

# Four Ingredients for New Approaches to Macroeconomic Modeling

Masanao Aoki

Department of Economics, University of California, Los Angeles  
aoki@econ.ucla.edu; fax 1-310-825-9528

## Abstract

This paper outlines some of the concepts and tools which, although not in the mainstream macroeconomics literature, have been effective either in providing new results, or insights to known results.<sup>1</sup>

Briefly put, the new approaches borrow concepts and tools from population genetics, condensed matter physics, and recently developed stochastic combinatorial analysis in statistics. Continuous-time Markov chains are constructed for clusters of heterogeneous types of interacting economic agents. We can then draw macroeconomic policy implications by examining solutions of master (Chapman-Kolmogorov) equations, Fokker-Planck equations or Langevin equations, as the needs call for them.

This paper attempts to introduce the reader to some of these new notions, and procedures to gain new insights and results.<sup>2</sup>

This paper reports on some of these by loosely organizing them into four sections.

## Introduction

In mid 1990's, new approaches to macroeconomic modeling have been proposed in Aoki (1996), and elaborated further in Aoki (2002).

His modeling approaches have been suggested by examples in population genetics, condensed matter physics, and in stochastic combinatorial analysis, and differ substantially from the mainstream macroeconomics in model constructions. Some of the similarities in models in condensed matter physics and biology have already been noted in Higgs (1995), Mekjian and Chase (1997), and in Derida and Flyvbjerg (1987). We extend similar approaches to modeling macroeconomics. For example, the notion of the relative sizes of basins of attractions in random map models in physics, the Herfindahl index as an economic idea of shares of markets, Aoki (2002, p.142, 173-174),

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<sup>1</sup>For details of the methods and some examples see Aoki (1996, 2002), and their corrected versions Aoki (1998, 2004), and recent Wehla conference proceedings.

<sup>2</sup>Some of the reported results have been obtained in cooperation with a few of like-minded economists, statisticians, and physicists. In particular the author gratefully acknowledges several important insights he obtained as the results of many discussions with H. Yoshikawa, and D. Costantini.

and Ewens distributions in population genetics are remarkably similar or identical.

Models we use are dynamic, that is, model states change over time. The model dynamics are described by the continuous time Markov chains. Instead of differential (difference) equations for the states, we use differential (difference) equations for the *probability* of state variables. They are the Chapman-Kolmogorov equations. We call them master equations following the physics usage.

We loosely classify our approaches and results into four groups or categories depending on what new "ingredients" or viewpoints are used in modeling or in describing the models. First, the notion of equilibria is extended to stationary or equilibrium stochastic distributions. Equilibria are stationary distributions. See Yoshikawa (2003) for elaboration.

Second, we do not use representative agents in our models. Instead, several types of agents are considered. Sets of agents are partitioned into subsets, called clusters. Clusters are composed of agents of the same characteristics, called type for short. In considering these partitions, combinatorial considerations naturally come into play in counting the number of different configurations that these partitions can assume. The notion of entropy and various distributions on the set of clusters of agents in different configurations also become necessary. Less well-known distributions such as Ewens, Poisson-Dirichlet, residual allocation models, and Lévy distributions are some of the examples.

These clusters are not treated symmetrically. Some are closer together than others. We introduce a notion of distance of clusters that is transitive. Correlations will not do since they are not transitive, as is well known from the literature on numerical taxonomy. We use the notion of ultrametrics.

The clusters are organized as leaves of trees and dynamics on trees are examined as in the physics literature by assuming that stochastic transition rates between clusters are functions of the ultrametric distances between clusters. Dynamics of states organized into trees are used to examine the effects of idiosyncratic shocks to one of the clusters spreading throughout the trees. We have shown that sluggish spread of the idiosyncratic shocks throughout the trees are one of the causes for slow responses of macroeconomic signals to these shocks. The tree structures help explain sluggish macroeconomic indices, and policy ineffectiveness under uncertainty which is touched on next.

Third, uncertainty also contributes to sluggish responses of macroeconomy. Uncertainty of the forecasts of the effects of current actions has been shown to make policy actions less effective. Uncertainty, moreover, has implications not fully explored in the existing mainstream macroeconomics, as has been demonstrated in Aoki and Yoshikawa (2005a,b,c), and in Aoki, Yoshikawa, and Shimizu (2005).

Fourth, dynamics of clusters lead us naturally to examine fat-tailed distributions, also known as power-laws, and (scale-invariant truncated) Lévy distributions. These distributions are well-known in finance but not in macroeconomics.

We have examined labor market dynamics as a vehicle of illustration of

some of the points touched on here. Unlike the traditional approach, our model of labor market dynamics dispenses with the traditional matching functions. We derive Okun's law and Beveridge curves in economies which respond to aggregate demands.

In this connection we mention new Schumpeterian perspective on long-run behavior as another example. We model interaction between innovation and imitation processes as birth-death with immigration models and examine long-run behavior of this model, by solving a model of two-sector economy composed of innovative and imitative sectors. Explicit stationary solutions of the first and second moments are obtained for the sizes of the two sectors, using cumulant generating functions for dynamically interacting two sectors. Distributions of relative sizes of technically efficient and inefficient sectors are quite similar to those we obtain in our labor market model. See Aoki, Nakano, and Yoshida (2004), and Aoki and Yoshikawa (2005 b) for detail.

## Stochastic Equilibria

Bellman was the first to identify probability distributions as the proper notion of state in stochastic dynamics, hence equilibria are stationary probability distributions, Bellman (1961), and Bellman and Dreyfus (1962). There usually are several basins of attractions. Models are not confined to some basins of attractions. They eventually wander out of the basins they currently occupy. The idea of equilibrium selections in macroeconomics loses its meaning in stochastic context.

## Sluggish Macroeconomic Behavior

Our approach in explaining sluggishness in macroeconomy is different from the well-known Taylor's explanation of staggered labor contract, Taylor (1980). His model and virtually all multi-sector models treat sectors symmetrically with equal distance between any two sectors. There is no notion of adjustment speeds as functions of some similarity measures among clusters.

Dynamics of trees have two aspects to it. There are multiplier lags or impulse or step responses. These are lags in responses at the output of dynamics when a *known* input, such as an impulse or a step input is applied to the input. There is another kind of lags related to the delay in exogenous disturbances to one of the leaves of a tree spreading throughout the tree as the input signals to other leaves or nodes on higher levels of trees. These are multiplier and information transmission lags. For further detail see Aoki and Yoshikawa (2005).

## Uncertainty Trap

To explain this notion simply, suppose that a large number,  $N$  of agents face a binary choice optimization problem. There is externality because the current number,  $n$  of the agents with one choice may influence how the rest

of the agents choose, and consequently the dynamics of how the size of the fraction evolve. The number of ways  $n$  agents out of  $N$  form one cluster turns out to be important. Here the entropy of this patterns matter as has been shown in Aoki (1996, pp. 137-147).

The same formulation can be used to conclude that in situations with a large degree of uncertainty policy effectiveness is greatly reduced. See Aoki and Yoshikawa (2005a) for detail.

## New Features of Multi-Sector Economy

In Aoki (2002, Sec. 8.6) a new multi-sector economy has been examined where sectors have different productivity coefficients to illustrate effectiveness of demand management. Despite the simplicity of the model, its output (GDP) has been shown to respond to demand management policies. Later in Aoki and Yoshikawa (2005) the model has been extended to examine Okun's law and the Beveridge curves, all without the traditional matching functions. They exhibit an unexpected effects of demand share switching when the model is not in equilibrium. Expanding demand shares of less productive sectors lead to the increase in size of the less productive sectors. When more demands are directed to more productive sectors, the sizes of the less productive sectors shrink faster than the sizes of the more productive sectors grow. This leads to decrease in GDP, contrary to our intuition. A similar phenomenon has also been observed in a more elaborate model in Schumpeterian spirit, Aoki, Nakano, and Yoshida (2004).

## Concluding Remarks

One area that requires further attention is the construction of macroeconomic model with asset markets. There are many proposals using representative agents, and some with heterogeneous agents where agents solve very complicated intertemporal optimization problems under ad hoc sets of assumptions.

Asset market behavior has been extensively modeled by the econophysicists, a group of physicists who turn their training to discover power-laws and scale invariant properties with almost no work being done in macroeconomics.

We try to match their efforts in modeling financial phenomena by focusing on the real phenomena such as consumption streams and GDP.

There are other results not included in this list. See the forthcoming book by Aoki and Yoshikawa (2005).

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