Recruiting Talent*

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Abstract

We propose a new model of a competitive labor market in which firms privately screen workers to identify talent (e.g. via interviews, referrals), and study its implications for inequality and inefficiency. The equilibrium exhibits dispersion in wages and productivity: firms with superior screening skills post higher wages, attract better applicants, and recruit more talented workers. This equilibrium leads to an inefficient selection of talent into the industry, and can be improved by policies that reduce wage dispersion. We also provide a micro-foundation for firms' heterogeneous screening skills. When talented workers are better at screening (e.g. via superior referrals), a dynamic version of the economy converges to a unique steady-state in which differences in talent, profits and screening skills persist forever.

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1 Introduction

The success of most firms is built upon hundreds of individuals who take thousands of decisions, making it critical to identify and recruit the best talent. For example, the Netflix human resource manual states "One outstanding employee gets more done and costs less than two adequate employees. We endeavor to have only outstanding employees" (Hastings and McCord, 2009). Similarly, Google's head of human resources writes "Hiring is the single most important people activity in any organization. [...] Our greatest single constraint on growth has always, always been our ability to find great people" (Bock, 2015). A large economics literature measures the importance of employee talent, from top executives (Bertrand and Schoar, 2003) to blue-collar workers (Lazear, 2000). For example, Benson et al. (2019) study 200 business-to-business sales firms and show that, on average, the 75th percentile manager is 58% more productive than the 25th percentile manager, while the 75th percentile salesperson is over 1000% percent more productive than the 25th percentile salesperson.

The market for talent is plagued by imperfect information. Unobservable measures of quality take time to filter into wages (Farber and Gibbons, 1996), and changes in employers' information, in the form of criminal convictions or certification, have an observable impact on employment (Agan and Starr (2017), Pallais (2014)). As a result, firms carefully screen applicants, conducting interviews and obtaining referrals. For example, in Behrenz's (2001) survey, employers report that their most important source of information when hiring are methods of "private screening" in the form of interviews (41%) and personal contacts (25%), as compared to "public information" like references from past employers (21%), references from schools (5%) and the application (3%).

We address these facts with a parsimonious model of a competitive labor market, where firms privately screen workers (e.g. interviews, referrals), and investigate its implications for inequality and inefficiency. In the short-run, equilibrium gives rise to dispersion in wages and productivity across firms, with lower wages attracting worse, adversely selected applicants. This dispersion increases as screening skills improve. In the long-run, the economy converges to an equilibrium featuring persistent dispersion in talent and profits, as firms with superior screening skills complement them with high wages that attract better applicants. Surprisingly, this positive assortative matching between firms' screening skills and applicants' talent is inefficient. Indeed, we show that it minimizes firms' aggregate effectiveness in sorting talented workers into the industry. Policies that reduce wage dispersion increase efficiency.

The paper consists of three parts. In Section 2, we introduce a static model of labor market competition in which firms have private information about workers' talent. Specifically, a continuum of identical firms competes for a continuum of workers who have high or low ability. Talented workers have positive value added, while untalented workers would be better employed outside the industry; *mismatch* arises when untalented workers are employed in the industry, and when talented are not. Firms attract applicants by posting wages, and then receive independent noisy signals about each applicant. The market is frictionless, so the highest-paying firm chooses from all workers while other firms hire from the remaining, adversely selected pool of workers. The applicant pool quality thus endogenously declines with the wage rank, giving rise to equilibrium dispersion of wages and productivity. We show that the dispersion and segregation of talented workers across firms increases with firms' screening skills, helping to explain trends documented by Barth et al. (2016). Improved screening skills also reduce mismatch, but adverse selection prevents effective information aggregation compared to a benchmark model based on Phelps (1972), in which all firms receive the same signal.

In Section 3, we suppose that firms differ in their skill at screening applicants because of differences in their referral networks or decision making ability. Assuming that talent is scarce, we show that firm-worker matching is *positive assortative*. That is, firms with more skilled recruiters post higher wages, attract better applicants, and hire more talented recruits. Intuitively, skilled firms have a comparative advantage in hiring from a high-wage applicant pool with a balance of talented workers, rather than hiring from a low-wage pool in which few talented applicants remain. In sharp contrast to classic matching models (e.g. Shapley and Shubik (1971), Becker (1973)), the positive assortative matching seen in equilibrium is inefficient. Intuitively, high-wage firms screen applicants first and exert a negative *compositional externality* on low-wage firms by extracting talent from the applicant pool. Positive assortative matching maximizes this externality since the high-wage firms are skilled at extracting talent; this outweighs the private gain from positive assortative matching. Indeed, we show that negative assortative matching, whereby low-skill firms offer high wages and screen first, minimizes the externality and optimally selects talent into the industry. Thus, welfare is increased by policies that reduce wage inequality, lowering the dispersion of talent and the segregation of workers across firms.

In Section 4 we introduce a dynamic version of our model in which talented workers are better at recruiting and show that persistent differences in firms' talent and recruiting skills arise endogenously. In the model, the talent of firms evolves as workers retire and today's recruits become tomorrow's recruiters. In the unique equilibrium, talented firms post high wages, attract the best applicants and hire talented recruits, reinforcing their initial advantage. If all firms start off with similar talent, the better endowed accumulate talent over time, while the worse endowed hire from poor, deteriorating applicant pools and lose talent. The economy converges to a steady-state with persistent talent differentials, balancing two countervailing forces: imperfect screening which leads to mean reversion and equalizes firms, and positive assortative matching that amplifies differences across firms. While low-quality firms could in principle catch up by posting higher wages and hiring more talented workers, it is not profitable for them to do so. Thus, talent becomes a source of sustainable competitive advantage.

1.1 Literature

The static model of sequential screening is most closely related to Montgomery (1991) and Kurlat (2016). In Montgomery's classic model of referrals, firms make individualized wage offers: They first post high wages for disproportionately talented workers in the referred market; if they strike out, they pay low wages in the adversely selected, non-referred market. In contrast, our firms compete in a single labor market, which each firm setting a single wage. This modeling difference matters empirically: Kramarz and Skans (2014) find referred workers are paid below-average wages within a firm, but work at firms that pay above-average wages. It also matters theoretically: talent differences across firms persists in our model, but they erode in a fully dynamic version of Montgomery's model, developed in Section 4.4. Additionally, our simpler model helps us to study mismatch in Sections 2.3 and 3.2, which Montgomery does not address.

Kurlat (2016) studies financial markets with adverse selection where buyers' signals about sellers' assets are nested in that a more informed buyer knows everything that a less informed buyer knows. In a labor market application (e.g. Kurlat and Scheuer (2020)), nested signals best capture information that is, in principle, publicly accessible, such as reference letters. In contrast, we follow the auction literature in assuming that firms' signals are conditionally independent; this captures truly private information, such as personal referrals or interviews. This modeling difference matters: Our firms post different wages, matching is positive assortative and inefficient; in contrast, Kurlat's buyers post the same price with ties broken in favor of the less-informed buyers, and equilibrium is efficient. Intuitively, when signals are nested, high-skill firms can screen out all applicants who failed tests of low-skill firms and so do not mind hiring last. See Appendix A for more details.[1]

[1]In a related paper, Lockwood (1991) considers a random search model where firms screen workers via independent signals and unemployment duration is a negative signal of quality.

Our model complements the wider literature on wage dispersion. In Albrecht and Vroman (1992) and Burdett and Mortensen (1998), dispersion derives from firms competing for more workers in an economy with search frictions, whereas our dispersion derives from firms competing for better workers in an economy with adverse selection. In many labor markets, the pertinent search friction is in evaluating the quality of applicants, rather than finding them in the first place. Van Ours and Ridder (1992) find that "76% of all vacancies are filled by applicants who arrived during an application period that lasts for about 2 weeks", leading them to write that "vacancy durations should be interpreted as selection periods and not as search periods for applicants." This view is consistent with the extensive evidence of imperfect information in the labor market (e.g. Farber and Gibbons (1996)), the widespread use of referrals (e.g. Holzer (1987)), and the significant amount firms spend on screening candidates (e.g. Barron et al. (1985)).[2]

Our heterogeneous-firm model contributes to the literature on firm-worker matching. Becker (1973) observed that if firms and workers are heterogeneous and complementary, then more productive firms hire more talented workers. In a dynamic model, Anderson and Smith (2010) and Anderson (2015) suppose agents match each period and evolve as a function of the match; they show that equilibrium is efficient, and derive sufficient conditions for matching to be positive assortative. In contrast to this literature that focuses on complementary production, our paper focuses on asymmetric information in the hiring process, and shows that complementarities arise endogenously, with more skilled firms posting higher wages. Our model has different implications from Becker. On the normative side, we prove that, the equilibrium is inefficient; surprisingly, this inefficiency still pertains even if we allow for production complementarities (see Section 3.3). On the positive side, we examine how the wage dispersion and market-level mismatch change in response to private screening. Our mechanism is also consistent with evidence that more productive managers hire more productive recruits, who remain highly productive even after they switch teams (Gupta, 2018).

Our dynamic model provides a theory of firm evolution in which a firm's stock of talent is its key strategic asset. In addition to Montgomery (1991), this relates to a broader set of papers on firm dynamics in which competitive advantages stem from technology (Lucas and Prescott (1971), Hopenhayn (1992)), reputation (Jovanovic, 1982) or the stock of labor (Hopenhayn and Rogerson, 1993). By focusing on talent and recruiting, our paper provides a new channel through which firms can sustain a competitive advantage that

[2] In recent years, online job search has further reduced search frictions: In one study, Davis and Samaniego (2017) report that 45% of job applications arrive in the first 48 hours and half the job postings are taken down in a week, whereas the mean vacancy duration exceeds 40 days.

is particularly relevant for industries ranging from technology to sales. It also gives rise to predictions concerning the inter- and intratemporal relationship between productivity, wages and employee quality.

2 A Static Model of Sequential Screening

We first describe our benchmark model of sequential screening. A unit mass of identical firms, each with one vacancy, competes for a unit mass of workers. Workers differ in their talent θ, with proportion $\bar{q} \in (0,1)$ talented, $\theta = H = 1$, and the remainder untalented, $\theta = L = 0$. Firms select among applicants by administering a pass/fail test to each applicant. Talented workers always pass the test, whereas untalented workers are screened out with probability $p \in (0,1)$.

The labor market is anonymous and perfectly competitive. Firms simultaneously post wages and offer their job to any worker who passes their test. Workers only care about wages, and so accept the highest wage offer they receive. To operationalize this, order firms in terms of the wages they post and suppose workers then apply to firms from highest to lowest wage. The highest firm screens the anonymous workers in a random order and hires the first who passes their test; the adversely selected remainder then apply to the "second" firm, and so on until all firms and workers are matched.[3]

When a recruiter screens an applicant pool with expected talent q, proportion $1-(1-q)p$ of the applicants pass the test. Bayes' rule implies that the fraction of recruits who are talented equals

$$\lambda(q,p) := \frac{q}{1-(1-q)p}. \tag{1}$$

Clearly, expected talent $\lambda(q,p)$ increases in both the applicant quality q and the screening skill p.

Payoffs are as follows. Workers only care about wages and so accept any job paying more than their outside option, $\underline{w} \geq 0$. We call this outside option "unemployment", but it could be a job in a different industry. Productivity is 1 for talented workers and 0 for untalented workers. Thus, when a firm posts wage w and attracts applicants q, its expected profits are

$$\pi := \lambda(q,p) - w. \tag{2}$$

We solve for Nash equilibrium in wages.

[3]This description assumes firms post different wages, as happens in equilibrium. For concreteness, assume that in case of a tie, all workers break the tie in the same way, as if the firms were infinitesimally differentiated.

We assume that talent is scarce. This assumption is motivated by industries with relatively few highly productive individuals, such as technology companies or sales. For example, Bock (2015, p. 62) writes that "Only 10% of your applicants (at best) will be top performers."

Assumption. *Talent is scarce,*

$$\lambda(\bar{q}, p) \leq 1/2 \tag{3}$$

This is a joint condition on the talent distribution and screening skills, and states that a worker from the unselected pool who passes the test is more likely untalented than talented. We use this assumption in Proposition 1(a), but the main role is to guarantee the positive assortative matching in Sections 3 and 4.

Remarks. The assumption that firms use binary, pass-fail tests and that talented workers pass the test with certainty is without loss. To see this, assume a more general information structure with finitely many signals s that arise with probability $p^\theta(s)$. Firms then hire the first applicant with the signal $\bar{s}$ that maximizes the odds-ratio $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s})$. Recruit quality is then $\frac{q\bar{\ell}}{q\bar{\ell}+(1-q)}$ which collapses to (1) when $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s}) = 1/(1-p)$ for $\bar{s} =$"pass".

We can reinterpret firms' screening as referrals. Assume that each firm is connected to each talented worker via a referral with probability ϵ, and to each untalented worker with probability $\epsilon(1-p)$. Each firm extends provisional wage offers to their referrals, and the market clears from the top with workers accepting their best offers and firms rescinding their remaining offers once their position is filled.

If $\underline{w} = 0$, then all workers are employed and the game is constant sum. Assuming $\underline{w} > 0$ introduces a welfare margin, in that talented workers should be employed in the industry, and untalented workers should take the outside option. One can also interpret $\underline{w}$ as an operating cost for the firm; under this formulation, allocations and payoffs are the same while wages are shifted down by $\underline{w}$.

The model assumes that it is free to screen a worker. In reality, reading applications, conducting interviews and checking references takes time. Barron et al. (1985) report that the average firm spends about 8 hours screening applicants, with more educated roles requiring more screening. Indeed, Bock (2015, p. 76) writes that Google spends 150-500 hours interviewing for each role. Our analysis can easily accommodate such screening costs: If there is a cost κ to screen each applicant and firms screen applicants until one passes their test, profit (2) becomes $\pi := \lambda(q,p) - \kappa/(1-p(1-q)) - w$. We study this extension in Section 3.3.

2.1 Equilibrium

To characterize equilibrium, we first study how the quality of the applicant pool depends on the firm's rank in the wage distribution. Suppose all firms post different wages, write $x \in [0,1]$ for the resulting wage quantiles, and $Q(x)$ for applicant quality at wage quantile x.[4] The highest ranked firm faces applicant pool $Q(1) = \bar{q}$; thus, proportion $\lambda(Q(1), p)$ of its recruits are talented. Since firms select talented workers disproportionately, lower-ranked firms face an adversely selected applicant pool, meaning that $Q(x)$ falls as the firm rank x declines. Specifically, at rank x there is a total of $xQ(x)$ talented workers, of which firms $[x, x+dx]$ hire $\lambda(Q(x), p)dx$; hence $d[xQ(x)] = \lambda(Q(x), p)dx$. Rearranging, the talent pool evolves according to the *sequential screening equation*

$$Q_x(x) = \phi(Q(x), p, x) := \frac{\lambda(Q(x), p) - Q(x)}{x}. \tag{4}$$

Since screening is imperfect, some talent remains, $Q(x) > 0$, for all $x > 0$. However, at the bottom, firms pick over the applicants so many times that no talent remains, $Q(0) = 0$.[5]

In equilibrium, wages are distributed continuously. To see this, observe that if an atom of firms offered the same wage, then a firm could attract discretely better job applicants with a marginal wage raise. Similarly, if there was a gap $[w, w']$ in the wage distribution, then the firm offering w' could attract the same applicants at the lower wage w. As a result, the lowest wage in the wage distribution equals the outside option $\underline{w}$.

Turning to equilibrium payoffs, the identical firms compete away all profits. Wages thus coincide with expected productivity, $w(x) = \lambda(Q(x), p)$, where applicant quality $Q(x)$ evolves according to (4). The worst applicant quality $\underline{q}$ that firms are willing to consider is given by $\lambda(\underline{q}, p) = \underline{w}$. The fraction of unemployed workers $\underline{x}$ is then implicitly given by $Q(\underline{x}) = \underline{q}$. Hence, there is unemployment, $\underline{x} > 0$, if and only if $\underline{q} > 0$, which is the case if and only if $\underline{w} > 0$. To summarize:

Theorem 1. *Equilibrium wages and productivity are continuously distributed with a minimum of $\underline{w}$. The distribution $w(x)$ is uniquely determined by Bayes' rule (1) and sequential screening (4).*

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[4]If the wage distribution $G(w)$ has an atom at w, our tie-break rule (footnote 3) implies that the rank x of a firm with wage w is drawn uniformly from $[\lim_{\epsilon \to 0} G(w - \epsilon), G(w)]$. Theorem 1, below, implies that this complication does not arise in equilibrium.

[5]To see $Q(x) > 0$ for $x > 0$, note that $(\log Q(x))_x = (\lambda(Q(x), p)/Q(x) - 1)/x = (1/(1-(1-Q(x))p) - 1)/x$ is bounded for any fixed $x > 0$ since $p < 1$. On the other hand, $(\log Q(x))_x$ is of order $1/x$, and so the integral $\log Q(x) = \log \bar{q} - \int_x^1 (\log Q)_x$ diverges as $x \to 0$, implying $\log Q(0) = -\infty$, or $Q(0) = 0$.

2.2 Productivity and Wage Dispersion

Labor economists have documented considerable dispersion in productivity and wages across firms, and a strong correlation between the two variables.[6] Theorem 1 is consistent with these findings; moreover, dispersion rises as screening improves and talent becomes more scarce. To see this, we use the demanding log-dispersive order (Shaked, 1982), so dispersion increases in p if the talent ratio $\lambda(Q(x'), p)/\lambda(Q(x), p)$ increases in p for all $x' > x$ (where applicant quality $Q(x)$ also depends on $\bar{q}, p$), and similarly for $\bar{q}$.

Proposition 1. *We have the following:*
(a) The dispersion of wages and productivity increases in screening skill, p.
(b) The dispersion of wages and productivity falls in average talent, $\bar{q}$.

Proof. See Appendix B.3. □

Part (a) means that as screening skills improve, the top-wage firms raise their productivity relative to the mean. Intuitively, these top firms extract more talented workers from the applicant pool, lowering the productivity and wages of lower-wage firms (even though their screening skill p increased by the same amount). This finding may help explain the increased dispersion of productivity and wages over the course of the information revolution of the last forty years (Barth et al., (2016)). For example, superior screening may have come from improved job tests (Autor and Scarborough (2008), Hoffman et al. (2017)) and social network data (Munshi (2003), Beaman (2011)).

Part (b) shows that a reduction in the number of talented workers, $\bar{q}$, perhaps driven by skill-biased technical change, also leads to an increase in the dispersion of productivity and wages. Intuitively, if the number of talented workers halves, the top firm's talent drops by less than half, as they are able to screen out many of the untalented workers, leaving proportionally less talent for lower firms, thereby raising inequality.

Finally, our model also speaks to the "AKM-decomposition" of wage dispersion into worker- and firm-effects (Abowd et al., 1999). In particular, it shows that differences between workers can look like firm-effects when firms imperfectly observe worker types. To capture such a decomposition in our model, consider a twice repeated version of our game where each firm and worker is rematched in the second period. Also assume that workers

[6] On the productivity side, Foster et al. (2008) find that the standard deviation of productivity is 20 log-points within homogeneous 7-digit industries. Residual wage inequality (after controlling for observables) is even greater, as documented by Katz and Murphy (1992) and Card et al. (2013). The relationship between the two is studied by Abowd and Kramarz (2005) and Card et al. (2018), who show that productivity is correlated with both firm- and worker-effects from an AKM regression (Abowd et al., 1999).

obtain proportion b of their output via a bonus payment, so the total compensation for worker $\theta \in \{0,1\}$ equals $w(x)+b\theta$. An AKM wage regression would interpret $w(x)$ as the firm-effect (even though all firms have identical fundamentals) and $b\theta$ as the worker-effect. The model generates correlation between the two, consistent with Card et al. (2013, 2018). Furthermore, an increase in screening skills increases segregation, thereby increasing the dispersion of firm-effects, worker-effects and their correlation, as seen in Card et al. (2013) and Song et al. (2019).

2.3 The Value of Information

Our model describes the labor market for one industry, where a worker's productivity equals θ in that industry and $\underline{w}$ elsewhere. The social value of screening then consists in sorting talented workers, $\theta = H$, into the industry and untalented workers, $\theta = L$, out of the industry. The loss of welfare relative to the first-best allocation due to mismatch is thus given by:

$$M = \underbrace{\underline{q}\underline{x}(1-\underline{w})}_{\text{“Unemployed talented”}} + \underbrace{(1-\bar{q}-(1-\underline{q})\underline{x})\underline{w}}_{\text{“Employed untalented”}} \tag{5}$$

This mismatch is empirically important: At a micro level, Fredriksson et al. (2018) show that mismatched workers, as judged by a pre-employment test, have lower earnings growth and higher separation rate. At a macro level, Hsieh et al. (2019) estimate that 20-40% of US economic growth over the last 50 years has come from a better allocation of talent to jobs.

Writing mismatch as a function of firms' screening skills,

Proposition 2. *Mismatch $M(p)$ is decreasing in screening skills p. But a little private information has no value: If $\bar{q} \neq \underline{w}$, then $M'(0) = 0$.*[7]

Proof. See Appendix B.4. □

The first statement is intuitive: Improved selection by individual firms aggregates across firms. But the second statement is surprising: Marginal screening skills that improve the selection of individual firms, $\lambda_p(q,0) > 0$, do not aggregate. This result sharply contrasts with the classic “wisdom of crowds” results (e.g. Pesendorfer and Swinkels (1997)): A continuum of imprecise, independent signals is perfectly informative when

[7] In fact, the proof of Proposition B.4 in Appendix B.4 shows more strongly that all higher-order derivatives of $M(p)$ also vanish at $p = 0$.

aggregated by Bayes' rule, but perfectly uninformative when aggregated by our firms in equilibrium. Intuitively, improved private screening by high-wage firms aggravates the selection problems of low-wage firms.

The impact of adverse selection also means that mismatch is higher in our model of private screening than in a classic *public screening* model (e.g. Phelps (1972), Pallais (2014)). Consider a version of our model in which all firms receive the same public signal that screens out bad workers with probability p.[8] Posterior expected talent then equals $\lambda(\bar{q}, p)$ for the $1-(1-q)p$ applicants who pass the test and 0 for the $(1-q)p$ applicants who fail the test. This distribution of expected talent is a mean-preserving spread of the distribution of expected talent in our model with private signals (Theorem 1), which consists of a continuous distribution from $\underline{w}$ to $\lambda(\bar{q}, p)$ for employed workers and an atom at $\underline{q}$ for the unemployed. Since hiring decisions are efficient conditional on the available information, the superior information with public signals implies higher welfare, i.e. lower mismatch. Intuitively, failing a public test black-lists an untalented job applicant, while failing a private test allows him to re-enter the pool and worsen the adverse selection for other firms.[9]

3 Heterogeneous Screening Skills

Some firms are better at screening than others. For example, Google obtains more than half its workers via referrals and, for other applicants, obtains "back door" references from current employees (Bock, 2015, p. 80). More broadly, Bloom and Van Reenen (2007) show that firms vary in their ability to manage human capital, and that this ability is positively correlated with firm productivity. Bender et al. (2018) further show that much of this relationship is due to managerial quality. Such managers may participate in recruiting directly, or build institutions to identify and hire the most talented applicants.[10]

In this section, we wish to understand the implications of such heterogeneous screening skills for wages, productivity, and mismatch. Formally, suppose firms' screening skills p are distributed according to some continuous, strictly increasing cdf F on $[\underline{\underline{p}}, \bar{p}]$, where $0 < \underline{\underline{p}} < \bar{p} < 1$, allowing us to identify each firm with its screening skill p. For a given

[8] Phelps (1972)'s original model differs by assuming normal quality and signals.

[9] In fact, mismatch under public information $\tilde{M}(p)$ is easily computed. Since it is solely due to untalented applicants with a positive signal, we have $\tilde{M}(p) = (1-p)(1-\bar{q})\underline{w}$ and so $\tilde{M}'(0) = -\bar{q}\underline{w} < 0$, in contrast to Proposition 2.

[10] Indeed, the Bloom and Van Reenen survey asks managers questions such as "Do senior managers discuss attracting and developing talented people?" and "Do senior managers get any rewards for bringing in and keeping talented people in the company?"

wage profile, denote the skill of the firm with wage-rank x by $P(x)$. Applicant quality $Q(x)$ evolves according to the sequential screening equation (4), where we replace p by $P(x)$. As in Theorem 1, equilibrium wages are distributed continuously with a minimum of $\underline{w}$. And as before, we assume that talent is scarce, $\lambda(\bar{q}, \bar{p}) < 1/2$. We then ask: Which firms post higher wages? How are profits and wages determined in equilibrium?

3.1 Positive Assortative Matching

We say there is *positive assortative matching* (PAM) between firms and applicants if $P(x)$ is increasing, meaning firms with high screening skills p are matched with applicant pools of high quality, q. From Becker (1973), we know that equilibrium features PAM if skilled recruiters have a comparative advantage in screening applicants with higher expected talent, i.e. if expected recruit quality $\lambda(q, p)$ is supermodular.

Theorem 2. *Equilibrium exists and is unique. Firms with skill exceeding a cutoff* $\underline{p}$ *enter; matching is then positive assortative.*

Proof. Note the partial derivatives

$$\lambda_p(q, p) = \frac{q(1-q)}{(1-p(1-q))^2} \quad \text{and} \quad \lambda_{qp}(q, p) = \frac{(1-p(1-q)) - 2q}{(1-p(1-q))^3}. \tag{6}$$

Since $\lambda_p \geq 0$, there exists a $\underline{p} \in [\underline{\underline{p}}, \bar{p}]$ such that firms $p \geq \underline{p}$ enter, and firms $p < \underline{p}$ do not. Scarce talent (3) then implies that $1 - p(1-q) \geq 2q$, so $\lambda_{qp} > 0$ for all $q \leq \bar{q}, p \leq \bar{p}$, and matching is positive assortative. That is, higher skill firms post higher wages, and so $P(x)$ increases.

We can now construct the equilibrium. Given PAM, a firm's rank in the skill distribution equals its equilibrium wage rank, $F(p) = x$, and we can identify a firm by this rank x. The skill of the firm with wage-rank x is then given by $P(x) = F^{-1}(x)$, its applicant quality $Q(x)$ is determined by the sequential screening equation (4), and the recruit quality $\lambda(Q(x), P(x))$ by Bayes' rule, (1).

From this we can derive the entry threshold $\underline{p}$, wages, and profits. Denote the equilibrium wage required to attract applicants of quality q by $W(q)$. The marginal firm pays the outside option, so employment $\underline{x}$ is given by $\lambda(Q(\underline{x}), P(\underline{x})) = \underline{w}$, which determines the entry threshold via $F(\underline{p}) = \underline{x}$. Wages are determined by firm x's first-order condition,

$$W_q(Q(x)) = \lambda_q(Q(x), P(x)). \tag{7}$$

Finally, profits $\Pi(p)$ follow by the envelope condition

$$\Pi_p(P(x)) = \lambda_p(Q(x), P(x)). \tag{8}$$

Intuitively, productivity $\lambda(q,p)$ depends on both the applicant pool quality and the screening ability; workers capture the marginal benefit of the former and firms capture the marginal benefit of the latter.

The so constructed wage profile $W(Q(x))$ is the only possible candidate for an equilibrium. To verify that the wages are indeed optimal, it suffices to note that marginal profits $\lambda_q(q,p) - W_q(q)$ are single-crossing in p and matching is positive assortative; hence the FOC (7) implies global optimality. □

Theorem 2 captures a natural complementarity in the recruiting function λ. Intuitively, recruiting skills do not matter if all applicants are either talented or untalented, but they do matter when applicant quality is intermediate. Since $Q(x)$ is bounded above by $\bar{q}$, our scarce-talent assumption (3) implies that skilled firms have a comparative advantage at screening better applicants. Thus, skilled firms pay high wages, attract high-quality applicants, recruit talented workers, and achieve high productivity and profits. The result also shows that heterogeneous screening skills raise both the dispersion of productivity and the level of profits in the economy.[11]

3.2 Equilibrium Inefficiency

Equilibria in matching models with transferable utility are typically efficient (e.g. Shapley and Shubik (1971), Becker (1973)). Surprisingly, this welfare theorem fails in our model. In particular, Theorem 3 shows that mismatch is maximized by positive assortative matching and minimized by negative assortative matching (NAM). The key difference to standard matching models is the compositional externality: The quality of applicant pool is endogenous and depends on the screening skill of higher-paying firms. Intuitively, high-skilled firms pick out more talented workers than low-skilled firms and introduce more adverse selection. This externality is maximized by PAM and minimized by NAM.[12]

[11] In a previous version of this paper, Board et al. (2017), we characterize equilibrium matching when talent is abundant using a slightly different parametrization of firms' screening skills. In contrast to the increasing wage function $w(p)$ seen in Theorem 2, equilibrium wages $w(p)$ are hump-shaped.

[12] Compositional externalities of a different nature are seen in models of directed search. In Albrecht et al. (2019), workers applying to recent job postings exert an externality on later applicants if employers fail to remove their filled "phantom vacancies" from the market. In Athey and Ellison (2011)'s model of position auctions, high-quality advertisers serve more searchers than low-quality advertisers when winning first position in the ad auction; this order minimizes search costs, meaning equilibrium is efficient.

To illustrate the idea, suppose firms are either skilled, $p > 0$, or unskilled, $p = 0$; to abstract from entry, assume firms are on the short side of the market and the minimum wage $\underline{w}$ does not bind, so all firms enter. Under PAM, first the skilled firms hire from the unselected pool $\bar{q}$; unskilled firms then hire from an adversely selected pool with quality $q < \bar{q}$. In contrast, under NAM, unskilled firms hire from the unselected pool $\bar{q}$; they do not change the quality of the talent pool, and so the skilled firms also hire from a pool of quality $\bar{q}$. Since the unskilled firm impose no compositional externality on the skilled firms, more talented workers are hired under NAM than under PAM, raising social surplus.[13]

To argue the inefficiency of the competitive equilibrium more generally, consider a planner who can direct all firms' entry and wage decisions but is subject to the same informational frictions.[14] Her problem is to choose employment level $1 - \underline{x}$ and a matching function, or equivalently *screening order*, $P : [\underline{x}, 1] \to [\underline{p}, \bar{p}]$ to minimize mismatch, or equivalently, maximize surplus

$$\int_{\underline{x}}^{1} [\lambda(Q(x), P(x)) - \underline{w}] dx. \tag{9}$$

For example, in equilibrium, matching is positive assortative $P^{\text{PAM}}(x) = F^{-1}(x)$, with entry cutoff $\underline{x}$ satisfying $\lambda(Q(\underline{x}), P^{\text{PAM}}(\underline{x})) = \underline{w}$; assuming the same entry behavior, NAM is characterized by $P^{\text{NAM}}(x) = F^{-1}(\underline{x} + 1 - x)$. In contrast to the standard assignment model, the surplus of the x-ranked firm in the integrand of (9) depends on $P(x')$ for $x' > x$ via the applicant quality $Q(x)$.

We first argue that the planner wants the highest skilled firms to enter, and hence the matching function P is measure-preserving with range $[\underline{p}, \bar{p}]$, where $F(\underline{p}) = \underline{x}$. Increasing skills $P(x)$ at wage rank x increases the surplus at that rank, $\lambda(Q(x), P(x))$, but reduces applicant quality $Q(\check{x})$ at lower ranks $\check{x} \in [\underline{x}, x]$. This negative indirect effect diminishes but does not overturn the positive direct effect. Formally, we show in the proof of Theorem 3 that the indirect effect scales down the direct effect by a factor

$$\exp\left(- \int_{\underline{x}}^{x} \frac{\lambda_q(Q(\check{x}), P(\check{x}))}{\check{x}} d\check{x} \right). \tag{10}$$

[13]This argument is incomplete. In fact, skilled firms are worse off under NAM. While unskilled firms do not reduce pool quality, they do reduce the pool size x. And while x does not directly affect skilled firms' recruiting, it magnifies the adverse selection induced by high-wage skilled firms on low-wage skilled firms since the former extract the same amount of talent from a smaller pool, as captured by the denominator x in (4). Despite this complication, Theorem 3 implies that NAM minimizes mismatch.

[14]In particular, the planner cannot communicate firms' test results to each other. With a continuum of firms and independent tests, allowing such communication would trivially solve any mismatch.

Turning to the screening order $P(\cdot)$, we claim that surplus (9) increases whenever a low-skill firm p is promoted from wage rank x to $x' > x$ past firms with higher skills $P(\hat{x}) > p$ for $\hat{x} \in [x, x']$. Thus PAM minimizes surplus, and NAM maximizes it. In a discrete analogue of our model, consider two firms with screening skills p and $p' = p + dp$ at adjacent wage ranks x and $x' = x + dx$ under PAM. The corresponding applicant quality equals $q = q' - \phi(q', p', x')dx$ and q', recalling the compositional externality ϕ from (4). How does switching their screening order affect their joint recruited talent and hence surplus? First, the low-skill firm p now screens the better pool; since $\lambda(q, p)$ is supermodular, this lowers total surplus of the two firms by $\lambda_{qp}\phi dpdx$. In Figure 1 this is represented by the vertical shift from white circles representing PAM to the black circles representing NAM. If the pool quality was exogenous as in Becker, this would be the end of the story. In our model, there is a second effect: firm p extracts $\phi_p dpdx$ less talent, increasing surplus by $\lambda_q \phi_p dpdx$. In Figure 1, this is represented by the applicant quality of firm p' rising from q' to q'', and the associated shift of the black circle to the black square. The next inequality shows that the second term outweighs the first, and so moving the low-skill firm p ahead of the high-skill firm p' raises surplus

$$\frac{\lambda_{qp}}{\lambda_q} = 2\frac{1-q}{1-p(1-q)} - \frac{1}{1-p} < \frac{1-q}{1-p(1-q)} = \frac{\lambda_p}{\lambda} < \frac{\lambda_p}{\lambda}\frac{\lambda}{\lambda - q} = \frac{\phi_p}{\phi}. \tag{11}$$

In words, marginal recruiting success λ_q is less sensitive to p than absolute recruiting success λ, which in turn is less sensitive to p than the compositional externality ϕ. Thus, total surplus of the two firms p, p' is maximized by NAM; the effect on aggregate surplus (9) must be discounted by (10), but the sign remains unchanged. The compositional externality overturns one of the most fundamental insights of the assignment model, namely the First Welfare Theorem.

Theorem 3. *For any level of unemployment $\underline{x} > 0$, PAM minimizes surplus (9) and NAM maximizes surplus.*

Proof. See Appendix C.2. □

Theorem 3 has practical implications. In some applications, the planner might be able to directly implement negative assortative matching. For example, in the NFL "reverse" draft the lowest-ranked football teams pick first. This lowers talent dispersion and makes games more competitive; Theorem 3 suggests that it also maximizes sorting of talent into the league.

If the planner cannot observe the skill of different firms, she can still improve welfare by restricting the set of admissible wages. In equilibrium, high-skill firms offer weakly

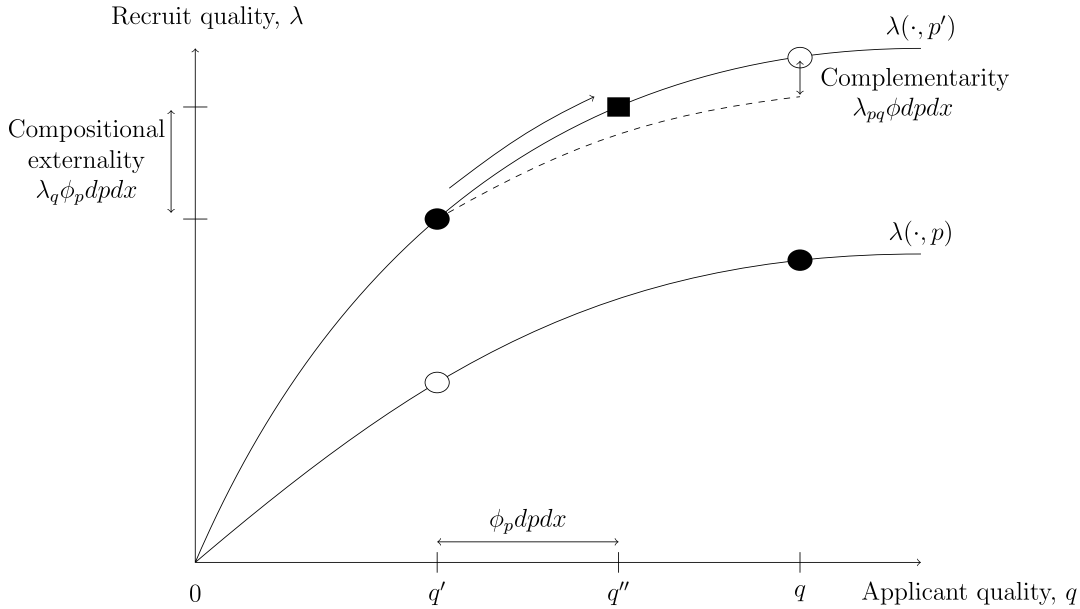

Figure 1: This figure illustrates the supermodularity and compositional externality that arise in Theorem 3. The empty dots represent PAM, the solid dots represent NAM if applicant quality were exogenous (the "Becker effect"), and the shift to the square comes from the endogeneity of the applicant pool (the "Akerlof effect").

higher wages than low-skill firms by Theorem 2. Thus, the planner's optimal policy is a single wage, inducing firms to select in a random order. We argue in Appendix C.3 that the same outcome can be achieved by a wage cap. This argument suggests that the NCAA's ban on paying athletes may raise talent in competitive college sports by preventing colleges with the best scouts bidding away the best athletes and lowering the quality of the marginal programs. Such a policy improves surplus in our model, but may also lower the utility of college athletes.

3.3 Discussion of the Compositional Externality

In the competitive equilibrium, all firms face the same equilibrium wage schedule $W(Q(x))$ even though the compositional externality on lower paying firms depends on their screening skills p. In contrast, Pigouvian wages that support the efficient, negative assortative matching would depend both on the screening rank x and the screening skills p. Here we discuss the foundations and implications of this economic force.

The Robustness of the Compositional Externality: In our setting, the positive assortative matching seen in equilibrium is inefficient from the planner's perspective. This result is robust to productive complementarities. To see this, consider a Becker-style model in which a firm's revenue is multiplicative in firm type and expected worker type $p \cdot \lambda(q, p)$, so high-type firms also have a higher marginal product. Since the (exogenous) productive-complementarity reinforces the (endogenous) screening-complementarity, equilibrium sorting remains positive assortative. More surprisingly, the compositional externality overcomes both the productive-complementarity and the screening-complementarity, and PAM remains inefficient. See Appendix C.4 for details.

Second, the result is robust to screening costs. Consider the screening cost model mentioned in Section 2, in which profit (2) becomes $\pi := \lambda(q, p) - \kappa/(1 - p(1 - q)) - w$. Since skilled firms are more selective, they must pay higher screening costs and are more sensitive to applicant quality; thus equilibrium continues to exhibit PAM. Moreover, this equilibrium continues to be inefficient, with the planner improving on PAM by swapping neighboring firms. See Appendix C.5 for details.

The Role of Perfect Competition: In a two-firm version of our model, Pigouvian wages arise endogenously. To see this, suppose firm i has mass n^i vacancies and screening skills p^i. Each firm consists of many independent divisions, who share screening skills and are bound by a common wage policy, but do not share information about applicants. To keep the wage-offer game simple, consider a second-price auction with outside option $\underline{w}$. Thus, the high-wage firm screens unselected applicants and pays the low firm's wage, and the low firm faces adverse selection and pays the workers' outside option $\underline{w}$.

We claim that the unique equilibrium of the two-firm model gives rise to negative assortative matching. When firm i screens first, applicant quality follows (4), with $P(x) = p^i$ for $x \geq 1 - n^i$ and $P(x) = p^j$ for $x \leq 1 - n^i$. Firm i's productivity is then given by $\bar{\Lambda}^i := \int_{1-n^i}^{1} \lambda(Q(x), p^i)dx$ and firm j's by $\Lambda^j := \int_{1-n^i-n^j}^{1-n^i} \lambda(Q(x), p^j)dx$. As usual in a second-price auction, it's a weakly dominant strategy to bid the value of choosing first, $w^i = \underline{w} + \bar{\Lambda}^i - \Lambda^i$, which equals the increase in productivity from avoiding adverse selection. When both firms bid their values, firm i offers the higher wage if and only if $\bar{\Lambda}^i - \Lambda^i \geq \bar{\Lambda}^j - \Lambda^j$; that is, if and only if having firm i screen first maximizes aggregate surplus. Since negative assortative matching maximizes surplus by Theorem 3, the low-skilled firm bids the higher wage. In contrast to the competitive equilibrium, wage schedules are firm-specific with firm i's wage offer w^i, which firm j's must pay in order to screen first, accounting for firm j's skill-specific externality on i.

Endogenous Recruiting Skills: Finally, we note that the compositional externality also leads firms to overinvest in their recruiting skills if firms can choose their recruiting skill p at cost $c(p)$. This is most clearly seen when the game is constant sum, $\underline{w} = 0$, where all investment is wasteful. More generally, firms that invest highly into screening skills also post high wages in equilibrium, and the social return of investment falls short of the private return $\lambda_p(Q(x), p)$ by the factor (10). This result contrasts with the finding of efficient investment found in classic matching models, e.g. Cole et al. (2001).

Overinvestment can be important in practice. Bock (2015, p. 60) urges companies to spend more on screening recruits, writing that "you can find a way to hire the very best, or you can hire average performers and try to turn them into the best. [...] At Google, we front-load our people investment. This means the majority of our time and money spent on people is invested in attracting, assessing, and cultivating new hires." Alas, not every firm can hire the best.

4 Talent as a Sustainable Competitive Advantage

We now embed our static labor market into a model of firm dynamics to endogenize firms' screening skills and study the evolution of talent over time. The key premise is that firms with more talented employees are more skilled at recruiting. For example, Gupta (2018) shows that more productive sales managers hire more productive salespeople, while Waldinger (2012, 2016) finds that the loss of star professors in Nazi Germany led to a permanent reduction in the quality of hires. Firms thus desire talented workers both for the immediate increase in productivity, and for the benefit of having skilled recruiters in the future. Indeed, Bock (2015, p. 85) writes "the first step to building a recruiting machine is to turn every employee into a recruiter."

There are two rationales for talented employees to be better at recruiting. First, they may provide better referrals. Referrals are a crucial source of information for firms, accounting for a third of US jobs (Holzer, 1987). Moreover, referrals are better than average applicants and those coming from high productivity workers are particularly high quality. In their field experiment, Beaman and Magruder (2012) find that productive workers have more and higher-performing referrals than unproductive workers. Similarly, Pallais and Sands (2016) find that "a referrer's performance is a strong predictor of her referral's performance."[15]

[15] In observational studies, Burks et al. (2015) find that in trucking, "there are large differences in profits between referrals from high-productivity referrers compared to low-productivity referrers", while Hensvik

Second, talented employees may be better at recruiting since it takes competence to assess competence. This idea underlies the famous Dunning-Krueger effect: "The skills you need to produce a right answer are exactly the skills you need to recognize what a right answer is."[16] This is seen in Beaman and Magruder (2012), where "high-ability participants are able to predict their referrals' ability [...] low-ability participants, on the other hand, are not systematically able to predict their referrals' performance." At a more systematic level, more talented employees may be able to design better technologies and institutions to better screen applicants.

Below we show that, as in the static model, equilibrium is unique with high-talent firms offering high wages that complement their superior screening skills; they thus attract superior applicants, amplifying their talent advantage. We characterize firm dynamics, and show that the economy converges to a steady state that exhibits persistent dispersion in talent, wages and productivity. In equilibrium, the positive assortative firm-applicant matching offsets the regression to mediocrity that results from imperfect screening. Importantly, steady-state talent differences are sustained by firms' endogenous wage choices and thus arise for any degree of correlation between productivity and recruiting skills, however weak.

This result complements Montgomery (1991)'s classic two-period referral model which has been influential since it "allows for positive profits from referrals, [...] presents a natural rationale for endogenous skill segregation across firms, [and...] can generate wage inequality" (Hensvik and Skans, 2016). However, in the fully dynamic version of Montgomery's model, that we develop in Section 4.6, firm heterogeneity in profits, skills and wage levels fades away in the long run. In our model, such differences persist.

4.1 Model

Time $t \geq 0$ is continuous. There is a unit mass of firms, each with a unit mass of jobs. At time t, a firm is described by its proportion of talented workers $r(t)$; initially, the distribution of $r(0)$ is exogenous. At every instant $[t, t + dt]$, proportion αdt workers retire, leaving firms with vacancies. In the job market, there are then αdt open jobs and

and Skans (2016) write that "the cognitive abilities of linked entrants are positively associated with the abilities of their linked incumbents" and that "firms with more productive employees rely more on social ties." This positive association is a natural consequence of homophily along traits like intelligence (e.g. McPherson et al. (2001)). Indeed, Brown et al. (2016) find that "most referrals take place between a provider and a recipient with similar characteristics in terms of age, gender, ethnicity, education, and division and staff level within the corporations."

[16]David Dunning in "The Anosognosic's Dilemma: Something's Wrong but You'll Never Know What It Is," *New York Times*, 10th June, 2010.

αdt applicants, of whom fraction $\bar{q}$ are talented. Analogous to the static model, firms compete for these applicants by posting life-time wages $w(t)$.

We assume that talented workers have an advantage in recruiting, either because of superior referral networks, or because of better judgment. We describe this relationship by a skill function $\psi(r)$ satisfying $\psi_r > 0$ and $\psi_{rr} \geq 0$. For example, if an employee with talent $\theta \in \{L, H\}$ has screening skill p^θ and firms ask random employees to act as recruiters, possibly by providing a referral, then the skill function is linear, $\psi(r) = p^L + r(p^H - p^L)$. Indeed, Pallais and Sands's (2016) oDesk experiment found that the average referred worker submitted work 13pp more often than referrals of the 20% least-productive workers. As before, we assume that talent is scarce for all firms, which holds if $\lambda(\bar{q}, \psi(1)) < 1/2$.[17]

If a firm with talent r posts wage w with rank x at time t, it attracts applicants with quality $Q(x,t)$ and hires recruits of quality $\lambda(Q(x,t), \psi(r))$. Writing $R(x,t)$ for the talent of the firm with wage rank x at time t, $Q(x,t)$ is determined by the sequential screening equation,

$$Q_x(x,t) = \phi(Q(x,t), \psi(R(x,t)), x) \text{ and } Q(1,t) = \bar{q}, \tag{12}$$

as in (4). In turn, the evolution of a given firm's talent $r(t)$ with wage rank $x(t)$ is given by the difference between its inflow λ and outflow r,

$$r_t(t) = \alpha\Big(\lambda(Q(x(t),t), \psi(r(t))) - r(t)\Big). \tag{13}$$

Turning to payoffs, workers maximize lifetime wages $w(t)$, while firms' revenue equals $r(t)$. To abstract from entry and exit, suppose that there is no outside option, $\underline{w} = 0$.[18] A firm's problem is to choose wages to maximize total discounted profits. Denoting the discount rate by $\beta > 0$, its value function is

$$V(r,s) = \max_{\{w(t)\}_{t\geq s}} \int_s^\infty e^{-\beta(t-s)}(r(t) - \alpha w(t))dt, \tag{14}$$

where $r(t)$ evolves according to (13) with initial condition $r(s) = r$.[19]

An equilibrium is given by a wage path $\{w(t)\}_{t\geq 0}$ for every firm,[20] so that given

[17] This assumption states that a firm exclusively endowed with talented workers has less that 50% talented recruits. In Section 4.3 we consider an example where firms are initially similar; the assumption can then be weakened to $\lambda(\bar{q}, 1/2) < 1/2$.

[18] With $\underline{w} = 0$, the game is constant-sum and there is no scope for inefficiency. In an earlier version of this paper, Board et al. (2017), we showed that a version of our inefficiency result, Theorem 3 extends to the dynamic model.

[19] Note that the "wages" $w(t)$ are really one-time payments to each of the α newly hired employees at time t. More realistically, one could model worker compensation as constant flow wage $(\beta + \alpha)w(s)$ until retirement, but it simplifies the accounting to have firms incur these costs up-front.

[20] As in the static model, the restriction to deterministic wages is without loss. In principle, a firm

the induced wage ranks $x(w,t)$ and applicant qualities $Q(x,t)$, every firm's wage path is optimal. We say an equilibrium is *essentially unique*, if the induced distribution over equilibrium trajectories $\{r(t)\}_{t\geq 0}$ is unique.[21]

4.2 Firm's Problem

First, we study a firm's optimal wage path $\{w(t)\}_{t\geq 0}$ for any given applicant function $Q(x(w,t),t)$ without imposing equilibrium restrictions on other firms. As in Section 3, it is convenient to write $W(q,t)$ for the wage required to attract applicants q at time t, and let the firm optimize directly over the applicant pools $\{q_t\}_{t\geq 0}$. After this change of variable, the firm's Bellman equation becomes

$$\beta V(r,t) = \max_q \{r - \alpha W(q,t) + \alpha(\lambda(q,\psi(r)) - r)V_r(r,t) + V_t(r,t)\}. \tag{15}$$

Firm value is determined by its flow profits plus appreciation due to talent acquisition and a secular trend. Assuming wages are differentiable, the first-order condition is

$$W_q(q,t) = \lambda_q(q,\psi(r))V_r(r,t). \tag{16}$$

Intuitively, the cost of attracting better applicants (the LHS) must balance the gains of a higher quality applicant pool which increases the recruit quality and thereby firm value (the RHS). The RHS of (16) is increasing in r, so firms with more talent have a higher marginal benefit from attracting better applicants, yielding positive assortative matching. Intuitively, firms with more talent have higher marginal benefit from better applicants $\lambda_q(q,\psi(r))$ because of the supermodularity of λ (see Theorem 2), and such firms have a higher marginal value of talent $V_r(r,t)$ because $V(r,t)$ is convex in r (see Appendix D.1). Hence, wages are dynamic complements: an increase in today's wage raises tomorrow's talent, and thereby tomorrow's optimal wage.

To compute equilibrium wages from the first-order condition (16), we write $r(u)$ and $q(u)$ for the equilibrium trajectory of talent and applicant quality, and apply the envelope theorem and the law of motion of firm talent (13) to compute

might mix between two wages by switching between them arbitrarily fast. To avoid measurability issues associated with such strategies, we allow for "distributional wage strategies" but show in Section 4.3 that equilibrium strategies are almost always pure.

[21] This definition avoids two spurious notions of multiplicity. First, in continuous time, any firm's optimal strategy $\{w_t\}_{t\geq 0}$ can be unique only almost always. Second, if two or more firms are initially identical but then drift apart, only the distribution of trajectories can be determined uniquely.

$$V_r(r(t),t) = \int_t^\infty e^{-\int_t^s \beta+\alpha\left(1-\lambda_p(q(u),\psi(r(u)))\psi_r(r(u))\right)du} ds, \tag{17}$$

as we formally show in Appendix D.1. Intuitively, the future benefit of better employees is discounted both at the interest rate β and the retirement rate α. But selective recruiting raises the persistence of firm talent or, equivalently, reduces the talent decay rate by a factor $1 - \lambda_p \psi_r$.

4.3 Equilibrium

Given the single-firm analysis, it is straightforward to characterize equilibrium. Firms with more talent post higher wages and attract better applicants. More strongly, even if firms share the same talent $r(0)$ initially, they post different wages (as in the static model), recruit different types of workers, and diverge immediately (see Appendix D.2). Thus, in equilibrium, each firm is characterized by a rank x, which describes the firm's position in the talent, applicant, and wage distribution at all times $t > 0$.

Equilibrium is then characterized in two steps

(1) Allocations. At time t, applicant quality $Q(x,t)$ is determined by sequential screening (12). The evolution of firm x's talent $R(x,t)$ is then given by the firm dynamics equation (13) with $x(t) \equiv x$.

(2) Payoffs. Firm x's marginal value of talent is determined by (17), with $q(u) = Q(x,u)$. Using this, wages $W(q,t)$ are given by the first-order condition (16), with $r = R(x,t)$, $q = Q(x,t)$, and $W(0,t) = \underline{w} = 0$.

Given these wages, the FOC (16) implies global optimality of the HJB (15) because the net benefit of attracting marginally better applicants $\lambda_q(q,\psi(r))V_r(r,t) - W_q(q,t)$ single-crosses in r. Standard verification theorems then imply that the policy functions are indeed optimal. To summarize:

Theorem 4. *Equilibrium exists and is essentially unique. Firm-applicant matching is positive assortative and the distribution of talent has no atoms at any $t > 0$.*

Proof. Only the last claim, about no atoms at $t > 0$, remains to be shown. See Appendix D.2. □

This result shows that even if firms start off with identical talent, some post higher wages than others, attract better applicants, and hire better recruits. These firms accumulate talent, continue to pay high wages, and the distribution of talent disperses over time.

To understand the evolution of talent $R(x,t)$ and applicant quality $Q(x,t)$, consider Figure 2. At $t=0$, all firms employ average workers, with quality $\bar{q}=0.25$. The "vertical" lines represent the cross-sectional distribution of (r,q) at different times, while the "horizontal" lines represent the sample-paths of selected firms. The top-ranked firm recruits from the constant applicant pool $Q(1,t)=\bar{q}$, and so (13) implies that its talent grows monotonically and converges to a steady state. For lower-ranked firms, the dynamics are more subtle. For instance, firm $x=0.5$ initially improves as its recruits are more talented than its retirees. However, as higher-ranked firms become better at identifying talent, its applicant pool deteriorates and its quality eventually falls back.

In this example, the top firms initially lose money as they post high wages and invest in talent. This ultimately raises both their productivity and their screening skill, giving them an advantage in the labor market, and delivering a steady stream of profits. Over time, rents shift from workers to firms: Firms earn zero lifetime value, with early workers paid more than their productivity and later workers paid less, as differentiated firms compete less intensely over workers. The way small initial differences are amplified over time resembles the dynamics in Giorcelli (2019), where a management training program in post-war Italy generated productivity gains that grew from 15% in the first year to 49% after 15 years. One channel was that "better managed firms paid higher average wages to their workers, which may indicate that trained managers were able to hire/retain better workers."

4.4 Steady State

Steady-state talent among recruits and applicants $\{R^*(x), Q^*(x)\}$ is easily characterized. First, the talent of each firm's recruits and retirees balance,

$$\lambda(q,\psi(r))=r. \tag{18}$$

This has a unique fixed point, $r=\rho(q)$, since $\lambda(q,\psi(r))-r$ is convex (see Appendix D.1), positive at $r=0$, negative at $r=1$, and hence crosses zero exactly once. Naturally, firms with better applicants have higher talent; formally, $\rho(q)$ increases since $\lambda(q,\psi(r))-r$ rises in q and single-crosses from above in r.

Second, substituting $R^*(x)=\rho(Q(x))$ into the sequential screening equation (12), steady-state applicant quality $Q^*(x)$ is given by

$$Q^*_x(x)=\phi\left(Q^*(x),\psi(\rho(Q^*(x))),x\right). \tag{19}$$

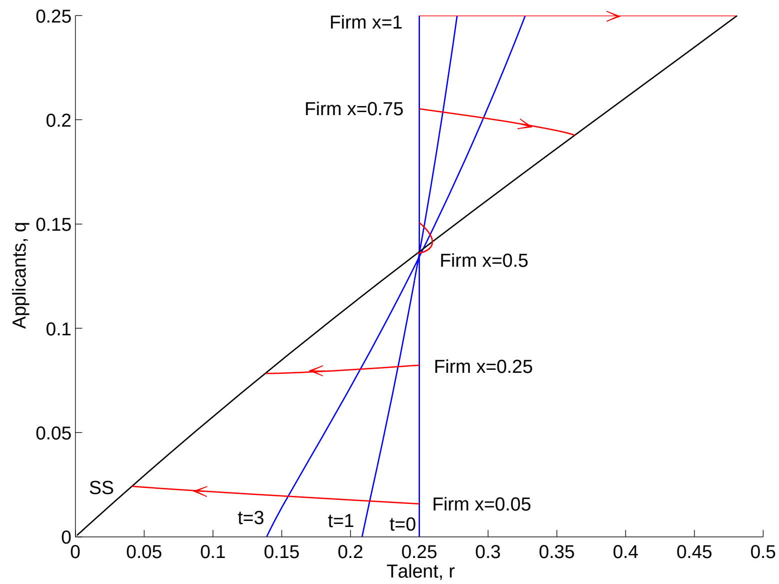

Figure 2: **Equilibrium Dynamics of Applicant and Recruit Quality.** All firms start with talent $\bar{q} = 0.25$, and choose wages optimally as characterized in the text. The skill function is $\psi(r) = 0.4 + 0.5r$ and (annual) turnover is $\alpha = 0.2$.

Together (18) and (19) pin down firm x's talent $R^*(x)$ and applicant quality $Q^*(x)$ in steady state, independent of turnover α and the discount rate β.[22] Differentiating, steady-state talent dispersion is given by[23]

$$R^*_x(x) = \rho_q(Q^*(x))Q^*_x(x) = \frac{\lambda_q(Q^*(x), \psi(R^*(x)))}{1 - \lambda_p(Q^*(x), \psi(R^*(x)))\psi_r(R^*(x))} Q^*_x(x). \tag{20}$$

We now show that from any initial condition, the economy converges to the steady state.

Theorem 5. *The steady-state talent distribution $R^*(x)$ is unique and has no gaps or atoms. For any distribution of initial talent $r(0)$, firm x's equilibrium talent $R(x,t)$ converges to $R^*(x)$.*

Proof. See Appendix D.3 □

To show convergence, Figure 2 suggests a "proof by induction." The top firm recruits

[22]These parameters still matter for equilibrium: The turnover rate α determines the rate at which talent converges to steady state, while the interest rate β determines how much firms care about the speed of convergence. As we see below, they both affect firm and worker shares in steady state.

[23]Note that the denominator in this expression is positive around the steady state: While $1 - \lambda_p(q, \psi(r))\psi_r(r)$ may be negative for arbitrary values of r, this cannot happen in steady state where $r = \rho(q)$, since we know that $\rho(q)$ increases in q and, by the implicit function theorem, $R_q(q) = \lambda_q(q, \psi(\rho(q)))/(1 - \lambda_p(q, \psi(\rho(q)))\psi_r(\rho(q)))$.

from a pool of constant quality, so equation (13) implies that its talent converges exponentially to steady state. Then consider firm x close to 1. As the talent of higher firms converges, firm x's applicant pool converges, and then firm x's talent converges, too, by equation (13). The formal proof is more complicated because x is continuous; it proceeds by showing that the steady state satisfies a contraction property, and then applies the contraction mapping theorem over a small interval, akin to the proof of the Picard-Lindelöf theorem.

In steady state, talent/productivity $R(x)$ are dispersed. If all firms were hiring from the same applicant pool q, talent differences from the steady state level $r - \rho(q)$ would decay exponentially. The link between talent and recruiting skills slows this decay but cannot stop it. Rather, what stops the decay is the effect of positive assortative matching: firms with skilled employees post higher wages and recruit from a better pool. As a result, the steady state supports permanent heterogeneity in firm quality, productivity and profits. Equation (20) highlights that *any* degree of correlation between productivity and recruiting skills $\psi_r > 0$ gives rise to non-vanishing talent dispersion. In particular, if we take the direct talent-skill relationship ψ_r to zero, (20) shows that the strategic effect in itself gives rise to dispersion $R^*_x = \lambda_q Q^*_x$, while the direct link merely amplifies this dispersion by a factor $1/(1 - \lambda_p \psi_r)$.

As discussed in Section 2, improved screening (e.g. referral quality) may help explain the increasing dispersion in productivity, wages and talent between firms. These forces are magnified in the dynamic model. In the short-run, if we fix firms' talent and increase signal accuracy $\psi(\cdot)$, productivity dispersion rises as high-paying firms fish out more of the talented workers from the applicant pool; over time, dispersion is further amplified as talent accumulates at the top firms, further raising their screening ability and the talent of their recruits.

Talent thus generates a sustainable competitive advantage. One might wonder why a firm with untalented workers doesn't compete more aggressively in wages to build its talent over time. While this is a feasible strategy, it is simply too expensive. High-talent firms have a higher marginal benefit from raising wages because recruiting skills and applicant quality are complements in the job market. The model captures a comparative advantage of Google which Paul Otellini, the CEO of Intel, called their "self-replicating talent machine" (Bock, 2015, p. 67).

4.5 Rent Sharing

The model generates predictions about how value is shared between workers and firms. In steady state the marginal value of talent (17) simplifies to

$$V_r^*(\rho(q)) = \frac{1}{\beta + \alpha\left(1 - \lambda_p \psi_r\right)}. \tag{21}$$

where we omit arguments for legibility. As in (17), the marginal product of talent is annuitized at the interest rate β and the talent decay rate $\alpha(1 - \lambda_p \psi_r)$. Substituting (21) into the first-order condition (16), flow wages in steady-state equilibrium $(\beta + \alpha)W^*(q)$ are given by

$$(\beta + \alpha)W_q^* = \frac{(\beta + \alpha)\lambda_q}{\beta + \alpha\left(1 - \lambda_p \psi_r\right)}. \tag{22}$$

Steady-state flow profits $\Pi^*(q)$ in turn are given by

$$\Pi_q^* = \rho_q - (\beta + \alpha)W_q^* = \frac{\beta \lambda_p \psi_r}{\left(1 - \lambda_p \psi_r\right)\left(\beta + \alpha\left(1 - \lambda_p \psi_r\right)\right)}. \tag{23}$$

Equations (22) and (23) show an increase in turnover raises both the level of flow wages and their dispersion. To see this, observe that an increase in turnover, α, has no impact on the steady-state distribution of talent $R^*(x)$, but raises the rate at which the economy converges to the steady state. Equations (22) and (23) then imply that flow wages $(\beta + \alpha)W^*(Q^*(x))$ rise and flow profits $\Pi^*(Q^*(x))$ fall for all firms x. Intuitively, when turnover is high, a firm's stock of talent quickly depletes and a low-talent firm can achieve almost the same profits as a high-talent firm by mimicking its wage policy; this intensifies competition and drives up wages. Higher turnover also increases wage dispersion in both the dispersive order and the log-dispersive order (see Appendix D.4). Intuitively, shifting surplus from firms to workers disproportionally benefits high-wage workers, while $W^*(Q^*(0)) = 0$, irrespective of α.

4.6 Is Talent Always a Sustainable Competitive Advantage?

Talent differences emerge and persist in Theorem 5 since positive assortative matching overcomes the mean-regression induced by noisy recruiting. Here we discuss the assumptions that underlie this argument, helping us understand in which industries talent may be a sustainable competitive advantage.

The first key assumption is that wages are set at the firm level, rather than being individually negotiated. In Montgomery (1991), employed workers refer applicants to their employer with talented workers being more likely to refer talented applicants. The firm then makes *applicant-specific* wage offers that condition on the talent of the referring employee. Since multiple firms may share a referred worker, wage competition gives rise to a symmetric mixed strategy equilibrium along the lines of Burdett and Judd (1983). We now argue that this equilibrium does not generate persistent dispersion. For an apples-to-apples comparison with our model, assume an infinite time horizon, continuous time, and that firms employ a unit mass of workers with aggregate talent $r(t)$ who retire at rate α. When a firm has a vacancy, a random current employee makes a referral. Write $\bar{\lambda}$ (resp. $\underline{\lambda}$) for the equilibrium expected talent of a recruit who was referred by a talented (resp. untalented) employee. Since wage offers are applicant specific, $\bar{\lambda}$ and $\underline{\lambda}$ do not depend on the talent of the firm's other employees. Talent evolution is then governed by $r_t(t) = \alpha(\underline{\lambda} + (\bar{\lambda} - \underline{\lambda})r(t) - r(t))$ and converges to $r^* = \underline{\lambda}/(1 - \bar{\lambda} + \underline{\lambda})$, irrespective of initial talent. In contrast, our firms have *firm-specific* wage policies. If we interpret the skill function $\psi(r)$ as the firm sampling its current employees for referrals, our model predicts that a talented worker recruits better referrals if their colleagues are also talented. This is because high-talent firms pay high wages, and help convert the recommendation of the talented worker into a talented recruit. This breaks the separability seen in Montgomery, implying that better firms retain their talent advantage in the long-run.

Second, we assume non-decreasing returns to talent, in that revenue is linear in talent r, while screening skills $\psi(r)$ are weakly convex in r; together, these assumptions generate positive assortative matching. The model provides strict incentives for positive assortative matching, so a little concavity does not affect equilibrium sorting and the distribution of steady-state talent. But sufficient concavity induces negative assortative matching and gives rise to convergence. For an extreme example, if flow revenue equaled $\max\{r(t), \bar{q}\}$, then all firms would randomize over wages to maintain talent at $\bar{q}$. Similarly, if it suffices to have 10% of talented employees to guarantee good screening outcomes, then firms below the 10% threshold would bid aggressively to improve their screening.[24]

Third, we assume that firms only differ in their talent. In Board et al. (2017) we allow firms to have different production technology in addition to different talent, and suppose the two are complements. The idea is illustrated by Netflix's HR manual: "In procedural work, the best are two times better than average. In creative/inventive work, the best are ten times better than average" (Hastings and McCord, 2009). Whether the

[24] As discussed in Section 3, positive assortative matching relies on a number of other assumptions. If talent is not scarce (3), the most skilled firms may pay lower wages than some firms with average skills (see Board et al. (2017)). And if there is imperfect competition, low-skilled firms may bid more to avoid the externalities imposed on them by high-skilled firms (see Section 3.3).

firms converge then depends on the relative persistence of these two state variables. If technology is immobile then talent ultimately lines up with technology, so high-tech firms eventually become high-talent. However, if the industry is fast moving and talented people make new inventions, then technology ultimately lines up with talent, so high-talent firms eventually become high-tech. Indeed, universities seem to fall into the latter category: Waldinger (2016) argues the loss of human capital at German universities had a large, persistent effect on output, whereas the loss of physical capital had a small, temporary effect.

Finally, our firms have a continuum of workers. This means that the firms' stock of talent moves slowly and firms do not leap-frog each other. If instead we modeled firms in terms of discrete individuals, a lucky hire would allow firm r to jump over firm $r + \epsilon$; it would then raise its wages, helping it to stay ahead. Formally, equilibrium would still exhibit dispersion in steady-state talent, but the randomness in the hiring process renders a firm's talent an irreducible Markov chain, so any firm cycles through all positions in the distribution eventually. Of course, if firms are large, the regression to the mean would be slow, and we can think of the perfect persistence in Theorem 5 as an idealization of this economic force.

5 Conclusion

In their survey on personnel economics, Oyer and Schaefer (2011) write that "This literature has been very successful in generating models and empirical work about incentive systems [. . .] The literature has been been less successful at explaining how firms can find the right employees in the first place." By studying the equilibrium interaction of firms' recruiting strategies, we hope this paper takes a step in this direction.

The paper has three major contributions. First it provides a simple framework to understand how private screening generates wage and productivity dispersion. It thus provides an alternative to Phelps's classic (1972) model of public screening, giving rise to substantially more mismatch. Second, the paper shows that skilled firms post higher wages, attract better applicants, hire better recruits and obtain higher productivity and profits. Crucially, this positive assortative matching is inefficient, in contrast to the classic result of Becker (1973) in which complementarities are exogenous. This compositional externality can be mitigated by policies that reduce wage dispersion. Finally, we propose a new model of firm dynamics based on the tenet that talented workers are better at identifying talented applicants. Initially similar firms diverge over time and the economy converges to a steady state featuring persistent dispersion in talent, wages and productivity. We thus complement the classic model of Montgomery (1991) by showing that referrals can generate across-firm inequality, even in the long run.

The model relates to a number of fields and can be taken in a number of directions. IO economists may wish to consider firms that interact in the product market. Workers' talent then affects both the quality and nature of the products offered. This is particularly relevant when two markets converge, like Netflix and Disney in streaming, or Uber and Ford in car services. Organizational economists may be interested in how the internal organization of firms affects the quality of its recruits. For example, an improving firm should place authority in the hands of more recent hires, but this may conflict with senior managers' goals. And labor economists may wish to allow for job-to-job transitions. These may stem from a firm learning its workers' quality and firing the worst, or from the firm learning about competitors' workers and poaching the best. How such transitions affect mismatch remains an open question. Finally, we hope the paper also stimulates empirical work into the "production function" of hiring. What makes some managers (or firms) better at recruiting than others? How does this skill interact with the firms' strategy (e.g. wage, advertising) in order to produce better recruits?

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Appendix

A Comparison with Kurlat (2016)

In this section we clarify the relationship between our model and that of Kurlat (2016) with "false positives," and two types of firms: skilled and unskilled. The most significant difference is that our model assumes that firms' tests are conditionally independent, whereas Kurlat assumes "nested" signals in that an untalented worker who passes the test of a skilled recruiter also passes the test of an unskilled recruiter.[25] Workers thus fall into one of four categories: A) talented workers, who automatically pass both tests, B) untalented workers who pass both tests, C) untalented workers who pass the test of the unskilled recruiter but fail the test of the skilled recruiter, and D) untalented workers who fail both tests.[26]

This seemingly small difference from our model results in very different equilibria. To see why, assume that $\underline{w}$ is small, so all skilled firms and some unskilled firms enter. Since unskilled firms hire proportionally from categories A, B, and C, they do not impose a compositional externality on other firms. But skilled firms, who hire proportionally from categories A and B, but screen out C workers, impose a negative externality on unskilled firms. Thus, unskilled firms have an incentive to outbid skilled firms but not vice versa (and neither type of firm has an incentive to outbid its own type). The equilibrium wage distribution is thus degenerate, with all entering firms offering some wage w^* and workers endogenously breaking ties in favor of unskilled firms. The wage w^* is determined by the entry condition of unskilled firms, and the proportion of unskilled firms that enter is determined by the condition that all workers in categories A and B are employed. This equilibrium has negative assortative matching and efficient aggregate sorting, in sharp contrast to the positive assortative matching and inefficient aggregate sorting of our model with independent signals.

[25]Other differences between our model and Kurlat's are superficial. His focus on asset markets and ours on labor markets is purely semantic; indeed, the follow-up paper Kurlat and Scheuer (2020) is phrased in terms of labor markets. The impatience of his distressed sellers would correspond to a reservation wage of talented workers in our model. Finally, his model allows for richer interaction across many markets, but the unique equilibrium described in his Proposition 1 features a single market and is equivalent to the equilibrium in an auction where buyers bid for assets, and then sellers apply from top to bottom bid with buyers screening applying sellers and accepting the first seller who passes their test, exactly as in our model.

[26]Category A corresponds to the "green assets" in Kurlat's table I, category C to "red assets," and category D to the "black assets."

B Proofs from Section 2

In this appendix we prove Proposition 1 (the effect of p and $\bar{q}$ on wage/productivity dispersion) and Proposition 2 (the effect of p on mismatch). As preliminary results, we establish how applicant quality and employment depend on p and $\bar{q}$.

B.1 Applicant Quality

Write the applicant quality for a firm with rank x as $Q(x, \bar{q}, p)$ to make explicit the effect of aggregate talent $\bar{q}$ and screening skills p. Recall from (1) and (4) that

$$\lambda(q,p) := \frac{q}{1-p(1-q)} \quad \text{and} \quad \phi(x,q,p) := \frac{\lambda(q,p)-q}{x}.$$

Lemma 1. *Applicant quality $Q(x, \bar{q}, p)$ is:*
(a) Increasing in wage rank x with derivative $Q_x(x, \bar{q}, p) = \phi(x, Q(x, \bar{q}, p), p)$.
(b) Increasing in aggregate talent $\bar{q}$ with derivative

$$Q_{\bar{q}}(x,\bar{q},p) = \exp\left(-\int_x^1 \phi_q(\hat{x}, Q(\hat{x},\bar{q},p),p)d\hat{x}\right).$$

(c) Decreasing in screening skills p with derivative

$$Q_p(x,\bar{q},p) = -\int_x^1 \exp\left(-\int_x^{\hat{x}} \phi_q(\check{x}, Q(\check{x},\bar{q},p),p)d\check{x}\right)\phi_p(\hat{x}, Q(\hat{x},\bar{q},p),p)d\hat{x}.$$

(d) Log-submodular in $(x, \bar{q})$.
(e) Log-supermodular in (x, p).

Proof. (a) is the sequential screening equation (4).

(b) and (c) follow from the theory of ordinary differential equations, e.g. Hartman (2002, Theorem 3.1), whereby the solution of the ODE $Q_x(x, \bar{q}, p) = \phi(x, Q(x, \bar{q}, p), p)$ with boundary condition $Q(1, \bar{q}, p) = \bar{q}$ satisfies $Q_{x\bar{q}} = \phi_q Q_{\bar{q}}$ and $Q_{xp} = \phi_q Q_p + \phi_p$ with boundary conditions $Q_{\bar{q}}(1, \bar{q}, p) = 1$ and $Q_p(1, \bar{q}, p) = 0$.

(d) and (e) follow because

$$(\log Q(x,\bar{q},p))_x = \frac{\lambda(Q(x,\bar{q},p),p) - Q(x,\bar{q},p)}{Q(x,\bar{q},p)x} = \frac{1}{x}\left(\frac{1}{1-p(1-Q(x,\bar{q},p))} - 1\right)$$

falls in $\bar{q}$ by (b) and rises in p by (c). $\square$

B.2 Employment

To analyze equilibrium employment, we first establish that the inverse of applicant quality $Q(x, \bar{q}, p)$ is given in closed form by

$$X(q, \bar{q}, p) = \left(\frac{1-\bar{q}}{1-q}\right)^{\frac{1}{p}} \left(\frac{q}{\bar{q}}\right)^{\frac{1-p}{p}}.$$

Indeed, differentiating,

$$X_q = \left(\frac{1}{p}\frac{1}{1-q} + \left(\frac{1}{p} - 1\right)\frac{1}{q}\right) X = \left(\frac{q + (1-p)(1-q)}{pq(1-q)}\right) X = \frac{X}{\lambda(q,p) - q}$$

which is the inverse of (4).

Turning to employment, as long as there is any employment, applicant quality at the marginal firm solves $\lambda(\underline{q}, p) = \frac{q}{1-p(1-\underline{q})} = \underline{w}$. Hence, the cutoff applicant quality equals $\underline{q} = \underline{Q}(p) := \min\left\{\frac{\underline{w}(1-p)}{1-\underline{w}p}, \bar{q}\right\}$. Thus, equilibrium unemployment is given by

$$\begin{aligned}\underline{X}(p) = X(\underline{Q}(p), \bar{q}, p) &= \left(\frac{1-\bar{q}}{1-\underline{Q}(p)}\right)^{\frac{1}{p}} \left(\frac{\underline{Q}(p)}{\bar{q}}\right)^{\frac{1-p}{p}} \\ &= \exp\left(\frac{1}{p}\log\left(\frac{1-\bar{q}}{1-\underline{Q}(p)}\right) + \left(\frac{1}{p} - 1\right)\log\left(\frac{\underline{Q}(p)}{\bar{q}}\right)\right).\end{aligned}$$

Differentiating and dropping the argument p from the notation,

$$\underline{X}_p = \left(\left(\frac{1}{p}\cdot\frac{1}{1-\underline{Q}} + \frac{1-p}{p}\cdot\frac{1}{\underline{Q}}\right)\underline{Q}_p + \frac{1}{p^2}\log\left(\frac{\bar{q}}{\underline{Q}}\cdot\frac{1-\underline{Q}}{1-\bar{q}}\right)\right)\underline{X}. \tag{24}$$

Lemma 2. *Assume $\underline{w} \neq \bar{q}$.*
(a) If $\bar{q} < \underline{w}$, then $\underline{X}(p) \equiv 1$ for $p \in [0, \underline{p}]$, where $\underline{p}$ solves $\lambda(\bar{q}, \underline{p}) = \underline{w}$.
(b) If $\bar{q} > \underline{w}$ then $\underline{X}(0) = \underline{X}_p(0) = 0$.

Proof. Part (a) is obvious. If $\bar{q} < \underline{w}$, and so firms would not hire without screening, employment also vanishes for small $p \in [0, \underline{p}]$.

The first equation in part (b), $\underline{X}(0) = 0$, is obvious, too. If $\underline{w} < \bar{q}$, firms are willing to hire without screening, so when $p = 0$ and there is no selection, all workers are hired. The second equation in part (b) $\underline{X}_p(0) = 0$ follows from (24) by l'Hopital's rule, since $\underline{X} \propto \exp(-1/p)$ term goes to zero much faster than the $1/p$-terms go to ∞. In fact, all higher-order p-derivatives of $\underline{X}$ vanish at $p = 0$, too. □

B.3 Wage/Productivity Dispersion: Proof of Proposition 1

We wish to show that $\lambda(Q(x,\bar{q},p),p) = \frac{Q(x,\bar{q},p)}{1-p+pQ(x,\bar{q},p))}$ is (a) log-supermodular in (x,p) and (b) log-submodular in $(x,\bar{q})$. For part (a) we compute

$$(\log \lambda(Q(x,\bar{q},p),p))_p = \frac{\lambda_q(Q(x,\bar{q},p),p)Q_p(x,\bar{q},p) + \lambda_p(Q(x,\bar{q},p),p)}{\lambda(Q(x,\bar{q},p),p)}$$

and, omitting arguments for legibility,

$$(\log \lambda(Q(x,\bar{q},p),p))_{px} = \frac{1}{\lambda^2}\Big[\lambda\lambda_{qq}Q_xQ_p + \lambda\lambda_{pq}Q_x - \lambda_q Q_x \lambda_q Q_p + (\lambda\lambda_q Q_{px} - \lambda_q\lambda_p Q_x)\Big]$$

To see that this is positive, recall $Q_x = \phi > 0, Q_p < 0$ from Lemma 1(a,c), and $\lambda_{qq} < 0$ and $\lambda_{qp} > 0$ from (6), using (3). Then the first three terms are positive. Using $Q_x = \phi$, the term in brackets then equals $\lambda_q(\lambda\phi_p - \lambda_p\phi) = \lambda_q(\lambda\lambda_p - \lambda_p(\lambda - q))/x > 0$.

For part (b) we compute

$$\begin{aligned}(\log \lambda(Q(x,\bar{q},p),p))_{\bar{q}} &= (\log Q(x,\bar{q},p))_{\bar{q}} - \frac{pQ_{\bar{q}}(x,\bar{q},p)}{1-p+pQ(x,\bar{q},p))} \\ &= (\log Q(x,\bar{q},p))_{\bar{q}}\left(\frac{1-p}{1-p+pQ(x,\bar{q},p)}\right)\end{aligned}$$

which falls in x since $Q(x,\bar{q},p)$ rises in x and is log-submodular in x and $\bar{q}$ by Lemma 1(a,d). Intuitively, the larger proportional decline of applicant quality at lower-ranked firms is aggravated by the concavity of recruit quality $\lambda(q,p)$ in applicant quality q.

B.4 Mismatch: Proof of Proposition 2

To see that mismatch falls with screening skills, we note that mismatch is inversely related to aggregate surplus $M = \bar{q}(1-\underline{w}) - S$, where the latter is defined in the proof of Theorem 3 in Appendix C. Equation (39) shows that the marginal effect of screening skills at any firm rank x' on aggregate surplus equals

$$\lambda_p(Q(x',\bar{q},p),p)\exp\left(-\int_{\underline{X}(\bar{q},p,\underline{w})}^{x'} \frac{\lambda_q(Q(x,\bar{q},p),p)}{x}dx\right) > 0.$$

Thus, the effect of rank-x' screening skills on mismatch is negative for any x', and hence also in aggregate when all firms' screening skills p rise simultaneously, as we assume here.

We next consider infinitesimal screening skills, and show that $M'(0) = 0$ if $\bar{q} \neq \underline{w}$.

Recalling from (5) that

$$M(p) = \underline{Q}(p)\underline{X}(p)(1-\underline{w}) + (1-\bar{q}-(1-\underline{Q}(p))\underline{X}(p))\underline{w},$$

the result follows from Lemma 2. For $\bar{q} < \underline{w}$, the result is obvious since Lemma 2(a) implies that $\underline{X}(p) \equiv 1$ and $\underline{Q}(p) \equiv \bar{q}$ are flat for $p \in [0, \underline{p}]$, and thus so is $M(p) \equiv \bar{q}(1-\underline{w})$. For $\bar{q} > \underline{w}$, the result is surprising. Differentiating, we get $M_p = -\underline{X}_p(\underline{w}-\underline{q}) + \underline{X}(\underline{w}-\underline{Q}_p)$, and the result follows by Lemma 2(b). More strongly, as in Lemma 2(b), all higher-order derivatives of M in p also vanish at $p = 0$.

Intuitively, with $\bar{q} > \underline{w}$ and no information, everyone is employed and the point of information is to weed out some untalented applicants. While every single test achieves that, workers get so many attempts at different firms that asymptotically everyone passes one of the tests, resulting in full employment and no reduction in mismatch.

C Proofs from Section 3

In this section we prove our main inefficiency result, Theorem 3, and extend it to two model variants with production complementarities and screening costs, introduced in Section 3.3. To accommodate these variants, we first formulate a more general model where the surplus when a firm with skills p hires from an applicant pool with quality q is given by a general function $\omega(q,p)$. The baseline model corresponds to $\omega(q,p) = \lambda(q,p) - \underline{w}$, production complementarities correspond to $\omega(q,p) = h(p) \cdot \lambda(q,p) - \underline{w}$, and screening costs to $\omega(q,p) = \lambda(q,p) - \kappa/(1-p(1-q)) - \underline{w}$.

We develop the apparatus to analyze aggregate surplus for general surplus functions $\omega(q,p)$ in C.1, specialize to the baseline model and prove Theorem 3 in C.2, prove some auxiliary claims from Section 3 in C.3, and then extend the inefficiency result to models with production complementarities and screening costs in C.4 and C.5.

C.1 Aggregate Surplus for General Surplus Functions $\omega(q,p)$

Fix the number of entering firms, and thereby the cutoff $\underline{x}$. Aggregate surplus under matching $P(\cdot)$ is given by

$$S(P(\cdot)) = \int_{\underline{x}}^{1} \omega(Q(x), P(x))dx \tag{25}$$

Consider adding a new firm p at rank x. This has three effects on surplus. First, firm p hires from a applicant pool with quality $Q(x)$ and so generates surplus $\omega(Q(x),p)$.

Second, firm p pushes out the marginal firm, $P(\underline{x})$. Third, it lowers the ranking of intermediate firms, $\check{x} < x$, thereby lowering their applicant quality, $Q(\check{x})$. To quantify the latter externality, note that firm p's hiring reduces applicant quality just below x by $dQ(x) = \phi(Q(x), p, x)dx$. For lower ranking firms $\check{x}$, this effect is mitigated since firm p pushes intermediate firms $\hat{x} \in [\check{x}, x]$ to lower wage ranks and thereby reduces their externality by $-\phi_x(Q(\hat{x}), p(\hat{x}), \hat{x})$. By standard results on ODEs in the proof of Lemma 1, firm p's total effect on applicant quality $Q(\check{x})$ is given by

$$\chi(\check{x}; p, x, P(\cdot)) = -\phi(Q(x), p, x) \exp\left(-\int_{\check{x}}^{x} \phi_q\right) - \int_{\check{x}}^{x} \left[\exp\left(-\int_{\check{x}}^{\hat{x}} \phi_q\right) (-\phi_x(Q(\hat{x}), P(\hat{x}), \hat{x}))\right] d\hat{x} \tag{26}$$

where we dropped the arguments in the integrand $\phi_q = \phi_q(Q(\tilde{x}), P(\tilde{x}), \tilde{x})$ to enhance legibility. Putting these three effects together, firm p's (infinitesimal) net-contribution to surplus when assigned to wage rank x in matching $P(\cdot)$ is thus given by

$$s(p, x, P(\cdot)) = \omega(Q(x), p) + \int_{\underline{x}}^{x} \chi(\check{x}; p, x, P(\cdot))\omega_q(Q(\check{x}), P(\check{x}))d\check{x} - \omega(Q(\underline{x}), P(\underline{x})). \tag{27}$$

We wish to evaluate the effect of changing a matching function $P(\cdot)$ into a second matching function $P'(\cdot)$. To do this, suppose we move one firm p from rank $\underline{x}(p)$ to $\bar{x}(p)$ given matching $P(\cdot)$. The (infinitesimal) change in surplus equals

$$\int_{\underline{x}(p)}^{\bar{x}(p)} s_x(p, x, P(\cdot))dx.$$

More generally, let us sequentially move all firms $p \in [\underline{p}, \bar{p}]$ in order of increasing p. The matching function changes over the course of this transformation, and we write $P^p(\cdot)$ for the matching function after firms $p' < p$ have been shifted; thus $P^{\underline{p}}(\cdot) = P(\cdot)$ and $P^{\bar{p}}(\cdot) = P'(\cdot)$.[27] The aggregate change in surplus is given by

$$S(P'(\cdot)) - S(P(\cdot)) = \int_{\underline{p}}^{\bar{p}} \left[\int_{\underline{x}(p)}^{\bar{x}(p)} s_x(p, x, P^p(\cdot))dx\right] dF(p). \tag{28}$$

[27] There are multiple ways to transform $P(\cdot)$ into $P'(\cdot)$ in increasing order of p. In a discrete analogue, if $p_1 < p_2 < p_3$, we can transform $\{p_1, p_3, p_2\}$ into $\{p_3, p_1, p_2\}$ by either moving p_1 to second position, or by moving p_1 to the top and then moving p_2 above it. Formally, a transformation is fully specified by the original matching $P(\cdot)$ and the intermediate target positions $\bar{x}(\cdot)$; the intermediate matchings $P^p(\cdot)$ as well as $\underline{x}(\cdot)$ are generated endogenously. In the two applications used in the proofs of Lemma 4 and Theorem 3 we move each firm immediately into its final position, $P'(\bar{x}(p)) = p$, but this need not be the case in general.

We first establish a general formula for the integrand in (28).

Lemma 3. *The marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x) = \hat{p}$ equals*

$$s_x(p,x,P(\cdot)) = \Big[\omega_q(Q(x),p)\phi(Q(x),\hat{p},x) - \omega_q(Q(x),\hat{p})\phi(Q(x),p,x)\Big] \tag{29}$$

$$-\Big[\phi_q(Q(x),p,x)\phi(Q(x),\hat{p},x) + \phi_x(Q(x),p,x) - \phi_q(Q(x),\hat{p},x)\phi(Q(x),p,x) - \phi_x(Q(x),\hat{p},x)\Big]\gamma$$

where

$$\gamma = \gamma(x,P(\cdot)) = \int_{\underline{x}}^{x} \exp\left(-\int_{\check{x}}^{x} \phi_q(Q(\hat{x}),P(\hat{x}),\hat{x})d\hat{x}\right)\omega_q(Q(\check{x}),P(\check{x}))d\check{x}. \tag{30}$$

To understand these equations, consider swapping p and $\hat{p}$, and let us drop the arguments $Q(x)$ and x to enhance legibility. The first term in the first line of (29) $\omega_q(p)\phi(\hat{p})$ is the increased surplus contribution of firm p when selecting before $\hat{p}$ from a pool with $dQ(x) = \phi(\hat{p})dx$ higher quality. Conversely, the second term $\omega_q(\hat{p})\phi(p)$ is $\hat{p}$'s decreased surplus when selecting after p. The term in the second line of (29) is the effect of switching p and $\hat{p}$'s screening order on residual pool quality $Q(x - dx)$. By screening applicants $q + \phi(\hat{p})dx$ from a pool of size $x + dx$ (rather than applicants q from pool size x), firm p's selection increases by $[\phi_q(p)\phi(\hat{p}) + \phi_x(p)]dx$; conversely firm $\hat{p}$'s selection decreases by $[\phi_q(\hat{p})\phi(p) + \phi_x(\hat{p})]\,dx$. This change in applicant quality $dQ(x)$ affects revenue $\omega(Q(\check{x}),P(\check{x}))$ of all lower-ranking firms $\check{x} \in [\underline{x},x]$, where the exponential term in (30) equals $dQ(\check{x})/dQ(x)$ and captures differential selection by intermediate firms $\hat{x} \in [\check{x},x]$.

Note that

$$s_x(P(x),x,P(\cdot)) = 0; \tag{31}$$

intuitively, switching the screening order of two identical firms $p = \hat{p} = P(x)$ makes no difference.

Proof. Differentiating (27)

$$\begin{aligned} s_x(p,x,P(\cdot)) =& \omega_q(Q(x),p)Q_x(x) + \chi(x;p,x,P(\cdot))\omega_q(Q(x),P(x)) \\ &+ \int_{\underline{x}}^{x} \chi_x(\check{x};p,x,P(\cdot))\omega_q(Q(x),P(\check{x}))d\check{x}. \end{aligned}$$

Since $Q_x(x) = \phi(Q(x),P(x),x)$ and $\chi(x;p,x,P(\cdot)) = -\phi(Q(x),p,x)$, and recalling $P(x) = \hat{p}$, the first two terms correspond to the first line of (29).

As for the last term, $\chi_x(\check{x}; p, x)$ equals

$$-\Big[\phi_q(Q(x),p,x)Q_x(x)+\phi_x(Q(x),p,x)-\phi(Q(x),p,x)\phi_q(Q(x),P(x),x)-\phi_x(Q(x),P(x),x)\Big]\exp\left(-\int_{\check{x}}^{x}\phi_q\right)$$

where the term is square-brackets equals the square-bracket term in the second line of (29), while integration over $\check{x} \in [\underline{x}, x]$ yields $\int_{\underline{x}}^{x} \exp\left(-\int_{\check{x}}^{x}\phi_q\right)\omega_q(Q(\check{x}), P(\check{x}))d\check{x} = \gamma$. $\square$

We cannot determine the sign of (29) in general. But at the lowest wage rank $x = \underline{x}$, the analysis simplifies because the integral domain in (30) collapses and so $\gamma = 0$, yielding a necessary condition for optimality of PAM that is easy to check. We will check that this condition is violated for the model variants in Propositions 3 and 4, below.

Lemma 4. *Assume that $1 - \underline{x}$ firms $p \in [\underline{p}, \bar{p}]$ enter. If*

$$\frac{\omega_{qp}(Q(\underline{x}), \underline{p})}{\omega_q(Q(\underline{x}), \underline{p})} < \frac{\phi_p(Q(\underline{x}), \underline{p}, \underline{x})}{\phi(Q(\underline{x}), \underline{p}, \underline{x})} \tag{32}$$

then PAM does not maximize surplus.

Proof. Let us transform $P(\cdot) = P^{PAM}(\cdot)$ into another matching $P'(\cdot)$ with NAM for $x \in [\underline{x}, \underline{x} + \epsilon]$ and PAM for $x \in (\underline{x} + \epsilon, 1]$. Intuitively, if there were 10 firms with skill $p_1 < \ldots < p_{10}$ then this would mean swapping p_1 and p_2, so firms are ranked $\{p_2, p_1, p_3, \ldots p_{10}\}$, from lowest to highest. Formally $P'(x) = F^{-1}(\underline{x} + \epsilon - (x - \underline{x}))$ for $x \in [\underline{x}, \underline{x} + \epsilon]$, and $P'(x) = F^{-1}(x)$ for $x > \underline{x} + \epsilon$, where $\epsilon > 0$ is small. We transform $P(\cdot)$ into $P'(\cdot)$ by shifting firms $p \in [\underline{p}, F^{-1}(\underline{x} + \epsilon)]$ (in rising order of p) to their $P'(\cdot)$-wage rank $\bar{x}(p) = \underline{x} + \epsilon - [F(p) - F(\underline{p})]$. Since lower firms $p' < p$ have already been shifted to $\bar{x}(p') > \bar{x}(p)$ at firm p's "turn," firm p starts at rank $\underline{x}(p) = F(\underline{p}) < \bar{x}(p)$ and is shifted exclusively past firms with higher screening skills $P^p(x) = F^{-1}[F(p) + x - \underline{x}] \geq p$, recalling the definition of the matching function $P^p(\cdot)$ at p's "turn."

We now argue that given (32) the net value of this transformation (28) is positive. Using (31), the integrand in (28) equals

$$s_x(p, x, P^p(\cdot)) = s_x(P^p(x), x, P^p(\cdot)) - \int_p^{P^p(x)} s_{xp}(\check{p}, x, P^p(\cdot))d\check{p} = -\int_p^{P^p(x)} s_{xp}(\check{p}, x, P^p(\cdot))d\check{p}. \tag{33}$$

To see that this is positive, differentiate (29) with respect to p, and evaluate for firm $\underline{p}$ at the marginal wage rank $\underline{x}$. The derivative of the second line is zero because $\gamma = 0$ for $x = \underline{x}$. Thus,

$$s_{xp}(\underline{p}, \underline{x}, P^{PAM}(\cdot)) = \omega_{qp}(Q(\underline{x}), \underline{p})\phi(Q(\underline{x}), \underline{p}, \underline{x}) - \omega_q(Q(\underline{x}), \underline{p})\phi_p(Q(\underline{x}), \underline{p}, \underline{x})$$

which is negative by (32). Now observe that $(\check{p}, x, P^p(\cdot))$ converges to $(\underline{p}, \underline{x}, P^{PAM}(\cdot))$ for all $\check{p}, p \in [\underline{p}, F^{-1}(\underline{x}+\epsilon)]$ and $x \in [\underline{x}, \underline{x}+\epsilon]$ as $\epsilon \to 0$, using for instance the topology of uniform convergence on matching functions $P(\cdot)$. Thus, the integrand of (33) is negative, and so the integral (28) is positive, so the transformation from $P(\cdot)$ to $P'(\cdot)$ raises welfare.

$\square$

C.2 Proof of Theorem 3

We now return to the baseline model where surplus is given by $\omega(q,p) = \lambda(q,p) - \underline{w}$. The comparison of the elasticities (11) implies (32), and so Lemma 4 already implies that PAM is inefficient. In particular, the proof of Lemma 4 shows that locally near the cutoff $\underline{x}$ shifting low-skill firms ahead of high-skill firms increases surplus.

Theorem 3 claims more strongly that surplus is minimized by PAM and maximized by NAM. We show this by arguing that for the baseline surplus function $\omega(q,p) = \lambda(q,p) - \underline{w}$ our local argument at the bottom of the wage distribution in fact holds globally.

Indeed, (29) simplifies considerably:

Lemma 5. *When $\omega(q,p) = \lambda(q,p) - \underline{w}$, the marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x) = \hat{p}$ equals*

$$s_x(p, x, P(\cdot)) = [\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p)]\,(1 - \gamma/x) \tag{34}$$

where we dropped arguments $Q(x)$ and x for legibility, and $1-\gamma/x = \exp\left(-\int_{\underline{x}}^{x} \frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}} d\check{x}\right) \in (0,1)$.

The term in square-brackets corresponds to the incremental talent hired by firms p and $\hat{p}$ when the former is shifted ahead of the latter. The effect on aggregate surplus is scaled down by the factor $1-\gamma/x$ as in (10) since incremental talent hired by the marginal firms p and $\hat{p}$ reduces the talent hired by lower-ranking firms.

Proof. We will show that the marginal surplus (29) equals

$$s_x(p, x, P(\cdot)) = \Big[\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p)\Big] - \Big[\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p)\Big]\frac{\gamma}{x} \tag{35}$$

and thus collapses to (34). Using the definition of $\omega(q,p)$, the first line in (29) equals the first square-bracket term in (35). Turning to the second line in (29) and recalling $\phi = \frac{\lambda - q}{x}$, elementary algebra implies

$$\phi_q(p)\phi(\hat{p}) + \phi_x(p) - \phi_q(\hat{p})\phi(p) - \phi_x(\hat{p}) = \frac{1}{x}\left[\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p)\right]. \tag{36}$$

Multiplying by γ, the second line in (29) equals the second square-bracket term in (35), as required.

To evaluate γ observe that

$$\exp\left(-\int_{\check{x}}^{x}\phi_q\right)=\exp\left(-\int_{\check{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))-1}{\hat{x}}d\hat{x}\right)=\frac{x}{\check{x}}\exp\left(-\int_{\check{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right).$$

Equation (30) thus simplifies to

$$\begin{aligned}\gamma&=x\int_{\underline{x}}^{x}\exp\left(-\int_{\check{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right)\frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}}d\check{x}\\&=x\int_{\underline{x}}^{x}\frac{d}{d\check{x}}\exp\left(-\int_{\check{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right)d\check{x}=x\left[1-\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}}d\check{x}\right)\right],\end{aligned}\tag{37}$$

as required. □

In the proof sketch in the body of the paper we argued that (34) is positive when $\hat{p}=p+dp$ by noting that $\lambda_q(p)\phi(p+dp)-\lambda_q(p+dp)\phi(p)=(\lambda_q\phi_p-\lambda_{qp}\phi)dp>0$, as shown in (11). To prove Theorem 3, we now generalize this local argument to show that transforming an arbitrary matching $P(\cdot)$ into $P'(\cdot)=P^{NAM}(\cdot)$ raises surplus. Intuitively, if there were 10 firms with skill $p_1<\ldots<p_{10}$, we would shift firm p_1 to the highest position, then shift firm p_2 to the second-highest position, and so on. Formally, we shift type-p firms in rising order of p to their NAM-rank $\bar{x}(p)=1-[F(p)-F(\underline{p})]$. At firm p's turn, i.e. in matching $P^p(\cdot)$, lower-skill firms $p'<p$ have already been shifted to their NAM rank in $\bar{x}(p')>\bar{x}(p)$, and so firm p starts at $\underline{x}(p)\le\bar{x}(p)$ and is shifted past higher-skill firms $P^p(x)\ge p$ for all $x\in[\underline{x}(p),\bar{x}(p)]$.

Recalling $s_x(P^p(x),x,P^p(\cdot))=0$ from (31), the marginal surplus along this transformation is given by

$$s_x(p,x,P^p(\cdot))=-\int_{p}^{P^p(x)}s_{xp}(\check{p},x,P^p(\cdot))d\check{p}.\tag{38}$$

To sign the integrand of this expression, we differentiate (33) with respect to p

$$s_{xp}(p,x,P^p(\cdot))=\left[\lambda_{qp}(p)\phi(\hat{p})-\phi_p(p)\lambda_q(\hat{p})\right](1-\gamma/x)<0.$$

Substituting back into (38) and recalling that $p<P^p(x)$, marginal surplus (38) is positive. Finally, integrating (38) over $p\in[\underline{p},\bar{p}]$ and $x\in[\underline{x}(p),\bar{x}(p)]$, we conclude that the aggregate change of surplus (28) from this transformation is positive, and NAM maximizes aggregate

surplus. The analogue argument implies that PAM minimizes aggregate surplus.

Finally we establish that the planner indeed wants the firms with the highest screening skills to enter the market. To see this, differentiate firm p's contribution to surplus (27) with respect to p

$$\begin{aligned} s_p(p,x,P(\cdot)) &= \lambda_p(Q(x),p) + \int_{\underline{x}}^{x} \chi_p(\check{x};p,x,P(\cdot))\lambda_q(Q(\check{x}),P(\check{x}))d\check{x} \\ &= \lambda_p(Q(x),p) - \phi_p(Q(x),p,x) \int_{\underline{x}}^{x} \exp\left(-\int_{\check{x}}^{x} \phi_q\right) \lambda_q(Q(\check{x}),P(\check{x}))d\check{x} \\ &= \lambda_p(Q(x),p) - \left(\frac{\lambda_p(Q(x),p)}{x}\right) x \left[1 - \exp\left(-\int_{\underline{x}}^{x} \frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}} d\check{x}\right)\right] \\ &= \lambda_p(Q(x),p) \exp\left(-\int_{\underline{x}}^{x} \frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}} d\check{x}\right) > 0, \end{aligned} \tag{39}$$

where the second equality uses the definition of the externality χ in (26), and the third equality uses the definition of γ (30) and its evaluation in (37).

C.3 Wage Caps

Restricting firms to a single wage $\check{w}$ implements a random screening order, which is constrained efficient by the proof of Theorem 3. The wage level $\check{w}$ does not affect the screening order, but rather pins down the marginal entering firm $\check{p} = \check{p}(\check{w})$ via the indifference condition $\int_{F(\check{p})}^{1} \lambda(Q(x),\check{p})dx = \check{w}$. The optimal wage level $\check{w}$ is the one that maximizes surplus $\int_{\check{p}}^{\bar{p}} [\int_{F(\check{p})}^{1} \lambda(Q(x),\check{p})dx - \underline{w}]dF(p)$. Since the marginal firm $\check{p}$ exerts a negative externality on firms $p > \check{p}$, we know that firms pay workers more than their outside option, $\check{w} > \underline{w}$.

Here we argue that firms have no incentives to underbid, offering $w < \check{w}$; hence, making $\check{w}$ a wage cap that firms are free to underbid also implements the second-best outcome. To see this, we need to check that the marginal firm $\check{p}$ (which is most tempted to underbid) does not want to cut its wage to $\underline{w}$ (which is the most profitable deviation). Indeed, note that at $w = \underline{w}$ firm $\check{p}$ exerts no externality on other firms or workers, so captures its full contribution to social surplus. By definition of $\check{p}$, this social surplus is zero when the firm offers $\check{w}$ and screens at a random rank, as instructed by the planner. When the firm disobeys the planner and posts $w = \underline{w}$, the contribution to social surplus is thus negative, meaning the firm's profits are also negative.

C.4 Production Complementarities

Theorem 3 is in stark contrast to the efficiency of equilibrium in the standard assignment model, where equilibrium also features PAM when the production function is supermodular. We now show that equilibrium continues to be inefficient even with exogenous complementarities.

Proposition 3. *Suppose firm p's surplus equals $\omega(q,p) = h(p)\lambda(q,p) - \underline{w}$ with $h, h_p > 0$.*
(a) Equilibrium exists, and is unique. All firms above some threshold $p \geq \underline{p}$ enter and sort according to PAM.
(b) If $h(p) = p$, PAM is inefficient.

Proof. (a) Surplus ω is increasing in p and supermodular, $\omega_{qp} = \lambda_{qp}h + \lambda_q h_p > 0$, so in equilibrium firms $p \geq \underline{p}$ enter and matching is PAM. Equilibrium existence then follows as in Theorem 2. Indeed, applicant quality $Q(x)$ and recruit quality $\lambda(Q(x), p)$ are identical to the baseline model, and the complementary production function only affects equilibrium wages and profits.

(b) To check the necessary condition for efficiency (32), note first that

$$\frac{\lambda_{qp}}{\lambda_q} = -\frac{1}{1-p} + 2\frac{1-q}{1-p(1-q)} = \frac{2(1-p)(1-q) - (1-p(1-q))}{(1-p)(1-p(1-q))}$$

and

$$\frac{\phi_p}{\phi} = \lambda_p \cdot \frac{1}{\lambda - q} = \frac{q(1-q)}{(1-p(1-q))^2} \cdot \frac{1-p(1-q)}{qp(1-q)} = \frac{1}{1-p(1-q)} \cdot \frac{1}{p}.$$

Thus, equilibrium is inefficient (and could be improved by re-ordering low-wage firms) if

$$\frac{\phi_p}{\phi} - \frac{\omega_{qp}}{\omega_q} = \left(\frac{\phi_p}{\phi} - \frac{\lambda_{qp}}{\lambda_q}\right) - \frac{h'}{h} = \frac{q}{(1-p)(1-p(1-q))} + \frac{1}{p} - \frac{h'}{h} \tag{40}$$

is positive. This depends on the degree of supermodularity. For the standard specification with $h(p) = p$, it is positive. That is, the effect of reducing the compositional externality outweighs two sources of supermodularity: one in the recruiting function $\lambda(q,p)$, and another one in the production function $\omega = \lambda p - \underline{w}$. □

C.5 Screening Costs

Here we show that Theorems 2 and 3 extend to a model with screening costs, where net surplus is given by $\omega(q,p) = \lambda(q,p) - \kappa/(1-p(1-q)) - \underline{w}$. Observe that screening costs $\kappa/(1-p(1-q))$ fall in applicant quality q but increase in screening skills p since more skillful firms interview more candidates.[28]

Proposition 4. *Suppose there is a cost $\kappa \geq 0$ to screen each applicant.*
(a) Equilibrium exists, and is unique. All firms above some threshold $p \geq \underline{p}$ enter and sort according to PAM.
(b) PAM is inefficient.

Proof. (a) As in Theorem 2, skilled firms post higher wages since

$$\omega_{qp} = \lambda_{qp} - \left(\frac{\kappa}{1-p(1-q)}\right)_{pq} = \frac{1-p(1-q)-2q}{(1-p(1-q))^3} + \frac{1+p(1-q)}{(1-p(1-q))^3}\kappa > 0.$$

As before, the first term is positive since $\lambda = q/(1-p(1-q)) < 1/2$. Additionally, the second term is always positive. Thus $\omega(q,p)$ is supermodular on $q \in [0,\bar{q}]$, as required. As for entry, note that $\omega_p = \omega(1-q)/(1-p(1-q)) \geq 0$. Equilibrium existence and uniqueness then follow as in Theorem 2.

(b) We apply Lemma 4 to show that PAM is inefficient. Indeed

$$\frac{\omega_{qp}}{\omega_q} = 2\frac{1-q}{1-p(1-q)} - \frac{1-\kappa}{1-p(1-\kappa)} < \frac{1-q}{1-p(1-q)} = \frac{\lambda_p}{\lambda} < \frac{\lambda_p}{\lambda}\frac{\lambda}{\lambda-q} = \frac{\phi_p}{\phi}$$

where the first inequality follows because $\kappa < q$, which in turn follows from $\omega(q,p) = (q-\kappa)/(1-p(1-q)) - \underline{w} \geq 0$. $\square$

Proposition 4 shows that the main insights of our paper carry over to a model with screening costs. This extension also creates a role for different information structures because our "perfect bad news" screening is no longer without loss. We can show that Proposition 4 extends to "symmetric signals" where high (resp. low) types pass the screening tests with probability p (resp. $1-p$), and firms differ in their screening skill $p \in (0.5, 1)$.

However, if firms receive "perfect good news" information whereby untalented applicants fail all tests, while firms differ in their probability p of identifying talented workers, then equilibrium matching is NAM and equilibrium is efficient. Intuitively, all firms are equally effective at screening and hire only talented workers, $\lambda = 1$, but skilled firms are more efficient and need not search as long to find a talented worker. We dislike this signal structure because of its counter-factual predictions that all firms hire workers of the same quality, and high-quality firms have lower search expenditure. Indeed, high-wage

[28]By adopting this surplus function, we implicitly assume that firms prefer to screen candidates rather than hiring a random, unscreened candidate. This mild condition is satisfied if (i) the minimum wage $\underline{w}$ is sufficiently high such that $\omega(q,p) > q$ for any firm p willing to hire a worker, $\lambda(q,p) \geq \underline{w}$, or (ii) screening additionally screens out unmodelled "terrible" types of workers.

firms have more applications (Belot et al. (2018), Banfi and Villena-Roldán (2019)) and interview more applicants (Barron et al. (1985)).

D Proofs from Section 4

D.1 Derivation of Equation (17) and Proof that $V(r)$ is Convex

The envelope theorem applied to (14) implies

$$V_r(r(t),t) = \int_t^\infty e^{-\beta(s-t)} \frac{\partial r(s)}{\partial r(t)} ds.$$

To compute the integrand, we write the solution of the talent evolution (13) as a function of its initial condition $r_s(s,\underline{r}) = \zeta(r(s,\underline{r}),s)$ where $\zeta(r,s) = \alpha(\lambda(Q(s),\psi(r)) - r)$ and $r(t,\underline{r}) = \underline{r}$. As in the proof of Lemma 1, Hartman (2002, Theorem 3.1) implies $r_{s\underline{r}} = \zeta_r r_{\underline{r}}$ with boundary condition $r_s(t,\underline{r}) = 1$. Hence

$$\frac{\partial r(s)}{\partial r(t)} = r_{\underline{r}}(s,r(t)) = \exp\left(\int_s^t \zeta_r(r(u),u)du\right) = \exp\left(-\alpha\int_s^t [1-\lambda_p(Q(u),\psi(r(u)))\psi_r(r(u))]du\right)$$

implying (17).

To see that $V(r,t)$ is convex in r, we differentiate again to obtain

$$\begin{aligned} V_{rr}(r(t),t) &= \int_t^\infty e^{-\beta(s-t)} \frac{\partial^2 r(s)}{\partial r(t)^2} ds \\ &= \int_t^\infty e^{-\beta(s-t)} \frac{\partial r(s)}{\partial r(t)} \alpha \int_s^t \left[\lambda_{pp}(Q(u),\psi(r(u)))\psi_r(r(u))^2 + \lambda_p(Q(u),\psi(r(u)))\psi_{rr}(r(u))\right] \frac{\partial r(u)}{\partial r(t)} duds \end{aligned}$$

which is positive since all four derivatives $\lambda_p = \frac{q(1-q)}{(1-p(1-q))^2}, \lambda_{pp} = \frac{2q(1-q)^2}{(1-p(1-q))^3}, \psi_r, \psi_{rr}$ are positive, the first three strictly. For an intuition consider the random recruiter example where $\psi(r) = p^L + r(p^H - p^L)$. Intermediate levels of recruiting skills r have the draw-back that the firm's wage must strike a compromise between the firm's low-skill and high-skill recruiters, while a firm with homogeneous recruiters, $r = 0$ or 1, can choose the optimal wage for all.

D.2 Proof of Theorem 4

Here we complete the proof of Theorem 4 by arguing that if there is an atom of initially identical firms, these firms diverge immediately. Assume to the contrary, that at time

$t > 0$ an atom of firms has the same worker quality $r(t)$ and write $[x^0, x^1]$ for the talent-ranks of these firms. Since optimal wages rise in talent and hence talent differences never vanish, firms in the atom must have identical talent $r(s)$ for all $s < t$. At any time $s < t$ the wage distribution must be smooth by the arguments in Section 2. If firms in the atom post different wages, they drift apart. Hence the firms must employ non-degenerate distributional strategies,[29] posting both high and low wages to attract good and bad applicants; they must thus be indifferent across a range of applicants $[q^0(s), q^1(s)]$ for all $s < t$. Thus, the first order condition (16) must hold with equality on $[q^0(s), q^1(s)]$ for all $s < t$ and the atom quality $r(s)$.

To see that such distributional strategies cannot be optimal, consider a firm that deviates by always attracting the best applicants in the atom $q^1(s)$, rather than mixing over good and bad applicants. At time $s = 0$, the choice $q^1(0)$ is optimal. Moreover, over time the firm's quality rises above $r(s)$ since it attracts better applicants. Since the marginal benefit of attracting better applicants, the RHS of (16), strictly increases in r, this deviation strictly improves on the posited distributional strategy. This proves that initially identical firms diverge immediately.

D.3 Proof of Theorem 5

Here we show that firm x's talent $R(x,t)$ and applicant quality $Q(x,t)$ converge to their steady state levels $R^*(x)$ and $Q^*(x)$. For constant applicants $Q(x,t) \equiv q$, talent drifts towards $\rho(q)$. The complication is that firm x's applicant quality $Q(x,t)$ also changes over time, with $Q_x(x,t)$ given by (12) and $r_t(x,t)$ by (13).

First, we establish a contraction property. Define the limits $\underline{Q}(x) := \liminf_t Q(x,t)$, $\bar{Q}(x) := \limsup_t Q(x,t)$, $\underline{r}(x) := \liminf_t R(x,t)$, and $\bar{R}(x) := \limsup_t R(x,t)$ and interpret (12) as an operator $\mathcal{Q}$, mapping firm quality functions $R(\cdot,t)$ into applicant quality functions $Q(\cdot,t) = \mathcal{Q}[R(\cdot,t)](\cdot)$. We claim that:

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x) \leq \underline{Q}(x) \leq \bar{Q}(x) \leq \mathcal{Q}[\rho(\underline{Q}(\cdot))](x). \tag{41}$$

To understand (41), first observe that that $\mathcal{Q}$ is antitone: if $R(x) \geq \hat{R}(x)$ for all x then $Q(x) = \mathcal{Q}(R(\cdot))(x) \leq \mathcal{Q}(\hat{R}(\cdot))(x) = \hat{Q}(x)$, since $Q(1) = \hat{Q}(1) = \bar{q}$ and $\phi(q, \psi(r), x)$ increases in r. Intuitively, better recruiters introduce more adverse selection. Inequalities (41) then state that if applicant quality was equal to one of its limits, $\underline{Q}$ and $\bar{Q}$, and talent

[29] When using a distributional strategy, a firm posts an entire distribution of wages $\nu = \nu(w,t)$ of wages at any time t; we then interpret $\psi(R(x,t))$ as the weighted-average skill of firms posting the x-ranked wage, and solve for the firm's evolution of talent by taking expectations over the RHS of (13).

r was in steady state, then the induced difference in applicant pools is larger than the original difference.

We prove (41) in two steps. First, since $R(x,t)$ drifts towards $\rho(Q(x,t))$, which is asymptotically bounded by $\rho(\underline{Q}(x))$ and $\rho(\bar{Q}(x))$, we have

$$\rho(\underline{Q}(x)) \leq \underline{R}(x) \leq \bar{R}(x) \leq \rho(\bar{Q}(x)) \tag{42}$$

for all x. Second,

$$\underline{Q}(x) = \lim_{t\to\infty} \inf_{t'>t} Q(x,t') = \lim_{t\to\infty} \inf_{t'>t} \{\mathcal{Q}[R(\cdot,t')](x)\} \geq \lim_{t\to\infty} \mathcal{Q}[\sup_{t'>t}\{R(\cdot,t')\}](x) = \mathcal{Q}[\bar{R}(\cdot)](x)$$

where the first equality is the definition of the lim inf, and the second the definition of the operator $\mathcal{Q}$. The inequality uses the antitonicity of $\mathcal{Q}$: since $R(\hat{x},t') \leq \sup_{t'>t}\{R(\hat{x},t')\}$ for all $t' > t$ and $\hat{x}$, we know that $\mathcal{Q}[R(\cdot,t')](x)$ (weakly) exceeds $\mathcal{Q}[\sup_{t'>t}\{R(\cdot,t')\}](x)$ for all $t' > t$ and x, and hence so does $\inf_{t'>t} \mathcal{Q}[R(\cdot,t')](x)$. The last inequality uses the dominated convergence theorem to exchange the limit $t \to \infty$ and the operator $\mathcal{Q}$, as well as the definition of the limsup, $\bar{R}(x) = \lim_{t\to\infty} \sup_{t'>t} R(x,t')$. Together with the analogue argument that $\bar{Q}(x) \leq \mathcal{Q}[\underline{R}(\cdot)](x)$, and applying the antitone operator $\mathcal{Q}$ to (42) we get

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x) \leq \mathcal{Q}[\bar{R}(\cdot)](x) \leq \underline{Q}(x) \leq \bar{Q}(x) \leq \mathcal{Q}[\underline{R}(\cdot)](x) \leq \mathcal{Q}[\rho(\underline{Q}(\cdot))](x)$$

establishing (41).

To complete the proof of convergence, suppose "inductively" that applicant and firm quality converge above some $\hat{x} \in (0,1]$, i.e. $\underline{Q}(x) = \bar{Q}(x)$, and hence $\underline{R}(x) = \bar{R}(x)$, for all $x \in (\hat{x},1]$. Fix ϵ, and let $\delta(\epsilon) := \max_{x\in[\hat{x}-\epsilon,\hat{x}]} \left|\bar{Q}(x) - \underline{Q}(x)\right|$ be the maximum distance between the liminf and limsup on $[\hat{x}-\epsilon,\hat{x}]$. Since $\rho(q)$ is locally Lipschitz in q with constant K,[30] we have

$$\max_{x\in[\hat{x}-\epsilon,\hat{x}]} \left|\rho(\underline{Q}(\cdot))(x) - \rho(\bar{Q}(\cdot))(x)\right| \leq K\delta(\epsilon).$$

Next, since $\mathcal{Q}[R(\cdot)](x)$ solves (12), the RHS of which is locally Lipschitz in q and r with constant K', and choosing $\epsilon < 1/K'(1+K)$ we get

$$\max_{x\in[\hat{x}-\epsilon,\hat{x}]} \left|\mathcal{Q}[\rho(\underline{Q}(\cdot))](x) - \mathcal{Q}[\rho(\bar{Q}(\cdot))](x)\right| \leq K'\epsilon(1+K)\delta(\epsilon) < \max_{x\in[\hat{x}-\epsilon,\hat{x}]} \left|\bar{Q}(x) - \underline{Q}(x)\right|.$$

[30]Indeed, recall that $\rho(q) = r \in [0,1)$ solves $\lambda(q,\psi(r)) - r = 0$. Thus, by convexity of the LHS we have $\lambda_p(q,\psi(\rho(q)))\psi_r(\rho(q)) - 1 < (\lambda(q,\psi(1)) - 1)/(1-\rho(q)) < 0$ and so $R_q(q) = -\frac{\lambda_q(q,\psi(\rho(q)))}{\lambda_p(q,\psi(\rho(q)))\psi_r(\rho(q))-1} < \frac{\lambda_q(q,\psi(\rho(q)))(1-\rho(q))}{1-\lambda(q,\psi(1))} =: K$.

contradicting (41). Hence we must have $\underline{Q}(x) = \bar{Q}(x)$ and hence $\underline{R}(x) = \bar{R}(x)$, for all $x \in [\hat{x} - \epsilon, 1]$, and thus for all $x \in [0, 1]$.

D.4 How Turnover affects Wage and Productivity Dispersion

Equation (22) implies that $(\beta + \alpha)(W^*(Q^*(x')) - W^*(Q^*(x)))$ rises in α for all $x' > x$, and hence steady-state wage variation grows in the dispersive order. Here we argue that $W^*(Q^*(x'))/W^*(Q^*(x))$ and $\Pi^*(Q^*(x'))/\Pi^*(Q^*(x))$ increase in α for all $x' > x$, and hence wage and profit variation also grows in the more demanding log-dispersive order.

Formally, we argue that steady-state wages and profits as functions of applicant quality and turnover $(\beta + \alpha)W^*(q, \alpha)$ and $\Pi^*(q, \alpha)$ are log-supermodular in q and α. This implies that they are also log-supermodular as functions of rank x and turnover α since applicant quality $q = Q(x)$ is a monotone transformation of x that does not depend on turnover α.

Recall that a function $\xi(q, \alpha)$ with $\xi(0, \alpha) = 0$ is log-supermodular in q, α if its partial derivative $\xi_q(q, \alpha)$ is log-supermodular: For then

$$(\log \xi(q, \alpha))_q = \left(\log \int_0^q \xi_q(\hat{q}, \alpha) d\hat{q}\right)_q = \frac{\xi_q(q, \alpha)}{\int_0^q \xi_q(\hat{q}, \alpha) d\hat{q}}$$

rises in α, since $\xi_q(q, \alpha)/\xi_q(\hat{q}, \alpha)$ rises in α for all $\hat{q} < q$.

Thus, it suffices to show that marginal wages and profits (22) and (23) are log-supermodular in q and α. The only factor that depends on both q and α is

$$1/\eta(q, \alpha) := 1/\left(\beta + \alpha\left(1 - \lambda_p\left(q, \psi(\rho(q))\right)\psi_r(\rho(q))\right)\right),$$

and so,

$$(\log \eta(q, \alpha)^{-1})_{q,\alpha} = \frac{\eta_\alpha \eta_q - \eta_{\alpha q}\eta}{\eta^2} = \frac{-(1 - \lambda_p\psi_r)\alpha(\lambda_p\psi_r)_q + (\lambda_p\psi_r)_q(\beta + \alpha(1 - \lambda_p\psi_r))}{\eta^2} = \frac{(\lambda_p\psi_r)_q\beta}{\eta^2} > 0$$

since $\left(\lambda_p\left(q, \psi(\rho(q))\right)\psi_r(\rho(q))\right)_q = (\lambda_{pq} + \lambda_{pp}\psi_r\rho_q)\psi_r + \lambda_p\psi_{rr}\rho_q > 0$.