Anarchy and Its Breakdown

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Anarchy, defined as a system in which participants can seize and defend resources without regulation from above, is not chaos but rather a spontaneous order. However, anarchy is fragile and may dissolve either into formless "amorphiy" or into a more organized system such as hierarchy. Under anarchy, each contestant balances between productive exploitation of the current resource base and fighting to acquire or defend resources. Anarchy is sustainable only when there are strongly diminishing returns to fighting effort (the "decisiveness parameter" is sufficiently low) and incomes exceed the viability minimum. These considerations explain many features of animal and human conflict.

What do the following have in common? (1) international struggles for control of the globe's resources, (2) gang warfare in Prohibition-era Chicago, (3) miners versus claim jumpers in the California gold rush, (4) animal territoriality, and (5) male elephant seals who fight to sequester "harems" of females. Answer: They are all anarchic situations.

Anarchy is not chaos. At least potentially, anarchic relationships can constitute a stable system. But not all environments are capable of sustaining an anarchic order. Anarchy can break down, to be replaced by another pattern of relationships.

Anarchy is a natural economy (Ghiselin 1978), or spontaneous order in the sense of Hayek (1979). Various forms of spontaneous order

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emerge from resource competition among animals, among them territoriality and dominance relationships (as surveyed in Wilson [1975, chaps. 11–13]). As for humans, while associations ranging from primitive tribes to modern nation-states are all governed internally by some form of law, their external relations with one another remain mainly anarchic. Yet intertribal or international systems also have their regularities and systematic analyzable patterns (see, e.g., Waltz 1959; Snyder and Diesing 1977; Bernholz 1985).

The term “anarchy” in ordinary usage conflates two rather different situations that the biological literature carefully distinguishes: “scramble” versus “interference” competition (Nicholson 1954) or, in an alternative terminology, “exploitation” versus “resource defense” (Krebs and Davies 1987, p. 93). Under scramble competition, which might be termed amorphy\(^1\) (absence of form), resources are not sequestered but consumed on the move. In the open sea, for example, resources are so fugitive that fish do not attempt to defend territories. Jean-Jacques Rousseau evidently had amorphy in mind when he described man in the state of nature as “wandering up and down the forests, without industry, without speech, without home, an equal stranger to war and to all ties, neither standing in need of his fellow-creatures nor having any desire to hurt them” (1950, p. 230).

Although amorphic competition poses a number of interesting modeling issues, the present analysis is limited to environments in which durable resources such as land territories or movable capital goods are captured and defended by individuals or by groups. (I shall generally treat groups as unitary actors that somehow managed to resolve the internal collective-action problem.) So, as defined here, anarchy is a social arrangement in which contenders struggle to conquer and defend durable resources, without effective regulation by either higher authorities or social pressures.\(^2\)

Given the possibility of sequestering resources, anarchic competitors have to divide their efforts between two main types of activities: (1) productively exploiting the assets currently controlled and (2) seizing and defending a resource base. Correspondingly, there are two separate technologies: a technology of production and a technology of appropriation, conflict, and struggle (Hirshleifer 1991b). There are ways of

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\(^2\) Since regulation can vary from total to zero effectiveness, anarchy is typically a matter of degree. In gold rush California the U.S. Army, though decimated by desertion to the goldfields, did maintain a limited presence (Sherman [1885] 1990, chaps. 2–3). And during the bootlegging wars in Prohibition-era Chicago (Allsop 1968), the local police, while notoriously corrupt, were still a factor. In fact, an element of anarchy persists even in the most normal of times: law and order being imperfect, some provision for self-defense of person and property is almost always advisable.
tilling the land, and quite a different set of ways of capturing land and securing it against intruders.

While I shall be using military terminology such as "capturing" and "fighting," they are to be understood as metaphors. Falling also into the category of interference struggles are political campaigns, rent-seeking maneuvers for licenses and monopoly privileges (Tullock 1967), commercial efforts to raise rivals' costs (Salop and Scheffman 1983), strikes and lockouts, and litigation—all being conflictual activities that need not involve actual violence.

A decision maker's chosen balance between productive and conflictual efforts may be influenced in the peaceful direction by an element of productive complementarity. Management and labor, since they need one another, are less motivated to engage in destructive struggles within the firm. Similarly, mutual interdependence within the polity may moderate international, regional, and other interest group conflicts. Exchange relationships, in particular, increase mutual interdependence and thus partially harmonize diverging interests. But I shall be assuming here a starker environment in which productive opportunities are entirely disjoint and the exchange option is excluded, so that competitors have to fight, or at least be prepared to fight, if they are to acquire or retain resources.

The economic theory of conflict, like economic modeling generally, involves two analytical steps: (i) Optimization: Each competitor chooses a preferred balance of productive effort versus conflictual effort. (ii) Equilibrium: On the social level, the separate optimizing decisions interact to determine levels of production and the extent of fighting activity, together with the distribution of product among the claimants. While the economic literature on conflict theory remains relatively sparse, in recent years a number of models employing such an analytical structure have been offered. But, as far as I know, none of these earlier writings has analyzed the viability of anarchy as a spontaneous social order.\(^5\)

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\(^5\) For analyses of conflict as moderated by a cooperative element in production, see Hirshleifer (1988) and Skaperdas (1992).

\(^4\) Fighting is of course Pareto-inefficient. All parties could always benefit from an agreed peaceful resolution, but under anarchy there is no superior authority to enforce any such agreement. (In some cases threats may suffice to deter conflict, but that possibility is not modeled here.)

\(^5\) I shall briefly review some related analytical contributions: (1) In Bush and Mayer (1974), production is costless (manna-like), but competitors may also steal, generating a "natural equilibrium." (2) Skogh and Stuart (1982) allowed for three types of activities: production, transfers (i.e., stealing, or offensive activity), and protection against transfers (defensive activity). (3) Usher (1989) modeled an alternation between despotism and anarchy. In anarchy there are two professions: farmers and bandits. The possible anarchic equilibria include a mixed population of farmers and bandits, an all-farmer
Among the specific issues to be considered here are the following.

1. *When is there a stable anarchic solution?*—Under what conditions can two or more anarchic contestants retain viable shares of the socially available resources in equilibrium? Or put the other way, in what circumstances does the anarchic system “break down” in favor of amopy on the one hand or, alternatively, in favor of tyranny or some other form of social control?

2. *Equilibrium allocations of effort.*—In a stable anarchic equilibrium, what fractions of resources will be devoted to fighting? What levels of incomes will be attained?

3. *Numbers.*—If the number of contenders $N$ is exogenously given, how are the equilibrium fighting efforts and attained levels of income affected as $N$ changes? Alternatively, if $N$ is endogenous, how many contenders can survive?

4. *Technology and comparative advantage.*—How do the outcomes respond to parametric variations, one-sided or two-sided, in the technology of production or in the technology of struggle?

5. *Strategic position.*—How do the outcomes respond to positional asymmetries, for example where one side is a Stackelberg leader?

The analysis here employs standard (though possibly still highly arguable!) economic postulates such as rationality, self-interested motivations, and diminishing returns. Certain other assumptions are designed to achieve analytical simplicity in ways familiar to economists; for example, only steady-state solutions are considered. But to push ahead I have also at times made more special modeling choices, for example, about the conflict technology. Whenever possible I shall try to flag the results of such “nongeneric” assumptions and discuss the analytical implications.

For the simplest symmetric case of two competitors ($N = 2$), Section I describes the conditions for a stable anarchic equilibrium. Section II analyzes the optimizing decision and final outcomes. Section III considers both exogenous and endogenous variation in the number of contenders $N$, and Section IV examines the consequences of various types of asymmetries between the rival parties. Section V relates the analysis to important features of animal and human conflict. Section VI summarizes the results and limitations. Finally, Section VII asks “After anarchy, what?”

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outcome, and a (nonviable) all-bandit outcome. (4) Closest to the present paper in terms of modeling approach are Hirshleifer (1988, 1991a), Skaperdas (1992), and Grossman and Kim (1994). However, in these articles agents have inalienable resource endowments or, at most, only a one-time reallocation is allowed. In contrast, the *continuing* struggle for resource endowments is the central phenomenon addressed in the present paper.
I. Stability of Conflict Equilibrium \((N = 2)\)

Each of two rival claimants aims solely to maximize own income. Neither benevolent nor malevolent preferences play a role, nor is there any taste for leisure or other non-income-generating activity.

At any moment of time, each contender \(i = 1, 2\) divides his or her current resource availability \(R_i\) between productive effort \(E_i\) (designed to extract income from resources currently controlled) and fighting effort \(F_i\) (aimed at acquiring new resources at the expense of competitors, or repelling them as they attempt to do the same):\(^6\)

\[
R_i = a_i E_i + b_i F_i. \tag{1}
\]

The aggregate resource base, \(R = R_1 + R_2\), is assumed constant and independent of the parties’ actions.\(^7\) The \(a_i\) and \(b_i\) can be interpreted as unit conversion costs (assumed constant) of transforming resources into productive effort or into fighting effort, respectively. In a military metaphor, \(b_i\) is a logistics cost coefficient quantifying the resource burden per fighting unit supported. Similarly, \(a_i\), the production cost coefficient, measures the resources required to maintain a worker or machine in civilian production.\(^8\) In the decades preceding the American Civil War, inventions such as the steamboat and railroad sharply reduced \(a_i\) (since workers could be fed and machines built more cheaply) and also \(b_i\) (since supplies could more easily be delivered to fighting troops). In consequence, vastly larger armies were able to take the field in the Civil War than in the Revolutionary War or the War of 1812.

It will sometimes be more convenient to deal with the corresponding “intensities” \(e_i\) and \(f_i\):

\[
e_i = \frac{E_i}{R_i}, \quad f_i = \frac{F_i}{R_i}. \tag{2}
\]

The \(e_i\) and \(f_i\) will be the crucial decision variables on each side, subject of course to:

\[
a_i e_i + b_i f_i = 1. \tag{3}
\]

\(^6\) I do not distinguish here between offensive and defensive activities. On this see Skogh and Stuart (1982) and Grossman and Kim (1994).

\(^7\) This crucial assumption—implying that fighting, while a diversion of resources, is nondestructive—will be discussed further in Sec. VI.

\(^8\) Taking \(a_i\) and \(b_i\) as constants implies a constant marginal rate of substitution between productive effort and fighting effort. Diminishing returns enter at another stage: the translation of productive effort \(E_i\) into income and of fighting effort \(F_i\) into contest success.
Under the assumption of steady-state conditions, each side makes an optimal once-and-for-all choice of $e_i$ and $f_i$.\footnote{More generally, instead of a once-and-for-all choice of $f_i$, and the implied $e_i$, side $i$'s choice could vary with the level of resources on hand. For example, it might pay to devote a larger fraction of resources to fighting when one is poor and a smaller fraction when rich (Hirshleifer 1991a). However, finding the optimal function $f_i(R_i)$ as a best reply to the opponent's corresponding $f_i(R_i)$, and vice versa, poses a fearsome analytic problem that I do not attempt to address here.}

The steady-state fighting intensity $f_i$ can allow for time averaging. A tribe choosing $n f_i$ such that half its resources are devoted to fighting need not have half its human and material capital engaged in war night and day, season in and season out. More likely, the tribe as a whole will be alternating between periods of war and periods of peace. Similarly, although a labor union may alternate between periods of strike and periods of work, its long-term strategy could be interpreted as choice of a steady-state average fighting intensity $f_i$.

With income to side $i$ symbolized as $Y_i$, let the production function take the simple form:

$$\text{production function: } Y_i = E_i^b = (e_i R_i)^b.$$ \hfill (4)

Resource control is achieved only by fighting, the outcome being the success fractions $p_1$ and $p_2$ (where of course $p_1 + p_2 = 1$). Thus:

$$\text{resource partition equation: } R_i = p_i R.$$ \hfill (5)

The technology of conflict is summarized by the Contest Success Function (CSF), which in the form employed here determines the success ratio $p_1/p_2$ as a function of the ratio of the fighting efforts $F_1/F_2$ and (what plays a crucial role in the analysis) a decisiveness parameter $m > 0$:\footnote{This form of the CSF, in which the success fractions are determined by the ratio of the fighting efforts, was proposed in Tullock (1980). If instead the outcome were to depend on the difference between the fighting efforts, the CSF would be a logistic function (Hirshleifer 1988). The question of the appropriate form for the CSF will arise again below.}

$$\text{Contest Success Function: } \frac{p_1}{p_2} = \left( \frac{F_1}{F_2} \right)^m$$ \hfill (6a)

or, equivalently:

$$p_1 = \frac{F_1^m}{F_1^m + F_2^m},$$ \hfill (6b)

$$p_2 = \frac{F_2^m}{F_1^m + F_2^m}.$$
Fig. 1.—Contest Success Function (CSF)

responds to changes in fighting effort $F_1$. Evidently, the sensitivity of $p_1$ to $F_1$ grows as the decisiveness parameter $m$ increases.

In military struggles, low $m$ corresponds to the defense having the upper hand. On the western front in World War I, entrenchment plus the machine gun made for very low decisiveness $m$. Throughout 1914–18, attacks with even very large force superiority rarely succeeded in doing more than move the front lines back a few miles, at enormous cost in men and materiel. But in World War II the combination of airplanes, tanks, and mechanized infantry allowed the offense to concentrate firepower more rapidly than the defense, thus intensifying the effect of force superiority. On the other hand, high decisiveness on the battlefield does not necessarily translate into correspondingly high decisiveness in a war as a whole. In 1870, Prussia won complete battlefield supremacy over France. But whereas Rome had razed Carthage to the ground, Prussia settled for very moderate peace terms: France had only to pay an indemnity and surrender the frontier provinces of Alsace and Lorraine. Prussian moderation was presumably due, in part at least, to fear of a guerrilla resistance against which its battlefield supremacy would be much less decisive.

The decisiveness factor is by no means limited to strictly military struggles. In democratic constitutions, features such as separation of powers and bills of rights reduce the decisiveness of majority supremacy, thereby tending to moderate the intensity of factional struggles.

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11 Of course, differences in ability to employ newer technologies are also often crucial, as demonstrated in the German victory over France in 1940. (This and other asymmetries will be addressed in Sec. IV.)
If the political system were winner take all, decisiveness $m$ would be very high and all politics would be a fight to the death.\footnote{Constitutions that are observed and last for a long time are those that reduce the stakes of political battles (Przeworski 1991, p. 36).}

From (5) and (6a):

$$\frac{R_1^n}{R_2^n} = \left(\frac{F_1}{F_2}\right)^m = \left(\frac{f_1R_1}{f_2R_2}\right)^m.$$  

This reduces to:

$$f_1^nR_1^{n-1} = f_2^nR_2^{n-1}. \quad (7a)$$

So, finally:

$$\text{equilibrium success ratio (steady state):} \quad \frac{p_1}{p_2} = \left(\frac{f_1}{f_2}\right)^{m/(1-m)}. \quad (7b)$$

Equations (7a) and (7b) describe the logically required steady-state relationships between the parties’ chosen fighting intensities $f_i$ and the equilibrium success ratio $p_1/p_2$ or resource ratio $R_1/R_2$. Figure 2 plots different values of $m$. Note that as $m \to 1$, the curve approaches a limiting step function such that $p_1/p_2 = 0$ for all $f_1 < f_2$ and jumps to $p_1/p_2 = \infty$ for $f_1 > f_2$. Without explicit proof, it will be evident that for an interior stable equilibrium, the decisiveness parameter must lie in the range $0 < m < 1$.

The preceding discussion has brought out one way in which anarchy could break down: an excessively large decisiveness parameter $m$ leads to dynamic instability, that is, movement toward a corner solution (see numerical example 1 in the Appendix). A second source of breakdown is income inadequacy. Suppose that some minimum income $y$ is required to sustain life for an individual actor or for a group to preserve its institutional integrity. Then anarchy cannot be stable if the equilibrium of the dynamic process implies income $Y_i < y$ for either contender. The following result summarizes this discussion.

**Result 1.** The conditions for sustainability of a two-party anarchic system include (i) a sufficiently low decisiveness parameter $m$ and (ii) sufficiently high attained incomes $Y_i$:

\begin{align*}
\text{condition for dynamic stability:} & \quad m < 1 \\
\text{condition for viability:} & \quad Y_i \geq y, i = 1, 2. \quad (8)
\end{align*}

Note that these are necessary, not sufficient, conditions for anarchy to be sustained. As will be seen below, anarchy may be fragile even when the conditions are satisfied.
II. Optimization and Equilibrium in Symmetrical Conflict \((N = 2)\)

Figure 2 did not illustrate the solution of the anarchic system for \(N = 2\) but only the relations that must hold, in equilibrium, among the dependent variables \(R_1\) and \(R_2\) and the decision variables \(f_1\) and \(f_2\). The actual solution involves optimizing behavior on each side. Under the traditional Cournot assumption, each contender \(i\) chooses between steady-state \(e_i\) and \(f_i\), on the assumption that the opponent’s corresponding choices will remain unchanged. In the maximization of income \(Y_i\), a larger fighting effort \(f_i\) captures more resources or territory whereas a larger productive effort \(e_i\) generates more income from the territory controlled. Thus player 1’s optimal \(f_1\) is given by:

$$\max Y_1 = E_1^h = (e_1 R_1)^h = (e_1 R f_1^M)^h = \left( \frac{e_1 R f_1^M}{f_1^M + f_2^M} \right)^h$$

subject to \(a_1 e_1 + b_1 f_1 = 1\), and defining for compactness \(M = m/(1 - m)\).

Straightforward steps then generate player 1’s reaction curve \(RC_1\), showing his optimal \(f_1\) as the opponent varies her \(f_2\).\(^{13}\) A correspond-

\(^{13}\) The first-order conditions, where \(\lambda\) is the Lagrangian multiplier, are:

$$h(e_i R f_1^M) \frac{R f_1^M}{f_1^M + f_2^M} - \lambda a_1 = 0$$
ing analysis leads the opponent to her reaction curve $RC_2$:

$$\text{reaction curve } RC_1: \quad \frac{f_1^M}{f_2^M} = \frac{M}{b_1 f_1} - (M + 1) \quad (10a)$$

and

$$\text{reaction curve } RC_2: \quad \frac{f_2^M}{f_1^M} = \frac{M}{b_2 f_2} - (M + 1). \quad (10b)$$

The reaction curve for player $i, RC_i$, depends only on the decisiveness parameter $m$ and on the decision maker's own logistics cost coefficient $b_i$. From the analytical form of the equations, and as illustrated in figure 3, the reaction curves have positive slopes throughout. Thus, if player 1 chooses higher $f_1$, it pays player 2 to respond with higher $f_2$. And note that, as required for stability, in the neighborhood of equilibrium the matching is less than one for one.

Equations (10a) and (10b) may be solved for $f_1$ and $f_2$, thus determining the equilibrium of the entire system. Unfortunately, there is no convenient general analytic solution. However, this section deals with the symmetric case in which $a_1 = a_2 = a$ and $b_1 = b_2 = b$. Hence $f_1 = f_2$ at equilibrium, and (10a) and (10b) reduce to:

symmetrical conflict equilibrium ($N = 2$):

$$f_1 = f_2 = \frac{M}{b(M + 2)} = \frac{m}{b(2 - m)}. \quad (11)$$

Symmetrical solutions for $b = 1$ are illustrated by the intersections of the paired $RC_1$ and $RC_2$ curves in figure 3. If $m = \frac{1}{2}$, the inner pair of curves apply and the solution is $f_1 = f_2 = .333$. With a higher decisiveness parameter $m = \frac{5}{6}$, the intersection occurs at $f_1 = f_2 = .5$.

The results below follow from the form of equation (11).

**Result 2.** When the conditions for dynamic stability and viability both hold, in symmetrical conflict larger values of the decisiveness parameter $m$ imply higher equilibrium fighting intensities $f_1$ and $f_2$ and thus higher fighting levels $F_1$ and $F_2$. And similarly, the lower the common value $b$ of the logistics cost coefficient, the greater the equilibrium $f_i$ and $F_i$.

and

$$h(e, p_1 R)^{e-1} e R M f_1^{M-1} f_2^M \left( f_1^M + f_2^M \right)^2 - \lambda b_1 = 0.$$  

Routine steps lead to eq. (10a), the reaction curve for player 1.
For the underlying intuition recall that, as $m$ increases, any given disparity between the fighting efforts $F_1$ and $F_2$ comes to have an increasingly powerful effect on the partition of resources. So as $m$ grows, each side is motivated to “try harder”—to choose a higher fighting intensity $f_i$ than before. And similarly for the logistics cost coefficient: a reduction in $b$ makes fighting effort cheaper, and hence more of it comes to be generated on each side.

What is possibly disturbing, equation (11) implies that $f_i$ cannot be zero in equilibrium. There can never be total peace in the sense of devoting zero resources to conflict. This is a “nongeneric” result, since there are alternate forms of the CSF that could be consistent with total peace (Hirshleifer 1988; Skaperdas 1992). On the other hand, the implication might be regarded as quite realistic in many or most anarchic contexts.

Since $p_1 = p_2 = \frac{1}{2}$ in the symmetrical conflict situation, direct substitutions lead to the equilibrium per capita incomes:

$$Y_i = (e_i p_i R)^h = \left[ \frac{1 - m}{a(2 - m)} R \right]^h.$$  \hfill (12)

**RESULT 3.** In the symmetrical conflict situation, when the conditions for dynamic stability and viability both hold, the incomes achieved (i) rise in response to increases in aggregate resource availability $R$ and the productivity parameter $h$, but (ii) fall in response to increases
in the decisiveness parameter $m$ and the production cost coefficient $a$.\footnote{A possibly puzzling feature of eq. (12) is that, although a lower logistics cost coefficient $b$ was shown above as increasing the fighting efforts $f$, the ultimate incomes $Y_i$ end up independent of $b$. The reason is that lower $b$ has two countervailing effects. On the one hand it implies lower $e_i$, smaller productive efforts on each side. But on the other hand, a smaller $b$ means that the opportunity cost burden of any given $f_i$ is less. That these two effects exactly cancel out is, however, also a “nongeneric” feature of the model and hence is not insisted on here. (Specifically, explorations indicate that the result would not be robust to changes in the form of the CSF that would make it sensitive to the difference in the respective fighting efforts.)} (See also numerical example 2 in the Appendix.)

III. Number of Competitors—Exogenous vs. Endogenous Variation

\textit{Exogenously Varying N}

Suppose that a fixed number of competitors $N$ engage in a mêlée—a Hobbesian struggle of each against all, coalitions being ruled out.\footnote{“During the time men live without a common power to keep them in awe, they are in that condition which is called war; and such a war as is of every man against every man” (Hobbes, \textit{Levithan}, chap. 13).} The Cournot solution has each contender $i$ choosing a fighting intensity $f_i$ on the assumption that every opponent $j$ will be holding $f_j$ fixed. Generalizing equation (7a) yields

$$f_1^n R_1^{n-1} = f_2^n R_2^{n-1} = \ldots = f_N^n R_N^{n-1}$$ \hspace{1cm} (13a)

or, equivalently:

$$p_1 : p_2 : \ldots : p_N = (f_1 : f_2 : \ldots : f_N)^M. \hspace{1cm} (13b)$$

Once again, for dynamic stability it is necessary to have $M > 0$, that is $m < 1$. Of course, the viability condition $Y_i \geq y$ must also hold.

Contender 1’s optimizing problem is:

$$\text{max } Y_1 = (e_1 R_1)^b = (e_1 p_1 R)^b = \left( \frac{e_1 R_1^M}{f_1^M + f_2^M + \ldots + f_N^M} \right)^b \hspace{1cm} (14)$$

subject to $a_1 e_1 + b_1 f_1 = 1$. The analogue of equation (10a), the generalized reaction curve for the first among $N$ competitors, is:

$$\text{reaction curve } RG_1: \frac{f_1^M}{f_2^M + \ldots + f_N^M} = \frac{M}{b_1 f_1} - (M + 1). \hspace{1cm} (15)$$

And similarly for the other decision makers from contender 2 on.

If we assume symmetrical logistics cost coefficients $b_i = b$ and productive cost coefficients $a_i = a$ and use the fact that in symmetrical
equilibrium all the \( f_i \) are equal, the solution is:

symmetrical conflict equilibrium (general \( N \)):

\[
f_1 = f_2 = \ldots = f_N = \frac{M/b}{M + 1 + [1/(N - 1)]} = \frac{m(N - 1)}{b(N - m)}. \tag{16}
\]

As before, the fractions of resources devoted to fighting increase as the decisiveness parameter \( m \) rises and as the logistics cost coefficient \( b \) falls. And we see now that these fighting intensities also increase with larger numbers. That is, as \( N \) rises parametrically, each contender has to waste more effort in fighting even to retain his new (reduced) pro rata share. The equilibrium incomes are:

\[
Y_i = (e_i p_i R)^h = \left[ \frac{1 - m}{a(N - m)} R \right]^h \tag{17}
\]

provided as always that \( m < 1 \) and \( Y_i \geq y \).

**RESULT 4A. Parametrically varying \( N \), fixed \( R \).**—Under the assumption that the conditions for sustainability of anarchy hold, with symmetrical production cost coefficients \( a_i = a \) and logistics cost coefficients \( b_i = b \), if aggregate resources remain fixed, then as \( N \) rises exogenously the equilibrium fighting intensities \( f_i \) increase. Individual incomes fall as \( N \) rises, owing to (i) smaller pro rata resource shares \( p_i = 1/N \) and (ii) larger \( f_i \).

It follows immediately that, as \( N \) increases, the attained incomes under anarchy are not only smaller *per capita* but smaller *in aggregate*.

Now consider instead a friendlier environment in which the aggregate resource base is not fixed but grows in proportion to the number of claimants. We can imagine that each entrant brings in a resource quantum \( r \), so that \( R = Nr \). Evidently, the expanding resource base exactly cancels out the adverse effect of increased \( N \) associated with the reduced pro rata share. But the adverse effect of the larger fighting efforts \( f_i \) remains. Under this more optimistic assumption the equilibrium incomes become:

\[
Y_i = (e_i p_i R)^h = \left[ \frac{1 - m}{a(N - m)} Nr \right]^h \tag{18}
\]

**RESULT 4B. \( R \) and \( N \) rising in proportion.**—Even if aggregate resource availability \( R \) increases in proportion to numbers \( N \), individual incomes still fall as \( N \) rises, owing to the higher equilibrium fighting intensities \( f_i \).

Figure 4 illustrates how fighting intensity \( f_i \) rises with numbers \( N \), and the implications of that fact for per capita income \( Y_i = (e_i p_i R)^h \) under both the more and the less favorable assumptions about the
relation of aggregate resources to the number of contenders. (The parameter values for the diagram are as stated in numerical example 3 in the Appendix.)

**Endogenous N**

If population numbers are subject to Malthusian increase/decrease or to immigration/emigration, the equilibrium $N$ will be determined by the viability limit $\gamma$—a kind of zero-profit condition:

$$\text{condition for equilibrium } N: \quad Y_i(N) = \gamma. \quad (19)$$

Once again, the actual viable population will depend on whether aggregate resources $R$ are fixed or alternatively grow in proportion to $N$. (See numerical example 4 in the Appendix.)

**RESULT 5.** If $N$ is endogenously determined, a zero-profit condition will establish the viable number of contestants, the number being of course smaller when aggregate resources remain constant and larger when each added entrant brings in a resource increment.

**IV. Three Types of Asymmetries**

So far only symmetrical solutions have been analyzed. In this section three different kinds of asymmetries are considered: cost differences, functional differences, and positional differences.

**Cost differences.**—A lower production cost coefficient ($a_1 < a_2$) or logistics cost coefficient ($b_1 < b_2$) would of course give side 1 a corre-
sponding advantage. (Since these are absolute comparisons, it is quite possible for one side to have the advantage in both directions at once.)

Figure 5\textsuperscript{16} shows that a reduced production cost coefficient $a_1$ for player 1 leaves all the equilibrium solutions unchanged except for raising 1’s own income $Y_1$.\textsuperscript{17} In contrast, as the logistics cost coefficient $b_1$ falls in figure 6, contender 1’s fighting intensity $f_1$ and income $Y_1$ both rise. And, since contender 2 will respond with less than a one for one increase in $f_2$, she suffers reduced income $Y_2$.

*Functional differences.*—Equation (4) for the production function postulated a common productivity parameter $k$. More generally, there could be differing $h_i$. If $h_1 > h_2$, side 1 has a productive advantage yielding him higher income $Y_1 > Y_2$. (No diagram is provided for this simulation, since—apart from a left-right reversal—such a picture would closely parallel fig. 5. That is, a rise in $h_1$, with $h_2$ held fixed, is very like a fall in the productive cost coefficient $a_1$, with $a_2$ held fixed.) Similarly, equation (6) could be generalized to allow for differing decisiveness parameters $m$. Figure 7 indicates that as $m_1$ rises, with $m_2$ held constant, contender 1’s optimal $f_1$ always increases. Contender 2 at first replies with a smaller increase in $f_2$, but eventually

\textsuperscript{16} Figures 5–8 each represent a large number of simulations using variations of the base case parameters given in numerical example 2.

\textsuperscript{17} This also needs to be flagged as one of the “nongeneric” results adverted to in Sec. 1. The special assumption most implicated here is the total disjunction of the productive efforts on the two sides. Given a degree of productive interaction, a reduction in one side’s production cost coefficient $a_1$ would generally affect the opponent’s $f_2$ and hence redound back on player 1’s optimal choice of fighting intensity $f_1$. 

she retreats from the unequal struggle and devotes more effort to production instead.

**Positional differences.**—Under the Cournot assumption, the parties are symmetrically situated. Among the many possible positional asymmetries, only the Stackelberg situation will be considered here. As first mover, the Stackelberg "leader" chooses a fighting intensity to which the opponent then optimally responds. Ability to move first is often advantageous, for example taking the high ground as a military
tactic. But the second mover, able to optimize in the light of the opponent's known choice, always has a countervailing informational advantage. So it is not clear a priori whether, in the present context, a Stackelberg leader can be expected to come out ahead. 18

Figure 8 shows that, in comparisons of the Stackelberg with the Cournot equilibrium, the fighting efforts $f_1$ have become smaller and the incomes $Y_i$ consequently higher on both sides. But note that the follower does better than the leader! Is this a general result? Recall that the reaction curves (see fig. 3) have positive slopes throughout. So if player 1 as leader were to choose a smaller than Cournot $f_1$, player 2 would respond with a smaller $f_2$, implying higher aggregate income for the two together. However, as already pointed out, in the neighborhood of equilibrium the best reply to an increase in the opponent's $f_i$ is always less than one for one. So while the leader gains absolutely, he loses out relatively. 19 In international affairs, for example, suppose that nation 1 were to take the initiative in a disarmament move, reducing $f_1$ in the hope that nation 2 will reciprocate.

18 A Stackelberg leader is quite different from a hierarchical leader. The latter is someone who, in order to influence a subordinate's behavior, can issue a credible prior threat or promise as to how he or she will react to the latter's choice. Thus the hierarchical leader is somehow able to commit in advance to a reaction curve, in the light of which it is up to the subordinate to make the first action move (see Thompson and Faith 1981; Hirschleifer 1988).

19 More generally, in an otherwise symmetrical situation with sequential moves, if the reaction curves are positively sloped, the relative advantage always goes to the second mover (Gal-Or 1985).
The model suggests that only partial reciprocation would occur, leaving the first disarmer at a relative disadvantage.

Result 6. In the Stackelberg equilibrium, in comparison with the Cournot outcome, both sides' fighting efforts $f_i$ are smaller and incomes $Y_i$ higher. But the follower gains relative to the leader.

V. Discussion and Applications

The model presented above, while more of a framework of analysis than a tightly specified theory, suggests new ways of understanding diverse yet logically parallel phenomena arising in entirely separate domains. I shall illustrate its bearing for some observed patterns of animal territoriality and human warfare.

Animal Territoriality\(^{20}\)

The biologists' textbook approach to the problem of territoriality is termed the economic defense model. Ecological theorists ask, When does it pay to defend territories, and if it does pay, what are the determinants of territory size and the level of conflict? I shall list only a few points of contact with the previous theoretical development.

i) If resources are unpredictable or nondefendable, organisms do not appropriate territories but compete by "scrambling"; that is, the social system is amorphism rather than anarchy in the sense of this paper. Territoriality (anarchy) tends to emerge when resources are defendable and predictable, and also dispersed. When resources are predictable and defendable but are geographically concentrated instead, dominance hierarchies tend to replace territoriality. (Explanation: Struggles for control of tightly concentrated resources approach "winner-take-all" battles [high decisiveness $m$]. High $m$ makes anarchy dynamically unstable [from result 1 above], leading to dictatorship by the strongest.)

ii) In a territorial system, increased population, even if sustainable in terms of the viability limit, reduces per capita territory size (smaller $p$ and $R$). Less obviously, larger $N$ raises the intensity of aggressive interactions (higher $f_i$, from result 4A above). As population pressure increases further, proprietors have to spend so much time fighting intruders that the system eventually breaks down. (Per capita incomes $Y_i$ fall below viability limit $y$ [result 1].) Under these conditions, territoriality is commonly succeeded by a dominance hierarchy in which at

\(^{20}\) This discussion is based mainly on McNaughton and Wolf (1973, chaps. 11–12), Wilson (1975, chap. 12), Morse (1980, chaps. 9–10), and Krebs and Davies (1987, chap. 5).
least a few stronger animals retain access to the resource (Barash 1977, p. 262).21

iii) While dominance systems lie outside the domain of the present model, the steepness of the hierarchical gradient—the disproportionality between incomes of dominants versus subordinates—tends to be minimized when there are ecological opportunities for subordinates to exit the group and when fighting abilities are not too dissimilar.22 These conditions correspond to a low value of the decisiveness parameter (small $m$) and relatively modest decisiveness asymmetry (not too disparate $m_i$’s). Thus the same qualitative factors that conducive to survival of anarchy also serve to mitigate the exploitive features of dominance systems.

**Human Warfare**

The model here, with its necessarily severe simplifications, can hardly be expected to “predict” all the subtleties and complications of human social arrangements. Still, it sheds light on some patterns of human social conflict, of which warfare is the most obvious.

Among the Enga tribesmen of New Guinea, at least up to quite recently, warfare was the regular means of redistributing territories (Meggit 1977). Contrary to assertions that primitive war is largely a ritualized show with few casualties, warfare in New Guinea was a serious matter. Deaths in battle or from wounds accounted for around 35 percent of lifetime mortality among male adults.23 The factor driving warfare has been increasing population density. (*Explanation:* As Malthusian pressures depress per capita incomes, it comes to a choice between fighting and starving. Yet, owing to low decisiveness $m$, no single tribe has been able to take over. The anarchic system appears to be stabilized by war casualties that bring per capita incomes $Y_i$ back in line with the viability limit $y_i$.)

In ancient Greece24 the persistence of small city-states was associated with relatively indecisive warfare patterns (low $m$). The phalanx was the dominant tactical formation, missile weapons were largely

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21 An experiment with Norway rats indicates that, if overcrowding becomes extremely severe, even hierarchy can break down in favor of a “pathological” (i.e., amorphous) state (Calhoun 1962).

22 See Vehrencamp (1983) and, for analogous results in terms of human hierarchical structures, Betzig (1992).

23 For the Yanomamo tribesmen of South America ("the fierce people"), Chagnon (1988) provides a similar estimate: 30 percent of adult male mortality is the result of violent conflict. An interesting comparison: for Prohibition-era Chicago, Allsop (1968, p. 41) reports 703 gangland fatalities in the course of 14 years. Given the number of active gangster-fighters, the proportion of deaths may not be too dissimilar.

24 This discussion is based largely on Fuller (1954, chaps. 1–3), Preston and Wise (1979, chaps. 1–2), and of course Thucydides.
ineffective, and cavalry almost absent—factors that combined to preclude the deadly pursuit that makes victory truly decisive.

However, with advancing wealth and commerce, sea power became increasingly important. Naval conflict tends of its nature to be militarily more decisive; the stronger force gains command of the seas. Athens, which was the wealthiest state and had a large and skilled navy, reduced many smaller city-states to dependencies within its empire. (*Explanation:* Higher decisiveness \( m \) implied higher fighting intensities \( f_i \) and thus a smaller number of militarily viable contenders \( N \).) But Athens was ultimately defeated by a countercoalition led by Sparta (Aegospotami, 404 B.C.). Sparta in turn failed to achieve sole hierarchical dominance (Leuctra, 371 B.C.), and a period of shifting alliances followed. (The available conflict technology was characterized by a decisiveness parameter \( m \) too high for independent city-states to survive without allies, but not high enough for a single hegemon to defeat countercoalitions.)

Eventually Macedon gained military predominance (Charonea, 338 B.C.), owing in large part to Philip II’s successful integration of cavalry, missile weapons, and siege apparatus with infantry in a disciplined force. (These military innovations led to higher \( m \) in land combat. And, of course, Macedon had the asymmetric advantage of being first in the field with them.) In the ensuing conflict between the united Greek forces under Alexander versus the Persian Empire, cavalry (high \( m \)) was again crucial to the decisive victory (Gaugamela, 331 B.C.). But none of Alexander’s successors was able to achieve sole control. (Owing mainly to the huge land masses involved, conflict decisiveness \( m \) was still not high enough for hegemony.) Thus an anarchic system returned in the form of a shifting pattern of three or four successor states, each based on combined sea and land power. This pattern lasted for some 150 years, ending when Rome finally did achieve hegemony in the Mediterranean (Pydna, 168 B.C.). (Rome benefited, it seems, from asymmetrically higher \( m \) due more to superior organization than to any special weaponry or tactics.)

A number of other historical periods or episodes also illustrate implications of our model.

1. Cannons.—In the early fifteenth century, the introduction of cannons made it possible to batter down old-style castle walls, ending a long historical period of indecisive siege warfare. A major consequence was a sharp reduction in the number of independent prinicipalities in western Europe. (Higher \( m \) implies higher \( f_i \), which implies smaller viable \( N \).) Actually, this technological predominance of the

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offense was only temporary, being shortly reversed by improvements in the art of fortification. But the economic effect remained much the same, since their enormous cost put modern fortifications beyond the reach of smaller political units\textsuperscript{20} (asymmetrically lower logistic cost coefficient $b$ favoring the larger states, given returns to scale in producing and transporting cannons).

2. Gang wars.—In Prohibition-era Chicago, the Capone mob ultimately achieved hegemonic control, owing perhaps to superior ruthlessness as evidenced by the St. Valentine’s Day massacre (asymmetrically higher $m$). As movie and television viewers know, it took decisive intervention by an outside power, the federal government, to put Capone away.

3. California gold rush.—In contrast, even though the official organs of law were impotent, no Capone-type hegemony over the “forty-niners” ever developed. Highly dispersed resources (widely separated goldfields in difficult mountainous country) made it difficult for a gang to achieve effective control (low decisiveness $m$). Another factor, falling outside the model here, is that despite the collective-action problems involved, mining camp communities were surprisingly effective in setting up “social contracts” for resisting invaders (Umbeck 1981).

VI. Conclusions and Limitations

It will be convenient to summarize by responding briefly to the specific questions raised in the Introduction.

1. *When is there a stable anarchic solution?* — An anarchic system, to be sustained, must be *dynamically stable* and *viable*. The former condition holds when, most important, the decisiveness of conflict (measured by the parameter $m$ in the model) is sufficiently low; else the most militarily effective contender would become a hegemon. Viability requires sufficiently high income $Y_i$ for survival on the individual level or, in the case of larger contending units, for maintaining group integrity.

2. *Equilibrium allocations of effort.* — In the symmetrical Cournot solution with $N = 2$ contestants, the crucial result is that as the decisiveness parameter $m$ rises, each side is forced to fight harder ($f_1$ and $f_2$ both increase). The consequences include reduced incomes on both sides.

\textsuperscript{20} In 1555 the city of Siena undertook modernization of its fortifications. But the costs were so high that, when attack came, not only were the defense works still incomplete but funds to hire a supporting mercenary army or fleet were lacking. So in 1555 Siena surrendered to Florence and permanently lost its independence (see Parker 1988, p. 12).
3. Numbers.—As $N$ grows exogenously, equilibrium fighting intensities $f_i$ rise. With fixed aggregate resources $R$, per capita incomes $Y_i$ fall for two reasons: first, because each party’s pro rata share $p_i = 1/N$ is less and, second, because $f_i$ is higher. That is, a contestant has to fight harder just to obtain a pro rata share. This second reason continues to apply even in a more generous environment in which resources grow in proportion to $N$. If $N$ is endogenous, the equilibrium number of contenders is determined by the viability condition $Y_i \geq y$; that is, entry occurs up to the point of zero profit.

4. Technology and comparative advantage.—An asymmetrical productive improvement (a decrease in the production cost coefficient $a$, or an increase in the productivity parameter $h_i$) increases own income $Y_i$ but within the model here does not otherwise affect any of the results. (However, I have flagged this as a “nongeneric” result deriving from special features of the model, in particular, the total disjunction of productive opportunities.) On the conflict side, corresponding one-sided improvements (i.e., a reduction in the logistics cost coefficient $b$, or an increase in the decisiveness parameter $m_i$) generally increase own income while reducing opponent income.

5. Strategic position.—The Stackelberg solution, as compared with the symmetric Cournot equilibrium, involves reduced fighting on both sides, but the follower gains relative to the leader. This evidently tends to stabilize the anarchic system. Although all could benefit from the change, each single participant is motivated to hold back and let the opponent become the leader.

The analytic results here depend on a particular way of modeling anarchy that omits many possibly important elements. To mention only a few: (1) Full information was assumed throughout, so that factors such as deception have been set aside (see, e.g., Tullock 1974, chap. 10; Brams 1977). (2) Apart from opportunity costs in the form of forgone production, fighting was assumed nondestructive. (This assumption biases our results in the direction of conflict.) (3) Distance and other geographical factors (see, e.g., Boulding 1962, chaps. 12–13) were not explicitly considered, though they entered implicitly as determinants of the logistics cost and decisiveness parameters. (4) The steady-state assumption rules out issues involving timing, such as arms races, economic growth, or (on a smaller time scale) signaling resolve through successive escalation. (5) Finally, I have not at-

27 The model of Grossman and Kim (1994) allows for damage due to fighting. The extent of “collateral damage” has been influenced by two opposed technological trends: greater destructive power and improved aiming precision. In a nonmilitary context, Becker’s (1983) analysis of pressure group competition shows how incidental damage to the economy (“deadweight loss”) tends to limit the extent of conflict.
tempted to model the problems of group formation and collective action (but see the next section).

The justification for these and other omissions is that one must begin somewhere. The model illustrates a method of analysis. In many contexts, for example, it might be unacceptable to omit the element of collateral damage (qualification 2 above). Still, that effect could be incorporated by means of an adjustment within the general analytical framework.

VII. After Anarchy, What?

Though this topic lies outside the bounds of the model, the analysis here insistently suggests the following question: Supposing that anarchy does break down, what happens next?

Theoretical considerations, as well as the historical and other applications described in Section V, combine to suggest that anarchic systems are fragile. Anarchy is always liable to “break down” into amorphy or “break up” into organization! First of all, exogenous changes may lead to violation of the necessary conditions of result 1. Military technology (very often, though not always) has moved in the direction of higher decisiveness in threatening dynamic stability. And Malthusian pressures are at work to dilute per capita incomes, threatening viability.

But even if the necessary conditions are met, making anarchy in principle sustainable, the system may be undermined by “the urge to merge.” Benefits from group formation may include (1) reduced fighting within, (2) complementarities in production, and (3) enhanced ability to fight outsiders. The other side of the coin, the factor hampering mergers, is the collective-action problem: how to get agreement on a social contract and, even more important, how to enforce it.

It is useful to distinguish vertical from horizontal social contracts. The vertical alternative, Thomas Hobbes’s version, would be represented by arrangements such as hierarchical dominance in the biological realm or dictatorship on the human level. John Locke’s version, the horizontal alternative, corresponds to more egalitarian arrangements in either sphere.28

Of the two major sources of breakdown—dynamic instability and

28 Chicago gangland history (Allsop 1968) provides nice instances of both arrangements. Johnny Torrio, a “statesman-like” leader, attempted to bring all the gangs together in a Lockian solution with profit sharing and allocation of territories. However, the intransigent South Side O’Donnells resisted confederation. Torrio’s more ruthless successor, Al Capone, ultimately succeeded in imposing a vertical Hobbesian solution.
income inviability—the former is likely to lead to a vertical social contract. An excessively high decisiveness coefficient \(m\) implies a range of increasing returns to fighting effort. At the extreme, this may imply a “natural monopoly” in fighting activity; that is, the struggle is likely to end up with all the resources under one party’s control. In contrast, the mere fact of low income under anarchy, since it may be the consequence of many different forces, of itself provides no clear indication as to what is likely to happen next.

Owing to closer sympathies, better monitoring of shirkers, and so forth, the collective-action problem is more readily solved in small groups. But these small groups in turn come into anarchic competition at the group level. This of course provides a cascading motivation for unification one level higher up. In modern times this process has led to a sharp reduction in the number of independent states and principalities: in Europe alone, from hundreds or even thousands to around a dozen or two after the unifications of Germany and Italy. Still, there never has been an all-European state. Nor should we assume that the process can go only in the direction of agglomeration, as the fall of the Roman Empire and the recent dissolution of the Soviet Union demonstrate.

The upshot is that, even if anarchy breaks up into organization on one level, anarchic conflict may be sharpened at the higher level. If the clans within a tribe agree on a social contract, peace among the clans may be only the prelude to more violent struggles against other tribes.

Appendix

Numerical Examples

All these numerical examples are connected and can be read together as a running illustration of the model.

Numerical Example 1

Let the decisiveness parameter be \(m = \frac{1}{3}\). Then equations (7a) and (7b) simplify to \(p_1/p_2 = R_1/R_2 = (f_1/f_2)^2\). If the total resources available are \(R = 100\) and the fighting intensities on each side have been chosen (not necessarily optimally) to be \(f_1 = 0.1\) and \(f_2 = 0.2\), respectively, then \(p_1/p_2 = (0.1/0.2)^2 = \frac{1}{4}\), implying that, in equilibrium, \(R_1 = 20\) and \(R_2 = 80\).

To illustrate convergence when \(m = \frac{1}{5}\), suppose that the initial resource vector is set at \((R_1, R_2) = (60, 40)\). The first-period conflict outcome is \(p_1/p_2 = (6/8)^{2/3} = 0.825\), implying an end-of-period revised resource allocation \((R_1', R_2') = (45.2, 54.8)\). After one more period of conflict the resource allocation becomes \((R_1'', R_2'') = (35.7, 64.3)\). Evidently, the equilibrium \((R_1, R_2) = (20, 80)\) is being approached asymptotically.
In contrast, for a decisiveness parameter in the range \( m > 1 \), say \( m = 2 \), starting from the same initial resource vector (60, 40), the first- and second-period reallocations would be (36.0, 64.0) and (7.3, 92.7). The process rapidly diverges in favor of the side with the higher \( f_i \).

**Numerical Example 2**

In the previous numerical example the contenders were arbitrarily assumed to have chosen fighting intensities \( f_1 = .1 \) and \( f_2 = 2 \), leading to the equilibrium resource distribution \( R_1 = 20 \) and \( R_2 = 80 \). But if the parties choose optimally instead under the Nash-Cournot assumption, with \( m = \frac{1}{b} \) and symmetrical logistics cost coefficients \( b_1 = b_2 = b = 1 \), from equation (11) the equilibrium choices are \( f_1 = f_2 = .5 \), implying an equal equilibrium resource division \( R_1 = R_2 = 50 \). From (12), and with production cost coefficients \( a_1 = a_2 = a = 1 \) and productivity parameter \( h = 1 \) (constant returns in production), the associated incomes are \( Y_1 = Y_2 = 25 \). (In contrast, had no conflict occurred—that is, if the choices had been \( f_1 = f_2 = 0 \)—the per capita incomes would have been 50 each.)

**Numerical Example 3**

With aggregate resources \( R = 100 \) and \( N = 2 \), equilibrium fighting intensities in the previous example were \( f_1 = f_2 = .5 \), yielding per capita incomes \( Y_1 = Y_2 = 25 \). If we use the same parameter values and hold aggregate resources fixed at 100, for \( N = 3 \) the equilibrium fighting efforts rise to \( f_1 = f_2 = .571 \) and the per capita incomes fall to \( Y_i = 14.3 \), approximately. (Note that the aggregate income is lower as well.) If, on the other hand, resources were to rise in proportion to numbers—specifically here, if \( R = Nr \), where \( r = 50 \)—then in equilibrium \( Y_i = 21.4 \), approximately. Thus, even when the resource base expands with \( N \), there is a per capita income loss owing to the larger optimal \( f_i \).

**Numerical Example 4**

With the same parameter values, for aggregate resources fixed at \( R = 100 \), suppose that the viability threshold is \( y = 4 \). From equation (17), the equilibrium incomes are \( Y_i = 4 \) at \( N = 9 \). So this fixed resource magnitude will support a population of \( N = 9 \) competitors. If instead resources expand with population so that \( R = 50N \), the situation is much more favorable. In such an environment, equation (19) indicates that an equilibrium population of \( N = 9 \) could be supported even at a much higher viability threshold \( y = 18 \).

**Numerical Example 5**

Under the quantitative assumptions of numerical example 2, the Cournot equilibrium was \( f_1 = f_2 = .5 \) and \( Y_1 = Y_2 = 25 \). When the same numerical assumptions are used, side 1 as Stackelberg leader does best by choosing a somewhat lower \( f_1 = .41 \), approximately, and his income rises slightly to
about 25.7. The second mover optimally responds by cutting back her fighting intensity only to about .466, reaping a considerably higher income of around 30.1.

References


