A New Model of Labor Dynamics: Ultrametrics, Okun's Law, and Transient Dynamics

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Abstract

This paper adds a labor dynamics sector to the model of outputs and fluctuations of Aoki and Yoshikawa (2003) to produce a new model of labor dynamics which behaves quite differently from those in the literature. Briefly put, the concept of ultrametrics is introduced to model differences in groups of unemployed workers with different geographical locations and/or work experiences, and human capitals. We keep our assumption in the previous model that marginal products of labor does not equalize quickly across sectors in our model.

This model share the property with our earlier one that larger GDP is produced with more demands on more productive sectors. In addition, the current model exhibits a relation between unemployment rates and GDP similar to that of the Okuns' law in its business cycle fluctuations. Our simulation results reveal that this Okun's coefficient increases as the average GDP increases. Our model also reaches stationary business cycles faster as more demands are put on more productive sectors of the model.

Introduction

There exists a large body of literature devoted to labor markets, such as Blanchard and Diamond (1989, 1992), Mortensen (1989), Pissarides (2000) and many others. In addition, empirical works by Davis and Haltiwanger (1992) on flows of job creations and job destructions point to their importance as determinants of labor market dynamic behavior.

Existing models in labor dynamics, however, do not successfully incorporate these empirical findings. For one thing, differences in geographical locations, human capitals, jobs experiences and the like are stressed in verbal discussions of labor markets, but are not formally modeled very convincingly. Unemployed workers are differentiated at most by their reservation wages in search models, or by the length of spell of unemployment in the literature.

In this paper, we augment the model of Aoki and Yoshikawa (2003) by adding a labor sector. We treat clusters of different types of unemployed

workers as forming a tree structure, and use ultrametrics to measure similarities of workers in different clusters. When a sector hires a worker it does so randomly from a pool of workers of different clusters suitably weighted by the ultrametric distance as we show in a later section. See Aoki (1996, Sec.2.5) on ultrametrics.

Our earlier model has exhibited that GDP responds to the demand patterns. This continues to be true with our new model. This chapter examines two other manifestations of the model: the Okun's law, and a new finding that dynamics of transient response also responds to demand patterns. See Yoshikawa (2000, 2003), Aoki (2002, Chapt. 8), and Aoki and Yoshikawa (2003).

The Okun's law refers to a stable empirical relation between unemployment rates and rate of changes in GDP: one percent increase (decrease) in GDP corresponds to x percent decrease (increase) in unemployment, where x is about 4 in the United States. It is well known that if labor market is a homogeneous single market operating under neoclassical setup, then the Okun's law does not hold. This numerical value of x is much larger than what one expects under the standard neoclassical framework. Take, for example, the Cobb-Douglas production function. Then, GDP is given by $Y = K^{1-\alpha}L^\alpha$ with α of about 0.7, where the total population is $N = L + U$ of which U is the number of unemployed.

We have $\Delta U = -\Delta L$, where ΔK and ΔN are assumed to be negligible in short run. The production function implies then that $\Delta Y/Y = \alpha \Delta L/L$ in short run. That is, one percent decrease in Y corresponds to an increase of $\Delta U/N = -(1/\alpha)(\Delta Y/Y)(1 - U/N)$, i.e., an increase of a little over 1 percent of unemployment rate. To obtain the number 3, as in the Okun's law, we need some other effects, such as increasing marginal product of labor or some other nonlinear effects. See Yoshikawa (2000). In the simulation studies of our new model we obtain numbers larger than 4, even though the linear production functions for all sectors in our model may lead us to expect numbers closer to 1.

Finally, potentially most intriguing finding we report here is that the model exhibits faster dynamics of approaching stationary equilibrium distribution when larger shares of demand fall on more productive sectors than otherwise.

The Model

Aoki and Yoshikawa (2003) and Aoki (2002, Sec. 8.6) presented a new model of fluctuations and growth. This paper adds a labor dynamics to that model. To briefly summarize that model, the economy has K sectors, and sector i employs some number n_i , $i = 1, \dots, K$ of workers.

We keep the constant coefficient structure of the model, where c_i is the productivity coefficient, and n_i denotes the number of employees of sector i . Sectors are either in normal time or in overtime, that is, there are two capacity utilization regimes. In normal time, n_i workers produce

$$y_i = c_i n_i,$$

and in overtime n_i workers produce output equal to

$$y_i = c_i(n_i + 1).$$

Demand for good i is given by $s_i Y$, with $Y = \sum_{i=1}^K y_i$, where s_i is a positive share of the total output Y , which falls on sector i goods, where $\sum_i s_i = 1$.

Each sector has the excess demand defined by

$$f_i = s_i Y - y_i, \quad (1)$$

for $i = 1, 2, \dots, K$.

We denote the sets of sectors with positive and negative excess demands by $I_+ = \{i : f_i \geq 0\}$, and $I_- = \{i : f_i < 0\}$. Changes in Y due to changes in any one of the sectors affect the excess demands of all sectors. That is, there exists an aggregate externality through demands among all sectors. Changes in the patterns of s 's also affect these sets.

The time evolution of the economy is modeled as a continuous-time Markov chain, as described in Aoki (1996, 2002), for example. In this model at each point in time, the sectors of economy belong to one of these two subgroups; one composed of sectors with positive excess demands for their products, and the other of sectors with negative excess demands. These two groups are used as proxies for groups of profitable and unprofitable sectors, respectively. All profitable sectors wish to expand their production. All unprofitable sectors wish to contract their production. A novel feature of our model is that only one sector succeeds in adjusting its production up or down by one unit of labor at any given time. The sector that has the shortest holding or sojourn time is the sector that jumps first. We call it as the jumping or active sector. Denoting the active sector variables with subscript a , we have the transition from n_a to $n_a + \text{sgn}(f_a)$. See Lawler (1995) or Aoki (2002, p. 28) for the notion of holding or sojourn time of a continuous-time Markov chain.

Transition rates

Dynamics are determined uniquely by the transition rates in continuous-time Markov chains.

Sectors adjust their outputs by hiring or firing workers in response to the signs of excess demands which are used as proxies for profitability of the sectors. We assume that economy has enough number of unemployed that sectors do not hoard workers. To increase outputs the sectors call back workers who were laid-off by various sectors.

To implement a simple model dynamics we assume the following. Other arrangements of the detail of the model behavior is of course possible.

Each sector has three state vector components: the number of employed, n_i , the number of laid off workers, u_i , and a binary variable v_i , where $v_i = 1$ means that it is in overtime status producing $c_i(n_i + 1)$ output with n_i employees. This sector posts one vacancy sign while in overtime status. When $v_i = 0$, it is in normal time producing $c_i n_i$ output with n_i workers. There is no vacancy sign posted in this state.

When $f_a < 0$, n_a is reduced by one, and u_a is increased by one, that is one worker is immediately laid off. We assume that the model is in an environment of "high" unemployment state, and cost of firing or hiring workers are negligible. We also assume that v_a is reset to zero.

When f_a is positive, we assume that it takes a while for the sector to hire one worker if it has not been in overtime status, i.e., v_a is not 1.

If sector a had previously posted vacancy sign, then sector a now hire one worker, and cancel the vacancy sign, i.e., reset v_a to zero. If it has not previously posted a vacancy sign, then, it now posts a vacancy sign, i.e., set v_a to 1, and increases its production with existing number n_a of workers by going into over-utilization state. The transition path may be stated as $\mathbf{z}$ to $\mathbf{z}'$, where $(n_a, u_a, v_a = 0) \rightarrow (n_a, u_a, v_a = 1)$, and $(n_a, u_a, v_a = 1) \rightarrow (n_a + 1, u_a - 1, v_a = 0)$. In either case $y'_a = y_a + c_a$.

Ultrametric trees

We endogenize job destructions and creation differently from Pissarides (2000). Signs of excess demands are used as proxies for profitability signals for sectors which want to change the sizes of labor forces accordingly. However, at any time only one sector out of all sectors wishing to adjust the sizes of their labor forces actually can act. The sector that acts is the sector with the shortest holding or sojourn time. To present a simple model we ignore quits and on-the-job searches, and assume that only unemployed get jobs.

In this model, sectors are differentiated with different 'distances' between each other. These 'distances' reflect such factors as geographical differences, differences in technology, and educational qualifications. Workers in different sectors are different in job experiences and human capitals, and their differences affect probabilities of being hired of different sectors by using the concept of ultrametrics. The stochastic process of filling vacancies of sector i by unemployed workers from the pool of sector j depends on the ultrametric distance $d(i, j)$ between the two sectors of the economy.

Transitions of the active sector depends on the sign of the excess demand, f_a as indicated above. When $f_a < 0$, then one unit of labor is fired immediately, and n_a become $n_a - 1$ as indicated at the end of the previous section.

Hiring a new unit occurs only with $f_a > 0$. Here we explain how the active sector employs one additional unit of labor. We need to distinguish u_a , which denotes the size of sector a 's laid-off workers, from the total pool of unemployed from which sector a randomly hire one unit of labor. This pool is composed of u_a and separate pools of laid-off workers from sector j $u_j, j \neq i$ suitably weighted by ultrametric distance. We denote the latter by $\tilde{u}_a$, that is $u_a + \tilde{u}_a$ is the total size of the pool of the unemployed units of labor for sector a . These separate sub-pools are organized as a hierarchical tree with ultrametric distance.

The clusters or sub-pools of unemployed have different probabilities of being picked. The highest probability is for the pool of the workers who are laid-off from that sector. Its size is u_a . This reflects the empirical observation

that often firms recall laid-off workers first as they become profitable again. Then pools of laid-off workers from other sectors are arranged in increasing order of the ultrametric distance from the pool of size $\tilde{u}_a$.

We illustrate this notion and its use in the case of $K = 3$ where sector 1 and 2 are at ultrametric distance 1, $d(1, 2) = d(2, 1) = 1$, and $d(1, 3) = d(2, 3) = 2$. Suppose that sector 1 is active. It draws from pools u_2 and u_3 after deflating them by $1 + d(1, 2)$ and $1 + d(1, 3)$ respectively. Thus a vacancy at sector 1 is filled from u_1 with probability u_1/U_1 , with $U_1 = u_1 + \tilde{u}_1$, where $\tilde{u}_1 = u_2/[1 + d(1, 2)] + u_3/[1 + d(1, 3)]$. When sector 2 is the active sector, it hires one unit of labor from u_2 with probability u_2/U_2 , with $U_2 = u_2 + \tilde{u}_2$, with $\tilde{u}_2 = u_1/[1 + d(1, 2)] + u_3/[1 + d(2, 3)]$. Similarly when sector 3 is active.

In this example vacancy in sector 1 will be filled from own laid-off pool with probability $6/11$, from u_2 with probability $3/11$, and from u_3 with probability $2/11$. A vacancy in sector 3 will be filled from u_1 with probability $1/5$, from u_2 also with probability $1/5$, and from u_3 with probability $3/5$.

The probability that a vacancy in sector 1, v_1 , is filled from its own pool of unemployed, given that sector 1 jumps first is

$$\Pr(v_1 \text{ is reduced by 1 to zero} | \text{sector 1 jumps}) = \frac{u_1}{U_1},$$

where

$$U_1 = u_1 + \tilde{u}_1.$$

Similarly v_1 is reduced by one from pool of unemployed of sector 2 with probability

$$\Pr(v_1 \text{ is reduced by 1 from sector 2} | \text{sector 1 jumps}) = \frac{u_2/[1 + d(1, 2)]}{U_1}.$$

Okun's Law

In simulation described below we note that after a sufficient number of time elapses, the model is in or near equilibrium distribution. Then, Y and U are nearly linearly related with a negative slope i.e.,

$$\frac{\Delta Y}{Y} = -\beta \frac{\Delta U}{N} \frac{N}{Y},$$

where U is the total number of unemployed, so that U/N is the fraction of unemployed of the total number N of workers. The total output Y is related to N by $Y = \sum_i c_i n_i$. In the equilibrium we have $c_i n_i = s_i Y$ or $n_i = (s_i/c_i)Y$, hence $Y = \hat{c}N$, with $(\hat{c})^{-1} = \sum_i s_i/c_i$, so that we may express the Okun's law as $\Delta Y = -\beta \Delta U$, where $\beta = x\hat{c}$. This relation is not exactly true in business cycles, but may be used as an approximation. Alternatively we can use the average GDP value together with the average employed number to approximate c in business cycles.

Simulation results suggests that the value of x is around 59.1 for case 1, 47.2 for case 2, 7.5 for case 5, and 5.2 for case 6. These figures are larger than Okun's. There are other empirical studies on the coefficients. For example,

Hamada and Kurosaka (1984) examined the Japanese economy from 1953 to 1995, with the numbers ranging from 10.5 to 32, depending on the time spans and whether the economy was in high unemployment rate period or in low unemployment period.

Our simulation results also show that the amplitudes of GDP and unemployment rates are higher when larger demand shares fall on more productive sectors.

As a final remark we record that the simulations vary somewhat with the initial conditions. This indicates that the dynamics are indeed nonlinear. Most results are for the initial setting of $n_i(0) = 85$.

Simulation

Our model behave randomly because the jumping sectors are random due to holding times being randomly distributed. This is different from the models in the literature which behave randomly by the technology shocks which are exogenously imposed. Apparently, the model states have many basins of attractions each with near equal "potential energy" levels, much as spin glasses are.

Since the model is nonlinear and possibly possess multiple equilibria, we resort to simulations to extract some of the properties of the models. We pay attention to the phenomena of trade-offs between GDP and unemployment, and the scatter diagrams of GDPs vs. unemployments to gather information on business cycle behaviors

A large number of simulations have been run. We state the qualitative results of these simulation runs as follows:

1) larger shares of demands on more productive sectors result in larger average values of GDP.

2) Under the same circumstances, the systems with larger GDP reach 'near equilibrium' conditions faster.

3) Amplitudes of business cycles are larger, the larger the average GDP.

4) Relations between unemployment and average GDP are described by the Okun's law as we give details in the next section.

Of these four qualitative conclusions, No.2 seems to be most interesting. In the existing literature this dynamic aspects of labor market characteristics has not been observed or commented on.

Case studies

All the cases presented here have $K = 8$.

Case 1

Demand shares $= [5, 4, 3, 2, 1, 1, 1, 1]/18$; The top four sectors have 78 per cent of total demand. We use three initial conditions, $n_i(0) = 90$, $n_i(0) = 85$, and $n_i(0) = 75$.

The coefficients of the Okun's laws are 7.0, 6.6, and 5.4 with the three initial conditions. The average GDPs are $Y_{av} = 386.5$, 364.7, and 321.9 respectively. The amplitudes of the business cycles are 0.26 per cent, .4 per cent, and .59 per cent respectively of the average GDP respectively.

In this case, consistent with demands, fewer number of workers are needed to meet the demands in productive sectors. The sizes of the sectors shrink before setting near the equilibrium levels of production after about 600 basic time steps. This is shown in Fig.1.1 which plots the average output over 8 sectors, Y_{av} , and can be seen also from Fig.1.2, which is the plot of Y_{av} vs $unav$.

The level of output average after 600 steps is about 386 units.

Case 2

The demand pattern of case 1 is reversed [1,1,1,1,2,3,4,5]. About 22 percent of demands fall on the top 4 sectors. Here we report results with the initial condition $n_i(0) = 85$. $x = 5.6$, $Y_{av} = 328.7$, and the amplitude of the business cycle is about .16 per cent of the average GDP.

All other settings are the same as case 1. Fig.2.1 shows that the model settles near stochastic equilibrium after about 1100 time steps. Fig. 2.2 is the scatter diagram of Y_{av} vs $unav$. This case took a lot longer to settle near stochastic equilibrium than case 1.

The value of Y_{av} after 1200 steps is about 178 units.

case 3

Effects of initial starting conditions are examined in case 3 and 4. In case 3, the initial condition, $n_i(0) = 75$, is used for all sectors. The model settles down to a stochastic equilibrium region after 500 steps or so, as in case 1. Fig.3.1 shows the trajectory of Y_{av} towards business cycles. Fig. 3.2 shows a scatter plot of Y_{av} vs $unav$. The average level of output is about 321.1, and is less than that of case 1.

case4 This case is the same as case 2 with the only difference in $n_i(0) = 75$ for all sectors. It takes about 1000 steps to approach a stationary distribution. Y_{av} is about 145, as shown in Fig.4.1. (The scale is too large to show the fluctuations in GDP in this plot.)

The next two cases, case 5 and 6, compare less concentrated demand pattern of [3,3,4,4,1,1,2,2] with [3,3,1,1,2,2,4,4]. The initial employees are set at $n_i(0) = 85$ for all sectors in both cases. The top 4 sectors occupy 70 per cent, and 40 per cent of total demand.

As expected, Y_{av} of case 6 is less than that of case 5.

case 5

Fig.5.1 shows that the model settles down after 700 steps or so. Fig.5.2 is the scatter plot of Y_{av} and $unan$.

case 6

Fig.6.1 shows that it takes about 1000 steps for the model to settle down. Fig.6.2 is the scatter plot of Y_{av} vs $unav$.

case 7

This case is tried with the initial condition $n_i(0) = 85$. $x = 5.6$, $Y_{av} = 328.7$, and the amplitude of business cycle is about .16 per cent of Y_{av} .

Conclusions

We have demonstrated by simulation that higher percentages of demands falling on more productive sectors produce three new results: Average GDPs are higher; the Okun's coefficients x is larger; and transient responses are

faster and in near stationary states, excursions of GDP (hence of U) are larger as well. In addition the amplitudes of the business cycles are higher as well.

It is remarkable that we obtain the relation like Okun's law with linear constant coefficient production functions. The fact that we obtain larger numbers than Okun may indicate the importance of capacity utilization as well as nonlinear interactions among sectors. Capacity utilization of capital is crudely incorporated in our model by the fact that sectors go into overtime under positive excess demand conditions before it can fill the vacancy position.

Unlike the Cobb-Douglas production or linear production function which lead to x values of less than one, we observe the values well over 1 in our simulation. This indicates the importance of stochastic interactions among sectors introduced through the device of stochastic holding times.

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Fig 1.1

Fig. 1.2

Fig 2.1

22%
[1, 1, 1, 1, 2, 3, 4, 5]

$$w(0) = 100$$

$$n(0) = 90$$

$$\theta = 0$$

Fig 2.2

Fig 3.1

$$W(0)=100, \mu(0)=75$$

Fig 3.2

Fig 4.1

Fg 5.1

Fig 5.2

Fig 1

last 600

Fig 6.2