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THE PARADOX OF POWER*

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In power struggles, the strong might be expected to grow ever stronger and the weak weaker still. But in actuality, poorer or smaller combatants often end up improving their position relative to richer or larger ones. This is the paradox of power. The explanation is that initially poorer contenders are rationally motivated to fight harder, to invest relatively more in conflictual activity. Only when the decisiveness of conflict is sufficiently high does the richer side gain relatively in terms of achieved income. Among other things, the paradox of power explains political redistributions of income from the rich to the poor.

IN CONFLICTUAL interactions (broadly interpreted to include not only hot and cold wars but also lawsuits, strikes and lockouts, redistributive politics, and even family rivalries) we might ordinarily expect the strong to grow ever stronger and the weak weaker. Nevertheless, surprisingly often, initially weaker or poorer contenders end up gaining on initially stronger or wealthier opponents. This is the paradox of power.[1]

In wars, smaller or poorer nations have often fought much larger ones to a standstill, the recent Vietnam conflict being but one instance. As a more general historical phenomenon, from earliest times poor nomadic tribes have successfully preyed upon more affluent cities and empires. Turning to the modern political redistributive struggle, although in democratic states the wealthy would normally be presumed to have more political power than the poor, we almost always observe income being transferred from upper to lower fractiles of the wealth distribution.[2] As another contemporary example, the farmers, although steadily

[1] "Power" here is taken to mean the ability to achieve one's ends in the face of rivals. For an interpretation of the traditional literature on power from an economic point of view, see Bartlett [1989, Ch. 1-2].

[2] In 1976 the lowest-income quintile of U.S. families initially received only 0.3% of aggregate family income but redistribution through the tax-transfer system raised their share to 7.2%. Correspondingly, the highest quintile began with 50.2% of aggregate family income but ended up with 41.4% after taxes and transfers (Browning and Browning [1979], pp. 203-205). Such redistribution is evidently not simply a matter of relative numbers, i.e., of masses of the poor at the bottom of the pyramid outvoting a few rich at the top. In fact, since on average the wealthy have larger family sizes, the highest-income quintile of families comprises a larger number of people than the lowest quintile.

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diminishing in numbers and therefore in voting strength have nevertheless been winning ever-increasing subsidies and benefits. Some analysts have even pointed to a paradoxical "Law of the Few" as a general characteristic of majoritarian politics.[3]

On the other hand, history is replete with examples to the opposite effect, where the strong have exploited the weak. Over the centuries the Jews of Europe, certainly a weaker group in political and military terms, were repeatedly despoiled of their wealth—at times by rulers, at times by the masses, often by both. In despotic societies, tyrants commonly drain the wealth of their subjects.[4] And on occasions too numerous to cite in detail, more powerful nations or tribes have subjugated weaker neighbors.

The key to the paradox of power is a surprisingly simple economic point, yet one that has been inadequately appreciated. While wealth certainly provides the wherewithal for successfully exploiting a poorer opponent, *the initially disadvantaged group is typically rationally motivated to fight harder.*[5] Or, put the other way, non-conflictual or cooperative strategies tend to be relatively more rewarding for the better-endowed side.

In the analysis here, it is postulated that the stakes are limited: one side cannot totally exterminate or enslave the other. Constitutional rules, for example, normally prevent redistributive politics from going on to total confiscation. Industrial, legal, and family conflicts are evidently also subject to strict limits. And even in warfare, one side—owing possibly to technological limitations, to the presence of third parties, or to moral restraints—might not aim at total destruction of its opponent. Bounds upon what can be done in the way of conflict increase the attractiveness of cooperative techniques for achieving one's goals.

War and peace, or more generally conflict and settlement, are usually regarded as mutually exclusive. Rival nations are said to be at war or else at peace; a trade union may call a strike or else sign a collective-bargaining contract; a lawsuit may be settled or litigated in court. But it will be convenient here to employ a paradigm in which the choice is not between "going to war" and "making peace." Instead, the parties choose a steady-state strategy along a continuum ranging between the extremes of struggle and accommodation. To some extent this is a matter of perspective. At any single moment, members of a labor union are either on strike or at work. Taking the long view, however, we can regard the

[3] The Law of the Few suggests, for example, that as the numbers of the elderly continue to grow, their political power will decrease—so that redistributive benefits to the aged will tend to fall rather than rise (McKenzie [1990]).

[4] "In Rwanda . . . the masses of the people were peasants who were forced to contribute goods and services to the support of a vast and complex political administration. . . . Although they are said to have gloried in their subjugation, which is a matter in doubt, they received little beyond the minimum reallocations in return for almost the entirety of their production over and above what was needed for their own bare subsistence."—Codere [1968], p. 242. And see also the Biblical tale of Naboth's vineyard (1 Kings 21).

[5] Only two-party interactions are considered here. This assumption rules out cases in which a small group might gain disproportionate power by being able to swing the outcome between two larger contenders.

union as having adopted a strategy of going on strike a certain proportion of the time. Similarly, a primitive tribe may be observed to alternate between peace and war with its neighbors, but over the long run its actions can be interpreted as a chosen division of effort between productive exploitation of its own territory versus appropriative struggles with other tribes.

Apart from the central issue of the relative success achieved by richer versus poorer contenders, the analysis will also cast light upon questions such as:

(1) Will progress in productive technology promote cooperation? As the returns from productive investments improve, will the rival contestants tend to shift their resource allocations more toward producing or more toward fighting?
(2) What about corresponding progress in the technology of conflict?
(3) As economies become more intertwined so that trade linkages across boundaries become stronger, will international conflict decrease?[6] In the first decade of the twentieth century, increasingly close productive ties among the national economies of Europe led to a widespread belief that large-scale war had become out of the question.[7] The first World War dashed these hopes. Since we are now seeing a revival of such beliefs, [8] it will be important to understand why productive complementarity is less effective a force for peace than might initially have been anticipated.

1. MODELLING CONFLICTUAL EQUILIBRIUM

Just as there is a general equilibrium of a market economy, there is also an equilibrium outcome when rivals may also compete by conflict and struggle. The analytical elements of such an equilibrium include:

The contenders and their intentions: I abstract here from the within-group incentive problem and assume that decision-makers on each side make collectively rational choices aimed solely at maximizing group income.

Capabilities: Each side has resources that can be utilized for productive activity or conflictual activity. In accordance with the limited-stakes assumption, the underlying resources themselves are supposed invulnerable to destruction or capture. Only the income generated by productive use of resources is at issue.

Technology: In parallel with the familiar *technology of production* that translates productive efforts into income, there is a *technology of conflict* that translates commitments of resources to struggle into distributive success.

Rules of the game and solution concept: In competition through struggle, the equilibrium may depend upon the sequence of moves, the informational

[6] On this see, for example, Polachek [1980].
[7] Angell [1911].
[8] See, for example, Mueller [1989].

assumptions, whether or not threats are permitted, and so forth. I will be assuming that the Cournot solution concept is applicable.[9]

Two broad, almost self-evident generalizations apply to mixed interactions in which the parties are simultaneously cooperating yet competing with one another: (i) The resources devoted to productive activity mainly determine the aggregate income available. (ii) The contenders' relative commitments to conflictual activity mainly determine how the aggregate income will be distributed between them.

The equation system reflecting these considerations has four classes of logical elements.

First, each side $i = 1,2$ must divide its exogenously given resources R_i between productive effort E_i and fighting effort F_i:

$$\begin{aligned} E_1 + F_1 &= R_1 \\ E_2 + F_2 &= R_2 \end{aligned} \qquad \text{Resource Partition Equations} \quad (1)$$

Second, the productive technology is summarized by an Aggregate Production Function (APF) showing how the productive efforts E_1 and E_2 combine to determine income I—the social total available for division between the two parties. A convenient form for our purposes is:

$$I = A({E_1}^{1/s} + {E_2}^{1/s})^s \qquad \text{Aggregate Production Function.} \quad (2)$$

This type of production function is characterized by constant returns to scale and constant elasticity of subsitution. Parameter A is a *total productivity index*: as the overall yields of the resource inputs rise over time, owing to technical progress, A increases. Parameter s, which plays a crucial role in the analysis, is a *complementarity index*: as nations become more closely and synergistically linked by international trade, for example, s rises. As can be seen in Figure 1, higher values of s are reflected in increasingly convex curvature of the productive isoquants. At the lower limit, when $s = 1$, the APF takes on a linear (additive) form.[10]

The third element, the Contest Success Function (CSF), summarizes the technology of conflict—whose inputs are the fighting efforts F_1 and F_2 and whose outputs are the distributive shares p_1 and p_2 (where of course $p_1 + p_2 = 1$).

[9] I have considered Stackelberg and hierarchical solution concepts elsewhere (Hirshleifer [1988]).

[10] Values of s below 1 have the unacceptable implication that the marginal products of productive input are increasing throughout.

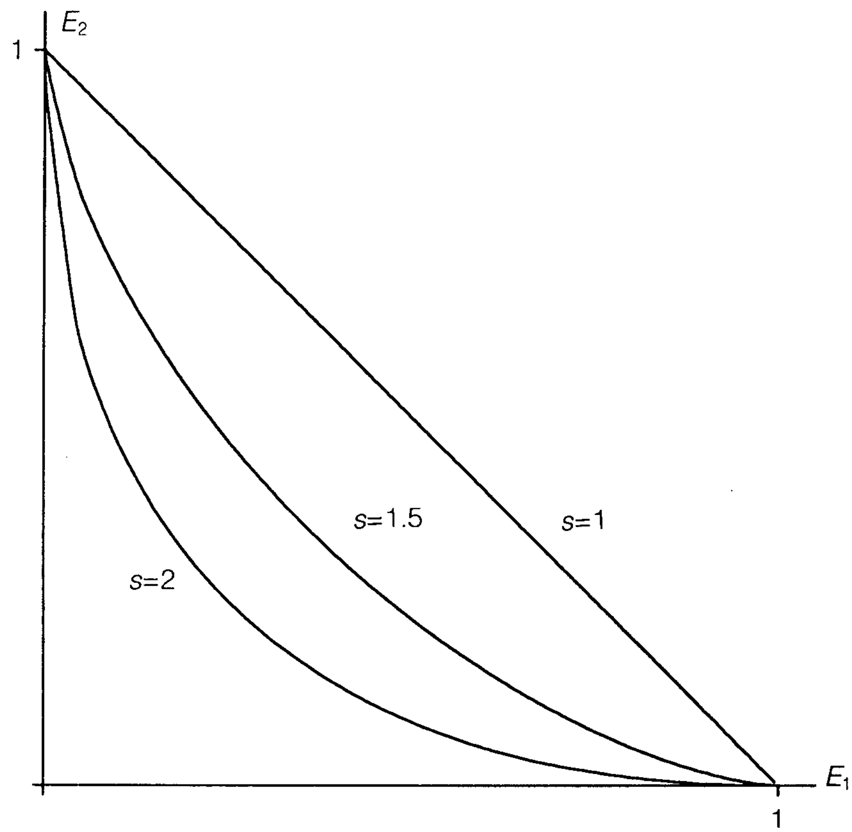

Figure 1. Unit isoquants of the Aggregate Production Function (APF), for different values of the complementarity parameter s.

There are significantly different ways of formulating the Contest Success Function.[11] However, for analytical tractability I will assume here that the outcome of the struggle depends only upon the *ratio* of the parties' conflictual efforts F_1 and F_2, indexed by a single *mass effect parameter* m:[12]

$$p_1 = F_1^m/(F_1^m + F_2^m) \qquad \text{Contest Success Functions.} \quad (3)$$
$$p_2 = F_2^m/(F_1^m + F_2^m)$$

As illustrated in Figure 2, the mass effect parameter m scales the *decisiveness of conflict*, that is, the degree to which a superior input ratio F_1/F_2 translates into a superior proportionate success ratio p_1/p_2.

[11] I have explored elsewhere some of the implications of making fighting success a function of the *numerical difference* between the commitments (Hirshleifer [1988, 1989]). It is also possible, for example, to adjust the equations so as to make one side militarily more effective than the other (see for example Grossman [1991]) or to make defense less costly than offense.

[12] See Tullock [1980].

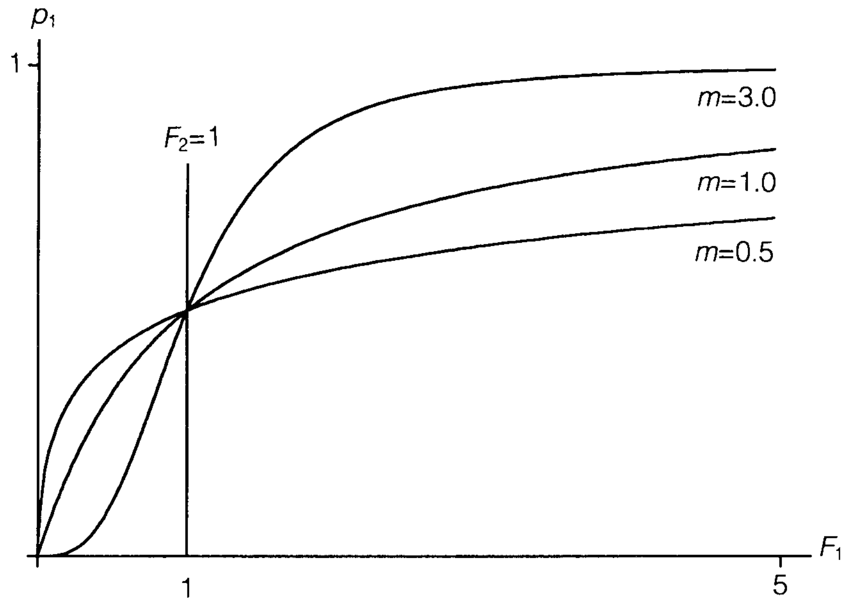

Figure 2. The Contest Success Function (CSF), for different values of the mass effect parameter *m*.

Finally, there are Income Distribution Equations defining the achieved income levels I_1 and I_2:

$$I_1 = p_1 I$$
$$I_2 = p_2 I \qquad \text{Income Distribution Equations.} \quad (4)$$

Equations (3) and (4) together imply that all income falls into a common pool available for capture by either side. More generally, the contenders might also have opportunities for generating invulnerable[13] income, but this consideration is set aside here.

The equation system (1) through (4) is illustrated by the four-way diagram of Figure 3. The upper-right quadrant shows the range of contender # 1's choices between productive effort E_1 and conflictual effort F_1, within his initial resource endowment R_1. The diagonally opposite quadrant shows the corresponding options for contender # 2. The upper-left quadrant shows how the respective

[13] In a model allowing for some fraction of invulnerable income, each side would be deciding among three rather than only two options: resources R_i could be allocated to conflictual effort, to productive effort that generates contestable income I, or to productive effort yielding invulnerable income. More generally still, income on either side could be scaled along a vulnerability dimension: for example, crops grown along a country's borders are more vulnerable to capture than production taking place in the interior.

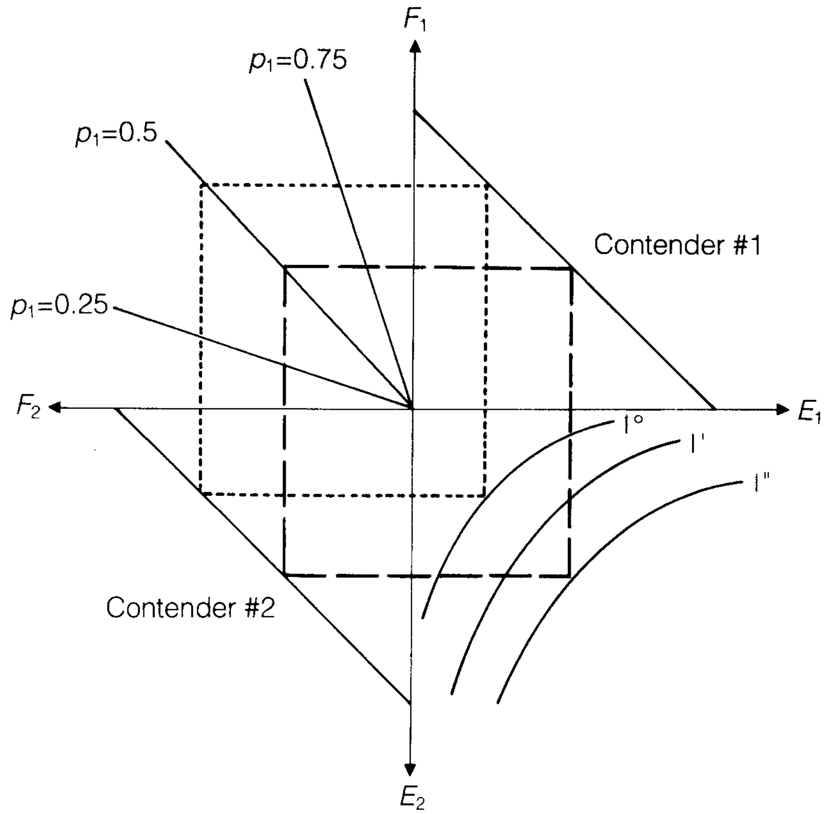

Figure 3. Productive technology determines income I while conflict technology determines fractional share p_1.

fighting efforts F_1 and F_2 determine p_1, the share of aggregate income won by #1, where of course $p_2 \equiv 1 - p_1$. (The p_1 contours are straight lines emerging from the origin, which follows from the assumption stated above that the distributive shares are functions only of the ratio F_1/F_2.) Finally, the lower-right quadrant shows how the productive efforts E_1 and E_2 combine to generate different overall totals of income I.

The dashed rectangle in Figure 3 illustrates one possible outcome of the postulated interaction: for given initial choices E_1,F_1 on the part of decision-maker #1 and E_2,F_2 on the part of #2, the productive activity levels E_1 and E_2 determine aggregate income I while the conflictual commitments F_1 and F_2 determine the respective shares p_1 and p_2. The dotted rectangle shows what happens when the two sides choose instead to devote more effort to fighting. As drawn here, the increases in F_1 and F_2 have cancelled one another out so that p_1 and p_2 remain unchanged. Thus, the only effect of *symmetrically* increased fighting efforts may be to reduce the amount of income available to be divided.

2. REACTION CURVES AND COURNOT EQUILIBRIUM

On the assumption that the underlying strategic situation justifies the Cournot solution concept, the Reaction Curves RC_1 and RC_2 show each side's optimal fighting effort given the corresponding choice on the part of the opponent. The Cournot solution occurs at the intersection where each party's decision is a best response to the opponent's action.

Decision-maker #1's optimizing problem can be expressed:

$$\text{Max } I_1 = p_1(F_1|F_2) \times I(E_1|E_2) \text{ subject to } E_1 + F_1 = R_1 \tag{5}$$

and similarly for side #2. Using equations (2) and (3), by standard constrained-optimization techniques we can solve for the Reaction Curves RC_1 and RC_2:

$$\begin{aligned} \frac{F_1 E_1^{(1-s)/s}}{F_2^m} &= \frac{m(E_1^{1/s} + E_2^{1/s})}{F_1^m + F_2^m} \\ \frac{F_2 E_2^{(1-s)/s}}{F_1^m} &= \frac{m(E_1^{1/s} + E_2^{1/s})}{F_1^m + F_2^m} \end{aligned} \qquad \text{Reaction Curves.} \tag{6}$$

Note that the parameter A of the Aggregate Production Function has cancelled out and does not enter into the Reaction Curve equations at all. Thus, responding to a question raised in the introduction, *an increase in overall economic productivity leaves the proportionate allocation of resources between producing and fighting unchanged.* Intuitively, an increase in A raises the marginal profitability of productive activity and of conflictual activity in the same proportion (see below).

These equations are simultaneously valid only when both sides' choices are in the interior range, with $F_i < R_i$. It can be shown that for any $s > 1$ (that is, except for the limiting case $s = 1$ where productive complementarity is absent), only interior solutions exist. While there is no convenient general analytical solution, in the special case where $m = 1$ and the resource endowments are equal (that is, when $R_1 = R_2$), for any s the equilibrium is simply:

$$F_1 = F_2 = E_1 = E_2 = (R_1 + R_2)/4 \quad \text{Symmetrical Cournot solution } (m = 1). \tag{7}$$

Here, exactly half of the available resources are dissipated in mutually wasteful fighting effort. (As will be seen, higher values of m lead to still greater dissipation of resources, lower values to lesser dissipation.)

To begin with, it will be of interest to consider the limiting case where productive complementarity is absent ($s = 1$). While a corner solution is then possible, the solution will nevertheless always be in the interior whenever the parties' endowments are symmetrical so that equation (7) applies. In Figure 4, the parameters are set at $A = m = s = 1$. The RC_1° and RC_2° Reaction Curves in

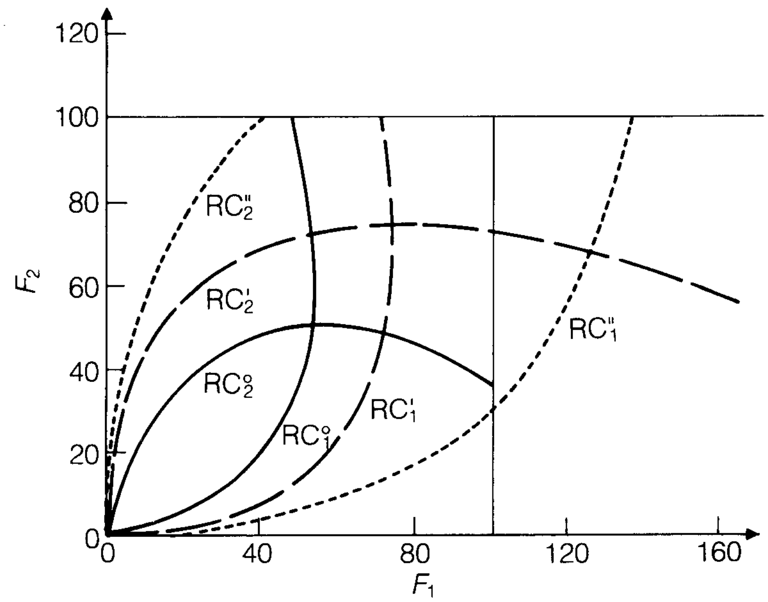

Figure 4. Reaction Curves and Cournot equilibrium—Interior and corner solutions.

the diagram apply for equal initial resources, specifically $R_1 = R_2 = 100$. The interior intersection of the Reaction Curves is the Cournot equilibrium,[14] as summarized in Numerical Example 1.

Numerical Example 1: With resources $(R_1,R_2) = (100,100)$ and assuming parameter values $A = s = m = 1$, equation (7) indicates that exactly half the resources on each side are dissipated in conflict: $(F_1,F_2) = (50,50)$. The remaining resources on each side are put to productive use: $(E_1,E_2) = (50,50)$. These generate an aggregate income $I = 100$, which is then equally divided between the contenders: $(I_1,I_2) = (50,50)$.

3. RESOURCE DISPARITIES AND INCOME: THE PARADOX OF POWER

The paradox of power (POP) emerges when a preponderant resource ratio $R_1/R_2 > 1$ is not reflected in a correspondingly large achieved income ratio I_1/I_2. I shall be examining "strong" versus "weak" forms of the paradox:

POP (strong form): In mixed conflict-cooperation interactions, the contending parties will end up with exactly identical incomes ($I_1/I_2 = 1$) regardless of the initial resource distribution.

[14] The Reaction Curves also appear to intersect at $F_1 = F_2 = 0$, but the zero-zero intersection is not a Cournot equilibrium. If (say) player #1 chooses $F_1 = 0$, then player #2 would rationally respond by setting F_2 equal to any small positive magnitude, since doing so discontinuously improves his fighting success from 50% to 100%. Thus, strictly speaking, the Reaction Curves are defined only over the open interval that does *not* include the singular point at the origin.

POP (weak form): In mixed conflict-cooperation interactions, the final distribution of income will have lesser dispersion than the initial distribution of resources. Thus, assuming # 1 is the better-endowed side: $R_1/R_2 > I_1/I_2 > 1$.

Considering once again the limiting case where $s=1$ (where there is no complementarity between the productive activities on the two sides), the Reaction Curve equations (6) reduce to:

$$\frac{F_1}{F_2^m} = \frac{m(E_1+E_2)}{F_1^m+F_2^m} \qquad \frac{F_2}{F_1^m} = \frac{m(E_1+E_2)}{F_1^m+F_2^m} \qquad \text{Reaction Curves } (s=1). \quad (6a)$$

Of course, these equations are valid only for interior solutions. On this assumption, at equilibrium we must have $F_1 = F_2$. Thus, regardless of initial resource disparities, so long as the solution remains in the interior, the equilibrium fighting efforts will always be exactly equal. This implies, of course, equal incomes $I_1 = I_2$—the *strong form of the POP applies*.

In Figure 4, under the same parameter assumptions, the *dashed* curves RC_1' and RC_2' represent a situation where # 1's resources R_1 have doubled in size while # 2's are held constant. Despite this asymmetry, at the intersection of the RC_1' and RC_2' curves the fighting efforts F_1 and F_2—though both larger than before—remain equal to one another! It follows, of course, that the richer party must now be devoting absolutely and relatively more resources to productive effort.

Numerical Example 2: With $A=m=s=1$ as before, let the initial resources be $(R_1,R_2)=(200,100)$. At the Cournot equilibrium, once again half of the social aggregate of resources is dissipated in conflictual effort: $(F_1,F_2)=(75,75)$. The wealthier side devotes more effort to production so that $(E_1,E_2)=(125,25)$. Nevertheless, the equality of F_1 and F_2 dictates that the final incomes must also be equal: $(I_1,I_2)=(75,75)$.

An intuitive interpretation is as follows. With an increase in his endowment, contender # 1 (he) will surely want to spend more on each of the two types of activity: his E_1 and F_1 will both be greater. Knowing this, side # 2 (she) then has both offensive and defensive incentives to shift toward spending *more* than before on fighting (choosing a larger F_2), and therefore less on production (E_2 must be smaller). Her offensive incentive for making F_2 larger is that, E_1 being greater, there is more social income available to be seized. Her defensive incentive is that, F_1 being greater, she has to make F_2 larger even if only to maintain her previous level of income.

The key to the paradox of power is that, when a contender's resources are small relative to the opponent's, *the marginal yield of fighting activity is higher to begin with than the marginal yield of productive activity*. This is transparently easy to see for the special case of $m=s=1$. Then, supposing that #2 is the poorer-endowed side, differentiation of $I_2 = p_2 I$ leads to:

$$\frac{\partial I_2}{\partial E_2} = \frac{AF_2}{F_1 + F_2} \quad \text{and} \quad \frac{\partial I_2}{\partial F_2} = \frac{AF_1(E_1 + E_2)}{(F_1 + F_2)^2} . \tag{8}$$

When R_2 is very small then of course E_2 and F_2 must be small as well. As both E_2 and F_2 go toward zero, the partial derivative $\partial I_2 / \partial E_2$ (the marginal payoff of productive activity) also goes to zero, while the partial derivative $\partial I_2 / \partial F_2$ (the marginal payoff of fighting effort) remains positive. Thus, as R_2 approaches zero, the poorer side will find that any productive expenditure E_2 will largely be wasted, since the wealthier opponent will be capturing almost all of whatever is produced. But any positive F_2, however small, will win some share of the enlarged total product available.

Conflict is therefore a relatively more attractive option for the poorer side.[15] Fighting effort permits you to "tax" the opponent's production, while your own production is "taxed" by his fighting effort. When your rival is richer it becomes relatively more profitable to tax him (to capture part of his larger production) and relatively more burdensome to be taxed by him (to devote effort to production which will be largely captured by him anyway). Thus rational behavior in a conflict interaction, under the assumptions here, is for the poorer side to specialize more in fighting, the richer side more in production.

What about the possibility of corner solutions? In the special case of $s=1$, there will be some critical resource ratio $R_1/R_2 = \rho^*$ at which the poorer side has already devoted *all* of its resources to fighting (has become a pure predator, so to speak). It can be shown that the critical ratio, beyond which the poorer side will be forced to a corner solution, is given by:

$$\rho^* = R_1/R_2 = (2+m)/m. \tag{9}$$

Resource ratios more extreme than ρ^* lead to corner solutions where the Reaction Curve for the poorer side (#2, say) reduces simply to $F_2 = R_2$. At corner solutions, since the fighting efforts are no longer equal, the *strong form of the POP* cannot hold. Nevertheless, the *weak form of the POP* may apply—that is, attained incomes, while no longer exactly equalized, could remain less unequal than the initial resource endowments. In Figure 4 the dotted Reaction

[15] Compare Becker ([1983], p. 385): "Politically successful groups tend to be small relative to the size of the groups taxed to pay their subsidies". However, as will be noted below, Becker employs an entirely different line of reasoning to arrive at this result.

Curves RC_1'' and RC_2'' illustrate such a case, where #1's resources have doubled again relative to #2's. These Reaction Curves intersect at a corner equilibrium for #2.

Numerical Example 3: Let the resource endowments be $(R_1,R_2)=(400,100)$. It can be verified that, for the assumed parameter values $A=m=s=1$, the critical resource ratio ρ^* is 3. Since $R_1/R_2=4$, side #2 will have to devote all its resources to fighting ($F_2=100$). The solution is $(F_1,F_2)=(123.6,100)$. The better-endowed side now obtains a larger proportionate share $(p_1,p_2)=(0.553,0.447)$ and higher level of income $(I_1,I_2)=(152.8,123.6)$. Nevertheless, the attained income ratio $I_1/I_2=1.236$ is considerably smaller than the resource ratio $R_1/R_2=4$.

Another point of interest: within any given resource total R_1+R_2, corner equilibria involve less overall wastage of resources in fighting.

Figure 5 provides a more general picture of how incomes respond to the resource ratio, given the parameter values $A=m=s=1$. (Henceforth, the total productivity index A of the Aggregate Production Function will always be assumed equal to unity.) In the diagrams here, #2 is always the poorer side with resources arbitrarily fixed at $R_2=1$. We have seen that the absolute allocations F_1 and F_2 to fighting effort are equal in the interior range, up to the critical resource ratio $R_1/R_2=\rho^*=3$. Figure 5a shows how this is reflected in the *proportionate* allocations, the curve F_2/R_2 rising in this range for the poorer side while F_1/R_1 is declining for the richer side. (Of course, the equal *absolute* allocations imply $p_1=p_2$ in this range.) Once the critical ratio $\rho^*=3$ is exceeded, for the poorer side $F_2/R_2=1$. So #1's fractional share p_1 begins to rise steadily relative to p_2, even though the optimal F_1/R_1 for the richer side continues to decline. Correspondingly, Figure 5b shows that the achieved incomes I_1 and I_2 remain exactly equal in the range of interior solutions before the critical ratio is reached. Only in the range of corner solutions does the better-endowed side achieve an income advantage. (In Figure 5b the curve labelled I is the aggregate income produced, while $\bar{I}$ represents the ideal level of aggregate income that could be generated if no resources were devoted to fighting effort on either side.)

Summarizing, for the parameter values $m=s=1$ *the strong form* of the paradox of power (POP) continues to hold so long as the resource ratio is only moderately unequal, so that the equilibrium falls within the interior-solution range. When the resource asymmetry becomes sufficiently great, the parties enter a corner-solution range where only *the weak form* of the POP applies.

However, as already indicated $s=1$ is a limiting case. In social interactions we normally expect to observe a certain degree of productive complementarity. Nations at war with one another can, alternatively or even at the same time, also benefit from trade that leads to an improved international division of labor.[16]

[16] War may or may not be accompanied by a total cessation of trade between the contending sides. "Cold" wars, where only minor actual combats take place, are of course quite consistent with ongoing trade. And substantial trade across the lines sometimes occurs even in quite hot wars, a notable example being the American Civil War.

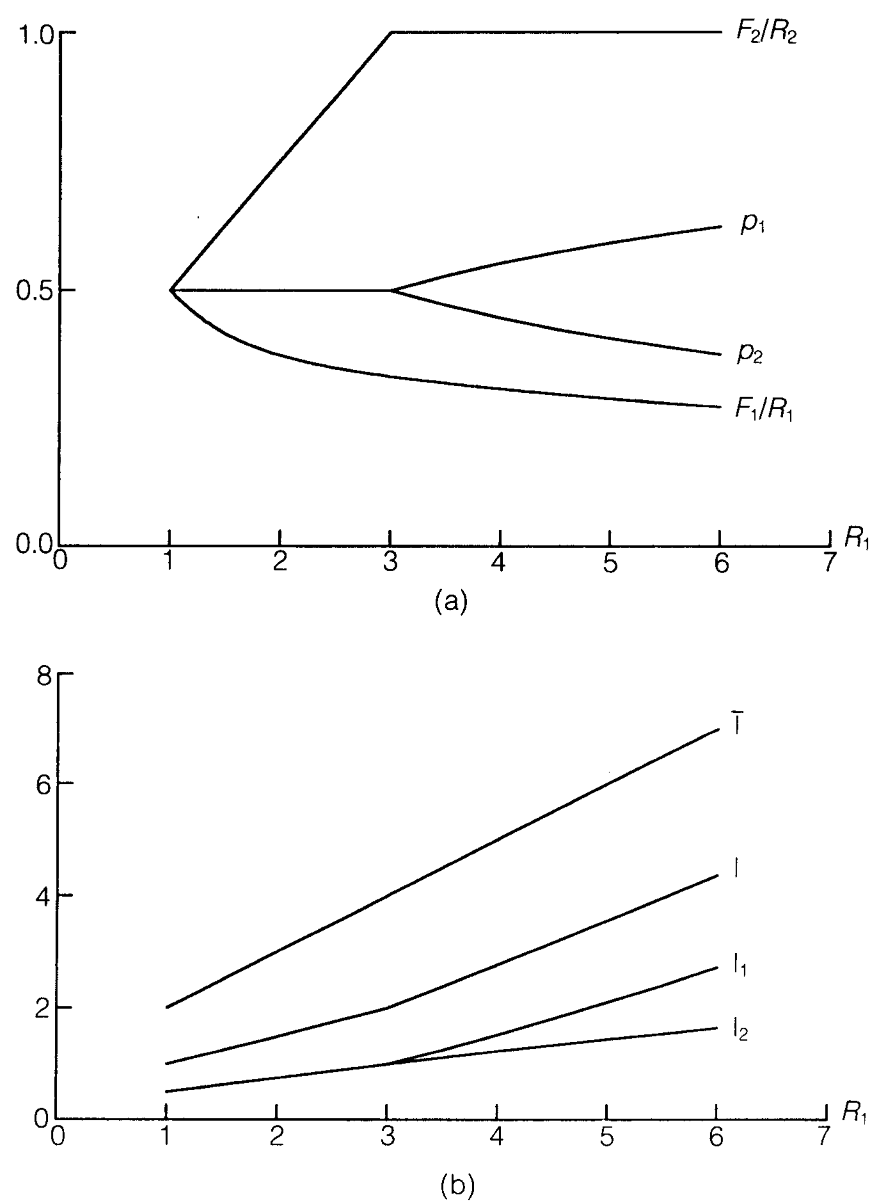

Figure 5. Proportionate allocations to fighting, fractional shares, and achieved incomes—$m = 1$, $s = 1$.

This is of course even clearer in industrial relations, where labor and capital are productively vital to one another even as the two sides struggle over shares of the firm's revenues.

As the complementarity index s rises, for either contender the marginal payoff of productive effort E_i will rise in comparison with the marginal payoff of fighting effort F_i. Thus, as s increases we might be inclined to anticipate "an era of better feeling" in which each side redirects its activities so as to devote more effort to increasing the size of the pie, and less to grabbing a bigger slice. However, the actual outcome is somewhat more complex.

Figure 6 is the analog of Figure 5, except that the productive complementarity parameter is set at $s = 1.25$ in place of $s = 1$. As already indicated, there are no corner solutions whenever $s > 1$. Apart from the smoothing due to this change, the broad picture is similar to the previous diagram. Of course, the potential income $\bar{I}$, the actual aggregate income I, and the achieved incomes on each side are all larger than before. However, closer comparison of the F_i/R_i curves reveals that while the poorer side is tilting somewhat away from fighting, the wealthier side shifts a bit in the opposite direction. The consequence is a greater vertical spread between p_1 and p_2. In other words, while productive complementarity provides absolute benefits to both sides, it operates comparatively in favor of the wealthier side. Not only is there more income to fight over, but since the poorer side is now holding back somewhat from fighting, the wealthier side will find it relatively easier to obtain a larger share. Nevertheless, it remains true that $I_1/I_2 < R_1/R_2$. So, given that the mass effect parameter m (measuring the decisiveness of conflict) is set at $m = 1$, *the weak form of the paradox of power continues to apply*—even when a greater degree of complementarity is postulated.

Numerical Example 4: Under the conditions of Numerical Example 2, with initial resources $(R_1,R_2) = (200,100)$ and $A = m = s = 1$, the solution was in the range of interior equilibria, with $(F_1,F_2) = (75,75)$, $(p_1,p_2) = (0.5,0.5)$ and incomes $(I_1,I_2) = (75,75)$—i.e., the strong form of the POP applied. Holding all conditions the same except for an increase in s to 1.25, at the new equilibrium the fighting efforts become $(F_1,F_2) = (77.1,67.5)$. The proportionate shares change somewhat in favor of the better-endowed side to $(p_1,p_2) = (0.533,0.467)$ and similarly for the attained incomes $(I_1,I_2) = (94.9,83.1)$. Increased productive complementarity benefits both sides, but differentially favors the wealthier contender. Nevertheless, $I_1/I_2 = 1.42$ remains less than $R_1/R_2 = 2$: the weak form of the POP applies.

Thus, responding to the second numbered question in the introductory section, the numerical simulation suggests that a higher degree of productive complementarity (such as might result from increased international trade and greater integration of national economies over time) does tend to induce some net reorientation of resources away from fighting. Actually, the better-endowed

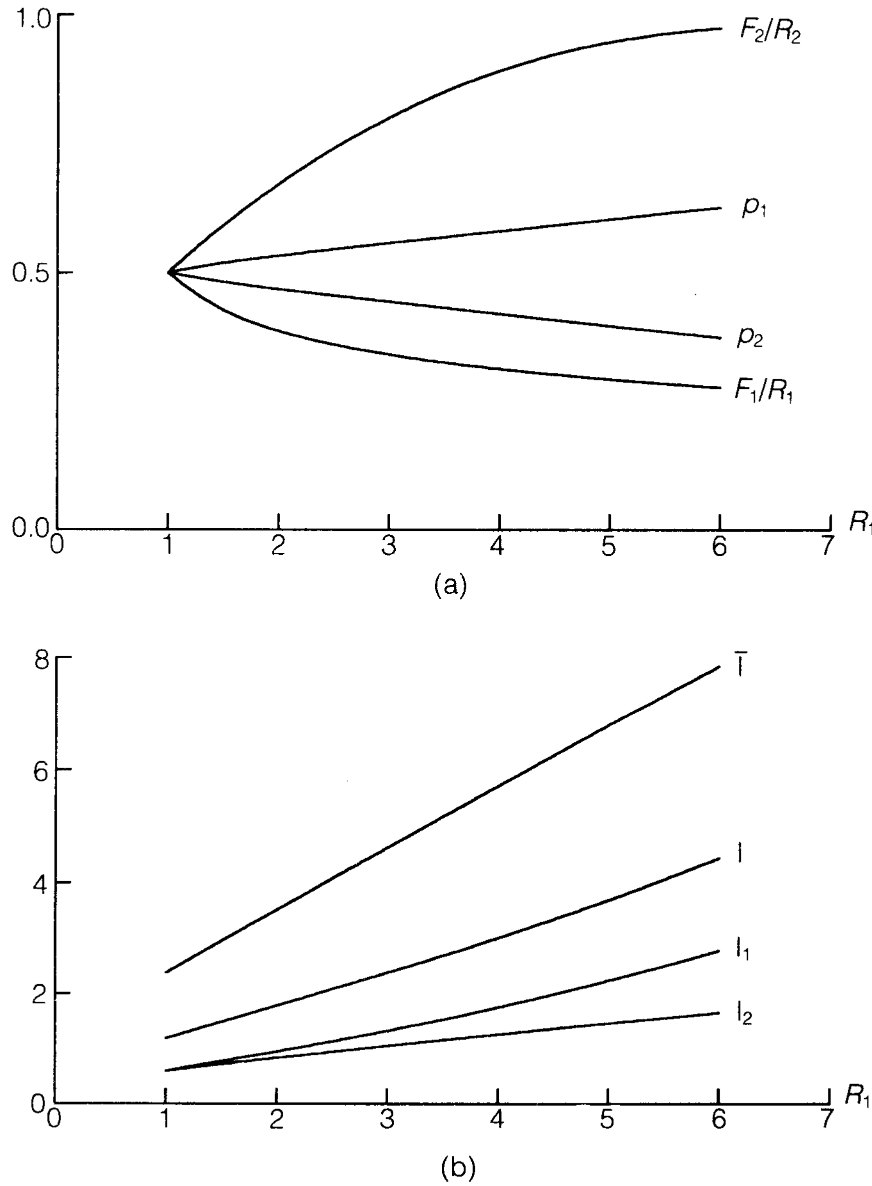

Figure 6. Proportionate allocations to fighting, fractional shares, and achieved incomes— $m = 1$, $s = 1.25$.

side fights somewhat more, but the poorer side's fighting effort falls by a larger amount. But owing to these countervailing influences, the overall effect is not as great as might be expected. And the increased value of the complementarity parameter confers a somewhat greater absolute and relative benefit upon the better-endowed side.

4. WHEN CONFLICT BECOMES MORE DECISIVE

This section considers the implications of improvements in conflict technology. Specifically, what happens when the mass effect parameter m in equation (3) rises? That is, when the "decisiveness" of conflict increases, meaning that any given preponderance of fighting effort F_1/F_2 will be reflected in a disproportionately larger ratio of the achieved success proportions $p_1/p_2 = I_1/I_2$.

In Figure 7, which corresponds to the limiting $s = 1$ case, the solid curves (for p_1 and p_2 in the upper panel, and I_1 and I_2 in the lower panel) are repeated from Figure 5 (based on $m = 1$) while the dashed curves show the solution values for a higher mass effect parameter ($m = 2$). Notably, as m increases *both* sides are motivated to invest more than before in fighting effort. Consequently, the poorer side #2 hits its boundary constraint $F_2 = R_2$ earlier; the range of interior solutions applies here only up to $\rho^* = 2$. Beyond this point, as the dashed curves in Figure 7a show, p_1 rises dramatically relative to p_2, meaning that the *improvement in fighting technology operates strongly in favor of the better-endowed side*. Correspondingly, Figure 7b shows that, despite the greater diversions from productive efforts on both sides, over a considerable range the better-endowed side does not only relatively but even absolutely better than before.

Numerical Example 5: In Numerical Example 3, with $s = 1$ and initial resources $(R_1,R_2) = (400,100)$ the solution was in the region of corner equilibria, with fighting efforts $(F_1,F_2) = (123.6,100)$, income shares $(p_1,p_2) = (0.553,0.447)$, and incomes $(I_1,I_2) = (152.8,123.6)$. When $m = 2$ so that fighting becomes more decisive, the less well-endowed side #2 can do nothing about it, being already at a corner optimum. So F_2 remains at 100, but side #1 can now advantageously increase its F_1. The new solution values are $(F_1,F_2) = (151.3,100)$, $(p_1,p_2) = (0.696,0.304)$, and $(I_1,I_2) = (173.1,75.6)$. So, in comparison with Numerical Example 3, the increase in the mass effect parameter has substantially reduced aggregate income I but considerably improved the outcome for the better-endowed side. Nevertheless, the weak form of the POP still holds here: $p_1/p_2 = I_1/I_2$ equals 2.288, considerably less than the resource ratio $R_1/R_2 = 4$.

A change in conflict technology that raises the mass effect parameter m, so as to make conflict more decisive than before, provides a widened opportunity for the better-endowed side. A richer contestant, evidently, will be more able

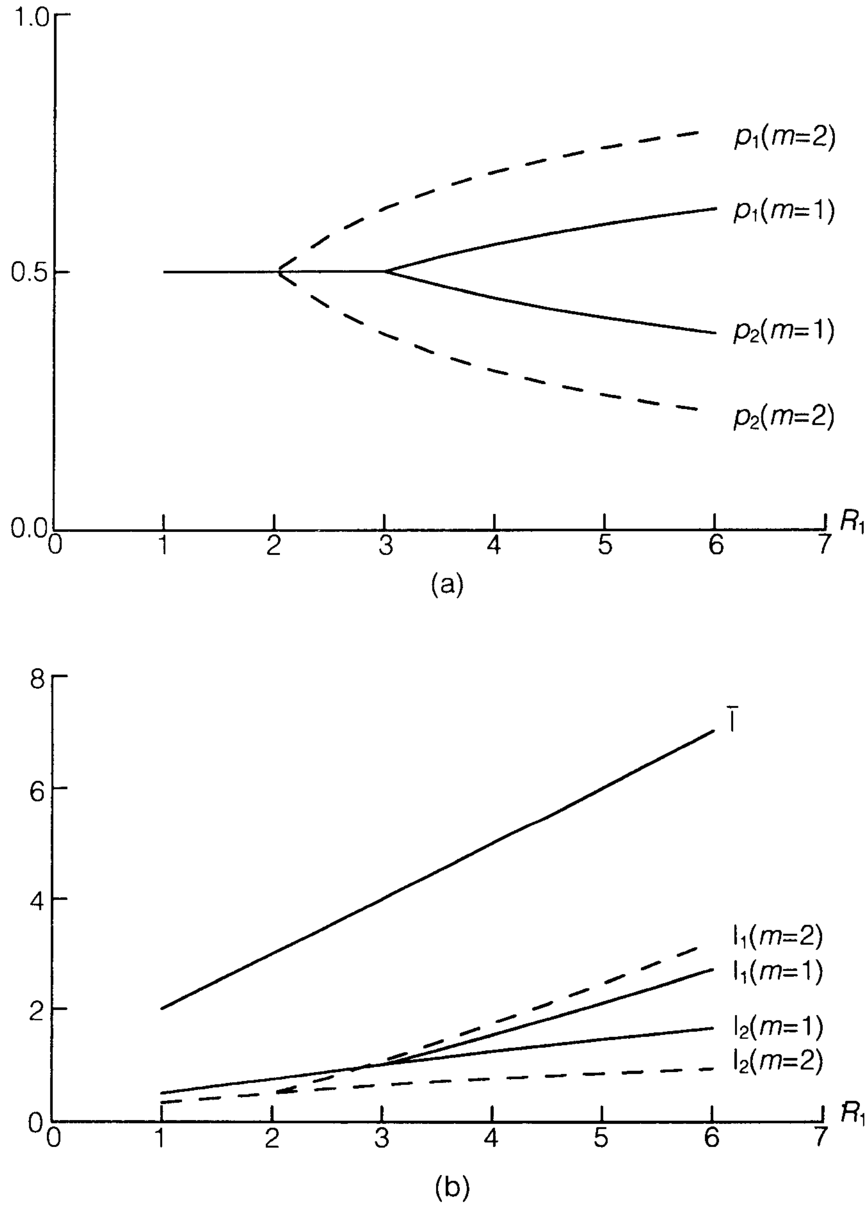

Figure 7. When fighting becomes more decisive: $m = 2$ versus $m = 1$ (for $s = 1$).

to afford the large investments in fighting effort needed to take advantage of the enhanced decisiveness of conflict. But, the question is, in equilibrium will a better-endowed contender actually find it optimal to utilize this opportunity to the extent of reversing the paradox of power? In other words, can greater

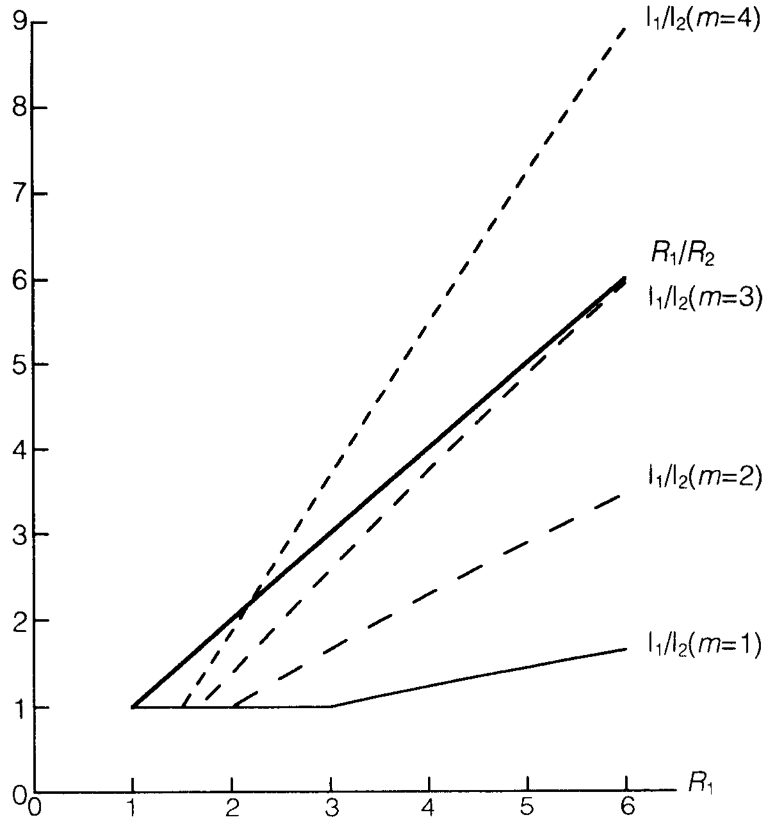

Figure 8. Income ratio versus resource ratio, varying m (for $s=1$).

decisiveness of conflict ever outweigh the previously emphasized proposition—that the poorer side does relatively better ($I_1/I_2 < R_1/R_2$) in mixed cooperative-conflictual situations, because it has more to gain and less to lose by fighting rather than producing?

In the numerical example just above, with $s=1$ and given the resource ratio $R_1/R_2=4$, an increase in the mass effect parameter from $m=1$ to $m=2$ was not sufficient to overcome the paradox of power. However, the more general picture provided in Figure 8 suggests a different result. With the complementarity index set at $s=1$, the diagram shows curves picturing the income ratios I_1/I_2 as a function of the resource ratio R_1/R_2, for levels of m ranging from $m=1$ to $m=4$. As can be seen, there is an initial range along each of the I_1/I_2 curves where $I_1/I_2=1$, that is, where the strong form of POP applies. Beyond the critical resource ratio $R_1/R_2=\rho^*$ for each such curve, there is a range of corner solutions where I_1/I_2 rises above unity.

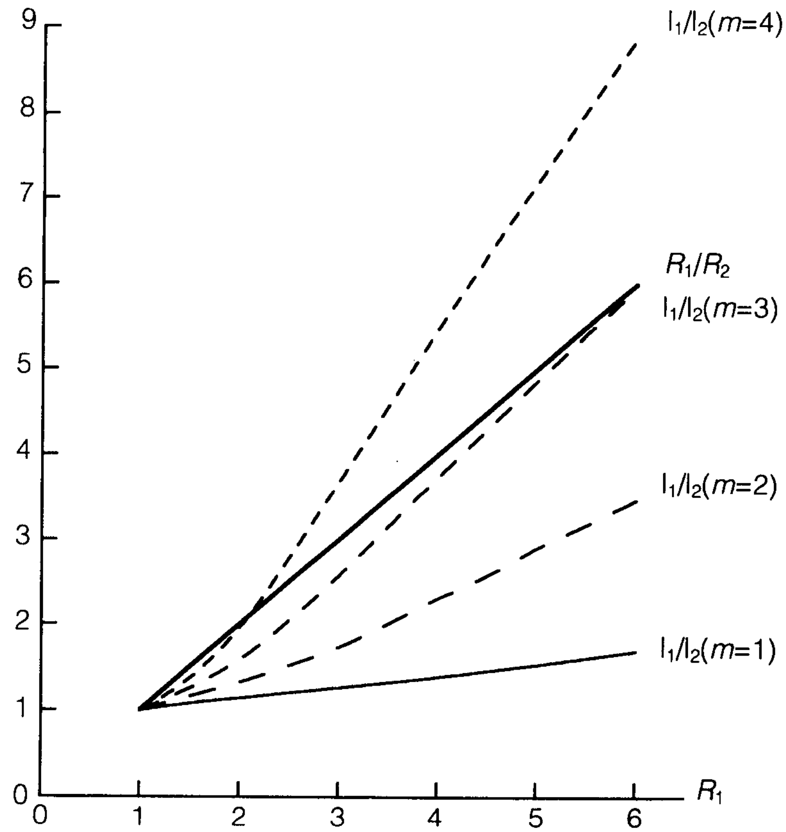

Figure 9. Income ratio versus resource ratio, varying m (for $s = 1.25$).

The picture suggests, and carrying out the simulations over a larger range of resource ratios confirms, that for $m = 1$ and $m = 2$ the POP will always hold, at least in its weak form. But for $m = 3$ it begins to look as if the POP might be violated for sufficiently large resource ratios, and the curve for $m = 4$ conclusively establishes that such a violation may well occur—that is, that I_1/I_2 may exceed R_1/R_2. Figure 9, the corresponding diagram for $s = 1.25$, reveals a similar picture, though without any initial range where $I_1/I_2 = 1$.

Numerical Example 6: Given $s = 1$ and resources $(R_1,R_2) = (400,100)$, in the previous Example we saw that, for $m = 2$, the income ratio I_1/I_2 equalled 2.288, less than the resource ratio $R_1/R_2 = 4$. Now setting $m = 4$, the corner equilibrium has fighting efforts $(F_1,F_2) = (152.9,100)$, success fractions $(p_1,p_2) = (0.845,0.145)$, and achieved incomes (208.9,38.2). So I_1/I_2 now equals 5.465, which *exceeds* the resource ratio $R_1/R_2 = 4$.

If the model of this paper had implied that the paradox of power (at least in its weak form) is *universally* applicable, it would have proved too much. For history reveals that although the less well-endowed side often does surprisingly well in conflict situations, nevertheless the strong and rich do often succeed in exploiting the weak and poor. What this paper does is to explain when we can expect to observe each type of outcome—sometimes the poor doing relatively better, sometimes the rich. The balance of relative advantage is the result of two countervailing factors. On the one hand, poorer contenders have a comparative advantage, so to speak, in fighting. That is, they are motivated to devote relatively more effort to conflictual as opposed to productive effort. But on the other hand, when the decisiveness of conflict is large (that is, when a preponderance of fighting effort has a disproportionate effect upon the outcome of conflict), the better-endowed side is better placed to take advantage of this fact.

What are the circumstances that make fighting more or less decisive? According to Lanchester [1916], under "ancient" conditions fighting was man-for-man, so the larger army could not bring all its forces to bear—hence military strength was linear in the forces committed ($m = 1$). Under "modern" conditions, long-range weapons allow concentration of fire, making strength proportional to the square of the forces engaged ($m = 2$). Turning away from military to political contests within nations, in democratic politics a high m corresponds to "majority tyranny." Put the other way, winning a majority is less *decisive* to the extent that checks and balances, civil rights, and other constitutional provisions limit a dominant faction's ability to use the machinery of government for oppression of minorities. Thus, requiring super-majority approval for crucial actions like amending the U.S. constitution limits the decisiveness of ordinary majority control. The consequence, very likely intended by the Founding Fathers, is that all factions have a reduced incentive to escalate the domestic political struggle.

6. DISCUSSION AND SUMMARY

The analysis here dealt with struggles over a common pool of income. The parties' opposed interests as to division of the pool lead them to engage in conflictual activity, but they are also motivated to engage in productive activity with the aim of making the social total of income as large as possible. The chosen levels of these two activities determine the outcome of a steady-state process in which contenders divide their efforts between productive activity and appropriate struggle.

Management and labor, for example, ordinarily cooperate in production while struggling over the factor shares. The same applies for contests taking place within families or clubs: individual members may want to do as well as possible for themselves, but still must take care not to totally subvert the common goals. We can think of such struggles as conducted under constitutional rules limiting the stakes at issue. Specifically, it was assumed here that the resource bases of the competitors are invulnerable: only *income* is in contention. Thus, within the firm, workers do not ordinarily try to seize the factory or machinery, nor does management aim at enslaving the employees (although under abnormal or revolutionary conditions, such escalation has been known to occur).

Conflicts among tribes or nations are perhaps more likely to take the form of intensified struggles for resources. Nevertheless, nations and tribes still have a common interest in furthering trade and avoiding destructive warfare. Domestic politics of the familiar type fit the model rather well: taxation may be used to redistribute income, but there are constitutional protections against massive seizures of property. (Once again, however, revolutionary circumstances sometimes occur in which the redistribution game is played for higher stakes.)

Substantively, the main theme of the analysis concerned the *paradox of power*—the observation that, in many though not all conflictual contexts, the relatively less well-endowed side improves its position compared with its better-endowed rival. As a leading example, in modern political redistributive struggles the rich end up transferring income to the poor. The underlying explanation derives from a comparison of the marginal payoffs of productive versus conflictual activities, which reveals that the less well-endowed side has a comparative advantage in fighting, the richer side in producing. Appropriative effort allows you to place a tax upon your opponent's productive effort, and it is more profitable to tax a rich opponent than a poor one. When the paradox of power is applicable, the conflict process, while dissipating income in aggregate, also tends to bring about a more equal distribution of whatever income remains.

However, it would be absurd to claim that the paradox of power holds universally—that, in mixed conflictual-cooperative interactions poorer contenders always gain relative to their richer opponents. In war and politics both, sometimes the rich do get richer and the poor poorer. The analysis here revealed that the comparative advantage of the poor in conflictual processes can be overcome when the *decisiveness* of conflict is sufficiently great, that is, when a given ratio of fighting efforts is very disproportionately effective in determining the outcome of conflict. High decisiveness operates to the advantage of better-endowed parties, who are always able to invest more heavily in fighting should it be profitable to do so. Over past centuries military technology, it appears, has tended in the direction of greater decisiveness, which roughly translates into an advantage of the offense over the defense. This factor helps explain the extinction of so many smaller states in the modern era. When it comes to the more limited contests that take place within nation-states (class struggles) or firms (labor-management conflicts) or families (sibling and generational rivalries), no such tendency is evident and the paradox of power does seem mainly to hold.

The paradox of power aside, among the other results of interest are:

1. Generalized progress in productive technology, though of benefit to both sides, has essentially no effect upon the parties' *proportionate* allocations of resources between conflict and production. Fighting activity and productive activity tend to increase more or less in parallel.

2. Greater productive complementarity between the parties (as might result from increased international trade) does tend to induce some shift of resources away from conflictual activity, but the effect need not be large. In addition, the better-endowed side reaps a disproportionate share of the benefit.
3. Apart from their tendency to favor the better-endowed side, improvements in the technology of conflict cause all contenders to tilt in the direction of greater conflictual activity. Since the increased fighting efforts largely cancel one another out, the net effect is typically a substantial social loss.

As in all attempts to model complex phenomena, a variety of simplifying assumptions were employed here. To comment briefly on a few of these: (i) Only two-party interactions were examined, ruling out issues like alliances and the balance of power.[17] (ii) Full information was assumed throughout, so that factors like deception have been set aside.[18] (iii) The simplified mathematical form of the Contest Success Function does not allow for differences between offensive and defensive weapons, between ground and naval forces, between battle-seeking and Fabian tactics, and so on.[19] (iv) The steady-state assumption rules out issues involving timing, such as arms races, economic growth, or (on a smaller time-scale) signalling resolve through successive escalation.[20] (v) In the model here, all income falls into a common pool available for capture. More generally, each side might have some income secure from capture, and in fact would be making an optimizing choice between resources devoted in that way versus resources generating income in a common pool. (vi) The underlying *resources* on each side were assumed invulnerable to seizure. (vii) Apart from opportunity costs in the form of foregone production, fighting was assumed non-destructive. (vii) The effects of distance and other geographical factors were not considered. (viii) The Nash-Cournot solution concept was employed; no allowance was made for Stackelberg leadership or for the use of threats and promises.

Even when generalized in the various ways suggested by the list above, the steady-state model will remain inappropriate for the analysis of conflicts dominated by single overwhelming or irreversible events like a Pearl Harbor attack. In the military domain it is more applicable to protracted cold wars or to continuing low-level combats like those between city-dwellers and nomads in early times, or among the small states of pre-imperial China (as described in Sun Tze's *The Art of War*). Outside the military domain, it is particularly relevant for analyzing the ongoing cooperative-conflictual processes we observe in capital-labor relations, in politics, and within families.

[17] There is of course a vast literature on these questions. I shall cite here only Blainey (1973) and Bernholz (1985).

[18] On this see, for example, Tullock (1974, Ch. 10) and Brams (1977).

[19] See footnote 11 above.

[20] See, for example, Intriligator [1975], Wolfson [1985], and Garfinkel [1990].

I will expand briefly on only one application, political redistribution of income. In modern politics, at least, redistribution is overwhelmingly from the rich to the poor. This might seem surprising. After all, starting from their initial resource advantage the rich could, it appears, make themselves richer still by appropriating what others have produced. And on the other hand, if a presently poor group is powerful enough to achieve redistribution in its own favor, why stop at equality when it can go on further and expropriate the rich?

Several explanations have been put forward, among them: (1) altruism on the one side and envy on the other imply essentially unanimous support for equalization, but not for reversal of the positions of rich and poor; (2) the deadweight costs of the transfer process set limits upon how far any beneficiary group can advantageously push for redistribution.[21] The alternative explanation offered here is the paradox of power. Since the marginal payoff of conflictual effort (i.e., of redistributive political activity) tends to be higher for the less well-endowed side, *popularist politics are profitable for the poor.*

As a further implication, any exogenous change in relative wealths will motivate newly deprived groups to shift their energies toward redistributive politics. When tuition fees rise, college students will organize demonstrations; if disaster strikes, impacted areas demand assistance from government; when the business cycle trends downward, industries seek tariff protection; and in times of low agricultural prices, Kansas farmers "raise less corn and raise more hell." Since the newly enriched groups will be shifting their energies more toward production, the political equilibrium changes. So government will lean against the wind[22] to moderate the impact of *losses*, even when suffered by individuals who are by no means *poor*. After a Bel-Air fire, affluent householders are made eligible for Federal disaster relief; when car sales fall, wealthy shareholders of Ford and GM are permitted to share the benefits of import quotas upon Japanese autos.

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[21] This is the factor playing the key role in Becker [1983].
[22] Hirshleifer [1976].

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