

# IMPERFECT COMMON KNOWLEDGE OF PREFERENCES IN GLOBAL COORDINATION GAMES\*

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## Abstract

I study a simple global game, in which I relax the assumption that preferences are common knowledge. I show that with higher-order uncertainty regarding preferences, players can at best coordinate on the risk-dominant equilibrium, regardless of whether information about fundamentals is common or private. The example also suggests that the strength of the coordinating effect of public information depends positively on the degree of common knowledge regarding preferences. I conclude by reinterpreting recent experimental results in the light of this example, and suggest alternative experimental designs to test for the effects of informational assumptions, as well as the importance of common knowledge regarding preferences.

## 1 Introduction

The purpose of this note is to highlight the importance of the assumption made commonly in the context of global coordination games and its applications that the preferences of all players are common knowledge. A series of papers have used this assumption to study the effects of incomplete information in coordination settings. This entire strand of literature has started out from Carlsson and van Damme's (1993) uniqueness result, which states that if players observe the payoffs of a coordination game with idiosyncratic noise, then the game has a unique equilibrium, in which players play the risk-dominant equilibrium. This uniqueness result has been extensively

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used in applications, such as Morris and Shin's (1998) discussion of currency crises, or in bank runs (Goldstein and Pauzner, 2000; Rochet and Vives, 2002). The literature has further examined the effects of various sources of information on the degree of coordination that is achieved; hence Morris and Shin (2001) and Hellwig (2002) emphasize the coordinating effect that is associated with public information, and recent work by Angeletos, Hellwig and Pavan (2002) discusses issues of signaling and endogenous revelation of information through policy choices in coordination games. Morris and Shin (2000) provide a detailed overview of many of these and related papers.

This entire literature takes common knowledge of preferences as given. Here, I study a simple example to discuss the effects of a lack of common knowledge of preferences. My main objective is to show that higher-order uncertainty regarding preferences leads to similar effects as higher-order uncertainty regarding payoffs: since players face higher-order uncertainty about their respective willingness to coordinate on the riskier payoff-dominant equilibrium, coordination will be less than the efficient level, even if payoffs are common knowledge. To draw this conclusion, I study a simple coordination game in four different treatments regarding information (common or private) about either fundamentals or the degree of risk aversion. It turns out that in order to achieve full coordination on the payoff dominant equilibrium, common knowledge of both fundamentals and preferences is necessary; if there is higher-order uncertainty about either, play reverts to risk-dominant strategies, however, the risk-dominant strategies do depend on the underlying risk that individuals are facing (and which varies from one environment to another). In a more general environment, where players have access to a mix of information regarding either dimension of uncertainty, a complementarity seems to exist in the sense that the higher the degree of common knowledge of *preferences*, the higher the sensitivity of equilibrium strategies to informational parameters regarding *fundamentals* (and vice versa).

The discussion of higher-order uncertainty regarding preferences depends on the assumption that play is subject to small trembles; in the absence of trembles, players effectively do not face any risk in equilibrium, and hence could in principle coordinate on the payoff-dominant equilibrium; only in the presence of trembles does higher-order uncertainty regarding preferences have an effect on the players behavior.

This short paper was initially motivated by a careful reading of experimental results which tried to test for the coordinating effect of common information (Heinemann, Nagel and Ockenfels,

2002). Their study, as well as another one by Cabrales, Nagel and Armenter (2000), comes to the conclusion that there is little difference in the degree to which players coordinate between an environment with public and an environment with private information. While such a conclusion seems at first sight contrary to the existing theory, these experimental results are perfectly in line with the theoretical predictions, when common knowledge of preferences is violated. After discussing my main theoretical example, I therefore briefly reinterpret the results of Heinemann, Nagel and Ockenfels in the light of this example, before suggesting a modification of the experimental setting that might control for information regarding preferences, and hence might serve as a true test for the coordinating effect of public information that is suggested by the theory. I conclude by discussing some issues that arise when the informational conclusions are carried into economic applications.

## 2 The model

Consider the following two-player, two-action game, where monetary payoffs are given by:

	$R$	$S$	
$R$	$\theta \quad \theta$	$\theta - 1 \quad 0$	
$S$	$0 \quad \theta - 1$	$0 \quad 0$	

where  $R$  denotes the risky action, and  $S$  the safe action. Depending on values of  $\theta$ , this game has either a unique or multiple equilibria; if  $\theta$  is common knowledge among the players, one observes immediately that, if  $\theta \in [0, 1]$  both  $(R, R)$  and  $(S, S)$  are equilibria, while if  $\theta < 0$ ,  $(S, S)$  is the unique equilibrium, and if  $\theta > 1$ ,  $(R, R)$  is the unique equilibrium.

In this note, I am concerned with the equilibrium outcomes, when preferences are not common knowledge (and when  $\theta$  may or may not be common knowledge). To be specific, I assume that players maximize expected utility. A realized payoff of  $w$  yields utility

$$U_i(w) = 1 - e^{-\gamma_i w}$$

To provide a specific structure, assume that some  $\hat{\gamma}$  is drawn from a uniform distribution over the positive real numbers  $[0, \infty)$ , and that  $\gamma_i = \hat{\gamma} + \eta_i$ , where  $\eta_i$  is uniformly distributed over  $[-k, +k]$ . Here, I consider two different informational assumptions: In both cases,  $\hat{\gamma}$  is unobserved. In the first case, however, both players observe  $\gamma_1$  and  $\gamma_2$ , that is the degrees of risk aversion are

common knowledge. This treatment is contrasted with the private information treatment, where each player observes only his own degree of risk aversion. The distributional assumptions are made for simplicity; nothing important hinges on the distributional assumptions concerning  $\gamma_i$ , other than the lack of common knowledge about risk aversion in the second treatment.

To complete the description, suppose that each player has access to a signal  $x_i$  concerning  $\theta$ ; i.e.  $x_i = \theta + \xi_i$  for  $i \in \{1, 2\}$ . Suppose further that  $\xi_1$  and  $\xi_2$  are independently, normally distributed with mean 0 and variance  $\sigma^2$ . When  $\theta$  is taken to be common knowledge, then  $\sigma = 0$  and  $x_1 = x_2 = \theta$ .

For further analysis, it will be useful to derive the dominance function  $p(x, \gamma)$  for this game; a player with degree of risk aversion  $\gamma$  will find it optimal to play  $R$  after observing a signal  $x$ , whenever he believes that the other player will play  $R$  with a probability at least as large as  $p(x, \gamma)$ , and he will prefer  $S$ , whenever his belief that the other player plays  $R$  falls below  $p(x, \gamma)$ . To derive  $p(x, \gamma)$ , let  $f(x, \gamma, p)$  denote the expected utility of playing  $R$ , given  $x$  and  $\sigma$ , and a belief  $p$  that the other player plays  $R$ ;  $f(x, \gamma, p)$  is given by

$$\begin{aligned} f(x, \gamma, p) &= pE(U_i(\theta) | x) + (1-p)E(U_i(\theta-1) | x) \\ &= 1 - (p + (1-p)e^\gamma)e^{-\gamma(x - \frac{1}{2}\gamma\sigma^2)} \end{aligned}$$

$p(x, \gamma)$  is then defined implicitly by  $f(x, \gamma, p) = 0$ ; explicitly, this leads to

$$p(x, \gamma) = 1 - \frac{e^{\gamma(x - \frac{1}{2}\gamma\sigma^2)} - 1}{e^\gamma - 1}$$

Note that  $p(x, \gamma)$  is increasing in  $\gamma$  and decreasing in  $x$ ; in words, the more risk averse a player is the higher has to be his belief that the other player plays  $R$ , in order to induce him to play risky. And the higher his ex post expected return to playing  $R$ , the lower the threshold probability at which  $R$  becomes optimal. To find the signs of the partial derivatives of  $p(x, \gamma)$  with respect to  $x$  and  $\gamma$ , note that

$$\frac{\partial p(x, \gamma)}{\partial \gamma} = (1 - p(x, \gamma)) \left[ \frac{e^\gamma}{e^\gamma - 1} - \frac{(x - \frac{1}{2}\gamma\sigma^2)e^{\gamma(x - \frac{1}{2}\gamma\sigma^2)}}{e^{\gamma(x - \frac{1}{2}\gamma\sigma^2)} - 1} \right]$$

and since

$$\begin{aligned} \frac{\partial}{\partial w} \left( \frac{we^{\gamma w}}{e^{\gamma w} - 1} \right) &= \frac{we^{\gamma w}}{e^{\gamma w} - 1} \left[ \frac{1 + \gamma w}{w} - \frac{\gamma e^{\gamma w}}{e^{\gamma w} - 1} \right] \\ &= \frac{e^{\gamma w}}{e^{\gamma w} - 1} \left[ 1 - \frac{\gamma w}{e^{\gamma w} - 1} \right] \\ &> 0, \end{aligned}$$

it follows that  $\frac{\partial p(x, \gamma)}{\partial \gamma} > 0$ . Furthermore,

$$\frac{\partial p(x, \gamma)}{\partial x} = -\frac{\gamma e^{\gamma(x - \frac{1}{2}\gamma\sigma^2)}}{e^\gamma - 1} < 0.$$

I conclude the introduction of this example by noting that nothing material hinges on the assumptions regarding payoffs and preferences. All of the results below qualitatively rely on the two monotonicity properties of  $p(x, \gamma)$  that I derived above. In principle, any coordination game, in which the dominance function  $p(x, \gamma)$  exhibits the same properties will lead to identical results. In this sense, the ordering of preferences by risk aversion could be replaced by an ordering of more complex behavior regarding uncertainty, in order to account for regret, loss aversion or other psychological biases - all of these could in principle be modelled by some mapping of pay-offs into a corresponding utility function  $U_i(\theta, \gamma; a)$  that is defined as the ex post utility given preferences  $\gamma$ , a payoff parameter  $\theta$ , and an action  $a \in \{R, S\}$  of the other player.

## 2.1 Common Knowledge of Risk Aversion and Fundamentals

If the degrees of risk aversion are common knowledge, the previous statement regarding multiplicity when  $\theta$  is common knowledge is not affected by the introduction of risk aversion, since players are not facing any risk in equilibrium.  $(R, R)$  and  $(S, S)$  remain pure strategy equilibria, whenever the strategies  $R$  (resp  $S$ ) are not strictly dominated, and multiple equilibria exist for any  $\theta \in [0, 1]$ . Note also for further reference that these conclusions are not affected when trembles are introduced.

## 2.2 Higher-order uncertainty about fundamentals

In this subsection, I briefly review the main result of Carlsson and van Damme, as it applies in this context. For this purpose, assume that the degrees of risk aversion are common knowledge,

but players have conditionally independent, noisy observations about  $\theta$ . Under these assumptions, Carlsson and van Damme have shown that there exists a unique equilibrium, in which player  $i$  plays risky, if and only if his signal falls above a certain threshold  $\tilde{x}_i$ . The equilibrium thresholds satisfy the pair of equations

$$\begin{aligned}\Pr(x_1 > \tilde{x}_1 \mid \tilde{x}_2) &= p(\gamma_2, \tilde{x}_2) \\ \Pr(x_2 > \tilde{x}_2 \mid \tilde{x}_1) &= p(\gamma_1, \tilde{x}_1)\end{aligned}$$

If, in addition,  $\gamma_1 = \gamma_2 = \gamma$ , we can use symmetry, as well as the fact that the information structure implies  $\Pr(x_{-i} > \tilde{x} \mid x_i = \tilde{x}) = \frac{1}{2}$  for all  $\tilde{x}$ , to further characterize the equilibrium. The equilibrium threshold  $\tilde{x}$  then satisfies

$$\frac{1}{2} = p(\gamma, \tilde{x}) = 1 - \frac{e^{\gamma(\tilde{x} - \frac{1}{2}\gamma\sigma^2)} - 1}{e^\gamma - 1}$$

or

$$1 + e^\gamma - 2e^{\gamma(\tilde{x} - \frac{1}{2}\gamma\sigma^2)} = 0$$

This condition implicitly defines an equilibrium threshold  $\tilde{x}(\gamma)$  as a function of  $\gamma$ . Solving for  $\tilde{x}$  yields

$$\tilde{x}(\gamma) = \frac{1}{2}\gamma\sigma^2 + \frac{1}{\gamma} \log\left(\frac{1 + e^\gamma}{2}\right)$$

The first component in the expression for the threshold measures the adjustment of the certainty equivalent of  $\theta$  for the risk due to signal noise. The second component measures the ability to coordinate - note that as in the standard treatment without risk aversion, the unique equilibrium leads to the play of risk-dominant strategies, i.e. each player maximizes his expected utility assuming that the other player will play each strategy with probability  $\frac{1}{2}$ . From the discussion of the partial derivatives of  $p(\gamma, \tilde{x})$ , note that  $\tilde{x}(\gamma)$  is increasing in  $\gamma$ . I conclude by characterizing the limiting properties of this threshold function, as  $\gamma \rightarrow 0$ , and  $\gamma \rightarrow \infty$ . Applying Bernoulli-L'Hospital's rule, as  $\gamma \rightarrow 0$ , yields

$$\lim_{\gamma \rightarrow 0} \tilde{x}(\gamma) = \lim_{\gamma \rightarrow 0} \frac{e^\gamma}{1 + e^\gamma} = \frac{1}{2}$$

As  $\gamma \rightarrow \infty$ ,  $1 + e^\gamma - 2e^{\gamma w} = 1 + e^{\gamma w} [e^{\gamma(1-w)} - 2] \rightarrow \infty$  whenever  $w < 1$ , and  $1 + e^\gamma - 2e^{\gamma w} \rightarrow -\infty$ , as  $\gamma \rightarrow \infty$ , whenever  $w > 0$ . It follows that  $\lim_{\gamma \rightarrow \infty} (\tilde{x}(\gamma) - \frac{1}{2}\gamma\sigma^2) = 1$ . Thus, the certainty equivalent of the signal converges to 1; consequently  $\lim_{\gamma \rightarrow \infty} \tilde{x}(\gamma) = \infty$ .

Starting from this uniqueness result, the global games literature has derived a series of results concerning the effects of public and private information, in particular that better public information improves coordination, in the sense that it enables players to coordinate on the risky strategies, for signal values (and consequently fundamental realizations) that are closer to the lower bound 0, above which  $(R, R)$  payoff-dominates  $(S, S)$ . The degree to which the players are able to coordinate on the payoff-dominant equilibrium depends continuously on the composition of the information structure.

### 2.3 Higher-order uncertainty about risk aversion, common knowledge of fundamentals

In this subsection, I remove the assumption that the degrees of risk aversion are commonly known, and assume instead that each player only observes his own  $\gamma_i$ . However, I revert to the assumption that  $\theta \in (0, 1)$  is common knowledge among players. If each player has perfect control over his actions, then both  $(R, R)$  and  $(S, S)$  remain Nash equilibria, since it is common knowledge that for both players, the best response to  $R$  is  $R$ , and the best response to  $S$  is  $S$ , *independently of their respective degrees of risk aversion*. Since in equilibrium, no player faces any uncertainty, higher-order uncertainty about risk aversion has no effects.

However, the set of equilibria changes dramatically, if we add small trembles into the model. Suppose that whenever a player intends to play  $R$ , he plays  $R$  with probability  $1 - \varepsilon$ , and  $S$  with probability  $\varepsilon$ . One immediately observes that the "safe" equilibrium continues to exist, at least for values of  $\theta \leq 1 - \varepsilon$ . To see this, note that if all  $\gamma_i$  intend to play  $S$ , the expected payoff to  $R$  is  $\theta - 1 + \varepsilon$ , which is negative whenever  $\theta \leq 1 - \varepsilon$ . Hence, the certainty equivalent payoff to  $R$  also has to be strictly negative for any  $\gamma_i \geq 0$ , and all  $\gamma_i$  strictly prefer  $S$ .

The same does not apply to the "risky" equilibrium: clearly, for any  $\varepsilon > 0$ , there exists  $\gamma^*$ , such that  $\gamma \geq \gamma^*$  implies  $f(\theta, \gamma, p) < 0 \forall p \in [\varepsilon, 1 - \varepsilon]$ , i.e. playing  $R$  is a strictly dominated strategy for any  $\gamma \geq \gamma^*$ . We have thus established the existence of an upper dominance region in the space of coefficients of absolute risk aversion.

In any equilibrium, let  $(\tilde{\gamma}_1, \tilde{\gamma}_2)$  denote a pair of thresholds for the degrees of risk aversion, such that player  $i \in \{1, 2\}$  never plays risky when  $\gamma_i > \tilde{\gamma}_i$ . It immediately follows from the monotonicity of  $p(\theta, \gamma)$  with respect to  $\gamma$ , coupled with the existence of the upper-dominance region, that  $(\tilde{\gamma}_1, \tilde{\gamma}_2)$  has to satisfy the following pair of inequalities:

$$\begin{aligned} (1 - \varepsilon) \Pr(\gamma_1 > \tilde{\gamma}_1 \mid \tilde{\gamma}_2) + \varepsilon \Pr(\gamma_1 \leq \tilde{\gamma}_1 \mid \tilde{\gamma}_2) &\geq p(\tilde{\gamma}_2, \theta) \\ (1 - \varepsilon) \Pr(\gamma_2 > \tilde{\gamma}_2 \mid \tilde{\gamma}_1) + \varepsilon \Pr(\gamma_2 \leq \tilde{\gamma}_2 \mid \tilde{\gamma}_1) &\geq p(\tilde{\gamma}_1, \theta) \end{aligned}$$

In addition, if  $(\tilde{\gamma}_1, \tilde{\gamma}_2)$  satisfy both with equality, then they characterize a "threshold equilibrium", in which player  $i$  plays risky, if and only if  $\gamma_i \leq \tilde{\gamma}_i$ . Using symmetry, as well as the fact that the information structure regarding the coefficients of risk aversion implies  $\Pr(\gamma_{-i} \leq \tilde{\gamma} \mid \tilde{\gamma}) = \frac{1}{2}$ , we find that this threshold equilibrium solves

$$(1 - \varepsilon) \Pr(\gamma_{-i} \leq \tilde{\gamma} \mid \tilde{\gamma}) + \varepsilon \Pr(\gamma_{-i} > \tilde{\gamma} \mid \tilde{\gamma}) = p(\tilde{\gamma}, \theta)$$

which gives

$$\frac{1}{2} = p(\tilde{\gamma}, \theta) = 1 - \frac{e^{\tilde{\gamma}\theta} - 1}{e^{\tilde{\gamma}} - 1}$$

or

$$1 = 2e^{\tilde{\gamma}\theta} - e^{\tilde{\gamma}}.$$

Again, this defines a threshold curve between  $\tilde{\gamma}$  and  $\theta$ , such that a player is willing to attack, given  $\theta$ , if and only if  $\gamma_i \leq \tilde{\gamma}$ . The curve is identical to the previous one, except that  $\theta$  replaces  $\tilde{x} - \frac{1}{2}\gamma\sigma^2$ . Note that the equilibrium threshold does not depend on the trembling probability, nor does it depend on the "noise" in the information regarding  $\gamma_i$  and  $\gamma_{-i}$ . Applying the previous limit results, we immediately find that  $\lim_{\tilde{\gamma} \rightarrow 0} \theta(\tilde{\gamma}) = \frac{1}{2}$  and  $\lim_{\tilde{\gamma} \rightarrow \infty} \theta(\tilde{\gamma}) = \frac{1}{2}$ .

As  $\theta$  converges to 1 (i.e. when  $R$  becomes strictly dominant), all players start playing risky. On the other hand,  $\frac{1}{2}$  serves as a lower bound for the equilibrium threshold, whenever players are risk-averse, and with higher-order uncertainty about the degree of risk aversion, no higher level of coordination on the risky equilibrium can be sustained, *even when it is common knowledge that  $(R, R)$  strictly dominates  $(S, S)$* . We thus conclude that in the presence of higher-order uncertainty regarding the degree of risk aversion, full coordination on the payoff dominant equilibrium is no longer feasible. Instead, the threshold equilibrium, in which players play the risky action most aggressively corresponds to the risk-dominant strategy profile, which maximizes the expected return under the assumption that the other player plays each action with probability  $\frac{1}{2}$ .

## 2.4 Higher-order uncertainty about fundamentals and preferences

I now consider briefly the environment in which neither  $\theta$  nor the degrees of risk aversion are common knowledge. In that case, a threshold equilibrium is implicitly defined by a pair of threshold functions  $\tilde{x}_1(\gamma)$ ,  $\tilde{x}_2(\gamma)$  such that the following pair of conditions holds pointwise:

$$\begin{aligned} p(\gamma_2, \tilde{x}_2(\gamma_2)) &= (1 - \varepsilon) \Pr(x_1 > \tilde{x}_1(\gamma_1) \mid \tilde{x}_2(\gamma_2), \gamma_2) \\ &\quad + \varepsilon \Pr(x_1 \leq \tilde{x}_1(\gamma_1) \mid \tilde{x}_2(\gamma_2), \gamma_2) \\ p(\gamma_1, \tilde{x}_1(\gamma_1)) &= (1 - \varepsilon) \Pr(x_2 > \tilde{x}_2(\gamma_2) \mid \tilde{x}_1(\gamma_1), \gamma_1) \\ &\quad + \varepsilon \Pr(x_2 \leq \tilde{x}_2(\gamma_2) \mid \tilde{x}_1(\gamma_1), \gamma_1) \end{aligned}$$

Symmetry implies that  $\tilde{x}_1(\gamma) = \tilde{x}_2(\gamma) = \tilde{x}(\gamma)$ , and it follows that the equilibrium condition can be reduced to

$$\begin{aligned} p(\gamma_i, \tilde{x}(\gamma_i)) &= (1 - \varepsilon) \Pr(x_{-i} > \tilde{x}(\gamma_{-i}) \mid \tilde{x}(\gamma_i), \gamma_i) \\ &\quad + \varepsilon \Pr(x_{-i} \leq \tilde{x}(\gamma_{-i}) \mid \tilde{x}(\gamma_i), \gamma_i) \end{aligned}$$

Since  $\tilde{x}(\gamma)$  is generally not linear, the left-hand side can, at best, be approximated, and closed form solutions are generally not available. It follows, however, from the main theorem in Carlsson and van Damme (1993) that, if the level of noise in  $x_i$  and  $\gamma_i$  is small,  $\Pr(x_{-i} \leq \tilde{x}(\gamma_{-i}) \mid \tilde{x}(\gamma_i), \gamma_i) \approx \frac{1}{2}$ . In that case, one finds that in the presence of higher-order uncertainty about fundamentals, the threshold curve is approximated by

$$\tilde{x}(\gamma) = \frac{1}{2}\gamma\sigma^2 + \frac{1}{\gamma} \log\left(\frac{1 + e^\gamma}{2}\right)$$

i.e. the same curve that described the equilibrium thresholds in an environment with private signals, without higher-order uncertainty concerning fundamentals.

In conclusion, one therefore finds that once players are exposed to higher-order uncertainty regarding either the fundamental or the preferences towards risk, the highest degree of coordination on the risky strategies (approximately) corresponds to the risk-dominant equilibrium. Adding another source of uncertainty only affects payoffs through its effect on fundamental risk, i.e. the risk premium.

### 3 Some comments on the design of experimental tests

#### 3.1 Existing experimental results

Some recent contributions have tried to test the informational comparative statics of coordination games in experiments. Heinemann, Nagel and Ockenfels (2002) for instance compare the laboratory results of a herding game when subjects have common knowledge of the state with an environment where subjects receive noisy private signals. Their overwhelming conclusion is that while players seem to play slightly more risky strategies in the common knowledge game as compared with the private information game, this difference is dwarfed by the gaps between the experimental outcomes and the respective theoretical predictions. Hence, since the coordinating effect of public information emphasized in Morris and Shin (2000, 2001) and Hellwig (2002) does not appear in the observations of a simple experiment, they interpret their results as a rejection of the informational predictions of the global games theory.

The example discussed above suggests that the experimental results may not be as contrary to the theory as the authors claim; indeed since the experimental design takes no steps to control the players' information about each others preferences, it seems plausible to assume the respective degree of risk aversion are not common knowledge in the experimental setting. According to the example, this aspect should swamp the concerns regarding information and common knowledge about fundamentals; indeed the small difference in the outcomes between the two treatments seem to be reconcilable with the risk premium associated with the noise in the private information.

Heinemann, Nagel and Ockenfels also report on the evolution of coordination in their experiments: in their setting, players are asked to play 16 rounds of the coordination game, and in each round make 10 independent decisions. The payoff parameters are held the same during the first eight rounds, and then changed to a different configuration for the last eight rounds. It is interesting to observe how coordination evolved over time: in the common information treatment, one observes that 'regrettable decisions', where a player could have increased his pay-off by playing the other strategy, decrease over time to virtually zero, i.e. the players learn to coordinate on a common threshold. However, the threshold on which they learn to coordinate is below the first-best level, and depends among others, on the thresholds played in the initial rounds of coordination. In contrast, in the private information treatment, the learning effect is much less pronounced, occurs only during the initial two periods, and thereafter, the proportion of regrettable decisions remains

at a level that is significantly higher than the level predicted on the basis of private information only.

Arguably, these experimental results roughly conform to the theoretical predictions when higher-order uncertainty about preferences exists. The dynamic element inherent in their set-up implies that at the beginning of each round, the players have common information regarding the outcomes of previous rounds; over time, they can use this information to draw some inference about the preferences of the other participants; in this respect, the results seem to suggest that with common information, learning regarding fundamentals is much more efficient, and is better able to create common information regarding preferences. The observation that there is a high degree of inertia in the thresholds also conforms to the theoretical predictions, since the observation of a threshold indicates a level up to which playing  $R$  can be viewed as fairly safe, where as above the threshold, there is little information available as to how the other players will react. Summing up, it seems like the evolution of coordination in their experiment could be explained by the dynamics of learning about the players' preferences.

### 3.2 An Alternative Design

While the experimental results of Heinemann, Nagel and Ockenfels seem to loosely conform to the predictions of the model with imperfect common knowledge about preferences, they fail as an experimental test of the coordinating effect associated with public information. This raises the problem of designing the experiment in such a way that the experimenter controls for the information that players have about each others attitudes toward risk. This problem appears to be tricky - short of asking players to communicate with each other on what they would choose under given conditions, it is not clear, how this information may be indirectly solicited, so as to become common knowledge among players. The problem is exacerbated by the fact that individual behavior when facing risk is much more complex than what is predicted expected utility theory, and many of the aspects not taken into account in this simple model here may even be difficult to parametrize in a simple way. Thus, the experimenter who wishes to control for common knowledge regarding preferences faces the additional problem that in practice, the space of possible preferences is undescribable.

Even though perfect common knowledge of preferences appears to be an impossible objective

to achieve in an experimental lab, the following set-up may at least partially control for the degree of higher-order uncertainty regarding preferences: Divide the experimental set-up into two stages. During the second stage, a coordination game takes place, where the experimenter controls the information structure regarding the fundamental. The first stage serves to elicit information regarding preferences. Suppose the players are each asked to make a series of choices between a "risky" and a "safe" alternative, with monetary payoffs as follows:

$R$	$\theta$
$S$	$0$

where players observe the value of theta with some noise, i.e. they observe  $x = \theta + \xi$ . At the end of the first stage, but before the coordination game, the players' choices, as well as the realized payoffs are revealed to all players, i.e. become common knowledge before the start of the coordination game.

If preferences were common knowledge, the information revealed in the first stage should be irrelevant for the outcome of the subsequent coordination game. However, if preferences are not common knowledge, the first stage should enable players to learn something about each other's behavior, and consequently about their preferences. Most importantly, this non-interactive choice settings convey information about whether one player is more or less inclined to play aggressively than another. Also, by making this information common, the experimenter generates some degree of common beliefs regarding preferences. Presumably, by altering the initial choice settings to which the individuals are subject, the experimenter can also vary the degree to which players share information about their preferences. In any case, the example suggests that in such an experiment, the more information about preferences is shared by the players initially, the more sensitive their outcomes ex post are to common information.

## 4 Discussion

This brief example intends to highlight the importance of the common knowledge assumption regarding preferences. The results suggest that the informational comparative statics that are highlighted in the literature are valid only under the assumption that preferences are common knowledge. More generally, higher-order uncertainty along a single dimension is sufficient to reduce

the ability of market participants to coordinate, and while I haven't formally worked this out here, it seems to be the case that the informational comparative statics exhibit a complementarity across dimensions, i.e. *the higher the degree of common knowledge of preferences, the more responsive equilibrium strategies are to changes in the information structure regarding fundamentals.*

It should be noted that while the model took the degree of risk aversion as the dimension along which preferences differ, individual attitudes towards risk vary substantially along many dimensions. What determines the degree to which the players are able to coordinate on the "risky" equilibrium, is how the various sources of higher-order uncertainty are translated into *actions*, roughly speaking, what matters for coordination is the degree of common knowledge about their respective degrees of "aggressiveness" in their inclination to play the risky strategy; in the language of the model, this is the degree of common knowledge of the players' dominance level of the risky action.

Much of the literature following Morris and Shin (1998, 2000) has focused on the role of higher-order uncertainty regarding fundamentals, and has consequently discussed the welfare effects of information disclosure in coordination settings. This discussion is entirely based on the assumption that the player's objectives are common knowledge. The results here are meant to serve as a cautionary note, especially when regarding applications: Presumably, the motives of players are much better known (and much closer to be *commonly known*), when few players are involved in the coordination game, and when these players engage in communication, as compared to a large population playing the game without much communication. Similarly the *nature* of the "players" and the "game" will have an effect: typically the objectives of a large investment bank or fund in the context of a currency crises are much better defined and observed than the motivations of a small household who withdraws his funds from a bank in the fear of a bank run and subsequent failure. Finally, in repeated or dynamic environments, the precise nature of interaction and communication will have an effect on how players learn about each others' objectives, and consequently about how this coordination takes place over time.

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