A Simple Quantity Adjustment Model of Economic Fluctuations and Growth

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Abstract¹

This paper analyzes a new model of economy with several sectors which face quantity constraints, and show that business cycles and growth are present in this simple model. The sizes of sectors either increase or decrease stochastically, depending on the signs of the sectoral excess demands. We assume that allocation of resources is not instantaneous, and rely on the notion of holding time of continuous-time, discrete-state Markov processes to select the sector that changes its size. Not only the output of this model fluctuates, but also, and more importantly, the level of the aggregate economic activity depends on the pattern of demands of the sectors of the economy. The greater is demands for high productivity sectors, the higher is the expected values of the model outputs.

1 introduction

Fluctuations of aggregate economic activities or business cycles have long attracted economists' attention. The real business cycle (RBC) theory by Kydland and Prescott (1982) may arguably be the most influential current theory of the mainstream economists. As typified by the RBC, economists explain business cycles as a direct outcome of the behavior of *individual agents*. This approach has been the standard in the mainstream economics in the last twenty years or so. The more strongly one wishes to interpret aggregate fluctuations as something 'rational' or 'optimal', the more likely one is led to this essentially microeconomic approach. The mission of this approach is to explain fluctuations as responses of individual agents to changes in their economic environments. The consumer's intertemporal substitution, for example, is a device to achieve this goal.

We take a different approach to fluctuations. We start with the fact that the economy consists of a large number of agents or sectors, and adjustments

¹This is an abridged version of Aoki and Yoshikawa 2001.

leading to equalization of productivities among sectors take time. Although agents attempt intertemporally to maximize their respective objective functions, they are subjected to environmental constraints, and idiosyncratic shocks that are usually all different among agents. Plainly, an outcome of interactions of a large number of agents facing such incessant idiosyncratic shocks cannot be described by a response of the representative agent. It calls for a model of stochastic processes.

We use a simple model to accomplish two main objectives. First, we show that fluctuations of the aggregate economy arise as an outcome of interactions of many agents/sectors. Second, we demonstrate that the *level* of the *aggregate* economic activity depends on the structure of demand. In the standard neoclassical equilibrium, where the marginal products of production factors such as labor are equal in all activities and sectors, demand determines only the composition of goods and services to be produced, but *not* the level of the *aggregate* economic activity.

Demands affect the aggregate level of economic activities in several ways. One is externality associated with demand which might produce multiple equilibria such as in Diamond (1982).² Another is the differences in productivity across sectors/activities. Recent works by Murphy, Shleifer and Vishny (1989), and Matsuyama (1995), for example, emphasize the importance of increasing returns in order to demonstrate the role of demand in determining the level of the aggregate production. They, in effect, allow differences in productivity across sectors to draw their conclusions. We assume that productivities differ across sectors in the economy, and the differences in productivities persist.³ In the standard analysis, resources are assumed to be instantly reallocated so as to attain equality of productivity across sectors.

Although studies of macroeconomy with many heterogeneous agents are not new, dynamic behavior of economies in disequilibrium is not satisfactorily analyzed.⁴ Clower (1965) and Leijonhufvud (1968) pointed out that quantity adjustment might be actually more important than price adjustment in economic fluctuations.⁵

The model in this paper has three new features which distinguishes it from the standard business cycles or growth models. First, we introduce a new stochastic adaptive scheme which is different from many adaptive models in economics in general, and from agent-based computational models in particular. Second, the notion of holding, or sojourn, time is used to

²Also see Aoki and Shirai (2000).

³Yoshikawa (1995, 2000) taking the Japanese economy as an example, shows that productivity differences across sectors actually persist.

⁴The traditional Walrasian economy is the egregious example. It focuses on price adjustment with the help of the non-existent auctioneer.

⁵Here, as Leijonhufvud (1968) puts it, quantities determine quantities. This insight spawned a vast literature of the so-called non-Walrasian or disequilibrium analysis; see, for example, Bénassy (1975), Malinvaud (1977), and Dréze (1991). This approach, however, suffers from its basically static nature of analysis, because their purpose is primarily to show the existence of non-Walrasian quantity-constrained equilibrium. Dréze (Chap.10) is an exception, but it is a traditional deterministic model. In contrast, our model is dynamic and stochastic in order to examine possible fluctuations of the quantity-constrained economy.

select stochastically the sector or agents which acts next. Third, we apply the Ewens distribution in connection with growth processes, which is the first such applications in economics of the notion originally developed in the literature of population genetics.⁶ Specifically, the higher is the share of demand for high productivity sectors, the higher is the *average* GDP.

In the first part of the paper we keep the number of sectors fixed. Then, we allow the number of sectors to grow in a stochastic way. Here we employ a scheme analogous to that of the Ewens sampling formula as discussed by Costantini and Garibaldi (1999, 2000) to introduce new sectors.

2 The model

Suppose that there are K sectors in the economy. We keep the value of K fixed for now. We later drop this assumption. Sectors adjust their outputs by varying the utilization rates of the factor of production in response to the excess demands for their goods. We later show that the sectors are weakly coupled, that is, there is externality when any one sector changes its output.

We model the state of this economy as a continuous-time Markov chain to avoid the phenomena associated with lock-step adjustments by several different sectors in the economy. We use the notion of holding (sojourn) times to select a sector which adjusts first.

Let sector i have productivity coefficient, c_i , which is exogenously given and fixed. We assume that changes in utilization of a production factor are much faster than the processes for equalization of productivity. Assume, for convenience, that sectors are arranged in the decreasing order in productivity. Sectors only observe own productivities and excess demands, but not of other sectors. Sector i employs N_i units of factor of production. It is a non-negative integer-valued random variable. We call its value as "size" of the sector. When $N_i(t) = n_i$, $i = 1, 2, \dots, K$, the output of sector i is $c_i n_i$, and the total output (GDP) of this economy is

$$Y(t) := \sum_{i=1}^K c_i n_i(t). \quad (1)$$

Demand for the output of sector i is denoted by $s_i Y(t)$, where $s_i > 0$ is the share of sector i , and $\sum_i s_i = 1$. The shares are also assumed to be exogenously given and fixed.

We denote the excess demand for goods of sector i by

$$f_i(t) := s_i Y(t) - c_i n_i(t),$$

⁶In this sense, the model may be thought of a generalized or modern version of quantity adjustment model. Leijonhufvud (1974, 1993) described a Marshallian quantity adjustment model. He envisioned a representative firm which adjusts outputs to partially narrow the gap between the supply and demand prices of the good produced by the firm. Since the demand price schedule is unknown to the firm, the market clearing price is substituted for it. Aoki analyzed his model in Aoki (1976, 193ff., 319 ff.) In this paper sectors are not the same, and sectors adopt stochastic response rules to gaps between the demand and supplies. Sectors are subject to aggregate externalities as we discuss in the text.

$i = 1, 2, \dots, K$. Denote the set of sectors with positive excess demands by $I_+ = \{i; f_i > 0\}$, and similarly for the set of sectors with negative demands by $I_- = \{j; f_j \leq 0\}$.

To shorten notation, summations over these subsets are denoted as $\sum_+$ and $\sum_-$. Denote by n_+ the number of n s in the set I_+ , that is, we write $n_+ := \sum_+ n_i$, where the subscript $+$ is a short-hand for the set I_+ , and similarly $n_- := \sum_- n_j$, for the sum over the sectors with negative excess demands. Let $n = n_+ + n_-$.

Sectors with non-zero excess demands attempt to reduce the sizes of excess demands by adjusting their sizes, up or down, depending on the signs of the excess demands. This response pattern implies that firms increase their (positive) profits by raising the level of production when they face excess demand, while they reduce their losses by lowering the production level when they face excess supply. The adjustment, however, is not instantaneous. Furthermore, the same excess demand or supply does not necessarily bring about the same response. For example, the same excess demand or supply may be interpreted by firms to be either permanent or temporary. Firms would also hold different inventory stocks.

It is simply impossible for us to obtain enough information on how sectors (firms) react to a given level of excess demand or supply. Also, as we show later, there is externality. Excess demands or supplies of all sectors are changed when a single sector reacts to its own excess demand or supply condition. The only assumption we make is that a sector which raises the level of production faces excess demand, while a sector which lowers the production level faces excess supply. We do not attempt to build additional details into the model.

The model is a jump Markov process, and as such it is completely specified by the transition rates. The sizes are adjusted up or down by a unit (in a suitably chosen unit). They are

$$\Pr(N_i(t+h) = n_i + 1 | N_i(t) = n_i) = \gamma_i h + o(h),$$

for $i \in I_+$, and

$$\Pr(N_i(t+h) = n_i - 1 | N_i(t) = n_i) = \rho_i h + o(h),$$

for $i \in I_-$, where the transition rates, γ and ρ , of the jump Markov process are specified later.

Holding times

We assume that the time it takes for sector i to adjust its size by one unit, up or down, T_i , is exponentially distributed, $\Pr(T_i > t) = \exp(-b_i t)$, where b_i depends on the sign of the excess demand, as given later. This time is called sojourn time or holding time in the probability literature. We assume that the random variables T s of the sectors with non-zero excess demand are independent.

The same amount of excess demand or supply brings about a different reaction by sector (or firm), because each sector faces idiosyncratic economic

environment or constraint. For the same reason, we do not know when the sector facing disequilibrium "reacts". It is stochastic. The notion of "holding time" models the timing of stochastic reaction. We assume that the reaction times are random, and are independent for each sectors with non-zero excess demands.

The sector that adjusts first is determined by the sector with the shortest holding time. Let T^* be the minimum of all the holding times of the sectors with non-zero excess demands. Lawler calculates that for $a \in I_+$, $\Pr(T_a = T^*) = \gamma_a/(\gamma_+ + \rho_-)$, where $\gamma_+ = \sum_+ \gamma_i$, and $\rho_- = \sum_- \rho_j$, and if $a \in I_-$, then the probability of the jump in sector a is given by $\rho_a/(\gamma_+ + \rho_-)$. See Lawler (1995, 56) or Aoki (1996, Sec.4.2).

Transition rate specifications

We assume that γ s and ρ s depend on the total number of sizes and the current size of the sector that adjusts, $\gamma_i = \gamma_i(n_i, n)$, and $\rho_j = \rho(n_j, n)$. This is an example of applying W. E. Johnson's sufficientness postulate, see Zabell (1992). We have discussed specifications of entry and exit probabilities in Aoki (2000). See also Costantini and Garibaldi (1979, 1989), who give clear discussions on reasons for these specifications. As explained clearly by Zabell, there is a long history of statisticians who have considered this type of problems. There are good reasons for γ_i to depend only on n_i and n , and similarly for ρ_i . See Zabell for further references on the statistical reasons for this specification.

We specify the entry rate, that is, the rate of size increase by $\gamma_a(n_a, n) = (\alpha + n_a)/(K\alpha + n)$, where α is a non-negative parameter, and that of the exit rate, namely, the rate of size decrease by $\rho_a(n_a, n) = n_a/n$.

If α is much smaller than K , then $\gamma_i \approx n_i/(\theta + n)$, where we set $\theta := K\alpha$, that is,⁷ $b\gamma_i(n_i, n) \approx [n/(\theta + n)](n_i/n)$.

So long as θ is kept constant, the above expression implies that the choice of K and α do not matter, provided α is much smaller than K . It is also clear that γ_i is nearly the same as the fraction n_i/n , which is the probability for exit. Then, time histories of n_i are nearly those of tosses of a nearly fair coin. We have K such coin tosses available at each jump. The sector that jumps determines which coin toss is selected from these K coins.

It is well known, at least in the population genetics and statistics literature, that a positive-valued θ implies stochastic emergence of a new sector. We set $\alpha = 0$ to discuss economies with fixed numbers of sectors, and set it to a positive number to allow for new sectors to emerge. In the latter case, a new sector emerges with probability $\theta/(\theta + n)$, while a size of sector i increases by one when the sector has positive excess demand with probability $(\alpha + n_i)/(\theta + n)$. See Ewens (1972).

⁷There is an obvious interpretation of this approximate expression in terms of the Ewens sampling formula, Ewens (1972).

3 The Model Behavior

First, we describe simple patterns of interactions among excess demands of the sectors.

Aggregate output and demand

After a change in the size of a sector, the total output of the economy changes to

$$Y(t+h) = Y(t) + \text{sgn}\{f_a(t)\}c_a, \quad (2)$$

where a is the sector that jumped first by the time $t+h$.

After the jump, this sector's excess demand changes to

$$f_a(t+h) = f_a(t) - c_a(1-s_a)\text{sgn}\{f_a(t)\}. \quad (3)$$

Other, non-jumping sectors have the excess demand changed to

$$f_i(t+h) = f_i(t) + \text{sgn}\{f_a(t)\}s_i c_a, \quad (4)$$

for $i \neq a$.

These two equations show the effects of an increase of size in one sector. An increase by c_a of output increases GDP by the same amount. However, sector a experiences an increase of its demand by only a fraction s_a of it, while all other sectors experience increase of their demands by $s_i c_a$, $i \neq a$. Eq. (3) shows a source of externality for this model that affects the model behavior significantly. The index sets I_+ and I_- also change in general.

Defining $\Delta Y(t) := Y(t+h) - Y(t)$, and $\Delta f_i(t) = f_i(t+h) - f_i(t)$, rewrite (1) through (3) as $\Delta Y(t) = \text{sgn}\{f_a(t)\}c_a$, $\Delta f_a(t) = -(1-s_a)\Delta Y(t)$, and $\Delta f_i(t) = s_i \Delta Y(t)$, respectively for $i \neq a$.

Equilibrium sizes of the sectors (Zero excess demand conditions)

When excess demands of all sectors are zero, no section changes its output, and therefore, total output or GDP does not change either. We call this state "equilibrium". We solve K equations of zero excess demands $f_i = 0$, $i = 1, 2, \dots, K$, and obtain the equilibrium sizes, denoted by superscript e , of the fractions of sector sizes, n_i^e/n^e for $i = 1, \dots, K$, and the ratio of the total output to the total number of units, Y^e/n^e .

Define K dimensional column vectors $\mathbf{c} := (c_1, c_2, \dots, c_K)'$, and $\mathbf{s} := (s_1, s_2, \dots, s_K)'$. A diagonal $K \times K$ matrix $\mathbf{C} := \text{diag}(c_1, c_2, \dots, c_K)$ is introduced to simplify our discussion.

The output is $Y = \langle \mathbf{c}, \mathbf{n} \rangle$, and the set of zero excess demand conditions is expressed by $s_i Y = c_i n_i$, $i = 1, 2, \dots, K$, which is rewritten compactly as $\mathbf{C}\mathbf{n} = \mathbf{s}Y = \mathbf{s}\mathbf{c}'\mathbf{n}$, or $\Phi\mathbf{n} = \mathbf{0}$, with $\Phi = \mathbf{C} - \mathbf{s}\mathbf{c}'$.

Noting that the shares sum to one, matrix Φ does not have full rank, because $|\Phi| = |\mathbf{C}|(1 - \mathbf{c}\mathbf{C}^{-1}\mathbf{s}) = 0$. It has rank $K-1$, and its null space has dimension one which is spanned by the solutions we give next.

The solution is $\mathbf{n} = C^{-1}\mathbf{s}Y$, or

$$n_i^e/n^e = (s_i/c_i)/\sum (s_i/c_i), i = 1, \dots, K. \quad (5)$$

That is, the relative size of sectors in equilibrium n_i^e/n^e is uniquely determined by the productivity coefficients c_i , and demand shares s_i . The absolute levels of the sizes are indeterminate, however. Multiply (5) by c_i and sum over i to obtain the relation between Y and n as

$$Y^e = n^e / \sum_i s_i/c_i. \quad (6)$$

Equation (6) shows that Y^e and n^e corresponds to each other. The value of Y^e is indeterminate because that of n^e is. When $c_i = c$ for all i , (4) and (5) reduce to $n_i^e = s_i n^e$, and $Y^e = c n^e$. With n^e fixed, Y^e is maximized by $s_i = 1/K$ for all i , that is, by equal demands on all sectors.

Note that $Y_i^e = s_i Y^e$, for all i , as it should.

The behaviour out of equilibrium: Two-sector model

The expected level of total output or GDP is indeterminate in equilibrium in a deterministic model. In this subsection, we explore the behavior of the economy out of equilibrium in a stochastic two-sector model.

After a jump by a sector, patterns of signs of the excess demands in sectors change. The changes are rather complicated to analyze in generality. To gain some insight, we analyse a simple two sector model, and comment on how the results of the two sector model may generalize. Note that $s_2 = 1 - s_1$ in the two-sector model. This model is thus characterized by two parameters s_1 and c_2/c_1 . (If you wish, c_1 may be set to one with a suitable choice of unit to measure n_1 .)

Eq.(5) shows that $n_1^e/n_2^e = (s_1/c_1)/(s_2/c_2)$, that is, the sign of $(s_2/s_1 - c_2/c_1)$ determines the relative sizes of the two sectors at equilibrium. Hence it does matter in the details of stochastic evolution whether n_1^e is larger than n_2^e or not. We describe the model behavior assuming that this sign is positive, that is, $s_2/c_2 > s_1/c_1$, which implies that $n_2^e \geq n_1^e$. The other case may be examined by switching the subscripts. We suppress time arguments.

The non-negative quadrant of the plane for n_1 and n_2 , with the horizontal axis labelled by n_1 , and the vertical by n_2 , is divided into six regions, denoted by R_k , $k = 1, 2, \dots, 6$. They are bounded by $n_i \geq 0$, $i = 1, 2$, and by five other straight lines with a common slope $\beta := (c_1/c_2)(s_2/s_1)$. This slope is larger than one for our choice of the parameter values. The intercepts of the five lines are β , 1, 0, -1, and $-\beta$. These five lines are denoted by $L_1, L_2, \dots, L_5$, respectively. Line 3 cuts the n_1 axis at 0, Line 4 at 1, and Line 5 at β .

In different regions, either the signs of the excess demands, or those after size changes by sector 1 or 2 are different, as detailed below. We note first that the two sector model is special in that $f_1 + f_2 = 0$. Further, denoting by $f_i'(\pm)$, the value of the excess demand of f_i after a change of n_1 by ± 1 , and similar by $f_i''(\pm)$ that of f_i after a change in n_2 by ± 1 , we note that and that $f_1'(\pm) + f_2'(\pm) = 0$ and similarly $f_1''(\pm) + f_2''(\pm) = 0$ from (3) and

(4). Recall that only an increase in n_1 is possible when $f_1 > 0$ in sector 1. Similarly with $f_2 < 0$, $f_1''(+)$ does not happen. To see these we note, for example, that in R_1, R_2 and R_3 which are above L_3 , $f_1 > 0$. Hence $f_2 < 0$ in these regions.

After a change in n_1 by ± 1 , the excess demands change to $f_1'(\pm) = s_1 c_2 [n_2 - \beta(n_1 + 1)] > 0$, above L_1 , and so on. In this way signs of the excess demands and how the sign changes by a change in size in sector 1 and 2 are easily tracked. Note that signs are reversed in regions above and below Line 3.

From a state (n_1, n_2) in R_1 , consecutive jumps in sector 1 will bring the state to the boundary L_1 by increasing n_1 and the model state enters R_2 , and the nature of dynamics changes. This is so because f_1 continues to be positive after jumps in sector 1. Similarly, consecutive jumps in sector 2 from a state in R_1 also eventually bring the state to the same boundary by decreasing n_2 . For that matter, we can calculate the various combinations of jumps in sector 1 and sector 2 to bring the state to the boundary, L_1 . We thus see that the state leaves R_1 with probability 1. From a state in R_2 consecutive decreases in n_2 are possible until the state enters R_3 . From a state on L_2 a jump in sector 2 bring the state on L_3 . Similar arguments can be made to see that a state (n_1, n_2) in R_6 eventually enters R_4 with probability 1. In this manner, we can show that states oscillate, possibly asymmetrically, near L_3 between $R_2 \cup R_3$ and $R_4 \cup R_5$, and so does the total output, Aoki (2002, Sec. 8.6) for further detail.

4 Stationary probability distribution for the two sector model

Next, we consider the stationary probability distribution for the total output or GDP in the two sector model. Our goal is to establish that the expected value of GDP depends on the demand pattern. A general discussion of dynamics is conducted via the master (backward Chapman-Kolmogorov) equation. Here we just derive the stationary probability distribution near the equilibrium represented by L_3 .

For simpler presentation we just treat the case with $\alpha = 0$. Suppose that the system enters R_3 . In the preceding subsection, we have shown that the state will oscillate in the region $R_3 \cup R_4$. Take the initial point b which is on or just below L_3 . Let $n(b) = (n_1(b), n_2(b))$ be the state of this point, and define two adjacent positions e and c by $n_1(e) = n_1(b) + 1$, $n_2(e) = n_2(b) + 1$, $n_1(c) = n_1(b) - 1$, and $n_2(c) = n_2(b) - 1$.

By repeated applications of the detailed balance conditions between successive states, starting with e and b , and those between b and c , we derive the relations for the stationary probabilities $\pi(b + (k, k))/\pi(b) = \gamma^{-\mu k}$, for $k = 1, 2, \dots$, where we use $n_2 = \beta n_1$ on or near L_3 , and similarly for $\pi(b - (l, l))/\pi(b) = \gamma^{\mu l}$, for $l = 1, 2, \dots, \bar{l} - 1$, with $\gamma = \exp(2/n_1(b))$, and $\mu = \beta/(1 + \beta)$.

From now on we write b for $n_1(b)$ since there is no ambiguity. The stationary distribution is $\pi(b + k) = A\gamma^{-\mu k}$ for $k = 1, 2, \dots$, and $\pi(b - l) =$

$A\gamma^{\mu l}$, where A is the normalizing constant $A^{-1} = \sum_{l=0}^{\bar{l}-1} \gamma^{\mu l} + \sum_{k \geq 1} \gamma^{-\mu k} = \gamma^{\mu \bar{l}} / [\gamma^{\mu} - 1]$. See Aoki (2002, Sec.8.6) for more detail of derivation.

Now, the expected value of the total output in the economy, or GDP, is $E(Y) = (c_1 + c_2\beta)E(n_1)$, where the expected value is calculated using the generating function, $E(n_1) = \frac{1}{1-\gamma^{-\mu}} + b - \bar{l} = b - \bar{l} - \frac{b}{2}(\beta - 1)$.

For the value of $\beta = 1$, this is clearly positive. With $\beta > 1$, it is positive if $1/(1 - \gamma^{-\mu}) > (b/2)(\beta - 1)$, which is satisfied for $\beta > \beta^*$ for some β^* . We assume that this condition is satisfied. Then, $EY = (c_1 + c_2\beta)E(n_1)$.

Obviously, $E(Y)$ depends on the initial state b . Its derivative with respect to s is positive, because of $dE(y)/ds = \{dE(Y)/d\beta\}d\beta/ds$, and $d\beta/ds = -(c_1/c_2)(1/s^2) \leq 0$. Hence, $E(Y)$ increases with a small increase in s if and only if $dE(Y)/d\beta < 0$.

We state this as

Proposition Given the initial state, the expected value, $E(Y)$, increases as the share of demand for sector 1 (high productivity sector) increases in the range of $\beta \geq 1$.

Analogous proposition may be established for $\beta \leq 1$ in similar manner.

This proposition means that the expected value of aggregate economic activity depends on the pattern of demand. More specifically, the higher is the share of demand for high productivity sector, the higher is the expected value of aggregate economic activity. Note that the level of aggregate economic activity is indeterminate in equilibrium of a corresponding deterministic model. In a stochastic model, the higher the share of demand for high productivity sector is, the more likely the sector faces excess demand and raises its production level. Therefore, the higher is the share of demand for high productivity sector, the greater would be the externality generated by an increase in the size of the sector. In this way we obtain the proposition.

4.0.1 Emergence of new sectors

Next, suppose that new sectors appear at the rate proportional to $\theta/(\theta + n_+)$. This transition rate may be justified as a limiting case in which parameter α goes to zero, while $K\alpha$ approaches a positive value θ . More in detail, we assume that either one of the sectors with positive excess demand increases size by one with probability $(\alpha + n_j)/(K_+\alpha + n_+)$, where K_+ denotes the number of sectors with positive excess demand, and n_+ is the total size of such sectors, or a new sector emerges with rate proportional to $(K_+ - 1)\alpha/(K_+\alpha + n_+)$.

We assume that a new sector, when it emerges, inherits characteristics, that is c and s of one of the existing sectors with equal probability. That is, if there are L sectors, then with probability $1/L$, the value of c and s of the randomly selected sector is inherited. The s s are then renormalized so that they sum to one, including the newly born sector. This is merely for convenience. Other schemes are certainly possible and may be informative to try.

As an example, suppose that $\mathbf{s} = (2, 2, 2, 2, 2, 1, 1, 1, 1, 1)/15$. This is one of the patterns of demands we simulate. We take $c_j = (K - j + 1)/K$, $j = 1, \dots, 10$, with $K = 10$. Then, the model has $\sum_i s_i/c_i = 2.38$. The

equilibrium sizes are: $n_1^e/n^e = .056$, $n_2^e/n^e = .06$, $n_3^e/n^e = .07$, $n_4^e/n^e = .08$, $n_5^e/n^e = .09$, $n_6^e/n^e = .056$, $n_7^e/n^e = .14$, $n_8^e/n^e = .09$, $n_9^e/n^e = .14$, and $n_{10}^e/n^e = .28$.

The simulation results for this case with 450 periods, with the initial sector sizes of 10 each, averaged over 200 runs, show that the total output cycles in the range of 99.5 to 100.3. Therefore, the average is about 99. Taking this value as a proxy for n^e , we obtain $n_1^e = 5.6$, $n_2^e = 6$, $n_3^e = 7$, $\dots$, $n_7^e = 14$, $n_8^e = 9$, $n_9^e = 14$, $n_{10}^e = 28$. These numbers agree with the numbers from the simulation very well. They are approximately $\mathbf{n}^e = (5, 6, 7, 8, 9, 5, 14, 9, 14, 28)$.

The reason for less efficient sectors to become larger than the initial starting values is that they must grow to meet the demands, while the efficient sectors shrink in size, because being efficient the initial sizes are too large for the demands.

Theoretical values of Y^e/n^e is about 0.42. The corresponding simulation number is also about 0.42. Here we also note that the maximum excursion $(Y_{max} - Y_{min})/Y_{mean}$ is about 2.5 per cent of the mean output.

5 Simulation runs for multi-sector model

The previous section analyzed the two sector model to show that the higher is the expected value of aggregate economic activity, the greater is the share of demand for high productivity sector. It is rather complicated, however, to analyze the multi-sector model explicitly. In this section we verify by simulation the robustness of the proposition obtained in the previous section for the multi-sector model.

We keep the total number of sectors to $K = 10$ in most of the simulation.⁸ As our discussion above indicates, for small value of θ , which ranges from 0.2 to .6 in our experiments, there is not much loss of generality in keeping the value of K fixed. We also keep fixed the order of the productivities from $c_1 = 1$ to $c_K = 1/K$ at equal interval. We start the simulation runs with the initial condition $n_i = 10$ for all sectors, $i = 1, 2, \dots, 10$.

In the graphs below, we skip the first 150 or 200 periods to avoid transient responses, and use the following five demand patterns for the outputs of the sectors with the vectors of share denoted by $\mathbf{s}^i$, $i = 1, 2, \dots, 5$:

$\mathbf{s}^1 = (5, 5, 4, 4, 3, 1, 1, 1, 1, 1)/26$; $\mathbf{s}^2 = (5, 3, 2, 1, 1, 1, 1, 1, 1, 1)/17$; $\mathbf{s}^3 = (2, 2, 2, 2, 2, 1, 1, 1, 1, 1)/15$; $\mathbf{s}^4 = (1, 1, 1, 3, 3, 3, 3, 1, 1, 1)/18$; and $\mathbf{s}^5 = (2, 2, 2, 1, 1, 1, 1, 1, 1, 1)/13$.

The sum of the shares of the top five sectors are 0.8, 0.7, .66, .5, and .61 respectively. By lumping the top five sectors and the bottom five sectors separately to produce a two sector model, the results of our analysis of the two sector model may be used to obtain some intuitive feel for the behavior of models with these five patterns. The simulation confirms what the analysis predicts, that is, the output is the largest for P_1 followed by those of P_2 , P_3 , and so on.

⁸We have done simulations with $K = 15$, and 20, but do not report them, since we have not observed any substantive differences.

All patterns are run 200 times for 500 periods with $\theta = .6$ except as we note below. Pattern $P2$ has also been run with $\theta = .2$. Pattern $P5$ is also run for 1000 periods 400 times.

Runs of 200 are small for Monte Carlo experiments. Our interest here, however, is not accurate estimates of any statistical properties of output variations, but rather in exhibiting possibilities of cyclical behavior in this simple model, and that output levels respond to demand pattern shifts.

One of the clear effects of different patterns is the dependence of average output levels on the patterns. Mean outputs are approximately in the order of $P1$ to $P5$. By putting larger demand shares at higher productivity sectors, the average output shifts up. Because standard deviation of outputs are still large due to small numbers of runs, effects of different demand patterns on the statistical features of cycles are not so clear cut. Peak-to-peak swings are about 2 per cent of the mean levels of outputs.

Due to space limitations we can show only two sets of simulation runs in this paper. The five panels of Fig. 1 show total outputs Y of five demand patterns, each averaged over 200 runs. The equilibrium value is $y^e/n^e = .4196$. This value is independent of θ values.

The four panels of Fig. 2 show a sample of how the number of sectors increases, together with the total number of sizes, total output, and output per unit size, for the demand pattern $P3$ with $\theta = .3$. As the value of θ is increased, the number of new sectors increases more quickly. For small values such as $\theta = 0.01$, new sectors come in much more slowly. See Aoki 1992 or Aoki and Yoshikawa (2001) for more simulation results.

6 Concluding remarks

This paper has demonstrated that sectoral adjustments and reallocations of resources generates fluctuations of aggregate economic activity, and that the average level of aggregate production depends on the sectoral demand pattern. By a careful examination of the U. S. job creation/destruction data, Davis, Haltiwanger, and Schuh (1996, p. 83) were led to the conclusion that a kind of approach we took in this paper was, in fact, important.

Economists routinely assume that resources are instantaneously reallocated so as to make marginal productivities in all the sectors equal. Instead of this standard assumption, we assume that allocation of resources takes time. This process is fundamentally stochastic. The model calculates the holding times of all sectors which determine the probability of the sector which actually increases output in response to positive excess demands of goods of sectors of the economy. As parts of this calculations probability of new sector emerging is also determined. The model solves the conceptual problem of which agent moves first and by how much in the usual agent-based simulation models.

Using a simple model, we have demonstrated that fluctuations of aggregate economic activity appear, and that the greater is demand for high productivity sectors, the higher the *expected* level of GDP. This result provides a new perspective to the *principle of effective demand*. Since the model is simple, we believe that the results are generic.

A couple of final comments. First, we have taken the entry and exit probabilities to depend on the sizes of the sectors. An alternative specification will specify them to depend on the excess demands themselves. This possibility is definitely worth pursuing.

Second, we note that changing the outputs from linear ones in (1) to concave ones $c_i n_i^\gamma$, with $0 < \gamma \leq 1$ does not change the patterns of the sign changes of excess demands in response to changes in n_i in the two sector model if we replace β by $\beta^{1/\gamma}$. For example, the inequality $n_2 > \beta(n_1 + 1)$ is replaced with $(n_2)^\gamma > \beta(n_1 + 1)^\gamma$, that is with $n_2 > (\beta)^{1/\gamma}(n_1 + 1)$. The regions R_1 through R_6 are analogously defined by lines L_1 through L_5 with slope $(\beta)^{1/\gamma}$. Arguments to derive the stationary distribution go through with β replaced by $\beta^{1/\gamma}$. Since Proposition holds for all values of β , it also holds for economies with $c_i n_i^\gamma$, $i = 1, 2, \dots, K$.

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