

MONEY, INTERMEDIARIES, AND CASH-IN-ADVANCE CONSTRAINTS*

CHRISTIAN HELLWIG[†]

FEBRUARY 2003

FIRST DRAFT: MAY 1999

Abstract

I study a search economy in which intermediaries are the driving force coordinating the economy on the use of a unique, common medium of exchange for transactions. If search frictions delay trade, intermediaries offering immediate exchange opportunities can make arbitrage gains from a price spread, but they have to solve the search market's allocation problem. Intermediaries solve this problem best by imposing a common medium of exchange to other agents, and a Cash-in-Advance constraint arises in equilibrium: Agents trade twice in order to consume, once to exchange their production against the medium of exchange, and once to purchase their consumption. By studying the evolutionary stability of equilibria, I discuss which equilibria are likely to arise as long run outcomes. I extend my analysis to the study of fiat currencies and free banking systems.

JEL classification numbers: D51, E40

Keywords: Monetary Exchange Economy, Intermediation, Search, Cash-in-Advance Constraints

*This paper grew out of my dissertation submitted for the MSc. in Econometrics and Mathematical Economics at the LSE, 1999, and was previously presented to audiences at LSE, UCSD, MIT, UCLA, Chicago, Northwestern, Boston University, as well as the Young Economists' 2000 Conference and the Econometric Society's 2000 World Congress. I am grateful for many discussions with my supervisor, Nobu Kiyotaki, as well as helpful comments from George-Marios Angeletos, Martin Hellwig, Peter Howitt, Godfrey Keller, Thomas Mariotti, Michele Piccione and Rob Townsend. Financial support through LSE and the Economic and Social Research Council (UK) is gratefully acknowledged. All remaining errors are mine.

[†]Department of Economics, U.C.L.A., Email: chris@econ.ucla.edu

1 Introduction

What accounts for the use of money in economic transactions in competitive markets? This simple, seemingly trivial question has been the cause of much debate and a rich tradition of research in economics. The answer that is typically given today starts from Jevons' (1875) suggestion that the use of a medium of exchange eliminates the need for a "double coincidence of wants", if market participants trade bilaterally, and have to spend time and resources to find suitable trade partners. If all market participants instead agree on the use of a common medium of exchange, they will sell their production for the medium of exchange, and will use the medium of exchange to buy what they consume. In a large economy, this reduces the resources and time that consumers spend trading in the market. In the purest statement of this idea, Kiyotaki and Wright (1989) show that the use of a common medium of exchange may be the equilibrium outcome of individually rational transaction decisions in an economy where encounters between market participants are random.¹

This paper attempts to provide an alternative explanation for the transactions role of money in an economy that appears close to the frictionless, competitive benchmark. The observation of transactions in markets will suggest that we rarely have to search randomly to find the goods that we want to consume, or to find a buyer for the products that we want to offer; indeed, in most instances, we don't even face a delay in the transactions we carry out. For most products, we know where we can buy or sell them, and we just go and buy them whenever we want to, and we expect to find them at that time and place. In other words, the same frictions that account for the use of money do not enter into the theoretical model for which we are trying to provide a foundation, nor do they seem to be relevant for the use of money in most transactions. The challenge of providing microfoundations thus lies not so much in providing an explanation for the transactions role of money, but in providing this explanation *in the context of an exchange economy that is perceived as frictionless and competitive*. This implies going outside the Walrasian framework and leads to a broader underlying question: How do individuals interact in a decentralized market, so that the market outcome appears to be virtually without frictions; in other words, *how do markets evolve?*

A natural way of dealing with search frictions is the centralization of transactions through a system of intermediaries of known location and specialization. Of course, money and intermedi-

¹See Aiyagari and Wallace (1991) for a general discussion of this statement in the context of the model of Kiyotaki and Wright (1989).

aries are both essential features of transactions in markets. However, insofar as they are studied separately, they are implicitly regarded as substitutes in dealing with the frictions in the market. In other words, if intermediaries are capable of alleviating market frictions, what would be the role of money (or vice versa)? In order to fully account for the transactions role of money, one therefore has to ask how money interacts with intermediation in alleviating market frictions.

I discuss this interaction of money and intermediation in the context of a decentralized exchange economy where trade is bilateral and potentially subject to delay. Individuals can modify the trading environment by acting as intermediaries, thereby reducing the frictions to which other agents are subject. Intermediaries are immediately accessible, and delays in trade with them only depend on their ability to accommodate the transactions demanded. They centralize transactions more easily, if a common medium of exchange is used by the agents with whom they trade. On the other hand, they have the possibility to introduce it to all other agents, who, in turn, are willing to use it, if it allows them to buy from the intermediary whatever good they want to consume. The analysis thus points to a *complementarity* between the use of a medium of exchange and the centralization of transactions by intermediaries that roughly matches historical facts: Throughout history, intermediaries were often the ones who developed more efficient ways of exchanging goods, and they were particularly important in introducing and using money. On the other hand, they were also the primary beneficiaries of the introduction of a common medium of exchange.²

Intermediaries have been introduced into models with trade frictions in the past.³ These models usually focus on the exchange of a single good with a given number of buyers and sellers who trade off the delay in the transaction against the price at which they trade. Intermediaries act as arbitrageurs who offer immediate transactions, but charge a mark-up for their services. With many commodities,

²A particularly neat example of these effects, that also highlights the mechanisms in this paper, is Radford's (1945) description of exchanges in a Prisoner of War camp. He describes how economic institutions and markets developed within the completely unorganized environment of a PoW-camp, driven mainly by the scope for trade arising from differences in endowments (Red-Cross packages) and tastes. In the early days of the camp, some individuals who exploited the price margins between different parts of the camp ("intermediaries") promoted and established the cigarette as common money. This was fundamental for the later development of more sophisticated market institutions, such as a store, and even the introduction of a paper money, backed by the store's inventories of goods.

³Rubinstein and Wolinsky (1987) explore intermediation in a search-theoretic model in which one good is traded. The present analysis is closer in spirit to Gehrig (1993).

the success of intermediaries depends on their ability to match buyers and sellers *for each good*. This becomes a problem, if the number of goods that an intermediary can trade is restricted, and consumers may not always be willing to consume what a given intermediary would be willing to offer them for their production. This transfers the double coincidence problem from the search market to the intermediaries. They solve it by introducing and promoting a common medium of exchange that enables consumers to transfer purchasing power from transactions with one intermediary to transactions with another. That an intermediary cannot trade with all commodities at once is an important part of the argument: Otherwise, one intermediary would be able to perfectly eliminate the frictions, and there would be no need for a common medium of exchange. Consumers could simply trade their excess demand in all goods at once with an intermediary, at the prices set by the latter. If the intermediary fixes market-clearing prices, then no medium of exchange is needed to buy some goods from a different intermediary. Similarly, if the medium of exchange solved the allocations problem perfectly, there would be no role for the intermediary. It is precisely the fact that each of them on its own is unable to perfectly alleviate frictions that makes them complementary.

The emergence of intermediaries alters the way in which transaction decisions are made by other agents. Trade with intermediaries enables consumers and producers to direct their search towards particular transactions, as opposed to the random search in economies without intermediation. By limiting their clients' choices to the use of a unique, common medium of exchange, intermediaries introduce its use to the entire economy. In an equilibrium of the economy considered here, all agents trade twice to acquire what they want to consume: once to obtain the medium of exchange (sell their production), and once to buy their consumption good. Effectively, a *Cash-in-Advance constraint* for transactions with intermediaries is introduced, i.e. market participants have to use the common medium of exchange to be able to trade with intermediaries. Since the medium of exchange enables intermediaries to match buyers and sellers, the latter face no waiting time to perform the desired transaction. As a result, the search market empties, since producers and consumers take advantage of the intermediaries' services. The constraint is observed in all intermediated exchange, but is not binding for exchange outside intermediation. Formally, I do not assume away the possibility that two agents exchange "goods" for "goods" outside intermediated transactions, but in equilibrium, they never incur a situation in which they agree to exchange "goods" for "goods". Equilibrium allocations bear the characteristics of Walrasian allocations, and the resulting transaction patterns

resemble trade in frictionless Walrasian markets: At any time, almost all agents are able to carry out their desired transactions immediately, at the prices posted by the intermediaries.

Intermediation also provides a mechanism by which the economy can coordinate on the common use of an *efficient* medium of exchange. If a small set of agents coordinates their activities and offers some new organization of transactions, they may induce other agents, and eventually the entire economy, to adopt their innovation. This can be assimilated to the historical role of intermediaries in developing more efficient means of exchange. Formally, I study which of the resulting equilibria are evolutionarily stable. In contrast to the standard search model, evolutionary stability implies Pareto efficiency in an environment with intermediaries.⁴ I also allow for the circulation of fiat money. Under a general set of conditions, *the unique evolutionarily stable steady-state is then a Cash-in-Advance equilibrium in which fiat money circulates as the common medium of exchange.*

Finally, I study how fiat money may come into circulation, and again illustrate the coordinating role of the intermediaries: assuming that these intermediaries can write out demandable debt certificates ("notes"), I discuss under what conditions they become perfect substitutes in a "free banking equilibrium". I illustrate how the clearing mechanism serves to monitor the note issue of banks. In practice, a free banking regime has to rely on (i) the clearing mechanism to monitor the competitive issue of notes, and (ii) reliable punishment mechanisms in case of default. It is important to note that with free entry into note issue, i.e. in a truly competitive environment, the loss of the banking licence is insufficient to prevent overissue and strategic defaults, since no rents are directly associated with being an intermediary. Historically, it seems that the most successful free banking regimes were the ones that effectively used the note clearing, and used harsh punishments in case of default. But free banking systems also faced difficulties, even when those conditions were met: the model illustrates a coordination problem arising in the clearing market, i.e. if notes are entirely safe, and costs are associated with clearing, banks may prefer to hold notes in reserve, or bring them back into circulation rather than return them to the issuer.

⁴In large population matching games, such as the search model of money, evolutionary stability considerations have little effect on equilibrium selection, since the "mutants" have no possibility to interact with each other to explicitly coordinate their actions. Intermediation provides such a channel.

2 Related Literature

The results in this paper have various implications for existing equilibrium models of monetary exchange. Money is, of course, one of the essential elements of our understanding of macroeconomic fluctuations. Since the competitive Arrow-Debreu framework does not endogenously account for such a transactions demand for money, its existence is usually assumed into the model by way of a restriction on the transactions in which individuals engage: money must be used to buy consumption goods.⁵ Money then becomes a short-term store of value, and the demand for real balances will depend on the availability of other assets for short- or longer term savings, and on their liquidity when they are to be sold to satisfy consumption needs. While such a restriction has proven successful for the purposes of macroeconomic analysis, the same observations of market transactions that make the constraint empirically appealing also suggest that the main purpose of money is *not* its use as a store of value, but as a convenient medium for transactions, and money is used as a short-term store of value *only because it has a primary purpose as a medium of exchange*. Precisely such a Cash-in-Advance constraint is the result of equilibrium trading strategies in the present model, where market interaction is viewed as an ongoing evolutionary process,⁶ and this paper may therefore be viewed as providing a microfoundation for macroeconomic applications that exogenously impose such a constraint. The microeconomic efficiency of the constraint is in stark contrast with its macroeconomic counterpart - in fact, viewing such a constraint as efficiency enhancing seems contradictory. Efficiency follows from the strategic interaction of intermediaries, as the consequence of an evolutionarily stable steady-state in a deterministic environment.

⁵Examples where such a Cash-in-Advance constraint is made explicit are Svensson (1987) and Lucas and Stokey (1987). The constraint that money is used to buy goods also appears in Romer's (1986) general equilibrium treatment of Baumol's (1952) and Tobin's (1956) inventory demand for Cash. An alternative approach assumes that the transactions services of money enter directly into the market participants' utility functions, following Sidrauski (1967). The overlapping generations model, introduced by Samuelson (1958), provides one example where money is essential in improving allocations, (without such a constraint that it must be used in transactions, or a direct effect on the utility function) - however, it is used to transfer wealth between generations, and it loses its role once its rate of return is dominated by other assets.

Hellwig (1993) provides a detailed, critical discussion of the recent and not-so-recent literature on monetary equilibrium theory, on which some of the ideas in this paper are based.

⁶Such an evolutionary view of markets has a long tradition in the Austrian school. For example, in his classical article on the origin of money, Menger (1892) views money as the determinate outcome of an evolutionary process; however his analysis does not recognize the potential for multiplicity inherent in coordination problems, nor does he explicitly refer to intermediaries as a coordinating force in the market.

The paper also responds to some of the weaknesses of existing search models of money that follow Kiyotaki and Wright (1989). While they succeed in explaining why it may be individually rational and socially efficient that all agents use a common medium of exchange, they cannot account for the fact that the vast majority of transactions involves the exchange of goods for money - indeed, one of the conclusions from the literature following Kiyotaki and Wright is that such a Cash-in-Advance constraint where “goods” are only traded for “money” fails to materialize (Aiyagari and Wallace, 1991), since the delays in trade provide a sufficient incentive to accept “goods” for further exchange, instead of immediate consumption. The same search frictions which motivate the use of a medium of exchange render the existence of a Cash-in-Advance constraint impossible.⁷ A second weakness of search models is the multiplicity of equilibria. The strategic complementarity that exists between players for using a single good as a common medium of exchange also implies that players may coordinate on *any* good as the common medium of exchange in equilibrium; in other words, the model remains silent about the choice of a medium of exchange. Similarly, while the search model can be used to show that a fiat money, which no one consumes and no one produces, may be valued and traded in equilibrium, the very same set-up always implies that this need not be the case in equilibrium. Hence, the search model is unable to say anything about how a fiat money comes into circulation in a decentralized exchange economy. In contrast, intermediation arguably provides a natural framework for studying these selection issues, as well as the emergence of fiat money.

It should be noted that the general equilibrium as well as the search models of money have multiple equilibria. As discussed in Hahn 1965, this multiplicity of equilibria is the manifestation of an intertemporal coordination problem: the acceptance of money today is based on the acceptance of money tomorrow. The evolutionary approach taken here resolves the multiplicity. It should be noted, however, that the solution relies on the assumption that individuals are able to coordinate their strategies explicitly not only within a single period, but also *across time*, at least on a small scale.

Formally, this paper is most closely related to, and shares much of its motivation with, a series

⁷While this result obviously clashes with the observation of Cash-in-Advance constraints in quasi-perfect markets, it has some intuitive appeal with respect to the importance of barter trade in environments, in which markets are far from frictionless.

of contemporaneous papers that discuss monetary trade in a "trading-post" environment.⁸ In such an environment, markets are in separate locations, and typically each location represents a different pair of goods that can be traded at that location. Iwai (1988) studies such an environment with search frictions. In Starr (1999), as well as Howitt (2000) and Howitt and Clower (2000), these trading posts are run by intermediaries similar to the ones encountered here.⁹ In all these papers, a combination of increasing returns to scale in the intermediary's transaction technology and the double coincidence problem lead to a concentration on the smallest possible number of trading posts and the thereby the use of a common medium of exchange. In another paper that uses a trading-post environment, Matsui and Shimizu (2001) discuss the emergence of money in a market place environment, where the location, rather than an intermediary defines the trading post. As in this paper, they study the evolutionary stability of equilibria, and show that a unique "single-price equilibrium" survives, in which the supply of fiat money is equal in value to its demand for market transactions. All these papers take the trading post structure as given, and are more concerned with the properties of the resulting equilibria, i.e. when a monetary equilibrium exists and what its properties are, as well as the relation of money and prices.

In contrast, this paper abstracts from the problems of price formation, and instead concentrates on the evolutionary aspects of the emergence of money and markets.¹⁰ In order to embed intermediation into the search framework of Kiyotaki and Wright (1989), I restrict transactions to one-for-one swaps, emphasizing the role of the double coincidence problem in the exchange process. The trading posts generate exchange opportunities only insofar as intermediaries become active, the choice of becoming an intermediary is itself endogenous in this model. As will become clear from the results, this leads to interesting insights regarding the coexistence of intermediated with "random" transactions, in particular that the latter follow the same patterns as the intermediated transactions. From a much less structured trading environment, we therefore obtain the same transaction patterns, but using the search-theoretical framework as a background, we give a strategic account as to how intermediation develops and induces improvements in the transaction process until at some point, transaction patterns and allocations closely resemble Walrasian equilibrium allocations.

⁸Although very similar in design, to the best of my knowledge, these papers were all developed independently from each other.

⁹See also Corbae, Temzelides and Wright (2000) for an endogenous matching environment with similar outcomes.

¹⁰In this sense, I view the afore-mentioned papers very much as complementary to this one.

I conclude this section by relating the ideas in this paper to other microfoundational approaches towards money, in particular those based on asymmetric information and limited enforcement; both have recently played a major role in theories of liquidity provision and banking. Banerjee and Maskin (1996) study a Walrasian economy, in which asymmetric information (a lemons problem between buyers and sellers regarding the quality of goods traded) as the source of frictions. In this environment, money endogenously arises as a trading arrangement that minimizes the losses due to asymmetric information. Kocherlakota (1998) argues that "Money is Memory", i.e. in an environment where imperfect record-keeping limits the possibility of writing and enforcing contracts in the future, money serves as a substitute for a record of past trading history, and thereby implements allocations that would otherwise require some explicit record-keeping. These approaches towards money remain silent about how money interacts with other ways to overcome these frictions. On the other hand, Dixit (2001) recently emphasized the role of intermediaries specialized in dealing with asymmetric information or contract enforcement issues. In an environment similar to the ones cited above, he shows that there is room for an information and enforcement intermediary, mentioning the mafia as a prime example. Another example of how intermediaries deal reduce contract enforcement issues, and in the process increase the liquidity in the market, is Diamond and Rajan (2000). In the conclusion, I briefly discuss how the arguments at work in the search and matching set-up of this paper apply more generally to the respective roles of money and intermediation in dealing with other types of frictions.

The rest of this paper is organized as follows: Section 3 describes the basic economic environment and introduces the notion of steady-state equilibrium. I then derive some preliminary results, to provide conditions that a steady-state has to satisfy. Section 4 considers one type of equilibrium, in which a particular good is used as a common medium of exchange. I contrast the findings of the economy with intermediation with the monetary equilibria resulting from pure search. Section 5 introduces evolutionary stability, and shows that any evolutionarily stable equilibrium must be Pareto efficient. Section 6 extends the initial set-up to allow for the circulation of fiat money. Under general conditions, it is then shown that the unique evolutionarily stable equilibrium has a Cash-in-Advance constraint for fiat money. I also discuss the implementation of this equilibrium in a free banking environment. I conclude the paper with some remarks on how the mechanism described here may be extended, or apply to other contexts.

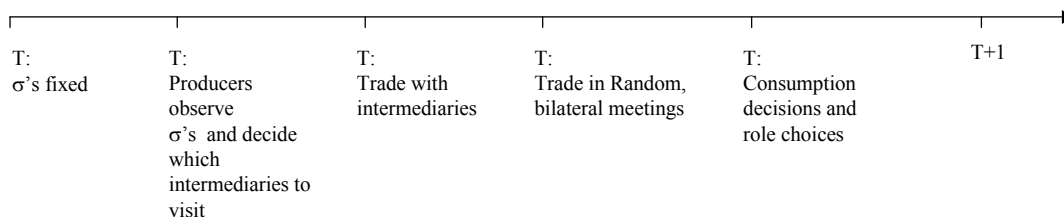


Figure 1: Timing of actions within each period

3 The Model

3.1 The physical environment

I consider a continuum of measure 1 of infinitely-lived agents. There are $N \geq 3$ different goods and N types of agents in the economy. Type i agents always consume good i . There is a measure of $\frac{1}{N}$ of each type.

Time is discrete and infinite, and all goods are perfectly durable. In order to consume, an agent chooses to act either as a *producer* or as an *intermediary*. A producer always holds one unit of a good, and tries to obtain, after a sequence of one-for-one exchanges, a unit of his own consumption good. He then consumes and immediately thereafter produces a unit of his production good, which for type i is good $i + 1$ (good N consumers produce good 1). An intermediary does not produce, but instead holds one unit of her own consumption good, and has a shop, where she trades.¹¹¹² In each period, the intermediary uses her inventory to offer a one-for-one exchange between her own consumption good i and some other good j and vice versa. She can immediately be located by all other producers and intermediaries. Whenever she gives out her consumption good and acquires the other good, the unit she gives out is reduced by a fraction σ_{ij} . Whenever she acquires her consumption good and gives out the other good, she sells a full unit for a full unit. The intermediary lives off this mark-up.

Within a period, the sequence of actions is described in figure 1. At the end of each period, every intermediary decides on the size of the unit of her consumption good that she offers during

¹¹I use male pronouns to refer to producers, and female pronouns to refer to intermediaries.

¹²An earlier version of this paper allowed intermediaries to accumulate inventories for transactions. The main results of this paper go through almost identically, but the restriction to one unit substantially simplifies the analysis.

the next period, and consumes the residual. She thus starts with a unit of her own consumption good that is reduced by a fraction σ_{ij} . She then lets all producers know that during the period, she is willing to exchange one unit of size $1 - \sigma_{ij}$ of good i against an integer unit of good j , and an integer unit of good j against an integer unit of good i . Producers observe these, and then decide if they want to trade with an intermediary, and which one they want to trade with. Then, transactions with intermediaries take place. To visualize this transactions process, it is useful to order all intermediaries for a given ij -transaction according to the size of their mark-up. All producers who intend to carry out a transaction of i for j then form a queue (the position of each individual within the queue being random), and each one chooses the intermediary with the lowest mark-up available to him to carry out the transaction. Furthermore, producers cannot use the intermediary to coordinate on a location for carrying out their transactions without going through the intermediary (i.e. producers cannot coordinate amongst each other to "meet in front of the shop"). Once producers are allocated to intermediaries, each intermediary first sells her unit of good i for an integer unit of good j , and then sells the unit of good j to acquire an integer unit of good i . I assume that an intermediary is willing to trade only if she is able to carry out a two-way transaction, that is, if she is allocated one producer for each side of the transaction. If there is an insufficient number of intermediaries to carry out all desired two-way transactions, or if there is an excess of producers on one or the other side of the market, then some producers are not allocated to an intermediary and are unable to carry out the desired transaction. On the other hand, if there are too many intermediaries compared to the total number of two-way transactions demanded by producers, then some intermediaries are unable to trade. Within each period, the total measure of two-way transactions between any pair of goods i and j is thus bounded by the total measure of ij -intermediaries, as well as by the number of agents who want to complete the transaction in each direction.

If the intermediaries' total inventory is insufficient to accommodate all transactions, or if there is a difference between the total demand for exchanging good i for good j and the demand for the opposite exchange, some producers are unable to acquire their desired good from an intermediary. In a second stage of the period, after all transactions with intermediaries are completed, all producers, who were either unable or unwilling to trade with an intermediary, are randomly matched into pairs, and thus have a second opportunity for a transaction. In such a random match, each producer observes which good his trading partner holds, and decides whether or not to swap his inventory

good for the good held by the other agent. An exchange takes place if and only if both agree to it. Since the matches are random, the probability of encountering a type i producer who holds good j (henceforth called ij -agent) in such a meeting is given by the proportion of ij -agents in the random matching stage.

After trade in random meetings has taken place, all agents decide on their consumption, and on their role during the following period. A producer can consume only if he has acquired a full or reduced unit of his own consumption good. In this case, he can also choose to become an intermediary, simply by using his consumption good unit as inventory for intermediation. An intermediary decides what proportion of her own consumption good unit to consume, and thereby, with what size of a unit she wants to enter the following period. An intermediary has the option to become a producer in any period, simply by consuming her entire inventory and producing one unit of the production good. To complete the description of trading, I assume that no agent (intermediary or producer) ever accepts a reduced unit of a good in a transaction. Reduced units are therefore acceptable only for immediate consumption, and are never held in inventory.

Preferences are symmetric across types. Consumption utility is linear: An intermediary obtains an instantaneous utility cU from consuming a fraction c of her inventory unit. A producer obtains utility $U(1 - \sigma)$ from consuming a unit of his consumption good of size $1 - \sigma$. Consuming any other good yields 0 utility. All agents discount time by a constant rate δ smaller than but close to 1. Whenever an agent trades, he incurs a direct transaction cost. Producers incur a cost of β_i , whenever they accept good i in a one-for-one exchange, or in a transaction with an intermediary. Goods are strictly ranked by transaction costs, and for further reference, good 1 is defined as the good which has the lowest cost of acceptance. Intermediaries incur a cost of $\beta_i + \beta_j$ from carrying out a two-way transaction between good i for good j . In addition, there is no fixed cost involved in setting up, maintaining, or abandoning intermediation.

The main innovation with respect to the original search-theoretic model of money by Kiyotaki and Wright (1989) is the formal introduction of intermediation. As in their framework, this paper's aim is to analyze transaction patterns and the emergence of a common medium of exchange within a decentralized economy. This requires an environment in which all goods are durable, and no commodity is predestined by its storability qualities to become a medium of exchange. Many of the seemingly ad hoc modelling choices are motivated by the objective to allow for trade in random

matches as well as through intermediaries within the same environment. This, however, comes at the expense of a formal modelling of price setting by intermediaries. Related papers that abstract from search trade (Howitt 2000, Starr 1999, Matsui and Shimizu 2001) suggest that the conclusions presented here for intermediated transactions apply to settings, where intermediaries set prices and act as "market-makers", and producers determine the quantities that they want to trade.

3.2 Strategies and equilibrium

I now introduce the notation for strategic variables to describe individual behavior, as well as the distribution of individual inventories to describe the evolution of the entire economy, given individual behavior. Throughout the paper, I restrict attention to symmetric, stationary strategy profiles, in which (i) all producers of the same type choose the same mixed-strategy profile for transaction strategies, and (ii) in each period, all intermediaries who offer the same transaction set the same markup. This is a weak restriction, but simplifies our notation considerably: In a stationary environment, i.e. one where the distribution of individual inventories across the population remains identical over time, identical and stationary markups will naturally come as a consequence of Bertrand competition among intermediaries.

Individual strategies and the aggregate state of this economy are described as follows: Trading strategies for a producer consist of two (probabilistic) decision rules, one that relates the current inventory to the choice of visiting an intermediary, and one that indicates the probability with which a good is accepted in exchange for another one in a bilateral meeting. I denote the decision rule for transactions with intermediaries by a vector of functions $\{\phi_{ij}\}_{j \neq i} : \{1, 2, \dots, N\}^2 \rightarrow [0, 1]$, where $\phi_{ij}(k, l)$ denotes the probability that an ij -agent (type i producer who holds good j) visits a kl -intermediary.¹³ The residual probability $1 - \sum_{k, l=0}^N \phi_{ij}(k, l)$ is assigned to the event that he chooses not to visit an intermediary. We observe immediately that $\phi_{ij}(k, l) = 0$, whenever $j \neq k$ and $j \neq l$, since a kl -intermediary does not accept good $j \neq k, l$. Also, $\phi_{ij}(k, j) = 0$, whenever $k \neq i$, since no producer is willing to acquire a reduced unit of a good other than his consumption good. This leaves as possible choices the visit of any jk -intermediary, to trade j for a full unit of k (either for further exchange, or for consumption, if $k = i$ is the producer's consumption good),

¹³Under the given assumptions concerning the matching between intermediaries and producers, these decision rules only need to indicate which transaction a producer intends to carry out, but not the intermediary's identity.

and the visit of an ij -intermediary to acquire a reduced unit of good i for consumption. Trading rules for bilateral meetings are described by a collection of functions $\{\tau_{ij}\}_{j \neq i} : \{1, 2, \dots, N\} \rightarrow [0, 1]$, where $\tau_{ij}(k)$ indicates the probability that an ij -agent accepts good k for good j . When an ij -agent meets an lk -agent, trade occurs with probability $\tau_{ij}(k)\tau_{lk}(j)$.

Each intermediary chooses his mark-up σ_{ij} , however note that Bertrand competition will imply that intermediaries of the same type set identical mark-ups. Finally, the aggregate state of the economy is given by the distribution of inventories and role choices, i.e. (i) the measures of type i intermediaries who trade good i for good j (henceforth: ij -intermediaries), denoted by ν_{ij} , and (ii) the distribution of inventories across producers, where we let μ_{ij} denote the measure of ij -agents. This notation leaves aside a formalization of the decision problem for role choices, which is given by an indifference condition between the two activities: any type who holds one unit of his own consumption good has to be indifferent between either becoming an intermediary or consuming and becoming a producer.

Introducing the strategies in this way implicitly assumes symmetric and stationary behavior, but this will also be the outcome of optimizing behavior in a symmetric, stationary environment. Since each agent has no direct effect on the aggregate state, we can consider the optimization problem for each type of intermediaries and producers separately, taking the behavior of others and the aggregate states as given. Furthermore, our set-up implies that mark-ups are determined by Bertrand competition, and an open-entry condition is at work. In a steady-state equilibrium, mark-ups are at a level where (i) no producer has an incentive to become an intermediary, and no intermediary has an incentive to become a producer, and (ii) no intermediary has an incentive to slightly undercut all other intermediaries to offer the same transaction, in order to increase his trading volume. Finally, a stationarity condition determines the equilibrium distribution of inventories and role choices, $\{\nu_{ij}, \mu_{ij}\}_{i,j=1}^N$. Summing up, this leads to the following informal definition of a symmetric stationary equilibrium:

Definition 1 *A symmetric, stationary equilibrium is a vector $\{\mu_{ij}, \nu_{ij}, \phi_{ij}, \sigma_{ij}, \tau_{ij}\}_{i,j=1}^N$, such that*

- (i) *for all ij -intermediaries, it is optimal to set their mark-up equal to σ_{ij} in each period,*
- (ii) *ϕ_{ij} and τ_{ij} are optimal trading strategies for producers,*
- (iii) *no intermediary wants to become a producer, and no producer wants to become an intermediary, and*

(iv) the distribution of inventories and role choices $\{\nu_{ij}, \mu_{ij}\}_{i,j=1}^N$ remains constant over time, given these strategy choices.

3.3 Preliminary Results

In this section, I derive some preliminary results that formalize the definition of a symmetric steady-state equilibrium given above. For given values of $\{\mu_{ij}, \nu_{ij}, \phi_{ij}, \sigma_{ij}, \tau_{ij}\}$, it is straight-forward to express a producer's optimization problem by a set of Bellman equations.¹⁴ The aggregate state and the strategy profile determine conditional trading probabilities for trade with intermediaries: for this purpose, define $\pi_{ij}(k)$ as the probability that an ij -intermediary is able to deliver a unit of good $k \in \{i, j\}$ to a producer who wants to trade with her. Define $V_i(j)$ as the life-time discounted utility of a producer of type i who holds good j at the end of a period. Then, $V_i(j)$ solves the following set of Bellman equations:

$$\begin{aligned}
 (1 - \delta) V_i(j) = & \delta \max_{\phi_{ij}, \tau_{ij}} \left\{ \sum_{l=1}^N (V_i(l) - \beta_l - V_i(j)) \phi_{ij}(j, l) \pi_{jl}(l) \right. \\
 & + (V_i(i) - \sigma_{ij}U - \beta_i - V_i(j)) \phi_{ij}(i, j) \pi_{ij}(i) \\
 & + \left(1 - \sum_{l=1}^N \phi_{ij}(j, l) \pi_{jl}(l) - \phi_{ij}(i, j) \pi_{ij}(i) \right) \\
 & \left. \frac{\sum_{k,l} \mu'_{kl} (V_i(l) - \beta_l - V_i(j)) \tau_{ij}(l) \tau_{kl}(j)}{\sum_{k,l} \mu'_{kl}} \right\} \tag{1}
 \end{aligned}$$

and

$$V_i(i) = U + V_i(i + 1); V_N(N) = U + V_N(1)$$

where

$$\mu'_{ij} = \mu_{ij} \left(1 - \sum_{l=1}^N \phi_{ij}(j, l) \pi_{jl}(l) - \phi_{ij}(i, j) \pi_{ij}(i) \right)$$

¹⁴Standard results imply that under stationarity, the solution to this set of Bellman equations is equivalent to the corresponding sequential optimization problem.

denotes the measure of ij -agents who did not visit an intermediary, and as a result enter a bilateral match.¹⁵ Lemma 1 discusses the properties of optimal trading strategies for intermediated and random transactions:

Lemma 1 *If $\{\tau_i, \phi_i\}_{j \neq i}$ is an optimal trade strategy for a producer of type i , then the following must be true:*

(i) *If $\phi_{ij}(j, k) > 0$, then $k \in \arg \max_l (V_i(l) - \beta_l - V_i(j)) \pi_{jl}(l)$, and*

$$\max_l (V_i(l) - \beta_l - V_i(j)) \pi_{jl}(l) \geq (V_i(i) - \sigma_{ij}U - \beta_i - V_i(j)) \pi_{ij}(i)$$

If k is a unique maximizer, then $\phi_{ij}(j, k) = 1$

(ii) *If $\phi_{ij}(i, j) > 0$, then*

$$(V_i(i) - \sigma_{ij}U - \beta_i - V_i(j)) \pi_{ij}(i) \geq \max_l (V_i(l) - \beta_l - V_i(j)) \pi_{jl}(l)$$

where $\phi_{ij}(i, j) = 1$, if the inequality is strict

(iii) *If $\tau_{ij}(k) > 0$, then $V_i(k) - \beta_k - V_i(j) \geq 0$; and $V_i(k) - \beta_k - V_i(j) > 0$ implies $\tau_{ij}(k) = 1$*

Lemma 1 highlights the difference between trade with intermediaries and random bilateral trade: Strategies for the latter amount to simple decision rules that indicate whether one good is accepted in exchange for another, and agents might be willing to accept more than one good in exchange for their current inventory. Hence, trading patterns remain indeterminate, as there may be many possible sequences of exchanges which lead a producer from his current inventory to his consumption good. In contrast, trading with an intermediary enables a producer to target the transaction that maximizes his expected surplus. The producer can follow a predetermined sequence of intermediated exchanges in order to eventually receive his consumption good, and generically, only one such sequence is optimal. Thus, intermediation replicates the structure of models with deterministic trading zones, where agents need to visit an “ ij -island” in order to trade good i for good j . However, in contrast to those models, the structure here arises *endogenously* from the activity of intermediaries. Consequently, any delay in trade results from the inability of intermediaries to accommodate all the transactions demanded by producers.

¹⁵(1) implicitly assumes that a consumer never holds out or trades away his own consumption good. It is straightforward to show that this is strictly dominated by immediate consumption.

One further observes that not visiting an intermediary is a weakly dominated strategy.

Next consider the requirements for stationarity of the distribution of inventories. Taking as given $\{\nu_{ij}, \phi_{ij}, \tau_{ij}\}$, $\{\mu_{ij}\}$ has to satisfy:

$$\begin{aligned} \mu_{ij} = & \sum_{l=1}^N \phi_{il}(l, j) \pi_{lj}(j) \mu_{il} + \frac{1}{\sum_{k,l} \mu'_{kl}} \sum_l \mu'_{il} \sum_k \mu'_{kj} \tau_{il}(j) \tau_{kj}(l) \\ & + \mu'_{ij} \left(1 - \frac{1}{\sum_{k,l} \mu'_{kl}} \sum_{k,l} \mu'_{kl} \tau_{ij}(l) \tau_{kl}(j) \right) \end{aligned} \quad (2)$$

whenever $j \neq i, i+1$, and

$$\begin{aligned} \mu_{i,i+1} = & \sum_{l=1}^N [\phi_{il}(l, i) \pi_{li}(i) + \phi_{il}(i, l) \pi_{il}(i)] \mu_{il} + \frac{1}{\sum_{k,l} \mu'_{kl}} \sum_l \mu'_{il} \sum_k \mu'_{ki} \tau_{il}(i) \tau_{ki}(l) \\ & + \mu'_{i,i+1} \left(1 - \frac{1}{\sum_{k,l} \mu'_{kl}} \sum_{k,l} \mu'_{kl} \tau_{ij}(l) \tau_{kl}(j) \right) \end{aligned} \quad (3)$$

An i -agent's production good is treated separately from all other goods he may hold as an inventory. Condition (2) can be explained as follows: $\sum_{l=1}^N \phi_{il}(l, j) \pi_{lj}(j) \mu_{il}$ is the measure of i -agents who acquire good j from an intermediary. μ'_{ij} is the set of ij -agents who are unsuccessful in trading with an intermediary, and was already defined above. A fraction $1 - \frac{1}{\sum_{k,l} \mu'_{kl}} \sum_{k,l} \mu'_{kl} \tau_{ij}(l) \tau_{kl}(j)$ of μ'_{ij} is unsuccessful in bilateral exchange as well, which gives the third term. Finally, a measure of $\frac{1}{\sum_{k,l} \mu'_{kl}} \sum_l \mu'_{il} \sum_k \mu'_{kj} \tau_{il}(j) \tau_{kj}(l)$ acquires good j through a bilateral match. Similarly, $\mu_{i,i+1}$ can be decomposed into those agents who were able to consume after visiting an intermediary, or after a successful bilateral meeting, and those who held good $i+1$ at the start of the period, but were unable to trade. Since holding one's own production good stands at the beginning of any sequence of trades, no agent will trade in his inventory for good $i+1$.

Trading probabilities for trade with intermediaries can be derived from the distribution of inventories and role choices as

$$\pi_{ij}(i) = \frac{\min \left\{ \nu_{ij}, \mu_{ij} \phi_{ij}(i, j), \sum_l \mu_{li} \phi_{li}(i, j) \right\}}{\mu_{ij} \phi_{ij}(i, j)} \quad \text{and} \quad \pi_{ij}(j) = \frac{\min \left\{ \nu_{ij}, \mu_{ij} \phi_{ij}(i, j), \sum_l \mu_{li} \phi_{li}(i, j) \right\}}{\sum_l \mu_{li} \phi_{li}(i, j)}$$

respectively, i.e. the maximum possible measure of two-way transactions divided by the measure of agents wishing to perform the same transaction. The next lemma summarizes the equilibrium implications of competition among intermediaries:

Lemma 2 *Bertrand competition and open entry into and exit from intermediation imply:*

(i)

$$V_i(i) = \frac{1}{1-\delta} [\sigma_{ij}U - \delta(\beta_i + \beta_j)] \quad (4)$$

(ii)

$$\nu_{ij} = \min \left\{ \mu_{ij} \phi_{ij}(i, j), \sum_l \mu_{li} \phi_{li}(i, j) \right\}. \quad (5)$$

Proof. (i) by the open entry condition, any agent who holds a unit of size 1 or $1 - \sigma_{ij}$ of his consumption good i has to be just indifferent between consuming everything and remaining a producer, and retaining $1 - \sigma_{ij}$ to become an intermediary.¹⁶ (ii) If $\nu_{ij} > \min \left\{ \mu_{ij} \phi_{ij}(i, j), \sum_l \mu_{li} \phi_{li}(i, j) \right\}$, then some intermediaries would not be able to trade within the period, and would be better off either lowering their mark-up, or leaving intermediation altogether. If $\nu_{ij} < \min \left\{ \mu_{ij} \phi_{ij}(i, j), \sum_l \mu_{li} \phi_{li}(i, j) \right\}$, then the demand for transactions exceeds the intermediaries' capacity, and every intermediary would be free to raise his price. ■

¹⁶A problem possibly arises in a non-stationary environment, if σ_{ij} decreases from one period to the next. A producer would then be unable to use his unit of consumption to start as an intermediary. However, this concern is irrelevant for steady-state analysis.

(5) states that ν_{ij} has to equal the measure of two-way transactions demanded between goods i and j . The transaction probabilities can then be rewritten as

$$\pi_{ij}(i) = \min \left\{ 1, \frac{\sum_l \mu_{li} \phi_{li}(i, j)}{\mu_{ij} \phi_{ij}(i, j)} \right\} \text{ and } \pi_{ij}(j) = \min \left\{ 1, \frac{\mu_{ij} \phi_{ij}(i, j)}{\sum_l \mu_{li} \phi_{li}(i, j)} \right\}.$$

Hence, in any steady-state equilibrium, lemma 1 provides necessary and sufficient conditions for the optimality of trading strategies ϕ_{ij} and τ_{ij} , (2) and (3) have to hold for stationarity of $\{\mu_{ij}\}$, and lemma 2 states that (4) and (5) determine σ_{ij} and ν_{ij} .

4 Commodity Money

4.1 Characterization

In this section, I discuss the emergence of a common medium of exchange as an equilibrium property of the economy outlined above. The concept of money referred to is commodity money, i.e. a good which is used by all producers for indirect exchange. For an economy with intermediaries, it is straight-forward to conjecture the existence of a type of equilibrium, where an arbitrary good m circulates as money. Any producer chooses to first trade his production good for a full unit of good m , and, once he has acquired m , exchanges m for a reduced unit of his own consumption good. Type m producers trade their production good $m + 1$ directly for good m , and producers of good m trade directly for their consumption good $m - 1$. In such an equilibrium, $\nu_{im} > 0$, for all $i \neq m$, and $\nu_{ij} = 0$ otherwise, i.e. for each good different from m , there exists an active set of im -intermediaries. I call this equilibrium a Cash-in-Advance equilibrium for good m , because transactions with intermediaries necessarily involve the use of good m as a medium of exchange.

Definition 2 *A stationary equilibrium exhibits a Cash-in-Advance constraint for some good m , if and only if $\{\phi_{ij}\}_{i=1}^N$ satisfies $\phi_{ij}(j, m) = 1$, and $\phi_{im}(i, m) = 1$, for all i, j .*

Proposition 1 *If transaction costs are sufficiently small, then for any good m , there exists a stationary equilibrium in which $\{\phi_i\}_{i=1}^N$ exhibits a Cash-in-Advance constraint for m .*

Proof. I proceed by guessing and verifying. Given the sets of intermediaries active in equilibrium, any producer has only one possible choice for transactions with intermediaries. If this transaction sequence leaves him with positive lifetime utility, it will therefore be optimal. Now, let μ_{im} be the measure of type i producers who hold good m , and $\mu_{i,i+1}$ the measure of type i producers who hold their production good $i+1$. Conjecture further that intermediaries are able to carry out all the transactions demanded by producers with probability 1, for all producer types different from $m-1$ or m . This requires $\nu_{im} = \mu_{im} = \mu_{i,i+1} = \frac{1}{2} \left(\frac{1}{N} - \nu_{im} \right)$, or $\nu_{im} = \mu_{im} = \mu_{i,i+1} = \frac{1}{3N}$, and one easily confirms the conjecture that trading probabilities for types $i \neq m-1, m$ are indeed equal to 1. Type $m-1$ acquires good $m-1$ with probability $\frac{1}{2}$ in each period, and type m acquires his consumption good with probability $\frac{1}{3}$. These types are the only ones that enter random matches, but they will never agree to swap their inventories in a random meeting, since type $m-1$ would never be willing to give up good m for a good that is not his own consumption good - we therefore conclude that no trade will ever take place as a result of a random meeting.

I next determine the mark-up for each intermediary. Whenever a consumer of type $i \neq m$ holds a full unit of his consumption good, he has the choice of becoming an intermediary in the following period, in which case he consumes σ_{ij} now, and uses the remainder to trade in the following period, or consuming the entire unit to become a producer in the following period. In equilibrium, he must be indifferent between the two. For types $i \neq m-1, m$, the life-time utility of an im -intermediary is $\frac{1}{1-\delta} (\sigma_{im}U - \delta(\beta_i + \beta_m))$, where $\sigma_{im}U$ is the utility from consuming the proceeds of one two-way transaction that the intermediary consumes at the end of each period, and $\delta(\beta_i + \beta_m)$ is the discounted cost of the next period's two-way transaction. The lifetime utility of a type i producer before consuming his unit is $U + \frac{\delta^2}{1-\delta^2} \left(U(1 - \sigma_{im}) - \beta_i - \frac{\beta_m}{\delta} \right)$: The type i producer consumes a unit of size $1 - \sigma_{im}$ in every other period, and during the intermediate periods he first incurs a transaction cost β_m for acquiring the medium of exchange, and then a cost β_i of acquiring his own consumption good. Equating the two and solving for σ_{im} yields

$$\sigma_{im}U = \frac{1}{1 + \delta + \delta^2} [U + \delta\beta_i + \delta^2\beta_m]. \quad (6)$$

The life-time utility of a type $m-1$ intermediary and a type $m-1$ producer are

$$\frac{1}{1-\delta} (\sigma_{m-1,m}U - \delta(\beta_{m-1} + \beta_m))$$

and

$$U + \frac{\frac{1}{2}\delta}{1-\delta} (U(1 - \sigma_{m-1,m}) - \beta_{m-1}),$$

respectively. In this case,

$$\sigma_{m-1,m}U = U \frac{2-\delta}{2+\delta} + \delta\beta_{m-1} \frac{1}{2+\delta} + \delta\beta_m \frac{2}{2+\delta}. \quad (7)$$

To complete the proof, I derive the equilibrium welfare levels denoted by W_i , which must be positive at the point right after a producer has consumed:

$$\begin{aligned} (1-\delta)W_i &= \frac{\delta^2}{1+\delta+\delta^2} \left[\delta(U-\beta_i) - \beta_i - \beta_m \frac{1+\delta^2}{\delta} \right], \text{ if } i \neq m-1, m \\ (1-\delta)W_{m-1} &= \frac{\delta}{2+\delta} [\delta(U-\beta_{m-1}) - \beta_{m-1} - \delta\beta_m], \\ (1-\delta)W_m &= \frac{1}{3}\delta(U-\beta_m). \end{aligned} \quad (8)$$

As long as transaction costs are sufficiently small, these life-time utility levels are strictly positive, and hence an equilibrium with good m as medium of exchange exists. ■

In this equilibrium, the medium of exchange is the result of the intermediaries' strategies: Their implicit coordination favors one good for the use as a medium of exchange. Intermediaries can deliver this good much more quickly than the search market. If transaction costs are small enough, Bertrand competition among intermediaries guarantees that the benefits of intermediation exceed its costs, so that producers have no incentive to deviate from the proposed trading sequence.

The characterization of Cash-in-Advance equilibria leads to several immediate observations. First, (8) gives an upper bound on transaction costs that must be satisfied so that a Cash-in-Advance equilibrium for good m exists. If transaction costs are sufficiently small and the discount rate is close to 1, $(1-\delta)W_i$ is close to $\frac{1}{3}U$ for all types. However, the producer and consumer of good m enjoy a kind of "rent" in equilibrium: If δ is close to 1, these two types will always prefer to be in a Cash-in-Advance equilibrium for good m , rather than in a Cash-in-Advance equilibrium in which they have to trade twice.

We further observe that the trading patterns in a Cash-in-Advance equilibrium exhibit a form of "market-clearing": for all types except m and $m-1$, trading probabilities for transactions with intermediaries are equal to 1, i.e. apart from the money producers and money consumers, no one faces delays in carrying out the desired transactions. Thus, in a Cash-in-Advance equilibrium, almost all desired transactions are carried out at the prices at which a Walrasian market would

clear. *Complete market-clearing* is impossible due to a disequilibrium in transaction sequences: The producers and consumers of the commodity money only trade once in order to consume, while all other types trade at least twice between the time they produce and the time they consume. Thus, the commodity money equilibrium distorts demand and supply of goods m and $m + 1$ away from equality at the prices at which Walrasian markets for these goods would clear. Obviously, this result would not be robust, if the environment were altered in such a way that intermediaries could change prices to equate the aggregate quantities demanded and supplied for each transaction. Nevertheless, this discussion leads to an important insight: the liquidity demand for the commodity money distorts such market-clearing prices away from underlying Walrasian prices.

4.2 Intermediated vs. Random Transactions

These observations about the Cash-in-Advance equilibria with intermediaries are in contrast with the characteristics of commodity money in a pure search economy.¹⁷ Since these properties have been studied extensively by Kiyotaki and Wright (1989) and Aiyagari and Wallace (1991, 1992), their main results will only briefly be reviewed here. In a pure search economy, a commodity money is defined as a good that is accepted by all producers, whenever it is offered in an exchange. A strategy profile entails a Cash-in-Advance constraint, if the commodity money is part of every transaction that takes place in equilibrium.

The aforementioned papers on pure search economies show that while commodity money equilibria do exist in pure search economies for some or all goods, these equilibria typically do not entail a Cash-in-Advance constraint: A Cash-in-Advance constraint implies that any agent must first acquire the medium of exchange, before he can acquire his own consumption good. However, due to asymmetries across goods in the steady-state inventory distribution, goods are endogenously characterized by different qualities for indirect exchange, and since agents cannot direct their search towards a direct trade for money, they may be willing to accept another "good" in an attempt to

¹⁷Note that intermediation widens the possible set of equilibria of this economy from the one originally studied in Kiyotaki and Wright. If no agent acts as an intermediary, it is weakly optimal for producers not to visit an intermediary. But then, no agent has an incentive for becoming intermediary and trade will only take place in bilateral meetings. Thus, any steady-state equilibrium of the original Kiyotaki-Wright economy can be supported as an equilibrium of this economy with intermediation, setting $\nu_{ij} = \sigma_{ij} = 0$ and $\phi_{ij}(k, l) = 0$, for all i, j, k, l . This reduces the equilibrium definition to the distribution of inventories and to the search strategy profile $\{\mu_{ij}, \tau_{ij}\}_{i,j=1}^N$.

increase the probability of trading for money (or one's consumption good). In contrast, this incentive is missing when intermediaries successfully eliminate waiting times, and producers are able to direct their transaction strategy towards a predetermined sequence of trades. If the intermediaries are efficient in carrying out transactions, producers are able to carry out the exchange proposed by the trade sequence (almost) immediately. Holding a particular good at time t becomes equivalent in value to exchanging it against the next good of the trading sequence at time $t + 1$. In the Cash-in-Advance equilibrium, any good can almost directly be exchanged against the commodity money, so that there is no incentive to reduce search frictions by goods-for-goods trade, as in the pure search model.

I conclude this discussion by considering the influence of intermediation on random transactions: In the Cash-in-Advance equilibrium, transactions cease to occur in bilateral meetings. This observation follows from the fact that the search market empties, but the conclusion continues to hold even if intermediated transactions were only approximately able to clear the search market, i.e. trading probabilities for intermediated transactions are close to (but smaller than) 1 and some agents enter the matching stage.¹⁸ If a producer enters the random matching phase holding the medium of exchange, he is only willing to exchange it against his own consumption good. On the other hand, if a producer enters a random match holding a good other than the medium of exchange, he will only exchange it against the medium of exchange or his own consumption good in a random meeting.¹⁹ The only possible trade is then between a pair of agents where one acquires the medium of exchange for his production good, while the other acquires his own consumption good against the medium of exchange, but those two types would have visited the same intermediary earlier in the period, and hence, given the random matching assumptions, cannot both enter into a random match (one of the two must have been successful in trading with the intermediary). Under the more plausible, yet technically intractable alternative assumption that all agents enter into random meetings (independently of whether they were successful in trading with an intermediary or not), we would have come to the conclusion that *if intermediation leads to approximate market-clearing, then transactions occurring in random meetings have to enable the trading parties to save*

¹⁸In an earlier version of this model, that allows for inventory accumulation by intermediaries, markets clear only approximately. The discussion here is based on the formal results there, which can be found in Hellwig (2000).

¹⁹Trading for a different good has to be dominated, since this can only be motivated by a reduction in future search costs, but with approximate market-clearing, expected future trading probabilities are already close to 1, and hence any reduction in search cost is more than outweighed by the direct cost of an additional transaction.

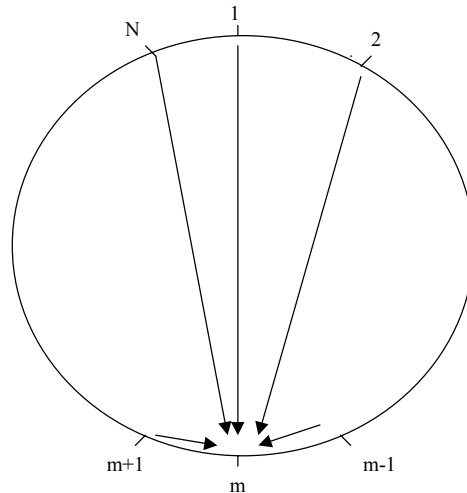


Figure 2: Cash-in-Advance equilibrium

on the costs of intermediation. In other words, had the two trade partners not met by chance, they would have chosen to carry out the same transaction indirectly through an intermediary. The existence of market institutions that successfully eliminate search frictions and approximately clear markets therefore has a deep impact on the transactions that arise out of random meetings, and the Cash-in-Advance property of intermediated transactions also extends to random meetings.

4.3 Other Equilibria

The following simple representation of the Cash-in-Advance equilibrium will also be useful to characterize other equilibria: In figure 2, we represent the activity of ij -intermediaries by an arrow leading from i to j . A feasible trading strategy for some producer type is represented by a sequence of arrows that lead from the producer's production good to his consumption good, and only for the last arrow in the sequence, when he acquires his consumption good, the producer can move against the direction of the arrow (that is: accept a good in reduced units from the intermediary).

In addition to the Cash-in-Advance equilibrium, other equilibria with intermediation exist. Any network of intermediaries that gives every producer type a positive welfare level and exactly one

trading sequence by which he can acquire his consumption good can be supported as an equilibrium. In figures 3 through 5, I consider just a few alternative examples of equilibrium intermediation networks, without attempting to provide an exhaustive description of all steady-states.

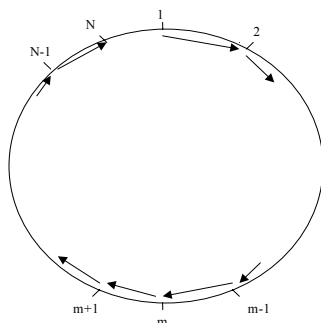


Figure 3: “Trade-one-up” equilibrium: for $i = 1, \dots, N - 1$, there exist $i, i + 1$ -intermediaries. All agents trade their production good directly against their consumption good, except for type N , who trade good 1 for good 2, then 3, etc. until they receive good N .

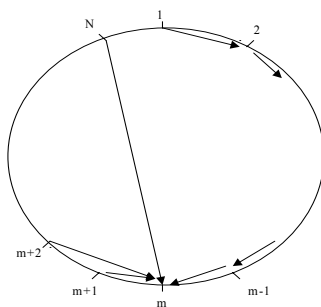


Figure 4: This equilibrium combines a Cash-in-Advance constraint with the previous case: Types 1 to $m - 1$ trade their production good directly against their consumption good, type N trades good 1 for good 2, then good 3 and so on, until he receives the medium of exchange m , which is used as a medium of exchange by types m to $N - 1$.

Among these alternative equilibria, the two-money equilibria are the most interesting ones, and

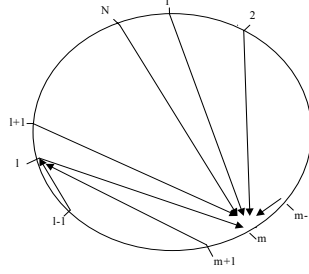


Figure 5: Two-money equilibrium: for $i = l + 1, \dots, m - 1$, there exist im -intermediaries, and for $i = m, \dots, l - 1$, there exist il -intermediaries. In this case, goods l and m are both locally used as medium of exchange, l by types m to $l - 1$, and m by types l to $m - 1$.

it will be useful for further analysis to provide a characterization. Consider an arbitrary two-money equilibrium with goods l and m as media of exchange, and suppose that type m never becomes an intermediary. In this equilibrium, every producer type trades at most twice between the time when he produces, and the time when he consumes. Again, a simple guess-and-verify procedure shows that each set of intermediaries is of measure $\frac{1}{3N}$, probabilities of trade with intermediaries are equal to 1, with the exception of types $l - 1$ and $m - 1$, who only trade once, and type m , who does not enter into intermediation. Types $m - 1$ and $l - 1$ trade their production good directly for their consumption good with probability $\frac{1}{2}$ in each period. For type m , the equilibrium inventory distribution (and hence the trading probabilities) are indeterminate, with possible solutions $(\mu_{m,m+1}, \mu_{m,l}) = (\frac{1}{3N} + \zeta, \frac{2}{3N} - \zeta)$, for any $\zeta \in [0, \frac{1}{3N}]$. Thus, on average, type m consumes every third period. For all types different from m , the indifference condition for each type is equivalent to those obtained in the Cash-in-Advance equilibrium, (when adjusting the indices for the good which each type uses as a medium of exchange). Thus, for $i \neq l - 1, m - 1, m$ the equilibrium mark-ups are

$$\sigma_{ij}U = \frac{1}{1 + \delta + \delta^2} [U + \delta\beta_i + \delta^2\beta_j]. \quad (9)$$

where $j \in \{l, m\}$ represents the medium of exchange for which i is traded. For $i \in \{l - 1, m - 1\}$, the mark-ups are

$$\sigma_{i,i+1}U = U \frac{2 - \delta}{2 + \delta} + \delta\beta_i \frac{1}{2 + \delta} + \delta\beta_{i+1} \frac{2}{2 + \delta}. \quad (10)$$

and welfare levels are

$$\begin{aligned} (1 - \delta) W_i &= \frac{\delta^2}{1 + \delta + \delta^2} \left[\delta (U - \beta_i) - \beta_i - \beta_j \frac{1 + \delta^2}{\delta} \right], \text{ if } i \neq m - 1, l - 1, m \\ (1 - \delta) W_i &= \frac{\delta}{2 + \delta} [\delta (U - \beta_i) - \beta_i - \delta \beta_{i+1}], \text{ if } i = m - 1, l - 1 \end{aligned} \quad (11)$$

The equilibrium welfare level for type m is indeterminate, since it depends on the equilibrium inventory distribution, i.e. on ζ . In the simplest case where $\zeta = \frac{1}{3N}$ (i.e. type m trades good $m + 1$ for l with probability $\frac{1}{2}$, and then l for m with probability 1), his welfare level is²⁰

$$(1 - \delta) W_m = \frac{\delta^2}{2 + \delta} \left[\delta (U - \beta_m) - \frac{1}{\delta} \beta_l \right]$$

The same observations that applied to the Cash-in-Advance equilibria also apply to any two-money equilibrium. In particular, there are equilibrium "rents" accruing to three types: those who produce the two media of exchange, and the one type who is able to consume in integer units. I conclude this section by a brief discussion of the Welfare properties of equilibria. For small levels of transaction costs, it is immediate that any equilibrium, in which all types trade at most twice with intermediaries, Pareto-dominates all other equilibria. The next lemma shows that (i) any equilibrium, in which producers trade at most twice in order to consume is either a Cash-in-Advance equilibrium or a two-money equilibrium, and (ii) these two classes of equilibria also dominate any mixed strategy equilibrium is Pareto-dominated. For δ close to 1, and a small level of transaction costs, it then follows that these two classes Pareto-dominate any other stationary equilibrium profile. On the other hand, the existence of equilibrium "rents", as previously discussed, prevents a Pareto ranking between equilibria within these two classes.

Lemma 3 (i) *Any pure-strategy equilibrium, in which producers trade at most twice in order to consume is either a Cash-in-Advance equilibrium or a two-money equilibrium.*

(ii) *Any mixed-strategy equilibrium is Pareto-dominated by some Cash-in-Advance equilibrium or two-money equilibrium.*

²⁰For other values of ζ , the welfare level is slightly lower, but for any ζ , this discrepancy vanishes, if δ is close to 1.

Proof. (i) In any pure-strategy equilibrium, an intermediation network consists of exactly $N - 1$ sets of intermediaries.²¹ $N - 2$ types trade twice, while the remaining two types trade once. If type i and $i + 1$ both trade twice, they use the same good as a medium of exchange, and in equilibrium, at most two goods are used as commodity money. (ii) Consider an arbitrary mixed-strategy equilibrium, in which all types trade at most twice between the moment when they produce, and the moment, when they consume. The equilibrium transactions network has to connect all N types so as to enable them to trade their production good for their consumption good in some sequence of transactions. This implies that the transaction structure of some Cash-in-Advance or two-money equilibrium (which are the minimal transactions networks) has to be included in any equilibrium transactions network; moreover, if the equilibrium is mixed, the inclusion is strict, and since some types follow two different transaction patterns in equilibrium, there must be at least two different Cash-in-Advance or two-money transactions networks that are embedded in the mixed strategy network. But since for each type, the welfare attained in the mixed strategy equilibrium has to be lower than either of the two pure-strategy networks, it follows that the mixed strategy equilibrium has to be Pareto-dominated. ■

5 Efficiency and Evolutionary Stability

The previous discussion of Cash-in-Advance equilibria in intermediated and pure search economies has highlighted the existence of multiple equilibria, which leads to the question of how an equilibrium, or a medium of exchange, is selected. In this section, I show that evolutionary forces lead to the selection of an *efficient* equilibrium. Loosely speaking, I study whether an arbitrarily small set of agents (containing both intermediaries and producers) can, by explicitly coordinating their actions, improve their welfare level and consequently induce a large measure of other agents to alter their strategies. From a historical perspective, the idea of small deviating coalitions is meant to capture the innovating role of intermediaries, whether it comes through explicit innovation and coordination, or through arbitrary "experimenting": Someone introduces a new system for organizing transactions. If others find that this arrangement is efficient, they will also start using it.

²¹ $N - 1$ is actually the minimum to sustain a complete intermediation network that enables everyone to carry out all transactions through intermediaries. If there are more sets of intermediaries active, at least one type must have at least two alternatives to trade from his production good to his consumption good, and hence must be indifferent and mix in equilibrium.

Since media of exchange, and more generally trading strategies are complementary across agents, everyone will start using the new system, if it leads to a Pareto-improvement. Clearly, intermediation is essential in promoting an innovation in the transactions system, since it provides a channel through which explicit coordination can take place. Moreover, innovations in among intermediaries become immediately accessible to the rest of the population.

Using a standard continuity notion, the definition of evolutionary stability below formulates the requirement that the coordination of a small set of players should not have a large (discontinuous) effect on the equilibrium. However, note that in contrast to its traditional application to large population matching games, evolutionary stability here acts through the coordination of strategies among *various types of players*, and hence is related to the concept of *coalition proofness*. Below, I briefly comment on the relation between the two concepts in the present environment.

Definition 3 *A Stationary Equilibrium is evolutionarily stable, if for every $\varepsilon > 0$, there exists $\eta^* > 0$, such that whenever the strategies of a set of players of measure $\eta \leq \eta^*$ is exogenously fixed (i.e. a set of measure η of players "explicitly coordinates their strategies" or "deviates"), there exists a stationary equilibrium in this modified game, whose Euclidean distance from the original equilibrium does not exceed ε .*

In any kind of decentralized trading economy, explicit coordination of several *types* of players is necessary for any successful deviation from an equilibrium. Trading environments, however, differ in *how many* agents need to coordinate to break out of a given equilibrium. In a pure search economy a la Kiyotaki-Wright, equilibrium payoffs are continuous in the strategy profile and the equilibrium inventory distribution, and since the "mutants" have no way to directly trade with each other, they have only a marginal impact on the payoffs of the non-mutant population. Hence any equilibrium where payoffs are strictly higher than the next-best alternative is immune to the invasion by a *small* set of mutants.

This conclusion is fundamentally altered in the intermediated economy that we study here: To be specific, suppose that some equilibrium network of intermediaries is dominated by another one. A small set of agents may then coordinate their actions as intermediaries on the new intermediation network with some small set of producers. If, by doing so, both the intermediaries and the producers increase their life-time utility, other agents have an incentive to take advantage of the new

intermediation network. Thus, the old equilibrium is no longer stable and will be replaced by the new one. This type of coordination is more explicit than the one resulting from Nash equilibrium strategies, yet it only requires coordination of an arbitrarily small, positive measure of agents. The key insight here is that intermediation enables the mutants not only to coordinate their trading strategies, but also to coordinate on *trading with each other* so as to take advantage of the explicit coordination. But this also gives others the ability to join the mutants' strategy profile.

In a trading economy with intermediaries, it turns out that evolutionary stability is actually equivalent to a general form of coalition proofness, where players coordinate their strategies with respect to intermediation. To see this, suppose that a large coalition could deviate from an existing equilibrium and leave all its participants better off by proposing a new network of intermediation. Then, this change could also be implemented by an arbitrarily small coalition that starts to form the new intermediation network. Other agents will be induced to switch their strategies, until the entire large coalition has deviated from the existing network to the new one.

It follows immediately from the above discussion that a steady-state equilibrium is unstable, if it is Pareto-dominated by an alternative intermediation network. In the next proposition, I formulate this result as an equivalence of evolutionary stability and Pareto-efficiency for the given environment. For this purpose, I use a definition of Pareto efficiency that only compares changes in the strategy profile for intermediated transactions, $\{\phi_{ij}\}_{i,j=1}^N$. This excludes inefficiencies resulting from the transactions in bilateral search meetings, however, in an equilibrium, in which all agents trade with intermediaries with very high probability, the resulting strategies τ_{ij} are prescribed by the network of intermediaries, and have only minor welfare implications.

Definition 4 *A stationary equilibrium $\{\mu_{ij}, \nu_{ij}, \phi_{ij}, \sigma_{ij}, \tau_{ij}\}$ is constrained Pareto-efficient, if there does not exist $\{\mu_{ij}^\circ, \nu_{ij}^\circ, \phi_{ij}^\circ, \sigma_{ij}^\circ, \tau_{ij}^\circ\}$, such that*

- (i) $\phi_{ij} \neq \phi_{ij}^\circ$ for at least one pair i, j .
- (ii) For all i , τ_{ij}° is optimal given $\{\mu_{ij}^\circ, \nu_{ij}^\circ, \phi_{ij}^\circ, \sigma_{ij}^\circ, \tau_{ij}^\circ\}$.
- (iii) $\{\mu_{ij}^\circ, \nu_{ij}^\circ, \phi_{ij}^\circ, \sigma_{ij}^\circ, \tau_{ij}^\circ\}$ is a Pareto improvement over $\{\mu_{ij}, \nu_{ij}, \phi_{ij}, \sigma_{ij}, \tau_{ij}\}$.

Proposition 2 *A stationary equilibrium is evolutionarily stable, if and only if it is constrained Pareto-efficient.*

Proof. If a Pareto improvement exists, then a small coalition of agents can increase their welfare by coordinating their strategies on a Pareto-superior intermediation network. Everyone else then has an incentive to play the Pareto-superior strategies, and hence the equilibrium is not stable.

To show the converse, note that by virtue of lemma 3, in any constrained Pareto-efficient steady-state, at most two different goods are used as media of exchange, and if type i uses good m as a medium of exchange, then type $i + 1$ either consumes good m or also uses m as a medium of exchange. Similarly, in a successful deviation, types i and $i + 1$ use the same medium of exchange (different from m). It follows that all N types have to participate in a successful coalition of mutants, and hence be made no worse of than initially. But that is impossible if the equilibrium is constrained Pareto efficient. ■

This result diverges from the main conclusions about search economies without intermediaries, where the continuity of objective functions with respect to strategies implies that small deviations change overall utility only marginally. Changes in the intermediation network may lead to discontinuous changes in payoff, and thus to strategy changes by a large fraction of the population. The second half of proposition 2 critically relies on the assumption of full specialization of production, i.e. good $i+1$ is produced *only* by type i . In the next section, we relax this assumption. In that case, a deviating coalition does not have to rely on the participation of all producer and consumer types, and hence an equilibrium may be Pareto-efficient, but not evolutionarily stable, if the implemented changes lead to welfare losses for agents who do not participate in the change. Some agents may strictly prefer the old equilibrium over the innovation, but once the innovation is introduced, they will change, because their trade partners also start using the new medium of exchange. Loosely speaking, different media of exchange are substitutes, but there are complementarities in using a medium of exchange.

What are the implications for the steady-state equilibria considered in the previous section? As an immediate consequence of lemma 3 and proposition 2, we find:

Proposition 3 *If transaction costs are sufficiently low, the set of evolutionarily stable equilibria is a subset of the Cash-in-Advance and two-money equilibria.*

We thus come to the conclusion that the monetary structure of transactions appears in any evolutionarily stable equilibrium of the previous section. In either case, trading probabilities equal

1, i.e. markets clear. Within the class of Pareto-efficient equilibria, however, evolutionary stability remains silent as a selection criterion: In all minimally coalition-proof equilibria, one type i of agents does not offer any intermediation, and as a consequence, consumes his consumption good in integer units. Due to the assumption of full specialization of consumption and production in this model, a deviating coalition can impose a good as the universal medium of exchange, only if this type participates in the deviation. Among this remaining set of equilibria, an equilibrium is unstable, only if it is dominated for all types, *including the types who benefit from a rent as producers or consumers of a medium of exchange*. Precisely the existence of such rents makes it impossible to break away from some of the two-money and Cash-in-Advance equilibria.

6 General Results

6.1 Less than full Specialization

In this section, I discuss how the previous results are affected by a generalization of the assumptions concerning consumption and production. The point of departure for this discussion is the observation that full specialization of production and consumption choices, i.e. the assumption that type i is the only type to produce good $i + 1$, protects equilibrium rents to money producers and consumers and thereby induces multiple evolutionarily stable equilibria. The following set of assumptions departs from full specialization of production and consumption patterns:

(A0) *There are N goods and $M \geq N$ types of measure $\frac{1}{M}$ of agents. Each type i is characterized by a production good $p(i)$ and a consumption good $c(i)$.*

(A1) *Each good is produced by at least two types.*

(A2) *For each good, the total number of types consuming the good equals the total number of types who produce it.*

(A3) *For every pair of types i, j , if $c(i) = p(j)$, then $p(i) \neq c(j)$.*

(A4) *For every triple of types i, j, k , if $c(i) = p(j)$ and $c(j) = p(k)$, then $c(k) \neq p(i)$.*

(A1) rules out full specialization. Under (A2), this market would clear at relative prices of one for one, if the environment was Walrasian. (A3) introduces the well-known "double coincidence problem", that there are no two types of agents who could just produce for each other. (A4) excludes the possibility of "three-way coincidences", i.e. situations, where a single type could successfully

act as a middleman between two other types. To get exchange off the ground, at least two types must coordinate their transaction activities and agree on one good as a medium of exchange.

Under these assumptions, one notes that the only candidate for an evolutionarily stable pure strategy equilibrium is the Cash-in-Advance equilibrium, in which good 1 is used as a common medium of exchange. All other pure strategy equilibria are destabilized by a small group of players comprising a strict subset of types, who coordinate on using good 1 as a medium of exchange, but don't have to rely on the participation of a type who enjoys an equilibrium rent (formally, the converse of proposition 2 no longer applies). If transaction costs are small, these rents are small, and eventually all types will prefer the more efficient equilibrium trading network.

It remains to be shown that the Cash-in-Advance equilibrium for good 1 exists, and is evolutionarily stable. While existence is immediate, the properties of Cash-in-Advance equilibria do not automatically carry over: As can be shown by example, the transaction probabilities for transactions with intermediaries need not equal 1, and hence the equilibrium may fail to exhibit market-clearing. Intuitively, the imbalance in transaction sequences induced by the use of one good as a common medium of exchange now affects trading probabilities throughout all markets, and the overall frequency of consumption then remains suboptimal. But since some small deviation that uses a more costly good as a medium of exchange can offer its members a higher frequency of consumption, the Cash-in-Advance equilibrium for good 1 will not be evolutionarily stable, if it fails to lead to market-clearing.

6.2 Fiat Money

I now extend the model to allow for the circulation of a fiat money, labelled good 0, and traded with a transaction cost β_0 . Following the search literature, I assume that a fraction S of the population each holds one indivisible unit of fiat money at any point in time. The next proposition discusses the existence and evolutionary stability of a Cash-in-Advance equilibrium for fiat money:

Proposition 4 *Suppose that (i) $S = \frac{1}{3}$ and (ii) $\beta_0 < \beta_1$. Then, under assumptions (A0) – (A4), the Cash-in-Advance equilibrium for fiat money clears markets and is the unique evolutionarily stable steady state equilibrium.*

Proof. Proceeding along the lines of proposition 1, it is straight-forward to show the existence of a Cash-in-Advance equilibrium for fiat money. If $S = \frac{1}{3}$, markets clear exactly, and mark-ups and welfare levels for consumers of good i are given by

$$\sigma_{i0}U = \frac{1}{1 + \delta + \delta^2} [U + \delta\beta_i + \delta^2\beta_0]$$

and

$$(1 - \delta)W_i = \frac{\delta^2}{1 + \delta + \delta^2} \left[\delta(U - \beta_i) - \beta_i - \beta_0 \frac{1 + \delta^2}{\delta} \right].$$

Since no type produces or consumes fiat money, there are no rents associated with its production or consumption. Note that this equilibrium is evolutionarily stable, if and only if $\beta_0 < \beta_1$: In that case, (A2) and (A3) together imply that any coalition that tries to deviate from the fiat money Cash-in-Advance equilibrium has to include one type who is willing to accept a higher-cost good as a medium of exchange, without enjoying a rent as a money producer or consumer. But then he must be made worse off, and no one will be willing to follow his strategy - on the other hand, if $\beta_0 \geq \beta_1$, such a coalition may exist, and successfully deviate from the proposed equilibrium.

Finally, note that in any other equilibrium, (A0) – (A4) imply that there has to exist a subset of types i_1, i_2, \dots, i_n who form a circle, i.e. $c(i_1) = p(i_2)$, $c(i_2) = p(i_3), \dots$, $c(i_n) = p(i_1)$, where neither of these types produces or consumes a medium of exchange in equilibrium. This subset of types can successfully mutate to start using fiat money in a steady-state. ■

This proposition states the central theoretical result of this paper and provides a foundation for a Cash-in-Advance equilibrium with fiat money as the unique evolutionarily stable equilibrium in a decentralized trading economy with intermediaries. The result is tied to a series of conditions, which are arguably of a technical nature. The assumptions (A3) - (A4) rule out possibilities for double or three-way coincidences. Their role is to rule out coordination among a small number of types²². (A1) assumes less than full specialization, which implies that deviations do not have to rely on the participation of commodity money-producer or -consumers, who enjoy equilibrium rents. The condition $S = \frac{1}{3}$ states that the supply of real balances has to equal the demand for transactions purposes. Matsui and Shimizu (2001) show that such a condition arises as the unique evolutionarily

²²With three-way coincidences three types could coordinate a deviation towards a "local" commodity money, in such a way that one type trades twice, but consumes in integer units, while the other two types trade only once. Each type then gets to enjoy a small rent, which might be enough to offset the loss of using a higher-transaction cost medium of exchange. An even simpler argument applies, of course, for double coincidences.

stable steady-state in a related model that makes stronger assumptions about the structure in which transactions take place and notably allows for nominal price adjustments. Finally, the condition that $\beta_0 < \beta_1$ states that it has to be desirable from an individual perspective to use fiat money as a medium of exchange - alternatively, individuals always have an incentive to use commodities for indirect exchange to save on transaction costs.

Proposition 4 should not be read as a statement that rules out the observation of anything but a fiat money equilibrium in a steady-state. Rather, it states that as the long-run outcome of an evolutionary process, in which intermediaries play a central role in coordinating transaction strategies, one should expect the most efficient transactions arrangement (in this case a fiat money equilibrium) to prevail. Since the proposition emphasizes the uniqueness of the long-run outcome, it also contrasts with the multiplicity of equilibria within Walrasian and search models of monetary exchange, a 'problem' that was first discussed by Hahn (1965). This multiplicity of equilibria is a general feature of infinite horizon economies, in which the optimality of current trading strategies depends on the expectation of future trading opportunities. Evolutionary stability implicitly assumes that intermediaries can resolve this intertemporal coordination problem, so that every producer expects to be able to trade money for consumption goods in the future, and hence is willing to acquire money in the current period. Proposition 4 remains silent about how the long-run equilibrium is reached, or what is observed in the interim stages. In this respect, the multiplicity of equilibria retains its relevance, as many of the observations made within the context of the search literature, or earlier in this paper with respect to commodity money, remain relevant as descriptions of intermediate stages of the evolutionary process, or as the consequence of aggregate shocks leading to a temporary break-down of market institutions and intermediation.

Nor should this proposition be viewed as stating that the final stage of the evolutionary process of market organization will be a fiat money equilibrium as the one described here. Indeed, one of the constants of the history of market organization and transactions is innovation and change, and virtually every innovation is promoted or coordinated by some kind of intermediary. The introduction of credit cards and other cashless means of transactions for example can be viewed as a move away from cash towards more efficient alternatives.

6.3 Free Banking

While the previous section discussed the existence and uniqueness of a fiat money Cash-in-Advance equilibrium as an evolutionarily stable steady-state, it does not consider the *emergence* of fiat money. Existing search-theoretic models also remain silent about this questions, since they consider steady-state equilibria, where fiat objects have been around forever in the past, and are valued, because they are expected to be valued forever into the future. The purpose of this section is to illustrate how fiat money may *come into circulation* in a "free banking" equilibrium, and to further discuss the conditions under which a fiat object becomes a generally accepted medium of exchange. For this purpose, I adapt the model by enabling intermediaries to issue debt certificates on which they promise to pay a unit of physical goods in the future.

To be specific, suppose that every intermediary has the ability to write out demandable debt claims, i.e. notes that are backed by her inventory of goods, and that are sold in transactions. Under what conditions do these notes start to circulate as media of exchange, and become perfect substitutes from the producers' perspective? To make such a system viable, it is necessary that in steady-state, intermediaries have an incentive to discipline their note issue, and don't overissue notes to default in the future. This incentive compatibility requires that notes eventually return to their issuer. Again for the sake of concreteness, I start by assuming that this occurs at the end of each period, when all intermediaries participate in a clearing market, where they return any notes to the initial issuer. Below, I will also take into consideration other clearing mechanisms, as well as different assumptions concerning note-issuing privileges.

Given a transactions cost of β_0 for accepting notes (and assuming that there are no costs involved in the clearing process), proposition 4 characterizes the Cash-in-Advance equilibrium, *provided that intermediaries have an incentive to refrain from overissuing*. The equilibrium behavior of intermediaries and the circulation of notes is characterized as follows: Each period begins with half of the producers holding their production good (those who previously consumed) and half of the producers holding fiat object, i.e. a note issued by the intermediary to whom they sold their production in the previous period. Intermediaries begin each period with one note outstanding and a reduced unit of their consumption good. They then sell this unit in exchange for the bank notes held by producers who purchase their consumption good, and withdraw this note from circulation. Afterwards, they purchase an integer unit of their consumption good from some producer who

wishes to sell his production, and pay for it by issuing a new note. After all transactions with producers are completed, intermediaries meet in a clearing market and exchange the notes they withdrew from circulation. Since every intermediary had one note outstanding, the market clears, and the following period begins with each intermediary having one note outstanding.

Under the conditions of proposition 4, such a free banking equilibrium is evolutionarily stable and may account for the emergence of a fiat object, if it provides intermediaries with the right incentives to participate in the clearing mechanism and not overissue. These incentives depend on the comparison between the short-term gains from overissuing, and the potential long-run punishment in case of default. In the present case, if an intermediary decides to overissue and default, she can issue one note during one period, and not accept someone else's note in return for her consumption good; she is then found out at the end of the period, when she fails to clear her note in the clearing market. Her short-term benefit is then equal to $\beta_0 + (1 - \sigma_{i0})U$. The cost of default depends on the punishment structure. In the most lenient case, this punishment might simply involve the loss of her note-issuing privileges; given the open entry condition, the intermediary could become a producer, and continue without any welfare losses.²³ We hence observe that the post-default punishment must involve the loss of market access privileges beyond a simple loss of note-issuing. The most severe punishment might involve total exclusion from the market (as a producer or an intermediary) for all future periods - which would amount to the loss of all future consumption. Whatever the punishment mechanism, the cost of punishment must exceed the short-term gains of over-issuing.

Under alternative note-issuing and clearing arrangements, notes may not be redeemed immediately, but circulate for several periods. This may happen, for instance, if the clearing market opens only infrequently, and instead notes are returned to the producers within the same period. Alternatively, only a limited number of types may issue and clear notes, and hence notes stay with the public or non-issuing intermediaries for several periods until they are returned to the issuer. To provide a specific example, return to the environment studied initial, where type i produces

²³This statement relies critically on the existence of open entry into intermediation, i.e. note-issuing. A related discussion by Cavalcanti, Erosa and Temzelides (1999) of private note-issuing in a search model discusses a stable private money equilibrium relying solely on the withdrawal of note-issuing privileges as an incentive mechanism, but in their environment there is no open entry into the banking sector, i.e. note-issuing privileges are exogenously fixed, and default leads to the loss of seignorage (i.e. essentially scarcity) rents. See also Hellwig and Lorenzoni (2003), who discuss reputational mechanisms for sustaining note circulation in a Walrasian equilibrium with borrowing constraints.

good $i + 1$. If notes are issued only by type 1 intermediaries (and only type 1 agents participate in the clearing), these notes circulate for N periods before they return to the initial issuer (From type 2 to type 3 to type 4 to... until they reach type N producers, and then type 1). In that case, a single producer might be willing to become a "rogue" intermediary, and start issuing notes, without ever exchanging back the consumption units. Since the notes take N periods until they return to the issuer, the intermediary does not have to redeem any notes immediately. Hence, it takes N periods to detect someone who overissues notes, and the short-run benefit of over-issuing increases. Similarly, the time of circulation of a note increases, when note clearing takes place less often, or not at all. Since an overissuing intermediary is detected only once the note is redeemed, the short-run benefits of over-issuing are proportional to the expected time of circulation of the notes.

While far from complete as a theory of free banking, this discussion points to some of the features that determine the stability of a free banking system. Clearing arrangements and the length of time a note circulates determine the short-run gains from over-issuing. These short-run benefits are contrasted with the long-run costs of default, determined by the harshness of punishment, as well as the potential loss of seigniorage or monopoly rents. Note that the clearing arrangements have no direct allocational purpose, but simply serve to decentralize the monitoring of the intermediaries' note-issuing activities. In this sense, the note-issuing and clearing has the purpose of providing a "memory" of economic transactions, for both intermediaries and producers. Finally, the model points to a coordination problem that arises in the clearing of notes: the equilibrium described above relied on the participation of intermediaries in the clearing market, and this participation was individually rational, since in order to clear her note, the intermediary had to return a note she collected to the initial issuer. There is, however, an alternative equilibrium, in which all intermediaries, instead of returning the notes they collected to the issuer, decide to return them to the producers within the same period, or withdraw them without clearing them, keeping them as reserves. Since no intermediary is clearing any notes, there is no reason for any intermediary to participate in the clearing, and if clearing notes is associated even with a small cost, intermediaries collectively prefer the no-clearing equilibrium. But then, the clearing market ceases to perform its monitoring role, and some intermediaries may find it optimal to default. Summing up, the model suggests that free banking regimes are stable, when:

- (i) Notes circulate only for short periods, and quickly return to the issuer through a well-

functioning clearing system, and

(ii) The loss of seigniorage or monopoly rents, or legal punishments provide incentives not to default.

In this respect, one notes the specific role of a government in promoting monetary exchange: the previous section highlighted the necessity for intertemporal coordination in sustaining a unique stable fiat money equilibrium. This coordination critically relies on the provision of incentives that commit a note issuer to honor her promises in the future, and not hold note-holders up by defaulting. In a free banking environment, the government's role consists precisely in providing an institutional framework that solves this hold-up problem and creates this commitment through legal provisions that punish default on privately issued notes.

I conclude this discussion with a brief review of some historical free banking episodes. Proponents of free banking typically point to Scotland as a country where free banking was extremely stable throughout several centuries. As is extensively discussed by Smith (1936) in her classic analysis of free banking, the Scottish banking system indeed fulfills the conditions laid out here, providing the most clear-cut example: Although labelled as "free" banking, the banking sector really had an oligopolistic, almost cartell-like structure with a small number of large players. These bankers met on a very regular (weekly) base to clear notes, and notes stayed in circulation for short time periods only. In addition, they were subject to unlimited liability in case of a default. Over a stretch of approximately three centuries, Scotland had virtually no banking panics or defaults. Another example of free-banking success was the Suffolk bank system in nineteenth century New England, described for example by Smith and Weber (1999). The Suffolk Bank, one of the biggest note-issuing banks in the area, internalized the cost of running a clearing market by accepting the notes of other banks at parity, if these banks made a large up-front deposit. Other regions in America did not have as sophisticated clearing mechanisms during the free banking era in the nineteenth century, and thus had longer times of note circulation, and coupled with a legal system that made default more acceptable than in most European countries and free entry into banking, this lead to a higher degree of instability, banking panics, and defaults.

Neldner's (1998) description of Switzerland during the nineteenth century provides an intriguing example for the ultimate failure of a free banking system despite initial success. Although the system performed reasonably well by all conventional accounts (even though highly competitive, it

was unusually safe, didn't lead to bank panics or failures throughout almost the entire century, and no noteholder ever incurred a loss due to default), it ultimately failed and was replaced by the Swiss National Bank in 1907. According to Neldner, while the system initially was very successful, during the last third of the nineteenth century, it suffered from overissuing of notes and a malfunctioning of the clearing market, even though very sophisticated clearing arrangements did exist. During this time period, the position of note-issuing banks was weakened by the arrival of non-issuing commercial banks, who held a competitive advantage in the market for loans. This gradually led note-issuing banks to reduce the clearing of notes, in fear of receiving their own notes in return, and having to pay in species. Within a very competitive environment, this led to an overissue of notes, and ultimately the deterioration of the exchange rate towards the French franc and an outflow of species from the country. The banks, however, did not return the notes which they received in exchange for the species to the issuer, preferring to return them directly to the market, and thus further slowing down the clearing process. While the exact causal link between these events is not entirely clear from Neldner's analysis, one possible interpretation might be that a weakening of the note-issuing banks led them to gradually reduce the clearing activities, i.e. switch from one equilibrium in the clearing market to another, in a collective attempt to maintain their economic viability.²⁴

7 Conclusion

This paper studied a decentralized trading economy in which intermediaries induce the use of a common medium of exchange. As such, intermediation and money are complementary phenomena. Strategic interaction of intermediaries leads to a Cash-in-Advance constraint, such that trade sequences with intermediaries follow the observed pattern that “goods buy money and money buys goods, but goods don't buy goods” (Clower, 1965). As opposed to many other models of monetary exchange, this pattern comes as an equilibrium outcome and not an assumption of the model. The second central result is that the characteristics of a monetary equilibrium with intermediaries differ fundamentally from those of equilibrium models without intermediaries. By coordinating its deviations, a small coalition of intermediaries can induce producers to use transaction strategies

²⁴Remarkably, the note-issuing banks were willing to collectively restrain from clearing notes, but they were unable to coordinate to limit the amount of note-issuing, even though this would have improved their competitive position relative to the commercial banks.

that ultimately lead to an efficient equilibrium. Under some additional conditions, the unique evolutionarily stable equilibrium leads to a Cash-in-Advance constraint for a fiat currency. I further study how this equilibrium can be implemented in a free banking equilibrium, in which fiat money is brought into circulation as debt certificates issued by intermediaries.

A series of questions are not properly addressed within this framework. Most importantly, I have abstracted from the problem of embedding a theory of prices into this decentralized trading economy, assuming that transactions take place at the implicit Walrasian prices. Under what conditions these prices prevail in a search or otherwise non-walrasian economy is an open question, since within each transaction, prices would be determined through some bilateral bargaining process, and hence also depend on the trading partners' outside options, which in turn depend on the trading environment. As discussed in the context of proposition 1, the liquidity demand for the medium of exchange creates some inherent price distortions away from the Walrasian equilibrium. Furthermore, as in many related models, production and consumption choices remain outside the model, since they are exogenously given in such a way that in a frictionless economy, all markets would clear at the relative price of 1. This assumption is problematic in an environment where trade is subject to frictions, since decision-makers would take into account their opportunities for trade when they decide what goods to produce. They may decide to produce one good because it is easy to trade, even though they are more efficient at producing a different, less marketable good. In a companion paper, I address this issue and show that the elimination of market frictions and asymmetries in marketability between goods by intermediaries in a fiat money equilibrium provides incentives for efficient production decisions.

Finally, the model presented here relies on some ad hoc assumptions about intermediation. An earlier working paper version of this paper augmented the model by an explicit inventory accumulation problem for intermediaries, but arrived at almost identical conclusions. Related papers by Howitt (2000) and Starr (1999) in an environment without search further show that increasing returns to scale in intermediation may also lead to Cash-in-Advance equilibria, even when the lack of double- or triple coincidence is not complete. Another technological assumption is the restriction of intermediaries to carry out transactions between exactly a pair of goods. Again, Howitt (2000) shows that this assumption can be relaxed. It is essential, however that the activity of an intermediary is affected by the need of some traders to carry purchasing power from one intermediary to another. Most importantly, the evolutionary stability arguments rely on the assumption that

intermediaries are immediately accessible to producers; this effectively enables them to coordinate on an arbitrarily small scale. While this paper takes the stand that such coordination will eventually occur, whenever it is feasible, questions arise as to how this coordination takes place, and, in relation to that, about the government's role in promoting an efficient trading arrangement.

Despite these technical shortcomings, the results presented here provide some general insights into the role of intermediation. The complementarity of the medium of exchange and intermediation and the non-stability of inefficient transaction patterns both follow from three basic assumptions about the environment:

- (i) A Pareto-optimal, market-clearing allocation, which would result from a competitive equilibrium in perfect markets, cannot be attained because of a form of market imperfection,
- (ii) some agents have a technology to alleviate the imperfection by offering intermediation, and by offering this technology to the economy, they can make arbitrage profits from a price spread, and
- (iii) the success of intermediaries depends on how they can deal with their own constraints.

In general, there are many reasons for frictions in a competitive economy, and the many facets and different forms of intermediation all respond to these imperfections. Here, I have considered search frictions as the underlying imperfection, however, one might try to apply the same logic to study how intermediation interacts with decentralized market instruments to alleviate other frictions, such as the lack of public memory, informational asymmetries, contracting constraints or other forms of credit market frictions. When these forms of market imperfections arise, intermediation performs a matching service between both sides of the market, for which a price spread is charged. The success of intermediaries depends mostly on appropriating a large volume of transactions, and on establishing a repeated, credible interaction with their customers. This transfers the problems of price-setting and market allocation to the intermediation sector. Many features traditionally attributed to competitive markets, such as market clearing, the use of money and Cash-in-Advance constraints, can thus be explained as being in the interest of intermediaries who organize the market so as to alleviate an imperfection and take arbitrage gains from it.

The results in this paper also have some implications for existing Walrasian macroeconomic and monetary theory. The intermediation model combines frictionless market transactions a la Walras

with an explicit, bilateral structure of exchanges. It thereby provides a channel, by which price-setting and information transmission can plausibly be discussed (although this is beyond the scope of this paper). The model further provides an evolutionary approach towards the development and structure of competitive markets. Extensions and simplifications of the intermediation model may thus prove useful in analyzing questions in monetary and macroeconomic theory for which the existing theory has come to its limits due to the ad hoc structure of markets and monetary exchange.

References

- [1] S. R. Aiyagari and N. Wallace, "Existence of Steady-States with Positive Consumption in the Kiyotaki-Wright Model", *Review of Economic Studies* **58** (1991), 901-916.
- [2] S. R. Aiyagari and N. Wallace, "Fiat Money in the Kiyotaki-Wright Model", *Economic Theory* **2** (1992), 447-464.
- [3] A. Banerjee and E. Maskin "A Walrasian Theory of Money and Barter", *Quarterly Journal of Economics* **111** (1996), 955-1005.
- [4] W. Baumol, "The Transactions Demand for Cash", *Quarterly Journal of Economics* **67** (1952), 545-556.
- [5] R. Cavalcanti, A. Erosa, and T. Temzelides, "Private Money and Reserve Management in a Random Matching Model", *Journal of Political Economy* **107** (1999), 929-945.
- [6] R. W. Clower, "A Reconsideration of the Microfoundations of Monetary Theory", *Western Economic Journal* **6** (1965), 1-9.
- [7] D. Corbae, T. Temzelides, and R. Wright, "Matching and Money", University of Pennsylvania, 2000.
- [8] D. Diamond and R. Rajan, "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking" *Journal of Political Economy* **109** (2000), 287-327.
- [9] A. Dixit, "On Modes of Economic Governance", Presidential address given at the Econometric Society European Meeting, Lausanne, August 2001.

- [10] T. Gehrig, "Intermediation in Search Markets", *Journal of Economics and Management Strategy* **2** (1993), 97-120.
- [11] F. Hahn, "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy", in Hahn and Brechling, eds., "Theory of Interest Rates", London: MacMillan, 1965.
- [12] C. Hellwig, "Money, Intermediaries, and Cash-in-Advance constraints", *FMG discussion paper* **349**, London School of Economics, 2000.
- [13] C. Hellwig, "Production and Exchange in a decentralized trading economy", U.C.L.A., work in progress.
- [14] C. Hellwig and G. Lorenzoni, "Bubbles and Private Liquidity", Princeton University, 2002.
- [15] M. Hellwig, "The Challenge of Monetary Theory", *European Economic Review* **37** (1993), 215-242.
- [16] P. Howitt, "Beyond Search: Fiat Money in Organized Exchange", Brown University, 2000.
- [17] P. Howitt and R. W. Clower, "The Emergence of Economic Organization", *Journal of Economic Behavior and Organization* **41** (2000), 55-84.
- [18] K. Iwai, "The Evolution of Money: A Search-Theoretic Foundation of Monetary Economics", *CARESS working paper* **88-03**, University of Pennsylvania, 1988.
- [19] W. Jevons, "Money and the Mechanism of Exchange", London: Appleton, 1875.
- [20] N. Kiyotaki and R. Wright, "On Money as a Medium of Exchange", *Journal of Political Economy* **97** (1989), 927-954.
- [21] N. Kocherlakota, "Money is Memory", *Journal of Economic Theory* **81** (1998), 231-251.
- [22] R. Lucas, Jr., and N. Stokey, "Money and Interest in a Cash-in-Advance Economy", *Econometrica* **53** (1987), 491-514.
- [23] A. Matsui and T. Shimizu, "A Theory of Money with Market Places", University of Tokyo, 2001.

- [24] K. Menger, "On the Origin of Money", *Economic Journal* **2** (1892), 239-255.
- [25] M. Neldner, "Lessons from the Free Banking Era in Switzerland: The Law of Adverse Clearings and the Role of the Non-issuing Banks", *European Review of Economic History* **2** (1998), 289-308.
- [26] R. Radford, "On the Economic Organisation of a P.O.W. Camp", *Economica* **12**, 189-201.
- [27] D. Romer, "A Simple General Equilibrium Version of the Baumol-Tobin Model", *Quarterly Journal of Economics* **101** (1986), 663-686.
- [28] A. Rubinstein and A. Wolinsky, "Middlemen", *Quarterly Journal of Economics* **102** (1987), 581-593.
- [29] P. Samuelson, "An Exact Consumption Loan Model of Interest with or without the Social Contrivance of Money", *Journal of Political Economy* **66** (1958), 1002-1011.
- [30] M. Sidrauski, "Rational Choice and Patterns of Growth in a Monetary Economy", *American Economic Review* **57** (1967), 534-544.
- [31] B. Smith and W. Weber, "Private Money Creation and the Suffolk Banking System", *Journal of Money, Credit, Banking* **31** (1999), 624-659.
- [32] V. Smith, "The Rationale of Central Banking and the Free Banking Alternative", Ph.D. Dissertation, London School of Economics, 1936, reprinted by Liberty Press, 1990.
- [33] R. Starr, "Why is there Money? Convergence to a Monetary Equilibrium in a General Equilibrium Model with Transaction Costs", *discussion paper 99-23*, Dept. of Economics, U.C. San Diego, 1999.
- [34] L. Svensson, "Money and Asset Prices in a Cash-in-Advance Economy", *Journal of Political Economy* **93** (1985), 919-944.
- [35] J. Tobin, "The Interest Elasticity of the Transactions Demand for Cash", *Review of Economics and Statistics* **38** (1956), 241-247.