no MP (isolation), only trade, and only MP. The increase in the real wage as we move from isolation to the calibrated version of the model with trade and MP yields the gains from openness, GO. Similarly, the increase in the real wage as we move from isolation to only trade yields the gains from trade, GT, while moving from isolation to only MP yields the gains from MP, GMP. Finally, the increase in the real wage from only MP to trade and MP yields gains from trade in the presence of MP, GT', and analogously, the increase in the real wage from only trade to trade and MP yields gains from MP in the presence of trade, GMP'. We also calculate the gains from trade coming from a calibrated version of the model with only trade, GT^*, and the gains from MP coming from a calibrated model with only MP, GMP^*. [23] Finally, we show gains separately for the intermediate goods' sector, where both trade and MP are possible, and for the final goods' sector where only MP is possible.[24]

Table 5 shows average the calculated gains across OECD countries, and Table 6 presents them by country.

	$(w_n/P_{fn})\,/(w_n^{\prime}/P_{fn}^{\prime})$						
Avg. OECD (19)	GO	GT	GT'	GT^*	GMP	$GMP^{\prime}$	GMP^*
All	1.15	1.03	1.05	1.03	1.09	1.11	1.11
Intermediate Tradable Sector	1.06	1.03	1.05	1.03	1.01	1.03	1.04
Final Non-Tradable Sector	1.08	-	-	-	1.08	1.05	1.07

Table 5: Gains from Openness, Trade, and MP. Average for nineteen OECD countries.

The implied average gains from openness (GO) are 15%. This is large specially compared to gains coming from models with only trade –3%. MP seems to be an important source for

[23]Details of these calibrations are presented in the Appendix.

[24]GMP in the intermediate goods sector are calculated by comparing real wages under isolation and a situation where MP is only allowed in the tradable sector (analogous calculation for GMP in the final goods' sector). GMP^{prime} in the intermediate goods' sector are calculated by comparing real wages under only trade and under trade and MP only in intermediates (analogous for the final goods' sector).

gains from openness. While the gains from trade (GT) implied by the model are 3%, the gains from MP (GMP) are 9%. The reason is that as MP flows are higher than trade flows in the data (for example, total inward MP flows are more than double total imports in the data), the calibrated MP costs are lower than trade costs. Additionally, MP is feasible in the non-tradable final goods' sector. In fact, if we shut down MP in the non-tradable sector, GMP lowers to just 1%. The extra gains from adding MP in the final non-tradable sector are 8%. Further, GO drops to 6% if MP were shut down in the non-tradable sector.

The calculated gains suggest that $GO > GT \times GMP$, indicating that trade and MP are complements. Or alternatively, $GT^{'} > GT$ and $GMP^{'} > GMP$. Notice that this complementarity comes, by construction, from the intermediate tradable goods' sector, where $GO^g > GT^g \times GMP^g$.

It is interesting to compare $GT^{'}$ to the gains from trade generated by a model with only trade, GT^*, calibrated to match bilateral trade shares and income per capita. Gains from trade calculated this way are 3%, almost half GT'. With complementarity between trade and MP, a model with only trade needs a lower trade cost to match the data than a model with both trade and MP. Thus, the gains from trade in such a model are higher than GT, but still lower than GT'. The analogous exercise for MP suggests that gains arising from a model with only MP, GMP^*, calibrated to match bilateral MP shares and income per capita, gives very similar results to the model with trade and MP; GMP^* is almost identical to $GMP^{'}$. However, even if most gains from MP are realized in the final non-tradable goods' sector as in the model with trade and MP, a model with only MP would overestimate gains from MP in both sectors (4% against 3% for tradable goods, and 7% against 5% for non-tradable goods.)

In Table 6, we show GO, GT, GMP, GT', GMP', GT^*, and GMP^*, by country. Countries are ordered by their size in terms of our measure of equipped labor. Indeed, gains from openness decrease with size. Trade and MP in all countries behave as complements ($GO > GT \times GMP$, or, $GT^{'} > GT$, or $GMP^{'} > GMP$), even if the gap is different across countries. For instance, a country like Belgium, which represents around 1% of OECD(19)'s equipped labor, has $GO = 1.30$, while $GT \times GMP = 1.26$; the United States, the largest country in the sample, has $GO = 1.05$ and $GT \times GMP = 1.04$.

	GO	GT	GMP	GT'	GMP'	GT^*	GMP^*	L_i/L_{us}
			$(w_n/P_{fn})/(w_n'/P_{fn}')$					(%)
New Zealand	1.08	1.01	1.05	1.02	1.07	1.01	1.05	1.1
Finland	1.21	1.05	1.13	1.07	1.16	1.05	1.15	1.7
Norway	1.19	1.05	1.11	1.07	1.14	1.05	1.16	1.8
Denmark	1.18	1.05	1.09	1.08	1.12	1.05	1.11	1.8
Portugal	1.21	1.04	1.14	1.07	1.17	1.04	1.12	1.9
Greece	1.20	1.03	1.14	1.06	1.17	1.03	1.16	2.3
Austria	1.21	1.06	1.11	1.09	1.14	1.06	1.12	2.3
Belgium	1.30	1.09	1.16	1.13	1.19	1.09	1.16	2.8
Sweden	1.18	1.04	1.11	1.06	1.14	1.04	1.17	3.1
Netherlands	1.17	1.04	1.10	1.07	1.12	1.04	1.12	4.5
Australia	1.02	1.00	1.02	1.01	1.02	1.00	1.02	6.1
Spain	1.12	1.02	1.08	1.04	1.10	1.02	1.09	8.4
Canada	1.17	1.04	1.11	1.06	1.13	1.04	1.13	11
Italy	1.08	1.01	1.05	1.03	1.07	1.02	1.06	13
France	1.14	1.02	1.09	1.04	1.11	1.03	1.12	16
Great Britain	1.12	1.02	1.09	1.04	1.10	1.02	1.11	17
Germany	1.18	1.02	1.13	1.05	1.16	1.02	1.21	27
Japan	1.02	1.00	1.02	1.00	1.02	1.00	1.04	52
United States	1.05	1.00	1.04	1.01	1.04	1.00	1.05	100

Countries are sorted by equipped labor.

Table 6: Gains from Openness, Trade, and MP, by country.

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A Summary Statistics

	Mean	Standard Deviation	Observations
Distance (in km)	6,006	6,099	342
Common Language	0.11	0.31	342
Common Border	0.09	0.28	342
bilateral trade share	0.019	0.035	342
bilateral MP share	0.021	0.043	219
bilateral intra-firm share†	0.074	0.072	34
bilateral MP share in manufacturing†	0.48	.13	33

†: from/to the United States.

Table 7: Summary Statistics.

B Additional results: Alternative Calibrations

C Proofs

Proof of Proposition 1. Consider first the case of intermediate goods and let $p_{gni} \equiv \min_l c_{gli} d_{nl} / z_{gli} p_{gnli}$. The probability that p_{gni} is lower than p is

$$\begin{aligned} G_{gni} \quad (p) = &\quad 1 - \Pr\left(z_{gli} \leq c_{gli} d_{nl}/p \text{ for all } l\right) \\ = &\quad 1 - \exp\left[-\left(\sum_l \left(T_{li}\left(c_{gli} d_{nl}/p\right)^{-\theta}\right)^{\frac{1}{1-\rho}}\right)^{1-\rho}\right] \\ = &\quad 1 - \exp\left[-\Phi_{gni} p^{\theta}\right] \end{aligned}$$

Since $G_{gni}(p)$ is independent across i, then the reasoning in Eaton and Kortum (2002) can be immediately applied to show that country n will buy goods produced with country $i's$ technologies for a measure of goods equal to $\phi_{gni} = \Phi_{gni} / \sum_j \Phi_{gnj}$. The corresponding result for final goods in point (a) of Proposition 1 is derived simply by letting $d_{nl} \to \infty$ for $n \neq l$.

Moving on to part (b), consider the intermediate goods that country n buys that are produced with country i technologies. What is the share of these goods that are produced in country l? This is equal to the probability that, for a specific good, country l is the cheapest location for i to produce for n with its technology. This is equivalent to $\frac{c_{gli} d_{nl}}{z_{gli}} \leq \frac{c_{gji} d_{nj}}{z_{gji}}$ or

	Data			Model	
	Equipped Labor	Population	real GDP pc	real GDP pc	λ_i
		(relative to US)			
Australia	0.061	0.0688	0.7774	0.7455	0.678
Austria	0.0231	0.0304	0.7385	0.7212	0.5937
Belgium	0.0281	0.0385	0.7341	0.7026	0.5134
Canada	0.1084	0.1114	0.8079	0.7993	0.4452
Denmark	0.0183	0.0199	0.8207	0.8409	0.5854
Spain	0.0837	0.1493	0.5457	0.5003	0.6173
Finland	0.0167	0.0194	0.6901	0.7009	0.4008
France	0.1597	0.2254	0.708	0.7113	0.8174
Great Britain	0.1647	0.223	0.6828	0.6902	0.6965
Germany	0.269	0.3092	0.7293	0.7954	0.5795
Greece	0.023	0.0396	0.4441	0.4018	0.2654
Italy	0.1342	0.2177	0.7039	0.6496	1.0536
Japan	0.5218	0.4768	0.8127	0.9048	0.6446
Netherlands	0.0449	0.0587	0.7389	0.73	0.6676
Norway	0.0176	0.0166	0.827	0.898	0.4598
New Zealand	0.0114	0.0139	0.591	0.5198	0.278
Portugal	0.0192	0.0377	0.4733	0.3743	0.3064
Sweden	0.0309	0.0333	0.7285	0.7829	0.4699
United States	1	1	1	1	1

Table 8: Country's Statistics: Data and Model.

$z_{gji} \leq \left[\frac{c_{gji}d_{nj}}{c_{gli}d_{nl}}\right] z_{gli}$ for all $j \neq l$. Without loss of generality, assume that $l = 1$. The probability that $z_{ji} \leq a_{ni,j} z_{1i}$ for all $j \neq 1$ where $a_{ni,j} \equiv \frac{c_{gji}d_{nj}}{c_{g1i}d_{n1}}$ is given by $\int_0^\infty F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz$. But

$$F_1(z, a_{ni,2}z, ..., a_{ni,I}z) = \left((c_{g1i}d_{n1})^{\frac{1}{\theta}}\,\Phi_{ni}\right)^{1-\frac{1}{1-\rho}} T_{1i}^{\frac{1}{1-\rho}} \theta z^{-\theta-1} \exp\left[-\left(c_{g1i}d_{n1}\right)^{\frac{1}{\theta}}\Phi_{ni} z^{-\theta}\right]$$

and

$$\int_0^\infty \theta c_{g1i}d_{n1}\Phi_{ni} z^{-\theta-1} \exp\left[-c_{g1i}d_{n1}\Phi_{ni}z^{-\theta}\right] dz = 1$$

This implies that

$$\int_0^\infty F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz = \frac{\left(T_{1i}\left(c_{g1i}d_{n1}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}}{\sum_j \left(T_{ji}\left(c_{ji}d_{nj}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}}$$

and the result follows immediately.

as % of GDP	Data				Model			
	Exports	Imports	MP		Exports	Imports	MP	
			inward	outward			inward	outward
Australia	0.05	0.10	0.28	0.10	0.02	0.02	0.08	0.18
Austria	0.18	0.24	0.29	0.13	0.32	0.32	0.45	1.75
Belgium	0.45	0.48	0.46	0.22	0.38	0.38	0.61	1.62
Canada	0.23	0.22	0.46	0.26	0.22	0.22	0.48	0.74
Denmark	0.23	0.19	0.12	0.17	0.27	0.27	0.38	1.63
Spain	0.11	0.15	0.25	0.03	0.13	0.13	0.37	0.41
Finland	0.22	0.16	0.23	0.48	0.25	0.25	0.51	1.13
France	0.13	0.14	0.20	0.18	0.16	0.16	0.40	0.39
Great Britain	0.12	0.15	0.35	0.32	0.12	0.12	0.38	0.36
Germany	0.16	0.13	0.29	0.29	0.13	0.13	0.54	0.16
Greece	0.09	0.17	0.07	0.01	0.16	0.16	0.55	0.52
Italy	0.12	0.12	0.15	0.07	0.10	0.10	0.25	0.39
Japan	0.05	0.02	0.06	0.16	0.01	0.01	0.09	0.02
Netherlands	0.32	0.27	0.50	1.00	0.24	0.24	0.41	0.94
Norway	0.11	0.17	0.17	0.18	0.25	0.25	0.45	1.24
New Zealand	0.14	0.18	0.25	0.04	0.07	0.07	0.28	0.53
Portugal	0.18	0.26	0.58	0.04	0.22	0.22	0.53	0.82
Sweden	0.23	0.19	0.32	0.36	0.21	0.21	0.48	0.77
United States	0.04	0.05	0.18	0.16	0.03	0.03	0.17	0.06

Table 9: Trade and MP Statistics: Data and Model.

	trade and MP	only trade	only MP
	(I)	$h_{gni} \to \infty$	$d_{ni} \to \infty$
δ^d_{dist}	0.13	0.13	N/A
δ^h_{dist}	0.05	N/A	0.05
δ^d_{border}	0.75	0.75	N/A
δ^h_{border}	0.79	N/A	0.79
$\delta^d_{language}$	0.70	0.70	N/A
$\delta^h_{language}$	0.68	N/A	0.68
δ^d_0	0.47	0.46	N/A
δ^h_0	0.45	N/A	0.49
a	0.20	N/A	N/A
ρ	0.16	0.16	0.16
θ	7.2	7.2	7.2

Table 10: Three calibrations: trade and MP, only trade, and only MP.

	trade and MP (I)	only trade $h_{gni} \to \infty$	only MP $d_{ni} \to \infty$
"Explanatory power" for:			
bilateral trade shares	0.69	0.69	N/A
bilateral MP shares	0.40	N/A	0.24
bilateral intra-firm trade shares	0.38	N/A	N/A
Correlations model data:			
Bilateral Trade Shares	0.82	0.82	N/A
Bilateral MP Shares	0.49	N/A	0.35
Total Exports Shares	0.82	0.80	N/A
Total Imports Shares	0.84	0.81	N/A
Total Outward MP Shares	0.17	N/A	0.08
Total Inward MP Shares	0.45	N/A	0.29
Implied Costs:			
Avg. trade costs (d_{ni})	2.1	2.1	N/A
Avg. MP costs (h_{gni})	1.7	N/A	1.7

Bilateral MP = sales of affiliates from country i in n; Total Outward MP = total sales of foreign affiliates from country i; Total Inward MP = total sales of foreign affiliates in country n.

Table 11: Goodness of Fit: benchmark, only trade, and only MP.

To prove part (c), focus on intermediate goods and condition on market n and technologies from country i. The probability that $p_{gni} \leq p$ and that l is the least cost production location to reach n is the probability that $d_{nl}c_{gli}/z_{gli} \leq p$ and $d_{nj}c_{gji}/z_{gji} \geq d_{nl}c_{gli}/z_{gli}$, or $z_{gli} \geq d_{nl}c_{gli}/p$ and $z_{gji} \leq \frac{d_{nj}c_{gji}}{d_{nl}c_{gli}} z_{gli}$. Without loss of generality, assume that $l = 1$ and again let $a_{ni,j} \equiv \frac{d_{nj}c_{gji}}{d_{n1}c_{g1i}}$. We want to compute $\int_{d_{n1}c_{g1i}/p}^{\infty} F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz$. But simple math establishes that

$$\int_{d_{n1}c_{g1i}/p}^{\infty} F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz = \left((d_{n1}c_{g1i})^{\theta} \Phi_{gni} \right)^{-\frac{1}{1-\rho}} T_1^{\frac{1}{1-\rho}} \left(1 - \exp\left(-\Phi_{gni} p^{\theta}\right)\right)$$

The distribution of prices in market n conditional on the good on the good produced in 1 with technology i, we need to divide by $\pi_{ni,1}\phi_{gni}$. This yields a probability equal to

$$1-\exp\left(-\Phi_{gni}p^{\theta}\right)=G_{gni}(p)$$

Since this does not depend on 1, it implies that for market n and conditioning on country i technologies, the distribution of p that actually are produced in l is the same for $l=1,2,...,I$. But independence across i allows us to apply the results from Eaton and Kortum (2002) to establish that the distribution of prices for goods that n actually buys from i is $G_{gn}(p)=1-\exp\left(-\Phi_{gn}p^{\theta}\right)$ for all i. This implies that the average price of goods is the same irrespective of where they are produced and irrespective of the origin of the technology. The proof for final goods follows immediately from independence across the i.

Finally, part (d) follows from the fact that $G_{gn}(p)=1-\exp\left(-\Phi_{gn}p^{\theta}\right)$ and $G_{fn}(p)=1-\exp\left(-\Phi_{fn}p^{\theta}\right)$ and the results in Eaton and Kortum (2002).□

Proof of Proposition 2. First note that P_nQ_n is the total cost of the intermediate goods *used in production* in country n. We first calculate the total cost of the intermediate goods *produced* in country n. This is $w_nL_{gn}+P_{gn}Q_{gn}$, plus the intra-firm imports of foreign multinationals located in n, $\sum_{i\neq n}\omega_{ni}Y_{gni}$, minus the exports of the domestic input bundle for intermediates to country $n's$ subsidiaries abroad, $\sum_{i\neq n}\omega_{in}Y_{gin}$. Hence, the total cost of intermediate goods produced in country n is

$$w_nL_{mn}+P_{gn}Q_{gn}+\sum_{i\neq n}\omega_{ni}Y_{gni}-\sum_{i\neq n}\omega_{in}Y_{gin}.$$

In equilibrium, this must be equal to the value of intermediate goods produced in country n, hence

$$w_nL_{mn}+P_{gn}Q_{gn}+\sum_{i\neq n}\omega_{ni}Y_{gni}-\sum_{i\neq n}\omega_{in}Y_{gin}=\sum_{i}Y_{gni}. \tag{31}$$

But

$$\sum_{i}Y_{gni}=\sum_{i}\sum_{j}\phi_{gji}\pi_{gji,n}P_{gj}Q_j,$$

and from () we then have

$$\sum_{i}Y_{gni}=\sum_{i}\phi_{gni}\pi_{gni,n}P_{gn}Q_n+\sum_{j\neq n}\left(X_{jn}-\omega_{jn}Y_{gjn}\right)$$

Substituting into (31) and simplifying we get

$$w_nL_{mn}+P_{gn}Q_{gn}+\sum_{i\neq n}\omega_{ni}Y_{gni}=\sum_{i}\phi_{gni}\pi_{gni,n}P_{gn}Q_n+\sum_{i\neq n}X_{in}$$

Using the trade balance condition (9) to substitute $\sum_{i\neq n} X_{in}$ for $\sum_{i\neq n} X_{ni}$, using (), and then simplifying yields

$$w_n L_{mn} + p_{mn} Q_{gn} = \left(\sum_i \sum_j \phi_{gnj} \pi_{gnj,i} \right) P_{gn} Q_n$$

But $\sum_i \sum_j \phi_{gnj} \pi_{gnj,i} = \sum_j \phi_{gnj} \sum_i \pi_{gnj,i} = 1$, hence,

$$w_n L_{gn} + P_{gn} Q_{gn} = P_{gn} Q_n. \tag{32}$$

We know that

$$\frac{L_{fn}}{Q_{fn}} = \left(\frac{\alpha}{1-\alpha} \right) \frac{P_{gn}}{w_n}, \tag{33}$$

and

$$\frac{L_{gn}}{Q_{gn}} = \left(\frac{\beta}{1-\beta} \right) \frac{P_{gn}}{w_n}. \tag{34}$$

Plugging 34 into 32 we get

$$\left(\frac{\beta}{1-\beta} \right) P_{gn} Q_{gn} + P_{gn} Q_{gn} = P_{gn} Q_n,$$

from which we get $Q_{gn} = (1-\beta)\, Q_n$. Using $Q_{fm} + Q_{gn} = Q_n$, we have

$$Q_{fn} = \beta Q_n. \tag{35}$$

Plugging $Q_{gn} = (1-\beta)\, Q_n$ back into (32), we get

$$w_n L_{mn} = \beta P_{gn} Q_n.$$

Using $L_{gn} + L_{fn} = L_n$, we have

$$w_n (L_n - L_{fn}) = \beta P_{gn} Q_n. \tag{36}$$

From (33) and (35), we get

$$w_n L_{fn} = \left(\frac{\alpha}{1-\alpha} \right) \beta P_{gn} Q_n.$$

Using (36), we then have $L_{fn} = \left(\frac{\alpha}{1-\alpha} \right) (L_n - L_{fn})$, and hence $L_{fn} = \alpha L_n$. Plugging into (36), we get finally get $(1-\alpha) w_n L_n = \beta P_{gn} Q_n$, or $P_{gn} Q_n = \eta w_n L_n$.■

Proof of Proposition 3. Under symmetry , $w_i = w$, for all i. The price index for intermediate goods, in Proposition 1, collapses to:

$$\begin{aligned} P_g &= \gamma \Psi_g^{-1/\theta} \\ &= (\gamma B)^{1/\beta} \cdot [\Delta_0 + (I-1)\Delta_1]^{-\frac{1}{\beta\theta}} \cdot T^{-\frac{1}{\beta\theta}} \cdot w. \end{aligned}$$

where

$$
\begin{aligned}
\Delta_0 &\equiv \left(1+(I-1)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho} \\
\Delta_1 &\equiv \left(d^{-\frac{\theta}{1-\rho}}+m^{-\frac{\theta}{1-\rho}}+(I-2)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}.
\end{aligned}
$$

The price index for final goods is then:

$$
\begin{aligned}
P_f &= \gamma\Psi^{-1/\theta} \\
&= \gamma\left[1+(I-1)h^{-\theta}\right]^{-1/\theta}\cdot T^{-1/\theta}\cdot c_f \\
&= \gamma\left[1+(I-1)h^{-\theta}\right]^{-1/\theta}\cdot T^{-1/\theta}\cdot Aw^{\alpha}P_g^{1-\alpha} \\
&= (\gamma A)\,(\gamma B)^{\eta}\left[1+(I-1)h^{-\theta}\right]^{-\frac{\eta}{\theta}}\cdot\left[\Delta_0+(I-1)\Delta_1\right]^{-\frac{\eta}{\theta}}\cdot T^{-\frac{\eta}{\theta}}\cdot w
\end{aligned}
$$

Then, the real wage is:

$$
\frac{w}{P_f} = \widetilde{\gamma}^{-1}\left[1+(I-1)h^{-\theta}\right]^{\frac{1}{\theta}}\left[\Delta_0+(I-1)\Delta_1\right]^{\frac{\eta}{\theta}}T^{\frac{1+\eta}{\theta}}.
$$

where $\widetilde{\gamma}\equiv(\gamma A)\,(\gamma B)^{\eta}$.■

Characterization of Symmetric Equilibrium

Under symmetry, we can derive explicit expressions for the trade and MP shares as well as for the real wage. MP in final goods from any other country as a share of a country's total income is given by

$$
\widetilde{Y}_f \equiv \frac{Y_{fni}}{w_nL_n} = \frac{h^{-\theta}}{1+(I-1)h^{-\theta}}
$$

for $i\neq n$. The term $h^{-\theta}$ captures the impact of the efficiency loss of MP on equilibrium MP flows; it acts as a "discount" on the importance of foreign technologies.

Turning to intermediate goods, we have

$$
\widetilde{Y}_g \equiv Y_{gni}/\eta w_nL_n = \widetilde{Y}_{g1}+\widetilde{Y}_{gB,0}+\widetilde{Y}_{gB,1}
$$

for $i\neq n$. The term $\widetilde{Y}_{gD}$ captures MP for goods destined to stay in the domestic market, while $\widetilde{Y}_{gB,0}$ is MP for goods that go back to the country where the technology originates, and $\widetilde{Y}_{gB,1}$ goes to a third market. Both $\widetilde{Y}_{gB,0}$ and $\widetilde{Y}_{gB,1}$ take place through BMP. The respective formulas are

$$
\widetilde{Y}_{gB,0} = \frac{\Delta_0^{1-\frac{1}{1-\rho}}(md)^{-\frac{\theta}{1-\rho}}}{\Delta_0+(I-1)\Delta_1},\;\; \widetilde{X}_1 = \frac{\Delta_1^{1-\frac{1}{1-\rho}}m^{-\frac{\theta}{1-\rho}}}{\Delta_0+(I-1)\Delta_1},\;\; \widetilde{Y}_{gB,1} = \frac{(I-2)\Delta_1^{1-\frac{1}{1-\rho}}(md)^{-\frac{\theta}{1-\rho}}}{\Delta_0+(I-1)\Delta_1}.
$$

The equilibrium trade share is given by:

$$\widetilde{X} \equiv X_{nl}/\eta w_n L_n = \widetilde{X}_{0,B} + \widetilde{X}_1 + \widetilde{X}_{1,B} + \omega \widetilde{Y}_g$$

for $l \neq n$. The term $\widetilde{X}_{0,B}$ captures the imports of goods produced abroad with a country's own technologies through BMP; the term $\widetilde{X}_1$ is the standard component associated with imports from a country that used that country's technology for production; the term $\widetilde{X}_{1,B}$ captures the imports of goods produced with foreign technologies through BMP; and the term $\omega\widetilde{Y}_g$ captures the imports of the input bundle for domestic operations of foreign multinationals. The formulas for $\widetilde{X}_{0,B}$ and $\widetilde{X}_{1,B}$ are the same as the formulas for $\widetilde{Y}_{gB,0}$ and $\widetilde{Y}_{gB,1}$, respectively, while $\widetilde{X}_1 = \widetilde{Y}_{g1}(d/m)^{-\theta/(1-\rho)}$.

It is easy to see from these results that the total value of BMP as a share total MP is

$$BMP = \frac{\widetilde{Y}_{gB,0} + \widetilde{Y}_{gB,1}}{\widetilde{Y}_g} \cdot \frac{\widetilde{Y}_g}{\widetilde{Y}_f + \widetilde{Y}_g}$$

Assume $\rho \to 1$, that is, technology draws are the same across production locations. In this case, $BMP \to 0$. Further, when $h > d$, $\widetilde{Y}_g \to 0$, and there is only trade, $\widetilde{X} = d^{-\theta}/(1+(I-1)d^{-\theta})$. On the contrary, when $h < d$, trade is just associated with MP flows, $\widetilde{X} = \omega m^{-\theta}/(1+(I-1)m^{-\theta}) = \omega\widetilde{Y}_g$.

Proof of Proposition 4. The real wage under under isolation is obtained by letting $d \to \infty$ and $h \to \infty$ in (13):

$$\left(\frac{w}{P_f}\right)^{ISOL} = \widetilde{\gamma}^{-1} \cdot T^{\frac{1+\eta}{\theta}}.$$

By setting $h \to \infty$ in 13, the real wage when there is only trade is:

$$\left(\frac{w}{P_f}\right)^{T} = \widetilde{\gamma}^{-1} \cdot \left[1 + (I-1)d^{-\theta}\right]^{\frac{\eta}{\theta}} \cdot T^{\frac{1+\eta}{\theta}}.$$

Similarly, the real wage when there is only MP is:

$$\left(\frac{w}{P_f}\right)^{MP} = \widetilde{\gamma}^{-1} \cdot \left[1 + (I-1)h^{-\theta}\right]^{1/\theta} \cdot \left[1 + (I-1)\widetilde{m}^{-\theta}\right]^{\frac{\eta}{\theta}} \cdot T^{\frac{1+\eta}{\theta}},$$

where

$$\widetilde{m} \equiv \lim_{d\to\infty} m = (1-a)^{\frac{1}{1-\xi}}\, h.$$

Thus, GO are just P_f^{ISO}/P_f, $GT = P_f^{ISO}/P_f^{T}$, $GMP = P_f^{ISO}/P_f^{MP}$, and $GT_f'^{MP}/P_f$.□

Lemma 1 *GO is decreasing in ρ.*

Proof of Lemma 1. Assume $I \geq 2$, $m, d \geq 1$, $\theta > 1$ and $\rho \in [0,1)$. We want to show that

$$GO^g(\rho) = \left[1 + (I-1)(md)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho} + (I-1)\left[d^{-\frac{\theta}{1-\rho}} + m^{-\frac{\theta}{1-\rho}} + (I-2)(md)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}$$

is decreasing in ρ. Note that if $m \to \infty$ or $d \to \infty$ then $dGO^g(\rho)/d(1-\rho) \to 0$. Also note that $\left[1 + (I-1)(md)^{-\theta/(1-\rho)}\right]^{1-\rho}$ is clearly increasing in $1-\rho$ because $1 + (I-1)(md)^{-\theta/(1-\rho)}$ is higher than one. It is left to show that $\left[d^{-\theta/(1-\rho)} + m^{-\theta/(1-\rho)} + (I-2)(md)^{-\theta/(1-\rho)}\right]^{1-\rho}$ is increasing in $1-\rho$. Without loss of generality, we assume that $m > d \geq 1$, then $\frac{m}{d} > 1$. We have

$$\begin{aligned}
&\left[d^{-\frac{\theta}{1-\rho}} + m^{-\frac{\theta}{1-\rho}} + (I-2)(md)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho} \\
= &\left[d^{-\frac{\theta}{1-\rho}}\left\{1 + \left(\frac{m}{d}\right)^{-\frac{\theta}{1-\rho}} + (I-2)m^{-\frac{\theta}{1-\rho}}\right\}\right]^{1-\rho} \\
= &\; d^{-\theta}\left\{1 + \left(\frac{m}{d}\right)^{-\frac{\theta}{1-\rho}} + (I-2)m^{-\frac{\theta}{1-\rho}}\right\}^{1-\rho}
\end{aligned}$$

Since $1+\left(\frac{m}{d}\right)^{-\theta/(1-\rho)}+(I-2)m^{-\theta/(1-\rho)} > 1$, $\frac{m}{d} > 1$, $m > 1$; $\left\{1 + \left(\frac{m}{d}\right)^{-\theta/(1-\rho)} + (I-2)m^{-\theta/(1-\rho)}\right\}^{1-\rho}$ is increasing in $1-\rho$. In the case of $d > m$, the logic is the same.$\square$

Lemma 2 *For $\rho \to 1$, if $\xi > 1 + \theta$, $GO^g < GT \times GMP^g$.*

Proof of Lemma 2. Recall that $GO^g = [\Delta_0 + (I-1)\Delta_1]^{\eta/\theta}$. For $\rho \to 1$, Δ_0 in equation (14) collapses to:

$$\lim_{\rho \to 1} \Delta_0 = \frac{1}{\min[1; (md)^\theta]} = 1,$$

and Δ_1 in equation (15) collapses to:

$$\lim_{\rho \to 1} \Delta_1 = \frac{1}{\min[d^\theta; (m)^\theta; (md)^\theta]} = \begin{cases} d^{-\theta} & for \quad h > d \\ m^{-\theta} & for \quad h < d \end{cases}$$

Hence:

$$\lim_{\rho \to 1} GO^g = \begin{cases} 1 + (I-1)d^{-\theta} & for \quad h > d \\ 1 + (I-1)m^{-\theta} & for \quad h < d. \end{cases}$$

Recall that $GT \times GMP^g = 1 + (I-1)\left[d^{-\theta} + \widetilde{m}^{-\theta} + (I-1)(d\widetilde{m})^{-\theta}\right]$. Then, with $h > d$, $GO^g < GT \times GMP^g$, and $GT' = GO^g/GMP^g < GT$.

With $h < d$, $GO^g < GT \times GMP^g$, and $GT' < GT$, only if

$$\left(m^{-\theta} - \widetilde{m}^{-\theta}\right) < d^{-\theta}\left[1 + (I-1)\widetilde{m}^{\theta}\right].$$

Hence, it is sufficient that:

$$\left(\frac{m}{d}\right)^{-\theta} - \left(\frac{\widetilde{m}}{d}\right)^{-\theta} < 1.$$

Replacing $m = \left[ad^{1-\xi+(1-a)h^{1-\xi}}\right]^{\frac{1}{1-\xi}}$, and $\widetilde{m} = (1-a)^{\frac{1}{1-\xi}}h$, the sufficient condition is:

$$\left[(1-a)(\frac{h}{d})^{1-\xi} + a\right]^{-\frac{\theta}{1-\xi}} - \left[(1-a)(\frac{h}{d})^{1-\xi}\right]^{-\frac{\theta}{1-\xi}} < 1,$$

that it is satisfied if $\xi - 1 > \theta$. $\square$

Proof of Proposition 5. Let $GO = GO^f \cdot GO^g$, that is, decompose gains from openness into a term for gains in the final goods' sector, $GO^f \equiv \left[1 + (I-1)h^{-\theta}\right]^{1/\theta}$, and another term for gains in the intermediate goods' sector, $GO^g \equiv \left[1 + (I-1)\left(d^{-\theta} + m^{-\theta} + (I-1)(md)^{-\theta}\right)\right]^{\eta/\theta}$. Similarly, gains from MP can be decompose in an analogous way, $GMP = GMP^f \cdot GMP^g$, where $GO^f = GMP^f$. Thus, GT'^g/GMP^g.

(a) $a > 0$, $\widetilde{m} > m$. Thus, for $\rho = 0$, it is straightforward that $GO > GT \times GMP^g$:

$$\begin{aligned} GO^g &= \left[1 + (I-1)\left(d^{-\theta} + m^{-\theta} + (I-1)(md)^{-\theta}\right)\right]^{\eta/\theta} \\ &> \\ GT \times GMP^g &= \left[1 + (I-1)\left(d^{-\theta} + \widetilde{m}^{-\theta} + (I-1)(\widetilde{m}d)^{-\theta}\right)\right]^{\eta/\theta}. \end{aligned}$$

It follows that $GT' = GO^g/GMP^g > GT$.

(b) For $a = 0$, $\widetilde{m} = m$. Hence, for $\rho = 0$, $GO^g = GT \times GMP^g$, and $GT' = GT$.

(c) For $a = 0$, $\widetilde{m} = m$. By Lemma 1, we know that $dGO^g/d\rho < 0$. Then, $GO^g_{\rho=0} > GO^g$ for any $0 < \rho < 1$. By 2), $GO^g_{\rho=0} = GT \times GMP^g$. It follows that for any $\rho \in (0,1)$, $GO^g < GT \times GMP^g$, and hence, $GT'^g/GMP^g < GT$.

(d) For $a > 0$, $\widetilde{m} > m$. By Lemma 1, we know that $dGO^g/d\rho < 0$. Also, $d(GT \times GMP^g)d\rho \to 1$. By 1), we know that for $\rho = 0$, $GO^g > GT \times GMP^g$. By Lemma 2, for $\rho \to 1$, if $\xi > 1 + \theta$, $GO^g < GT \times GMP^g$. By continuity, it follows that there exits $\widehat{\rho} \in (0,1)$ such that for $\rho < \widehat{\rho}$, $GO^g > GT \times GMP^g$ and $GT'^g/GMP^g > GT$; and for $\rho > \widehat{\rho}$, $GO^g < GT \times GMP^g$ and $GT' = GO^g/GMP^g < GT$.

Moreover, for $a > 0$, $\xi \to 1$ implies that $\widetilde{m} \to \infty$, and hence $GMP \to 0$. It follows then that $GO > GT \times GMP$. Conversely, $\xi \to \infty$ implies that $m, \widetilde{m} \to \min[h, d]$. This implies that $GO < GT \times GMP$. $\square$

D The Dynamic Model

Imagine that each country is composed of a number of locations. Let lv denote location v in country l and let N_l be the number of locations in country l. An idea in country i has a productivity q_{lvi} in location v of country l. Assume that the vector $\mathbf{q}_i = (q_{11i}, q_{12i}, ..., q_{1N_1i}, ..., q_{I1i}, q_{I2i}, ..., q_{IN_Ii})$ is drawn from the following multivariate distribution:

$$H(\mathbf{q}_i) = 1 - \left(\sum_{l=1}^{I} \sum_{v=1}^{N_l} \left(\frac{q_{lvi}}{\underline{\varepsilon}} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}$$

with $\sum_{l=1}^{I} \sum_{v=1}^{N_l} q_{lvi}^{-\theta/(1-\rho)} < \underline{\varepsilon}^{-\theta/(1-\rho)}$ for $\rho \in (0,1)$ and $\theta > 1$. Note that the marginal distribution of $q_{lvi} \geq \underline{\varepsilon}$ for any lvi is $1 - (q_{lvi}/\underline{\varepsilon})^{-\theta}$, so we can think of $H(\cdot)$ as a multivariate Pareto distribution.

Research is modeled as the creation of ideas, although for simplicity here we assume that this is exogenous. In particular, we assume that there is an instantaneous (and constant) rate of arrival $\overline{\varepsilon}\zeta_i$ of new ideas per person in country i. The parameter ζ_i varies across countries and captures differences in the "research" productivity across countries, while $\overline{\varepsilon}$ is a common parameter that will be normalized below. Ideas are specific to goods, and the good to which an idea applies can be an intermediate good or a final good with equal probability. If the idea applies to an intermediate (final) good the identity of the good is drawn from a uniform distribution in $v \in [0,1]$ ($u \in [0,1]$). This implies that at time t there is a probability $R_i(t) = \overline{\varepsilon}\zeta_i L_i(t)$ of drawing an idea for any particular (intermediate or final) good. The arrival of ideas is then a Poisson process with rate function $\overline{\varepsilon}R_i(t)$, so the number of ideas that have arrived for a particular good by time t is distributed Poisson with rate $\overline{\varepsilon}\lambda_i(t)$, where $\lambda_i(t) \equiv \int_0^t R_i(s)ds$. (From here onwards, we suppress the time index as long as it does not cause confusion.)

The technology frontier for country i is the upper envelope of all the vectors $\mathbf{q}_i$. That is, letting Ω_{lvi} denote the set of all q_{lvi} associated with ideas existing in country i at a certain point in time, then the technology frontier for country i is $\mathbf{x}_i \equiv (\max\{q_{11i} \in \Omega_{11i}\}, \max\{q_{12i} \in$

$\Omega_{12i}\}, ..., \max\{q_{1N_1i} \in \Omega_{1N_1i}\}, ..., \max\{q_{I1i} \in \Omega_{I1i}\}, ..., \max\{q_{IN_Ii} \in \Omega_{IN_Ii}\})$. This is distributed according to

$$\begin{aligned}
\Pr(X_{11i} &\leq x_{11i}, ..., X_{IN_li} \leq x_{IN_li}) = \sum_{k=0}^{\infty} \frac{e^{-\bar{\varepsilon}\lambda_i}(\bar{\varepsilon}\lambda_i)^k}{k!} H(\mathbf{x}_i)^k \\
&= e^{-\bar{\varepsilon}\lambda_i} \sum_{k=0}^{\infty} \frac{\left[\bar{\varepsilon}\lambda_i H(\mathbf{x}_i)\right]^k}{k!} = e^{-\bar{\varepsilon}\lambda_i(1-H(\mathbf{x}_i))} \\
&= \exp\left[-\bar{\varepsilon}\underline{\varepsilon}^{\theta}\lambda_i \left(\sum_{l=1}^{I}\sum_{v=1}^{N_l} x_{lvi}^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}\right]
\end{aligned}$$

for $\sum_{l=1}^{I}\sum_{v=1}^{N_l} x_{lvi}^{-\theta/(1-\rho)} < \underline{\varepsilon}^{-\theta/(1-\rho)}$. Letting $z_{li} \equiv \max\{x_{l1i}, ..., x_{lN_li}\}$ be the highest productivity among all country i ideas as they apply to locations of country l, then

$$F_i(z_i) = \exp\left[-\bar{\varepsilon}\underline{\varepsilon}^{\theta}\lambda_i \left(\sum_{l=1}^{I} N_l z_{li}^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}\right]$$

for $\sum_{l=1}^{I} N_l z_{li}^{-\frac{\theta}{1-\rho}} < \underline{\varepsilon}^{-\theta/(1-\rho)}$. Letting $T_{li} \equiv \lambda_i N_l^{1-\rho}$, setting $\bar{\varepsilon}\underline{\varepsilon}^{\theta} = 1$, and taking the limit as $\underline{\varepsilon}^{\theta} \to 0$, then we get the multivariate Fréchet distribution in 5.

Assuming that L_i grows at the constant rate g_L (assumed to be common across countries) then in steady state we must have $\lambda_i = R_i/g_L$, so λ_i and hence T_{li} grow at rate g_L for all l, i. This implies that the static equilibrium described in Section 2.2 is replicated at all dates, and that the real wage in all countries is increasing at rate $g = \frac{1}{\theta}(1+\eta)g_L$.