Reconstructing Macroeconomics: A Perspective from Statistical Physics and Combinatorial Stochastic Processes

Reconstructing Macroeconomics is an ambitious, informative, challenging, if not fully realized, book that addresses fundamental questions of economic theory on many levels. It sets forth a powerful critique of the attained-equilibrium representative-agent rational-expectations paradigm that dominates contemporary mainstream macroeconomics. It proposes an alternative philosophy of modeling and conception of equilibrium and disequilibrium, founded on the highly successful theoretical and empirical methods of statistical physics. The authors go beyond these foundational issues to demonstrate how their methodological ideas can work in practice by developing a series of detailed, specific, testable models that address classic problems of macroeconomics, including the determination of income, unemployment, inflation, business cycles, long-run economic growth, and volatility and efficiency of financial markets. In all these areas the authors propose technically sophisticated models that explain important and well-established features of macroeconomic data that remain anomalous from the point of view of conventional mainstream theory. These models are unified methodologically to a remarkable extent, but have not yet been shaped into a unified theoretical synthesis. They rest to a considerable degree on highly plausible assumptions bound to be regarded as "ad hoc" by adherents of conventional mainstream theory. The greatest lacuna in the argument is the authors' failure to come to grips with traditional economic arguments that market prices introduce pervasive stabilizing feedback into the economic system.

This book is notable also for its careful and wide-ranging discussion of important papers in the macroeconomic literature of the last fifty years, and the familiarity of the authors with "heterodox" contributions to this literature, especially from post-Keynesian scholars. In the tradition of physics, the authors make a serious effort to discuss the relevance of their models to real economic events, particular episodes in the history of the Japanese economy, and to compare the predictions of their models to available data.
Representative agent models and statistical physics

From the point of view of statistical physics, the traditional economic theory of equilibrium is an aberration. Thermodynamics incorporates a theory of fluctuations as an inherent feature of physical systems in equilibrium. The macro-behavior of complex physical systems does not directly reflect the behavior of the smaller systems that constitute it; a confined gas in equilibrium with a well-defined temperature, pressure, volume and energy does not behave at all like the individual molecules that constitute it. The molecules are engaged in a complex chaotic motion involving collisions among themselves and with the boundary of the container, but the gas settles down to an equilibrium in which fluctuations of pressure and temperature are small and statistically predictable. Statistical mechanics explains these macro-regularities as the result of statistical averaging over an equilibrium distribution of molecules in phase space (describing their positions and momenta). Equilibrium distributions can be calculated for specific physical systems either by the formalism of maximizing entropy subject to macro-constraints or by calculating detailed balance conditions through the "master equation" that accounts for the movement of particles among various available states.

What puzzles a physicist is that economic theories of equilibrium make no allowance for the difference between the micro-behavior of constituents (households and firms, for example) and the macro-behavior of the system. This gap is closely related for a physicist to the failure of these theories to address the problem of fluctuations. Economic theory equilibrium conceptualizes prices as specific values, not a statistically distributed phenomenon that allows for fluctuation, and the constituent parts of economic systems are imagined to be in static equilibrium when the economic system is in equilibrium. Because of unresolved problems with aggregation in economic systems with heterogeneous agents, economists center their attention on "representative agent" economies, which behave as if the macro-economy is simply a scaled-up version of an individual. No heterogeneity; no chaotic motion; no fluctuations; no statistics. This is particularly puzzling to physicists who look at economic and financial data, which are rife with what appear to be statistical fluctuations.

Aoki and Yoshikawa (hereafter AY) hammer home this point, which is the starting-point for their various models of macro-economic phenomena.
"Micro behaviors of the representative agent do not mimic the behavior of the macroeconomy. Macroeconomic phenomena are the outcomes of interactions of a large number of economic agents such as households and firms. Equilibrium in the macroeconomy is better described by a probability distribution than by a 'point' in some space or set." (page 26)

AY believe, as do many econophysicists, that these methodological shortcomings of conventional macroeconomics are reflected in the inability of the attained-equilibrium, representative agent, rational expectations to deliver relevant and correct explanations of macroeconomic phenomena such as price fluctuations in financial markets, business cycles, aggregate demand shortfalls, and unemployment. This book sets forth a systematic research program based on alternative methods to create a new macroeconomics. In some areas these new methods actually lead back to reformulations of traditional Keynesian theory; in others they open up unexplored issues and questions.

AY's analytical strategy for addressing the problems with mainstream macroeconomics is to shift the focus of theory from the behavior of individual agents, which in their view is at best of secondary importance in determining macro outcomes due to the heterogeneity of economic agents and the chaotic and unpredictable character of their actions.

To provide micro foundations for macroeconomics, one need not explicitly analyze optimizing behavior of individual agents. Instead one must use proper statistical methods suitable for the study of the macro system consisting of a large number of fluctuating micro units. We maintain that the so-called micro foundations for macroeconomics are not true micro foundations, but simply misguided. (page 12, italics in original)

This strategy leads AY to develop macroeconomic models by applying sophisticated distributional and combinatorial mathematical analysis to systems consisting of very simple heterogeneous agents. A considerable proportion of the book is devoted to exposition of tools, such as the master equation, for the analysis of combinatorial and stochastic models that are familiar to physicists but not much taught to graduate students of economics. The book in this respect is a harbinger of a mathematics gap developing between economics, where graduate mathematical training centers on real analysis, fixed-point and separating hyperplane theorems, and static and recursive optimization theory, and physics, where graduate mathematical training centers on stochastic dynamical systems theory, Markov chains, and information theory. There are, of course, some important areas of overlap, since constrained optimization appears in physics in
the least action principle formulation of mechanics, and in thermodynamics and statistical mechanics through the principle of entropy maximization. Nonetheless the mathematical worlds of the economist and physicist have a different "feel" and tone, as this book illustrates.

**An example: aggregate supply**

AY's treatment of aggregate supply in chapter 3 of this book is a useful example of their use of these methods. AY study the distribution of a given population, \( N \), over sectors \( i = 1, ..., S \) (which may include household production and job search), each with given "productivity" (presumably real GDP per worker) \( c, c_2 < ... < c_S \). For a given assignment of population to sectors \( n_i \), real output will be

\[
Y = \sum_{i=1}^{S} c_i n_i.
\]

For a given aggregate demand, \( D \), equilibrium requires \( Y = D \). AY assume, in line with typical statistical models from physics, that in equilibrium the entropy of the distribution of population across sectors \( (n/N) \), is maximized subject to the demand constraint. AY give a clear exposition of the derivation of the exponential Boltzmann-Gibbs distribution which solves this problem, in which aggregate demand, \( D \), plays the role of temperature in a physical system. A higher aggregate demand leads to higher real output by inducing a flatter distribution which puts a higher proportion of the population in higher-productivity sectors. Aggregate real supply, which is maintained by conventional economic theory to be determined entirely by resources, tastes, and technology, becomes endogenous and determined by aggregate demand in AY's model.

Further pursuit of the comparison of these two simple models of aggregate supply sheds considerable light on what is going on. According to conventional economic theory, when there is only one resource (in this case labor) to be allocated, the market, in the absence of capacity constraints, will allocate it to its highest productivity use, in this case sector \( S \), so aggregate supply will be \( Nc_S \) independent of aggregate demand. On this assumption the only way that aggregate demand can be accommodated to this given aggregate supply is through changes in money prices that will (through Keynes or Pigou effects) adapt aggregate demand. The conventional assumption is consistent with the basic idea that the market leads to an efficient allocation of resources (through the decentralized decisions of agents pursuing their self-interest). The average productivity of the economy under AY's assumption of a Boltzmann-Gibbs
distribution of labor over all sectors is obviously lower than if all labor were assigned to the highest-productivity sector. On the other hand, from a statistical physics point of view the conventional assignment of labor is extremely unlikely, to say the least; the entropy of the distribution that assigns all the labor to the most productive sector is zero. How, physicists (and AY) might reasonably ask, is the market supposed to achieve a state which requires effectively an infinite amount of information, and is analogous to the physically unreachable limit of absolute zero temperature? Conventional macroeconomics has trouble formulating an answer to this question, because the theory, as discussed above, has no room for fluctuations and statistical elements of equilibrium. Nonetheless, the conventional macroeconomist might equally well ask what constraints stop labor from flowing to its highest productivity uses in AY's model.

**Time and change**

AY do address this question somewhat obliquely. In their conception equilibrium has to be the outcome of a well-defined dynamical process. (It is in this respect that their approach departs sharply from the assumption of "attained equilibrium" that dominates mainstream macroeconomics.) Their workhorse for this task is the "jump Markov process" (explained carefully in chapter 2), a formalism that has had great success in unraveling the dynamical behavior of systems studied in statistical physics. The setup for a jump Markov process is a system consisting of a collection of component sub-systems, each of which can be in a particular state (from a finite or countable set of possible states). The list of states of the components (in economics think households and firms) constitutes the state of the system. This system state changes when the state of one or more of the component sub-systems changes. Using the plausible assumption underlying Poisson stochastic processes, that the probability of a change in more than one component in a given period goes to zero as the period length goes to zero, the jump Markov model dynamics are governed by a "master equation" that describes the dynamic evolution of the probability distribution of components over states as a function of the state of the system. The state of the whole system changes at discrete epochs and each discrete change affects only one component of the system. Equilibrium distributions are stationary under the transformations of the master equation. Another way to think of this is that at
equilibrium the probability of component parts of the system moving into and out of each state are equal. Thus in the example of allocation of labor to sectors we have been discussing, the dynamics would involve movement of a worker from one sector to another. According to the jump Markov model at any one moment only one worker would move, though the time scale of jumps might be such as to closely approximate continuous flows of workers between sectors. In contrast to their developments of some other models, AY do not present a master equation analysis of the aggregate supply model, but move directly to the entropy-maximizing formalism that leads to the Boltzmann-Gibbs distribution. If a conventional macroeconomist argued that all transitions in the jump Markov model would lead workers to move from lower to higher productivity sectors, it is not hard to see that having all the workers in the highest productivity sector would be an "absorbing state" to which the system would inevitably move. There are, of course, many reasons to think that in the real world workers do not always move to higher productivity sectors (including the evidence AY present for persistent productivity differences between sectors). AY also use jump-Markov processes to examine the dynamics of prices in chapter 5. In this setting the interactions of the component agents of the economy are described by their positions on a connected graph with a tree structure. AY treat price dynamics as the diffusion of exogenous shocks through the tree, as each agent responds to changes in its neighbors according the a jump-Markov dynamics. The key finding is that the average diffusion time for a shock depends critically on the number of levels of the tree structure, with more complex tree structures giving rise to slower price-adjustment dynamics. AY conclude that there is no "mystery" to the phenomenon of slow price adjustment (and hence no need for the somewhat mystical notion of "menu costs" which has flourished in the "New Keynesian" literature); it is a predictable feature of tree-structured individual agent interactions. This argument has intuitive appeal, since agents often experience inflationary pressures as changes in the prices of their suppliers. It raises some questions, however. First, AY take the tree structure of interactions as exogenous in their discussion. But if the tree structure is supposed to represent supplier-purchaser links in the economy, it probably reflects economic decisions on the part of agents. Second, the representative agent model argues that pricing decisions reflect simultaneous agent revaluations of stocks of commodities and other assets as the result of their reception of public, commonly
temperature is completely absent from conventional macroeconomics. The perspective of this book has the very constructive aspect of allowing for an explicit, modeled, treatment of this idea. On the other hand, this striking insight is not very well synthesized with the other models explored in the book. For example, the role of physical temperature is taken by aggregate demand in other chapters, but by "uncertainty" in this discussion.

**Financial markets**

Financial markets and prices play a central role in the "econophysics" literature to which AY's work relates closely. While attained equilibrium, representative agent, rational expectations models have dominated mainstream macroeconomic journals, the same ideas have been buffeted to the point of being discredited by financial market data showing persistent excess volatility, equity premia, and fat-tailed distributions of price fluctuations. In chapters 9 and 10 of this book, AY put forward some of their own ideas on these subjects. Chapter 9, which struck me as less completely explained and argued than the rest of the book, uses jump-Markov methods together with combinatorial reasoning to develop a model of "clustering" of financial investors according to behavior. This clustering (for example, between fundamentalists and technical traders) leads to kind of tug-of-war theory of financial prices, which can give rise endogenously to high volatility of prices. This model is one contribution (and perhaps not the simplest or most convincing) to a growing literature of behavioral agent-based financial models (see Tesfatsion, et al 2008).

In chapter 10 AY put forward a rather complicated and parametrically fragile model of fluctuations to explain why some macroeconomic variables, like the rate of growth of GDP, have exponentially distributed tails, while many financial variables have power-law tails. In AY's view this difference is explained by the frequency of the micro-growth events that contribute multiplicatively to the aggregate movements of the different variables per period. In the model they develop, low frequency of micro-growth events leads to exponential tails, while high frequency can lead to power-law tails. There are other perspectives from statistical physics that provide alternative explanations of this phenomenon. For example, exponential distributions are maximum entropy distributions subject to a constraint on the average level of the variable, while power-law distributions are maximum entropy distributions subject to a constraint on the average logarithm of the variable. Thus exponential distributions might be expected
where increments are additive, and power-law distributions where they are multiplicative, as is the case in financial markets if large and small portfolios have the same rate-of-return opportunities.

**Demand and economic growth**

Chapter 8 of this book, a demand-based theory of economic growth, is something of an outlier. The perspective is the explanation of long-run growth, not fluctuations, although AY adapt some of their basic tools of stochastic process analysis to this problem. Their starting point is the observation that the growth of individual sectors of the economy almost universally follows a logistic, S-shaped, pattern. An innovative sector (such as electronics, or automobiles in an earlier era) starts by growing more rapidly than the economy, but eventually reaches a point of "maturity" where its growth slows and in many cases levels out at a lower rate than the economy as a whole. (This phenomenon leaves room for the emergence of even newer fast-growing sectors.) The basic phenomenon that explains the logistic pattern is the diffusion of innovations through the economy. A new product has many potential buyers, but as more and more households acquire it, the potential for market growth diminishes. AY, following a long and vigorous tradition that goes back at least to the work of Schumpeter, view the emergence of sectors based on innovation as the basic mechanism of economic growth and development.

The model AY present in this book is notable as a serious attempt to formalize the insights of the demand-driven growth concept. They argue that it has substantial explanatory advantages over supply-side models, even of the Schumpeterian quality-ladder type, that focus exclusively on the impact of innovation on costs and productivity. They mobilize such mathematical wizardry as non-Poisson Polya urn models to surmount the substantial conceptual difficulties a demand-based theory of sustained growth presents. Whatever growth economists may eventually come to think of the specific model AY present here, I think this general line of thinking will turn out to be seminal for the development of growth theory. Aggregate growth models, even of the quality-ladder type, simply do not reflect well enough the crucial role structural change and product innovation play in economic development. Integrating concepts such as the saturation and creation of demand into the growth story seems to be an important research problem for economic growth theory.
Reconstructing macroeconomics

How much success will AY have in "reconstructing macroeconomics" by slaying the dragon of representative-agent, attained equilibrium, rational-expectations macroeconomics? Among the wealth of issues raised by this stimulating book three stand out.

Equilibrium fluctuations

It seems unlikely that economics in general and macroeconomics in particular can continue indefinitely to operate with a theory of equilibrium so much at odds with conceptions in other closely-related sciences as at present. If AY's effort does no more than firmly put the need for an economic theory that includes a theory of fluctuations and reflects both the diversity of the economic agents and the complex relation between aggregate outcomes and agent-level behavior, it will have a salutary impact.

At one level, one might see the possibility of a synthesis between AY's stochastic-process based approach to macroeconomics and more conventional market-based theories. Perhaps, as AY's arguments may lead some to accept, the type of Walrasian equilibrium that underlies conventional macro-economics is a kind of rough approximation to the complexity of real-world economic interactions, resting on simplifying abstractions, like the full attainment of equilibrium and the absence of endogenous fluctuations. In the real world it could be the case that the dynamic processes AY model are operating to bring about something like the attained equilibrium of Walrasian theories, and that macro-economic fluctuations reflect the complex dynamics of these processes. In this view we might see the Walrasian equilibrium as a kind of central point around which the real economy fluctuates. But no one seems to have a good handle on how this type of theory would deal with the behavior of heterogeneous economic firms and households out of (even though perhaps near) equilibrium.

Demand-driven models

AY are "heterodox" in their view that aggregate and sectoral demands (and hence policies designed to influence demand) play a central role in determining macro-economic outcomes. The Hayek-Friedman reaction against what they saw as politically dangerous features of Keynesian economics largely took the analytical form of insisting on regarding supply constraints as the ultimate determinants of economic outcomes.
To some degree this is ideological shadow-boxing. The distinction between "demand" and "supply" is difficult to maintain rigorously in economic theory (witness Ricardo's and Mill's insistence that supply and demand were two faces of the same economic activity). Perhaps it makes most sense in some very short-run perspective where we can take not only the division of labor and the organization of production, but financial states of households and firms as given. But over any substantial time period, it is clear that demand shifts have substantial and lasting impacts on supply and investment decisions. AY's theory of growth, for example, has to explain how production is organized to meet the demand for innovative products of new sectors.

But the self-imposed dogma of supply-side determination has hobbled and diminished macro-economic analysis. AY make powerful arguments in their critique of the "natural rate of unemployment". This is not to argue for the resurrection of the "good, old-fashioned" Phillips curve in pristine form, but in practice the project of identifying a robust natural rate to use for policy analysis has proved elusive or self-defeating empirically. It is possible that some may be able to re-frame the forces of innovation AY label "demand" in their model of economic growth as "supply" factors, but there is no doubt that these forces play a central role in real economic development.

**Markets and self-regulation of the economy**

Perhaps the weakest aspect of AY's impressive analytical acrobatics is their failure to introduce the regulating role of market prices into their story. At some level this is puzzling, because price equilibration is a type of informational feedback that seems well-suited to analysis using the tools of stochastic dynamics. It is not necessary to retreat all the way to attained equilibrium, which seems to assume that market prices can instantaneously regulate economic interactions without instabilities or frictions of any kind, to acknowledge that price adjustments do play an important real-world role.

This lacuna is also bound to be a big obstacle to mainstream macroeconomists, even those who may be restless within the constraints of attained-equilibrium, representative-agent, rational-expectations dogma, engaging scientifically with the substance of AY's arguments. The technical demands AY put on their readers are far from trivial; they deploy a dazzling but unfamiliar range of analytical models and ideas which they handle familiarly but are likely to be terra incognita for many macroeconomists. It might be tempting for mainstream
macroeconomists to ignore this book on the grounds of its failure to go farther in representing market price equilibration as a kind of excuse for not confronting the hard questions it raises about the foundations of economic theory and the mathematical challenges it presents.

Who should read this book? In my view mainstream macroeconomists can only benefit by becoming familiar with the point of view AY represent so ably here, even if they find it unconvincing as a whole. Those who aspire to create a new macroeconomics, including the various schools of "heterodox" economics, can also learn a lot from this book. In its imaginative, if not completely realized, reconstruction of macroeconomics it challenges unquestioned presumptions in all schools of economics.

References


Duncan K. Foley

Leo Model Professor of Economics
New School for Social Research
79 Fifth Avenue,
New York, NY 10003
Tel.: +1- 212-229-5717
E-mail address: foleyd@newschool.edu (D. Foley)