A New Stochastic Framework for Macroeconomics: Some Illustrative Examples

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Abstract

We need a new stochastic approach to study macroeconomy composed of a large number of stochastically interacting heterogenous agents. We reject the standard approach to microfoundation of macroeconomics as misguided, mainly because the framework of intertemporal optimization formulation for representative agents is entirely inadequate to serve as microfoundations of macroeconomics of stochastically interacting microeconomic units.

Given that economies are composed of many agents of different types, fundamentally differernt approaches are needed. This paper illustrates our proposed approaches by brief summaries of examples drawn from four separate problems areas: Stochastic equilibria, uncertainty trap and policy ineffectiveness, stochastic business cycle models, and a new approach to labor market dynamics.

1 Introduction

In cooperation with some like-minded macroeconomists and physicists, we have advocated modeling approaches to macroeconomics based on continuous-time Markov chains, coupled with random combinatorial analysis. The proposed approaches differ substantially from those commonly used by the macroeconomic profession.

Briefly put, we construct continuous-time Markov chains for several types of interacting economic agents to study macroeconomic problems. Stochastic dynamics are described by master (i.e., backward Chapman-Kolmogorov) equations, and stochastic dynamic behavior of clusters of agents of various types are examined to describe macroeconomic phenomena and policy implications. Instead of the usual notion of equilibrium as a deterministic concept, we use stationary distributions on a set of states as our definition of stochastic equilibria.

Needs for stochastic analysis of models composed of a large number of interacting agents of different types have been slow to be recognized by macroeconomists. For example, the notion of power-law distributions has been extensively applied to model financial events, but not to model macroeconomic phenomena.

By now we have obtained several new results, and gained new insights on macroeconomic behavior that are different, more informative, or not available in the mainstream macroeconomic literature. We also have some new perspectives on some macroeconomic policy effectiveness questions.

This paper surveys four topics. They are stochastic equilibria, uncertainty trap and policy ineffectiveness, stochastic business cycle models, and new approaches to labor market dynamics. More detailed accounts are available in Aoki (1996, 2002), and Aoki and Yoshikawa (2002, 2003, 2005), and Aoki, Yoshikawa, and Shimizu (2003). Here are their short summaries:

1. Stochastic equilibrium as probability distributions, and probability distribution of productivities: In stochastic framework we need a notion of equilibrium that is broader than that described in standard economic textbooks. In standard framework demand is determined by technology and factor endowments. There is no microeconomic fluctuations. We give simple examples to contradict these statements. In standard arguments, unequalized productivities across sectors imply unexploited profit opportunities, and contradict the notion of equilibrium. Heterogeneous stochastic agents behave differently. All agents and production factors do not move instantaneously and simultaneously to the sector with the highest productivity. Their moves are governed by the transition rates of the continous-time Markov chains. Value marginal products are not equalized across sectors and productivities of sectors have a Boltzman distribution. Section 2 covers this area.
2. Uncertainty and policy ineffectiveness: A new insight on macroeconomic policy ineffectiveness is described in section 3. We focus on effects of uncertainty on decision-making processses, and show that policy actions become less effective as degree of uncertainty facing agents increases.
3. Stochastic model of business cycles: Business cycles are often explained as direct outcomes of the behavior of individual rational agents. The stronger is the desire to interpret aggregate fluctuations as something rational or optimal, the more likely is this microeconomic approach is adopted. We claim that microeconomic fluctuations persist because thresholds for actions among microeconomic agents differ across sectors/firms, and their actions alter macroeconomic environment and a new, and different microeconomic fluctuations result, and this processes continue. We use a simple quantity adjustment model to illustrate these points in Section 4.

4. A new approach to labor market dynamics: Without using the notion of matching function, we model labor market dynamics in section 5 to explain Okun's law and Beveridge curves. and how they shift in response to macroeconomic demand policies. We use a simple quantity adjustment model as a Markov chain, and apply the notion of the miminal holding times to select the sectors that jump first. The notion of holding time of Markov chain processes is a natural device for this random selection. This model is extended then to include labor dynamics.
   Pools of laid-off workers are heterogeneous, because their human capitals, types of job experience, durations of unemployment, and so on are different. These pools form hierarchical trees. A new notion of distance, called ultrametric distance, is used to model different probabilities for unemployed being rehired as functions of ultrametric distance between the sector that is hiring and the unemployed in some pool of unemmployed.

In addition to the uncertainty trap mentioned above, our analysis of multisector models reveal rather unexpected consequences of certain macroeconomic demand policies. This example illustrates an aspect demand policies not discussed in the existing macroeconomic literature.

2 Stochastic Equilibria and Probability Distributions of Sectoral Productivity

According to Richard Bellman, the inventor of dynamic programming, state is a collection of information sufficient to determine the future evolution of the model, given whatever exogenous disturbances or control of state vector. In stochastic context, probability distriubtions are the states in the sense of Bellman. Deterministic equilibrium points are now replaced by stationary probability distributions.

2.1 Stochastic Equilibria

As a simple example, we follow Yosikawa (2003), and consider an economy composed of K sectors producing K types of goods. Taking the goods of the least productive sector as numeraire, let p_i be the relative price and c_i be the productivity of sector i, i.e., the amount of goods i produced by one unit of labor of sector i.

Let D_i be the demand for good i. The total output of the economy is

$$Y = \sum_{i=1}^{K-1} p_i D_i + (L - \sum_{i=1}^{K-1} D_i/c_i)c_K$$

$$= Lc_K + \sum_{i=1}^{K-1} (p_i - c_K/c_i)D_i,$$

where L is the total number of labor.

In the neoclassical equilibrium the value marginal products are equal across sectors, and the above equation reveals that the total output is independent of demand:

$$Y = Lc_K$$

It is only the function of productivity factors and labor endowment. In reality, productivities differ across sectors, We have inequalities $c_1 p_1 > \cdots > c_i p_i > \cdots c_k$. Hence increase in D_i increases Y, and Y depends on demands.

2.2 A Boltzman Distribution for Productivity Coefficients

The aggregate demand affects distribution of productivities of the economy, and the level of total output. This can be seen by a simple entropy maximization argument, Aoki (1996, Sec. 3.2).

Suppose that sector i employs n_i workers, and that $L = \sum_i n_i$. There are $L!/n_1! \cdots n_K!$ numbers of configurations compatible with this constraint. The total output is $Y = \sum_i c_i n_i$. Assuming that all allocations are equi-probable, we define the entropy in analogy to the Boltzman entropy by[1]

$$S = -\sum_i \ln n_i! \approx -\sum_i n_i(\ln n_i - 1).$$

See Aoki (1996, p.47).

We maximize this subject to the constraints that n_i sum to L and D is the sum of $c_i n_i$ over i

$$H = S + \alpha(L - \sum_i n_i) + \beta(D - \sum_i c_i n_i)$$

where α and β are Lagrange multipliers.

Maximization of this expression with respect to n_i yields

$$n_i^* = \exp(-\alpha - \beta c_i),$$

and the expression for the two parameters as

$$L = \sum_I n_i = e^{-\alpha} \Xi(\beta),$$

where we define the partition function by

$$\Xi(\beta) = \sum_i e^{-\beta c_i},$$

and

[1] The $N!$ factor disappear because of the allocations are exchangeable in the technical sense of random combinatorial analysis.

$$D = e^{-\alpha} \sum_i c_i e^{-\beta c_i} = -e^{-\alpha} \frac{d\Xi(\beta)}{d\beta}.$$

From the equality

$$D/L = -\frac{\Xi(\beta)'}{\Xi(\beta)}$$

we obtain

$$\ln \Xi(\beta) = -\frac{D}{L}\beta + \text{const.},$$

or

$$\Xi(\beta) = K \exp(-\frac{D}{L}\beta),$$

where constant K is the number of sectors in the model. The average productivity of the economy is

$$\bar{c} = \sum_i c_i n_i / L = D/L.$$

To obtain the relation between β and $\bar{c}$ we measure productivities in unit of a sufficiently small positive unit θ, and assume that $c_i = i\theta$, $i = 1, 2, \ldots, K$.. Then we have an explicit expression for

$$\Xi(\beta) = \sum_i e^{-i\theta\beta} \approx \frac{e^{-\theta\beta}}{1 - e^{-\theta\beta}} \approx \frac{e^{-\theta\beta}}{\beta\theta},$$

and

$$\Xi'(\beta) = -\frac{e^{-\beta\theta}}{\theta\beta^2}.$$

In other words, we have

$$\frac{1}{\beta} = \bar{c} = D/L.$$

The fraction of workers of sector i is distributed as

$$\frac{n_i^*}{L} = \frac{e^{-c_i/\bar{c}}}{\Xi(\beta)}.$$

We note that D/L, the average GDP per sector, plays the role of "economic temperature" in analogy with the Boltzman statistics in statistical mechanics.

3 Policy Ineffectiveness: Uncertainty Trap

The Japanese economy has apparently come out of a long period of stagnation by 2004. Many explanations have been offered as to the causes of this long period of stagnation. Similarly, many suggestions have been offered to bring the economy out of the stagnation, such as by Krugman (1998), Blanchard (2000), and Girardin and Horsewood (2001) among others.

In this section we describe one probable source for macroceoconomic sluggishness which has not received the attention of economists in general, and those who specialize in Japanese economic performances in particular.[2]

We call this source as uncertainty traps. This effect has been pointed out in Aoki, Yoshikawa, and Shimizu (2003).[3]

The other source of sluggish behavior by Japanese economy is discussed in Section 4.

3.1 Uncertainty Trap

To explain this notion concretely, suppose that there are N agents (firms) in the economy. We keep N fixed for simpler exposition. Each agent has two choices, choice 1 and choice 2, in selecting its production level. Choice i means production at the rate y_i, $i = 1, 2$, where $y_1 > y_2 \geq 0$.

Let n be the number of firms with choice 1. The number of firms with choice 2 is then $N - n$.

The total output of the economy, GDP, is $Y = ny_1 + (N-n)y_2$. We express this in terms of the fraction $x = n/N$ of agents with choice 1 as

$$Y = N[y_1 x + (y_2(1-x)].$$

Note that x is a random variable between 0 and 1, since the number of firms with choice 1 is random.

We analyze how agents change their choices over time by modeling the process of changes in n as a continous-time Markov chain (also called jump Markov chain). Firms can change their mind any time. They constantly evaluate the two present values, $V_i, i = 1, 2$ associated with the two choices, where V_i be the random discounted present value for a firm with choice i,conditional on the number of fraction x. During a small time interval only one firm may change its production rate. Agents' stochastic switching between the two choices are described in terms of two transition rates which uniquely determine the stochastic process involves here. See Breiman (1968, Chapter 15) for example.

Denote the probability that choice 1 is better than choice 2, given fraction x by

$$\eta(x) := \Pr(V_1 - V_2 \geq 0|x).$$

When the random variable $\eta(x)$ is approximated by normal distribution with mean

[2] The sources we discuss in this paper are not necessarily specific to Japanese economy and could affect other macroeconomies when conditions described in this paper are present.

[3] Yoshikawa coined the term "uncertainty trap" to distinguish this from the liquidity trap.

$$g(x) := E[V_1 - V_2|x],$$

and variance σ^2 which is assumed constant for simpler explanation, it can be approximately expressed as[4]

$$\eta(x) = \frac{e^{\beta g(x)}}{X},$$

where $\beta = \sqrt{2}/\pi\sigma$, and $X = e^{\beta g(x)} + e^{-\beta g(x)}$. See Aoki (2002, Chapter 5) for further analysis.

Let the expected value of x be denoted by

$$\phi := E(x).$$

It is shown in Aoki (1996, p. 136), Aoki (1998), and Aoki (2002, p.44) that this mean value is goverened by the differential equation

$$\frac{d\phi}{dt} = (1 - \phi)\eta(\phi) - \phi[1 - \eta(\phi)].$$

Its stationary solution is obtained by setting its left-hand side to zero, and noting that $\eta(\phi)/(1 - \eta(\phi))$ is equal to $e^{2\beta g(\phi)}$, it is the solution of

$$g(\phi) = \frac{1}{2\beta}\ln(\frac{\phi}{1 - \phi}).$$

With little uncertainty about the consequences of choices, that is, with small σ, the value of the parameter β is large, and the above equation is approximately equal to

$$g(\phi) = 0.$$

This shows that the expected value of the fraction of firms with choice 1 is the zero of the $g(\cdot)$ function, that is, critical points of g,

$$g(\bar{\phi}) = 0.$$

This $\bar{\phi}$ is a locally stable equilibrium if g' is negative at $\bar{\phi}$.

In this case, x varies randomly in neighorhood of $\bar{\phi}$. Accordingly, GDP fluctuates randomly but mostly in neighborhood of $\bar{Y} = N((y_1 - y_2)\bar{\phi} + y_2)$. Standard comparative analysis holds with no problem here. If policy makers find this $\bar{Y}$ value too low, they can raise it by increasing $\bar{\phi}$ by shifting up $g(\cdot)$. In circumstances with small uncertainty about the relative merits of alternative choices, the zeros of g function basically determines the stationary Y values.

The situation is quite different when uncertainty is large, that is, when value of parameter β is close to zero. We turn to this case next.

[4] See Aoki (2002, p. 62) for Ingber's approximate expression of the error function which leads to the expression given here.

To understand the model with small values of β we obtain the stationary distribution of x, not just the mean as above. Using $n(t)$ as the basic variable, we solve the (backward) Chapman-Kolmogorov equation for the probability $P(n(t) = k)$, written as $P(k,t)$. Over a small interval of time $(t, t + \delta t)$, the number of firms with choice one increases by one at the rate

$$r_k := \lambda N(1 - k/N)\eta(k/N),$$

and decreases by one at the rate

$$l_k := \mu N(k/N)[1 - \eta(k/N)],$$

where λ and μ are constant parameters that do not concern us here.[5]

The time derivative of this function expresses the net increase in probability that k agents have chosen 1, which is the difference of the probability influx and outflux given as

$$\frac{dP(k,t)}{dt} = \text{Influx to } \{n_1(t) = k\} - \text{Outfux from } \{n_1(t) = k\},$$

where

$$\text{Influx} = P(k+1,t)l(k+1,t),$$

and

$$\text{Outflux} = P(k,t)r(k,t),$$

subject to boundary conditions at $k = 0$ and $k = N$, which we do not show here.

We solve this equation for a stationary distribution of k by setting the left-hand side to zero. The stationary distribution, written as $\pi(k)$ is given by

$$\pi(k) = \text{constant} \prod_{j=1}^{k} \frac{r_{j-1}}{l_j}, k \geq 1,$$

where we omit the arguments of r and l from now on.

After substituting the above into the expression for the equilibrium distribution above, we derive

$$\pi(k) = \text{constant } C_{N,k} \prod_{j=1}^{k} \frac{r_{j-1}}{l_j},$$

where $C_{N,k}$ is the combinatorial coefficient $N!/k!(N-k)!$. We write this distribution in the exponential function form

[5] They are called birth and death rate in random walk model in probability textbook. When $\eta(x)$ is replaced by 1, the model is a standard random walk model. The expression $\eta(x)$ introduces externalities of choices among agents.

$$\pi(k) = X^{-1} N \exp[-\beta N U(k/N)],$$

where we write the probability by introducing an expression $U(k/N)$, called potential. The expression for X is the one defined above. By replacing k/N by x which is now treated as a real number between 0 and 1, and replacing the sum by the integral, we see that

$$U(x) = -2 \int^x g(y) dy - \frac{1}{\beta} H(x),$$

where $H(x) = -x \ln(x) - (1-x)\ln(1-x)$ is the Shannon entropy.

This entropy expression arises from the combinatorial factor of the number of ways of choosing k out of N. This combinatorial factor is entirely ignored in the standard economic analysis, but is crucial in large uncertainty choice problems such as this one, since the entropy term is multiplied by $1/\beta$ which is the largest term in the expression for $\pi(x)$.

Locally stable ϕ is that which minimizes the potential

$$0 = U'(\bar{\phi}) = -2g(\bar{\phi}) + \frac{1}{\beta} \ln \frac{\bar{\phi}}{1-\bar{\phi}}.$$

When value of β is large (case of little uncertainty), this reduces to our earlier expression that showed that $\bar{\phi}$ is a critical point of $g(\phi)$.

With small values of β, (case of large uncertainty), $\bar{\phi}$ which minimizes the potential is not a critial point of g.

A straightforward variational analysis shows that if $g(x)$ is modified to $g(x) + h$ with some $h > 0$, then this $\bar{\phi}$ is moved by

$$\delta\bar{\phi} = \frac{2\beta h(\bar{\phi})}{[\bar{\phi}(1-\bar{\phi})]^{-1} - 2\beta g'(\phi)} > 0.$$

However, with values of β small, this is approximately equal to

$$\delta\bar{\phi} \approx 2\beta h(\bar{\phi})\bar{\phi}(1-\bar{\phi}) \approx 0.$$

This expression shows that the effect of increasing the expected advantage of choice 1 over 2 by $h > 0$ is nullified by the presence of small β. This is why we call this phenomenon uncertainty trap. The economy cannot move out of $\bar{\phi}$ even when g is shifted upwards to favor choice 1.

To summarize, an expansionary effort by shifting g function by $h > 0$ is equal to

$$\delta\bar{\phi} = -h(\bar{\phi})/g'(\bar{\phi} > 0,$$

with large β, but is nearly zero with small β.

4 A Stochastic Model of Business Cycles

In standard economic explanations business cycles are direct consequences of individual agents' choices to changing economic environments such as consumers' intertemporal substitutions.

We have three main objectives in this section. First, we demonstrate that aggregate fluctuations arise as an outcome of interactions of many sectors/agents in a simple model. Second, we show that that the average level of aggregate output depends on the patterns of demand across sectors. Third, our simple quantity adjustment model clearly shows how some (actually one in our model) sectors are randomly selected to act first, and their actions alter aggregate outputs and the interaction patterns among sectors/agents, thus starting the stochatic cycles all over again.

4.1 A Quantity Adjustment Model

Consider an economy composed of K sectors, and sector i employs n_i workers,[6] $i = 1, \dots K$.

To present a simple model we assume K and prices are fixed in this section.[7]

The output is assumed to be given by a linear production function

$$Y_i = c_i n_i,$$

for $i = 1, 2, \dots, K$, where c_i is the productivity coefficient.

The total output (GDP) is given by the sum of all sectors

$$Y = \sum_{i=1}^{K} Y_i.$$

Demand for good i is given by $s_i Y$, where s_i is a positive share of the total output Y which falls on sector i goods, with $\sum_i s_i = 1$. Here they are treated as exogenously fixed. In the next section we let them depend on the total output (GDP) in explaining Okun's law.

Each sector has the excess demand defined by

$$f_i = s_i Y - Y_i, \tag{1}$$

for $i = 1, 2, \dots, K$.

Changes in Y due to changes in any one of the sector outputs affect the excess demands of all sectors. That is, there exists an externality between

[6] The variable n_i needs not be the number of employees in literal sense. It should be a variable that represents 'size' of the sector in some sense. For example it may be the number of lines in assembly lines, and so on.

[7] Actually, K can change as sectors enter and exit. See Aoki (2002, Sec. 8).

aggregate output and demands for goods of sectors. Changes in the patterns of s's also affect these sets of excess demands.

The time evolution of the model is given by a continous-time Markov chain, as described in Aoki (2002, Sec. 8.6).

At each point in time, the sectors of economy belong to one of two subgroups; one composed of sectors with positive excess demands for their products, and the other of sectors with negative excess demands.

We denote the sets of sectors with positive and negative excess demands by $I_+ = \{i : f_i \geq 0\}$, and $I_- = \{i : f_i < 0\}$, respectively. These two groups are used as proxies for groups of profitable and unprofitable sectors, respectively. All profitable sectors wish to expand their production. All unprofitable sectors wish to contract their production.

A novel feature of our model is that only one sector succeeds in adjusting its production up or down by one unit of labor at any given time. We use the notion of shortest holding time as a random selection mechanism of the sectors. That is, the sector with the shortest holding or sojourn time is the sector that jumps first. Only the sector that jumps first succeeds in implementing its desired adjustment. See Lawler (1995) or Aoki (2002, p. 28) for the notion of holding or sojourn time of a continuous-time Markov chain. We call that sector that jumps first as the active sector. Variables of the active sector are denoted with subscript a.

4.2 Transition Rates

It is well known that dynamics of this continous-time Markov chain are determined uniquely by the transition rates, Breiman (1968, Chapter 15).

We assume that the economy has initially enough number of unemployed that sectors incur zero costs of firing or hiring, and do not hoard workers. We also assume no search on the job by workers. To increase outputs the active sector calls back one (unit of) worker from the pool of workers who were earlier laid-off by various sectors.[8]

When $f_a < 0$, n_a is reduced by one, and the number of unemployed pool of sector a, u_a, is increased by one, that is one worker is immediately laid off. When f_a is positive, n_a is increased by one. See the next section for more detailed explanation.

4.3 Continuum of Equilibria

The equilibrium states of this model are such that all excess demands are zero, that is,

[8] The actual rehired worker is determined by a probabilistic mechanism that involve measuring distances among clusters of heterogeneous layed-off workers by ultrametrics. See Sec. 5. We note merely that our model can incorporate idiosyncratic variations in profitability of sectors and frictions in hiring and firing.

$$s_iY_e = c_in_i^e,\ i = 1, 2, \ldots, K,$$

where subscript e of Y, and superscript e to n_i denote equilibrium values.

Denoting the total equilibrium employment by $L_e = \sum_i n_i^e$, we have

$$\kappa Y_e = L_e. \tag{2}$$

where $\kappa = \sum_i s_i/c_i$. This equation is the relation between the equilibrium level of GDP and that of employment. We see that this model has a continuum of equilibria.

In equilibria, the sizes of the sectors are distributed as being proportional to the ratio s_i/c_i for all i,

$$\frac{n_i}{n} = \frac{s_i/c_i}{\kappa}. \tag{3}$$

In the next section we see that the parameter κ plays an important role in the model behavior.

4.4 Model Behavior

Aoki (2002, Sec. 8.6) analyzes a simple version with $K = 2$ and show that as s_i are changed, so are the resulting aggregate output levels.

In simulations we have used $K = 10$, and several different demand share patters: some with more demand shares among more productive sectors and others more demand shares on less productive sectors. Simulations verify the sector size distribution formula given above. The aggregate outputs for all demand patterns initially decrease when we start the model with initial conditions with $n_i(0)$ too large for equilibrium values. The model quickly shed excess labors and settle down to oscillate around equilibrum level, i.e., business cycles. Loosely speaking, the more demands are concentrated among more productive sectors, the more quickly the model settles into business cycles. The more demands are concentrated on more productive sectors, the higher is the average levels of aggregate output.

An interesting phenomenon is observed when the demand patterns are switch from more productive to less productive demand patterns and conversely. See Aoki and Yoshikawa (2005) for detail.

5 New Model of Labor Dynamics

This section discusses Okun's law and Beveridge curve by augmenting the model in the previous section by a mechanism for hiring and firing, while keeping the basic model structure the same.

5.1 A New State Vector

Consider, as before, an economy composed of K sectors, and sector i employs n_i workers, $i = 1, \dots K$.

Sectors are now in one of two status; either in normal time or in overtime. That is, each sector has two capacity utilization regimes.

The output of the sector is now given by

$$Y_i = c_i(n + i + v_i),$$

where v_i take the value of 0 in normal time, and 1 in overtime.

More explicitly, in normal time

$$Y_i = c_i n_i,$$

for $i = 1, 2, \dots, K$, where c_i is the productivity coefficient, and n_i denotes the number of employees of sector i. In overtime, indicated by variable $v_i = 1$, n_i workers produce output equal to

$$Y_i = c_i(n_i + 1).$$

In overtime, note that the labor productivity is higher than in normal time because $Y_i/n_i = c_i(1 + 1/n_i) > c_i$. This setup may be justified due to possible underutilization of labor. The total output (GDP) is given by the sum of all sectors, as before, $Y = \sum_{i=1}^{K} Y_i$.

Recall that demand for good i is given by $s_i Y$ as in the previous section, where s_i is a positive share of the total output Y which falls on sector i goods, with $\sum_i s_i = 1$.

5.2 Transition Tates

To implement a simple model dynamics we assume the following. Other arrangements of the detail of the model behavior is of course possible.

Each sector has three state vector components: the number of employed, n_i, the number of laid off workers, u_i, and a binary variable v_i, where $v_i = 1$ means that sector i is in overtime status producing $c_i(n_i + 1)$ output with n_i employees. Sectors in overtime status all post one vacancy sign during overtime status. When one of the sectors in overtime status becomes active with positive excess demand, then, it actually hires one additional unit of labor and cancels the overtime sign. When a sector in overtime becomes active with negative excess demand, then it cancels the overtime and returns to normal time and vacancy sign is removed. When $v_i = 0$, sector i is in normal time producing $c_i n_i$ output with n_i workers. When one of the sectors, sector i say, in normal time becomes active with positive excess demand, then it posts one vacancy sign and v_i changes into one. If this sector has negative excess demand when it becomes active, it fires one unit of labor.

To summarize: When $f_a < 0$, n_a is reduced by one, and u_a is increased by one, that is one worker is immediately laid off. We also assume that v_a is reset to zero. When f_a is positive, we assume that it takes a while for the sector to hire one worker if it has not been in overtime status, i.e., v_a is not 1. If sector a had previously posted vacancy sign, then sector a now hires one worker, and cancels the vacancy sign, i.e., resets v_a to zero. If it has not previously posted a vacancy sign, then, it now posts a vacancy sign, i.e., sets v_a to 1, and increases its production with existing number n_a of workers by going into over-utilization state.

The transition path may be stated as $\mathbf{z}$ to $\mathbf{z}'$, where $(n_a, u_a, v_a = 0) \rightarrow (n_a, u_a, v_a = 1)$, and $(n_a, u_a, v_a = 1) \rightarrow (n_a + 1, u_a - 1, v_a = 0)$. In either case the output of the active sector changes into $Y_a' = Y_a + c_a,$.

We next describe the variations in the outputs and employments in business cycles near one of the equiibria.

5.3 Hierarchical tree of unemployment pools

In our model jobs are created or destroyed by changes in excess demand patterns. Pools of unemployed are heterogenous, becasuse of geographical location s of sectors, human capitals, length of unemployed periods and so on.

A given sector i, say, has associated with it a pool of unemployed who are the laid-off workers of sector i. They have the highest probability of being called back if sector i is active and can hire one worker. Pools of workers who are laid of from sector $j, j \neq i$ have lower probability of being hired by sector i, depending on the distance $d(i,j)$, called ultrametrics. These pools are organized into hierarchical trees with the pool of the laid-off workers from sector i at the root. The probabilities of being hired by a worker outside pool i is a decreasing function of $d(i,j)$.

The ultrametric distance between pool i and j are symmetric $d(i,j) = d(j,i)$ and satisfies what is called ultrametric condition

$$d(i,j) \leq max\{(d(i,k), d(k,j)\}.$$

See Aoki (1996, p.36, Chapter 7) for further explanation.[9]

5.4 Okun's Law

Okun's law is an empirical relationship between changes in GDP, Y, and the unemployment rate u We define the Okun's law by

[9] This ultrametric notion is used also in numerical taxonomy, Jardine and Sibson (1971), Spin glasses and other physics, Mezard and Virasoro (1985). See Schikhof (1984) for the mathematics involved. Feigelman and Ioffe (1991) has an example to show why the usual correlation coefficients between patterns do not work in hierarchical organization.

$$\frac{\Delta Y}{Y_e} = -\beta\{\frac{\Delta U}{N}\}, \tag{4}$$

where $N = L + U$ is the total population of which L is employed and U is unemployed. In this paper we keep N fixed for simpler presentation.

This numerical value of β is much larger than what one expects under the the standard neoclassical framework. Take, for example, the Cobb-Douglas production function with no technical progress factor. Then, GDP is given by $Y = K^{1-\alpha}L^{\alpha}$ with α of about 0.7.

We have $\Delta U = -\Delta L$, where ΔK and ΔN are assumed to be negligible in the short run. The production function implies then that $\Delta Y/Y = \alpha\Delta L/L$ in the short run. That is, one percent decrease in Y corresponds to an increase of $\Delta U/N = -(1/\alpha)(\Delta Y/Y)(1-U/N)$, i.e., an increase of a little over 1 percent of unemployment rate. To obtain the number 4, as in the Okun's law, we need some other effects, such as increasing marginal product of labor or some other nonlinear effects. See Yoshikawa (2000).

We assume that economies fluctuate about its equilibrium state, and refer to the relation (4) as Okun's law, where Y_e is the equilibrium level of GDP, approximated by the central value of the variations in Y in simulation. Similarly, ΔU is the amplitude of the business cycle oscillation in the unemployed labor force. U_e is approximated by the central value of the oscillations in U, and Y_e and L_e are related by the equilibrium relation (2).

The changes $\Delta Y/Y$ and $\Delta U/U$ are read off from the scatter diagrams in simulation after allowing for sufficient number of time to ensure that the model is in "stationary " state.

In simulations we note that after a sufficient number of time steps have elapsds, the model is in or near the equilibirum distribution. Then, Y and U are nearly linearly related with a negative slope, which can be read off from scatter diagrams i.e.,

$$\Delta Y = -x\Delta U,$$

and we derive the expression for β

$$\beta = \frac{\kappa x}{1 - U_e/N}.$$

We next see that the situation changes as the demand shares are made to depend on Y. We now assume that demand shares depends on Y, hence κ depends on Y.

Differentiate the continuum of equilibrium relation $L = \kappa Y$ (dropping superscript e from now on) with respect to Y to obtain

$$\Delta L = \kappa[1 + \frac{1}{\kappa}\frac{d\kappa}{dY}].$$

with

$$\kappa(Y) = \sum_i \frac{s_i(Y)}{c_i},$$

so that

$$\frac{d\kappa}{dY} = \sum_i \frac{1}{Y}\Theta,$$

with

$$\Theta = \sum_i \frac{s_i(Y)}{c_i} \frac{dlns_i(Y)}{dlny}.$$

Then the coefficent of the Okun's law becomes

$$\beta = -(1 + \frac{u}{L})(1 + \frac{1}{\kappa}\frac{d\kappa}{dY})]. \tag{5}$$

Okun's law in the economic literature usually refers to changes in gross domestic products (GDP) and unemployment rates measured at two different time instants, such as one year apart. There may therefore be growth or decline in the economies.

To avoid confusing the issues about the relations between GDP and unemployment rates during stationary business cycle fluctuations, that is, without growth of GDP, and those with growth, we run our simulations in stationary states assuming no change in the numbers of sectors, productivity coefficients, or the total numbers of labor force in the model.

The Okun's law refers to a stable empirical relation between unemployment rates and rate of changes in GDP: one percent increase (decrease) in GDP corresponds to β percent decrease (increase) in unemployment, where β is about 4 in the United States.

Example

Let the shares vary according to

$$s_i(Y) = s_{0i} - \gamma_i(Y - Y^e),$$

$i = 1, 2$, where s_{0i} is the equilibrium value $s_i(Y^e)$, and $\gamma_1 + \gamma_2 = 0$ to satisfy the condition that the sum equals 1. Here are some numbers. With $(s_1 s_2) = (1 - s, s)$ where $s_{0i} = 0.1$, $i = 1, 2$, $(1/c - 1) = 10^2, \gamma_1 = 10^{-1}$ we obtain $\beta = 2.1$. With $\gamma = 1.5 \times 10^{-2}$, $\beta = 4.3$. With $s_0 = 4 \times 10^{-2}$ $1/c - 1 = 10^2$, $\gamma = 3 \times 10^{-2}, \beta = 3.6$.

The next two digures are the Okun's coefficients derived from simulation with $K = 10$.[10]

[10] The original Matlab program was written by a UCLA Dept. Economics graduate student, L.Kalesnikov, which was later revised by two Univ. Tokyo Fac. Economics graduate student, Ikeda, and Hamazaki.

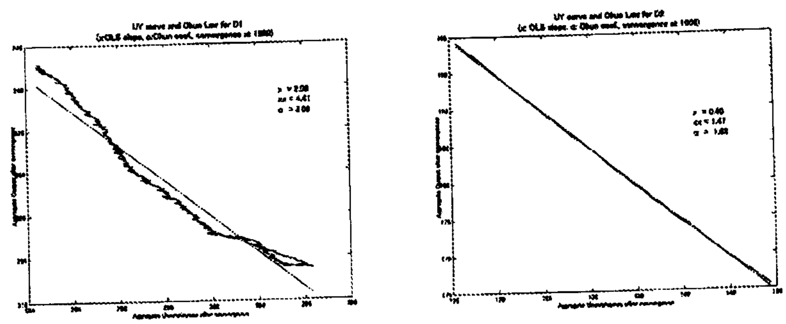

Fig. 1. Examples of Okun's law

5.5 Beveridge Curves

In real world unemployment and vacancies coexist. The relation between the two is called Beveridge curve. It is usually assumed that its position on the $u - v$ plane is independent of aggregate demand. In simulations, however, we observe that their loci will shift with Y.

Our model has a distribution of productivities. Demand affects not only Y but also the relation between unemployment and vacancy loci.

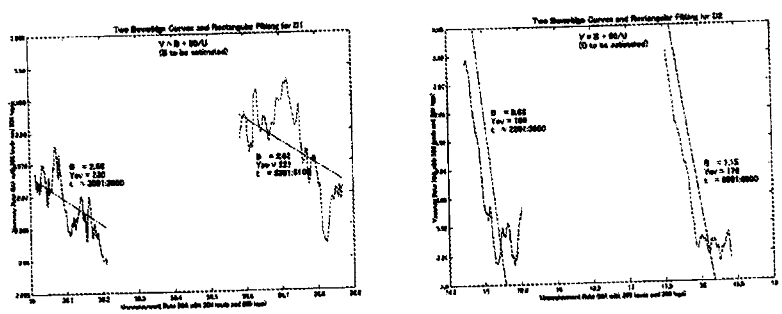

Fig. 2. Examples of Beveridge Curves

This result is significant because it means that structural unemployment cannot be separated from cyclical one due to demand deficiency. This implies that the notion of natural rate of unemployment is not well defined.

5.6 Simulation Studies

Since the model is nonlinear and possibly possesses multiple equilibria, we use simulations to deduce some of the properties of the models. We pay attention to the phenomena of trade-offs between GDP and unemployment, and the scatter diagrams of GDP vs. unemployment to gather informtion on business cycle behaviors.

Our model behaves randomly because the jumping sectors are random due to holding times being randomly distributed. This is different from the models in the literature which behave randomly by the technology shocks which are exoegnously imposed. As we indicate later the state space of the model have many basins of attractions each with near equal output levels.

Simulations are used to gather information on model behavior.[11]

Various cases with $K = 4$, $K = 8$ and $K = 10$ have been run. Four hundred Monte Carlo runs of duration 7000 elementary time steps each have been run. Fig. 2 is the average GDP of P_1. It shows that after 700 time steps the model is in the closed set. The lower panel shows the details of oscillations. More details on the simulations are found in Aoki and Yoshikawa (2005)

5.7 Effects of Demand Management on Sector Sizes

With some demand patterns such that the low productivity sectors share a major portiont of the aggregate demand, and as (2) shows suppose that the least productive sector has the largest equilibrium size to meet the demand. Suppose that the model has entered the closed set and exhibits stationary business cycle, and suppose that the demand pattern is switched so that the high productivity sectors now receive the major portion of the demand. One would conjecture that the stationary Y values will increase and the model will reach a new stationary state.

When the size of the least productive sector is very large, the model will start by shrinking the size of the least productive sector more often than increasing the size of the more productive sector. Under some conditions it is easy to show that the probability of size reduction by the least productive sector is much larger than that of the size increase of the productive sector, at least immediately after the switch of demand pattern. When the productivity coefficients demand shares satisfy certain condition, net reduction of Y is permanent, contrary to our expectation. See Aoki and Yoshikawa (2004) for detail.

5.8 Summary of Findings from Simulations

Simulation results may be summarized as follows:

1. Larger shares of demand on more productive sectors result in the higher average values of GDP.

[11] Simulation programs were written originally by V. Kalesnik, a graduate student at UCLA, and later modified by F. Ikeda, and M. Suda, graduate students of Faculty of Economics, Univ. Tokyo.

2. The relationship between unemployment and vacancy depends on demand. Our simulations show that Beveridge curves shift up or down when Y goes down or up, respectively. In other words, when Y declines (goes up) the Beveridge curve shifts outward (downward).
3. The relationship between unemmployment and the growth rate of GDP is described by a relation similar to the Okun's law.
4. The economy reaches the 'equilibrium' faster with larger shares of demand falling on more productive sectors. This indicates that demand effects not only the level of GDP but also adjustment speed toward equilibrium.

In other words, our simulations showthat higher percentages of demands falling on more productive sectors produce four new results: (1): average GDPs are higher; (2): the Okun's coefficients are larger; (3): transient responses are faster, and (4):timing of the demand pattern switches matter in changing GDP. Unlike the Cobb-Douglas production or linear production fucntion which lead to x values of less than one, we obtain the values from 2 to 4 depending on κ as the Okun's coefficient in our simulation.

It is remarkable that we can deduce these results from models with linear constant coefficient production functions. This indicates the importance of stochastic interactions among sectors introduced through the device of stochastic holding times. Using a related model Aoki and Yoshikawa (2003) allow the positive excess demand sectors to go into overtime until they can fill the vacancy. This model produces the Beveridge curse shifts. The details are to appear in Aoki and Yoshikawa (2005).

6 Summing Up

We have advanced the following propositions and perspectives by our new stochastic approach to macroeconomics.

1. Equilibria of macroeconomy is better described probability distribution. Master equations describe time evolutions of macroeconomy
2. Sectoral reallocations of resources generate aggregate fluctuations or business cycles. Given hetergeneous microeconomic objectives and constraints, thresholds for change in strategies differ across sectors/firms. It takes time for productivities across sectors equalize. In the mean time responding to excess demands or supplies the level of resource inputs in at least one sector changes, and macroeconomic situations also change initiating another rounds of changes.

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