Committee Decision with Multiple Votes*

Katalin Bognár

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Abstract

Under what circumstances can a static voting mechanism aggregate dispersed information of committee members? I argue that whenever the voters are able to cast multiple votes, the quality of the joint decision increases. However, voting mechanisms are intrinsically additive ways of aggregating private information. This, naturally, is not a binding constraint if the private information is conditionally independent. However, if the 'meaning' of the private information depends on other members' signals, i.e. the signals are conditionally correlated, then the joint decision by voting may be unsatisfactory. I relate this question to a representation problem in utility theory to derive abstract conditions on the joint signal distribution that are necessary and sufficient for efficient voting.

1 Introduction

When can a voting mechanism correctly aggregate the dispersed information of the committee members? The papers by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) illustrate how a voting procedure can induce committee

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members to vote insincerely. With this behavior, the committee members obscure and hence fail to convey their, otherwise valuable, private information through the decision process. Thus, the final verdict can be corrupted.

In this paper, I suggest a variation of the voting procedure discussed in the papers above. I allow more votes for the committee members. I show that whether the efficient decision rule arises in equilibrium depends on the structure of the committee members' private information. When the members have conditionally independent signals and are allowed sufficient number of votes, then the joint decision will be efficient. On the other hand, for a conditionally correlated signal distribution the above result may not be true. The main result of the paper introduces a necessary and sufficient condition on the signal distribution so that voting results in efficient decision.[1]

Why would a committee members ever vote against her information in a common interest setting? Consider a situation where each committee member receives a conditionally i.i.d. binary signal about the guilt of a defendant. The strength of the signals and the common preference is such that all jurors would prefer conviction if more than half of the private signals suggest guilt and similarly, acquittal if more than half of the private signals suggest innocence. However, the voting rule is such that acquittal is the status quo and a conviction requires a 2/3 majority. Now, if all committee members but one, say i, are voting sincerely, then the vote of i is pivotal only if approximately 2/3 of the others have voted for conviction, hence received guilty signal. In this case, the preferred verdict is conviction irrespective of i's signal. Thus, i should vote for conviction even if her signal suggests innocence.[2] The dilemma of the committee members is that they would like to transmit their information to the voting procedure but the latter is not flexible enough to let them do so accurately as two votes for conviction are required to compensate

[1] A significant part of the committee decision literature is concerned about the efficiency of information aggregation when the number of the committee members participating in the decision increases. However, assuming conditionally independent private information, more members means a refined information structure such that in the limit, the true state reveals. In this paper, I follow a different approach by keeping the structure of the private information fixed.

[2] Note that this insincere behavior of i is in everybody's interest as all agents and society are trying to maximize the same preferences.

one vote for acquittal. However, the problem disappears when a simple majority is required for either conviction or acquittal.

Now, consider a variant of this problem such that the signals for innocence and guilty are not equally informative, and the commonly preferred verdict is acquittal only if at least 2/3 of the signals point to innocence. In this case the simple majority rule works badly: If everybody else votes sincerely, the remaining committee member i is pivotal only if approximately half of the signals are for guilt. In which case, acquittal is the preferred verdict and i will vote for that alternative even if her signal suggest guilt. In this case the 2/3 majority rule for would incentivize sincere voting and lead to perfect information aggregation. The take-away from this variant is that a good voting procedure should let committee members report the intensity of their information accurately: In the original example they should be able to cast votes for acquittal and conviction that are weighted equally. In the variant they should be able to cast votes for acquittal that count for less than votes for conviction.

A possible way to address this issue would thus be to tailor the majority requirement of a voting rule to the information structure of the committee members. This is problematic for two reasons. First, we should think of voting mechanisms as widely applicable rules, like those outlined in a constitution, rather than specific mechanisms that need to be tailored to the situation at hand. Second, optimizing over the majority rule reaches its limits as soon as the signals of committee members can take on more than two possible values. Obviously, a committee member needs at least as many possible actions as she has possible signals to enable an efficient verdict (if each pair of two signals can be pivotal for some realizations of other signals).

The latter observation points to the approach of this paper: Allowing committee members to cast multiple votes rather than a single vote. If each committee member can cast two, say, instead of one vote the above problems can be resolved by casting only one vote for conviction if one's signal suggests guilt but both votes for acquittal if one's signal is suggests innocence in the original example and vice versa in the variant.

Formally, I extend the voting model in Austen-Smith and Banks (1996) in the following ways: (i) I allow committee members to cast multiple votes to express the intensity of their private information and (ii) I allow for conditional correlation of the private signals.[3] Focusing on efficient equilibria, I show the following:

- Increasing the number of votes available to the members improves the verdict for any non-unanimous voting rule (Propositions 2 and 3).

Whether or not the efficient decision is reached as the number of the votes becomes big, depends on the correlation of the underlying private information.

- If the private information of the committee members is conditionally independent then the efficient decision is possible with sufficiently many votes (Proposition 6).

- If the private information of the committee members is correlated then the efficiency of the verdict is not guaranteed. I give an intuitive necessary condition for efficient voting and a more abstract condition that is necessary as well as sufficient.

  The former essentially requires that each two signals s^i, s'^i of a committee member (and each two signal sub-profiles for groups of committee members) are uniformly ordered in the sense that there are no two realizations of others' signals such that in one case the verdict should be acquittal for s^i and conviction for s'^i and vice versa in the other case. The latter abstract condition requires additionally that this order extends to an irreflexive partial order on the formal sums of signal profiles.

The intuition for the first result was given above: introducing multiple votes allows committee members to increase the accuracy of their vote, align the effect of their action on the joint decision to the information content of their signals.

[3]Note that it is often the case that the committee members observe the same evidence, hence correlation among their private information can be quite natural.

Note that voting is an intrinsically additive method to aggregate private information. It is efficient if there is a way to transform the private signals to votes independently of everyone else's signal so that the sum of the votes represent all the relevant information dispersed among the committee members. Given a signal realization, the relevant information is summarized by the likelihood ratio. Whenever the signals are conditionally independent, the log-likelihood ratio of a signal profile, which is a monotone transformation of the likelihood ratio, is the sum of the log-likelihood ratios of the individual signals. Hence, there is a natural candidate for a voting strategy. However, committee members are usually presented the same evidence, therefore, it is plausible to assume that the private signals are conditionally correlated. In this case, the log-likelihood ratio does not have the above described additive property. Naturally, if there is another transformation of the signals to votes that has the above mentioned additive property then voting can be efficient. In the paper, I discuss when such a transformation exists.

The necessary condition above is straightforward: Committee member i must cast a certain number of votes $d^i(s^i)$ and $d^i(s'^i)$ (counting votes for conviction positively and votes for acquittal negatively) depending on her signal and as one of these numbers is greater than each other the efficient verdict cannot be reached if the above condition is not satisfied. It is tempting to think that this condition is sufficient as well as necessary for efficient voting. This is indeed the case for committees with two members. However, I give a counterexample with three voters where the condition is satisfied but efficient voting is not possible. Fortunately, I can draw a parallel to a mathematically identical question in utility theory. In this way, I can apply a theorem by Krantz, Luce, Suppes, and Tversky (1971) to derive the necessary and sufficient condition for efficient voting.

Related Literature. The quality of the decisions that are made by groups of decision makers has interested researchers for a long time. A seminal result on committee decisions, the Condorcet Jury Theorem, claims that a decision by the majority of a large group is better than the decision made by any of the individual members (Condorcet 1785). Moreover, if the committee is big enough, it can outperform the decision of even a highly competent individual. The early formal arguments to support this statement are of *statistical* nature. For Condorcet, committees are groups of people with limited decision skills (probability with which the member makes the right decision), who sincerely report their independent opinion. The sincere behavior and the independence of the occasional mistakes by the members allow the use of Law of Large Numbers to show that (i) the group is less likely to conclude a mistaken decision than any individual member and that (ii) the decision by the majority is almost certainly good when the committee is big.

The model of joint decision situations that I use in this paper builds on the work of Austen-Smith and Banks (1996). They consider privately informed committee members, such that the private information is conditionally independent and identical. They spell out the optimization problem of an individual committee member.[4] They show that in equilibrium, the individuals with binary signals may conclude the inappropriate verdict depending on the voting rule. This inefficiency is connected to insincere voting on the side of the committee members. Moreover, rational voters do not vote sincerely whenever the signal is more than binary and in committees with conflicting interests.

Further papers elaborate on the model by Austen-Smith and Banks (1996). Theorem 1 builds on the result of McLennan (1998) linking Bayes-Nash equilibria of the game to the cost minimizing voting profiles. Feddersen and Pesendorfer (1998) use the concept of pivotal voting to explain the behavior of the rational committee members. Their work demonstrates that a strategic individual considers not only her own information but the information content of the event that her vote is decisive in the process (pivotal voting). They show that the equilibrium verdict can be mistaken and this problem is the most severe if unanimity is required. Feddersen and Pesendorfer (1998), Coughlan (2000) and Duggan and Martinelli

[4]The literature on 'statistical voting' is silent on the origin of the voters' decision skills, whether the limited competence is due to a cognitive constraint or to lack of information. Miller (1986) links a mistaken vote to insufficient *information* and Ladha (1993) refers to a Bayesian updating process prior to voting. However, neither papers explicitly uses a state space and a signal distribution.

(2001) establish limit behavior of voting mechanisms as the committee becomes big, a question that is not in the focus of my work. All the papers above assume that only a single vote is available for each committee member and compare the performance of the different voting rules.

A few paper considers multiple votes for the committee members. Casella (2005) discusses a repeated joint-decision problem of individuals with independent private preferences. In her model, committee members may have multiple votes available in a decision round since they are able to transfer votes inter-temporally. She concludes that in this mechanism voters preference intensity can be expressed hence the quality of the decision is improved compared to a one vote / one decision problem mechanism. However, she investigates a situation with conflicting interests, where the goal is aggregating private preferences rather than private information. Li, Rosen, and Suen (2001) considers a two-member committee with conditionally independent private information. They show that whenever the interests of the members are aligned, efficient decision is possible if the number of votes is sufficiently big. However, with conflicting interests, in the equilibrium the information is garbled, in other words, there is no equilibrium in which the private information is fully revealed by any strategies. They state that with conflicting interests more votes can improve the decision but there are bounds on the quality of the verdict as the number of votes increases.

The independent work of Chakraborty and Ghosh (2003) is the closest to this paper. They establish that with perfectly divisible votes and conditionally independent signals efficient information aggregation is possible.

The paper is organized as follows. In Section 2, I introduce the formal model. In Section 3, I argue that increasing the number of the votes that are available for the committee members improves the joint decision. In Section 4, I state a condition on the joint signal distribution so that the private information can be effectively aggregated by a voting mechanism. Section 5 concludes the paper.

2 A Model of Committee Decision

Next, I introduce the model of a committee decision situation and define voting procedures. Juries are well-known examples of groups of people with the task of reaching a joint decision. Therefore, in the paper I use the terminology of a jury situation.

2.1 The Joint Decision Problem

Private Information. There are N jurors who have to come up with a joint decision. $\mathcal{N}$ denotes the set of jurors. There are two possible states of the world: innocence and guilt ($\theta \in \Theta = \{I, G\}$). The jurors' prior probability of each state is 0.5. Each juror i is endowed with a private signal $s^i \in S^i$ about the true state of the world. I assume that S^i is finite. The signal space is $S = S^1 \times S^2 \times \cdots \times S^N$ and a signal profile is $(s^1, s^2, \ldots, s^N) = s \in S$. The probability and conditional probability of the signal profiles are $P(s)$ and $P^\theta(s)$, respectively.

I assume that no signal profile is fully revealing. Hence for any signal profile the likelihood ratio $\ell_s \equiv \frac{P^G(s)}{P^I(s)}$ exists and is non-zero. It then follows that there is an upper limit on the informativeness of any signal profile.

Payoffs. There are two possible decisions to make: acquit or convict ($\omega \in \{A, C\}$). The jurors' *common* preference is characterized by a parameter $q \in [0, 1]$ such that q is the cost of convicting an innocent defendant and $1-q$ is the cost of acquitting a guilty defendant. The cost of reaching the appropriate verdict is zero. Formally, denote the payoff of decision ω in state θ by $u(\omega|\theta)$. Then $u(C|I) = -q,\ u(A|G) = -(1-q)$ and $u(C|G) = u(A|I) = 0$.

The *ex-post cost* of a decision ω if the signal profile is s is:

$$c(\omega, s) = \sum_{\theta} -u(\omega|\theta)P(\theta|s)$$

where $P(\theta|s)$ is the probability of the state θ when s is realized. Consider an *outcome function* $\Omega : S \rightarrow \{A, C\}$ that maps any signal profile into a decision. Then we can define the following costs:

- The ex-post cost for a signal realization s is $c(\Omega(s), s)$.
- The *interim cost* for a juror i with a signal s^i is:

$$\mathcal{C}^i(\Omega, s^i) \equiv \mathbb{E}_{(s^{-i}|s^i)} c(\Omega(s^{-i}, s^i), (s^{-i}, s^i)).$$

- The *ex-ante cost* of the mechanism with outcome function Ω is:

$$\mathcal{C}(\Omega) \equiv \mathbb{E}_s c(\Omega(s), s).$$

Efficiency. The jurors' goal is to acquit whenever the defendant is innocent and to convict whenever the defendant is guilty. However, no matter what the mechanism is, the decision they reach can never be the right one with certainty, simply because there is no fully informative signal profile. Given a signal realization s, the *efficient verdict* is $\arg\min_\omega c(\omega, s)$. An *efficient outcome function* is cost minimizing for any realized signal profile, i.e. it is ex-post efficient for any signal realizations.

Next, I show that the likelihood ratio of a signal realization is a convenient measure to characterize the efficient decision. Acquitting is costly only if the state is G and in this case it costs $1 - q$. Similarly, conviction is costly if the state is I and then it costs q. Hence, for a signal profile s, the cost of acquittal is: $c(A, s) = (1 - q)P(G|s)$ while the cost of conviction is: $c(C, s) = qP(I|s)$. Therefore, the efficient decision rule is:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad (1-q)P(G|s) \geq qP(I|s) \\ A & \text{otherwise.} \end{cases}$$

Intuitively, acquittal is better if the available information suggests that the state is rather I than G, and if acquitting a guilty defendant is not too costly compared to convicting an innocent defendant. Using that $\frac{P(G|s)}{P(I|s)} = \frac{P^G(s)}{P^I(s)}$ by the Bayes rule and the uniform prior assumption, the above decision rule can be conveniently rephrased as a threshold problem:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \ell_s \geq \ell_q \\ A & \text{otherwise.} \end{cases} \tag{1}$$

where $\ell_q \equiv \frac{q}{1-q}$ is the relative costs of the mistaken decisions and $\ell_s = \frac{P^G(s)}{P^I(s)}$ is the likelihood ratio of a signal profile s. This latter ratio expresses how likely guilt is relatively to innocence, given the signals. To summarize, if the likelihood ratio exceeds the ratio of the costs of mistaken decisions then conviction is the better verdict.

2.2 Voting Game (V, α)

Voting Procedure (V, α). In this paper, I focus on voting procedures to mediate the joint decision problem. I assume that the jurors reach a verdict in a one-shot game. Each juror is endowed with $V \in \mathbb{N}$ votes that she can fully or partly cast to either of the two possible decisions. The joint decision is convict if and only if a certain majority of the cast votes is for convict. The voting rule can be described by a parameter $\alpha \in [0, 1]$, so that the decision is convict if at least a proportion α of the cast votes is convict, i.e. if α(# votes cast for convict) $\geq$ (# votes cast for convict and acquit) then the joint verdict is convict.[5]

Strategies. A juror can determine the number of the votes she casts and which decision she supports. Her choice depends on her private signal about the state of

[5]For a given number of votes, V, any voting rule $\alpha < \frac{1}{NV}$ requires unanimity for acquittal, and similarly, any voting rule $\alpha \geq \frac{NV-1}{NV}$ requires unanimity for conviction. In the limit, as the number of votes increases, $\alpha = 1$ refers to the unanimity rule for conviction, however, $\alpha = 0$ does not mean unanimity for acquittal. This asymmetry follows from defining the tie breaking rule in an asymmetric manner. Recall, that if exactly α proportion of the cast votes is convict, the joint verdict is convict. It is possible to make the tie breaking rule dependent on the actual level of α and hence restore $\alpha = 0$ as a unanimous rule for continuous votes. This change would not alter my results except Proposition 5 in which I would need to make sure that I only compare voting rules that have the same tie breaking rule.

the world. Formally, a strategy of player i is $v^i = (v^i_A, v^i_C) : S^i \to \{0, 1, 2, \ldots, V\}^2$ such that either $v^i_A(s^i) = 0$ or $v^i_C(s^i) = 0$.[6] To simplify the analysis I alter the action space in two ways. I sign votes for acquittal as negative and votes for conviction as positive. Second, I normalize a juror's vote with the total number of votes that is available for her. Assume that the voting rule is simple majority, i.e. $\alpha = 1/2$, then define

$$X_V^{1/2} \equiv \left\{ \frac{k}{2V} | k \in \{1, 2, \ldots V\} \right\}.$$

Then there is a one-to-one correspondence between the intuitive action set and $X_V^{1/2}$. For any α-majority rules, I can generalize the method above and define:

$$X_V^\alpha \equiv \left\{ -\alpha \frac{k}{V} | k = \{1, 2 \ldots, V\} \right\} \cup \left\{ (1-\alpha) \frac{k}{V} | k = \{1, 2 \ldots, V\} \right\}.$$

Then a *normalized strategy* is $d^i : S^i \to X_V^\alpha$ such that

$$d^i(s^i) \equiv \frac{(1-\alpha) v^i_C(s^i) - \alpha v^i_A(s^i)}{V}. \tag{2}$$

Denote by $\Delta^i_{(V,\alpha)}$ the set of *feasible normalized voting strategies* for the juror i and $\Delta_{(V,\alpha)}$ the profiles of feasible normalized voting strategies.

This representation is convenient since the sum of d^i translates easily into the verdict. That is convict if $\sum_{\mathcal{N}} d^i(s^i) \geq 0$ and acquit otherwise, no matter the voting rule. However, the strategy space will depend on the voting rule.

I refer to the joint decision problem with a voting procedure as a *voting game*. Finally, denote by Ω_d the outcome function that is generated by a strategy profile d. An outcome function Ω is feasible in the voting game (V, α) if there is a feasible

[6] Notice that by formalizing the strategies in this way, I do not allow for abstention from voting. This assumption is standard in the models of Austen-Smith and Banks (1996), McLennan (1998) and Feddersen and Pesendorfer (1998), however others especially papers on elections allow for abstention. Assuming that both $v^i_A(s^i) = 0$ and $v^i_C(s^i) = 0$ is possible, I can easily formalize my model in a way that makes it possible for jurors to stay neutral. This modification would not change any of the result, however it would make it somewhat harder to present concise examples illustrating the advantages of introducing more votes into the voting procedure.

strategy profile that generates it.

Equilibrium Concept. I consider Bayes-Nash Equilibria of the voting game. I say that voting is efficient in a voting procedure if some equilibrium strategy profile in the actual voting game implements the efficient decision rule.

3 Committee Decisions with Finite Votes

I am interested in how well a voting procedure can aggregate the private information of the committee members. The papers by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) emphasize possible inefficiencies in the process. In this section, I argue that allowing more votes can improve the joint verdict for all voting rules except the unanimous rule. In all voting games there are potentially multiple equilibria. An important limitation of my analysis is that I state my results only with respect of one of those equilibria, namely the one with the highest welfare.[7] I neither investigate the effect of allowing more votes on equilibria with low welfare nor the question whether new equilibria with high ex-ante costs arise as the number of available votes increases. I cannot refer to any intuitive selection mechanisms that can support my treatment, however, given that the equilibrium I focus on is the one with the highest welfare a designer would want players to coordinate on that.

The first set of results shows that whenever the jurors have common interests, the equilibrium with the lowest ex-ante costs can be found by solving a constrained minimization problem. This result is convenient for two reasons. First, it simplifies the process of finding a particular equilibrium of the game. Second, it makes it easy to compare equilibria of games that only differ in the number of available votes. I argue that having more votes translates into a relaxed constraints of the optimization. This leads to the second set of results: allowing more votes for each juror raises expected welfare in equilibrium. I close the section with examples that

[7] The ordering of the equilibria according to the ex-ante costs is possible since the players have common interest.

illustrates this point.

3.1 Constrained Efficiency in Voting Games

Definition 1 (Constrained Efficiency)**.** *Given a voting procedure* (V, α)*, I refer to an outcome function* Ω *as* constrained efficient *if it is feasible with the voting procedure* (V, α) *and generates the lowest ex-ante costs among the feasible outcome functions in the voting game* (V, α)*.*

Next, note that there are finitely many signal profiles thus finitely many different outcome functions, which makes the feasible set finite as well given any voting game (V, α). Thus the infimum of the ex-ante costs over the set of feasible outcome functions is always attained and we have the following result.

Proposition 1. *A constrained efficient outcome function exists in any voting game* (V, α)*.*

The last result in this section shows that a voting profile that generates the constrained efficient outcome function is a Bayes-Nash Equilibrium in the voting game. It is easy to establish that if a deviation from the constrained efficient profile by juror i results in a lower interim cost for the juror then the original voting profile could not have been constrained efficient.

Theorem 1 (McLennan, 1998)**.** *If the voting profile* d_e *is such that*

$$d_e = \arg \min_{d \in \Delta_{(V,\alpha)}} \mathcal{C}(\Omega_d)$$

then d_e *is a Bayes-Nash Equilibrium in the voting game.*

Proof. Assume that all but juror i follow the prescribed strategy, d_e^{-i}. If there exists d^i and s^i such that $\mathcal{C}^i(\Omega_{(d_e^{-i}, d^i)}, s^i) < \mathcal{C}^i(\Omega_{d_e}, s^i)$ then $\mathcal{C}(\Omega_{(d_e^{-i}, d'^i)}) < \mathcal{C}(\Omega_{d_e})$. Contradiction. □

Corollary 1. *In any voting game* (V, α)*, an equilibrium exists.*

3.2 More Votes Are Better

In this section, I show that a better joint decision can be reached when more votes are available for the jurors. I have argued before that the constrained efficient equilibria are solutions of a constrained minimization problem. By allowing more votes, the constraint of the optimization problem is relaxed. Hence, the ex-ante cost in equilibrium cannot increase. To illustrate the point, I provide some examples after the formal results.

Proposition 2. *Consider a jury of size N and a voting game (V, α). Denote by d_V and d_{V+1} the constrained efficient equilibria of the voting games (V, α) and $(V+1, \alpha)$, respectively. Then $\mathcal{C}(\Omega_{d_V}) \geq \mathcal{C}(\Omega_{d_{V+1}})$.*

Proof. I argue that the feasible outcome set in the voting game $(V+1, \alpha)$ contains the one in the voting game (V, α). For any strategy profile in $\Delta_{(V,\alpha)}$ there is a strategy profile that is feasible in the voting game $(V+1, \alpha)$ and generates the same outcome. Consider a strategy profile $d_V \in \Delta_{(V,\alpha)}$ and define $d'_V \in \Delta_{(V+1,\alpha)}$ such that $d'^i_V(s^i) = d^i_V(s^i)\frac{V}{V+1}$. Then the set $\{s| \sum_{\mathcal{N}} d^i_V(s^i) \geq 0\}$ is identical to $\{s| \sum_{\mathcal{N}} d'^i_V(s^i) \geq 0\}$ since by definition $\sum_{\mathcal{N}} d'^i_V(s^i) = \frac{V}{V+1} \sum_{\mathcal{N}} d^i_V(s^i)$. Hence, the outcome functions implemented by d_V and d'_V are identical.

Therefore, the constrained efficient outcome function is at least as good in the voting game $(V+1, \alpha)$ as in the voting game (V, α). □

An additional vote may improve the decision if the voting rule is not too extreme. On the other hand, if the required majority is too strong, i.e. a single acquit vote of a single player can determine the verdict, then an additional vote is not useful. Whether or not this happens depends on the parameters of the model: the number of jurors involved in the decision process and the number of votes available for each juror. However, when unanimity is required, increasing the number of votes does not improve the equilibrium verdict. All voting games $(V, 1)$ (voting games $(V, 0)$) are equivalent in the sense that they generate the same equilibrium outcome.

Proposition 3. *If* Ω *is a feasible outcome function in the voting game* $(V, 1)$ *then* Ω *is a feasible outcome function in the voting game* $(1, 1)$*. Hence, in unanimous voting games, allowing more votes never strictly improves the verdict.*

Proof. Denote by d_V an equilibrium strategy profile in the voting game $(V, 1)$. For voting games such that unanimity is required for conviction, the set of normalized votes is $X_V^1 \subset [-1, 0]$. Define the following strategy profile d_1 in the voting game $(1, 1)$:

$$d_1^i(s^i) = \begin{cases} 0 & \text{if} \quad d_V^i(s^i) = 0 \\ -1 & \text{otherwise.} \end{cases}$$

The profile d_1 is valid in the voting game $(1, 1)$ since for every i it maps to $\{-1, 0\}$. Second, the profile d_1 induces the same outcome function as the profile d_V. In a voting game $(V, 1)$ for a signal realization s the verdict $\Omega_d(s) = C$ if and only if for all i, $d^i(s^i) = 0$. Therefore, for all $s \in S$, $\Omega_{d_1}(s) = \Omega_{d_V}(s)$. □

Notice that if the feasible votes are in $[-1, 0]$ then the only action supporting conviction is to assign all the available votes to convict. Therefore, even though multiple votes are available, there is no instrument to express signal strength in these procedures. This implies that the unanimity rule rarely allows for efficient information aggregation. The jurors are only able to express a binary partition of their signal space and that is sufficient only for very restricted information structures.

Finally, I give two illustrative examples. The first example shows that even with binary signals, a single vote can be insufficient for efficient voting if the voting rule is simple majority but the signals are not symmetric. In case of such biased signals, the information carried by one guilty signal does not cancel the information carried by one innocent signal. However, due to simple majority, casting the single available vote to acquit exactly balances a convict vote by an opponent. This implies that the intuitive strategy profile in which all the jurors vote according to their own signal is not an equilibrium strategy profile.[8] Thus, in the equilibrium,

[8]The other fully informative profile in which each juror vote against her signal cannot be an

which necessarily exists, a juror's action is non-responsive to her information with some probability. Therefore, the equilibrium cannot implement efficient voting.

As I discussed it in the introduction, there are two ways to address this problem. One solution is to adjust the voting rule so that the relative strength of the two kind of votes are aligned to the relative strength of the opposing signals. However, if one follows this route, then the adequate voting rule depends on the details across distributions. There is no rule that generally works well. A more robust solution for the problem is to allow more votes for the jurors. Having multiple votes to allocate for either of the two possible decisions enables the jurors to refine the effect of their actions on the final verdict. One can conclude that allowing multiple vote is a robust way to improve the quality of the joint decision while the optimal α would have to be tailored to the problem at hand.

The second example shows that for more than two signals, multiple votes are necessary to express different intensities of information carried by different signals.

Example 1. Consider a joint decision problem of $N = 5$ jurors, each with a preference parameter $q = 0.5$ ($\ell_q = 1$) and conditionally iid binary signals with the following distribution:

	$P^I(s^i)$	$P^G(s^i)$	ℓ_{s^i}
s_1	4/7	1/7	1/4
s_2	3/7	6/7	2

The efficient decision rule in this joint decision problem is:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \#\{i|s^i = s_2\} \geq 4 \\ A & \text{otherwise.} \end{cases}$$

In this example, the signals are initially biased in the sense that one innocent signal and one guilty signal do not balance out, i.e. in case of the same number of innocent and guilty signals the committee has a strict preference for acquittal. In other words, more than the simple majority of the signals have to be s_2 for a convict

equilibrium either.

verdict to be efficient. If the voting procedure is such that $V = 1$ and $\alpha = 1/2$ then there is *no equilibrium that implements the efficient decision rule.* First, a mixed strategy equilibrium cannot lead to the efficient outcome. For any signal profile, the efficient verdict is unique, either convict or acquit. However mixed strategies lead to a probabilistic outcome, that means inefficiency with positive probability. Second, the only strategy profile that may implement the efficient decision rule must be informative for all jurors and also, must assign a C vote, $(1/2)$ to s_2 and an A vote $(-1/2)$ to s_1. To see this consider the situation when the others received one s_1 and three s_2 overall. Then the jurors information is decisive, hence her vote must be decisive, it has to push the joint decision to be convict if her signal is s_2 and to be acquit if her signal is s_1. The only possible pure strategy of such is $(d^i(s_1), d^i(s_2)) = (-1/2, 1/2)$. However, this is not an equilibrium in this voting game since if all the others follow this strategy then the juror wants to vote acquit no matter. Her vote only counts if exactly two of the others received s_1 but then even if her signal is s_2, the efficient verdict is acquittal.

Notice that there is an equilibrium that implements the efficient decision rule if the voting procedure is such that $V = 1$ and $\alpha = 2/3$. The following strategies lead to the efficient outcome:[9]

$\alpha = 2/3$	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	1	0	$-2/3$
s_2	0	1	$1/3$

It is easy to check that the verdict is C exactly if at least 4 out of the 5 members received s_2.

Next, I demonstrate that allowing multiple votes for the jurors, can also improve on information aggregation. I show that with the voting procedure $(2, 1/2)$, the efficient decision rule in the above joint decision problem can be implemented as an equilibrium outcome. Consider the following voting profile:

[9] In this example I show the strategies with both the 'natural' votes and the normalize votes to help the understanding.

$\alpha = 1/2$	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	2	0	$-1/2$
s_2	0	1	$1/4$

If two jurors received s_1 and three jurors received s_2 then the sum of the votes are $(4, 3)$, for acquit and convict respectively, and the verdict is A according to simple majority. (The sum of the weighted votes is $-1/4 < 0$.) If one juror received s_1 and four jurors received s_2 then the sum of the votes are $(2, 4)$ so the verdict is C. (The sum of the vote difference is $1/2 > 0$.) Hence, the above voting profile implements the efficient decision rule and by the Theorem 1 it is an equilibrium in the voting game $(2, 1/2)$.

Example 2. Consider the joint decision problem of $N = 3$ jurors with preferences $q = 0.5$ ($\ell_q = 1$). Assume that the jurors have conditionally iid signals according to the following distribution:

	$P^I(s^i)$	$P^G(s^i)$	ℓ_{s^i}
s_1	5/12	1/12	1/5
s_2	4/12	2/12	1/2
s_3	2/12	4/12	2
s_4	1/12	5/12	5

If the voting procedure is such that $V = 1$ and $\alpha = 1/2$ then the efficient decision rule cannot be implemented. The argument in the previous example, such that mixed strategies cannot lead to the efficient decision rule, is equally valid here. Then notice that there are only two different actions to take: vote for convict $(1/2)$, vote for acquit $(-1/2)$. However, each juror can have four different signals, a strong and a weak signal supporting acquittal and a strong and a weak signal supporting conviction. Thus, in any pure strategy of a juror, at least two signals trigger the same action.

Next, I show that any two signals of a juror can be decisive in the sense that one of the signals with a possible realization of the others information suggests acquittal while the one signal with the same realization of the others suggests conviction. If the two signals of the juror support different alternatives then, whenever the two others received s_2 and s_3, the signal of the juror is decisive. If the two signals of the juror support the same alternative but with different strength then, whenever the others both have the opposite but weak signals, the signal of the juror is decisive

Nevertheless, if the juror's action is the same for the signals, the verdict is the same as well given any realization of the other. Therefore the resulting outcome cannot be efficient. To implement the ex-post decision rule it is important to follow different actions for different signals, which is impossible if every juror only has one vote.

If at least 3 votes are available for the jurors, then there is an equilibrium voting profile which implements the efficient decision rule, see for example, the strategy below.

	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	3	0	$-1/2$
s_2	1	0	$-1/6$
s_3	0	1	$1/6$
s_4	0	3	$1/2$

4 Committee Decisions with Continuous Votes

In the previous section, I argued that allowing the committee members to cast multiple votes enables them to communicate their information in a more accurate way. As the number of the votes grows, an obstacle is removed from the way of information aggregation. It is reasonable to ask if there are limits to this improvement. Can the jurors always conclude the efficient verdict if there are sufficiently many votes available? It turns out that the answer to this question is sensitive to the underlying signal structure.[10]

Focusing on voting mechanisms to reach a joint verdict restricts the feasible outcome functions in the joint decision problem. Simultaneous voting does not

[10]When answering the above question, I focus on the constrained efficient equilibria in the voting games, similarly to the treatment in Section 3.

allow the jurors' strategies to depend on each other's signals. As a consequence, it is possible that the ex-post efficient outcome rule cannot be implemented by a voting game, i.e. voting is not efficient. Consider, for example, a situation with two jurors and binary signals, such that matching signals suggest one decision while opposite signals suggest the other decision. In this case it is, indeed, impossible to always reach the efficient verdict by simultaneous voting such that the individual's vote only depends on the juror's own signal. In this situation not even increasing the number of available votes help. Example 3 formalizes this argument.

First, I define the limit game where each committee member can cast a continuous rather than a discrete number of votes. Proposition 4 shows that this game is not just the limit of a sequence of discrete voting games with growing number of votes but that for large enough V every decision function that is implementable in the limit voting game is also implementable in a discrete game with V or more votes. Then, I show that whenever the private signals of the jurors are conditionally independent, then one can construct an equilibrium voting profile in the limit game that leads to the efficient verdict for any signal realizations. Hence, having conditionally independent private information is sufficient for efficient voting. Using the intuition in the previous paragraph, I provide examples of correlated signals such that the private information cannot be perfectly aggregated. Then, I give a necessary and sufficient condition on the joint signal distribution for the voting to be efficient.

Definition 2 (Voting Game (∞, α))**.** *There are N jurors to make a joint decision. The private information and the preferences of the jurors are as characterized earlier. Define the voting procedure (∞, α) in the following way.*

- *A pure* strategy *for a juror i is a function mapping from the signal space to the interval $[-\alpha, 1-\alpha]$, formally $d^i : S^i \to [-\alpha, 1-\alpha]$.*
- *The outcome $\Omega_d(s)$ is convict if $\sum_{\mathcal{N}} d^i(s^i) \geq 0$ and it is acquit otherwise.*

I intend refer to the voting procedure (∞, α) as the limit of the procedures with finite votes as the number of the votes increases. Next, I show that if the number of the available votes is high enough then a decision rule is feasible in the limit game if and only if it is feasible in the finite games. This fact conveniently implies that (i) a constrained efficient decision rule exists in the limit game since it exists in the finite games by Proposition 1 and (ii) there is a $\overline{V}$ such that for all $V > \bar{V}$ the constrained efficient decision rule is the same as the one in the limit game. Therefore the limit game is informative about voting games with high but finite number of votes.

Proposition 4 (Properties of the Limit Game)**.** *For a given voting rule α there is a finite number $\overline{V}$ such that for all $V > \overline{V}$ the set of feasible decision rules in the voting game (V, α) is identical to the set of feasible decision rules in the voting game (∞, α).*

The difficulty lays in showing that every decision rule that is feasible in the limit game is feasible in the finite game. Notice, that an action that is available in the limit may be impossible in the finite game. However, if V high enough then for any voting strategy in the infinite game, one can define voting strategies in the finite game that are sufficiently close. Thus, for any signal profile the sums of the individual votes are close to each other, especially the sign of the sums are the same. Therefore the two voting profiles implements the same outcome function.

Proof. Since, the action set in the continuous game contains the action set of any finite game $X_V^\alpha \subset X_\infty^\alpha$, the set of the feasible decision rules in the limit game clearly contains the set of the feasible decision rules in any voting game (V, α). Next, fix a decision rule Ω that is implementable with the voting procedure (∞, α) and denote by d the voting profile that implements it. Note that if V votes are available for the jurors and the voting rule is α then the distance between two consecutive elements, x and x' of the action set, X_V^α is exactly $\frac{\alpha}{V}$ if $x, x' < 0$ and is exactly $\frac{\alpha}{V}$ if $x, x' > 0$. In any case there is a feasible action in any interval $\mathcal{I} \subset [-\alpha, 1-\alpha]$ of length $\frac{1}{V}$.

Next, due to the finite signal space, I can find the smallest margin by which acquittal is chosen with d, denote this margin by

$$m_\Omega \equiv \min \left\{ |\sum_{\mathcal{N}} d^i(s^i)| | s \in \Omega^{-1}(A) \right\}.$$

Fix a V such that $V > \frac{N}{m_\Omega}$ and define

$$d_V^i(s^i) = \min \left\{ \delta | \delta \in X_V^\alpha, \delta \geq d^i(s^i) \right\}.$$

For every i, $d^i(s^i) + \frac{m_\Omega}{N} \geq d_V^i(s^i) \geq d^i(s^i)$. Therefore, $\sum_{\mathcal{N}} d^i(s^i) + m_\Omega \geq \sum_{\mathcal{N}} d_V^i(s^i) \geq \sum_{\mathcal{N}} d^i(s^i)$ which implies that $\sum_{\mathcal{N}} d_V^i(s^i) \leq 0 \Longleftrightarrow \sum_{\mathcal{N}} d^i(s^i) \leq 0$. □

The next result shows that with continuous votes the voting rule does not influence the efficiency of the voting mechanism.

Proposition 5 (Neutrality of the Voting Rule)**.** *If a decision rule Ω is feasible in a voting procedure (∞, α') then it is feasible with a voting procedure (∞, α''), where $\alpha', \alpha'' \in (0,1)$.*

Proof. Assume that a decision rule Ω is implemented in the voting game (∞, α') by the voting profile d'. If $\alpha' > \alpha''$, then $\frac{\alpha''}{\alpha'} d'$ is a feasible strategy profile in the voting game (∞, α'') and implements Ω while if $\alpha' < \alpha''$ then $\frac{1-\alpha''}{1-\alpha'} d'$ is a feasible strategy profile in the voting game (∞, α'') and implements Ω. □

4.1 Conditionally Independent Signals - A Sufficient Condition for Efficient Voting

Next, I argue that the efficiency of a voting mechanism, as a method of reaching a verdict, hinges critically on the structure of the information possessed by the jurors.

If the jurors have conditionally independent information then it is possible to construct a voting profile that is feasible in the limit game and that generates the efficient verdict in the joint decision problem. Hence it is an equilibrium in the limit game. By Proposition 4 voting games with sufficiently many votes then have efficient equilibria as well.

Whenever the private information is conditionally independent across the committee members, it is possible to disentangle the information content of any individual signal from the rest of the signals. Formally, the log-likelihood ratio of any signal profile is the sum of the log-likelihood ratios of the individual signals. Hence, one can construct an efficient strategy such that the vote for any signal is a linear function of the log-likelihood ratio of the signal. The following theorem formalizes this argument.[11]

Proposition 6 (Efficient Voting Profile)**.** *Consider a joint decision problem such that the jurors have conditionally independent private information. In the voting game* (∞, α) *the efficient decision rule is implemented by the voting profile* d_o *such that a juror* i *with signal* s^i *casts*

$$d_o^i(s^i) = a\left(\log \ell_{s^i} - b^i\right) \tag{3}$$

where $\sum_i b^i = \log \ell_q$ *and* $a < \frac{\min\{\alpha, 1-\alpha\}}{\max_{i,s^i} |\log \ell_{s^i} - b^i|}$.[12] *Hence, the voting profile* d_o *is an equilibrium.*

If the committee members are more concerned with, say, convicting an innocent defendant than they are with acquitting a guilty defendant, i.e. $\log \ell_q > 0$, then the equilibrium votes should reflect this by concluding acquit whenever the ex-post probability of guilt and innocence are equal. The terms b^i take care of this. Second, a feasible strategy requires that $d^i(s^i) \in [-\alpha, 1-\alpha]$. The constant a rescales the value of $(\log \ell_{s^i} - b^i)$ making sure that it falls into this range.

Proof. By Theorem 1, if the outcome function implemented by d_o is efficient then it is an equilibrium in the voting game. An outcome function is efficient if the verdict is acquittal whenever the probability of innocence is high enough. Recall equation

[11] In the independent work of Chakraborty and Ghosh (2003) Theorem 4 demonstrates an equivalent result.

[12] Given that the signal space is finite and there is no perfectly informative signal, the bound on the likelihood ratio exists.

(1) that characterizes the efficient decision rule. With conditionally independent signals it simplifies to:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \sum_{\mathcal{N}} \log \ell_{s^i} \geq \log \ell_q. \\ A & \text{otherwise.} \end{cases}$$

Without loss of generality, pick a signal realization, s such that $\sum_{\mathcal{N}} \log \ell_{s^i} \geq \log \ell_q$, hence the efficient verdict for s is conviction. Then according to the strategy in (3) the sum of the votes is $\sum_{\mathcal{N}} d_o^i(s^i) = \kappa \sum_{\mathcal{N}} (\log \ell_{s^i} - a^i) = \kappa(\sum_{\mathcal{N}} (\log \ell_{s^i}) - \log q) \geq 0$. Hence, convict is concluded. For signal realization such that the efficient verdict is acquittal the proof is analogous. □

The construction of the efficient voting profile in Proposition 6 suggests that the additivity of the log-likelihood ratio is important for information aggregation by a voting procedure. Conditionally independent signals imply this property.

However, conditional independence is not necessary for additivity of the log-likelihood ratios. It is possible to tweak conditionally independent distributions in a way so that the log-likelihood ratio remains additive. For example, assume that $P(s, \theta)$ is a conditionally independent joint signal distribution and define $\tilde{P}(s, \theta) \equiv \prod_i P(s^i, \theta)\mu(s)$ where μ is not constant and picked appropriately so that $\tilde{P}$ is a probability distribution. The distribution $\tilde{P}$ implies the same likelihood ratios as the distribution P. Thus, signals according to $\tilde{P}$ also allow efficient information aggregation, although $\tilde{P}$ is not conditionally independent.

Below, I show that even additive log-likelihood ratios are not necessary for efficient voting when the signal space is finite.

4.2 Ordered Signals - A Necessary Condition for Efficient Voting

In the case of conditionally independent signals and the efficient voting strategies defined in (3), the sum of the individual votes is a strictly increasing function of the likelihood ratio of the realized signal profile. It is important to realize that this is not necessary for efficient voting. To reach an efficient outcome it is enough if the sign of sum of the individual votes is positive if the efficient verdict is conviction and is negative if acquittal is the efficient verdict.

Before coming to a necessary and sufficient condition for efficient voting in the next subsection, I first introduce a necessary condition. Namely, for every committee members there needs to be an unambiguous relation between every two signals of her, in the sense that one of the signals always makes conviction more favorable than the other.

Example 3. Consider a decision problem of a two-member committee. Prior to the voting, each member can observe the realization of a binary signal. The values s_j refer to the signal of the first and t_j to the signal of the second juror. The table below represents the efficient decision rule given each of the four possible signal profiles. A '$+$' indicates that the efficient verdict is conviction while a '$-$' refers to realizations such that the efficient verdict is acquittal.

ℓ	t_1	t_2
s_2	-	$+$
s_1	$+$	-

For example, the following conditional distributions with the preference parameter $q = 0.5$ generate this decision rule.

P^G	t_1	t_2
s_2	3/14	2/14
s_1	6/14	3/14

P^I	t_1	t_2
s_2	4/14	1/14
s_1	5/14	4/14

ℓ	t_1	t_2
s_2	3/4	2
s_1	6/5	3/4

Next, I show that there is no voting profile that leads to the efficient decision in this case. Assume, to the contrary, that there exist appropriate voting strategies d^1 and d^2. Then for (s_1, t_1) and (s_2, t_2) the sum of the votes must be positive while for (s_2, t_1) and (t_1, s_2) it must be negative. Therefore the following must be true:

$$\begin{aligned} d^1(s_1) + d^2(t_1) &> d^1(s_2) + d^2(t_1) \\ d^1(s_2) + d^2(t_2) &> d^1(s_1) + d^2(t_2). \end{aligned} \tag{4}$$

However, there are no numbers $d^1(s_2), d^2(t_2), d^1(s_1)$ and $d^2(t_2)$ that satisfy the above system. Adding up the two strict inequalities leads to a strict inequality with the same expression on both sides, which is a contradiction.

In this example, s_1 is more favorable for conviction than s_2 if the opponent has t_1 and less if the opponent has t_2. This feature makes it impossible to find good voting strategies. The signal s_j must be more or less favorable for conviction than an $s_{j'}$ independently of the opponents' realization.

Does excluding the above pattern always allow for efficient information aggregation? The answer is positive for two-member panels, no matter the number of possible signal values. The following definition generalize the notion that efficient voting fails in the above examples because there is no order on S^i such that Ω^e is monotone in s^i.

Definition 3. *For any subset of the committee members, $I \subset N$, denote the signal space by S^I, which is the product of the signal spaces S^i such that $i \in I$. Define a binary relation on S^I by $s^I \succ_I s'^I$ such that $s^I, s'^I \in S^I$ if there exists $t^{-I} \in S^{-I}$ such that $\Omega^e(s^I, t^{-I}) = C$ and $\Omega^e(s'^I, t^{-I}) = A$.*

The idea of this definition is that s^I is better news for conviction that s'^I whenever $s^I \succ_I s'^I$. It follows immediately from this definition that if the voting profile d is to implement the efficient decision rule then it needs to be that $\sum_{i \in I} d^i(s^i) > \sum_{i \in I} d^i(s'^i)$. This points straight to the next definition and the statement afterward.

Definition 4 (No flip-flop)**.** *The signal distribution satisfies* no flip-flop *if $\succ_I$ is a non-reflexive binary relation on S^I, i.e. $s^I \succ_I s'^I$ implies that $s'^I \succ_I s^I$ is not true.*

Lemma 1. *No flip-flop is necessary for efficient voting, and it is necessary and sufficient for efficient voting for a two-member committee.*

The proof of the second statement is constructive. I argue that for two jurors, no flip-flop allows for an intuitive order on the signals and based on this order one can construct an efficient voting profile.

Proof. PART 1: Whenever the binary relation is irreflexive, i.e. $s^I \succ_I s'^I$ as well as $s'^I \succ_I s^I$, efficient voting requires that $\sum_{i\in I} d^i(s^i) > \sum_{i\in I} d^i(s'^i)$ as well as $\sum_{i\in I} d^i(s'^i) > \sum_{i\in I} d^i(s^i)$, which is a contradiction.

PART 2: I denote by s_j the signals of juror 1 and by t_k the signals of juror 2. For any s_j, I define $T(s_j) = \{t_k|\Omega^e(s_j, t_k) = C\}$. The no flip-flop condition implies that for any s_j and $s_{j'}$ either $T(s_j) \subseteq T(s_{j'})$ or $T(s_{j'}) \subseteq T(s_j)$. Hence, it is possible to order the signals of the first juror such that $s_j \geq s_{j'}$ if $T(s_{j'}) \subseteq T(s_j)$. A similar property is true for the signals of the second juror and hence, there is an order on S^2 as well.

Rename the signals such that the indices now refer to the order in the above defined sense, i.e. such that $T(s_1) \subseteq T(s_2) \cdots \subseteq T(s_J) \subseteq S^2$, where J is the number of the signals in S^1. Find values $\phi^1(s_j)$ such that $\phi^1(s_j) < \phi^1(s_{j+1})$ for all $j \in \{1, 2, \ldots J-1\}$ and $\phi^1(s_1) < 0$ while $\phi^1(s_J) > 0$.

For a $t_k \in T(s_1)$ set $\phi^2(t_k) > -\phi^1(s_1)$. The set $T(s_1)$ includes all the t_k signals of the juror 2 such that the efficient verdict in case of (s_1, t_k) is conviction, and with this constriction the vote does conclude convict since, $\phi^1(s_1) + \phi^2(t_k) > 0$.

For all $j \in \{2, 3 \ldots J\}$, if $t_k \in T(s_{j+1}) \backslash T(s_j)$, set $\phi^2(t_k) \in (-\phi^1(s_{j+1}), -\phi^1(s_j))$. Note that a $t_k \in T(s_{j+1}) \setminus T(s_j)$ requires conviction if the juror 1 receives s_{j+1} but acquittal if the juror 1 receives s_j, and hence this construction ensures that $\phi^1(s^j) + \phi^2(t_k) < 0$ while $\phi^1(s^{j+1}) + \phi^2(t_k) > 0$.

For a $t_k \in S^2 \backslash T(s_J)$, let $\phi^2(t_k) < -\phi^1(s_J)$. Note that a $t_k \in S^2 \backslash T(s_J)$, requires acquittal for s_J which happens since $\phi^2(t_k) + \phi^1(s_J) < 0$.

Thus, we assigned ϕ^2 for every elements of S^2. Finally, depending on the voting rule α in the actual voting game, one can find $a > 0$ to make sure that $d^i \equiv a\phi^i$ is a valid voting strategy, i.e. it maps into $[-\alpha, 1-\alpha]$. □

I have argued that no flip-flop is the necessary and sufficient condition of efficient voting in two members committee. The next section shows that for more than two committee members, further restrictions are needed to ensure efficient voting. But before I discuss the relation of two orders on the individual signal space, the one implied by the efficient decision and the one implied by the likelihood ratio function.

Remark. It is tempting to think that the "no flip-flop" condition implies some sort of monotonicity of the likelihood ratio in the signals. This is not the case. Remember the earlier discussion that full information aggregation is sufficient but not necessary for efficient voting. The following example illustrates this:

P^G	t_1	t_2
s_1	3/7	2/7
s_2	1/7	1/7

P^I	t_1	t_2
s_1	1/7	1/7
s_2	3/7	2/7

ℓ	t_1	t_2
s_1	3	2
s_2	1/3	1/2

There is no order on the signals such that the likelihood ratio function is monotone and still efficient voting is possible for any value of q as for any value of q there is an order on the signals with respect to which the efficient decision rule Ω^e is monotone. The following tables represent the efficient decision for $\ell_q < 1/2$, $\ell_q \in [1/2, 2]$ and $\ell_q > 2$, respectively.

P^G	t_1	t_2
s_1	+	-
s_2	-	-

P^I	t_1	t_2
s_1	+	+
s_2	-	-

ℓ	t_1	t_2
s_1	+	+
s_2	-	+

Hence, the order on the signal space of the second juror, S^2 and hence the efficient vote will depend on q, i.e. if $\ell_q < 1/2$ then $t_2 \succ_1 t_1$ and if $\ell_q > 2$ then $t_1 \succ_1 t_2$ while for any other preference parameter either of the orders work.

4.3 A Necessary and Sufficient Condition for Efficient Voting

I start the section with an example that does not violate the flip-flop condition, however, does not allow for efficient voting. I discuss the property that blocks efficient voting in this example and suggest a sequence of conditions that are necessary for the existence of efficient voting profile. Then, I link the problem of efficient voting to a classic problem in utility theory. Finally, I present a necessary and sufficient condition for efficient voting. My formal argument relies on the work of Krantz, Luce, Suppes, and Tversky (1971).

Example 4. Consider a decision problem of a three-member committee. Prior to the voting each member can observe the realization of a private signal that has three possible values. The signals s_j, t_j and z_j refer to the signals of the juror 1, 2 and 3, respectively. The table below represents the efficient decision rule given all the possible signal profiles. Again, a '$+$' indicates that the efficient verdict is conviction while a '$-$' refers to the realizations such that the efficient verdict is acquittal.

z_1

	t_1	t_2	t_3
s_3	-	-	+
s_2	-	-	+
s_1	-	-	-

z_2

	t_1	t_2	t_3
s_3	+	+	+
s_2	-	-	+
s_1	-	-	-

z_3

	t_1	t_2	t_3
s_3	+	+	+
s_2	-	+	+
s_1	-	+	+

To see that this is a valid example, consider the conditional distributions below that generate this decision rule with the preference parameter $q = 0.5$.

z_1

P^G	t_1	t_2	t_3
s_3	$\frac{12}{376}$	$\frac{14}{376}$	$\frac{18}{376}$
s_2	$\frac{4}{376}$	$\frac{10}{376}$	$\frac{16}{376}$
s_1	$\frac{2}{376}$	$\frac{6}{376}$	$\frac{8}{376}$

z_2

P^G	t_1	t_2	t_3
s_3	$\frac{16}{376}$	$\frac{18}{376}$	$\frac{22}{376}$
s_2	$\frac{15}{376}$	$\frac{14}{376}$	$\frac{20}{376}$
s_1	$\frac{16}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_3

P^G	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_1

P^I	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_2

P^I	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{20}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{22}{376}$	$\frac{18}{376}$	$\frac{16}{376}$

z_3

P^I	t_1	t_2	t_3
s_3	$\frac{8}{376}$	$\frac{6}{376}$	$\frac{2}{376}$
s_2	$\frac{16}{376}$	$\frac{10}{376}$	$\frac{4}{376}$
s_1	$\frac{18}{376}$	$\frac{14}{376}$	$\frac{12}{376}$

z_1

ℓ	t_1	t_2	t_3
s_3	$\frac{12}{15}$	$\frac{14}{15}$	$\frac{18}{15}$
s_2	$\frac{4}{15}$	$\frac{10}{15}$	$\frac{16}{15}$
s_1	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{8}{15}$

z_2

ℓ	t_1	t_2	t_3
s_3	$\frac{16}{15}$	$\frac{18}{15}$	$\frac{22}{15}$
s_2	$\frac{15}{20}$	$\frac{14}{15}$	$\frac{20}{15}$
s_1	$\frac{16}{22}$	$\frac{15}{18}$	$\frac{15}{16}$

z_3

ℓ	t_1	t_2	t_3
s_3	$\frac{15}{8}$	$\frac{15}{6}$	$\frac{15}{2}$
s_2	$\frac{15}{16}$	$\frac{15}{10}$	$\frac{15}{4}$
s_1	$\frac{15}{18}$	$\frac{15}{14}$	$\frac{15}{12}$

One can check that there is no flip-flop in this example. However, efficient voting is still impossible. Assume, on the contrary, that there exist good voting strategies: d^1, d^2 and d^3. Then the sum of the votes for the signal profiles (s_2, t_3, z_1),(s_3, t_1, z_2) and (s_1, t_2, z_3) has to be positive while for (s_3, t_2, z_1),(s_1, t_3, z_2) and (s_2, t_1, z_3) it has to be negative. Therefore the following system of inequalities must have a solution.

$$\begin{aligned} d^1(s_2) + d^2(t_3) + d^3(z_1) &> d^1(s_3) + d^2(t_2) + d^3(z_1) \\ d^1(s_3) + d^2(t_1) + d^3(z_2) &> d^1(s_1) + d^2(t_3) + d^3(z_2) \\ d^1(s_1) + d^2(t_2) + d^3(z_3) &> d^1(s_2) + d^2(t_1) + d^3(z_3). \end{aligned} \tag{5}$$

However, one can see that there are no numbers $d^1(s_j), d^2(t_j)$ and $d^3(z_j)$ that satisfy the system. Adding up the three lines again, leads to a strict inequality with the same expression on both sides.

What goes wrong here? One can directly compare two signal sub-profiles if there is a profile of all the other jurors, such that the two signal sub-profiles are decisive. No flip-flopping occurs whenever directly comparison is not possible or if it is possible and the order is consistent. However, there are implicit ways of comparing signals.

Consider two pairs of signals (s_L, t_H) and (s_H, t_L) so that $s_H \succ_i s_L$ and $t_H \succ_j t_L$ and $(s_L, t_H) \succ_{i,j} (s_H, t_L)$. Then one can conclude that t_H signal of juror j relatively to t_L is stronger than the signal s_H is relatively to s_L, when comparing (s_H, t_L) to (s_L, t_H), juror i's information becomes more favorable for acquittal while juror j's information becomes more favorable for conviction. When these two effects are aggregated the one for conviction dominates. Now, consider an additional signal for both jurors, s_M and t_M such that $s_H \succ_i s_M$ and $s_M \succ_i s_L$ and also $t_H \succ_j t_M$ and $t_M \succ_i t_L$. Then, there is an implicit way toevaluate the relative strength of the above changes of the jurors information. First, one may compare (s_H, t_M) to (s_M, t_H) and then (s_M, t_L) to (s_L, t_M). In the example,

- If the third juror has the realization z_1, then the first inequality shows that having t_3 instead of t_2 is stronger news for conviction then having s_2 instead of s_3 is for acquittal.

- If the third juror has the signal realization z_2, then the second inequality shows that having t_3 instead of t_1 is weaker news for conviction then having s_1 instead of s_3 is for acquittal.

- And finally, with z_3, having t_2 instead of t_1 is stronger news for conviction then having s_1 instead of s_2 is for acquittal.

However, a problem occurs since the change from t_1 to t_2 dominates the change from s_2 to s_1 and the change from t_2 to t_3 dominates the change from s_3 to s_2, however the change from t_1 to t_3 is dominated by the change from s_3 to s_1.

If efficient voting exists, then the sum of votes for all signals profiles such that the efficient decision is convict is bigger than the sum of votes for all to the signal profiles such the efficient decision is acquit. By this requirement, any private information structure induces a system of inequalities that has to be solvable. The no flip-flop condition ensures that any two-element subset of the inequality system is consistent in the sense that it has solution. However, this condition does not guarantee a solution for the entire system. Example 4 presents an information structure such that, although, any two inequalities are solvable, there is system of three inequalities that is inconsistent. Hence, the whole system has no solution.

As the number of jurors and the possible signal values increases, the system becomes more and more difficult. Fortunately, there is an alternative way to represent the problem and an easily understandable condition is available which is equivalent to the set of inequality conditions.

There is a widely discussed question in utility theory, namely what are the properties of a preference relation that allows for an additive separable utility representation. This problem mathematically is very similar to the question whether there are voting profiles that represents the information content of the signals well, i.e. the sum of the votes are higher whenever the signal profile is better news for conviction.

However, an earlier remark suggested that efficient information aggregation is not necessary for efficient voting. The jurors have to get a binary decision right, so as long as the sum of the votes are positive whenever the efficient verdict is convict the voting is efficient.

Hence, I define a binary relation on the signal profiles that is implied by the efficient decision rule: A signal profile is 'bigger' then another whenever the efficient decision is convict for the first and acquit for the second profile. For $s, s' \in S$

$$s \succ_N s' \Longleftrightarrow \Omega^e(s) = C \text{ and } \Omega^e(s') = A. \tag{6}$$

Notice that this relation is not complete.

Then, I ask what characteristics of this binary relation ensure that there exist voting functions d^i that represent the binary relation in the sense that the sum of the votes is positive if and only if the efficient decision is convict, or formally, so that there exist functions $d^i : S^i \to \mathbb{R}$ such that:

$$\Omega^e(s) = C \Longleftrightarrow \sum_{\mathcal{N}} d^i(s^i) \geq 0. \tag{7}$$

Definition 5. *A function* $\phi : S \to \mathbb{R}$ is an additive separable representation of a binary relation *if there exist functions* $\phi^i : S^i \to \mathbb{R}$ *such that*

$$s \succ s' \Longleftrightarrow \phi(s) = \sum_{\mathcal{N}} \phi^i(s^i) > \sum_{\mathcal{N}} \phi^i(s'^i) = \phi(s').$$

The first result shows that the existence of the voting profile that is characterized by Equation (7) is equivalent to the existence of an additive separable representation of a binary relation on the signal space.

Lemma 2. *An efficient voting profile in a voting game* (∞, α) *exists if and only if there is an additive separable representation of the binary relation defined by Equation (6).*

Proof. A voting strategy itself is an additive separable representation since $s \succ_N s' \iff \Omega^e(s) = C$ and $\Omega^e(s') = A \iff \sum_{\mathcal{N}} d^i(s^i) \geq 0 > \sum_{\mathcal{N}} d^i(s'^i)$.

If there exist functions $\{\phi^i\}_{i\in\mathcal{N}}$ representing a binary relation then the functions $\{d^i | d^i = a\phi^i + b^i, a > 0\}_{i\in\mathcal{N}}$ represent the binary relation as well. $s \succ s'$ if and only if $\sum_{\mathcal{N}} \phi^i(s^i) > \sum_{\mathcal{N}} \phi^i(s'^i)$. Notice that $\sum_{\mathcal{N}} \phi^i(s^i) > \sum_{\mathcal{N}} \phi^i(s'^i) \iff \sum_{\mathcal{N}} a\phi^i(s^i) > \sum_{\mathcal{N}} a\phi^i(s'^i) \iff \sum_{\mathcal{N}} a\phi^i(s^i) + \sum_{\mathcal{N}} b^i > \sum_{\mathcal{N}} a\phi^i(s'^i) + \sum_{\mathcal{N}} b^i \iff \sum_{\mathcal{N}} d^i(s^i) > \sum_{\mathcal{N}} d^i(s'^i)$.

Hence, one can transform the functions ϕ^i representing $\succ_N$ into valid, efficient voting strategies. There are two conditions to satisfy: (i) valid voting functions map into $[-\alpha, 1-\alpha]$ and (ii) $\sum_{\mathcal{N}} d^i(s^i) \geq 0$ if and only if $\Omega^e(s) = C$.

By construction, there exists $\overline{\phi}$ with the property that for all s such that $\Omega^e(s) = C$, $\phi(s) \geq \overline{\phi}$ and for all s' such that $\Omega^e(s) = A$, $\phi(s') < \overline{\phi}$. Then setting $b^i = \frac{\overline{\phi}}{N}$ gives that $\Omega^e(s) = C$ if and only if $\sum_{\mathcal{N}} d^i(s^i) > 0$. Finally, a small enough a can make sure that the voting functions d^i map into $[-\alpha, 1-\alpha]$. □

Examples 3 and 4 indicated that we may need to consider combinations of signal profiles comparisons and to sum votes across these combinations. Therefore, I start with introducing formal sums of signal profiles. Since the sum of the signal profiles has no meaning in the signal space, it is more convenient to think in terms of the following vector representation.

Define $K^i = |S^i|$ and $K = \sum_{\mathcal{N}} |S^i|$. Then every signal realization can be written in the form of a vector of zeros and ones of length K, and hence there is a set $X \subset \{0,1\}^K$ such that each elements of X represents and element of S and all elements of S is represented in X. Consider the order $\succ_N$ on S and denote the inherited order on X by $\succ_X$.

Given the vectors in X, I define the set $Y \subset \mathbb{Z}^K$ as the additive span of X. A vector y is element of Y if and only if it is the finite sum of elements of X, i.e.

$y = \sum_{m \leq M} x_m$ for any $x_m \in X$ and $M \in \mathbb{N}$. The relation $\succ_X$ can be extended to Y in the following way: $y = \sum x_m$ is greater than $y' = \sum x'_m$, i.e. $y \succ_Y y'$ if for all m, $x_m \succ_X x'_m$.

Next, I state the main theorem. The proof is adopted from Krantz, Luce, Suppes, and Tversky (1971) Theorem 9.1

Theorem 2. *Efficient voting is possible if and only if the binary relation $\succ_Y$ is irreflexive.*[13]

In the proof I show that the existence of a voting profile is equivalent to the existence of a solution of a system of linear inequalities (Step 1) and that the reflexivity of the relation $\succ_Y$ is equivalent to the existence of a solution of an other system of linear inequalities (Step 3). I refer to a duality theorem stated in Krantz, Luce, Suppes, and Tversky (1971) Theorem 2.7 to demonstrate that the two systems are dual pairs and hence exactly one of the system has a solution (Step 2).

Proof. By Lemma 2 the existence of an efficient voting profile is equivalent to the existence of an additive separable representation $\phi^i(s^i)$ of $\succ_N$.

STEP 1: The set $\{\Omega^{-1}(C), \Omega^{-1}(A)\}$ is a partition of S. Denote by $\{X^C, X^A\}$ the respective partition of X. Then for any $x^c \in X^C$ and $x^a \in X^A$, $x^c \succ_X x^a$. Denote by$K^A = |X^A|$ and by $K^C = |X^C|$.

Any functions $\phi^i(s^i)$ imply a K-dimensional vector δ such that the first K^1 entries of δ equal to the K^1 values of $\phi^1(s^1)$, then the next K^2 entries of δ equal to the K^2 values of $\phi^2(s^2)$, and so on. At the same time, any K-dimensional vector imply a family of ϕ^i functions.

Thus, $\phi^i(s^i)$ is a representation of $\succ_N$ if and only if there is a δ such that $x^c\delta > x^a\delta$ for all $x^c \in X^C$ and $x^a \in X^A$.

Define $K^C K^A$ vectors $r_n = x^c - x^a \in \{0,1\}^K$ with $x' \in X^C$ and $x \in X^A$. Then, collect all these vectors into an integer matrix R of size $(K^C K^A) \times K$ such that

[13] The theorem in Krantz, Luce, Suppes, and Tversky (1971) originally allows for indifference between elements of the set. Here, I do not discuss this case, although the proof easily goes through.

r_n is the nth row of the matrix. Then there is a representation of $\succ_N$ if and only if $R\delta \gg 0$.[14]

Thus, I have shown so far that an efficient voting profile exists if and only if there exists a vector $\delta \in \mathbb{R}^K$ such that $R\delta \gg 0$.

Step 2: The Theorem 2.7 in Krantz, Luce, Suppes, and Tversky (1971, pp. 62) states that the system $R\delta \gg 0$ has a solution if and only if the system $R^T\rho = 0$ such that $\rho > 0$ has no solution.[15]

Moreover, immediately after the theorem they argue that if the elements of the matrix R are rational numbers then the system $R\delta \gg 0$ has a rational solution if and only if the system $R^T\rho = 0$ such that $\rho > 0$ has no rational solution. Also, if ρ is a solution of the second system, than for any $a > 0$, $a\rho$ is a solution as well. Therefore, the theorem implies that if $R\delta \gg 0$ has no solution then the system $R^T\rho = 0$ such that $\rho > 0$ has an integer solution.

Step 3: Now, I show that the existence of a K^AK^C-dimensional integer vector, $\rho > 0$ with $R^T\rho = 0$ is equivalent to $\succ_Y$ being reflexive.

$R^T\rho = 0$ is equivalent to saying that $\sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}(x^c - x^a) = 0$, where x^c are the elements of X^C, x^a the elements of X^A and $\rho^{x^a,x^c} \in \mathbb{N}$ the K^AK^C dimensions of ρ. Recall, that ρ is weakly positive and integer vector, hence each entries of it is a natural number.

Thus, if there exists $\rho > 0$ with $R^T\rho = 0$ then

$$\sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}x^c = \sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}x^a.$$

Since any ρ^{x^a,x^c} is a non-negative integer there are $x^c_1, x^c_2, \ldots, x^c_M \in X^C$ and $x^a_1, x^a_2, \ldots, x^a_M \in X^A$, not necessarily distinct vectors, such that $\sum_M x^c_m = \sum_M x^a_m \in Y$ which is the definition of $\succ_Y$ being reflexive.

On the other hand, if $\succ_Y$ is reflexive then there are $x^c_1, x^c_2, \ldots, x^c_M \in X^C$ and $x^a_1, x^a_2, \ldots, x^a_M \in X^A$, not necessarily distinct vectors, such that $\sum_M x^c_m =$

[14] $v \gg 0$ denotes that every component of the vector v is strictly positive.
[15] $v > 0$ denotes that every component of v is weakly positive and that $v \neq 0$.

$\sum_M x_m^a \in Y$. Then a vector $\rho \in \mathbb{N}^{K^A K^C}$ can be generated in the following way. For all $n \in K^A K^C$, the nth entry $\rho_n \equiv \{\#m | r_n = x_m^c - x_m^a\}$. Then $R^T \rho = 0$.

To summarize, I have shown that an efficient voting profile exists $\iff$ there exists an additive representation $\phi^i(s^i)$ of $\succ_N \iff$ there exists a K-dimensional vector δ with $R\delta \gg 0 \iff$ there exists no $K^A K^C$-dimensional vector $\rho > 0$ such that $R^T \rho = 0 \iff$ the relation $\succ_Y$ is irreflexive. □

Notice that while the elements of the set X represent signal profiles, the elements of the set Y represents collections of signal profiles. Recall the information structure in Example 3, there, $(s_1, t_1) \succ_{1,2} (s_2, t_1)$ and $(s_2, t_2) \succ_{1,2} (s_1, t_2)$. Hence, the representations of the collection $\{(s_1, t_1), (s_2, t_2)\}$ and the collection $\{(s_1, t_2), (s_2, t_1)\}$ in Y are related according to $\succ_Y$. Moreover, it is easy to see that they are equivalent in Y, so $\succ_Y$ is irreflexive.

Similarly, in Example 4 $(s_2, t_3, z_1) \succ_{1,2,3} (s_3, t_2, z_1)$, $(s_3, t_1, z_2) \succ_{1,2,3} (s_1, t_3, z_2)$ and $(s_1, t_2, z_3) \succ_{1,2,3} (s_2, t_1, z_3)$. Therefore the collection $\{(s_2, t_3, z_1), (s_3, t_1, z_2), (s_1, t_2, z_3)\}$ is related to the collection of $\{(s_3, t_2, z_1), (s_1, t_3, z_2), (s_2, t_1, z_3)\}$. Moreover, they are equivalent in Y.

Hence, an element y in the set Y such that $y \succ_Y y$ suggest that there are multiple ways - explicit or implicit - of comparing two signal sub-profiles and the implied relationship between them is ambiguous. Hence, it is impossible to assign efficient votes to those sub-profiles.

5 Conclusion

In this paper, I have studied the joint decision problem of a committee of privately informed individuals. I argued that allowing multiple votes for the members, improves the quality of the joint decision made by the committee. I also showed that for conditionally independent signals, full efficiency can be reached if there are sufficient number of votes available. I discussed that with correlated private information full efficiency may not be possible for any number of votes. Moreover, I provided conditions to ensure that full efficiency exists. To summarize, allowing multiple votes makes a voting mechanism better and is a remedy for a certain type of inefficiencies in the joint decision problem. However, the efficient information aggregation with correlated private signal would require a different class of decision mechanisms. In both cases, individuals want to express their private information but due to some institutional constraint are unable to do so.

One can think about an additional obstacle to aggregate private knowledge in a committee setting. As it is shown by Li, Rosen, and Suen (2001) if individual members have conflicting interests, they may not want to communicate their private signals even if that would be possible.

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