

Defense Policies Against Currency Attacks:

On the Possibility of Predictions in a Global Game with Multiple Equilibria^{*}

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Abstract

This paper studies defense policies in a global-game model of speculative currency attacks. Although the signaling role of policy interventions sustains multiple equilibria, a number of novel predictions emerge which are robust across all equilibria. (i) The central bank intervenes by raising domestic interest rates, or otherwise raising the cost of speculation, only when the value it assigns to defending the peg—its “type”—is intermediate. (ii) Devaluation occurs only for low types. (iii) The set of types who intervene shrinks with the precision of market information. (iv) A unique equilibrium policy survives in the limit as the noise in market information vanishes, whereas the devaluation outcome remains indeterminate. (v) The payoff of the central bank is monotonic in its type. (vi) The option to intervene can be harmful only for sufficiently strong types; and when this happens, weak types are necessarily better off. While these predictions seem reasonable, none of them would have been possible in the common-knowledge version of the model. Combined, these results illustrate the broader methodological point of the paper: global games can retain significant selection power and deliver useful predictions even when the endogeneity of information sustains multiple equilibria.

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1 Introduction

This paper investigates the properties of defense policies against speculative currency attacks: think of a central bank trying to prevent a run against the domestic currency by raising domestic interest rates, imposing a tax on capital outflows, or otherwise increasing the cost of speculation. The exercise is conducted within a global game that stylizes the role of coordination under incomplete information. Previous work has shown that the signaling role of policy interventions can lead to multiple equilibria (Angeletos, Hellwig and Pavan, 2006). Here we seek to understand what predictions, if any, one can deliver regarding policy choices and devaluation outcomes that do not rely on arbitrary equilibrium selections.

Understanding this point is important not only for the specific application under examination but also from a broader methodological perspective. The approach followed in most recent applications of global games is to assume certain exogenous information structures as a selection device—as a tool for achieving the convenience of unique-equilibrium comparative statics in models that were previously ridden with multiple equilibria. For certain questions, however, understanding the sources of information is the key to understanding the phenomenon under examination. Endogenizing the sources of information often brings back multiple equilibria. What we show here is that this multiplicity does not preclude concrete and testable predictions and is very different from the one that emerges under complete information.

The model features a large number of speculators deciding whether or not to attack the peg. Devaluation takes place if and only if the aggregate attack is sufficiently large. Speculators have heterogeneous information about the critical size of attack that triggers devaluation. Before speculators move, the central bank takes a costly action in an attempt to reduce the probability of devaluation. Such interventions convey information about the critical size of attack that the bank is willing, or able, to withstand. The “fundamentals” in this game thus coincide with the “type” of the policy maker.

The first part of the paper provides a complete characterization of the set of equilibrium outcomes. The multiplicity of equilibria follows from previous work. The challenge here is to provide an exhaustive characterization of *all* possible equilibrium outcomes—this is essential if one wishes to identify predictions that are not sensitive to equilibrium selection.

The result is achieved through a procedure of iterated deletion of strategies that cannot be part

of an equilibrium. This procedure is different from the one used in standard global games, because of the introduction of signaling. First, beliefs about payoffs in the coordination game played among the speculators (the receivers) are endogenous; they are a function of the strategy of the policy maker (the sender). Second, iterated deletion of strategies that cannot be part of an equilibrium imposes restrictions not only on coordination among the receivers but also on the information sent by the sender.

The second part of the paper then uses this characterization result to identify the following predictions about policy and devaluation outcomes that are *robust* in the sense that they hold true across all possible equilibria.

- The policy choice is non-monotonic in the type of the policy maker: only intermediate types raise the policy above the cost-minimizing level in order to avoid an attack. Intervention thus signals that the fundamentals are neither too weak nor too strong.
- The devaluation outcome, on the other hand, is monotonic in the policy maker's type: the peg is abandoned if and only if the fundamentals are sufficiently weak.
- Different equilibria can be indexed by the level of policy intervention necessary for preempting an attack, which can be interpreted as an index of the "aggressiveness of market expectations:" the higher this level, the larger the set of types that abandon the peg. For given aggressiveness, an increase in the precision of the speculators' information does not affect the set of types that maintain the peg but reduces the set of types that do so by raising the policy. In this sense, the quality of information does not affect the probability of devaluation, but reduces the need for policy intervention.
- As the noise in information vanishes, the set of types who intervene also vanishes. By implication, the equilibrium policy is essentially unique in the limit. Nevertheless, the devaluation outcome remains indeterminate for a non-vanishing set of types.
- The payoff of the policy maker is monotonic in his type.
- The option to intervene can be harmful only for sufficiently strong types. However, in any equilibrium in which some strong type is worse off, some weak type is necessarily better off. Hence, although the option to intervene leads to multiple equilibria, either the policy maker is better off no matter his type, or low types are better off at the expense of high types.

These predictions can be useful both for a policy maker and for an econometrician. For the policy maker, while the multiplicity result warns him that he may not have full control over the devaluation outcome, the aforementioned predictions give him a better understanding of how changes in the environment—e.g., reforms that improve the fundamentals or reduce the cost of intervention—affect the set of possible outcomes in the event of a crisis. For the econometrician, the aforementioned predictions provide empirical restrictions that can help him estimate and test the model.¹

Given the structure of the underlying environment, the aforementioned predictions seem reasonable. However, *none* of these predictions is shared by the complete-information version of the model. Under complete information, the devaluation outcome need not be monotonic in the type of the policy maker; intervention can occur for any arbitrary subset of the critical region (i.e., the region of fundamentals for which the peg is maintained if no speculator attacks but is abandoned if all speculators attack); the payoff of the policy maker need not be monotonic in his type; and the value of the option to intervene can be negative for all types in the critical region.

This observation highlights the key role that incomplete information plays in our model: even though it does not pin down a unique equilibrium, incomplete information puts significant restrictions on the mapping from primitives to outcomes, leading to predictions that otherwise would not have been possible. This is best illustrated in the limit as the noise in the speculators' information vanishes: even though multiplicity obtains for *any* level of noise, the limit of the set of incomplete-information outcomes is a zero-measure subset of the set of common-knowledge outcomes.

Combined, these results contain the methodological message of the paper. The game we consider here is an example of a global game with endogenous information and multiple equilibria. If the equilibrium outcomes obtained in this game were similar to those under common knowledge, then for practical purposes one could largely ignore both the endogeneity and the incompleteness of information, and go back to the earlier models that assumed common knowledge. What our results illustrate is that global games can retain a significant selection power even when the endogeneity of information sustains multiple equilibria.

Related literature. The global-games approach to equilibrium selection was pioneered by Carlsson and van Damme (1993) and was recently extended by Morris and Shin (2003) and Frankel,

¹The aforementioned predictions have been stated as if the econometrician knows which equilibrium is played. Nevertheless, because they are shared by all equilibria, these predictions remain true even when the econometrician is uncertain about which equilibrium is played. We show this by allowing for an arbitrary random selection over all equilibria and examining the implied distribution of equilibrium outcomes.

Morris and Pauzner (2003). By now they have been used in a variety of applications, including currency crises (Morris and Shin, 1998; Corsetti, Dasgupta, Morris and Shin, 2004; Guimaraes and Morris, 2006), bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), debt crises (Corsetti, Guimaraes and Roubini, 2006, Zwart, 2007), investment spillovers (Chamley, 1999; Dasgupta 2006), and liquidity crashes (Morris and Shin, 2004). Our approach differentiates from this literature in two respects. First, we see information as an integral part of the phenomenon under examination rather than a selection device; in this respect, we view the multiplicity that originates in the signaling role of policy as an important prediction by itself. Second, we show that global games can deliver useful predictions even when they fail to deliver uniqueness.²

The paper also departs from several strands of the literature that study the role of policy in crises. In common-knowledge coordination models of crises, policy analysis is by and large restricted to identifying policies that could make certain actions dominant for the market, thus removing the “bad” equilibrium and ensuring that a coordination failure never materializes (e.g. Cooper and John, 1988; Zettelmeyer, 2000; Jeanne and Wyplosz, 2001). More recently, Morris and Shin (2006b) and Corsetti, Guimaraes and Roubini (2006) use a global game to study how IMF interventions can, on the one hand, have a catalytic effect on crises and, on the other hand, exacerbate the moral-hazard problem for the governments of the countries in risk of a crisis. These papers, however, abstract completely from the signaling effects that are at the heart of our approach. Zwart (2007) examines a model in which IMF interventions convey information but in which the policy is uniquely determined because the IMF’s incentive to intervene depend only on the country’s fundamentals and not on the size of the attack in case of no intervention. Finally, Drazen (2000) and Drazen and Hubrich (2005) discuss signaling effects of policy interventions in currency crises but model the market as a single large player, thus completely abstracting from the coordination element of crises that is at the core of our analysis.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the set of equilibrium outcomes. Section 4 identifies predictions about policy and devaluation outcomes, while Section 5 identifies predictions about payoffs. Section 6 contrasts the multiplicity of the incomplete-information game with that of its common-knowledge counterpart. Section 7 concludes. All proofs omitted in the main text are in the Appendix.

²Somewhat related to this point is Chassang (2007). He finds that global games may retain significant selection power in a dynamic setting with exit even if they do not lead to uniqueness. Note, however, that in his model multiplicity is due to repeated play, not due to the endogeneity of information.

2 Model

The economy is populated by a policy maker and a measure-one continuum of speculators, indexed by i and uniformly distributed over $[0, 1]$. Each speculator can choose between two actions, either “attack” the peg (i.e., short-sell the domestic currency) or abstain from attacking. The policy maker has some privately-known value for maintaining the peg and controls a policy instrument that affects the speculators’ opportunity cost of attacking.

Let θ denote the policy maker’s type (his willingness to defend the peg, or his “strength”), $r \in [\underline{r}, \bar{r}] \subset (0, 1)$ the policy instrument (the speculators’ opportunity cost of attacking) and $D \in \{0, 1\}$ the devaluation outcome, with $D = 0$ when the peg is maintained and $D = 1$ otherwise.

The game evolves through three stages. In the first stage, the policy maker learns his type θ and sets the policy r . In the second stage, speculators decide simultaneously whether or not to attack, after observing the policy r , and after receiving private signals $x_i = \theta + \sigma \xi_i$ about θ ; the scalar $\sigma \in (0, \infty)$ parameterizes the quality of the speculators’ information, while ξ_i is noise, i.i.d. across speculators and independent of θ , with a continuous log-concave probability density function ψ strictly positive over the entire real line. The common prior about θ is assumed to be uniform over the interval $[-M, +M]$, for some $M > 0$; as it is standard in the literature, to simplify the analysis we consider the limit case where $M = +\infty$. In the third and final stage, the policy maker decides whether or not to maintain the peg after observing the mass of speculators who decided to attack.

The payoff for a speculator who does not attack is normalized to zero, whereas the payoff from attacking is $1 - r$ if the peg is abandoned and $-r$ otherwise. The policy instrument r can thus be interpreted as the interest rate differential between domestic and foreign bonds or as a tax on capital outflows.

The payoff for the policy maker, on the other hand, has two components: the gross value of maintaining the peg, and the cost of policy intervention. The cost of setting the policy at r is $C(r)$, where C is continuously increasing, with $C(\underline{r}) = 0$. The gross value of maintaining the peg is $V(\theta, A)$, where $A \in [0, 1]$ is the mass of speculators attacking; V is twice differentiable, with $V_\theta > 0 > V_A$, $V(\underline{\theta}, 0) = V(\bar{\theta}, 1) = 0$ for some $\underline{\theta} < \bar{\theta}$, $V_{\theta A} \geq 0$, and $\lim_{\theta \rightarrow \infty} [V(\theta, 0) - V(\theta, 1)] = 0$.³ The policy maker’s net payoff is thus $V(\theta, A) - C(r)$ if the peg is maintained and $-C(r)$ otherwise.

³ V_θ and V_A denote the partial derivatives of V with respect to θ and A ; $V_{\theta A}$ denotes the cross derivative.

Remark. The assumptions that the value of maintaining the peg, V , is increasing in θ and decreasing in A , and that there exist $\underline{\theta}$ and $\bar{\theta}$ such that $V(\underline{\theta}, 0) = V(\bar{\theta}, 1) = 0$ permit us to partition the policy maker’s type space in three regions: for $\theta < \underline{\theta}$ the peg is abandoned even if no speculator attacks; for $\theta \in [\underline{\theta}, \bar{\theta}]$ the peg is sound but vulnerable to a sufficiently large attack; and for $\theta > \bar{\theta}$ the peg survives even if everybody attacks. The interval $[\underline{\theta}, \bar{\theta}]$ thus identifies the “critical region” where multiple equilibria exist under common knowledge—a “good” equilibrium in which nobody attacks, along with a “bad” equilibrium in which everybody attacks.

The assumptions that $V_{\theta A} \geq 0$ and $\lim_{\theta \rightarrow \infty} [V(\theta, 0) - V(\theta, 1)] = 0$ mean that the benefit of avoiding an attack (or of reducing its size) is higher for weaker types, and eventually vanishes as $\theta \rightarrow \infty$. This last property guarantees that intervention is unnecessary for extremely high types.

Finally, the assumption that the policy maker’s payoff in the event of devaluation is independent of the size of the attack reflects the fact that the main costs of defending a peg, such as the cost of borrowing reserves from abroad, are costs that are not incurred if the peg is abandoned—see also Drazen (2000) for a discussion.

Equilibrium. We consider perfect Bayesian equilibria. Let $r(\theta)$ denote the policy chosen by type θ , $a(x, r)$ the action of a speculator who receives a private signal x and who observes a policy r , $A(\theta, r)$ the aggregate size of attack, and $D(\theta, r, A)$ the decision of whether to abandon the peg. Next, let $\mu(\theta|x, r)$ denote the cumulative distribution function of an speculator’s posterior belief about θ , conditional on x and r . Finally, let $U(\theta, r, A) \equiv \max_{D \in \{0,1\}} \{(1 - D)V(\theta, A) - C(r)\}$. The equilibrium definition can then be stated as follows.

Definition. A (symmetric pure-strategy) equilibrium consists of a policy function $r(\theta)$, a strategy for the speculators $a(x, r)$, a rule for devaluation $D(\theta, r, A)$ and a cumulative distribution function $\mu(\theta|x, r)$, such that

$$r(\theta) \in \arg \max_{r \in [\underline{r}, \bar{r}]} U(\theta, r, A(\theta, r)) \quad \forall \theta \tag{1}$$

$$a(x, r) \in \arg \max_{a \in \{0,1\}} a \left[\int D(\theta, r, A(\theta, r)) d\mu(\theta|x, r) - r \right] \quad \forall (x, r) \tag{2}$$

$$D(\theta, r, A) \in \arg \max_{D \in \{0,1\}} \{(1 - D)V(\theta, A) - C(r)\} \quad \forall (\theta, r, A) \tag{3}$$

$$\mu(\theta|x, r) \text{ is obtained from Bayes' rule using } r(\cdot) \text{ for any } x \in \mathbb{R} \text{ and any } r \in r(\mathbb{R}), \tag{4}$$

where $A(\theta, r) \equiv \int_{-\infty}^{+\infty} a(x, r) \sigma^{-1} \psi((x - \theta) / \sigma) dx$ is the equilibrium size of attack and $r(\mathbb{R}) \equiv \{r : r = r(\theta), \theta \in \mathbb{R}\}$ is the set of policy interventions that are played in equilibrium. The equilibrium devaluation outcome is $D(\theta) \equiv D(\theta, r(\theta), A(\theta, r(\theta)))$.

Conditions (1), (2) and (3) require that the policy maker's and the speculators' actions be sequentially rational, while condition (4) requires that, on the equilibrium path, the speculators' beliefs be pinned down by Bayes' rule.⁴

3 Equilibrium characterization

In this section we provide a complete characterization of the set of equilibrium outcomes. Let $\mathcal{E}(\sigma)$ denote the set of all possible equilibria in the game with noise σ . Next, let $\mathcal{E}(s; \sigma)$ denote the set of equilibria in which the range of the policy is $r(\mathbb{R}) = [\underline{r}, s]$ for some $s \geq \underline{r}$.

Proposition 1 (complete characterization) *There exist thresholds $\tilde{r} \leq \bar{r}$ and $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that the following are true for any $\sigma > 0$:*

(i) $\mathcal{E}(\sigma) = \cup_{s \in [\underline{r}, \tilde{r}]} \mathcal{E}(s; \sigma)$, with $\mathcal{E}(s; \sigma) \neq \emptyset$ for all $s \in [\underline{r}, \tilde{r}]$.

(ii) Any equilibrium in $\mathcal{E}(\underline{r}; \sigma)$ is such that

$$r(\theta) = \underline{r} \text{ for all } \theta \quad \text{and} \quad D(\theta) = \begin{cases} 1 & \text{for } \theta < \tilde{\theta} \\ 0 & \text{for } \theta > \tilde{\theta} \end{cases}$$

(iii) For any $r^* \in (\underline{r}, \tilde{r}]$, there exist unique thresholds $\theta^* \in (\underline{\theta}, \tilde{\theta}]$ and $\theta^{**} \geq \theta^*$ such that any equilibrium in $\mathcal{E}(r^*; \sigma)$ is such that

$$r(\theta) = \begin{cases} r^* & \text{for } \theta \in (\theta^*, \theta^{**}) \\ \underline{r} & \text{for } \theta \notin [\theta^*, \theta^{**}] \end{cases} \quad \text{and} \quad D(\theta) = \begin{cases} 1 & \text{for } \theta < \theta^* \\ 0 & \text{for } \theta > \theta^* \end{cases}$$

As mentioned in the Introduction, the challenge here is not to prove the existence of equilibria that satisfy the conditions in parts (ii) and (iii), but rather to show that *all* equilibria must satisfy these properties. In the rest of the section, we prove this result through a series of lemmas. Lemmas 1 to 6 iteratively eliminate strategy profiles that can not be part of an equilibrium, thus identifying

⁴The definition restricts attention to symmetric pure-strategy equilibria; as discussed after Proposition 1 this is without loss of generality.

a set of necessary conditions for equilibrium strategies. Lemma 7 completes the characterization by showing that these conditions are also sufficient. The reader interested only in how Proposition 1 translates into concrete testable predictions can jump to Section 4.

First, consider the set of equilibria in which all types pool on \underline{r} .

Lemma 1 (i) *There exist thresholds $\tilde{x} \in \mathbb{R}$ and $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that any equilibrium in $\mathcal{E}(\underline{r}; \sigma)$ is such that $a(x, \underline{r}) = 1$ if $x < \tilde{x}$, $a(x, \underline{r}) = 0$ if $x > \tilde{x}$, $D(\theta) = 1$ if $\theta < \tilde{\theta}$ and $D(\theta) = 0$ if $\theta > \tilde{\theta}$.*

(ii) The thresholds $\tilde{\theta}$ and \tilde{x} are the unique solutions to $V(\tilde{\theta}, 1 - \underline{r}) = 0$ and $1 - \Psi(\frac{\tilde{x} - \tilde{\theta}}{\sigma}) = \underline{r}$.

Proof. Consider the continuation game that follows \underline{r} . Because the observation of \underline{r} conveys no information, this game is identical to a standard global game (e.g. Morris and Shin, 1998, 2003), which proves the existence of a unique continuation equilibrium, as stated in Part (i). For part (ii), note that a speculator who expects devaluation to occur if and only if $\theta < \tilde{\theta}$ must be indifferent between attacking and not attacking when $x = \tilde{x}$, which gives

$$1 - \Psi\left(\frac{\tilde{x} - \tilde{\theta}}{\sigma}\right) = \underline{r}. \quad (5)$$

Similarly, a policy maker who faces an attack of size $A(\theta, \underline{r}) = \Psi((\tilde{x} - \theta)/\sigma)$ must be indifferent between abandoning and maintaining the peg when $\theta = \tilde{\theta}$, which gives

$$V\left(\tilde{\theta}, \Psi\left(\frac{\tilde{x} - \tilde{\theta}}{\sigma}\right)\right) = 0. \quad (6)$$

Finally, substituting (5) into (6) gives $V(\tilde{\theta}, 1 - \underline{r}) = 0$. ■

Next, consider equilibria in which some type raises the policy above \underline{r} . For any θ , let $\rho(\theta)$ denote the maximal policy that is not strictly dominated for θ : $\rho(\theta) = \underline{r}$ for $\theta < \underline{\theta}$, $\rho(\theta) = C^{-1}(V(\theta, 0))$ for $\theta \in [\underline{\theta}, \bar{\theta}]$, and $\rho(\theta) = C^{-1}(V(\theta, 0) - V(\theta, 1))$ for $\theta > \bar{\theta}$.⁵ Clearly in any equilibrium, the policy must satisfy $r(\theta) \leq \rho(\theta)$ for all θ .

Lemma 2 *In any equilibrium in which some type intervenes, there exists a single $r^* \in (\underline{r}, \rho(\bar{\theta})]$ such that $r(\theta) = r^*$ whenever $r(\theta) \neq \underline{r}$.*

⁵To simplify notation, we assume that $\rho(\bar{\theta}) < \bar{r}$. When this is not the case, all the results hold by replacing $\rho(\theta)$ with $\hat{\rho}(\theta) \equiv \min\{\rho(\theta), \bar{r}\}$.

Proof. Because the policy maker faces no uncertainty about $A(\theta, r)$, any type who raises the policy above \underline{r} must be spared from devaluation, for otherwise he would be strictly better off setting $r = \underline{r}$. Furthermore, because the noise in the speculators' information is unbounded, the observation of any equilibrium $r > \underline{r}$ necessarily signals that the peg will be maintained and thus induces all speculators not to attack no matter their signal x . But then the policy maker can always save on the cost of intervention by setting the lowest $r > \underline{r}$ among those that are played in equilibrium. Finally, that $r^* \leq \rho(\bar{\theta})$ follows from the fact that $\rho(\theta) \leq \rho(\bar{\theta})$ for all θ , which implies that any $r^* > \rho(\bar{\theta})$ is strictly dominated for all types. ■

The preceding lemma implies that any equilibrium that does not belong to $\mathcal{E}(\underline{r}; \sigma)$ necessarily belongs to $\mathcal{E}(r^*; \sigma)$ for some $r^* \in (\underline{r}, \rho(\bar{\theta})]$. The next three lemmas identify further properties of the set $\mathcal{E}(r^*; \sigma)$.

Lemma 3 *For any $r^* \in (\underline{r}, \rho(\bar{\theta})]$ and any equilibrium in $\mathcal{E}(r^*; \sigma)$, $D(\theta) = 0$ for all $\theta > \theta^*$, where $\theta^* \in (\underline{\theta}, \bar{\theta}]$ is the lowest solution to $\rho(\theta) = r^*$, or equivalently the unique solution to*

$$V(\theta^*, 0) = C(r^*). \quad (7)$$

Proof. Any type $\theta \geq \theta^*$ can guarantee himself a payoff $V(\theta, 0) - C(r^*) \geq 0$ (with strict inequality for $\theta > \theta^*$) by setting $r = r^*$; this follows directly from the fact that $A(\theta, r^*) = 0$ for any θ . But then no type above θ^* abandons the peg. ■

The threshold θ^* is thus an upper bound for the set of types who devalue across all equilibria in $\mathcal{E}(r^*; \sigma)$. Because r^* is strictly dominated for all $\theta < \theta^*$, θ^* is also a lower bound for the set of types who raise the policy at r^* . The next lemma identifies an upper bound θ^{**} for this set; it further establishes that the level of the policy r^* can not exceed $\tilde{r} \equiv \rho(\bar{\theta})$. The proof is based on iterated deletion of strategies that cannot be part of an equilibrium.

To state this lemma, and also for future use, we introduce the function $X(\theta', \theta'')$ implicitly defined, for $\theta'' \geq \theta'$, by

$$\frac{1 - \Psi\left(\frac{x - \theta'}{\sigma}\right)}{1 - \Psi\left(\frac{x - \theta'}{\sigma}\right) + \Psi\left(\frac{x - \theta''}{\sigma}\right)} = x. \quad (8)$$

Note that the left-hand-side of (8) is the probability that a speculator with signal x assigns to $\theta < \theta'$ conditional on $\theta \notin [\theta', \theta'']$. The function $X(\theta', \theta'')$ thus identifies the value of x that makes

a speculator indifferent between attacking and not attacking when that speculator observes \underline{r} and believes that $D(\theta) = 1$ if and only if $\theta < \theta'$ and $r(\theta) = \underline{r}$ if and only if $\theta \notin [\theta', \theta'']$. We can now state the lemma as follows.

Lemma 4 (i) For any $r^* \in (\underline{r}, \tilde{r}]$ and any equilibrium in $\mathcal{E}(r^*; \sigma)$, $r(\theta) = r^*$ only if $\theta \in [\theta^*, \theta^{**}]$, where θ^* is the unique solution to condition (7) and θ^{**} is the unique solution to

$$V(\theta^{**}, 0) - V(\theta^*, 0) - V\left(\theta^{**}, \Psi\left(\frac{X(\theta^*, \theta^{**}) - \theta^{**}}{\sigma}\right)\right) = 0. \quad (9)$$

(ii) For any $r^* > \tilde{r}$, $\mathcal{E}(r^*; \sigma) = \emptyset$.

Proof. For any $\theta'' \geq \theta^*$ and any $\theta \geq \underline{\theta}$, let

$$g(\theta^*, \theta'', \theta) \equiv V(\theta, 0) - V(\theta^*, 0) - \max\left\{0, V\left(\theta, \Psi\left(\frac{X(\theta^*, \theta'') - \theta}{\sigma}\right)\right)\right\}.$$

The function $g(\theta^*, \theta'', \theta)$ can be interpreted as the net payoff from raising the policy from \underline{r} to r^* for a type θ who expects no speculator to attack when $r = r^*$, no matter x , and all speculators to attack if and only if $x < X(\theta^*, \theta'')$ when $r = \underline{r}$. (Recall that $V(\theta^*, 0) = C(r^*)$).

Now consider the sequence $\{\theta^n\}_{n=0}^\infty$ constructed as follows. First, let $\theta^0 \in (\bar{\theta}, \infty)$ be the highest solution to $\rho(\theta) = r^*$; that this threshold exists follows from the fact that $\rho(\cdot)$ is continuous with $\rho(\bar{\theta}) > r^*$ and $\lim_{\theta \rightarrow \infty} \rho(\theta) = \underline{r}$.⁶ Next, for any $n \geq 1$, let

$$\theta^n = \sup\{\theta \geq \theta^* : g(\theta^*, \theta^{n-1}, \theta) \geq 0\}$$

if the set is non-empty, and $\theta^n = \theta^*$ otherwise.

This sequence has a simple meaning. Clearly, raising the policy at r^* is dominated for any $\theta \notin [\theta^*, \theta^0]$. Given so, a speculator who expects $D(\theta) = 0$ if and only if $\theta \geq \theta^*$ and $r(\theta) = \underline{r}$ if and only if $\theta \notin [\theta^*, \theta^0]$ finds it optimal to attack when observing \underline{r} if and only if $x < X(\theta^*, \theta^0)$. By

⁶The last property follows from the assumption that $\lim_{\theta \rightarrow \infty} \{V(\theta, 1) - V(\theta, 0)\} = 0$. Without this assumption, there exist equilibria in which the policy maker intervenes even for arbitrarily high types. These equilibria are sustained by the speculators threatening to attack no matter how favorable their information is. We find such a property implausible. Furthermore, these equilibria are not robust to the following perturbation. Pick any $K > \bar{\theta}$ and any $\delta > 0$ and suppose that with probability δ types $\theta > K$ are hit by a shock that forces them to set \underline{r} and assume that this shock is not observed by the speculators. The aforementioned equilibria are not robust to this perturbation, no matter how unlikely these shocks are (i.e. no matter δ, K). Instead of invoking such a refinement, we prefer to impose the aforementioned limit condition.

implication, a speculator who expects $D(\theta) = 0$ for all $\theta \geq \theta^*$ (but possibly also for some $\theta < \theta^*$) and $r(\theta) = \underline{r}$ for all $\theta \notin [\theta^*, \theta^0]$ (but possibly also for some $\theta \in [\theta^*, \theta^0]$) never finds it optimal to attack for $x > X(\theta^*, \theta^0)$. To see this, note that when the peg is maintained also for some $\theta < \theta^*$, the probability that a speculator assigns to devaluation when he observes \underline{r} is smaller than when devaluation occurs for *all* $\theta < \theta^*$. Similarly, when the policy maker sets the policy at \underline{r} also for some $\theta \in [\theta^*, \theta^0]$, the observation of \underline{r} is less informative of devaluation than when $r(\theta) = r^*$ for *all* $\theta \in [\theta^*, \theta^0]$. Hence, the incentives to attack after observing \underline{r} are maximal when $D(\theta) = 1$ for all $\theta < \theta^*$ and $r(\theta) = r^*$ for all $\theta \in [\theta^*, \theta^0]$, which explains why it is never optimal to attack for $x > X(\theta^*, \theta^0)$. But then a policy maker who expects no speculator to attack for $x > X(\theta^*, \theta^0)$ never finds it optimal to raise the policy at r^* for any $\theta > \theta^1$. Knowing this, no speculator finds it optimal to attack when $x > X(\theta^*, \theta^1)$ after observing \underline{r} , and so on.

In the Appendix (Lemma A1) we establish that the sequence $\{\theta^n\}_{n=0}^\infty$ is non-increasing. Because it is also bounded from below by θ^* , it has to converge. Clearly, the limit is the unique θ^{**} that solves condition (9) if such a solution exists; otherwise, the limit is θ^* . In the Appendix (Lemma A1) we further show that condition (9) admits a solution if and only if $\theta^* \leq \tilde{\theta}$ and that this solution is strictly above θ^* if and only if $\theta^* < \tilde{\theta}$. ■

The preceding lemma used an iteration “from above” to rule out strategies for which the policy is raised for $\theta > \theta^{**}$. The next lemma uses a similar iteration “from below” to rule out strategies for which the peg is maintained for $\theta < \theta^*$, which together with Lemma 3 completely characterizes the devaluation outcome.

Lemma 5 *For any $r^* \in (\underline{r}, \tilde{r}]$ and any equilibrium in $\mathcal{E}(r^*; \sigma)$, $D(\theta) = 1$ for any $\theta < \theta^*$.*

Proof. The result is established by comparing the speculators’ incentives to attack after observing \underline{r} with the corresponding incentives when they expect $r(\theta) = \underline{r}$ for all θ .

Let $\{\theta_n\}_{n=0}^\infty$ be the following sequence: $\theta_0 \equiv \underline{\theta}$ and for any $n \geq 1$, $\theta_n = \min\{\theta^*, \theta'_n\}$, where θ'_n solves $V(\theta'_n, \Psi(\frac{x_{n-1} - \theta'_n}{\sigma})) = 0$ with x_{n-1} implicitly defined by $1 - \Psi(\frac{x_{n-1} - \theta_{n-1}}{\sigma}) = \underline{r}$. This sequence also has a simple interpretation. When devaluation occurs at \underline{r} if and only if $\theta < \underline{\theta}$, a speculator who believes that $r(\theta) = \underline{r}$ for all θ finds it optimal to attack if and only if $x < x_0$. By implication, a speculator who expects $r(\theta) = \underline{r}$ for all $\theta < \theta^*$ (but possibly $r(\theta) > \underline{r}$ for some $\theta > \theta^*$) necessarily finds it optimal to attack for any $x < x_0$. This simply follows from the fact that the observation of

\underline{r} is most informative of devaluation when all types who devalue set $r = \underline{r}$, while some of the types who maintain the peg raise the policy at r^* . However, if all speculators attack whenever $x < x_0$, the peg is abandoned for all $\theta < \theta_1$. This in turn implies that there exists an $x_1 > x_0$ such that a speculator who expects the peg to be abandoned for all $\theta < \theta_1$ and who believes that $r(\theta) = \underline{r}$ for all θ , finds it optimal to attack for all $x < x_1$. By implication, a speculator who expects $r(\theta) = \underline{r}$ for all $\theta < \theta^*$ but possibly $r(\theta) > \underline{r}$ for some $\theta > \theta^*$, necessarily finds it optimal to attack for any $x < x_1$, and so on.

Because $\{\theta_n\}_{n=0}^\infty$ is increasing and bounded from above it necessarily converges. Note that V and Ψ are continuous and that the unique fixed point to $V(\theta, \Psi(\frac{x-\theta}{\sigma})) = 0$ and $1 - \Psi(\frac{x-\theta}{\sigma}) = \underline{r}$ is attained at $\theta = \tilde{\theta}$ and $x = \tilde{x}$. Because $\theta^* \leq \tilde{\theta}$, the limit of $\{\theta_n\}_{n=0}^\infty$ is clearly θ^* . It follows that $D(\theta) = 1$ for all $\theta < \theta^*$. ■

The results in the preceding lemmas identify the core restrictions on equilibrium outcomes: in any equilibrium in which the policy is raised, there exists at most one $r^* > \underline{r}$ played in equilibrium, the policy is raised to r^* only if $\theta \in [\theta^*, \theta^{**}]$, the peg is abandoned if $\theta < \theta^*$ and is maintained if $\theta > \theta^*$.

What these results leave open is the possibility that the policy is raised only for a strict subset of (θ^*, θ^{**}) . Although such a possibility would not affect the predictions we discuss in the subsequent sections in any serious way, it requires that the speculators' strategy after observing $r = \underline{r}$ be non-monotonic in x , which in turn requires that posterior beliefs about θ given \underline{r} be non-monotone in x . This possibility is ruled out by the following result (whose proof is rather technical and is thus in the Appendix).

Lemma 6 *For any $r^* \in (\underline{r}, \tilde{r}]$ and any equilibrium in $\mathcal{E}(r^*; \sigma)$, the following are true:*

(i) *If $a(x, \underline{r})$ is decreasing in x , then $r(\theta) = r^*$ if $\theta \in (\theta^*, \theta^{**})$, $a(x, \underline{r}) = 1$ if $x < x^*$, and $a(x, \underline{r}) = 0$ if $x > x^*$, where $x^* = X(\theta^*, \theta^{**})$.*

(ii) *If ψ is log-concave, then $a(x, \underline{r})$ is decreasing in x .*

So far we have identified sharp, but only necessary, conditions for the set of equilibrium outcomes. The next lemma completes the equilibrium characterization by showing that $\mathcal{E}(\underline{r}; \sigma) \neq \emptyset$ and $\mathcal{E}(r^*; \sigma) \neq \emptyset$ for any $r^* \in (\underline{r}, \tilde{r}]$. This last result follows from adapting Proposition 2 in Angeletos, Hellwig and Pavan (2006) to the different payoff structure assumed here.

Lemma 7 (i) *There exists an equilibrium in which $r(\theta) = \underline{r}$ for all θ , $a(x, r) = 1$ if and only if $x < \tilde{x}$, and $D(\theta) = 1$ if and only if $\theta < \tilde{\theta}$.*

(ii) *For any $r^* \in (\underline{r}, \tilde{r}]$, there exists an equilibrium in which $r(\theta) = r^*$ if $\theta \in [\theta^*, \theta^{**}]$, $r(\theta) = \underline{r}$ otherwise, $a(x, r) = 1$ if and only if $(x, r) < (x^*, r^*)$, and $D(\theta) = 1$ if and only if $\theta < \theta^*$.*

The strategies in Lemma 7 are particularly simple and permit us to identify r^* as the level of policy intervention at which the speculators switch from “aggressive” to “lenient” behavior. Although other strategies can also sustain the same equilibrium outcomes, Lemmas 2-6 ensure that these other strategies can differ only out of equilibrium (or for a zero-measure set of θ and x on equilibrium).⁷

Combining all the aforementioned results completes the proof of Proposition 1.

Remark. The equilibrium definition we have used rules out mixed strategies for either the policy maker or the speculators; it also imposes symmetry on the speculators’ strategies. However, from the arguments in the proofs of Lemmas 1-6, it should be clear that none of the necessary conditions identified in these lemmas depend on these restrictions. Indeed, the policy maker can find it optimal to randomize over r , or over D , only for a zero-measure subset of θ ; because this does not have any effect on the speculators’ posterior beliefs about policy and devaluation outcomes, it cannot affect their best-responses. Similarly, for any r , the speculators can find it optimal to randomize over a , or to play asymmetrically, only for a zero-measure subset of their signal space; because this does not have any effect on the aggregate size of attack, it does not affect the policy maker’s incentives. Proposition 1 thus characterizes the entire set of equilibrium outcomes, including those sustained by mixed-strategy or asymmetric equilibria.

4 Predictions about policies and devaluation outcomes

In this section, we show how the complete characterization of the set of equilibrium outcomes as given by Proposition 1 permits us to identify predictions regarding the structure of policy choices and devaluation outcomes that are not sensitive to equilibrium selection.

⁷Moreover, the strategies and the beliefs considered in the proof of Lemma 7 survive the intuitive criterion of Cho and Kreps (1987) and can be obtained as the limit to perturbations that introduce full-support noise in policy observations so that beliefs are always pinned down by Bayes’ rule. This follows from arguments similar to those in Angeletos, Hellwig and Pavan (2006).

Following Proposition 1, we henceforth index equilibria by $s \in [\underline{r}, \tilde{r}]$. To highlight the dependence of the equilibrium outcomes on the quality of information σ , for any $s \in [\underline{r}, \tilde{r}]$, we denote by $r_s(\theta; \sigma)$, $D_s(\theta; \sigma)$ and $\mathcal{D}_s(\sigma) \equiv \{\theta : D_s(\theta; \sigma) = 1\}$ respectively the equilibrium policy, the devaluation outcome, and set of types who abandon the peg, in any equilibrium in $\mathcal{E}(s; \sigma)$. For any $s \in (\underline{r}, \tilde{r}]$, we then let $\theta_s^*(\sigma)$ and $\theta_s^{**}(\sigma)$ denote the corresponding thresholds as defined in part (iii) of Proposition 1 and $\Delta_s(\sigma) \equiv \theta_s^{**}(\sigma) - \theta_s^*(\sigma)$ the (Lebesgue) measure of types who intervene. Finally, to save on notation, we also let $\theta_{\underline{r}}^*(\sigma) \equiv \theta_{\underline{r}}^{**}(\sigma) \equiv \tilde{\theta}$ for any $\sigma > 0$, where $\tilde{\theta}$ is the devaluation threshold in any of the pooling equilibria of part (ii) in Proposition 1.

Proposition 2 (policy choices and regime outcomes) *Equilibrium policies and devaluation outcomes satisfy the following properties.*

(i) **Non-monotonic policy.** *For any $\sigma > 0$ and any $s > \underline{r}$, $r_s(\theta; \sigma)$ is inverted U-shaped in θ , and $r_s(\theta; \sigma) \leq \tilde{r}$ for all θ .*

(ii) **Monotonic devaluation outcome.** *For any $\sigma > 0$ and any s , $D_s(\theta; \sigma)$ is decreasing in θ , and $\mathcal{D}_s(\sigma) \subseteq (-\infty, \tilde{\theta})$.*

(iii) **Impact of aggressiveness.** *For any $\sigma > 0$, $s' > s > \underline{r}$ implies $\Delta_{s'}(\sigma) < \Delta_s(\sigma)$, whereas $\mathcal{D}_{s'}(\sigma) \supset \mathcal{D}_s(\sigma)$.*

(iv) **Impact of noise.** *For any $s \in (\underline{r}, \tilde{r})$, $\sigma' > \sigma > 0$ implies $\Delta_s(\sigma') > \Delta_s(\sigma)$, whereas $\mathcal{D}_s(\sigma') = \mathcal{D}_s(\sigma)$. Moreover, $\lim_{\sigma \rightarrow 0} \Delta_s(\sigma) = 0$.*

The first two properties establish that, no matter which equilibrium is played, (i) the policy maker intervenes only for intermediate θ and never raises r above \tilde{r} ; (ii) the peg is abandoned if and only if θ is low enough and never for $\theta > \tilde{\theta}$. These predictions follow directly from the characterization result of Proposition 1.

Property (iii), on the other hand, can be interpreted as the impact of the “aggressiveness” of market expectations: the higher the level of the policy at which the speculators switch to lenient behavior (i.e., refrain from attacking), the higher the cost of intervention necessary for preventing an attack, and hence the smaller the set of types who find it optimal to intervene and the larger the set of types who abandon the peg.

Finally, property (iv) shows that, for given “aggressiveness,” the set of types who find it optimal to intervene shrinks as the speculators’ information becomes more precise, whereas the set of types who devalue is independent of σ . To understand this result, recall that the opportunity cost of

raising the policy at $s \in (\underline{r}, \tilde{r})$ is devaluing for $\theta = \theta_s^*(\sigma)$, which explains why $\theta_s^*(\sigma)$ is independent of σ . On the other hand, the opportunity cost for $\theta = \theta_s^{**}(\sigma)$ is facing a positive attack which however does not lead to devaluation; as $\sigma \rightarrow 0$, the size of attack vanishes for any $\theta > \theta_s^*(\sigma)$ and $\theta_s^{**}(\sigma)$ converges to $\theta_s^*(\sigma)$.

The predictions identified in Proposition 2 presume that the outside observer, say the econometrician, knows which equilibrium is played (although they are true for *any* equilibrium). We now turn to the predictions that the model delivers for an econometrician who is uncertain about which equilibrium is played. This uncertainty can be captured by introducing a distribution over the set of all possible equilibria and examining the implied distribution of equilibrium outcomes.

Because different equilibria within $\mathcal{E}(s, \sigma)$ lead to the same outcomes, any distribution over outcomes generated by a random selection over the equilibrium set $\mathcal{E}(\sigma)$ can be replicated by a random variable s with support $[\underline{r}, \tilde{r}]$ such that a pooling equilibrium is played when $s = \underline{r}$, while a semi-separating equilibrium in which $r^* = s$ is played when $s \in (\underline{r}, \tilde{r}]$. We then denote by \mathcal{F} the family of all possible cumulative distribution functions over $[\underline{r}, \tilde{r}]$. As far as outcomes are concerned, any beliefs the econometrician may have about which equilibrium is played simply corresponds to an element of \mathcal{F} .⁸

Note that, because θ is the policy maker's *private* information, the speculators' strategies cannot depend directly on θ ; they can only be functions of the information the speculators have about θ . It follows that the equilibrium being played cannot be a function of θ ; equivalently, the realization of the random variable s has to be independent of θ .

Now, let $I_{premise}$ denote the indicator function assuming value one if *premise* is true and zero otherwise. Next, for any $F \in \mathcal{F}$, let $D(\theta; F, \sigma) \equiv \int_{s \in [\underline{r}, \tilde{r}]} D_s(\theta; \sigma) dF(s)$ denote the probability that type θ abandons the peg, $P(r, \theta; F, \sigma) \equiv \int_{s \in [\underline{r}, \tilde{r}]} I_{\{r_s(\theta) \geq r\}} dF(s)$ the probability that type θ raises the policy at or above r , and $\Delta(r; F, \sigma) \equiv \int_{\theta \in \mathbb{R}} I_{\{P(r, \theta; F, \sigma) > 0\}} d\theta$ the (Lebesgue) measure of types who raise the policy at or above r with positive probability, when the selection is F . Finally, let $F' \gg F$ if and only if $F'(r) \leq F(r)$ for all r , with strict inequality for $r \in (\underline{r}, \tilde{r})$ and equality for $r \in \{\underline{r}, \tilde{r}\}$. The following result then translates the deterministic predictions of Proposition 2 into their probabilistic analogues for the case in which the econometrician is uncertain about which equilibrium is played.

⁸Note that s could also be interpreted as a sunspot that is used by the players to determine which equilibrium to play; the set \mathcal{F} can thus also be interpreted as the set of sunspot equilibria.

Proposition 3 (random equilibrium selections) (i) For any $r > \underline{r}$, any $F \in \mathcal{F}$ and any $\sigma > 0$, there exist θ° and $\theta^{\circ\circ}$, with $\underline{\theta} < \theta^\circ \leq \theta^{\circ\circ}$, such that $P(r, \theta; F, \sigma) > 0$ only if $r \leq \tilde{r}$ and $\theta \in [\theta^\circ, \theta^{\circ\circ}]$.

(ii) For any $\sigma > 0$ and any $F \in \mathcal{F}$, $D(\theta; F, \sigma)$ is non-increasing in θ , with $D(\theta; F, \sigma) = 1$ for $\theta \leq \underline{\theta}$ and $D(\theta; F, \sigma) = 0$ for $\theta \geq \tilde{\theta}$.

(iii) For any $\sigma > 0$, $F' \gg F$ implies $\Delta(r, F', \sigma) < \Delta(r, F, \sigma)$ for all $r \in (\underline{r}, \tilde{r})$, whereas $D(\theta; F', \sigma) > D(\theta; F, \sigma)$ for all $\theta \in (\underline{\theta}, \tilde{\theta})$ (unless $D(\theta; F, \sigma) = 1$).

(iv) For any $F \in \mathcal{F}$, $\sigma' > \sigma > 0$ implies $\Delta(r, F, \sigma') > \Delta(r, F, \sigma)$ for all $r \in (\underline{r}, \tilde{r})$ (unless $\Delta(r, F, \sigma) = 0$), whereas $D(\theta; F, \sigma') = D(\theta; F, \sigma)$ for all θ . Moreover, $\lim_{\sigma \rightarrow 0} \Delta(r, F, \sigma) = 0$.

Parts (i) and (ii) say that, no matter F , the probability of observing a policy above r is positive only for intermediate θ , and the probability of devaluation is monotonic in θ . Part (iii) says that, if the econometrician expects the players to coordinate on more aggressive equilibria, then he should also expect a smaller set of types to raise the policy and a higher probability of devaluation for any θ . Finally, part (iv) says that, holding F constant, more precise information does not affect the probability of devaluation but induces fewer types to intervene with positive probability.

One frequent criticism of common-knowledge models of crises, such as Obstfeld (1986, 1996) and Calvo (1986), is that they document the existence of a critical region of fundamentals over which there are multiple equilibria, but say little about the relation between fundamentals and equilibrium outcomes. For example, the probability of devaluation in Obstfeld need not be monotonic in the strength of the currency. In contrast, the result in part (ii) of Proposition 3 delivers a monotonic relation between the fundamentals (the policy maker's type) and the devaluation outcome, while at the same time allowing for "randomness" in this relation generated by the econometrician's uncertainty over the equilibrium selected.

The result in part (iv), on the other hand, is interesting because it suggests that the precision of information is not important for whether the peg is maintained, but it is crucial for whether this goal is achieved with or without intervention. Note, however, that this result presumes that F does not change with σ . Because the model imposes no relation between F and σ , this is possible, although not necessary.

Now suppose that one is completely agnostic on whether, or how, the equilibrium selection F changes with σ . Is it still possible to say anything about the relation between the equilibrium

outcomes and the quality of information? The answer is yes. It suffices to consider the bounds on the probability of devaluation and on the probability of intervention across all possible equilibria.

Let $D(\theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} D(\theta; F, \sigma) d\theta$ denote the probability of devaluation and $P(r, \theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} P(r, \theta; F, \sigma) d\theta$ denote the probability that the policy is raised at or above r , both conditional on the event that $\theta \in [\theta_1, \theta_2]$, for given selection F . Then let $\bar{D}(\theta_1, \theta_2; \sigma) \equiv \sup_{F \in \mathcal{F}} D(\theta_1, \theta_2; F, \sigma)$, $\underline{D}(\theta_1, \theta_2; \sigma) \equiv \inf_{F \in \mathcal{F}} D(\theta_1, \theta_2; F, \sigma)$, $\bar{P}(r, \theta_1, \theta_2; \sigma) \equiv \sup_{F \in \mathcal{F}} P(r, \theta_1, \theta_2; F, \sigma)$, and $\underline{P}(r, \theta_1, \theta_2; \sigma) \equiv \inf_{F \in \mathcal{F}} P(r, \theta_1, \theta_2; F, \sigma)$; these are bounds on the equilibrium probabilities of devaluation and policy interventions. Clearly, $\bar{D}(\theta_1, \theta_2; \sigma) \geq \underline{D}(\theta_1, \theta_2; \sigma)$ and $\bar{P}(r, \theta_1, \theta_2; \sigma) \geq \underline{P}(r, \theta_1, \theta_2; \sigma)$, with strict inequalities when $\underline{\theta} \leq \theta_1 < \theta_2 \leq \tilde{\theta}$ and $r \in (\underline{r}, \tilde{r})$. That these bounds do not coincide over a subset of the critical region reflects the equilibrium indeterminacy. The next proposition examines how these bounds depend on the quality of information.

Proposition 4 (bounds) *The bounds \underline{D} , \bar{D} , and \underline{P} are independent of σ . In contrast, \bar{P} is a nondecreasing function of σ , with $\lim_{\sigma \rightarrow 0} \bar{P}(r, \theta_1, \theta_2; \sigma) = 0$ for any $r > \underline{r}$ and any $\theta_1, \theta_2 \in \mathbb{R}$.*

Therefore, more precise information does not affect the range of equilibrium probabilities of devaluation but it reduces the range of equilibrium probabilities of intervention. What is more, in the limit as $\sigma \rightarrow 0$, the probability of raising the policy vanishes for all measurable sets of θ , whereas the probability of devaluation can take any value for any subset of $(\underline{\theta}, \tilde{\theta})$. In essence, the policy choices are uniquely determined in the limit, even though the devaluation outcomes remain indeterminate.

5 Predictions about payoffs

We now turn to the predictions the model delivers for the payoff of the policy maker. In contrast to predictions about policy choices and devaluation outcomes, predictions about payoffs need not be directly testable (the econometrician cannot directly observe the policy maker's payoffs). Nevertheless, these predictions are important for their policy implications. For example, they permit us to characterize the ex-ante value the policy maker may attach to the option to intervene once θ is realized.

Let $U_s(\theta; \sigma)$ denote the payoff that type θ obtains in any of the equilibria in $\mathcal{E}(s; \sigma)$. Next, consider the variant of our model in which r is exogenously fixed at \underline{r} for all θ , interpret this as

the game in which the option to intervene is absent, and let $\tilde{U}(\theta; \sigma)$ denote the payoff that type θ obtains in the unique equilibrium of this game.

Proposition 5 (payoffs) (i) For any s and any $\sigma > 0$, $U_s(\theta; \sigma)$ is increasing in θ .

(ii) For any $s \in (\underline{r}, \tilde{r})$, either it is the case that $U_s(\theta; \sigma) \geq \tilde{U}(\theta; \sigma)$ for all θ , with strict inequality for some θ , or there exists $\theta_s^\#(\sigma) > \tilde{\theta}$ such that $U_s(\theta; \sigma) \geq \tilde{U}(\theta; \sigma)$ if and only if $\theta \leq \theta_s^\#(\sigma)$, with strict inequality for $\theta \in (\theta_s^*(\sigma), \theta_s^\#(\sigma))$. Moreover, σ small enough ensures that the first case holds.

Part (i) establishes that payoffs, like devaluation outcomes, are monotonic in θ , no matter the equilibrium selected. This follows from the fact that, in any equilibrium, the speculators' response to $r \in \{\underline{r}, s\}$ is monotonic in x , which implies that higher types can always do as well as lower types by taking the same actions that the latter take.

Part (ii), on the other hand, establishes that the policy maker can be worse off with the option to intervene only when the equilibrium selected is such that $s \in (\underline{r}, \tilde{r})$ and both θ and σ are sufficiently high. Clearly, when $s \in \{\underline{r}, \tilde{r}\}$, $U_s(\theta; \sigma) = \tilde{U}(\theta; \sigma)$ for all θ .⁹ Thus consider an equilibrium in which $s \in (\underline{r}, \tilde{r})$ and let $\theta_s^*(\sigma)$ and $\theta_s^{**}(\sigma)$ be the unique solutions to (7)-(9) and $x_s^*(\sigma) \equiv X(\theta_s^*(\sigma), \theta_s^{**}(\sigma))$. That types $\theta \leq \tilde{\theta}$ can not be worse off follows from the fact that these types necessarily abandon the peg in the game without the option to intervene. Types $\theta > \tilde{\theta}$, on the other hand, can be worse off only if $x_s^*(\sigma) > \tilde{x}$, that is, only if the size of the attack they face when they set $r = \underline{r}$ is higher than the one they would have faced without the option to intervene. When this is the case, all types $\theta \geq \theta_s^{**}(\sigma)$ are clearly worse off. On the other hand, type $\tilde{\theta}$ is strictly better off; in fact, $s < \tilde{r}$ implies that $\theta_s^*(\sigma) < \tilde{\theta}$ and hence type $\tilde{\theta}$ can guarantee himself a strictly positive payoff by raising the policy at $r = s$, whereas he would have obtained a zero payoff absent the possibility to intervene. Together with the fact that $V(\theta, 0) - C(s) - V(\theta, \Phi(\frac{\tilde{x}-\theta}{\sigma}))$, the payoff differential between the two games for types $\theta \in [\tilde{\theta}, \theta_s^{**}(\sigma)]$ who find it optimal to intervene, is continuous and decreasing in θ (by the assumption that $V_{\theta A} \geq 0$), this ensures that there exists a $\theta_s^\#(\sigma) \in (\tilde{\theta}, \theta_s^{**}(\sigma))$ such that the policy maker is worse off if and only if $\theta > \theta_s^\#(\sigma)$. However, as we show in the Appendix, for any given $s \in (\underline{r}, \tilde{r})$, a sufficiently low σ ensures that $x_s^*(\sigma)$ is smaller than \tilde{x} , and hence that the policy maker is always better off, whatever his type. This follows from the fact that $\theta_s^{**}(\sigma) \rightarrow \theta_s^*(\sigma)$ and $x_s^*(\sigma) \rightarrow \theta_s^*(\sigma)$, whereas $\tilde{x} \rightarrow \tilde{\theta}$ as $\sigma \rightarrow 0$, which together with the fact that $\theta_s^*(\sigma) < \tilde{\theta}$ whenever $s \in (\underline{r}, \tilde{r})$ ensures that $x_s^*(\sigma) < \tilde{x}$ for σ small enough.

⁹Recall that when $s = \tilde{r}$, $\theta_s^*(\sigma) = \tilde{\theta} = \theta_s^{**}(\sigma)$ and $x_s^*(\sigma) = \tilde{x}$.

These results can easily be extended to arbitrary random equilibrium selections $F \in \mathcal{F}$. Indeed, part (i) directly implies that, for any F , the expected payoff $U(\theta; F, \sigma) \equiv \int_{s \in [\underline{r}, \bar{r}]} U_s(\theta, \sigma) dF$ is increasing in θ . Part (ii), on the other hand, implies that, if we fix an arbitrary set of types $[\theta_1, \theta_2] \subset \mathbb{R}$ and an arbitrary selection F and consider the implied probability that, conditional on $\theta \in [\theta_1, \theta_2]$, the policy maker is strictly worse off, then this probability is zero either for all σ , or at least for σ small enough. Notwithstanding the fact that, in general, the selection F may also depend on σ , this property suggests that the risk of being worse off with the option to intervene vanishes as market information becomes highly precise.

Furthermore, we can accommodate the case that F changes with σ by considering bounds on equilibrium payoffs across all possible equilibria. Let $\bar{U}(\theta; \sigma) \equiv \sup_{s \in [\underline{r}, \bar{r}]} U_s(\theta; \sigma)$ and $\underline{U}(\theta; \sigma) \equiv \inf_{s \in [\underline{r}, \bar{r}]} U_s(\theta; \sigma)$. The following proposition characterizes the relation between these bounds and the payoff obtained in the game without the option to intervene.

Proposition 6 (payoff bounds) $\bar{U}(\theta; \sigma) = V(\theta, 0) > \tilde{U}(\theta; \sigma)$ for all $\theta > \underline{\theta}$. On the other hand, there exists $\hat{\theta}(\sigma) \geq \tilde{\theta}$ such that $\underline{U}(\theta; \sigma) < \tilde{U}(\theta; \sigma)$ if and only if $\theta > \hat{\theta}(\sigma)$. Finally, $\lim_{\sigma \rightarrow 0} \underline{U}(\theta; \sigma) = \lim_{\sigma \rightarrow 0} \tilde{U}(\theta; \sigma)$ for all θ .

Consider first the supremum of the equilibrium payoffs. For any $\theta > \underline{\theta}$, the highest *feasible* payoff is $V(\theta, 0)$, the payoff enjoyed when no speculator attacks. This payoff can be approximated arbitrarily well in the game in which intervention is possible (it suffices to take any equilibrium in which s is sufficiently close to \underline{r}) but not in the game in which the option to intervene is absent.

Next consider the infimum of the equilibrium payoffs. Type θ can be worse off in *some* equilibria of the game with the option to intervene only if θ is above some threshold $\hat{\theta}(\sigma) \in [\tilde{\theta}, \infty)$. This is a direct implication of part (ii) of Proposition 5: no type $\theta \leq \tilde{\theta}$ can be worse off, and if a type $\theta > \tilde{\theta}$ is strictly worse off in some equilibrium, then any $\theta' > \theta$ is also strictly worse off in the same equilibrium. That $\hat{\theta}(\sigma) < \infty$ follows from the fact that, for any given σ , one can find an equilibrium with s close enough to \underline{r} such that $x_s^*(\sigma) > \tilde{x}$. In such equilibrium, any $\theta > \theta_s^{**}(\sigma)$ is strictly worse off.

Simulations suggest that $\hat{\theta}(\sigma) \rightarrow \bar{\theta}$ as $\sigma \rightarrow 0$, meaning that the subset of the critical region where the policy maker can be worse off with the option to intervene vanishes as information becomes infinitely precise. We have not been able to prove that this is true in general. However

we have proved that $\hat{\theta}(\sigma)$ is strictly higher than $\tilde{\theta}$ for σ small enough.¹⁰

Finally, to see why, for any θ , the difference between $\underline{U}(\theta; \sigma)$ and $\tilde{U}(\theta; \sigma)$ vanishes as $\sigma \rightarrow 0$, note that, for any $\theta \leq \tilde{\theta}$, this difference is clearly zero because the lower bound is simply the payoff obtained in any equilibrium in which type θ is forced to abandon the peg. For types $\theta > \tilde{\theta}$, on the other hand, the lower bound on possible payoffs is obtained by considering equilibria in which the size of attack the policy maker faces if he does not raise the policy is *higher* than the one he would have faced if he did not have the option to intervene.¹¹ When σ is small enough, this possibility (that $x_s^*(\sigma) > \tilde{x}$) requires that s be close enough to \underline{r} , and the closer so the smaller σ . But then, because any $\theta > \tilde{\theta}$ can always opt to raise the policy ensuring a payoff $V(\theta, 0) - C(s)$, the lower bound on possible payoffs necessarily converges to $V(\theta, 0)$ for any $\theta > \tilde{\theta}$. Because this is also the payoff that the policy maker obtains in the game without the option to intervene as $\sigma \rightarrow 0$, we conclude that $\lim_{\sigma \rightarrow 0} \underline{U}(\theta; \sigma) = \lim_{\sigma \rightarrow 0} \tilde{U}(\theta; \sigma)$ for all θ .

Now imagine that, before knowing his type, the policy maker decides whether to maintain or to give up the option to intervene after learning θ . The aforementioned results suggest that, in general, the policy maker need not be able to ensure that he will be better off with the option to intervene no matter the realized θ : he may get “trapped” in an equilibrium in which he is worse off when θ turns out to be sufficiently high. Even then, however, the policy maker is better off for low θ . Therefore, the option to intervene either is beneficial for all θ , or it implements a form of insurance across types.

6 Contrast to common knowledge

We now contrast the predictions of the incomplete-information game with those of its common-knowledge counterpart. We further show that, while multiplicity obtains in our model for any level of noise, the set of equilibrium outcomes becomes smaller (in an appropriate sense) the more precise the speculators’ private information, but explodes at zero noise. The purpose of these exercises is to highlight that the selection power of global games has significant bite also in our multiple-equilibria

¹⁰This follows from the fact that, when σ is small, $x_s^*(\sigma) > \tilde{x}$ is possible only for r^* bounded away from \tilde{r} ; but then in any such equilibrium type $\hat{\theta}$ and, by continuity, all types θ in a right neighborhood of $\hat{\theta}$ are necessarily strictly better off. See the Appendix for details.

¹¹This follows from the fact that $\underline{U}(\theta; \sigma) \leq \tilde{U}(\theta; \sigma)$ for all θ . Indeed, the game with the option to intervene always admits two equilibria in which all types obtain the same payoff as in the unique equilibrium of the game without this option; these are the pooling equilibrium ($s = \underline{r}$) and the semi-separating equilibrium in which $s = \tilde{r}$.

setting and to establish that the predictions we have identified, albeit quite intuitive, would not have been possible with complete information.

Proposition 7 (common knowledge) *Consider the game with $\sigma = 0$.*

(i) *A policy $r(\cdot)$ can be part of a subgame-perfect equilibrium if and only if $r(\theta) \leq \rho(\theta)$ for $\theta \in [\underline{\theta}, \bar{\theta}]$ and $r(\theta) = \underline{r}$ for $\theta \notin [\underline{\theta}, \bar{\theta}]$.*

(ii) *A devaluation outcome $D(\cdot)$ can be part of a subgame-perfect equilibrium if and only if $D(\theta) = 1$ for $\theta < \underline{\theta}$, $D(\theta) \in \{0, 1\}$ for $\theta \in [\underline{\theta}, \bar{\theta}]$, and $D(\theta) = 0$ for $\theta > \bar{\theta}$.*

This result contrasts sharply with the results in Propositions 1 and 2. None of the predictions in the game with incomplete information are valid in the game with common knowledge. In particular, the policy can now exceed \tilde{r} for $\theta \in (\tilde{\theta}, \bar{\theta}]$ and can take any shape in the critical region $[\underline{\theta}, \bar{\theta}]$. Similarly, the probability of devaluation can take any value within the critical region, it need not be monotonic in θ , and can be positive also for $\theta > \tilde{\theta}$.

Under complete-information, the only policy choices and devaluation outcomes that are ruled out by equilibrium reasoning are those that are ruled out by strict dominance. In essence, “almost anything goes” within the critical region under complete information.

The contrast between the complete- and incomplete-information versions of our model is most evident in the limit as $\sigma \rightarrow 0$. Let $\mathcal{G}(\sigma)$ denote the set of pairs (θ, r) such that, in the game with noise $\sigma \geq 0$, there is an equilibrium in which type θ sets the policy at r .

Proposition 8 (limit outcomes) *Under complete information,*

$$\mathcal{G}(0) = \{(\theta, r) : \text{either } \theta \in [\underline{\theta}, \bar{\theta}] \text{ and } \underline{r} \leq r \leq \rho(\theta), \text{ or } \theta \notin [\underline{\theta}, \bar{\theta}] \text{ and } r = \underline{r}\}.$$

In contrast, under incomplete information,

$$\lim_{\sigma \rightarrow 0^+} \mathcal{G}(\sigma) = \{(\theta, r) : \text{either } \theta \in [\underline{\theta}, \tilde{\theta}] \text{ and } r \in \{\underline{r}, \rho(\theta)\}, \text{ or } \theta \notin [\underline{\theta}, \tilde{\theta}] \text{ and } r = \underline{r}\},$$

which is a zero-measure subset of $\mathcal{G}(0)$.

This result is illustrated in Figure 1. The common-knowledge set, $\mathcal{G}(0)$, is given by the large triangular area. The incomplete-information set, $\mathcal{G}(\sigma)$ for $\sigma > 0$, is given by the dashed area. As long as $\sigma > 0$, the lower σ is, the smaller the set of policies that can be played by any given θ , and

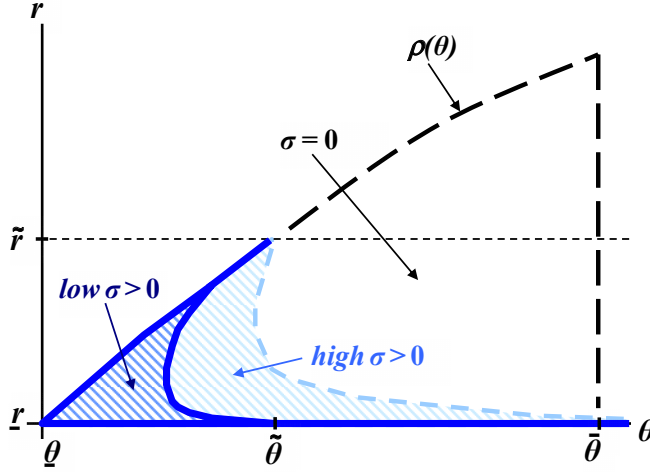


Figure 1: Pairs (θ, r) that can be observed as equilibrium outcomes.

hence the smaller the dashed area in Figure 1 (i.e., $\sigma' > \sigma > 0$ implies $\mathcal{G}(\sigma') \supset \mathcal{G}(\sigma)$). In this sense, the predictions of the model become sharper as the noise in the speculators' information becomes smaller. Indeed, the predictions are sharpest in the limit as $\sigma \rightarrow 0^+$. The set $\mathcal{G}(\sigma)$ then converges to the boundary points of the set of policies that would have been possible under complete information for any $\theta \leq \tilde{\theta}$, and to the cost-minimizing policy for $\theta > \tilde{\theta}$.

The restrictions that incomplete information poses on equilibrium outcomes are useful for identification when one estimates the model. For example, suppose the speculators' opportunity cost of attacking is $r + c$, for some $c \in (-\underline{r}, 1 - \bar{r})$. Further assume that the econometrician does not know c and tries to estimate it assuming that the data are generated from our model. The set of common-knowledge equilibrium outcomes is insensitive to c , implying that c cannot be identified (unless one makes "ad hoc" assumptions on equilibrium selection). In contrast, the set of incomplete-information equilibrium outcomes shrinks with c , in the sense that both $\tilde{\theta}$ and \tilde{r} decrease with c ; it is this kind of sensitivity that can help identification.

7 Conclusion

The approach followed in most recent applications of global games is to use incomplete information as a tool to select a unique equilibrium in coordination settings that admit multiple equilibria under common knowledge: to assume certain exogenous information structures that ensure uniqueness,

without investigating what determines information in the first place. For certain questions, however, understanding the endogeneity of information is essential for understanding the phenomenon under examination. This often brings back multiple equilibria. However, this multiplicity is very different from the one that emerges with complete information. More importantly, this multiplicity need not preclude concrete and testable predictions that are robust across all equilibria.

In this paper, we demonstrated these points in the context of defense policies against speculative currency attacks. However, these points are likely to be relevant also for other applications that endogenize information in global games. These include *learning* in dynamic settings (Angeletos, Hellwig and Pavan, 2007), aggregation of information through *prices* (Angeletos and Werning, 2006; Hellwig, Mukherji, and Tsyvinski, 2006; Morris and Shin, 2006a; Ozdenoren and Yuan, 2006; Tarashev, 2006), and manipulation of information through *propaganda* (Edmond, 2006). In Angeletos, Hellwig and Pavan (2007), for example, learning sustains multiplicity but all equilibria share the prediction that dynamics alternate between phases of tranquility, in which no attack is possible, and phases of distress, in which an attack is possible but does not necessarily take place.

Finally, in this paper we did not confront the predictions we delivered with the data. This is clearly an important next step for future research. The task is challenging, but recent advances in structural estimation of models with multiple equilibria seem to help in this direction.

Appendix: proofs omitted in the main text

Lemma A1. (i) *The sequence $\{\theta^n\}_{n=0}^\infty$ defined in the proof of Lemma 4 is non-increasing.*

(ii) *Condition (9) admits a solution if and only if $\theta^* \leq \tilde{\theta}$. Furthermore, this solution is strictly above θ^* for any $\theta^* < \tilde{\theta}$.*

Proof of Lemma A1. Part (i). Let

$$\tilde{g}(\theta^*, \theta^{n-1}, \theta) \equiv V(\theta, 0) - V(\theta^*, 0) - V\left(\theta, \Psi\left(\frac{X(\theta^*, \theta^{n-1}) - \theta}{\sigma}\right)\right) \quad (10)$$

and note that, by the assumptions that $V_A < 0 < V_\theta$ and $V_{\theta A} \geq 0$, $\tilde{g}(\theta^*, \theta^{n-1}, \theta)$ is decreasing in θ , with $\tilde{g}(\theta^*, \theta^{n-1}, \theta^*) = -V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^{n-1}) - \theta^*}{\sigma}\right)\right)$.

Take any $n \geq 1$. If $V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^{n-1}) - \theta^*}{\sigma}\right)\right) \geq 0$, then $g(\theta^*, \theta^{n-1}, \theta) = \tilde{g}(\theta^*, \theta^{n-1}, \theta) < \tilde{g}(\theta^*, \theta^{n-1}, \theta^*) \leq 0$ for all $\theta > \theta^*$, and hence $\theta^n = \theta^* \leq \theta^{n-1}$. But because $X(\theta^*, \theta')$ is increasing in θ' ,

$$V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^n) - \theta^*}{\sigma}\right)\right) \geq V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^{n-1}) - \theta^*}{\sigma}\right)\right) \geq 0$$

and hence $\theta^k = \theta^*$ for all $k \geq n$. If, instead, $V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^{n-1}) - \theta^*}{\sigma}\right)\right) < 0$, then $\theta^n > \theta^*$ is the highest solution to $\tilde{g}(\theta^*, \theta^{n-1}, \theta) = 0$, or equivalently $\theta^n = f(\theta^{n-1})$ where $f(\theta)$ is an increasing function defined implicitly by $\tilde{g}(\theta^*, \theta, f(\theta)) = 0$. To prove that the sequence $\{\theta^n\}_{n=0}^\infty$ is non-increasing, it thus suffices to show that $\theta^1 < \theta^0$. To see this, note that

$$\tilde{g}(\theta^*, \theta^0, \theta^0) < V(\theta^0, 0) - C(r^*) - V(\theta^0, 1) = 0,$$

which together with the fact that $\tilde{g}(\theta^*, \theta^0, \theta)$ is decreasing in θ proves that $\theta^1 < \theta^0$.

Part (ii). Note that the function

$$G(\theta^*, \theta) \equiv V(\theta, 0) - V(\theta^*, 0) - V\left(\theta, \Psi\left(\frac{X(\theta^*, \theta) - \theta}{\sigma}\right)\right) \quad (11)$$

is continuously decreasing in θ with $G(\theta^*, \theta^*) = -V(\theta^*, 1 - \underline{r})$ and $\lim_{\theta \rightarrow \infty} G(\theta^*, \theta) = -V(\theta^*, 0) < 0$. It follows that a solution to $G(\theta^*, \theta) = 0$ exists if and only if $G(\theta^*, \theta^*) \geq 0$, or equivalently $V\left(\theta^*, \Psi\left(\frac{X(\theta^*, \theta^*) - \theta^*}{\sigma}\right)\right) \leq 0$. Because the function $\tilde{V}(\theta) \equiv V\left(\theta, \Psi\left(\frac{X(\theta, \theta) - \theta}{\sigma}\right)\right)$ is increasing in θ with $\tilde{V}(\theta) < 0$ (resp. $\tilde{V}(\theta) > 0$) if and only if $\theta < \tilde{\theta}$ (resp. $\theta > \tilde{\theta}$), a solution to (9) exists if and only if $\theta^* \leq \tilde{\theta}$ and is strictly above θ^* for any $\theta^* < \tilde{\theta}$. *Q.E.D.*

Proof of Lemma 6. Part (i). When $r^* = \tilde{r}$, $\theta^* = \theta^{**} = \tilde{\theta}$, $x^* = X(\theta^*, \theta^{**}) = \tilde{x}$, and, by Lemma 4, $r(\theta) = \underline{r}$ for all $\theta \neq \tilde{\theta}$; the result then follows from the same argument as in the proof of Lemma 1. Thus consider $r^* < \tilde{r}$, in which case $\theta^* < \tilde{\theta}$ and $\theta^* < \theta^{**}$. From the proof of Lemma 4, $a(x, \underline{r}) = 0$ for all $x > x^*$, while from the proof of Lemma 5, $a(x, \underline{r}) = 1$ for all $x < X(\theta^*, \theta^*)$. It follows that $\Psi\left(\frac{\theta - x^*}{\sigma}\right) \geq A(\theta, \underline{r}) \geq \Psi\left(\frac{\theta - X(\theta^*, \theta^*)}{\sigma}\right)$ for all θ . By the fact that $V\left(\theta, \Psi\left(\frac{\theta - X(\theta, \theta)}{\sigma}\right)\right)$ is increasing in θ and equal to zero at $\theta = \tilde{\theta}$, $V\left(\theta^*, \Psi\left(\frac{\theta^* - X(\theta^*, \theta^*)}{\sigma}\right)\right) < 0$, while by the fact that θ^{**} solves (9), $V(\theta^{**}, \Psi\left(\frac{\theta^{**} - x^*}{\sigma}\right)) > 0$. Combining, we have that $V(\theta^*, A(\theta^*, \underline{r})) < 0 < V(\theta^{**}, A(\theta^{**}, \underline{r}))$, which together with the monotonicity of A in θ ensures that there exists a unique $\hat{\theta} \in [\theta^*, \theta^{**})$ such that $V(\theta, A(\theta, \underline{r})) < 0$ if and only if $\theta < \hat{\theta}$.

Now let $\theta'' = \sup\{\theta : r(\theta) = r^*\}$. Clearly, $\theta'' \geq \hat{\theta}$; if $\theta'' < \hat{\theta}$, types $\theta \in (\theta'', \hat{\theta})$ would be better off raising the policy. But then θ'' must solve the indifference condition $V(\theta'', 0) - C(r^*) = V(\theta'', A(\theta'', \underline{r}))$. This together with the monotonicity of A in θ and the assumption that $V_{\theta A} \geq 0$ ensures that all $\theta \in [\theta^*, \theta'']$ necessarily set r^* . The posterior probability that a speculator assigns to devaluation conditional on observing \underline{r} is then given by

$$\mu(\theta^* | x, \underline{r}) = \frac{1 - \Psi\left(\frac{x - \theta^*}{\sigma}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\sigma}\right) + \Psi\left(\frac{x - \theta''}{\sigma}\right)}.$$

Since $\mu(\theta^* | x, \underline{r})$ is decreasing in x , $a(x, \underline{r}) = 1$ if $x < x'$ and $a(x, \underline{r}) = 0$ if $x > x'$, where $x' \equiv X(\theta^*, \theta'')$. This in turn implies that $A(\theta'', \underline{r}) = \Psi\left(\frac{X(\theta^*, \theta'') - \theta''}{\sigma}\right)$ and therefore θ'' must solve $V(\theta'', 0) - C(r^*) = V\left(\theta'', \Psi\left(\frac{X(\theta^*, \theta'') - \theta''}{\sigma}\right)\right)$, which is the same as condition (9). That is, $\theta'' = \theta^{**}$ and hence $x' = x^*$.

Part (ii) We now show that if ψ is log-concave, $a(x, \underline{r})$ is decreasing in x . The probability of devaluation given x and \underline{r} is $\mu(\theta^* | x, \underline{r}) = (1 + 1/M(x))^{-1}$, where

$$M(x) \equiv \frac{1 - \Psi\left(\frac{x - \theta^*}{\sigma}\right)}{\int_{\theta^*}^{\infty} [1 - I(\theta)] \frac{1}{\sigma} \psi\left(\frac{x - \theta}{\sigma}\right) d\theta}$$

with $I(\theta) = 0$ when $r(\theta) = \underline{r}$ and $I(\theta) = 1$ when $r(\theta) = r^*$. It follows that $\mu(\theta^* | x, \underline{r})$ is decreasing in x if $d \ln M(x) / dx < 0$, or equivalently

$$\frac{\int_{-\infty}^{\theta^*} \frac{1}{\sigma^2} \psi'\left(\frac{x - \theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\theta^*} \frac{1}{\sigma} \psi\left(\frac{x - \theta}{\sigma}\right) d\theta} - \frac{\int_{\theta^*}^{\infty} [1 - I(\theta)] \frac{1}{\sigma^2} \psi'\left(\frac{x - \theta}{\sigma}\right) d\theta}{\int_{\theta^*}^{\infty} [1 - I(\theta)] \frac{1}{\sigma} \psi\left(\frac{x - \theta}{\sigma}\right) d\theta} < 0. \quad (12)$$

Using the fact that $I(\theta) = 0$ for all $\theta \leq \theta^*$, (12) is equivalent to

$$\mathbb{E}_\theta \left[\frac{\psi' \left(\frac{x-\theta}{\sigma} \right)}{\psi \left(\frac{x-\theta}{\sigma} \right)} \middle| \theta \leq \theta^*, x, \underline{r} \right] - \mathbb{E}_\theta \left[\frac{\psi' \left(\frac{x-\theta}{\sigma} \right)}{\psi \left(\frac{x-\theta}{\sigma} \right)} \middle| \theta > \theta^*, x, \underline{r} \right] < 0,$$

which holds true when ψ'/ψ is decreasing, i.e. when ψ is log-concave. The monotonicity of $\mu(\theta^*|x, \underline{r})$ in x then implies monotonicity of $a(x, \underline{r})$. *Q.E.D.*

Proof of Lemma 7. For both parts we let the strategy of the policy maker in stage 3 be such that $D(\theta, r, A) = 0$ when $V(\theta, A) = 0$, meaning that, when indifferent, the policy maker maintains the peg.

Part (i). Because $A(\theta, r)$ is independent of r , $r(\theta) = \underline{r}$ is clearly optimal for the policy maker; the optimality of $D(\cdot)$ is obvious. Thus consider the speculators. Because $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \tilde{\theta}$, a speculator finds it optimal to follow the equilibrium strategy if and only if, for any r , his beliefs satisfy

$$\mu(\tilde{\theta}|x, r) \geq r \text{ if } x < \tilde{x} \text{ and } \mu(\tilde{\theta}|x, r) \leq r \text{ if } x \geq \tilde{x}. \quad (13)$$

When $r = \underline{r}$, Bayes' rule imposes that $\mu(\theta|x, \underline{r}) = 1 - \Psi\left(\frac{x-\theta}{\sigma}\right)$; that these beliefs satisfy (13) follows directly from the definition of \tilde{x} . When, instead, $r > \underline{r}$, a deviation is detected whatever x . There then exists an arbitrarily large set of out-of-equilibrium beliefs that satisfy (13).

Part (ii). Because $A(\theta, r) = A(\theta, \underline{r}) = \Psi\left(\frac{x^*-\theta}{\sigma}\right)$ for any $r < r^*$ and $A(\theta, r) = A(\theta, r^*) = 0$ for any $r \geq r^*$, the policy maker clearly prefers \underline{r} to any $r \in (\underline{r}, r^*)$ and r^* to any $r > r^*$. Furthermore, \underline{r} is dominant for any $\theta \leq \underline{\theta}$. For $\theta > \underline{\theta}$, on the other hand, the payoff from setting r^* is $V(\theta, 0) - C(r^*)$, while the payoff from setting \underline{r} is $\max\{0, V(\theta, A(\theta, \underline{r}))\}$. Hence, r^* is optimal if and only if $C(r^*) \leq V(\theta, 0)$ and $C(r^*) \leq V(\theta, 0) - V(\theta, A(\theta, \underline{r}))$. From the definitions of θ^* , θ^{**} and x^* this is the case if and only if $\theta \in [\theta^*, \theta^{**}]$. Now consider the speculators. When $r < r^*$, $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \hat{\theta}$, where $\hat{\theta}$ solves $V(\hat{\theta}, \Psi\left(\frac{x^*-\hat{\theta}}{\sigma}\right)) = 0$ (note that $\theta^* \leq \hat{\theta} \leq \theta^{**}$, with strict inequalities for $r^* < \tilde{r}$). When, instead, $r \geq r^*$, $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \underline{\theta}$. A speculator thus finds it optimal to follow the equilibrium strategy if and only if his beliefs satisfy

the following two conditions:

$$\text{when } r < r^*, \mu(\hat{\theta}|x, r) \geq r \text{ if } x < x^* \text{ and } \mu(\hat{\theta}|x, r) \leq r \text{ if } x \geq x^*; \quad (14)$$

$$\text{when } r \geq r^*, \mu(\underline{\theta}|x, r) \leq r \text{ for all } x. \quad (15)$$

Beliefs are pinned down by Bayes' rule when either $r = \underline{r}$ or $r = r^*$. In the first case ($r = \underline{r}$),

$$\mu(\hat{\theta}|x, \underline{r}) = \mu(\theta^*|x, \underline{r}) = \frac{1 - \Psi\left(\frac{x - \theta^*}{\sigma}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\sigma}\right) + \Psi\left(\frac{x - \theta^{**}}{\sigma}\right)},$$

which is decreasing in x and equals \underline{r} at $x = x^*$, thus satisfying (14). In the second case ($r = r^*$), $\mu(\underline{\theta}|x, r^*) = 0$, which clearly satisfies (15). Finally, whenever $r \notin \{\underline{r}, r^*\}$, there exist an arbitrarily large set of out-of-equilibrium beliefs that satisfy (14) and (15). *Q.E.D.*

Proof of Proposition 2. Parts (i)-(ii) follow directly from Proposition 1. Thus consider parts (iii) and (iv). Let $\theta_s^*(\sigma)$ and $\theta_s^{**}(\sigma)$ denote the (unique) thresholds corresponding to the equilibria in which the policy is raised at $s \in (\underline{r}, \tilde{r}]$. The threshold $\theta_s^*(\sigma)$ is the unique solution to $V(\theta^*, 0) = C(s)$, whereas $\theta_s^{**}(\sigma)$ is the unique solution to $G(\theta_s^*(\sigma), \theta^{**}; \sigma) = 0$, where, for any $\theta \geq \theta^*$ and any $\sigma > 0$, the function $G(\theta^*, \theta; \sigma)$ is as defined in (11). Now let

$$B(\theta^*, \theta; \sigma) \equiv \Psi\left(\frac{X(\theta^*, \theta; \sigma) - \theta}{\sigma}\right) \quad (16)$$

where $X(\theta^*, \theta; \sigma)$ is implicitly defined by (8). Using (8), $B(\theta^*, \theta; \sigma)$ can also be implicitly defined by

$$B = \left(\frac{1 - \underline{r}}{\underline{r}}\right) \left[1 - \Psi\left(\Psi^{-1}(B) + \frac{\theta - \theta^*}{\sigma}\right)\right].$$

For any $\theta > \theta^*$, $B(\theta^*, \theta; \sigma)$ and hence also $G(\theta^*, \theta; \sigma)$ is increasing in σ . Because $G(\theta^*, \theta; \sigma)$ is decreasing in θ , by the Implicit Function Theorem, $\theta_s^{**}(\sigma)$ is increasing in σ . Moreover, for any $\theta > \theta^*$, $\lim_{\sigma \rightarrow 0^+} B(\theta^*, \theta; \sigma) = 0$ and hence $\lim_{\sigma \rightarrow 0^+} G(\theta^*, \theta; \sigma) = -V(\theta^*, 0) < 0$, which implies that $\theta_s^{**}(\sigma)$ converges to $\theta_s^*(\sigma)$ as $\sigma \rightarrow 0$. Finally, that $\mathcal{D}_s(\sigma) = (-\infty, \theta_s^*(\sigma))$ is independent of σ follows directly from the fact that $\theta_s^*(\sigma)$ is independent of σ . This completes the proof of part (iv).

For part (iii), let $\Delta_s(\sigma) \equiv \theta_s^{**}(\sigma) - \theta_s^*(\sigma)$. We seek to prove that $\Delta_s(\sigma)$ is decreasing in s , with

$\Delta_s(\sigma) \rightarrow 0$ as $s \rightarrow \tilde{r}$. Note that $\Delta_s(\sigma)$ solves $\hat{G}(\theta_s^*(\sigma), \Delta; \sigma) = 0$, where

$$\hat{G}(\theta^*, \Delta; \sigma) \equiv G(\theta^*, \theta^* + \Delta; \sigma) = V(\theta^* + \Delta, 0) - V(\theta^*, 0) - V\left(\theta^* + \Delta, \hat{B}(\Delta; \sigma)\right)$$

with $\hat{B}(\Delta; \sigma)$ implicitly defined by

$$B = \left(\frac{1-r}{r}\right) \left[1 - \Psi\left(\Psi^{-1}(B) + \frac{\Delta}{\sigma}\right)\right].$$

Because $G(\theta^*, \theta; \sigma)$ is decreasing in θ , $\hat{G}(\theta^*, \Delta; \sigma)$ is decreasing in Δ . To see that $\hat{G}(\theta^*, \Delta; \sigma)$ is also decreasing in θ^* , note that

$$\hat{G}_{\theta^*}(\theta^*, \Delta; \sigma) = V_{\theta}(\theta^* + \Delta, 0) - V_{\theta}(\theta^*, 0) - V_{\theta}\left(\theta^* + \Delta, \hat{B}(\Delta; \sigma)\right).$$

The above is negative because $V_{\theta} > 0$ and $V_{\theta A} \geq 0$. By the Implicit Function Theorem, $\Delta_s(\sigma)$ thus decreases with $\theta_s^*(\sigma)$ and hence with s . The continuity of $\Delta_s(\sigma)$ in s (which follows from the continuity of G in θ^* and θ), then implies that $\Delta_s(\sigma) \rightarrow 0$ as $s \rightarrow \tilde{r}$. Together with the fact that $\theta_s^*(\sigma)$ is increasing in s and hence that $\mathcal{D}_{s'}(\sigma) \supset \mathcal{D}_s(\sigma)$ for any $s' > s$, this completes the proof of part (iii). *Q.E.D.*

Proof of Proposition 3. Note that

$$\begin{aligned} D(\theta; F, \sigma) &= \int_{s \in [\underline{r}, \tilde{r}]} I_{\{\theta < \theta_s^*(\sigma)\}} dF(s) \\ P(r, \theta; F, \sigma) &= \begin{cases} 1 & \text{if } r = \underline{r} \\ \int_{s \in [r, \tilde{r}]} I_{\{\theta_s^*(\sigma) \leq \theta \leq \theta_s^{**}(\sigma)\}} dF(s) & \text{if } r \in (\underline{r}, \tilde{r}] \\ 0 & \text{if } r > \tilde{r} \end{cases} \\ \Delta(r; F, \sigma) &= \begin{cases} \infty & \text{if } r = \underline{r} \\ \int_{s \in [r, \tilde{r}]} \Delta_s(\sigma) dF(s) & \text{if } r \in (\underline{r}, \tilde{r}] \\ 0 & \text{if } r > \tilde{r} \end{cases} \end{aligned}$$

Consider first part (i). For $r > \tilde{r}$, $P(r, \theta; F, \sigma) = 0$ follows directly from part (i) in Proposition 2. For $r \in (\underline{r}, \tilde{r}]$, it suffices to let $\theta^{\circ} = \theta_r^*(\sigma)$ and $\theta^{\circ\circ} = \max\{\theta_s^{**}(\sigma) : s \in [r, \tilde{r}]\}$. Note that, for any $r \in (\underline{r}, \tilde{r}]$, $\theta_s^{**}(\sigma)$ is continuous over $s \in [r, \tilde{r}]$, ensuring that the maximum exists.

Part (ii) follows directly from part (ii) in Proposition 2.

Part (iii) follows from these properties: $\Delta_s(\sigma)$ is strictly decreasing in s , for any $s \in (r, \tilde{r})$; $D_s(\theta; \sigma)$ is nondecreasing in s , for any $s \in (r, \tilde{r})$; for any $\theta \in (\underline{\theta}, \tilde{\theta})$, $D(\theta; F', \sigma) = 1 - F'(C^{-1}(V(\theta, 0))) > 1 - F(C^{-1}(V(\theta, 0))) = D(\theta; F, \sigma)$.

Finally, consider part (iv). That $\Delta(r, F, \sigma') \geq \Delta(r, F, \sigma)$ for all $r \in (\underline{r}, \tilde{r})$, with strict inequality unless $\Delta(r, F, \sigma) = 0$ (i.e., unless F assigns all measure to $s \in \{\underline{r}, \tilde{r}\}$), follows from the fact that $\Delta_s(\sigma)$ is positive and strictly decreasing in σ for all $s \in (\underline{r}, \tilde{r})$. That $\lim_{\sigma \rightarrow 0} \Delta(r, F, \sigma) = 0$ then follows from Lebesgue monotone convergence theorem using the fact that $\lim_{\sigma \rightarrow 0} \Delta_s(\sigma) = 0$ for any $s \in (r, \tilde{r}]$. Finally, that $D(\theta; F, \sigma') = D(\theta; F, \sigma)$ follows from the fact that $D_s(\theta; \sigma)$ is independent of σ for any $s \in [r, \tilde{r}]$. *Q.E.D.*

Proof of Proposition 4 That \underline{D} , \bar{D} , and \underline{P} are independent of σ is immediate. Thus consider \bar{P} . Because for any $s \in (\underline{r}, \tilde{r}]$, $\theta_s^*(\sigma)$ is independent of σ , whereas $\theta_s^{**}(\sigma) = \theta_s^*(\sigma) + \Delta_s(\sigma)$ is strictly increasing in σ , it follows that, for any $F \in \mathcal{F}$, any $\theta_1 < \theta_2$, and any $r \in [\underline{r}, \tilde{r}]$,

$$\begin{aligned} P(r, \theta_1, \theta_2; F, \sigma) &\equiv \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} P(r, \theta; F, \sigma) d\theta \\ &= \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \int_{s \in [r, \tilde{r}]} I_{\{\theta \in (\theta_s^*(\sigma), \theta_s^{**}(\sigma))\}} dF(s) d\theta \end{aligned}$$

is weakly increasing in σ . By the envelope theorem, $\bar{P}(r, \theta_1, \theta_2; \sigma)$ is thus also weakly increasing in σ . Finally, note that for any $r > \underline{r}$, any $F \in \mathcal{F}$, and any $\theta_1 < \theta_2$,

$$\begin{aligned} P(r, \theta_1, \theta_2; F, \sigma) &= \frac{1}{\theta_2 - \theta_1} \int_{s \in [r, \tilde{r}]} \int_{\theta_1}^{\theta_2} I_{\{\theta \in (\theta_s^*(\sigma), \theta_s^{**}(\sigma))\}} d\theta dF(s) \\ &= \frac{1}{\theta_2 - \theta_1} \int_{s \in [r, \tilde{r}]} \max\{0, [\min\{\theta_2, \theta_s^{**}(\sigma)\} - \max\{\theta_1, \theta_s^*(\sigma)\}]\} dF(s) \\ &\leq \frac{1}{\theta_2 - \theta_1} \int_{s \in [r, \tilde{r}]} \Delta_s(\sigma) dF(s) \\ &\leq \frac{1}{\theta_2 - \theta_1} \Delta_r(\sigma), \end{aligned}$$

where the last inequality follows from the fact that $\Delta_s(\sigma)$ is decreasing in s for all $s \in (\underline{r}, \tilde{r}]$. By implication, $\bar{P}(r, \theta_1, \theta_2; \sigma) \leq \frac{1}{\theta_2 - \theta_1} \Delta_r(\sigma)$. That $\lim_{\sigma \rightarrow 0} \bar{P}(r, \theta_1, \theta_2; \sigma) = 0$ then follows from the fact that $\lim_{\sigma \rightarrow 0} \Delta_r(\sigma) = 0$. *Q.E.D.*

Proof of Proposition 5. All results follow from the arguments in the main text, once the following claim is proved.

Claim 1. For σ small enough, there exists $r_H(\sigma) < \tilde{r}$, with $r_H(\sigma) \rightarrow \underline{r}$ as $\sigma \rightarrow 0$, such that $x_s^*(\sigma) > \tilde{x}$ only if $s < r_H(\sigma)$.

Equivalently: there exists a $\theta_H(\sigma) < \tilde{\theta}$ with $\theta_H(\sigma) \rightarrow \underline{\theta}$ as $\sigma \rightarrow 0$, such that $x_s^*(\sigma) > \tilde{x}$ only if $\theta_s^*(\sigma) < \theta_H(\sigma)$.

Proof. For any $\theta_s^*(\sigma) \leq \tilde{\theta}$, let $\theta^{**}(\theta^*)$ be the unique solution to $G(\theta^*, \theta^{**}) = 0$, where $G(\theta^*, \theta)$ is the function defined in (11), and let $x^*(\theta^*) \equiv X(\theta^*, \theta^{**}(\theta^*))$. From (8),

$$X(\theta^*, \theta) = \theta^* + \sigma \Psi^{-1} \left[1 - \frac{r}{1-r} B(\theta^*, \theta) \right]$$

with $B(\theta^*, \theta)$ defined as in (16). Hence,

$$x^*(\theta^*) = \theta^* + \sigma \Psi^{-1} \left[1 - \frac{r}{1-r} B^*(\theta^*) \right]$$

where $B^*(\theta^*) \equiv B(\theta^*, \theta^{**}(\theta^*))$. Using again (8) and (16), $B^*(\theta^*) = \hat{B}(\Delta(\theta^*))$, where $\hat{B}(\Delta)$ is the continuous function implicitly defined by

$$B = \left(\frac{1-r}{r} \right) \left[1 - \Psi \left(\Psi^{-1}(B) + \frac{\Delta}{\sigma} \right) \right] \quad (17)$$

and $\Delta(\theta^*) \equiv \theta^{**}(\theta^*) - \theta^*$ is the continuous function implicitly defined by $\hat{G}(\theta^*, \Delta) = 0$, with $\hat{G}(\theta^*, \Delta) \equiv V(\theta^* + \Delta, 0) - V(\theta^*, 0) - V(\theta^* + \Delta, \hat{B}(\Delta))$.

Because $\Delta(\theta^*)$ is decreasing in θ^* (from Proposition 2), B^* is increasing in θ^* . Moreover, $B^* \rightarrow 0$ as $\theta^* \rightarrow \underline{\theta}$ and $B^* \rightarrow 1-r$ as $\theta^* \rightarrow \tilde{\theta}$. It follows that

$$x^* \rightarrow +\infty \text{ as } \theta^* \rightarrow \underline{\theta} \quad \text{and} \quad x^* = \tilde{x} \text{ when } \theta^* = \tilde{\theta},$$

where $\tilde{x} = \tilde{\theta} + \sigma \Psi^{-1} [1-r]$.

Now note that

$$\frac{dx^*}{d\theta^*} = 1 - \sigma \frac{1}{\psi \left(\Psi^{-1} \left(1 - \frac{r}{1-r} B^*(\theta^*) \right) \right)} \left(\frac{r}{1-r} \right) \frac{dB^*(\theta^*)}{d\theta^*}$$

Pick any $\delta_1 \in (0, 1)$ and any $\theta_1 \in (\underline{\theta}, \tilde{\theta})$. Because for any $\theta^* \in [\theta_1, \tilde{\theta})$, $dB^*(\theta^*)/d\theta^*$ is bounded away from infinity, there exists $\sigma_1 > 0$ such that, whenever $\sigma < \sigma_1$, $dx^*/d\theta^* > \delta_1$ for all $\theta^* \in [\theta_1, \tilde{\theta})$.

Together with the fact that $x^* = \tilde{x}$ at $\theta^* = \tilde{\theta}$, this implies that $x^* < \tilde{x}$ for all $\theta^* \in [\theta_1, \tilde{\theta}]$. Hence, for any $\sigma < \sigma_1$, there exists a $\theta_H(\sigma) \leq \theta_1 < \tilde{\theta}$ such that $x_s^*(\sigma) > \tilde{x}$ only if $\theta_s^*(\sigma) < \theta_H(\sigma)$. Furthermore, because the argument above holds for any $\theta_1 \in (\underline{\theta}, \tilde{\theta})$, this implies that $\theta_H(\sigma)$ can be made arbitrarily close to $\underline{\theta}$ by taking σ small enough. *Q.E.D.*

Proof of Proposition 7. For $\theta < \underline{\theta}$, it is dominant for the policy maker to set \underline{r} and abandon the peg and for private speculators to attack. Similarly, for $\theta > \bar{\theta}$, the peg is never abandoned, private speculators do not attack, and there is no need to undertake any costly policy measure. Finally, take any $\theta \in [\underline{\theta}, \bar{\theta}]$. The continuation game following any level of the policy r is a coordination game with two (extreme) continuation equilibria, no attack and full attack. Let $r(\theta)$ be the minimal r for which private speculators coordinate on the no-attack continuation equilibrium, i.e. they attack if and only if $r < r(\theta)$. Clearly, it is optimal for the policy maker to set $r(\theta) > \underline{r}$ if and only if $V(\theta, 0) - C(r(\theta)) \geq 0$, or equivalently $r \leq \rho(\theta)$. *Q.E.D.*

Proof of Proposition 8. The characterization of $\mathcal{G}(0)$ follows directly from Proposition 7. Thus consider $\mathcal{G}(\sigma)$ for $\sigma > 0$. Note that

$$\mathcal{G}(\sigma) = \{(\theta, r) : \text{either } r = \underline{r} \text{ and } \theta \in \mathbb{R}, \text{ or } r \in (\underline{r}, \tilde{r}] \text{ and } \theta_r^*(\sigma) \leq \theta \leq \theta_r^{**}(\sigma)\}.$$

For all $r \in (\underline{r}, \tilde{r}]$ and all $\sigma > 0$, $\theta_r^{**}(\sigma)$ is continuous in r and also continuous and nondecreasing in σ (strictly increasing when $r < \tilde{r}$), with $\lim_{\sigma \rightarrow 0} \theta_r^{**}(\sigma) = \theta_r^*(\sigma) = \rho^{-1}(r)$. It follows that, for any $\varepsilon > 0$, there exists $\bar{\sigma} > 0$ such that, for all $\sigma \in (0, \bar{\sigma})$ and all $r \in [\underline{r} + \varepsilon, \tilde{r}]$, $\theta_r^{**}(\sigma) < \rho^{-1}(r) + \varepsilon$. But then, for all $\sigma \in (0, \bar{\sigma})$,

$$\mathcal{G}(\sigma) \subset \{(\theta, r) : \text{either } r \in [\underline{r}, \underline{r} + \varepsilon] \text{ and } \theta \in \mathbb{R}, \text{ or } r \in [\underline{r} + \varepsilon, \tilde{r}] \text{ and } \theta \in [\rho^{-1}(r), \rho^{-1}(r) + \varepsilon]\}.$$

Together with the fact that, for all $\sigma > 0$,

$$\mathcal{G}(\sigma) \supset \{(\theta, r) : \text{either } r = \underline{r} \text{ and } \theta \in \mathbb{R}, \text{ or } r \in (\underline{r}, \tilde{r}] \text{ and } \theta = \rho^{-1}(r)\},$$

this establishes the result.

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