Nonparametric Discrete Choice Models with Unobserved Heterogeneity[1]

Richard A. Briesch
Cox School of Business
Southern Methodist University

Pradeep K. Chintagunta
Graduate School of Business
University of Chicago

Rosa L. Matzkin
Department of Economics
Northwestern University

February 2007

Rosa Matzkin gratefully acknowledges support from NSF for this research. The authors gratefully thank Information Resources, Inc., ESRI, and Edward Fox, Director of the Retailing Center at Southern Methodist University for providing the data used in this research.

[1] Corresponding author. Richard Briesch; Cox School of Business; Southern Methodist University; PO Box 750333; Dallas, Tx 75275-0333. Email: rbriesch@mail.cox.smu.edu, phone: 214-768-3180, fax: 214-768-4099.

Nonparametric Discrete Choice Models with Unobserved Heterogeneity

Abstract

In this research, we provide a new method to estimate discrete choice models with unobserved heterogeneity that can be used with either cross-sectional or panel data. The method imposes nonparametric assumptions on the systematic subutility functions and on the distributions of the unobservable random vectors and the heterogeneity parameter. The estimators are computationally feasible and strongly consistent. We provide an empirical application of the estimator to a model of store format choice. The key insights from the empirical application are: 1) consumer response to cost and distance contains interactions and non-linear effects which implies that a model without these effects tends to bias the estimated elasticities and heterogeneity distribution and 2) the increase in likelihood for adding non-linearities is similar to the increase in likelihood for adding heterogeneity.

Keywords: Random Effects; Heterogeneity; Discrete Choice; Nonparametric

1. INTRODUCTION

Since the early work of McFadden (1974) on the development of the Conditional Logit Model for the econometric analysis of choices among a finite number of alternatives, a large number of extensions of the model have been developed. These extensions have spawned streams of literature of their own. One such stream has focused on relaxing the strict parametric structure imposed in the original model. Another stream has concentrated on relaxing the parameter homogeneity assumption across individuals. This paper contributes to both these areas of research. We introduce methods to estimate discrete choice models where all functions and distributions are nonparametric, individuals are allowed to be heterogeneous in their preferences over observable attributes, and the distribution of these preferences is also nonparametric.

As is well known in discrete choice models, each individual possesses a utility for each available alternative, and chooses the one that provides the highest utility. The utility of each alternative is the sum of a subutility of observed attributes - the systematic subutility - and an unobservable random term - the random subutility. For example, in a model of a commuter choosing between various means of transportation, the alternatives may be car and bus, and the attributes observable to the researcher may be the cost and time associated with each alternative. The utility to the commuter for a means of transportation is the sum of a function (the systematic subutility) of the time and cost of that means of transportation and the effects of other attributes or factors that are not observed by the researcher. These latter effects are represented by an unobservable random term that represents the value of a subutility of unobserved attributes of the means of transportation, such as comfort.

Manski (1975) developed an econometric model of discrete choice that did not require specification of a parametric structure for the distribution of the unobservable random subutilities. This semiparametric, distribution-free method was followed by other semiparametric distribution-free methods, developed by Cosslett (1983), Manski (1985), Han (1987), Powell, Stock and Stoker (1989), Horowitz (1992), Ichimura (1993), and Klein and Spady (1993), Moon (2004), among others. More recently, Geweke and Keane (1997) and Hirano (2002), have applied mixing techniques that allow nonparametric estimation of the error term in Bayesian models. Similarly, Klein and Sherman (2002) propose a method that allows non-parametric estimation of the density as well as the parameters for ordered response models. These methods are termed semiparametric because they require a parametric structure for the systematic subutility of the observable characteristics.

A second stream of literature has focused on relaxing the parametric assumption about the systematic subutility. Matzkin (1991)'s semiparametric method freed the systematic subutility of a parametric structure, while maintaining a parametric structure for the distribution of the unobservable random subutilities. Matzkin (1992, 1993) also proposed fully nonparametric methods where neither the systematic subutility nor the distribution of the unobservable random subutility are required to possess parametric structures.

Finally, a third stream of literature has focused on incorporating consumer heterogeneity into choice models. Wansbeek, et al (2001) noted the importance of including heterogeneity in choice models to avoid finding weak relationships between explanatory variables and choice. However, they also note the difficulty of incorporating heterogeneity into nonparametric and semiparametric models. Further, Allenby and Rossi (1999) noted the importance of allowing heterogeneity in choice models to extend into the slope coefficients. This extension is required because "optimal marketing decisions must account for the substantial uncertainty on decision criteria which often involve non-linear functions of model parameters" (p. 58). Specifications that have allowed for heterogeneous systematic subutilities include those of Heckman and Willis (1977), Albright, Lerman and Manski (1977), McFadden (1978), and Hausman and Wise (1978). These papers use a particular parametric specification, i.e., a specific continuous distribution, to account for the distribution of systematic subutilities across consumers. Heckman and Singer (1984) propose estimating the parameters of the model without imposing a specific continuous distribution for this heterogeneity distribution. Ichimura and Thompson (1998) have developed an econometric model of discrete choice where the coefficients of the linear subutility have a distribution of unknown form, which can be estimated.

More recently, much empirical work has been done that allows some relaxation of the heterogeneity distributions. Lancaster (1997) allows for non-parametric identification of the distribution of the heterogeneity in Bayesian models. Taber (2000) and Park et al (2007) apply semiparametric techniques to dynamic models of choice. Briesch, Chintagunta and Matzkin (2002) allow consumer heterogeneity in the parametric part of the choice model while restricting the non-parametric function to be homogeneous. Dahl (2002) applies non-parametric techniques to transition probabilities and dynamic models. Finally, Pinkse, et al (2002) allow for heterogeneity in semiparametric models of aggregate-level choice.

The method that we develop here combines the fully nonparametric methods for estimating discrete choice models (Matzkin (1992, 1993)) with a method that allows us to estimate the distribution of unobserved heterogeneity nonparametrically (Heckman and Singer (1984)) as well. The unobserved heterogeneity variable is included in the systematic subutility in a nonadditive way (Matzkin (1999, 2003)). We provide conditions under which the systematic subutility, the distribution of the nonadditive unobserved heterogeneity variable, and the distribution of the additive unobserved random subutility can all be nonparametrically identified and consistently estimated from individual choice data. The method can be used with either cross sectional or panel data. These results improve upon Briesch, Chintagunta, and Matzkin (2002).

We apply the proposed methodology to study the drivers of grocery store-format choice for a panel of households. There are two main types of formats that supermarkets (i.e., grocery stores or chains) classify themselves into – everyday low price (EDLP) stores or high-low price (Hi-Lo) stores. The former offer fewer promotions of lower "depth" (i.e., magnitude of discounts) than the latter. The main tradeoff facing consumers is that EDLP stores are typically located farther away (longer driving distances) than Hi-Lo stores although their prices, on average, are lower than those at Hi-Lo stores leading to a lower total cost of shopping "basket" for the consumer. Since there is strong evidence that the value of time (associated with the driving distance) is heterogeneous across households, and there is no consensus across the previous empirical results about the particular shape of the utility for driving distance and expenditure, we think that the proposed method is ideally suited to understanding the nature of the tradeoff between distance and expenditure facing the consumer. To decrease the well-known dimensionality problems associated with relaxing parametric structures, we use a semiparametric version of our model. In particular, only the subutility of distance to the store and cost of refilling inventory at the store are nonparametric. We allow this subutility to be heterogeneous across consumers, and provide an estimator for both, the subutilities of the different types, the distribution of types, and the additional parameters of the model. Further, we assume that the unobserved component of utility in this application is distributed according to a type-I extreme value distribution.

In the next section we describe the model. In Section 3 we state conditions under which the model is identified. In Section 4 we present strongly consistent estimators for the functions and distributions in the model. Section 5 provides computational details. Section 6 presents the empirical application. Section 7 concludes.

2. THE MODEL

As is usual in discrete choice models, we assume that a typical consumer must choose one of a finite number, J, of alternatives, and he/she chooses the one that maximizes the value of a utility function, which depends on the characteristics of the alternatives and the consumer. Each alternative j is characterized by a vector, z_j, of the observable attributes of the alternatives. We will assume that $z_j \equiv (x_j, r_j)$, where $r_j \in R$ and $x_j \in R^K$ $(K \geq 1)$. Each consumer is characterized by a vector, $s \in R^L$, of observable socioeconomic characteristics for the consumer. The utility of a consumer with observable socioeconomic characteristics s, for an alternative, j, is given by

$$V^*(j, s, z_j, \omega) + \epsilon_j$$

where ϵ_j and ω denote the values of unobservable random variables. For any given value of ω, and any j, $V^*(j, \cdot, \omega)$ is a real valued, but otherwise unknown, function. The dependence of V^* on ω allows us to incorporate into the model the possibility that this systematic subutility be different for different consumers, even if the observable exogenous variables posses the same values for these consumers. We will denote the distribution of ω by G^* and we will denote the normalized distribution of the random vector $(\epsilon_1, \ldots, \epsilon_J)$ by F^*.

The probability that a consumer with socioeconomic characteristics s will choose alternative j when the vector of observable attributes of the alternatives is $z \equiv (z_1, \ldots, z_J) \equiv (x_1, r_1, \ldots, x_J, r_J)$ will be denoted by $p(j|s, z; V^*, F^*, G^*)$. Hence,

$$p(j|s, z; V, F, G) = \int \Pr(j|s, z; \omega, V, F) \quad dG(\omega)$$

where $\Pr(j|s, z; \omega, V, F)$ denotes the probability that a consumer with systematic subutility $V(\cdot; \omega)$ will choose alternative j, when the distribution of ϵ is F. By the utility maximization hypothesis,

$$\Pr(j|s, z; \omega, V, F)$$

$$= \Pr\left\{V(j, s, x_j, r_j, \omega) + \epsilon_j > V(k, s, x_k, r_k, \omega) + \epsilon_k \quad \text{for all } k \neq j\right\}$$

$$= \Pr\left\{\epsilon_k - \epsilon_j < V(j, s, x_j, r_j, \omega) - V(k, s, x_k, r_k, \omega) \quad \text{for all } k \neq j\right\}$$

which depends on the distribution F. In particular, if we let F_1^* denote the distribution of the vector $(\epsilon_2 - \epsilon_1, \ldots, \epsilon_J - \epsilon_1)$, then

$$\Pr(1 \mid s, z; \omega, V, F^*)$$

$$== F_1^*\big(V(1,s,x_1,r_1,\omega) - V(2,s,x_2,r_2,\omega),\ldots,V(1,s,x_1,r_1,\omega) - V(J,s,x_2,r_2,\omega)\big)$$

and the probability that the consumer will choose alternative 1 is then

$$p(1 \mid s, z; V, F^*, G)$$

$$= \int \Pr(1 \mid s, z; \omega, V, F^*) \; dG(\omega)$$

$$= \int F_1^*\big(V(1,s,x_1,r_1,\omega) - V(2,s,x_1,r_1,\omega),\ldots,V(1,s,x_1,r_1,\omega) - V(J,s,x_1,r_1,\omega)\big) \; dG(\omega)$$

For any j, $\Pr(j \mid s, z; \omega, V, F^*)$ can be obtained in an analogous way, letting F_j^* denote the distribution of $(\epsilon_1 - \epsilon_j, \ldots, \epsilon_J - \epsilon_j)$.

A particular case of the polychotomous choice model is the Binary Threshold Crossing Model, which has been used in a wide range of applications. This model can be obtained from the Polychotomous Choice Model by letting $J = 2$, $\eta = \varepsilon_2 - \varepsilon_1$, and $\forall (s, x_2, r_2, \omega)$ $V(2, s, x_2, r_2, \omega) \equiv 0$. In other words, the model can be described by:

$$y^* = V(x, r, \omega) - \eta$$

$$y = \left\{ \begin{array}{ll} 1 & if \quad y^* \geq 0 \\ 0 & \quad otherwise \end{array} \right\}$$

where y^* is unobservable. In this model, F_1^* denotes the distribution of η. Hence, for all x, r, ω

$$\Pr(1 \mid x, r; \omega, V^*, F^*) = F_1^*\big(V^*(x, r, \omega)\big)$$

and for all x, r the probability that the consumer will choose alternative 1 is

$$p(1 \mid s, z; V^*, F^*, G^*) = \int \Pr(1 \mid s, z; \omega, V^*, F^*) \; dG^*(\omega) = \int F_1^*\big(V^*(x, r, \omega)\big) \; dG^*(\omega)$$

3. NONPARAMETRIC IDENTIFICATION

Our objective is to develop estimators for the function V^* and the distributions F^* and G^*, without requiring that these functions and distributions belong to parametric families. It follows from the definition of the model that we can only hope to identify the distributions of the vectors

$\eta_j \equiv (\epsilon_1 - \epsilon_j, \ldots, \epsilon_J - \epsilon_j)$ for $j = 1, \ldots, J.$ Let F_1^* denote the distribution of η_1. Since from F_1^* we can obtain the distribution of η_j $(j = 2, \ldots, J),$ we will deal only with the identification of F_1^*. We let $F^* = F_1^*$.

DEFINITION: *The function* V^* *and the distribution* F^* *and* G^* *are identified in a set* $(W \times \Gamma_F \times \Gamma_G)$ *such that* $(V^*, F^*, G^*) \in (W \times \Gamma_F \times \Gamma_G)$ *if* $\forall (V, F, G) \in (W \times \Gamma_F \times \Gamma_G)$

$$\int \Pr(j|s,z; V(\bullet;\omega), F)\ dG(\omega) = \int \Pr(j|s,z; V^*(\bullet;\omega), F^*)\ dG^*(\omega) \quad \text{for } j = 1, \ldots, J,\ a.s.$$

implies that

$$V = V^*,\ F{=}F^*\ \&\ G = G^*.$$

That is, (V^*, F^*, G^*) is identified in a set $(W \times \Gamma_F \times \Gamma_G)$ to which (V^*, F^*, G^*) belongs, if any triple, (V, F, G) that belongs to $(W \times \Gamma_F \times \Gamma_G)$ and is different from (V^*, F^*, G^*) generates, for at least one alternative j, and a set of (s, z) that possesses positive probability, choice probabilities $p(j|s,z; V, F, G)$ that are different from $p(j|s,z; V^*, F^*, G^*)$.

We next present a set of conditions that, when satisfied, guarantee that (V^*, F^*, G^*) is identified in $(W \times \Gamma_F \times \Gamma_G)$.

ASSUMPTION 1: *The support of* $(s, x_1, r_1, \ldots, x_J, r_J, \omega)$ *is a set* $(S \times \prod_{j=1}^J (X_j \times R_+) \times Y)$*, where* S *and* X_j $(j = 1, \ldots, J)$ *are subsets of Euclidean spaces and* Y *is a subset of* R.

ASSUMPTION 2: *The random vectors* $(\epsilon_1, \ldots, \epsilon_J)$, $(s, z_1, \ldots, z_J)$ *and* ω *are independent.*

ASSUMPTION 3: *For all* $V \in W$ *and* j, *there exists a real valued function* $v(j, \bullet, \bullet, \omega) : int(S \times X_j) \to R$ *such that* $\forall (s, x_j, r_j, \omega) \in (S \times X_j \times R_+ \times Y)$,

$$V(j, s, x_j, r_j, \omega) = v(j, s, x_j, \omega) - r_j.$$

ASSUMPTION 4: $\exists\ (\bar{s}, \bar{x}_1, \ldots, \bar{x}_J) \in (S \times \prod_{j=1}^J X_j)$ *and* $(\alpha_1, \ldots, \alpha_J) \in R^J$ *such that* $\forall j\ \forall \omega\ \forall V \in W$,

$$v(j, \bar{s}, \bar{x}_j, \omega) = \alpha_j$$

ASSUMPTION 5: $\exists \tilde{j},\ \beta_{\tilde{j}} \in R,$ *and* $\tilde{x}_{\tilde{j}} \in X_{\tilde{j}}$ *such that* $\forall s \in S\ \ \forall \omega \in Y\ \ \forall V \in W$,

$$v(\tilde{j}, s, \tilde{x}_{\tilde{j}}, \omega) = \beta_{\tilde{j}}$$

ASSUMPTION 6: $\exists \quad j^* \neq \tilde{j}$ *and* $(\hat{s}, \hat{x}_{j^*}) \in (S \times X_{j^*})$ *such that*

$$\forall V \in W \ \forall \omega \quad v(j^*, \hat{s}, \hat{x}_{j^*}, \omega) = \omega$$

ASSUMPTION 6': $\exists \quad j^* \neq \tilde{j}$, $(\hat{s}, \hat{x}_{j^*}, \hat{\omega}) \in (S \times X_{j^*} \times Y)$ *and* $\gamma \in R$ *such that* $\forall V \in W$

$$v(j^*, \hat{s}, \hat{x}_{j^*}, \hat{\omega}) = \gamma,$$

$$\text{and} \ \forall V \in W \ \forall \lambda \in R$$

$$v(j^*, \lambda\hat{s}, \lambda\hat{x}_{j^*}, \lambda\hat{\omega}) = \lambda\gamma$$

ASSUMPTION 6": $\exists \quad j^* \neq \tilde{j}$, $(\hat{s}, \hat{x}_{j^*}) \in (S \times X_{j^*})$, $\gamma \in R$, and there exists a real valued function $m(s, x_1, x_2 \omega)$ *such that* $\forall V \in W, \forall \omega \in Y$,

$$v(j^* s, x, \omega) = m(s, x_1, x_2 \omega),$$

$$\text{and } m(\hat{s}, \hat{x}_1, \hat{x}_2 \omega) = \gamma$$

ASSUMPTION 7: $\forall (s, x_{j^*}) \in (S \times X_{j^*})$

(i) *either it is known that* $v(j^*, s, x_{j^*}, \bullet)$ *is strictly increasing in* ω, *for all* $\omega \in Y$ *and* $V \in W$, *or*

(ii) *it is known that* $v(j^*, s, x_{j^*}, \bullet)$ *is strictly decreasing in* ω, *for all* $\omega \in Y$ *and* $V \in W$.

ASSUMPTION 8: $\forall \ k \neq j^*, \tilde{j}$ *either* $v(k, s, x_k, \omega)$ *is strictly increasing in* ω, *for all* (s, x_k), *or* $v(k, s, x_k, \omega)$ *is strictly decreasing in* ω, *for all* (s, x_k).

ASSUMPTION 9: $\exists j \neq \tilde{j}$ *such that* $\forall (s, x_j, x_{\tilde{j}}) \in (S \times X_j \times X_{\tilde{j}})$

(i) *either it is known that* $v(j, s, x_j, \cdot) - v(\tilde{j}, s, x_{\tilde{j}}, \cdot)$ *is strictly increasing in* ω, *for all* $\omega \in Y$ *and* $V \in W$, *or*

(ii) *it is known that* $v(j, s, x_j, \cdot) - v(\tilde{j}, s, x_{\tilde{j}}, \cdot)$ *is strictly decreasing in* ω, *for all* $\omega \in Y$ *and* $V \in W$.

ASSUMPTION 10: G^* *is a strictly increasing distribution on* Y .

ASSUMPTION 11: *The characteristic functions corresponding to the marginal distribution functions* $F^*_{\varepsilon_{\tilde{j}} - \varepsilon_j}$ $(j = 1, \ldots, J; j \neq \tilde{j})$ *are everywhere different from 0.*

An example of a model where Assumptions 4-9 are satisfied is a binary choice where each function $v(1, \bullet)$ is characterized by a function $m(\bullet)$, and each function $v(2, \bullet)$ is characterized by a function $h(\bullet)$ such that for all s, x_1, x_2, ω :

(i) $v(1,s,x_1,\omega) = m(x_1,\omega)$

(ii) $v(2,s,x_2,\omega) = h(s,x_2,\omega)$

(iii) $m(\bar{x}_1,\omega) = 0$

(iv) $h(s,\bar{x}_2,\omega) = 0$

(v) $h(\hat{s},\hat{x}_2,\omega) = \omega$

(vi) $h(s,x_2,\cdot)$. is strictly increasing when $x_2 \neq \bar{x}_2$, and

(vii) $m(x_1,\cdot)$ is strictly decreasing when $x_1 \neq \bar{x}_1$,

where $\bar{x}_1$, $\bar{x}_2$, $\hat{s}$, and $\hat{x}_2$ are given.

In this example, Assumption 4 is satisfied when $\bar{s}$ is any value and $\alpha_1 = \alpha_2 = 0$. Assumption 5 is satisfied with $\tilde{j} = 1$, $\beta_{\tilde{j}} = 0$ and $\tilde{x}_1 = \bar{x}_1$. By (v), Assumption 6 is satisfied with $j^* = 2$. Finally, Assumption 7-9 are satisfied by (vi) and (vii).

If in the above example, (v) is replaced by the assumption

(v') $h(\hat{s},\hat{x}_2,\hat{\omega}) = \alpha$ and $h(\cdot,\cdot,\cdot)$ is homogenous of degree one,

where, in addition to $\bar{x}_1$, $\bar{x}_2$, $\hat{s}$, and $\hat{x}_2$, $\hat{\omega}$ and $\alpha \in R$ are also given, then the model satisfies Assumptions 4, 5, 6', and 7-9.

Assumption 1 specifies the support of the observable explanatory variables and of ω. The critical requirement in this assumption is the large support condition on $(r_1,...,r_J)$. This requirement is used, together with the other requirements, to identify the distribution of $(\varepsilon_1-\varepsilon_2,...,\varepsilon_1-\varepsilon_J)$. Assumption 2 requires that the vectors $(\varepsilon_1,\varepsilon_2,...,\varepsilon_J)$, $(s,z_1,...,z_J)$, ω be jointly independent. That is, for all $(\varepsilon_1,\varepsilon_2,...,\varepsilon_J)$, $(s,z_1,...,z_J)$, ω,

$$f_{(\varepsilon_1,...,\varepsilon_J),((s,z_1,...,z_J),\omega}(\varepsilon_1,...,\varepsilon_J,s,z_1,...,z_J,\omega) = f_{(\varepsilon_1,...,\varepsilon_J)}(\varepsilon_1,...,\varepsilon_J) \bullet f_{(s,z_1,...,z_J)}(s,z_1,...,z_J) \bullet f_\omega(\omega)$$

where $f_{(\varepsilon_1,...,\varepsilon_J),((s,z_1,...,z_J),\omega}$ denotes the joint density of $(\varepsilon_1,...,\varepsilon_J,s,z_1,...,z_J,\omega)$, and $f_{(\varepsilon_1,...,\varepsilon_J)}, f_{(s,z_1,...,z_J)}(s,z_1,...,z_J)$, and $f_\omega(\omega)$ denote the corresponding marginals. A critical implication of this is that for all $(\varepsilon_1,...,\varepsilon_J),(s,z_1,...,z_J),\omega$, the vectors $(\varepsilon_1,...,\varepsilon_J),\omega$ and $(r_1,...,r_J)$ are jointly independent conditional on $(s,z_1,...,z_J)$, i.e.,

$$f_{(\varepsilon_1,...,\varepsilon_J),(r_1,...,r_J),\omega|(s,x_1,...,x_J)}(\varepsilon_1,...,\varepsilon_J,r_1,...,r_J,\omega) = f_{(\varepsilon_1,...,\varepsilon_J),(r_1,...,r_J),\omega}(\varepsilon_1,...,\varepsilon_J,s,z_1,...,z_J,\omega)$$

Assumption 3 restricts the form of each utility function $V(j,s,x_j,r_j,\omega)$ to be additive in r_j, and the coefficient of r_j to be known. A requirement like this is necessary even when the function $V(j,s,x_j,r_j,\omega)$ is linear in variables, to compensate for the fact that the distribution of $(\varepsilon_1,...,\varepsilon_J)$ is not specified. Since the work of Lewbel (2000), regressors such as $(r_1,...,r_J)$ have been usually called "special regressors." Assumptions 4 and 5 specify the value of the functions $v(j,\cdot)$, defined in Assumption 3, at some points of the observable variables. Assumption 4 specifies the values of these $v(j,\cdot)$ functions at one point of the observable variables, for all values of ω. This guarantees that at those points of the observable variables, the choice probabilities are completely determined by the value of the distribution of $(\varepsilon_2-\varepsilon_1,...,\varepsilon_J-\varepsilon_1)$. This, together with the support condition in Assumption 1, allows us to identify this distribution. Assumption 5 specifies the value of one of the $v(j,\cdot)$ functions at one value of x_j, for all s,ω. As in the linear specification, only differences of the utility function are identified. Assumption 5 allows to recover the values of each of the $v(j,\cdot)$ functions, for $j\neq\widetilde{j}$, from the difference between $v(j,\cdot)$ and $v(\widetilde{j},\cdot)$. Once the $v(j,\cdot)$ functions are identified, one can use them to identify $v(\widetilde{j},\cdot)$.

Assumptions 5-10 guarantee that the difference function $m(s,x_{j^*},x_{\widetilde{j}},\omega)=v(j^*,s,x_{j^*},\omega)-v(\widetilde{j},s,x_{\widetilde{j}},\omega)$ and the distribution of ω are identified from the joint distribution of (γ,s,x) where $\gamma=m(s,x_{j^*},x_{\widetilde{j}},\omega)$. Using the results in Matzkin (2003), identification can be achieved if (i) $m(s,x_{j^*},x_{\widetilde{j}},\omega)$ is either strictly increasing or strictly decreasing in ω, for all $(s,x_{j^*},x_{\widetilde{j}})$, (ii) ω is distributed independently of (s,x_{j^*}) conditional on all the other coordinates of the explanatory variables, and (iii) one of the following holds: (iii.a) for some $(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}})$, the value of m is known at $m(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}})$, for all ω; (iii.b) for some $(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}},\hat{\omega})$, the value of m is known at $(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}},\hat{\omega})$ is known and $m(s,x_{j^*},\widetilde{x}_{\widetilde{j}},\omega)$ is homogeneous of degree 1 along the ray that connects $(\hat{s},\hat{x}_{j^*},\hat{\omega})$ to the origin; (iii.c) for some $(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}},\hat{\omega})$, the value of m at $(\hat{s},\hat{x}_{j^*},\widetilde{x}_{\widetilde{j}},\hat{\omega})$ is known and m is strictly separable into a known function of ω and at least one coordinate of (s,x_{j^*}). Assumptions 6, 6', and 6'' guarantee, together with Assumption 5, that (iii.a), (iii.b), and (iii.c) are, respectively satisfied. Assumption 7 with Assumption 5 guarantees the strict monotonicity of m in ω. Assumption 2 guarantees the conditional independence between ω and (s,x_{j^*}). Hence, under our conditions, the function m and the distribution of ω are identified nonparametrically, as long as the joint distribution of (γ,s,x), where $\gamma=m(s,x_{j^*},x_{\widetilde{j}},\omega)$, is identified.

Assumption 10 and 11, together with Assumption 2, guarantee that the distribution of (γ, s, x), where $\gamma = m(s, x_{j^*}, x_{\tilde{j}}, \omega)$, is identified, by using results in Teicher (1961).

Assumptions 8 and 9 guarantee that all the functions $v(j,\cdot)$ can be identified when the distribution of ω, the distribution of $(\varepsilon_1 - \varepsilon_2, ..., \varepsilon_1 - \varepsilon_J)$, and the function $v(j^*,\cdot)$ are identified.

Using the assumptions specified above, we can prove the following theorem:

THEOREM 1: *If Assumptions 1-11, Assumptions 1-5,6',7-11 or Assumptions 1-5,6'',7-11 are satisfied, then* (V^*, F^*, G^*) *is identified in* $(W \times \Gamma_F \times \Gamma_G)$.

This theorem establishes that one can identify the distributions and functions in a discrete choice model with unobserved heterogeneity, making no assumptions about either the parametric structure of the systematic subutilities or the parametric structure of the distributions in the model. The proof of this theorem is presented in the Appendix.

The result of Theorem 1 can be extended to situations with a multidimensional heterogeneity vector, $\omega = (\omega_1, ..., \omega_K)$, by imposing some additional structure. For example, one can show identification of the distribution, G^*, of $\omega = (\omega_1, ..., \omega_K)$, by substituting Assumption 6 by the assumption that the coordinates of $\omega = (\omega_1, ..., \omega_K)$ are independently distributed and $\exists j^* \neq \tilde{j}$ *and, for k=1,…,K,* $(\hat{s}^k, \hat{x}^k_{j^*}) \in (S \times X_{j^*})$, *such that*

$$\forall V \in W \ \forall \omega \ \forall \kappa \ \ v(j^*, \hat{s}^k, \hat{x}^k_{j^*}, \omega) = \omega_\kappa .$$

This latter condition is satisfied when the function $v(j^*, \hat{s}, \hat{x}_{j^*}, \omega)$ is linear in $\hat{s}$ *and* $\hat{x}_{j^*}$, and when $\omega = (\omega_1, ..., \omega_K)$ is the vector of random linear coefficients, as in Ichimura and Thompson (1998), but it is also easily satisfied when the function $v(j^*, \hat{s}, \hat{x}_{j^*}, \omega)$ is additively separable into K nonparametric functions, each depending on only one of the coordinates of ω. By imposing restrictions on the way the $v(j, \cdot, s, x_{j^*}, \omega)$ functions depend on ω, one can use G^* to identify these functions as well as the distribution F^*.

Theorem 1 can also be modified to apply to situations where $(\varepsilon_1, ..., \varepsilon_J)$ and/or ω *are not distributed independently of* $(s, z_1, \ldots, z_J)$, by extending the methods in Matzkin (2004, 2005). (See Matzkin (2006) for a review of other existing methods that deal with endogeneity.)

4. NONPARAMETRIC ESTIMATION

Given N independent observations $\{y^i, s^i, z^i\}_{i=1}^{N}$ we can define the log-likelihood function:

$$L(V,F,G) = \sum_{i=1}^{N} \log \int \left[\Pr(j \mid s^i, z^i; V(\cdot;\omega), \underline{F}) \right]^{y_j^i} dG(\omega)$$

where $F = (F_1,...,F_J)$. We then define our estimators, $\hat{V}$, $\hat{F}$ and $\hat{G}$, for V*, F*, and G*, to be the functions and distributions that maximize L(V,F,G) over triples (V,F,G) that belong to a set $(W \times \Gamma_F \times \Gamma_G)$.

Let d_W, d_F, and d_G denote, respectively, metric functions over the sets W, Γ_F, and Γ_G. Let $d : (W \times \Gamma_F \times \Gamma_G) \times (W \times \Gamma_F \times \Gamma_G) \to R_+$ denote the metric defined by

$$d[(V,F,G),(V',F',G')] = d_W(V,V') + d_F(F,F') + d_G(G,G').$$

Then, the consistency of the estimators can be established under the following assumptions:

ASSUMPTION 12: *The metrics d_W and d_F are such that convergence of a sequence with respect to d_W or d_F implies uniform convergence over compact sets. The metric d_G is such that convergence of a sequence with respect to d_G implies weak convergence.*

ASSUMPTION 13: *The set $(W \times \Gamma_F \times \Gamma_G)$ is compact with respect to the metric d.*

ASSUMPTION 14: *The functions in W and Γ_F are continuous.*

The following theorem is proved in the Appendix:

THEOREM 2: *Under Assumptions 1-14 $(\hat{V},\hat{F},\hat{G})$ is a strongly consistent estimator of (V^*,F^*,G^*) with respect to the metric d. The same conclusion holds if Assumption 6 is replaced by Assumption 6' or 6''.*

In practice, one may want to maximize the log-likelihood function over some set of parametric functions that increases with the number of observations, in such a way that it becomes dense in the set $(W \times \Gamma_F \times \Gamma_G)$ (Elbadwai, Gallant, and Souza (1983), Gallant and Nychka (1987), Gallant and Nychka (1989).) (See X. Chen (2006) for a review.). Let W^N, Γ_F^N, *and* Γ_G^N denote, respectively, such sets of parametric functions, when the number of observations is N. Let $(\widetilde{V}_N, \widetilde{F}_N, \widetilde{G}_N)$ denote a maximizer of the log-likelihood function $L(V,F,G)$ over $(W^N \times \Gamma_F^N \times \Gamma_G^N)$. Then, we can establish the following theorem:

THEOREM 3: *Suppose that Assumptions 1-14 are satisfied, with the additional assumption that the sequence $\{(W^N \times \Gamma_F^N \times \Gamma_G^N)\}_{N=1}^{\infty}$ becomes dense (with respect to d) in $(W \times \Gamma_F \times \Gamma_G)$ as $N \to \infty$. Then, $(\widetilde{V}_N, \widetilde{F}_N, \widetilde{G}_N)$ is a strongly consistent estimator of (V^*,F^*,G^*) with respect to the metric d. The same conclusion holds if Assumption 6 is replaced by Assumption 6' or 6''.*

Note that Theorem 3 holds when only some of the functions are maximized over a parametric set, which becomes dense as the number of observations increases, and the other functions are maximized over the original set.

5. COMPUTATION

In this section, we introduce a change of notation from the previous sections. Let H be the number of households in the data (H ≤ N), and N_h be the number of observed choices for household h=1..h (N_h > 0, and $N = \sum_{h=1,H} N_h$). Therefore, the key difference in this and the following sections from the previous section is that we have repeated observations for the households. The above theorems still apply as long as the assumptions are maintained; including independence of choices (see, e.g., Lindsay (1983), Heckman and Singer (1984), Dayton and McReady (1988)).[2]

Let $(\hat{V}, \hat{F}, \hat{G})$ denote a solution to the optimization problem:

$$\underset{(V,F,G)\in(W\times\Gamma_F\times\Gamma_G)}{Max} L(V,F,G) = \sum_{h=1}^{H} \log \int \prod_{i=1}^{N_h} \prod_{j=1}^{J} \left[\Pr(j \mid s_h^i, z_j^i; V(\cdot,\omega), F)\right]^{y_{h,j}^i} dG(\omega)$$

It is well known that, when the set Γ_G includes discrete distributions, $\hat{G}$ is a discrete distribution with at most H points of support (Lindsay (1983), Heckman and Singer (1984)). Hence, the above optimization problem can be solved by finding a solution over the set of discrete distributions, G, that possess at most H points of support. We will denote the points of support of any such G by $\omega_1,...,\omega_H$, and the corresponding probabilities by $\pi_1,...,\pi_H$. Note that the value of the objective function depends on any function $V(\cdot,\omega)$ only through the values that $V(\bullet,\omega)$ attains at the finite number of observed vectors $\{(s_1^1, x_j^1, r_j)_{j=1,...,J},...,(s_H^{N_H}, x_j^{N_H}, r_j)_{j=1,...,J}\}$. Hence, since at a solution, $\hat{G}$ will possess at most H points of support, we will be considering the values of at most H different functions, $V(\cdot,\omega_c)$ $c = 1,...,H$, i.e., we can consider for each j $(j = 1,\ldots,J)$ at most H different subutilities, $V(j,\cdot;\omega_c)$ $(c = 1,...,H)$. For each i and c, we will denote the value of $V(j, s_h^i, x_j^i, r_j, \omega_c)$ by $V_{j,h,c}^i$. Also, at a solution, the value of the objective function will depend on any F only through the values that F attains at the finite number of values $(V_{j,1,c}^i - V_{1,1,c}^i,...,V_{j,H,c}^i - V_{1,H,c}^i)$ $i = 1,...,N_h, j = 1,.,.,J, h = 1,...,H, c = 1,...,H$. We then let $F_{j,h,c}^i$ denote,

[2] Note that this assumption excludes endogenous (including lagged endogenous variables). We leave it for futre research to determine the identification and consistency conditions for models with endogeneity.

for each j $(j = 1,\ldots,J)$, the value of a distribution function F_j at the vector $(V^i_{j,1,c} - V^i_{1,1,c},\ldots,V^i_{j,H,c} - V^i_{1,H,c})$. It follows that a solution, $(\hat{V},\hat{F},\hat{G})$ for the above maximization problem can be obtained by first solving the following finite dimensional optimization problem, and then interpolating between its solution:

$$\max_{\{V^i_{j,h,c}\},\{\pi_c\},\{F^i_{j,h,c}\}} \sum_{h=1}^{H} \log \sum_{c=1}^{H} \pi_c \prod_{i=1}^{N_h} \prod_{j=1}^{J} \left[F^i_{j,h,c}\right]^{y^i_{h,j}}$$

$$\textit{subject to} \qquad (\{V^i_{j,h,c}\},\{\pi_c\},\{F^i_{j,h,c}\}) \in K$$

where K is a set of a finite number of restrictions on $\{V^i_{j,h,c}\},\{\pi_c\},\{F^i_{j,h,c}\}$. The restrictions characterize the behavior of sequences $\{V^i_{j,h,c}\},\{\pi_c\},\{F^i_{j,h,c}\}$ whose values correspond to functions V in W, probability measures G in Γ_G, and distribution functions F in Γ_F.

To see what is the nature of the restrictions determined by the set K, consider for example a binary choice model where $x_1 \in R_+$, $v(1,s_h,x_1,\omega) = r(s_h) + \omega\, x_1, v(2,s_h,x_2,\omega) = h(x_2,\omega)$, $r(0) = 0$, $h(0,\omega) = 0$ for all ω, $r(\bullet)$ is concave and increasing, and $h(\bullet,\bullet)$ is concave and decreasing. Then, the finite dimensional optimization problem takes the following form:

$$\max_{\{V^i_{j,h,c}\},\{\pi_c\},\{F^i_{h,c}\},\{T^i_h\},\{D^i_{h,c}\}} \sum_{h=1}^{H} \log \sum_{c=1}^{H} \pi_c \prod_{i=1}^{N_h} \left[F^i_{h,c}\right]^{y^i_{1,h}} \left[1 - F^i_{h,c}\right]^{(1-y^i_{1,h})}$$

subject to

$$\begin{aligned}
&(a)\ \ F^i_{h,c} < F^k_{h1,d} \qquad if \quad r^i_h + x^i_{1,h}\ \omega_c - h^i_{h,c} < r^k_{h1} + x^k_{1,h1}\ \omega_d - h^k_{d,h1},\ \pi_c > 0,\ and\ \ \pi_d > 0\\
&\quad\ \ F^i_{h,c} = F^k_{h1,d} \qquad if \quad r^i_h + x^i_{1,h}\ \omega_c - h^i_{h,c} = r^k_{h1} + x^k_{1,h1}\ \omega_d - h^k_{h1,d},\ \pi_c > 0,\ and\ \ \pi_d > 0\\
&(b)\ \ 0 \le F^i_{h,c} \le 1,\\
&(c)\ \ r^i_h \le r^k_{h1} + T^k_{h1}\cdot(s^i_h - s^k_{h1})\\
&(d)\ \ h^i_{h,c} \le h^k_{h1,c} + D^k_{h1,c}\cdot((x^i_{h,2},\omega_c) - (x^k_{h1,2},\omega_c)) \qquad if\ \pi_c > 0\\
&(e)\ \ T^k_{h1} \ge 0,\quad r^{N+1} = 0,\quad s^{N+1} = 0,\\
&(f)\ \ D^k_{h,c} \le 0,\quad h^{N+1}_{h,c} = 0,\ and\ \ x^{N+1}_{h,2} = 0 \qquad if\ \pi_c > 0
\end{aligned}$$

for $i \in \{1,\ldots,N_h,N+1\};k \in \{1,\ldots N_j,N+1\};\ c,d,h,h1 = 1,\ldots,H.$

Constraints (a) and (b) guarantee that the $F^i_{h,c}$ values are those of an increasing function whose values are between 0 and 1. Constraint (c) guarantees that the r^i_h values correspond to those of a concave function. Constrains (d) guarantees that the $h^i_{h,c}$ values correspond to those of a concave function, as well. Constraints (e) and (f) guarantee that the r^i_h and the $h^i_{h,c}$ values correspond, respectively, to those of a monotone increasing and a monotone decreasing function, and that the r_h^i and the $h_{h,c}^i$ values correspond to functions satisfying $r(0) = 0$ and $h(0,\omega) = 0$ for all ω. Some additional constraints would typically be needed to guarantee the compactness of the sets W, Γ_F, and Γ_G and the continuity of the distributions in Γ_F.

A solution to the original problem is obtained by interpolating the optimal values obtained from this optimization (see Matzkin (1992, 1993, 1994, 1999)) for more discussion of a similar optimization problem).

To describe how to obtain a solution to this maximization problem, we let

$$\widetilde{L}[(r_1^1,...r_1^{N_1},...,r_H^{N_H},r^{N+1}),(T_1^1,...,T_1^{N_1}...,T_H^{N_H},T^{N+1}),(h_{1c}^1,...,h_{1c}^{N_1}...,h_{Hc}^{N_H},h_c^{N+1})(D_{1c}^1,...,D_{1c}^{N_1}...,D_{Hc}^{N_H},D_c^{N+1}),(\pi_1,...,\pi_H)]$$

denote the optimal value of the following maximization problem:

$$\text{(1)} \qquad \max_{\{V_{j,h,c}^i\},\{\pi_c\},\{F_{h,c}^i\},\{T_h^i\},\{D_{h,c}^i\}} \sum_{h=1}^{H} \log \sum_{c=1}^{H} \pi_c \prod_{i=1}^{N_h} \left[F_{h,c}^i\right]^{y_{1,h}^i} \left[1-F_{h,c}^i\right]^{(1-y_{1,h}^i)}$$

subject to

$$(a) \quad F_{h,c}^i < F_{h1,d}^k \qquad if \quad r_h^i + x_{h,1}^i\ \omega_c - h_{h,c}^i < r_{h1}^k + x_{h1,1}^k\ \omega_d - h_{h1,d}^k,\ \pi_c > 0,\ and\ \ \pi_d > 0$$

$$F_{h,c}^i = F_{h1,d}^k \qquad if \quad r_h^i + x_{h,1}^i\ \omega_c - h_{h,c}^i = r_{h1}^k + x_{h1,1}^k\ \omega_d - h_{h1,d}^k,\ \pi_c > 0,\ and\ \ \pi_d > 0$$

$$(b) \quad 0 \le F_{h,c}^i \le 1\,.$$

A solution to this latter problem can be obtained by using a random search over vectors $(F_{1,c}^1,...,F_{H,c}^{N_H})_{c=1,...,H}$ that satisfy the monotonicity constraint (a) and the boundary constraint (b).

Then, a solution to the full optimization problem can be obtained by using a random search over vectors $(r_1^1,...r_1^{N_1},...,r_H^{N_H},r^{N+1})$, $(h_{1c}^1,...,h_{1c}^{N_1}...,h_{Hc}^{N_H},h_c^{N+1})$, and $(\pi_1,...,\pi_H)$ that satisfy, respectively, constraints (c) and (e), constraints (d) and (f), and the following constraints:

$$\pi_j \ge 0 \quad (j = 1,...,H) \ \text{ and } \ \textstyle\sum_{j=1}^{H} \pi_j = 1.$$

Instead of estimating the distribution function F using (a) and (b), one could add alternative-specific random intercepts to the model and assume that ε has a known, parametric distribution. When the specified distribution for ε is smooth, this has the effect of smoothing the likelihood function.

In some cases, one may want to control for variables that may affect the distribution of heterogeneity by allowing them to enter the distribution of the heterogeneity variable, rather than the utility functions. Assume that the support of consumer heterogeneity (ω) is SC. One may specify the probabilities for each support point, π_c, c=1..SC, as known parametric functions of the demographic variables, *s*. For instance the function used by Kamakura and Russell (1989) is

$$\pi_c = \exp(s\theta_c) \Big/ \left[\sum_{i=1}^{SC} \exp(s\theta_i) \right], \text{ with } \theta_1 = 0 \text{ for identification}$$

Let $\Theta = \{\beta_1,...,\beta_{SC},\theta_1,...,\theta_{SC}\}$ be the set of parametric parameters, where $\{\beta_1,...,\beta_{SC}\}$ are the parameters of the utility function; let $\omega = \{\omega_1,...,\omega_{SC}\}$ be the vector of unobserved heterogeneity, and $H = \{h_1^1,...,h_{N+1}^1,...h_{N+1}^{SC}\}$ be the set of values for the non-parametric function. The computational problem then becomes estimating (Θ,ω,H) efficiently using the likelihood function described in equation (1). Clearly, a random search over the entire parameter space is infeasible as the parametric parameters are unconstrained and the heterogeneity parameters are only constrained to be positive (from Assumption 9). Therefore, we adapted the algorithm for concave functions developed in Matzkin (1999) and later used by Briesch, Chintagunta and Matzkin (2002) to monotone functions of the form described above.[3] This is a random search algorithm combined with maximum likelihood estimation.

The basic idea of the random search algorithm is the following: Suppose that one has to calculate the value of the H vector that maximizes a function L(V) subject to a set of linear inequalities AH ≤ 0, where A is a M x (N+1) matrix, and M is very large. Then, using the fact that the constraint set is convex (because the constraints are linear) and relatively small (because the number of restrictions is large), one may find a solution using a random search method. Each time a point $H^{(i)}$, in the constraint set is drawn, one calculates the objective function at that point and the points along the segment connecting $H^{(i)}$ with the point, H^*, that up to this stage, was found to achieve the highest value of the function L. If a point along this segment achieves a value larger than H^*, then that point takes the place of H^*, and a new random point is drawn continuing the procedure. The search terminates when a pre-specified, large number of consecutive number of draws results in no improvement. When the matrix M is sparse, as in the case where the matrix A is determined by monotonicity restrictions, one can speed up the process. Given a point in the constraint set, one can easily find the upper and lower bounds of this segment that is included in the constraint set and is parallel to the axis. This is obtained by fixing the values of all of the coordinates but one, and using the constraints where that coordinate appears.

[3] Fortran code is available from the authors upon request.

In terms of evaluating the likelihood function; note that the parametric parameters are optimized for each random search point and that the non-parametric portion of the utility function is fixed during the optimization. Therefore, maximum likelihood can be used to get the “optimal” parametric parameters.

6. EMPIRICAL APPLICATION

Our empirical application deals with the issue of store format choice. Specifically, we are interested in answering the research question of how consumers select between an Every Day Low Price (EDLP) format retailer (e.g., Wal-Mart) and a HiLo format (e.g., Kroger or many grocery stores) retailer.[4] The marketing literature has modeled store choice as a function of planned purchases. That is, consumers look in their household inventory, construct a list (written or mental) of needed items and quantities of these items, then determine which store to visit based upon the cost of refilling their inventory and the distance to the store, and potentially, interactions among them (Huff 1962, Bucklin 1971, Bell and Lattin 1998, Bell, Ho and Tang 1998).[5]

However, the nature of this relationship is unknown as many different functional forms have been used in the literature. For instance, distance enters the indirect utility function non-linearly (i.e., natural logarithm) in Smith (2004), whereas it enters linearly in Bell and Lattin (1998). Further, Rhee and Bell (2002) find that the cost of a format is not significant in the consumer’s decision to switch stores, so they conclude that the “price image” drives store choice, not contemporaneous cost to refill the inventory. This finding is at odds with Bell and Lattin (1998) and Bell, Ho and Tang (1998). Additionally, it is likely that the influence of these variables is potentially heterogeneous across consumers. Our objective is to apply our proposed method to shed some light on the nature of tradeoffs being made by consumers between distance to the store and the price paid for the shopping basket. To do this, we apply a semi-parametric version of the method described above (details on the specific restrictions placed follow) that allows us to (a) recover the appropriate functional form for the effects of inventory replenishing cost and distance on format choice; and (b) account for heterogeneity in this response function across consumers.

We model the utility of a format as a tradeoff between the consumer’s cost of refilling their inventory at a format and the distance of the format from the consumer as in equation (2)

[4] These formats are self-declared by the retailers, but an EDLP retailer generally has lower price variance over time than a HiLo retailer (Tang, et al 2001).

[5] We note that consumers may make planned purchases or unplanned purchases for other reasons than inventory levels. For instance, a consumer may be in a store and see a good price for an item in a category and make a purchase. This type of purchase is called an “impulse” purchase. These other purchase decisions are left for future research.

(2) $$V_{h,f}^{t} = h(S_{h,f}^{t}, D_{h,f}, \omega) + X_{h,f}^{t}\beta_{h} + \varepsilon_{h,f}^{t}$$

where: $V_{h,f}^{t}$ is the value of format *f* for household *h* in period (or shopping occasion) *t*, $S_{h,f}^{t}$ is the cost of refilling household *h*'s inventory at format *f* in period *t*, $D_{h,f}$ is the distance (in minutes) from household h to format f, h(.) is a monotonically decreasing function of its parameters, $X_{h,f}^{t}$ are other variables that need to be controlled for, ω is the unobserved heterogeneity and $\varepsilon_{h,f}^{t}$ is the error term. Note that the matrix of variables in $X_{h,f}^{t}$ can be used to provide identification of the non-parametric function h (S, D,ω), if required. Because each format has multiple stores, aggregation of the variables is discussed in more detail below.

In the gravity model literature, attractiveness of the price format incorporates service quality, parking, assortment, inherent preference and other fixed attributes of the price format (e.g., Bucklin 1971, Tang, et al 2001). These components are incorporated into a household-specific intercept term for the price format. In addition to the format-specific intercept, key consumer demographic variables are also included. Finally, several researchers (e.g., Bell and Lattin, 1998; Bell, Ho, and Tang, 1998) have found that consumers prefer to go to EDLP stores when the expected basket size is large or as time between shopping trips increases (e.g., Leszczyc, et al., 2000). Therefore, we can rewrite the utility of each format as in equation (3),

(3) $$V_{h,f}^{t} = \beta_{0f} + \beta_{1} EDLP * TS_{h}^{t} + \beta_{2} E_{h} + \beta_{3} HS_{h} + \beta_{4} I_{h} + \beta_{5} CE_{h} + \beta_{6} L_{h,f} + h(S_{h,f}^{t}, D_{h,f}, \omega) + \varepsilon_{h,f}^{t}$$

where: EDLP is a binary variable set to one when the format is EDLP and zero otherwise, TS_{h}^{t} is the elapsed time (in days) from the previous shopping trip to the current shopping trip, E_h is a binary indicator set to one if the household is classified as "Elderly" (head of household is at least 65 years old), HS_h is the size of the household, I_h is the household income, CE_h is a binary indicator of whether the head of household is college educated, and L_{hf} is a format-specific loyalty term.[6] We use the same measure of loyalty as Bell, Ho and Tang (1998) after adjusting for using store formats instead of stores:

$$L_{h,EDLP} = \left(NV_{h,EDLP}^{i} + 0.5\right) / \left(NV_{h,EDLP}^{i} + NV_{h,HiLo}^{i} + 1\right),$$
$$L_{h,HiLo} = 1 - L_{h,EDLP}$$

Where $NV_{h,EDLP}^{i}$ is the number of visits to EDLP stores by household *h* during an initialization period denoted by *i*, and $NV_{h,HiLo}^{i}$ is the number of visits to HiLo stores by household *h* during the same

[6] Our objective is to account fully for all observable sources of heterogeneity so any unobserved heterogeneity estimated from the data is to the extent possible, unobservable.

initialization period, *i*. The data from the initialization period temporally precede the data that we use for the estimation for each household included in our sample. In this way we are not directly using any information on our dependent variable as a model predictor.

We make the assumption that the error term, $\varepsilon_{h,f}^{t}$, has an extreme value distribution. We use a discrete model of consumer heterogeneity, where there are SC segments of consumers (or points of support). The parameter vector is allowed to be segment specific, so the utility function in equation (3) can be rewritten as equation (4):

$$(4)\, V_{h,f,s}^{t} = \beta_{0f,s} + \beta_{1,s} EDLP * TS_{h}^{t} + \beta_{2,s} E_{h} + \beta_{3,s} HS_{h} + \beta_{4,s} I_{h} + \beta_{5,s} CE_{h} + \beta_{6,s} L_{h,f} + h(S_{h,f}^{t}, D_{h,f}, \omega_{s}) + \varepsilon_{h,f,s}^{t}$$

where ω_s is the unobserved heterogeneity in consumer response to cost and distance. To guarantee that Assumption 3 is satisfied, we impose the restriction that for all segments s, $\beta_{1,s} = -0.02$.[7] To guarantee that Assumptions 4, 5, and 6'' are satisfied, we impose the restrictions that

$$h(S_{h,f}^{t}, D_{h,f}, \omega_{s}) = m(S_{h,f}^{t} \omega_{s}, D_{hf})$$

$$m(0, D_{N+1}) = \alpha \text{, and}$$

$$m(1, D_{N+1}) = \gamma \text{,}$$

with N being the number of observations in the data, D_{N+1}, α and γ known. Assumption 4 is then satisfied when $D = D_{N+1}$ and $S = S_{N+1} = 0$. We measure the variable S in differences from its average. To guarantee that Assumptions 7 and 8 are satisfied, we restrict the function m(.) to be decreasing in each coordinate.

We define the probabilities for the support points for each household, $\pi_{h,s}$, such that $\sum_{s=1}^{SC} \pi_{h,s} = 1, \forall h$. Following the literature on discrete segments (see Dayton and McReady 1988), we write the mass points as

$$\pi_{h,c} = \exp(\lambda_{c}) \Big/ \left[\sum_{i=1}^{SC} \exp(\lambda_{i}) \right]$$

with λ_1 set to zero for identification.

We next describe how the computational algorithm presented in the previous section can be modified to leverage our specific form of the nonparametric function. For expository ease, we drop the

[7] This value was determined from the single segment parametric model.

household subscript from the predictor variables and treat them as if they are independent observations. Note that because there are 2 alternatives, we have 2*N +2 pairs of cost and distance.[8] If a continuous, differentiable and constraint maintaining (i.e., does not violate any of the AH $\leq$ 0 constraints) approximation of $m(\omega S^i, D^i)$ for all ω>0, is used, then the dimensionality of the random search (or the size of the vector H) can be reduced from 2*(N+1)*SC to 2*(N+1) and maximum likelihood can be used to estimate ω as well as Θ.

A natural selection for this interpolation would be a multidimensional kernel, i.e.,

$$h(S^j, D^j, \omega_c) = \sum_{i=1}^{2N+2} w(i,j,\omega_c) h(S^i, D^i, 1) \Big/ \sum_{i=1}^{2N+2} w(i,j,\omega_c) \tag{5}$$

where $w(i,j,\omega_c)$ is the weight placed on observation *i*, and is inversely proportional to the Euclidian distance from $\{S^j \omega_c, D^j\}$ to $\{S^i, D^i\}$, defined as $E_{i,j,c} \equiv \left| \{\omega_c S^j, D^j\} - \{S^i, D^i\} \right|$. For instance, for a normal kernel, $w(i,j,\omega_c)$ is defined as

$$w(i,j,\omega_c) = \exp\left(-E_{i,j,c}{}^2 / 2h\right) \tag{6}$$

where h is the bandwidth parameter. We note that this estimate is a consistent estimate as h→0. The problem with using a multi-dimensional kernel is that is does not preserve the shape restrictions, and a stylized proof and example using two dimensions is provided below. Therefore, a single dimensional kernel is used to interpolate the function, where the function is only smoothed over the $S^j \omega_c$ values. Specifically, the weights in equation (6) are defined as:

$$w(i,j,\omega_c) = \exp\left(-E_{c,i,j}{}^2 / 2h\right) \text{ iff } D^i = D^j \text{, and 0 otherwise}$$

First, for the example and stylized proof, we adopt a two-dimensional space, like the empirical application. Next, we define three operators for a two-dimensional space: less than (≤), greater than (≥), and equivalence (≈).

$$(S^i, D^i) \leq (S^j, D^j) \Leftrightarrow (S^i \leq S^j).and.(D^i \leq D^j)$$
$$(S^i, D^i) \geq (S^j, D^j) \Leftrightarrow (S^i \geq S^j).and.(D^i \geq D^j)$$
$$(S^i, D^i) \approx (S^j, D^j) \Leftrightarrow .not.((S^i, D^i) \leq (S^j, D^j).or.(S^i, D^i) \geq (S^j, D^j))$$

From a monotonic function standpoint, equivalence of two points implies that neither one places constraints on the values of the other. For instance, let $\{S^i, D^i\}$=(0,1) and $\{S^j, D^j\}$=(1,0) then

[8] We note that we are assuming that the pairs are all unique. If there are repeated pairs, then we use the subset of unique pairs. Additionally, there are two constraints.

$\{S^i, D^i\} \approx \{S^j, D^j\}$. Now, what we want to show is that a continuous and differentiable kernel estimator can not be used if it has two dimensions. Define h(.) to be a monotonically decreasing function, $x^i \equiv (s,d)$, $x^j \equiv \{s+\partial_1, d-\partial_2)$, $\delta_1 > 0, \delta_2 > 0$, $h(x^i) = v^i$, and $h(x^j) = v^j$ with $v^i < v^j$. n.b. Because $x^i \approx x^j$, no constraints exists between v^i and v^j and these values could be switched while still maintaining h(.) as a monotonically decreasing function.

Now define $x^1 = (s+\partial_1/2, d-\partial_2/2)$, and $x^2 = (s+\partial_1/2+\phi, d-\partial_2/2)$ with $\delta_1/2 \geq \phi > 0$. These definitions imply $x^1 \leq x^2$, x^1 is closer to x^i than x^2 ($E_{2,i,1} > E_{1,i,1}$), and x^2 is closer to x^j than x^1 ($E_{2,j,1} < E_{1,j,1}$). Therefore, $h(x^1) < h(x^2)$ which violates the shape restrictions that h() is monotonically decreasing. A numerical example is let $v^i=1$, $v^j=2$, $\phi=.1$, s=0, d=1, $\delta_1=\delta_2=1$, and the normal kernel bandwidth set to 0.1. Then $x^1=(0.5,0.5)$ and $x^2=(0.6,0.5)$ and $h(x^1)=1.5$, and $h(x^2)=1.73$. A unidimensional kernel avoids this problem because, by definition, there are no equivalent points.

The final issue to address is identification of the ω's. Clearly, the segments are not uniquely defined as segments can be renumbered (which switches all of the coefficients between segments) while still maintaining the same likelihood function. Additionally, we note while all of the ω's are identified, the estimation is computationally inefficient as we estimate the base function for $\omega_0=1$, then interpolate for all SC segments. The computational efficiency can be improved by estimating ω\` instead of ω, where $\omega' \equiv \omega/\omega_i$ for some non-zero ω_i. This constraint then implies we can set $\omega_1=1$, so SC-1 segment values are estimated. The assumption that at least one ω_i, i=1..SC, is non-zero is not strong, as it implies that at least one segment of consumers respond to the parameter.

6.1 Data

We use a multi-outlet panel dataset from Charlotte, North Carolina that covers a 104-week period between September 2002 and September 2004. This panel dataset is different from panel data commonly used by marketing researchers because panelists recorded all packaged and non-packaged goods purchases by UPC using in-home scanning equipment. Thus, purchase records are not limited to a small sample of grocery stores; rather purchases made in all grocery and non-grocery stores are captured. This is important since packaged goods purchases are frequently made outside of grocery stores, for example at mass merchandisers.

Households were included in the sample we use for estimation if at least 95 percent of their grocery and mass purchases were at the 7 stores (five supermarket, two mass merchandisers) for which we have geolocation data, if they recorded at least one purchase (of at least $5) per month, and if they spent at least $20 per month in the panel. The last two criteria were used to ensure that the panelist was faithful in recording its purchases and remained in the panel for the entire 104 week period. On average, the panelists made 2.3% of their trips and 1.2% of their spending at non-focal grocery and mass merchandisers.

The resulting data set had 161 families with a total of 26,540 shopping trips. The first 25% of the weeks were used as our "initialization" period to compute the various weights and other quantities described below. After constructing the inter-temporal variables used in the model, the final 26 weeks were used as a "hold out" sample and remaining weeks were were used as the estimation sample The resulting estimation sample had 13,857 shopping trips while the holdout sample had 6,573 shopping trips. On average, each household made a shopping trip every 4.6 days. Descriptive statistics for the households are provided in Table 1.

< Put Table 1 about here>

Consistent with the extant literature, (see, e.g., Bell, Ho and Tang, 1998) we identified retailers as falling into either the EDLP or HiLo category based on their advertised pricing strategy. This resulted in three EDLP retailers and four HiLo retailers. The EDLP retailers had 58% of the shopping trips; while the HiLo retailers had 42% of the shopping trips. To determine distance between a panelist and a store, we use the travel time (in minutes) from a panelist's zip+4 to the store's location.[9]

We have detailed price information for 289 categories, of these categories we selected 150 categories based upon the following criteria. First, three common UPCs had to be carried by each retailer so that category price indices could be computed. Second, at least 5% of the selected households had to make at least three purchases in the category to ensure that the category is substantial. Third, the category had to account for at least 0.9% of total basket spending of the selected households. These categories together comprise more than 88% of the market basket on average (excluding fresh meat, fruit and vegetables). So, we use these categories to estimate the cost at each format. The descriptive statistics for the categories are provided in Table 2.

Table 3 shows specific statistics for the price formats with the standard deviations in parenthesis. Note that the description of cost to refill inventory is provided in the next section.

< Put Tables 2 and 3 about here>

[9] Note that for privacy reasons, the panelists actual street address is not included in the data. We thank ESRI for providing this data. The travel time is the shortest average travel time (i.e., time of day is not taken into account) across all routes.

6.1 Data Aggregation

Because our focus is on consumers selecting a type of store (EDLP vs. Hi-Lo, this is also called "store format") rather than selecting a specific store, we need to aggregate our store-level data to the format level. This aggregation is done based on the proportion of a household's visits to each store during a time period prior to that used in our subsequent analysis, i.e., the initialization period defined previously. Specifically, let F_E be the set of stores with an EDLP price format, F_H be the set of stores using a HiLo format and D_{hs} be the distance from household *h*'s home to store s.[10] The distance from household *h* to format *f* can be defined as

(7) $$D_{h,f} = \sum_{s \in F_f} w_{h,f,s} D_{h,s}$$

where $w_{h,f,s}$ is the proportion of visits to store *s* made by household *h* in the initialization period, and F_f is either F_E or F_H.

Since the cost to replenish the inventory (also called "cost") by a household, $S^t_{h,f}$ on a shopping occasion is not observed prior to the visit, we need to create a measure of cost for each trip. The general goal is to create a time-varying, household-specific index for each store. This index is then aggregated to the format level similar to distance in equation (7). Clearly, the cost at store *s* in period *t* by household *h*, denoted $S^t_{h,s}$, is the sum of the cost to the household in each category, c=1..C the household will purchase in period *t* (Bell, Ho and Tang 1999). Therefore, cost can be written as

(8) $$E[S^t_{h,s}] = \sum_{c=1}^{C} E[p^t_{h,s,c}] E[q^t_{h,c}]$$

where $E[p^t_{h,s,c}]$ is the expected price of category *c* at store *s* in period *t* for household *h*, and $E[q^t_{h,c}]$ is the quantity household *h* needs to purchase in category *c* in period *t* to replenish its inventory. For price, we construct "market average" price indicies for the EDLP and HiLo retailers based upon the retailers' long-run share of visits.[11] The second component on the right hand side of equation (8) is quantity that needs to be replenished, $E[q^t_{h,c}]$. Once again, since we do not observe household inventories, we need a mechanism to predict this quantity using data that we, as researchers, observe – quantities purchased on previous occasions in each of the categories. We use Tobit models (that account for household

[10] Due to privacy reasons, we have zip+5 values for both the households and the stores, but not the actual physical address of the household. Therefore, the distance is calculated using the centroid of the household's zip code to the location of the closest store in the chain.

[11] Ainsile and Rossi (1998) argue that long-run market share is not endogenous. Further several papers, e.g., Nevo, 2001, use average market prices as instruments, implicitly assuming that the average price in the market is independent of the error term.

heterogeneity) to predict each household's expected purchase quantity.[12] Using the arguments found in Nevo and Hendel (2002) and elsewhere, we define ($E[q^t_{h,c}]$) to depend upon previous quantity purchased, the amount of time since last purchase, and the interaction between these terms as in equation (9)

$$(9)\ E\left[q^t_{h,c}\right]/\bar{q}_{h,c} = \beta_{0,h} + \beta_{,1h}\left(q^{t-1}_{h,c}/\bar{q}_{h,c}\right) + \beta_{2,h}\left(d^t_{h,c}/\bar{d}_{hc}\right) + \beta_{3h}\left(\left(q^{t-1}_{h,c}/\bar{q}_{hc}\right)-1\right)\left(\left(d^t_{h,c}/\bar{d}_{hc}\right)-1\right) + \xi^t_{h,c}$$

Where $q^t_{h,c}$ is the quantity purchased in category *c* in period *t* by household *h*, $\bar{q}_{h,c}$ is the average quantity of category c purchased by household *h* conditional on purchase in the category, $d^t_{h,c}$ is the number of days since the category has been purchased, $\bar{d}_{hc}$ is the average number of days between purchases in category *c* by household *h*, and $\xi^t_{h,c}$ is an error term which has normal distribution and is independent between categories. In the interaction term we subtract one (which is the mean of both $q^{t-1}_{h,c}/\bar{q}_{hc}$ and $d^t_{h,c}/\bar{d}_{hc}$) from the quantity and time terms to allow clear definition of β_{1h} and β_{2h}: β_{1h} is the household response to quantity when the time between purchases is at the mean value, and β_{2h} represents household response to time when the average quantity was purchased on the prior occasion. Without this normalization, interpretation of the coefficients would be less clear. The interaction term then allows the household's consumption rate to be non-constant, i.e., depend upon both the quantity purchased on the prior occasion as well as the time since the last occasion.

The household coefficients in equation (9) are assumed to have a normal distribution with mean β and standard deviation of Σ, where Σ is a diagonal matrix. The distribution of tobit coefficients is shown below, where the expected sign for lag Quantity is negative, for time is positive, and for time*lag quantity is positive.

Distribution of Tobit Coefficients

	Positive		Negative	
	$p<0.05$	$p<0.10$	$p<0.05$	$p<0.10$
Lag Quantity (β_1)	3	4	71	77
Time (β_2)	95	99	13	13
Time*Lag Quantity (β_3)	46	52	0	2

At the 5% level, three categories (2%) had positive and significant coefficients for lag quantity, 71 categories (47%) had negative and significant coefficients for lag quantity, 95 (63%) categories had positive and significant coefficients for time since last category purchase, 13 (9%) had negative and

[12] Results of these estimations are available from the authors upon request.

significant coefficients for time since last category purchase, 46 (31%) has positive and significant coefficients for the interaction, and 0(0%) has negative and significant coefficients for the interaction. These results provide face validity for the models where the coefficients with incorrect signs are, on aggregate, approximately 5% (16 of 450), which would be expected by chance. Similar results are obtained at the 10% significance level. The significant value for the interaction of time and lag quantity implies that for these categories, the hypothesis of constant consumption is rejected.

The results of the Tobit models are then used to predict the expected quantity required by each household in each period. We then follow the same aggregation scheme from above to obtain the cost of visiting a specific format. Given the complexity and computational intensity of this method, we leave it for future research to determine methods that allow computationally feasible simultaneous estimation of quantity and store choice. However, we note that even in a parametric case, the problem of simultaneously estimating 150 category equations and one format choice equation is formidable.[13]

6.3 Results

In addition to the semiparametric model identified above, we estimated a parametric model to allow a comparison of the relative model fits. For the parametric model, the function h(.) is defined as a simple linear model:

$$h(S_{hft}, D_{hf}, \omega_s) = \gamma_1^s S_{hft} + \gamma_2^s D_{hf}$$

6.3.1 Model Selection

There are several well-known Information Criteria used in the selection of parametric models, AIC and Schwarz Criteria. One limitation of these information criteria is that they include degrees of freedom as part of the calculations, and it is unclear how to calculate the degrees of freedom for a semi-parametric model with constrained shape. Spiegelhalter, et al (2002) proposes the Deviance Information Criteria (DIC) which calculates a measure of effective degrees of freedom that is bounded for non-parametric models.

Table 4 provides the Maximum Likelihood results, Information Criteria for the estimation and hold-out samples for one to four discrete segments.[14] We calculate hits by assigning household to segments using the posterior probability of segment membership (see Kamakura and Russell 1989) then

[13] We account for the estimation error in the standard errors by estimating the tobit models for each bootstrap simulation.
[14] To get the parameter estimates, we use a combined estimation strategy to help minimize the change of finding a local minimum. We first use a Simplex algorithm (with starting coefficients of zero) to get close to the optimal solution. Once the simplex finishes, we switch to a quasi-Newton algorithm to get the final parameter estimates. This strategy is recommended by Nevo (2002).

calculating the correct number of predicted choices using only that segment's parameters to which the household is assigned. We use 25 bootstrap simulations to calculate the standard deviations of the model fit statistics across all models in both the parametric and semi-parametric estimation.

<Put Table 4 about here>

The Schwarz Criteria and the out-of-sample performance (log-likelihood and hit rate) indicate that the three segment model is the "best" parametric model. Interestingly, the DIC suggests that the four segment model is superior. However, we use the more conservative Swartz Criteria to avoid over-parameterization of the model. The effective degrees of freedom (from the DIC) is calculated as the difference between the likelihood of the estimation and the mean likelihood of the bootstrap simulations, and is related to the variability of the results.

Given a three-segment parametric solution, we then estimated one through three segment solutions of the semi-parametric model. The results of the estimation are provided in table 5, with the DIC indicating that the three segment model is superior. It is interesting to note that the improvement in the likelihood for using the one-segment semiparametric model versus the one-segment parametric model is similar to the improvement in the two-segment parametric model versus the one segment parametric model. Therefore, replacing a linear function with a monotone function has a similar impact on the likelihood as adding heterogeneity, and this relationship remains as more discrete segments are added. This result is consistent with the findings in Briesch, Chintagunta and Matzkin (2002).

<Put Table 5 about here>

6.3.2 Model Results

In this section, we examine why the semi-parametric model fits the data (and predicts out of sample better) than the parametric model. We first provide the coefficient estimates from the parametric and semi-parametric estimation to highlight differences. Next we examine the estimated response to cost and distance by the parametric and semi-parametric models, including heterogeneity distribution. Finally, we examine the demographic profiles of the segments.

Table 6 provides the MLE parameter estimates from the three segment models for the parametric and semi-parametric estimations, with the standard errors reported in parenthesis. The segments were matched based upon the size of the mass coefficient (which roughly translates to the number of households in the segment). Within Table 6, the significant coefficients, at $p < 0.05$, are in bold font.

<Put Table 6 about here>

We note that this matching is somewhat arbitrary as other criteria could be used, e.g. minimize the distance of the significant coefficients. Some of the key differences are:

1. The pattern of significance and signs for the demographics are very different between the parametric and semi-parametric models. In the parametric model, Household Size is significant and positive for all segments, while it is significant (and negative) for only one segment in the semi-parametric model. Conversely, college educated head of household is significant for all of the semi-parametric segments, while it is significant for only one of the parametric segments. This pattern of the demographics affecting the utility function is consistent for most of the demographic variables.
2. Loyalty is similar for two of three segments between the parametric and semi-parametric models. However, the parametric model suggests that for one parametric segment (number three), the households are variety seeking. Specifically, the coefficient is negative and significant, indicating that the household's proportion of visits in the initialization period is negatively correlated with the behavior in the estimation sample.
3. Finally, we have face validity for all of the distance and cost coefficients for the parametric model, where the coefficients are all negative and significant at $p<0.05$ (one-tailed test).

Now, we turn our attention to heterogeneity distribution of cost and distance sensitivities. Table 7 provides the elasticity estimates for both parametric and semi-parametric models as well as the percent of households assigned to each segment. First, we note that the heterogeneity distribution is somewhat similar between the methods in an ordinal manner, i.e., most of the families are in the moderate sensitivity segment (number 2), followed by the highest cost sensitive segment (one) then the low cost sensitivity segment (three). However, there is some bias in the parametric estimates, as the semi-parametric distribution is skewed more towards the higher cost sensitivity than the parametric results.

The differences in the elasticities are striking. We see large biases for both the cost and distance elasticities across all three segments. For both cost and distance elasticities, two of three parametric estimates are at least twice the semi-parametric estimate. In the third segment, the semi-parametric estimate is more than twice the parametric estimate.

These differences suggest that there are strong differences in the response surfaces. To examine this implication in more detail, the estimated response surfaces for the variables of interest (cost and distance) are shown in Figures 1-6. Bootstrap simulations are used to get the semiparametric confidence intervals. The response surface for distance is plotted holding cost fixed at the mean and +/- one standard deviation. Similarly, the response surface for cost is plotted holding distance fixed at the mean and +/- one standard deviation.

If we examine consumer response to distance (figures 1-3), we see that the semi-parametric function is convex and decreasing, where it is almost linear at large costs. This finding implies an interaction with the two variables would be required in a parametric representation of the model. We note that there are large biases in the parametric function, with much stronger slopes and different intercept.

<put figures 1-3 about here>

If we examine consumer response to cost (figures 4-6), we find bias in the parametric response surface as well. In the semi-parametric case, we see a large interaction with distance. At large distances, the response surface is almost flat. However, there are significant non-linearities as the distance decreases. The finding is consistent with the "tipping point" argument made in Bell, Ho and Tang (1998), although these results are much stronger.

<put figures 4-6 about here>

Finally, we examine the demographic profiles of the segments which are provided in table 8. The significant differences (at $p < 0.05$) between the parametric and semi-parametric models are in bolder font in the table. There are three significant differences between the parametric and semi-parametric results: percent of trips to EDLP retailer for segment one, average distance to selected formats in segments one and two and college education of segment one. Otherwise the profiles are very similar. Note that the average expected spending is similar for the segments but that the number of trips (and hence total cost) are different between the segments, similar to Bell, Ho and Tang (1998).

<Put Table 7 about here>

In summary, our key findings are:

1) There are significant differences between the parametric and non-parametric response surfaces, where the semi-parametric response to distance appears concave and the response to cost appears non-linear. Additionally, there exists significant interactions between cost and distance that change the shapes at different levels. This difference also suggests possible biases in the estimated cost and distance elasticities when using a parametric specification.

2) The parametric model provides a significantly different estimate of the heterogeneity distribution than the semiparametric model, with the parametric model classifying

fewer households with high cost elasticity, and more households with low cost elasticity.

3) The improvement in likelihood that stems from adding semi-parametric functions is similar to the improvement in likelihood from adding heterogeneity to parametric functions. This increase stays relatively constant as the number of discrete segments increase.

4) There was very little difference in the demographic profiles of the households assigned to each segment.

7. Conclusions

We have presented a method to estimate discrete choice models in the presence of unobserved heterogeneity. The method imposes weak assumptions on the systematic subutility functions and on the distributions of the unobservable random vectors and the heterogeneity parameter. The estimators are computationally feasible and strongly consistent. We described how the estimator can be used to estimate a model of format choice. Key insights from the application include:

1) the parametric model provides biased estimates of the heterogeneity distribution,
2) the semi-parametric suggests interactions between cost and distance that change the shape of the response function,
3) the benefit to adding semi-parametric estimation is roughly equal in magnitude to adding heterogeneity to parametric models. This benefit remains as the number of discrete segments increase.

One drawback of the method is the computational time. The amount of time required to estimate the model is proportional to how good of a starting point is used. We used the one-segment semi-parametric solution as a starting point for the three segment solution, and it took approximately two weeks for the first simulation to complete on a 1 GHz personal computer (the bootstrap simulations took a much shorter amount of time as they used the three-segment solution as a starting point). As computing power becomes cheaper, this should be less of a problem.

Some variations to the model presented in the above sections are possible. For example, instead of letting $V(j,s,x_j,r_j,\omega) = v(j,s,x_j,\omega) - r_j$, where $r_j \in R_+$, one can let r_j be an L dimensional vector, and specify $V(j,s,x_j,r_j,\omega) = v(j,s,x_j,\omega) - \beta \cdot r_j$. Assuming that one coordinate of β equals one, it is possible to identify β as well as all the functions and distributions that were identified in the original model. Another variation is obtained by eliminating the unobservable random variables ε_j. When $V(j,s,x_j,r_j,\omega) = v(j,s,x_j,\omega) - r_j,$ the distribution of ω and the functions $v(j,\cdot)$ are identified.

APPENDIX

PROOF OF THEOREM 1:

Let $\eta_j \equiv (\epsilon_1 - \epsilon_j, \ldots, \epsilon_J - \epsilon_j)$ ($j = 1, \ldots J$). To recover the distribution of η_k for all $k = 1, \ldots, J,$ it is enough to determine the identification of F_1^* (see Thompson (1988)). So, let $(t_2, \ldots, t_J)$ be given. Let $(r_1, \ldots, r_J)$ be such that $(t_2, \ldots, t_J) = (-r_1 + r_2 + \alpha_1 - \alpha_2, \ldots, -r_1 + r_J + \alpha_1 - \alpha_J).$ Then, $\forall V \in W, G \in \Gamma_G$

$$F_1^*(t_2, \ldots, t_J) = \int F_1^*(t_2, \ldots, t_J)\ dG(\omega)$$

$$= \int F_1^*(-r_1 + r_2 + \alpha_1 - \alpha_2, \ldots, -r_1 + r_J + \alpha_1 - \alpha_J)\ dG(\omega)$$

$$= p(1|\bar{s}, \bar{x}_1, r_1, \bar{x}_2, r_2, \ldots, \bar{x}_J, r_J)$$

where the last equality follows from Assumption 4. It follows that F_1^* is identified, since if for some F_1 and $(t_2, \ldots, t_J),$ $F_1(t_2, \ldots, t_J) \neq F_1^*(t_2, \ldots, t_J),$ then $\forall V, V', G, G'$

$$p(1|\bar{s}, \bar{x}_1, r_1, \bar{x}_2, r_2, \ldots, \bar{x}_J, r_J; V, F, G) \neq p(1|\bar{s}, \bar{x}_1, r_1, \bar{x}_2, r_2, \ldots, \bar{x}_J, r_J; V', F^*, G')$$

Next, assume w.l.o.g. that the alternative, $\tilde{j},$ that satisfies Assumption 5 is $\tilde{j} = 2,$ $\beta_2 = 0,$ and the alternative, j^* that satisfies either Assumption 6 or 6' or 6'' is $j^* = 1.$ To show that $v^*(1, \cdot)$ and G^* are identified, we transform the polychotomous choice model into a binary choice model by letting $r_j \to \infty$ for $j \geq 3.$ Let $\eta \equiv \epsilon_2 - \epsilon_1,$ and denote the marginal distribution of $\epsilon_2 - \epsilon_1$ by $F_\eta^*.$ Since F^* is identified, we can assume that F_η^* is known. $\forall (s, x_1, x_3, \ldots, x_J, r_1, r_2) \in (S \times X_1 \times (\prod_{j=3}^J X_j) \times R_+^2),$

$$p(1|s, x_1, \tilde{x}_2, \ldots, x_J, r_1, r_2, \ldots, r_J) = \int F_\eta^*(v^*(1, s, x_1, \omega) - r_1 + r_2)\ dG^*(\omega).$$

Let $\gamma = v^*(1,s,x_1,\omega)$ and let S^* denote the distribution of γ conditional on (s,x_1). Then, $\forall(r_1,r_2) \in R_+^2$

$$p(1|s,x_1,r_1,r_2) = \int F_\eta^*(v^*(1,s,x_1,\omega) - r_1 + r_2)\ dG^*(\omega)$$

$$= \int F_\eta^*(\gamma + r_1 - r_2)\ dS^*(\gamma)$$

$$= \int F_\eta^*(\gamma + r_1 - r_2)\ dS^*(\gamma)$$

It then follows by Assumption 11 and Teicher (1961) that $S^*(\cdot)$ is identified. Let $f(s,x_1)$ be the marginal pdf of (s,x_1); $f(\cdot)$ is identified since (s,x_1) is a vector of observable variables. Hence, since $S^*(\cdot)$ is the distribution of γ conditional on (s,x_1), we can identify the joint pdf, $f(\gamma,s,x_1)$, of (γ,s,x_1). $\gamma = v^*(1,s,x_1,\omega)$ is a nonparametric function, which by Assumptions 5-7 satisfies the requirements in Matzkin (2003) for identification of $\gamma = v^*(1,s,x_1,\omega)$ and the distribution of ω. Hence, $v^*(1,\cdot,\cdot,\cdot)$ *and* G^* are identified. Substituting i^* in the above argument by k, as in Assumption 8, it follows that the distribution of $\gamma^k = v^*(k,s,x_k,\omega)$ conditional on (s,x_k) is identified. Since by Assumption 8 the nonparametric function $v^*(k,s,x_k,\omega)$ is known to be either strictly increasing or strictly decreasing in ω, and the distribution of ω has already been shown to be identified, it follows by Matzkin (2003) that $v^*(k,\cdot,\cdot,\cdot)$ is identified. Finally, since for all $j \neq \tilde{j}$, $v^*(j,\cdot,\cdot,\cdot)$ is identified, it follows by Assumption 9, using similar arguments as above that $v^*(\tilde{j},\cdot,\cdot,\cdot)$ is identified. This completes the proof of Theorem 1.

PROOF OF THEOREM 2:

We show the theorem by showing that the assumptions necessary to apply the result in Kiefer and Wolfowitz (1956) are satisfied (see also Wald (1949)). For any $(V,G,F) \in (W \times \Gamma_G \times \Gamma_F)$, define

$$f(y,s,z;V,G,F) = \int \prod_{j=1}^{J} p(j\,|\,s,s,z;V(\cdot,\omega),F)^{y_j^i}\ dG(\omega)$$

and for any $\rho > 0$, define the function $f'(y,s,z;V,G,F,\rho)$ by

$$f'(y,s,z;V,G,F,\rho) = \sup_{d[(V,G,F),(V',G',F')]<\rho} f(y,s,z;V',G',F').$$

We need to show continuity, measurability, integrability, and identification.

CONTINUITY: $\forall\{(V_k,F_k,G_k)\}_{k=1}^{\infty},(V,F,G)$ *such that* $\{(V_k,F_k,G_k)\}_{k=1}^{\infty} \subset (W\times\Gamma_F\times\Gamma_G),(V,F,G)\in(W\times\Gamma_F\times\Gamma_G)$ *and* $d[(V_k,F_k,G_k),(V,F,G)]\to 0,$ *one has that* $\forall(y,s,z),$ *except perhaps on a set of probability 0,* $f(y,s,z;V_k,F_k,G_k)\to f(y,s,z;V,F,G).$

MEASURABILITY: $\forall(V,F,G)\in(W\times\Gamma_F\times\Gamma_G)$ *and* $\forall\rho>0,$ $f'(y,s,z;V,G,F,\rho)$ *is a measurable function of* $(y,z).$

INTEGRABILITY: $\forall(V,F,G)\in(W\times\Gamma_F\times\Gamma_G)$

$$\lim_{\rho\to 0} E\left[\log\left(\frac{f'(y,s,z;V,G,F,\rho)}{f(y,s,z;V^*,G^*,F^*)}\right)\right]^+ < \infty$$

IDENTIFICATION: $\forall(V,F,G)\in(W\times\Gamma_F\times\Gamma_G)$ *such that* $(V,F,G)\neq(V^*,F^*,G^*)$ *there exists a set* Ω *such that*

$$\int_\Omega f(y,s,z;V,G,F)\,d(y,z) \neq \int_\Omega f(y,s,z;V^*,G^*,F^*)\,d(y,z)$$

To show continuity, we note that $d[(V_k,F_k,G_k),(V,F,G)]\to 0$ implies that for all $j,$ $F_{j,k}(V_{j,k}(s,z,\omega))$ converges uniformly on all compact subsets of $(S\times\prod_{j=1}^{J}(X_j\times R_+))$ to $F_j(V_j(s,z,\omega)),$ where $V_j(s,z,\omega) = (V(j,s,z_j)-V(1,s,z_1),\ldots,V(j,s,z_j)-V(J,s,z_J)),$ and that G_k converges weakly to G over $Y.$ Let $h_k : Y\to R$ and $h : Y\to R$ denote, respectively, the functions $F_{j,k}(V_{j,k}(s,z,\bullet))$ and $F_j(V_j(s,z,\bullet)).$ Since h is continuous, it follows from Theorem 5.5 in Billingsley (1968) that $G_k h_k^{-1}$ converges weakly to $Gh^{-1}.$ Hence, using a change of variables, it follows that

$$p(j\,|\,s,z;V_k,F_k,G_k)$$

$$= \int F_{j,k}(V_{j,k}(s,z,\omega))\,dG_k(\omega) = \int h_k(\omega)\,dG_k(\omega) = \int t\,d(G_k h_k^{-1})(t)$$

$$\to \int t\,d(Gh^{-1})(t) = \int h(\omega)\,dG(\omega) = \int F_j(V_j(s,z,\omega))\,dG(\omega)$$

$$= p(j\,|\,s,z;V,F,G)$$

where the convergence follows because t is a continuous and bounded function on the supports of $G_k h_k^{-1}$ and Gh^{-1}. Hence, it follows that $f(y,s,z;V_k,F_k,G_k) \to f(y,s,z;V,F,G)$ for all z.

To show measurability, we first note that it suffices to show that for all j, $\sup_{d[(V,G,F),(V',G',F')]<\rho} p(j \mid s,z;V',F',G')$ is measurable in (s,z).

Now, since $(W \times \Gamma_F \times \Gamma_G)$ is a compact space, there exists a countable, dense subset of $(W \times \Gamma_F \times \Gamma_G)$. Denote this subset by $(\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)$. Then,

$$\sup\left\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (W \times \Gamma_F \times \Gamma_G)\right\} = \sup\left\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)\right\}$$

Since, suppose that the left hand side is bigger than the right hand side, then, there must exist $\delta > 0$ and $(V',G',F') \in (W \times \Gamma_F \times \Gamma_G)$ such that $\forall (V'',G'',F'') \in (\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)$,

(i) $p(j|s,z;V',F',G') > \delta > p(j|s,z;V'',F'',G'')$.

But, $(\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)$ is dense in $(W \times \Gamma_F \times \Gamma_G)$. Hence, there exists a sequence $\{(V_k,F_k,G_k)\} \subset (\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)\}$ such that $d[(V_k,F_k,G_k),(V',F',G')] \to 0$. As it was shown in the proof of continuity, this implies that $p(j \mid s,z;V_k,F_k,G_k) \to p(j \mid s,z;V',F',G')$, which contradicts (i).

Hence,

$$\sup\left\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (W \times \Gamma_F \times \Gamma_G)\right\} = \sup\left\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)\right\}$$

Since $\forall (V',G',F') \in (\tilde{W} \times \tilde{\Gamma}_F \times \tilde{\Gamma}_G)$, $p(j \mid s,z;V',F',G')$ is measurable in (s,z), it follows that

$$\sup\left\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (W \times \Gamma_F \times \Gamma_G)\right\}$$

is measurable in (s,z).

To show integrability, we note that $\forall j\ \forall (V',G',F')$, $p(j \mid s,z;V',F',G') \le 1$. Hence, $\forall j$, $\sup\{p(j \mid s,z;V',F',G') \mid d[(V,G,F),(V',G',F')] < \rho\,,\ (V',G',F') \in (W \times \Gamma_F \times \Gamma_G)\} \le 1$. It follows then that $\forall \rho > 0$,

$$E\left[\log\left(\tfrac{f'(y,s,z,V,G,F,\rho)}{f(y,s,z,V^*,G^*,F^*)}\right)\right]^+ \le E\left[\log\left(\tfrac{1}{f(y,s,z,V^*,G^*,F^*)}\right)\right]^+$$
$$= -\int_{(s,z)} \left[\sum_{j=1}^{J} p(j|s,z;V^*,F^*,G^*)\ \left(\log(p(j|s,z;V^*,F^*,G^*)\ \right)\right] dF(s,z)$$

Since the term in brackets is bounded, it follows that

$$E\left[\log\left(\tfrac{f'(y,s,z,V,G,F,\rho)}{f(y,s,z,V^*,G^*,F^*)}\right)\right]^+ < \infty.$$

Finally, identification follows from Theorem 1.

Hence, it follows by Kiefer and Wolfowitz (1956) that the estimators are consistent.

PROOF OF THEOREM 3:

The properties shown in the proof of Theorem 2 imply that the log-likelihood function converges a.s.uniformly, over the compact set $(W \times \Gamma_F \times \Gamma_G)$, to a continuous function that is uniquely maximized over $(W \times \Gamma_F \times \Gamma_G)$ at (V^*, F^*, G^*) (see Newey and McFadden (1994)). Since $\{(W^N \times \Gamma_F^N \times \Gamma_G^N)\}_{N=1}^{\infty}$ becomes dense in $(W \times \Gamma_F \times \Gamma_G)$ as the number of observations increases, it follows by Gallant and Nychka (1987, Theorem 0) that the estimators obtained by maximizing the log-likelihood over $(W^N \times \Gamma_F^N \times \Gamma_G^N)$ are strongly consistent.

REFERENCES

AINSILE, A., P.E. ROSSI (1998), "Similarities in Choice Behavior Across Product Categories," *Marketing Science*, 17 (2), 91-106.

ALBRIGHT, R.L., S.R. LERMAN, and C.F. MANSKI (1977) "Report on the Development of an Estimation Program for the Multinomial Probit Model." Report for the Federal Highway Administration. Cambridge Systematics, Inc.: Cambridge, Massachusetts.

ALLENBY, G.M. and P.E. ROSSI (1999), "Marketing models of consumer heterogeneity," *Journal of Econometrics*, 89, 57-78.

BELL, D.R., and J.M. LATTIN (1998), "Shopping Behavior and Consumer Preferences for Store Price Format: Why 'Large Basket' Shoppers Prefer EDLP," *Marketing Science*, 17(1), 66-88.

BELL, D.R., T. HO, and C.S. TANG (1998), "Determining Where to Shop: Fixed and Variable Costs of Shopping," *Journal of Marketing Research*, XXXV (August), 352-369.

BOZDOGAN, H. (1987), "Model Selection and Akaike's Information Criterion (AIC): The General Theory and Its Analytical Extensions," *Psychometrika* 52(5) 345-370.

BRIESCH, R.A., P.K. CHINTAGUNTA, and R.L. MATZKIN (1997), "Nonparametric Discrete Choice Models with Unobserved Heterogeneity," mimeo, Northwestern University.

BRIESCH, R.A., P.K. CHINTAGUNTA, and R.L. MATZKIN (2002), "Semiparametric Estimation of Brand Choice Behavior," *Journal of the American Statistical Association*, 97(460), Dec, 973-982.

BROWN, D.J. and R.L. MATZKIN (1995), "Nonparametric Estimation of Simultaneous Equations, with an Application to Consumer Demand," mimeo, Northwestern University.

BUCKLIN, L.P. (1971), "Retail Gravity Models and Consumer Choice: A Theoretical and Empirical critique," *Economic Geography*, 47 (October), 489-497.

CHEN, X. (2006) "Large Sample Sieve Estimation of Semi-Nonparametric Models," forthcoming in *Handbook of Econometrics,* Vol. 6, edited by J.J. Heckman and E.E. Leamer, Elsevier.

COSSLETT, S. R. (1983), "Distribution-free Maximum Likelihood Estimator of the Binary Choice Model," *Econometrica*, 51(3), 765-782.

DAHL, G.B., (2002), "Mobility and the Return to Education : Testing a Roy Model with Multiple Markets," *Econometricia*, 70(6), Nov, 2367-2420.

DAYTON, C.M. and G.B. MCREADY (1988), "Concomitant-Variable Latent-Class Models," *Journal of the American Statistical Association*, 83(401), 173-178.

ELBADAWI, I, A.R. GALLANT, and G. SOUZA (1983), "An Elasticity Can Be Estimated Consistently Without A Priori Knowledge of Functional Form," *Econometrica,* 51, 6, 1731-1751.

FLINN, C.J. and J.J. HECKMAN (1982) "Models for the Analysis of Labor Force Dynamics," *Advances in Econometrics*, 1, 35-95.

GALLANT, A.R. and D.W. NYCHKA (1987) "Seminonparametric Maximum Likelihood Estimation," *Econometrica*, 55, 363-390.

GALLANT, A.R. and D.W. NYCHKA (1989) "Seminonparametric Estimation of Conditionally

Constrained Heterogeneous Processes: Asset Pricing Applications," *Econometrica*, 57, 5, 1091-1120.

GEWEKE, J. and M. KEANE (1997), "An Empirical Analysis of Male Income Dynamics in the PSID: 1968:1989," Research Department Staff Report 233, Federal Reserve Bank of Minneapolis.

HAUSMAN, J.A. and D. A. WISE (1978) "A Conditional Probit Model for Qualitative Choice Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences," *Econometrica*, 46, 403-426.

HECKMAN, J.J. (1974) "The Effect of Day Care Programs on Women's Work Effort," *Journal of Political Economy*.

______________ (1981a) "Statistical Models for Discrete Panel Data," in C. Manski and D. McFadden (eds.), *Structural Analysis of Discrete Data with Econometric Applications*, M.I.T. Press.

______________ (1981b) "Heterogeneity and State Dependence," in S. Rosen (ed.) *Studies in Labor Markets*, University of Chicago Press.

HECKMAN, J.J. and G.J. BORJAS (1980) "Does Unemployment Cause Future Unemployment? Definitions, Questions and Answers from a Continuous Time Model of Heterogeneity and State Dependence," *Economica*, 47, 247-283.

HECKMAN, J.J. and B. SINGER (1984) "A Method for Minimizing the Impact of Distributional Assumptions in Econometric Models for Duration Data," *Econometrica*, 271-320.

HECKMAN, J.J. and C.R. TABER (1994) "Econometric Mixture Models and More General Models for Unobservables," University of Chicago.

HECKMAN, J.J. and J. WALKER (1990a) "The Relationship Between Wages and Income and the Timing and Spacing of Births: Evidence from Swedish Longitudinal Data," *Econometrica*, 58(6), 235-275.

___________________________ (1990b) "Estimating Fecundability from Data on Waiting Times to First Conceptions," *Journal of the American Statistical Association*, 84(408), 958-965.

HECKMAN, J.J. and R. WILLIS (1977) "A Beta-Logistic Model for the Analysis of Sequential Labor Force Participation by Married Women," *Journal of Political Economy*.

HIRANO, K. (2002), "Semiparametric Bayesian Inference in Autoregressive Panel Data Models," *Econometricia*, 70(2), Mar, 781-799.

HOCH, S.J., X. DREZE, and M.E. PURK (1994), "EDLP, Hi-Lo and Margin Arithmetic," *Journal of Marketing*, 58(4), 16-27.

HOROWITZ, J.L. (1992) "A Smooth Maximum Score Estimator for the Binary Choice Model," *Econometrica*, 60, 505-531.

HOROWITZ, J.L, and N.E. SAVIN (2001), "Binary Response Models: Logits, Probits and Semiparametrics," *The Journal of Economic Perspectives*, 15(4), Autumn, 43-56.

HUFF, D.L. (1962), "A Probability Analysis of Consumer Spatial Behavior," in William S. Decker (ed.), *Emerging Concepts in Marketing* (Chicago: American Marketing Association), 443-461.

ICHIMURA, H. (1993) "Semiparametric Least Squares (SLS) and Weighted SLS Estimation of Single

Index Models," *Journal of Econometrics,* Vol. 58, pp. 71-120.

ICHIMURA, H. and T.S. THOMPSON (1998) "Maximum Likelihood Estimation of a Binary Choice Model with Random Coefficients of Unknown Distribution," *Journal of Econometrics,* Vol. 86, No. 2, pp. 269-295.

KAMAKURA, W.A. and G.J. RUSSELL (1989), "A Probabilistic Choice Model for Market Segmentation and Elasticity Structure," Journal of Marketing Research, 26(4), Nov, 379-390.

KIEFER, J. and J. WOLFOWITZ (1956) "Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters," *Annals of Mathematical Statistics*, 27, 887-906.

KLEIN, R.W. and R.H. SPADY (1993) "An Efficient Semiparametric Estimator for Discrete Choice Models," *Econometrica*, 61, 387-422.

KLEIN, R.W., R.P. SHERMAN (2002), "Shift Restrictions and Semiparametric Estimation in Ordered Response Models," *Econometrica*, 70(2), Mar, 663-691.

LANCASTER, T. (1979) "Econometric Methods for the Analysis of Unemployment," *Econometrica*, 47, 939-956.

_________ (1997), "Orthogonal Parameters and Panel Data," Brown University Department of Economics working paper 97-32.

LESZCZYC, P.T., A. SINAH, and H.J.P. TIMMERMANS (2000), "Consumer Store Choice Dynamics: An Analysis of Competitive Market Structure for Grocery Stores," *Journal of Retailing*, 76(3), 323-345.

LEWBEL, A. (2000) "Semiparametric Qualitative Response Model Estimation with Unknown Heteroskedasticity and Instrumental Variables," *Journal of Econometrics,* 97, 145-177.

LINDSAY, B.G. (1983) "The Geometry of Mixture Likelihoods: A General Theory," *The Annals of Statistics*, 11(1), 86-94.

MANSKI, C. (1975) "Maximum Score Estimation of the Stochastic Utility Model of Choice," *Journal of Econometrics*, 3, 205-228.

MATZKIN, R.L. (1991) "Semiparametric Estimation of Monotone and Concave Utility Functions for Polychotomous Choice Models," *Econometrica*, 59, 1315-1327.

______________(1992) "Nonparametric and Distribution-free Estimation of the Binary Choice and the Threshold Crossing Models," *Econometrica*, 60, 239-270.

______________(1993) "Nonparametric Identification and Estimation of Polychotomous Choice Models," *Journal of Econometrics*, 58,137-168.

______________(1999) "Computation and Operational Properties of Nonparametric Shape Restricted Estimators," mimeo, Northwestern University.

______________(1994) "Restrictions of Economic Theory in Nonparametric Methods," in *Handbook of Econometrics*, Vol. 4, by McFadden, D. and R. Engel (eds.).

______________ (1999) "Nonparametric Estimation of Nonadditive Random Functions," mimeo, Northwestern University, presented at the 1999 Latin American Meeting of the Econometric Society.

______________ (2003) “Nonparametric Estimation of Nonadditive Random Functions," *Econometrica.*71, 5, 1339-1375.

______________ (2004) “Unobservable Instruments,” Mimeo, Northwestern University.

______________ (2005) “Identification in Nonparametric Simultaneous Equations,” Mimeo, Northwestern University.

______________ (2006) “Nonparametric Identification,” forthcoming in *Handbook of Econometrics,* Vol. 6, edited by J.J. Heckman and E.E. Leamer, Elsevier.MOON, H. R. (2004), “Maximum score estimation of a nonstationary binary choice model,” *Journal of Econometrics*, 122, 385-403.

NEVO, A. and I. HENDEL (2002), “Measuring the Implications of Sales and Consumer Stockpiling Behavior,” Mimeo, Berkeley, CA: University of California.

NEWEY, W. and D. McFADDEN (1994) “Large Sample Estimation and Hypothesis Testing,” in *Handbook of Econometrics,* Vol 4, edited by R.F. Engle and D.L. McFadden, Elsevier Science B.V.

PARK, B.U., R.C. SICKLES, and L. SIMAR (2007), “Semiparametric efficient estimation of dynamic panel data models,” *Journal of Econometrics*, 136, 281-301.

PINKSE, J., M.E. SLADE, C. BRETT (2002), “Spatial Price Competition: A Semiparametric Approach,” *Econometrica*, 70(3), May, 1111-1153.

RHEE, H., and D.R. BELL (2002), “The inter-store mobility of supermarket shoppers,” Journal of Retailing, 78, 225-237.

SMITH, Howard (2004), “Supermarket Choice and Supermarket Competition in Market Equilibrium,” *Review of Economic Studies*, 71, 235-263.

SPIEGELHALTER, D.J., N.G. BEST, B.P. CARLIN, and A. VAN DER LINDE (2002), “Bayesian measures of model complexity and fit,” Journal of Royal Statistical Society B, 64(4), 583-639.

TABER, C.R. (2000), “Semiparametric identification and heterogeneity in discrete choice dynamic programming models,” *Journal of Econometrics*, 96, 201-229.

TANG, C.S., D.R. BELL, and T. HO (2001), “Store Choice and Shopping Behavior: How Price Format Works,” California Management Review, 43(2), 56-74.

TEICHER, H. (1961) “Identifiability of Mixtures,” *Annals of Mathematical Statistics*.

THOMPSON, T.S. (1989) “Identification of Semiparametric Discrete Choice Models,” Discussion Paper No. 249, Center for Economic Research, University of Minnesota.

WALD, A. (1949) “A Note on the Consistency of the Maximum Likelihood Estimator,” *Annals of Mathematical Statistics*, 20, 595-601.

WANSBEEK, T., M. WEDEL, E. MEIJER (2001), “Comment on ‘Microeconometrics’ by J.A. Hausman,” *Journal of Econometrics*, 100, 89-91.

YACHEW, A. (1998), “Nonparametric Regression Techniques in Economics,” *Journal of Economic Literature*, 36(2 June), 699-721.

Table 1 – Descriptive statistics for the households

	Mean	Std Dev
Number of Households	161	
Average Monthly Spending	$233.9	85.4
Minimum Monthly Spending	$102.2	61.9
Number of Shopping Trips	184.2	83.4
Av Days Between Trips	4.6	1.8
Av Spending Per Trip	36.5	15.8
Elderly	13.7%	34.5%
Household Size	2.9	1.3
Income (,000)	$56.2	$25.0
College	41.0%	49.3%
Married	82.0%	38.5%

Table 2 – Descriptive statistics for selected categories.

Category	Pen. Rate[1]	Av Spend[2]	No Fams[3]	Percent Cat.[4]	Percent Cum.[5]	Purchases Av / Family[6]	Purchases Total[7]
CARBONATED BEVERAGES	98%	284	157	4.6%	4.6%	61	9781
CIGARETTES	22%	853	36	4.1%	8.7%	49	2319
MILK	100%	198	161	3.3%	12.0%	65	10409
BEER/ALE/ALCOHOLIC CIDER	39%	273	62	2.7%	14.6%	25	2353
FRESH BREAD & ROLLS	100%	155	161	2.6%	17.2%	66	10677
COLD CEREAL	96%	143	154	2.3%	19.5%	31	4982
FZ DINNERS/ENTREES	79%	134	127	2.1%	21.7%	22	3362
SALTY SNACKS	96%	122	155	2.0%	23.7%	40	6384
NATURAL CHEESE	93%	104	149	1.7%	25.4%	29	4627
DOG FOOD	52%	148	83	1.6%	27.0%	27	2905
ICE CREAM/SHERBET	83%	94	133	1.5%	28.5%	19	2965
LUNCHEON MEATS	87%	94	140	1.5%	30.0%	28	4326
CRACKERS	98%	91	157	1.5%	31.5%	29	4646
RFG JUICES/DRINKS	78%	95	125	1.5%	33.0%	28	4240
COOKIES	89%	87	144	1.4%	34.4%	27	4296
SOUP	99%	79	159	1.3%	35.7%	27	4416
BREAKFAST MEATS	84%	82	136	1.3%	37.0%	20	3096
LAUNDRY DETERGENT	84%	76	136	1.2%	38.2%	13	2015
TOILET TISSUE	85%	73	137	1.2%	39.3%	18	2787
VEGETABLES	96%	68	155	1.1%	40.4%	31	4905
TOTAL CHOCOLATE CANDY	88%	66	142	1.1%	41.5%	20	3120
COFFEE	75%	69	121	1.1%	42.5%	16	2415
FZ PIZZA	55%	80	88	1.0%	43.6%	14	1772
CAT FOOD	36%	127	58	1.0%	44.6%	31	2489
RFG SALAD/COLESLAW	85%	65	137	1.0%	45.7%	26	4098
BOTTLED JUICES - SS	79%	61	127	1.0%	46.6%	19	2916
FZ POULTRY	53%	66	86	1.0%	47.6%	10	1348
PROCESSED CHEESE	88%	57	141	0.9%	48.5%	19	3080
RFG FRESH EGGS	99%	50	159	0.8%	49.3%	31	4938
VITAMINS	39%	59	62	0.7%	50.0%	7	807
PET SUPPLIES	53%	54	85	0.7%	50.8%	9	1257
PAPER TOWELS	81%	47	131	0.7%	51.5%	18	2660
CANNED/BOTTLED FRUIT	86%	42	139	0.7%	52.2%	19	3080
FOOD & TRASH BAGS	88%	42	141	0.7%	52.9%	13	2120
DRY PACKAGED DINNERS	72%	45	116	0.7%	53.5%	16	2326
HOUSEHOLD CLEANER	85%	41	137	0.7%	54.2%	11	1809
DOUGH/BISCUIT DOUGH - RFG	76%	43	122	0.6%	54.8%	15	2191
FRANKFURTERS	68%	43	110	0.6%	55.5%	12	1691
YOGURT	53%	43	86	0.6%	56.1%	17	2339
FZ PLAIN VEGETABLES	70%	39	113	0.6%	56.7%	14	2161
INTERNAL ANALGESICS	57%	41	91	0.6%	57.2%	8	1071
TOTAL NON-CHOCOLATE CANDY	75%	38	121	0.6%	57.8%	15	2287
SNACK BARS/GRANOLA	48%	44	77	0.6%	58.4%	10	1267

BARS						
SALAD DRESSINGS - SS	77%	34	124	0.5%	58.9%	12	1888
COLD/ALLERGY/SINUS TABLETS	40%	43	64	0.5%	59.5%	7	859
DISH DETERGENT	84%	33	136	0.5%	60.0%	11	1827
BOTTLED WATER	49%	41	79	0.5%	60.6%	13	1691
PASTRY/DOUGHNUTS	58%	36	93	0.5%	61.1%	13	1851
GASTROINTESTINAL - TABLETS	30%	42	49	0.5%	61.6%	6	773
SOAP	71%	33	115	0.5%	62.1%	10	1475
SNACK NUTS/SEEDS/CORN NUTS	53%	35	86	0.5%	62.6%	10	1445
TOOTHPASTE	73%	32	117	0.5%	63.1%	9	1293
FZ BREAD/FZ DOUGH	62%	36	100	0.5%	63.6%	11	1470
FZ BREAKFAST FOOD	50%	37	80	0.5%	64.1%	12	1504
PICKLES/RELISH/OLIVES	76%	31	123	0.5%	64.6%	13	1961
MARGARINE/SPREADS/BUTT ER BLE	83%	32	133	0.5%	65.0%	18	2622
CANNED MEAT	54%	34	87	0.5%	65.5%	11	1540
BAKING MIXES	81%	29	130	0.5%	66.0%	14	2177
BATTERIES	54%	31	87	0.5%	66.4%	5	760
FACIAL TISSUE	68%	32	110	0.5%	66.9%	13	1827
SHORTENING & OIL	81%	28	131	0.5%	67.4%	10	1565
SPAGHETTI/ITALIAN SAUCE	74%	30	119	0.5%	67.8%	12	1727
DIAPERS	12%	101	20	0.5%	68.3%	9	382
SS DINNERS	60%	33	96	0.5%	68.7%	13	1731
SPICES/SEASONINGS	80%	28	128	0.5%	69.2%	10	1595
SUGAR	81%	28	130	0.4%	69.6%	16	2354
DINNER SAUSAGE	42%	34	68	0.4%	70.0%	8	937
SEAFOOD -SS	64%	28	103	0.4%	70.4%	11	1679
BAKING NEEDS	73%	26	117	0.4%	70.8%	10	1576
TEA - BAGS/LOOSE	53%	28	85	0.4%	71.2%	9	1280
PEANUT BUTTER	60%	24	96	0.4%	71.6%	9	1371
FZ DESSERTS/TOPPING	60%	25	96	0.4%	72.0%	9	1357
AIR FRESHENERS	48%	26	78	0.4%	72.3%	7	1024
MAYONNAISE	78%	22	126	0.4%	72.7%	9	1443
CUPS & PLATES	58%	25	93	0.4%	73.0%	10	1432
SANITARY NAPKINS/TAMPONS	41%	33	66	0.4%	73.4%	9	949
POPCORN/POPCORN OIL	56%	26	90	0.4%	73.7%	8	1077
TOASTER PASTRIES/TARTS	41%	32	66	0.3%	74.1%	12	1272
PASTA	84%	21	135	0.3%	74.4%	15	2431
CREAMS/CREAMERS	42%	28	68	0.3%	74.8%	13	1597
TOMATO PRODUCTS	76%	22	122	0.3%	75.1%	12	1821
SHAMPOO	60%	23	97	0.3%	75.4%	7	988
FZ APPETIZERS/SNACK ROLLS	34%	26	54	0.3%	75.8%	6	742
FZ POTATOES/ONIONS	50%	23	81	0.3%	76.1%	9	1185
BUTTER	42%	28	68	0.3%	76.4%	9	1013
DEODORANT	60%	21	96	0.3%	76.7%	7	1004

RICE	76%	20	122	0.3%	77.0%	11	1635
GRAVY/SAUCE MIXES	67%	22	108	0.3%	77.3%	11	1564
GELATIN/PUDDING MIXES	71%	20	115	0.3%	77.6%	11	1647
FABRIC SOFTENER LIQUID	32%	31	51	0.3%	77.9%	8	710
CAT/DOG LITTER	20%	55	32	0.3%	78.2%	10	519
CLEANING TOOLS/MOPS/BROOMS	31%	21	50	0.3%	78.5%	4	453
JELLIES/JAMS/HONEY	59%	18	95	0.3%	78.7%	7	1048
TOOTHBRUSH/DENTAL ACCESORIES	39%	21	63	0.3%	79.0%	5	647
FOILS & WRAPS	69%	17	111	0.3%	79.3%	7	1069
ASEPTIC JUICES	22%	37	35	0.3%	79.5%	12	826
BLADES	24%	23	39	0.3%	79.8%	4	389
MUSTARD & KETCHUP	73%	15	118	0.2%	80.0%	9	1439
BAKED BEANS/PORK & BEANS	54%	16	87	0.2%	80.3%	10	1480
MEXICAN SAUCE	44%	19	71	0.2%	80.5%	7	851
LIGHT BULBS	45%	16	72	0.2%	80.7%	4	611
JUICES - FROZEN	16%	30	25	0.2%	81.0%	9	672
BAKERY SNACKS	41%	18	66	0.2%	81.2%	11	1346
CREAM CHEESE/CR CHS SPREAD	54%	15	87	0.2%	81.4%	8	1114
LUNCHES - RFG	21%	37	34	0.2%	81.6%	10	556
FLOUR/MEAL	70%	13	112	0.2%	81.8%	8	1296
HOT CEREAL	45%	15	72	0.2%	82.0%	6	752
SOUR CREAM	63%	14	102	0.2%	82.2%	11	1517
SUGAR SUBSTITUTES	25%	22	41	0.2%	82.4%	7	614
WEIGHT CON/NUTRITION LIQ/PWD	9%	40	15	0.2%	82.6%	6	264
DRINK MIXES	27%	19	43	0.2%	82.8%	8	784
SPORTS DRINKS	14%	23	23	0.2%	83.0%	5	427
SYRUP/MOLASSES	39%	13	63	0.2%	83.2%	5	711
LAUNDRY CARE	33%	14	53	0.2%	83.4%	4	516
DENTURE PRODUCTS	11%	49	17	0.2%	83.6%	12	420
GUM	44%	15	71	0.2%	83.7%	9	1058
MOIST TOWELETTES	25%	21	40	0.2%	83.9%	7	569
KITCHEN STORAGE	27%	15	44	0.2%	84.1%	3	343
ALL OTHER SAUCES	46%	12	74	0.2%	84.3%	6	789
DRY FRUIT SNACKS	20%	22	32	0.2%	84.4%	7	523
PEST CONTROL	11%	15	18	0.2%	84.6%	3	299
PAPER NAPKINS	52%	13	83	0.2%	84.7%	6	768
BLEACH	44%	11	71	0.2%	84.9%	6	744
MEXICAN FOODS	39%	13	62	0.2%	85.0%	7	774
MOUTHWASH	26%	15	42	0.1%	85.2%	5	424
HAIR CONDITIONER	26%	15	42	0.1%	85.3%	5	460
FABRIC SOFTENER SHEETS	30%	13	49	0.1%	85.5%	4	451
COFFEE CREAMER - SS	21%	18	34	0.1%	85.6%	6	485
FZ POT PIES	22%	17	36	0.1%	85.7%	6	435
HAIR SPRAY/SPRITZ	29%	15	46	0.1%	85.9%	6	525
EYE/CONTACT LENS CARE	12%	19	19	0.1%	86.0%	3	229

PRODUC							
RUG/UPHOLSTERY/FABRIC TREATM	19%	13	31	0.1%	86.2%	3	312
MISC. SNACKS	19%	15	30	0.1%	86.3%	4	373
CANNED JUICES - SS	28%	12	45	0.1%	86.4%	5	526
SALAD TOPPINGS	25%	12	40	0.1%	86.6%	6	581
DRIED FRUIT	30%	12	49	0.1%	86.7%	5	536
FIRST AID ACCESSORIES	24%	11	38	0.1%	86.8%	3	340
COLD/ALLERGY/SINUS LIQUIDS	13%	17	21	0.1%	86.9%	4	238
STUFFING MIXES	36%	9	58	0.1%	87.0%	4	544
FIRST AID TREATMENT	22%	11	35	0.1%	87.2%	3	364
MISC HEALTH REMEDIES	19%	11	31	0.1%	87.3%	3	299
STEAK/WORCESTERSHIRE SAUCE	24%	10	39	0.1%	87.4%	3	366
MILK FLAVORING/COCOA MIXES	24%	12	39	0.1%	87.5%	4	402
GASTROINTESTINAL - LIQUID	10%	20	16	0.1%	87.6%	3	164
CANDLES	16%	12	25	0.1%	87.7%	3	293
BARBEQUE SAUCE	29%	9	46	0.1%	87.8%	4	505
EVAPORATED/CONDENSED MILK	36%	9	58	0.1%	87.9%	5	620
TEA - INSTANT TEA MIXES	9%	21	14	0.1%	88.0%	5	246
VINEGAR	29%	8	47	0.1%	88.1%	4	487
HAIR STYLING GEL/MOUSSE	17%	12	28	0.1%	88.2%	3	265

Notes: 1) Pen. Rate is “Penetration Rates” which is percentage of families purchasing in the category.
2) Av. Spend is average total dollars spent in category (averaged over families purchasing in category).
3) No. Fams is number of families purchasing in the category.
4) Percent Cat. is the percentage of all dollars spent by families in this category versus all dollars spent in all categories.
5) Percent Cum. Is the cumulative percentage spent in these categories.
6) Av Purchases/Family is the average number of purchases by families purchasing in this category.
7) Total purchases is the total number of purchases in this category by all selected families.

Table 3 – Price Format Statistics.

	EDLP	HiLo
Share of Trips	57.8%	42.2%
	35.5	12.4
Distance (minutes)	(22.6)	(10.7)
	34.3	37.2
Cost to refill inventory	(10.9)	(11.0)
Loyalty (percent of trips to format	0.57	0.43
during initialization period)	(0.30)	(0.30)

Table 4 – Estimation results for parametric models.

	Segments				
	Zero	One	Two	Three	Four
In-Sample					
-Log-Likelihood	9605	6398	6122	5904	5883
(std dev)[1]		(15)	(32)	(70)	(93)
Hits		10846	11032	11172	11167
(std dev)[1]		(14)	(47)	(58)	(72)
Hit Rate		78%	80%	81%	81%
No Parameters	0	9	19	29	39
AIC	19210	12814	12282	11866	11845
Swartz	19210	12882	12425	12085	12138
DIC		12845	12350	12123	11975
Effective DOF[2]		24	53	157	104
Out of Sample					
-Log-Likelihood	4556	3169	3010	2907	2968
(std dev)[1]		(11)	(24)	(30)	(58)
Hits		5032	5165	5219	5215
(std dev)[1]		(15)	(31)	(34)	(45)
Hit Rate		77%	79%	79%	79%

Notes: 1. Standard deviations calculated using 25 bootstrap simulations.

2. Effective Degrees of Freedom is defined as the difference between the mean of the bootstrap simulation liklihoods and the estimation sample likelihood Spiegelhalter, et al (2002) .

Table 5 – Semiparametric estimation results.

	Segments		
	One[1]	Two	Three
In-Sample			
-Log-Likelihood	6185	5816	5676
(std dev)[2]	(22)	(64)	(30)
Hits	10956	11112	11281
(std dev)[2]	(30)	(77)	(36)
Hit Rate	79%	80%	81%
No Parameters	6	14	22
DIC	12489	11896	11518
Effective DOF[3]	59	132	83
Out of Sample			
-Log-Likelihood	3224	2993	2892
(std dev)[2]	(56)	(50)	(20)
Hits	5093	5248	5270
(std dev)[2]	(24)	(33)	(26)
Hit rate	77%	80%	80%

Notes: 1. The one segment model is over-identified as the extant literature (Matzkin (1992), Briesch, Chintagunta and Matzkin (2002)) show that only one extra point and no parameter restrictions are required for identification of this model. We include this model for completeness.

2. Standard deviations calculated using 25 bootstrap simulations.

3. Effective Degrees of Freedom is defined as the difference between the mean of the bootstrap simulation liklihoods and the estimation sample likelihood Spiegelhalter, et al (2002) .

Table 6 – MLE parameter estimates for three-segment model.

	Parametric			Semiparametric		
Coefficients	Segment 1	Segment 2	Segment 3	Segment 1	Segment 2	Segment 3
Mass	**1.27**	0.12	0.00	0.00	-0.26	**-1.50**
	(0.24)	(0.29)			(0.18)	(0.34)
Intercept	**-0.20**	**3.23**	**-0.74**	**1.45**	-0.01	-2.93
	(0.07)	(0.48)	(0.15)	(0.12)	(0.07)	(1.52)
Loyalty	**2.61**	**3.31**	**-0.95**	**3.21**	**3.12**	**1.90**
	(0.03)	(0.15)	(0.19)	(0.07)	(0.04)	(0.31)
Elderly	**-0.32**	**0.84**	**-4.51**	**-2.18**	**1.28**	**3.13**
	(0.05)	(0.19)	(0.29)	(0.09)	(0.09)	(1.17)
HH Size	**0.05**	**0.38**	**0.28**	0.02	**-0.14**	0.58
	(0.01)	(0.12)	(0.02)	(0.02)	(0.02)	(0.31)
Income	**0.002**	**0.008**	**0.063**	**-0.007**	**-0.003**	**0.029**
	(0.001)	(0.003)	(0.004)	(0.002)	(0.001)	(0.011)
College Educated	0.06	1.56	**-4.01**	**0.07**	**0.10**	**-0.92**
	(0.03)	(7.92)	(0.26)	(0.07)	(0.05)	(0.40)
Distance	**-0.20**	**-0.85**	**-3.82**			
	(0.01)	(0.06)	(0.21)			
Cost	**-0.95**	**-2.33**	**-0.51**			
	(0.19)	(0.82)	(0.38)			
EDLP * Days since last trip	-0.01	***-0.07***	**-0.14**	-0.013	-0.013	-0.013
	(0.01)	(0.02)	(0.02)			
Theta				1.000	**2.09**	**0.53**
					(0.01)	(0.02)

Notes: 1. One segment's mass point set to zero for identification.

2. EDLP*Days since last trip set to -0.013 for identification in semiparametric model.

3. One semiparametric theta constrained to one.

4. Segments are matched based upon mass values.

5. Bold values are significant at $p < 0.05$.

Table 7 – Cost and Distance Elasticities by Segment and Model

	Percent of Families	Cost Elasticity	Distance Elasticity
Segment 1			
Parametric	22.4	-1.19	-0.57
		(1.60)	(0.45)
Semiparametric	38.5	-2.52	-0.21
		(0.35)	(0.06)
Segment 2			
Parametric	62.1	-0.96	-0.12
		(1.36)	(0.45)
Semiparametric	52.8	-0.53	-0.22
		(0.04)	(0.10)
Segment 3			
Parametric	15.5	-0.48	-1.39
		(0.98)	(0.32)
Semiparametric	8.7	-0.22	-0.19
		(0.09)	(0.07)

Table 7 – Demographic profiles of segments.

	Parametric			Semi-Parametric		
	Segment 1	Segment 2	Segment 3	Segment 1	Segment 2	Segment 3
Size (# families)	36	100	25	62	85	14
% Trips at EDLP	**82%** [a]	55% [b]	43% [c]	**63%** [a]	58% [a,b]	44% [b]
	(0.28)	(0.31)	(0.30)	(0.31)	(0.35)	(0.25)
Av Days between trips	5.57 [a]	5.20 [a]	4.28 [b]	4.86 [b]	5.46 [a]	4.41 [b]
	(2.20)	(1.75)	(1.79)	(2.01)	(1.73)	(1.99)
COST-Average Cost at selected format ($/10)	-0.07	0.18	0.10	0.14	0.11	0.00
	(0.80)	(0.93)	(0.71)	(0.83)	(0.85)	(1.17)
Distance to selected format/10	**3.28** [a]	***1.82*** [b]	1.38 [c]	**1.98**	***2.19***	1.85
	(2.32)	(1.07)	(0.91)	(1.44)	(1.70)	(1.20)
Income/10,000	*48.96* [b]	*59.22* [a]	*54.44* [a,b]	*50.23* [b]	*58.51* [a]	*68.39* [a]
	(19.89)	(26.10)	(25.37)	(26.83)	(21.46)	(30.72)
Family Size	2.67	2.85	3.20	2.82	2.93	2.64
	(1.17)	(1.29)	(1.63)	(1.34)	(1.33)	(1.28)
Elderly	14%	13%	16%	18%	11%	14%
	(0.35)	(0.34)	(0.37)	(0.39)	(0.31)	(0.36)
College Educated	**19%** [b]	48% [a]	44% [a]	**47%**	38%	36%
	(0.40)	(0.50)	(0.51)	(0.50)	(0.49)	(0.50)
Av. Trips per family	79.03 [b]	82.36 [b]	111.04 [a]	91.77 [a]	77.86 [b]	110.64 [a]
	(34.91)	(35.57)	(53.86)	(43.46)	(30.63)	(59.55)

Notes: 1) Bold implies comparisons (parametric vs. semi-parametric) are significant at $p < 0.05$.

2) Bold and Italic comparisons (parametric vs. semi-parametric) are significant at $p<0.10$.

3) a>b>c in paired comparisons at $p < 0.10$ (within parametric or semiparametric)

FIGURE 1. Segment Distance Response Surface at Mean Cost Minus One Standard Deviation.

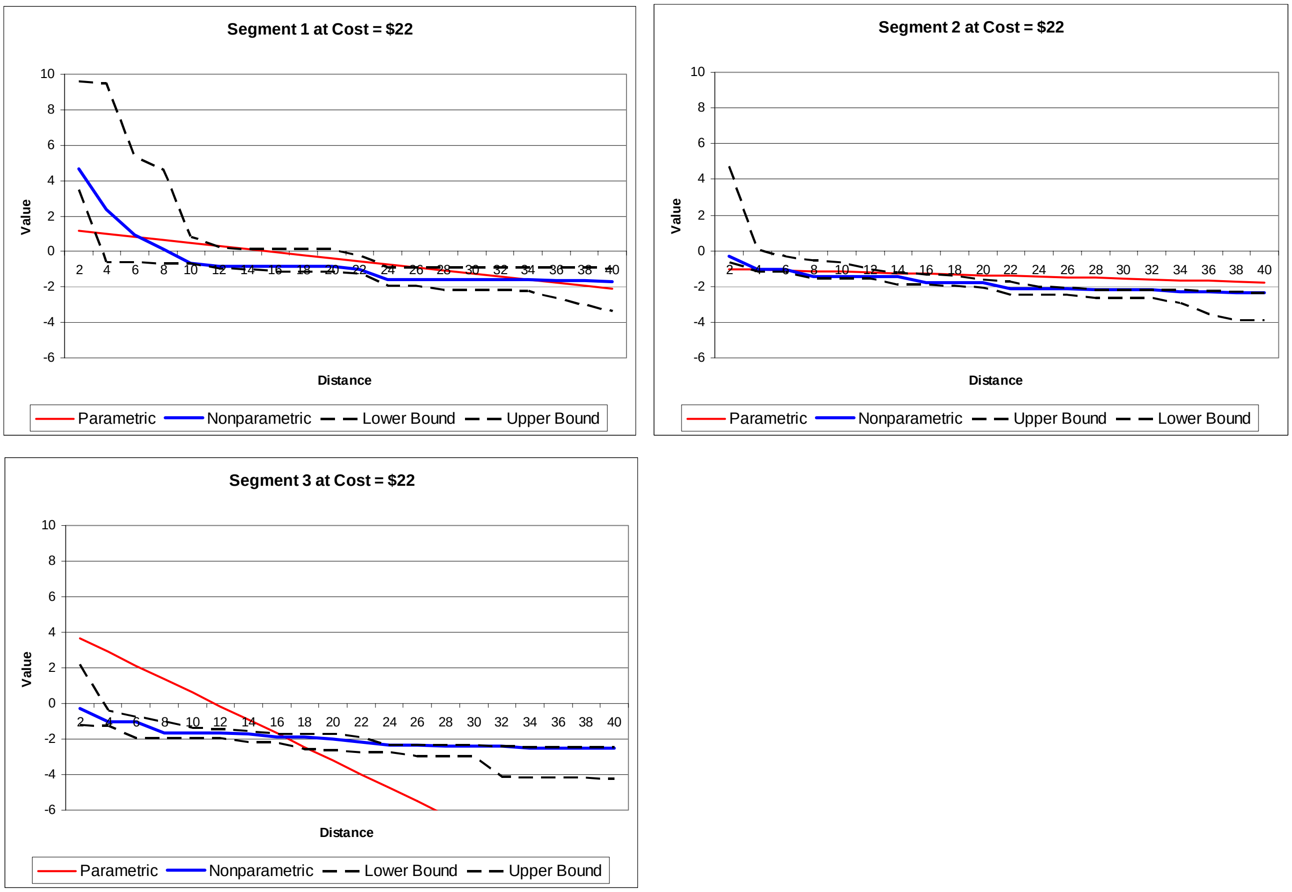


FIGURE 2. Segment Distance Response Surface at Mean Cost .

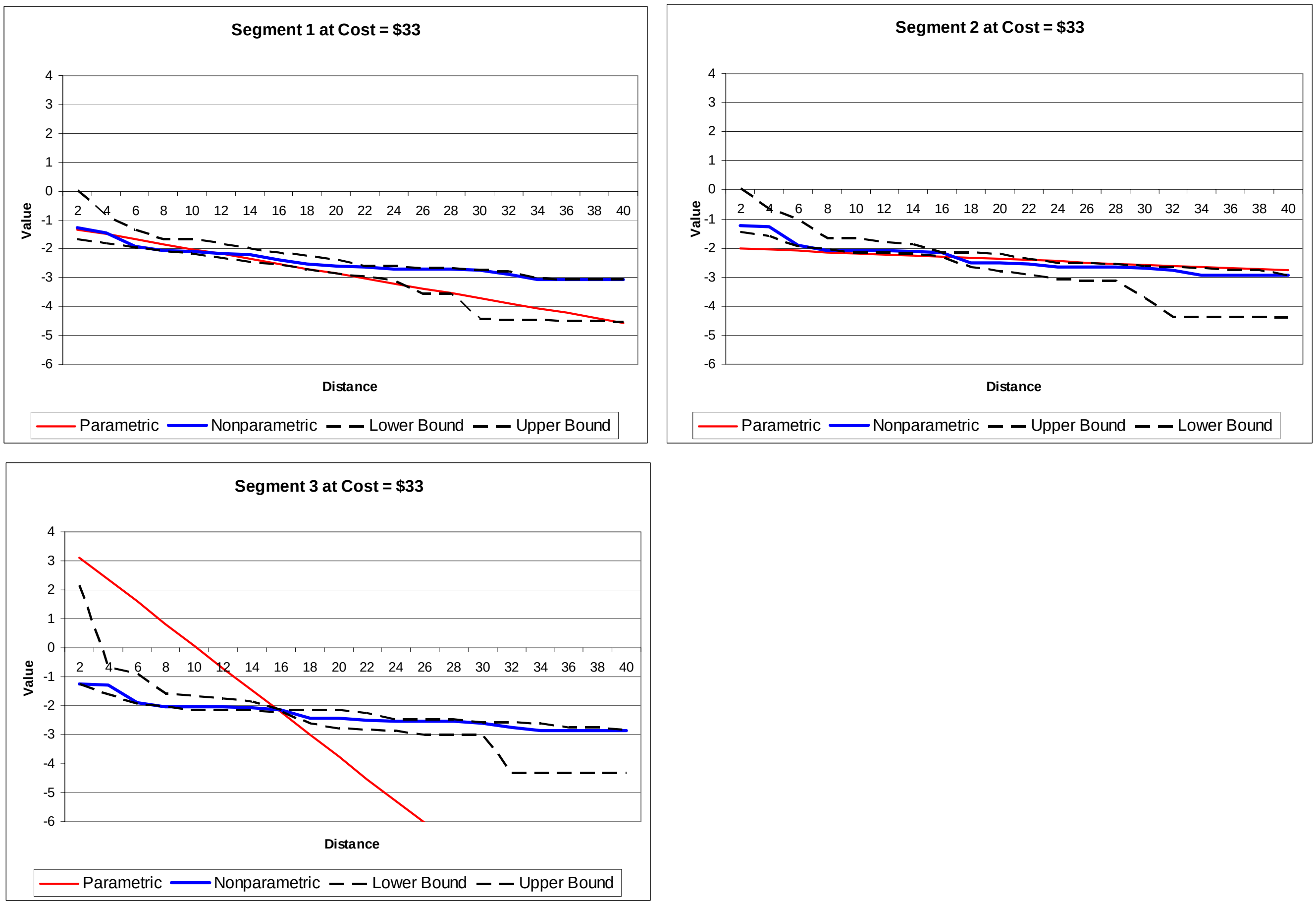
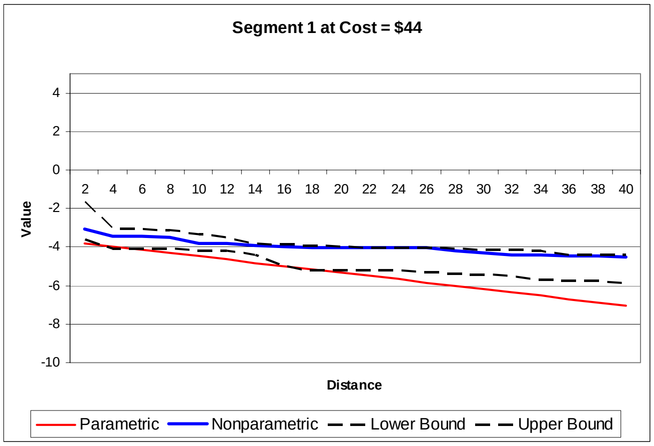
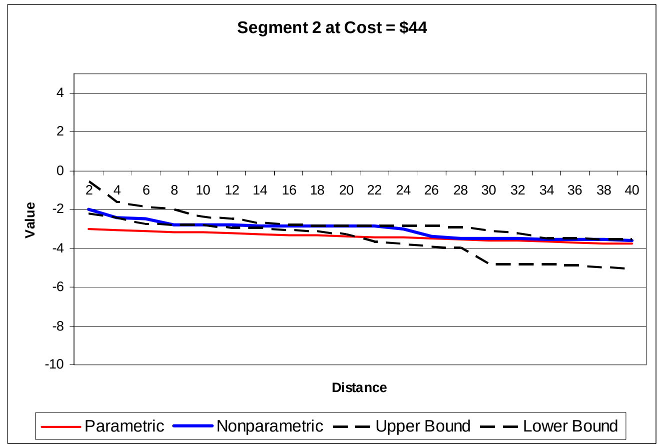


FIGURE 3. Segment Distance Response Surface at Mean Cost Plus One Standard Deviation.





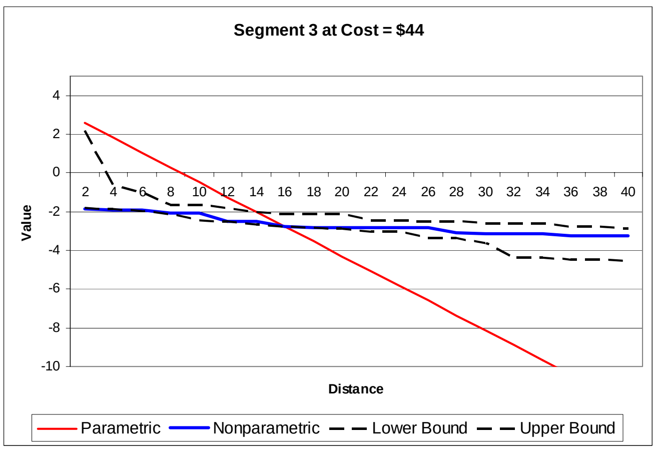


FIGURE 4. Segment Cost Response Surface at Mean Distance Minus One Standard Deviation.

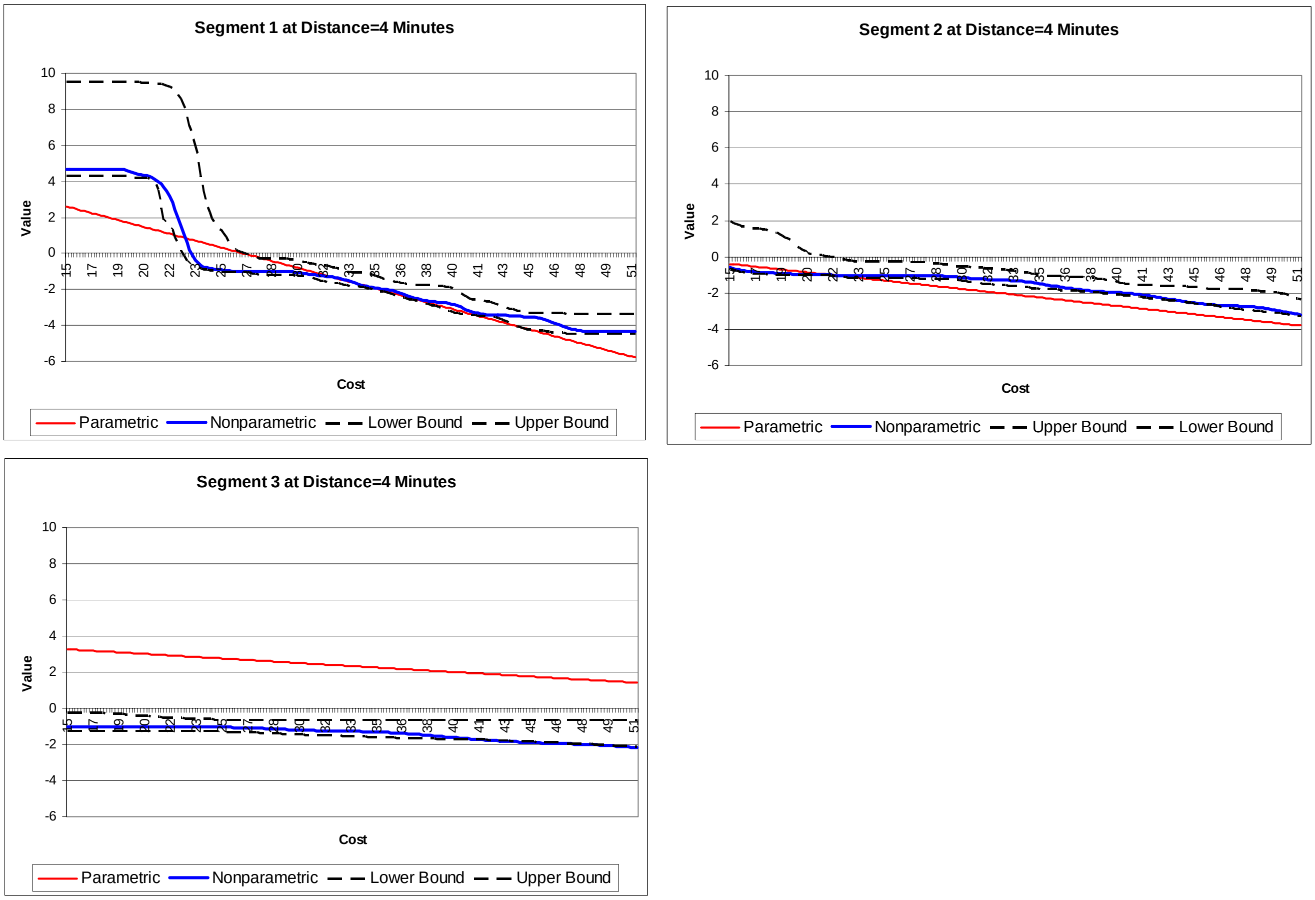


FIGURE 5. Segment Cost Response Surface at Mean Distance.

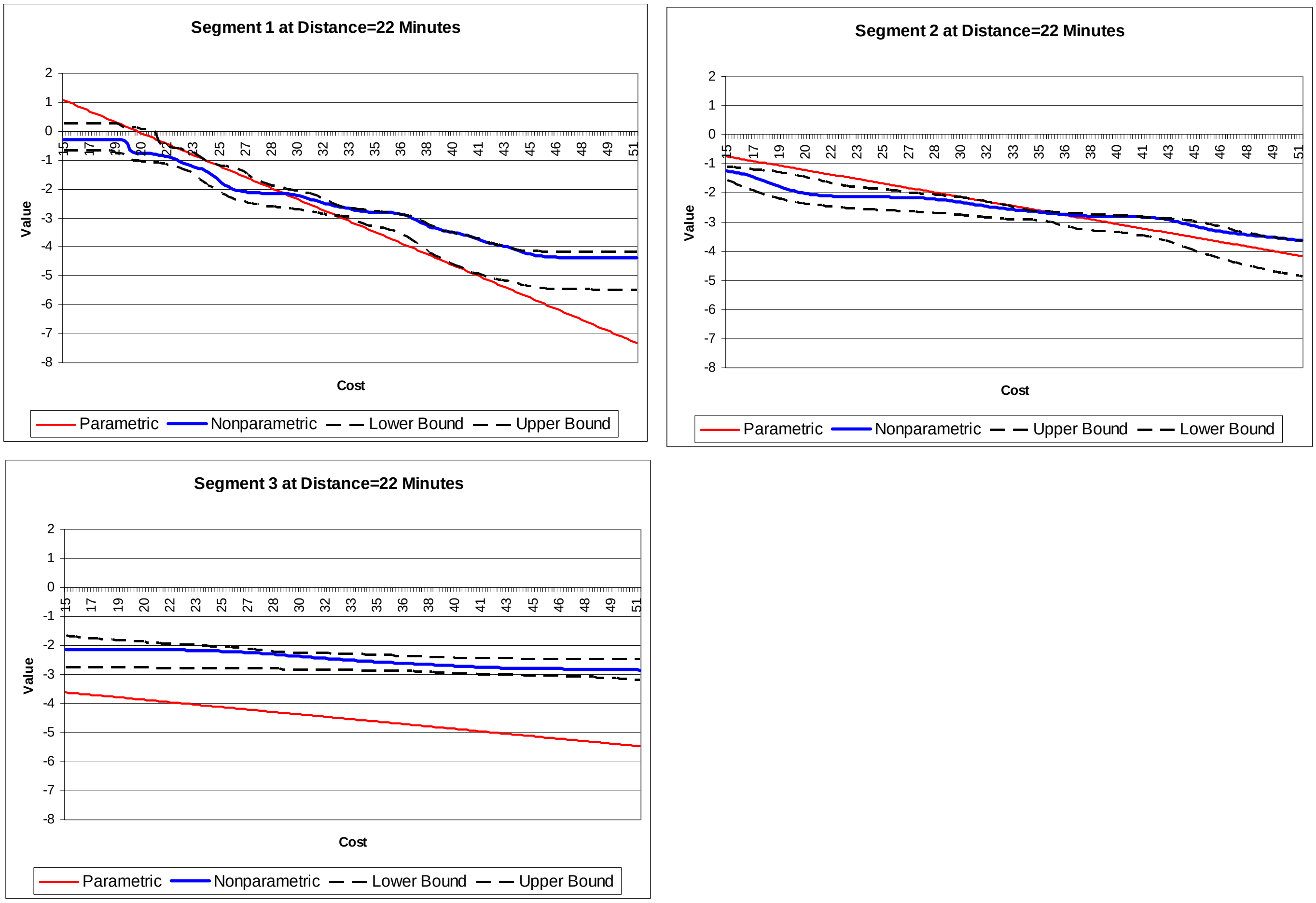


FIGURE 6. Segment Cost Response Surface at Mean Distance Plus One Standard Deviation.