

An Experimental Evaluation of Weakest Link/Best Shot Models of Public Goods

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In the supply of public goods, far less free-riding actually occurs than traditional theory predicts. As one explanation, the social composition function (SCF), which aggregates individual contributions into an available social total, may not always take the standard summation form. Theoretical considerations indicate that free-riding should be least for an SCF of the weakest-link type but greatest for the best-shot type. Using a sequential protocol, our experiments strongly confirmed theoretical anticipations under all three types of SCF. Even under the more onerous sealed-bid (simultaneous play) protocol, the experimental subjects were able to make some partial progress toward the theoretical ideal.

I. Introduction

Standard economic theory predicts that self-interested agents will undersupply public goods in comparison with efficient social totals. This inference derives from modeling the provision of public goods as a single-period Prisoners' Dilemma in a continuous-strategy space.

We are grateful to the Foundation for Research in Economics and Education, the Social Sciences and Humanities Research Council of Canada, and the UCLA Center for Managerial Economics and Public Policy for financial assistance. Helpful comments have been received from Joel Guttman, Stan Reynolds, E. E. Rutström, and Lynne Zucker, and extremely valuable suggestions were provided by two referees.

[*Journal of Political Economy*, 1989, vol. 97, no. 1]
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However, people in both natural and designed experimental settings appear to solve the underprovision problem to a surprising degree, so that far less "free-riding" occurs than anticipated. As an important historical example, it has been observed that disasters tend to elicit an extraordinary degree of seemingly unselfish behavior.¹ Outsiders rush in to contribute relief to the stricken community, while community members themselves pitch in to rescue neighbors in peril and to engage in other forms of voluntary mutual aid.² In experimental settings as well, the evidence is at least mixed with regard to the observed extent of free-riding behavior.³

While humanly gratifying, these results are somewhat disturbing for standard theory. Among the possible explanations are that (i) individuals may not be as selfish, or as rational, as standard theory assumes; (ii) public-goods provision often corresponds not to a single-period but to a repeated-interaction game, for which free-riding is not uniquely predicted by the theory; and (iii) even in a single-period situation, public-goods provision may not always be a Prisoners' Dilemma. In this study we will be making use of laboratory conditions that render the repeated-game explanation inapplicable. But employing a generalization of the theory of public goods along the lines suggested in Hirshleifer (1983), we will show that single-period public-goods provision is indeed not always a Prisoners' Dilemma. With due attention paid to the alternative decision situations involved—so that underprovision is predicted in some cases, but not in others—our results (subject to certain important qualifications) tend to confirm theoretical expectations under the maintained assumption of rational self-interested behavior.

In predicting the decisions of rational self-interested parties as to the voluntary provision of public goods, it is essential to attend to two different aspects of the game setting: the social composition function (SCF) and the protocol of play. The SCF refers to the way in which individual contributions are amalgamated into an available social aggregate of the public good. The protocol corresponds to the exten-

¹ Even if we assume a degree of benevolence, so that improvements to the well-being of disaster victims enter positively into other people's utility functions, there is a free-rider problem. Each potential donor is motivated to contribute less, in the expectation that others will also be providing the desired "good"—assistance to those in need.

² This "disaster syndrome" has been described and analyzed by a number of economists including Hirshleifer (1963), Dacy and Kunreuther (1969), and De Alessi (1975).

³ The experimental evidence ranges from Chamberlin (1978), Marwell and Ames (1979, 1980, 1981), and Schneider and Pommerehne (1981), whose results tend to refute versions of the free-rider hypothesis, to Dawes (1980), Isaac, McCue, and Plott (1985), and Dawes et al. (1986), who tend to confirm it. Isaac, J. Walker, and Thomas (1984) and Kim and M. Walker (1984) have explored various combinations of design features that might explain these seemingly divergent results.

sive form of the game, which specifies (among other things) the sequence of moves and the informational conditions under which play takes place. In designing our experiments we have been careful to specify both the SCF and the protocol so that comparisons could accurately be drawn between what theory predicts and what experimental subjects actually do.

Alternative Social Composition Functions

Public goods are defined in terms of a peculiar feature on the demand side: the amount produced is equally available for nonrivalrous consumption by all members of the community. But our standard models do not assume anything special on the supply side, specifically about the SCF, which converts individual provisions into a socially available aggregate amount of the public good. On the standard assumption, this aggregate is always the simple sum of the individual contributions.

But the observation that people often overcome the free-rider problem in disaster situations provides a clue that the standard assumption about the form of the SCF may not be applicable. Such situations often correspond to “weakest-link” environments, which are characterized by the fact that failure of any unit may be fatal to the whole. In desperate circumstances in which each person must do his or her duty (and even more) if the community is to survive, what appears to be self-sacrificing behavior may actually be selfishly optimal in swinging the balance between community viability and social collapse.

Table 1 displays the three simple social composition functions—relations between the individual contributions q_i and the socially available total Q of the public good—that our experiments cover. The SCF implicit in the usual theory, which of course leads to the usual predic-

TABLE 1
ALTERNATIVE SOCIAL COMPOSITION FUNCTIONS

Social Composition Function	Formula	Predicted Extent of Free-Riding
Standard Summation	$Q = \sum_i q_i$	Intermediate
Weakest Link	$Q = \min_i q_i$	Least
Best Shot	$Q = \max_i q_i$	Most

tion of free-riding, is called Standard Summation. But this is only one point along a spectrum of possibilities. At one extreme of the spectrum is Weakest Link, in which the available aggregate is the *minimum* of the individual provisions. Here, as suggested above, the generalized theory predicts that free-riding on the part of rational self-interested individuals will fall to low levels. (In fact, it should disappear entirely if the population is homogeneous.) At the other extreme in table 1 is Best Shot, in which the socially available amount is the *maximum* of the individual provisions. Under Best Shot, as will be shown, free-riding is predicted to be even more predominant and intractable than under Standard Summation.⁴

The Weakest Link model describes a variety of situations in which each member of a social group has a kind of veto power over the extent of collective achievement. Examples might include (i) townspeople manning sectors of a levee when the river is in flood (where any person's failure means that the water will break through and inundate the entire community), (ii) military units defending segments of the front against an enemy offensive, or (iii) a group of agents responsible for dredging successive stretches of a navigation channel (since the minimum depth dredged determines how much traffic can flow). All these cases are parallel to the disaster situation. Once the usual protections and redundancies supporting the social division of labor can no longer be relied on, breakdown is in prospect unless everyone cooperates. At the opposite extreme, the Best Shot model applies when different teams engage in a contest in which "victory" benefits the entire team as a public good, while the scoring rule depends solely on the best individual performance. Examples might include (i) antimissile batteries firing under local control at a single incoming intercontinental ballistic missile, (ii) gang wars in which each "family's" gunmen aim solely to assassinate the rival don, or (iii) the mice in the fable attempting to bell the cat.⁵

Of course, many intermediate and variant SCFs are also possible. Table 1 could be generalized to place arbitrary ascending or descending weights on individual contributions (Hirshleifer 1984). Or the

⁴ A number of other studies have dealt, implicitly at least, with social composition functions bearing a family resemblance to Weakest Link or Best Shot. Mueller (1979, pp. 13–14) uses the term "jointness of supply" to describe a weakest-link type of situation, in which a public good will not be provided at all unless everyone contributes, and correctly indicates that free-riding should then be minimal. Bliss and Nalebuff (1984) discuss "dragon slaying," which is a best-shot type of public good. Lipnowski and Maital (1983) also discuss a best-shot situation that is even closer to the way in which we operationalize it (viz., in terms of the treatment of contributions by agents who do not provide the "best" shot).

⁵ This is essentially the same as the "dragon slaying" problem in Bliss and Nalebuff (1984).

SCF could be made to depend on various parameters—for example, the mean, the median, or the average of the upper and lower quartiles—of the distribution of individual contributions. Or under either Weakest Link or Best Shot, there might be various provisions for “refunding” nondecisive contributions (see Bohm 1972, 1984; Van de Kragt, Orbell, and Dawes 1983; Palfrey and Rosenthal 1984, 1985). But we limit ourselves here only to the simplest polar alternatives to the Standard Summation composition function.

Alternative Protocols

A number of imaginative protocols have been designed by theorists and experimentalists to mitigate the free-riding problem in Standard Summation environments. Influential examples include Bohm (1972, 1984), Green and Laffont (1977), Groves and Ledyard (1977), Smith (1977, 1979a, 1979b, 1980), Ferejohn, Forsythe, and Noll (1979), and Ferejohn et al. (1982). These protocols demonstrate the important normative possibility of designing decentralized mechanisms that will induce optimal or near-optimal public-good provision levels.

Our experiments involved only two-person groups. We employed two basic protocols: sequential and sealed-bid. The sealed-bid protocol is a simultaneous-move arrangement, following the direct contribution procedure of Smith (1979a), Isaac et al. (1985), and Banks, Plott, and Porter (1986). Here in each two-person group the agents privately and concurrently specify their individual levels of provision for the public good. In contrast, the sequential protocol is an alternating-move arrangement. In this case one agent in each group declares his or her provision level first, that decision is then made public, and then the other makes a choice in response. As we shall see, the protocol employed importantly affects both the theoretical predictions and the behavioral outcomes.

II. Experimental Design

A summary of our experiments appears in table 2. All subjects were economics undergraduates at the University of Western Ontario.⁶ In the sequential protocol session there were 18 subjects segregated randomly into nine groups of two. Three of these groups were then randomly assigned to each of the social composition functions in each period. In the sealed-bid protocol session we had 26 subjects in 13 groups of two. The two sessions were run back to back, with subjects

⁶ The dollar payoffs to the subjects were adjusted using the exchange rate prevailing at the time of the experiments (C\$1.32 to US\$1.00).

TABLE 2
SUMMARY OF EXPERIMENTS

Experiment	Protocol	Social Composition Function	Periods	Replications per Period
SQ-1	Sequential	Standard Summation	6	3
SQ-2	Sequential	Weakest Link	6	3
SQ-3	Sequential	Best Shot	6	3
SB-1	Sealed bid	Standard Summation	10	5
SB-2	Sealed bid	Weakest Link	10	5
SB-3	Sealed bid	Best Shot	10	3

from a larger pool of volunteers randomly assigned to a particular session.

Nobody ever knew the identity of his or her partner within the larger population of subjects. Furthermore, the partners were changed each period. This feature was designed to prevent possible extraneous influences, such as the desire to make friends or build reputations, from contaminating the experiment. All subjects were given the same fixed valuation schedule for the public good, valid for each experimental period, as shown in table 3 and pictured in figure 1.

TABLE 3
REDEMPTION VALUE SHEET

Project Level (Units)	Redemption Value of Specific Units	Total Redemption Value of All Units
1	\$1.00	\$1.00
2	0.95	1.95
3	0.90	2.85
4	0.85	3.70
5	0.80	4.50
6	0.75	5.25
7	0.70	5.95
8	0.65	6.60
9	0.60	7.20
10	0.55	7.75
11	0.50	8.25
12	0.45	8.70
13	0.40	9.10
14	0.35	9.45
15	0.30	9.75
16	0.25	10.00
17	0.20	10.20
18	0.15	10.35
19	0.10	10.45
20	0.05	10.50
21	0.00	10.50

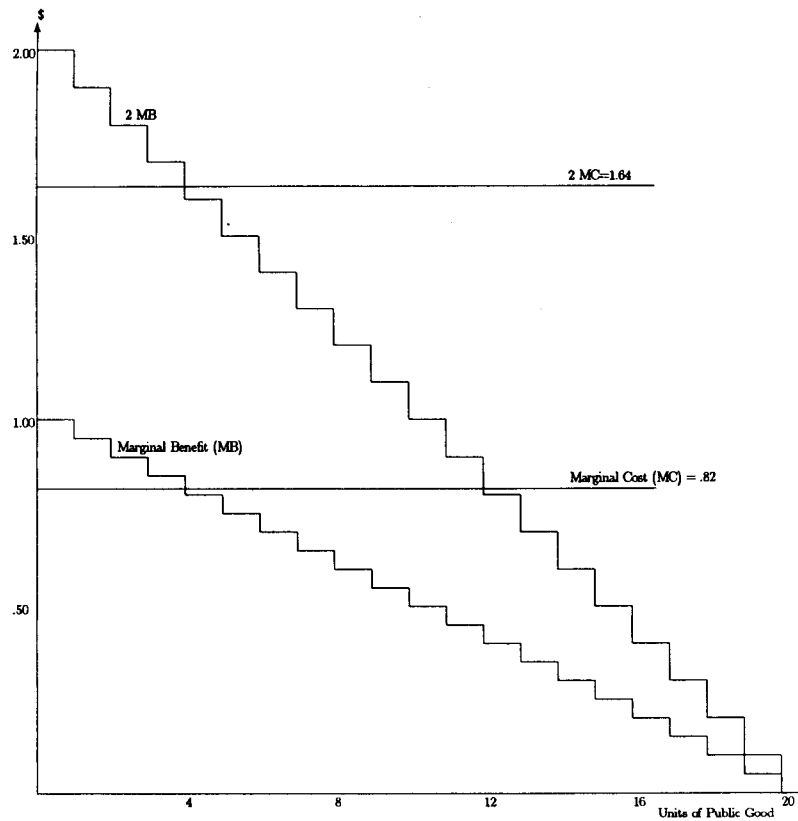


FIG. 1.—Marginal benefit and marginal cost schedules

No subject was informed of the payoffs of any other subject in our experiments, and in particular the fact that all valuation schedules were the same was not revealed. Our theoretical analysis, in contrast, presumes that the payoffs are common public knowledge. This informational discrepancy made our experiments quite a severe test of the underlying theory. We were in effect “predicting” on the basis of a compound hypothesis that (i) each subject was a rational, self-interested economic agent and believed that his partner was also, and (ii) the subjects correctly conjectured that their payoffs were identical. Owing to the possibility of learning, we would expect condition ii to describe the situation more accurately in later than in earlier experimental replications, and this in fact occurred (as will be explained in more detail below). However, we should point out that learning was far from trivially easy since subjects were told (as in fact occurred) that the partnership assignments were to be reshuffled each period. Thus

there was no chance of mutual education and accommodation between any given pair.⁷

Under each of the two different experimental protocols, a trio of experiments was conducted, one member of the trio corresponding to each of the social composition functions of table 1.

In the sequential protocol (experiments SQ-1, SQ-2, and SQ-3), one subject in each pair was randomly selected to have the first move. The first mover was required to declare his or her own irrevocable contribution to the pair's joint provision of the public good. The other member of the pair could then use that information in choosing a best response, in the form of a second-move decision on how much to contribute in turn. Although the last mover has an informational advantage over the first mover, he is also at a severe disadvantage with respect to his ability to commit to a certain strategy. We will see that the commitment asymmetry dominates any informational asymmetry.

In the second trio of experiments under the sealed-bid protocol (experiments SB-1, SB-2, and SB-3), each subject had to select a level of voluntary contribution in ignorance of the simultaneous choice being made by his or her partner. Here it might be thought that the poorer information (about the other side's moves) available to the pair as a group would make it more difficult for the subjects to arrive at their self-interested optimal choices. This inference was in fact supported, to a marked extent, by our experimental results.

All subjects in the sealed-bid experiments SB-1, SB-2, and SB-3 were made familiar with the instructions given in the Appendix.

The redemption value sheet referred to in the instructions is shown in table 3 and was common to each participant in all the experiments. Our instructions closely follow those used by Isaac et al. (1985, pp. 70–73) except for the references to alternative rules and the shuffling of subjects from group to group. For the sequential experiments SQ-1, SQ-2, and SQ-3, minor modifications were introduced in accordance with the changed protocol.

III. Efficient Outcomes under Alternative Social Composition Functions

To clarify the nature of the social composition functions and to illustrate the basis for our theoretical predictions of the outcomes, the three pairs of matrices shown in table 4 represent simplified versions of the decision processes involved. In this illustration (but *not* in the

⁷ We have subsequently conducted a "complete information" series of experiments, the results being virtually identical to those reported here (and somewhat closer to the Best Shot predictions).

TABLE 4
SIMPLIFIED ILLUSTRATION OF THE THREE SOCIAL COMPOSITION FUNCTIONS

Weakest Link:					
Algebraic			Numerical ($b = 2, c = 1$)		
	P	N		P	N
P	$b - c, b - c$	$-c, 0$	P	1, 1	-1, 0
N	0, $-c$	0, 0	N	0, -1	0, 0
Best Shot:					
Algebraic			Numerical ($b = 2, c = 1$)		
	P	N		P	N
P	$b - c, b - c$	$b - c, b$	P	1, 1	1, 2
N	$b, b - c$	0, 0	N	2, 1	0, 0
Standard Summation:					
Algebraic			Numerical ($B = 4, b = 2, c = 3$)		
	P	N		P	N
P	$B - c, B - c$	$b - c, b$	P	1, 1	-1, 2
N	$b, b - c$	0, 0	N	2, -1	0, 0

actual experiments), cooperation, in the form of a decision to contribute to the provision of the public good, is simply a yes or no affair: the individual is either a provider (P) or a nonprovider (N).

In the Weakest Link case, the matrix on the left shows the respective payoffs to the four possible combinations of P and N strategies, where b is the benefit received by each player should *both* contribute, and c is the cost to *either* of contributing. If the Weakest Link model is to apply, it is necessary that $b > c$. The matrix on the right is a numerical illustration for $b = 2$ and $c = 1$.

For the Best Shot case, b is the benefit received by each player should *either* contribute, while c remains the cost to either of contributing. Again, a necessary condition is $b > c$. As before, the matrix on the right is a numerical illustration assuming $b = 2$ and $c = 1$.

In the Standard Summation case, matters are somewhat more complex since we must now distinguish two possible levels of benefits obtained. Specifically, let B signify the benefit to each player when *both* contribute, and b the benefit to each when *only one* contributes. The necessary conditions here can be expressed as $B > c > b$, but $B - c < b$. The numerical illustration on the right assumes $B = 4, b = 2$, and $c = 3$. As is well known, this Standard Summation situation for the private provision of public goods is a Prisoners' Dilemma.

The *efficient* levels of provision of the public good are easily visualized in the simplified numerical illustrations of table 4. For Weakest Link the maximum joint payoff is $1 + 1 = 2$, achieved if the parties adopt the strategy pair $[P, P]$. Under the social composition function

TABLE 5
EFFICIENT OUTCOMES UNDER EXPERIMENTAL CONDITIONS

SOCIAL COMPOSITION FUNCTION	EFFICIENCY CONDITION	NUMERICAL		
		q_1	q_2	Q
Standard Summation	$MC_1 = MC_2 = MB_1 + MB_2$	6	6	12
Weakest Link	$MC_1 + MC_2 = MB_1 + MB_2$	4	4	4
Best Shot	$MC_1 = MB_1 + MB_2$ and $q_2 = 0$	12	0	12
	or $MC_2 = MB_1 + MB_2$ and $q_1 = 0$	0	12	12

represented by Weakest Link, since only one unit of the public good becomes available when each party chooses P , the efficient quantity of public good provided is one. For Best Shot the maximum joint payoff of $2 + 1 = 3$ is achieved at either of the off-diagonal cells. Here one player chooses P and the other N , which under Best Shot suffices to generate one unit of the public good. For Standard Summation, finally, the maximum joint payoff is $1 + 1 = 2$, achieved if the parties both choose P . Under this social composition function, the efficient quantity of the public good is two.

Let us turn from these simplified illustrations to the actual experimental situation, with benefits and costs as pictured in figure 1. The efficiency conditions and corresponding numerical results are shown in table 5. Notice that the *efficient* outcomes depend only on the social composition functions, and not at all on the protocols. It is also worth noting that under Weakest Link the Pareto-optimal allocation is unique. Under the two other SCFs, however, there are multiple Pareto-optimal allocations (under our assumption that the players' marginal cost functions are level and identical).

One can interpret table 5 as follows. Under Standard Summation, each individual's contribution goes toward purchasing units of the public good to be enjoyed by both. As shown in figure 1, each individual can always purchase a unit of the public good at a constant individual marginal cost (MC) equal to \$0.82. The individual marginal benefit (MB) schedule shown in the diagram corresponds, of course, to the benefits tabulated in table 3 as "Redemption Value of Specific Units." Since the social marginal benefit is simply twice the individual marginal benefit, inspection of the diagram reveals that the efficiency condition under Standard Summation, to wit, $MC_1 = MC_2 = MB_1 + MB_2$ (where the subscripts identify members of the participating pairs), is met when each individual provides six units of the public good. Thus $q_1 = q_2 = 6$, so that the efficient aggregate quantity is $Q = 12$.

Under Weakest Link, *both* members must contribute if a unit of the

public good is to be generated that both can enjoy. Here the efficiency condition is $MC_1 + MC_2 = MB_1 + MB_2$, which is met when $q_1 = q_2 = 4$. Since under Weakest Link the social amount provided is the *lesser* of q_1 and q_2 , the efficient social aggregate is $Q = 4$. Finally, under Best Shot, a unit of the public good is provided when *either* contributes. For efficiency here one party should contribute zero while the other should set his or her $MC_i = MB_1 + MB_2$. Numerically, the member contributing should provide $q_i = 12$ so that the social amount generated, the *greater* of q_1 and q_2 , is $Q = 12$.

IV. Predicted versus Actual Outcomes

We now come to the crucial point, comparing the experimental outcomes with those predicted under the assumption of rational self-interested behavior.

The Sequential Experiments

In these experiments the second mover, knowing his own benefit and cost schedule and having seen his partner's prior choice, could in principle always calculate his privately optimal contribution toward purchase of the public good. However, the rational choice for the first mover would depend on his partner's anticipated response. As already indicated, he does not know his partner's payoffs, nor can he be sure that the latter is a rational, self-interested player. But our prediction is that the first mover will correctly conjecture that the payoffs are identical and will presume that the partner will choose in accordance with rational self-interest when it is his turn to play. In game theory terms, this mutual rationality condition means that we are making use of the "subgame perfect equilibrium" concept due to Selten (1975).

On these assumptions, our predicted outcomes and profits for the sequential group of experiments can be read from table 6 and are pictured in figure 2. From figure 1, an individual's profit π_i is the sum of his marginal benefits for the number of units socially provided by both partners together, *less* the cost of whatever units he provides himself. The subscripts 1 and 2 here identify the first mover and second mover of each trial pair.

Under the Standard Summation social composition function, the predicted rational choice on the part of the first mover is to contribute nothing (i.e., to choose $q_1 = 0$). Should he do so, the second mover is then forced in his own self-interest to provide $q_2 = 4$ units, making the social aggregate also $Q = 4$. It would be foolish for the first mover to "generously" choose any positive q_1 . If, for example, the first mover

TABLE 6
 PREDICTED AND ACTUAL OUTCOMES: SEQUENTIAL EXPERIMENTS

SOCIAL COMPOSITION FUNCTION	PROVISION				PROFIT				Percentage of Efficient II
	q_1	q_2	Q	Percentage of Efficient Q	π_1	π_2	Π	Percentage of Efficient I	
Standard Summation (SQ-1):									
Efficient	I*	I	12	100	I	I	7.56	100	
Predicted	0	4	4	33.3	3.70	.42	4.12	54.5	
Observed	.436	3.778	4.204	35.0	3.493	.744	4.237	56.0	
Weakest Link (SQ-2):									
Efficient	4	4	4	100	.42	.42	.84	100	
Predicted	4	4	4	100	.42	.42	.84	100	
Observed	3.938	3.889	3.889	97.2	.377	.417	.794	94.5	
Best Shot (SQ-3):									
Efficient	12	0	12	100	8.70	-1.24	7.56	100	
Predicted	0	12	4	33.3	-1.24	8.70	4.12	54.5	
Observed	.629	3.501	4.062	33.8	3.227	.873	4.100	54.2	

* I stands for "indeterminate."

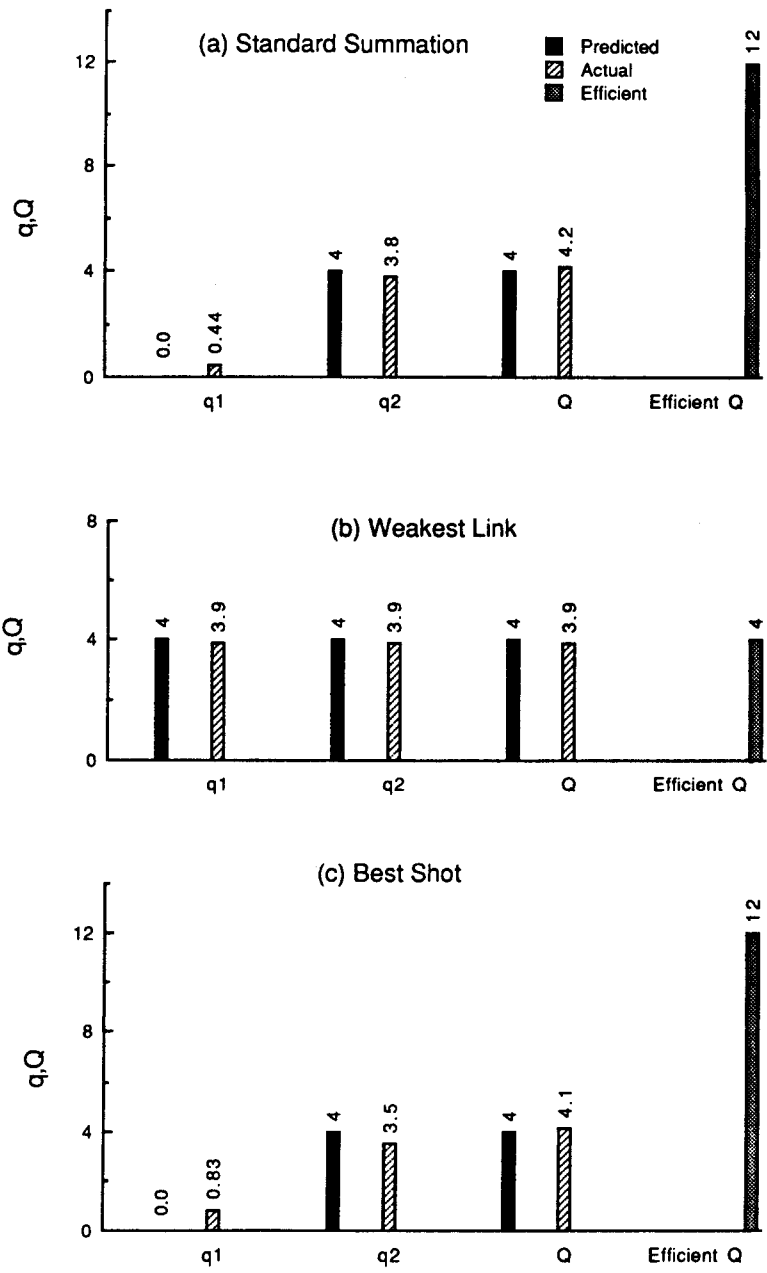


FIG. 2.—Outcomes of sequential experiments

set $q_1 = 1$, then a self-interested second mover would rationally respond by cutting his own provision back to $q_2 = 3$, leaving the total $Q = 4$ as before.⁸

Under the Weakest Link social composition function, the public good will be provided only to the extent that *both* contribute. The first mover, therefore, can be confident that a rational partner would exactly match his contribution, up to $q_1 = 4$. Our consequent prediction is $q_1 = q_2 = 4$, which means that the available social total is $Q = 4$ as well. Finally, Best Shot is like Standard Summation in that a rational first mover will contribute nothing ($q_1 = 0$), realizing that his partner would once again be left holding the bag and forced in his own self-interest to set $q_2 = 4$. Since Q under Best Shot is the larger of q_1 and q_2 , the social aggregate is once again $Q = 4$.

The predicted payoffs for each subject vary with the SCF and with assignment to first-mover or second-mover status. They range from a high of \$3.70 (to the first mover in SQ-1 and SQ-3) to a low of \$0.42 (to the second mover in all three cases). We assume that our experimental rewards always dominate the subjective costs to agents of computing the optimal solution.

In summary, $Q = 4$ is the predicted social aggregate in all three cases. Under Weakest Link, this corresponds to efficient provision of the public good, but in the other two instances it is only one-third of the efficient amount. The predicted distributions of the *individual* contributions differ drastically over the three cases, as indicated in table 6. It is also of interest to notice that in the two cases in which there is an advantage of one player over another (Standard Summation and Best Shot) the benefit goes to the first mover, despite the informational asymmetry in favor of the second mover.

The "actual" figures reported in table 6 and shown in figure 2 are the experimental results averaged over six periods and three replications, or 18 trial pairs for each of the three social composition functions. As can be seen, the observed results square remarkably with the theoretical predictions. Furthermore, detailed inspection of the trial-by-trial data reveals that essentially all discrepancies that appear in the pooled averages were due to mistaken choices of subjects in their very first or second decision periods. These discrepancies almost always took the form of an "excessive" contribution by the first

⁸ This is of course a standard proposition in public-good theory. As a slight qualification, there will in general be some "wealth effect" owing to the fact that each party's contribution enriches the other, thus making each of them reciprocally willing to purchase somewhat more of the public good. No wealth effect is allowed for in the experimental marginal benefit schedule given to the subjects. It has been shown, however, that in the provision of public goods, any such wealth effect will essentially always be of negligible magnitude (McGuire 1974; Margolis 1982, pp. 19–21).

mover in the Standard Summation and Best Shot cases or a “deficient” contribution in the Weakest Link case. Thus the parties were able to learn rapidly, despite the informational handicap to rationally optimal choice making. Hence our predicted results were satisfied quite remarkably under what we regarded as a somewhat severe test.

One possibly puzzling aspect of the data is why, since under Best Shot the relation $Q = \max(q_1, q_2)$ applies on any given trial, the average observed social aggregate $Q = 4.062$ was not identical with the average observed $q_2 = 3.501$, which is the larger of the averaged q_1 and q_2 . The reason is that although (as predicted) under Best Shot the second mover's q_2 was in fact almost always larger than his partner's q_1 , there were a few instances in early decision periods in which q_1 was larger than q_2 . Thus the overall averaged Q ended up higher than the averaged q_2 . A corresponding discrepancy in the other direction could have occurred under Weakest Link, where $Q = \min(q_1, q_2)$ for any given trial. But in fact it never did; in the Weakest Link experiments the second mover's contribution never exceeded the first mover's, and so the average of the Q provided was the same as the averaged q_2 . Of course, given this informational situation, it would *never* be rational under Weakest Link for the second mover to exceed the first mover's contribution. This difference between the Best Shot and Weakest Link outcomes is therefore another subsidiary confirmation of our rationality prediction.

The Sealed-Bid Experiments

The informational obstacles to rational decision making, already rather severe under the sequential protocol, are considerably more onerous under the sealed-bid protocol. In the sequential experiments, one of the players (the second mover) could always make his or her decision with all relevant information in the open. Under sealed-bid, in contrast, *each* of the players had to choose in the dark as to his partner's move. Not only did this ignorance make the decision at any moment more difficult, but it also limited what could be learned from experience. So in this group of experiments we anticipated a considerably less perfect match between theoretical and actual results. (Indeed, as will be shown shortly, the theoretical “predictions” themselves become somewhat problematic.) Because of the greater anticipated variability of results, in this group of experiments we generally allowed for more periods of learning and more replications as indicated in table 2. Table 7 and figure 3 summarize the predictions and actual observations under the sealed-bid protocol.

In the sealed-bid experiments we employed the Nash equilibrium solution concept. The Nash equilibrium, for present purposes, may

TABLE 7
 PREDICTED AND ACTUAL OUTCOMES: SEALED-BID EXPERIMENTS

SOCIAL COMPOSITION FUNCTION	PROVISION					PROFIT			Percentage of Efficient Π
	q_L	q_S	Q	Percentage of Efficient Q	π_L	π_S	Π		
Standard Summation (SB-1):									
Efficient	1*	1	12	100	1	1	7.56	100	
Predicted	2	2	4	33.3	2.06	2.06	4.12	54.5	
Observed	3.063	.782	3.845	32.0	.828	2.640	3.468	45.9	
Weakest Link (SB-2):									
Efficient	4	4	4	100	.42	.42	.84	100	
Predicted	4	4	4	100	.42	.42	.84	100	
Observed	4.287	3.290	3.290	82.3	-.386	.381	-.005	-.6	
Best Shot (SB-3):									
Efficient	12	0	12	100	8.70	-1.24	7.56	100	
Predicted	0	12	.821	6.8	-1.24	8.70	.842	11.1	
Observed	3.129	.944	3.129	26.1	.288	2.080	2.368	31.3	

* 1 stands for "indeterminate."

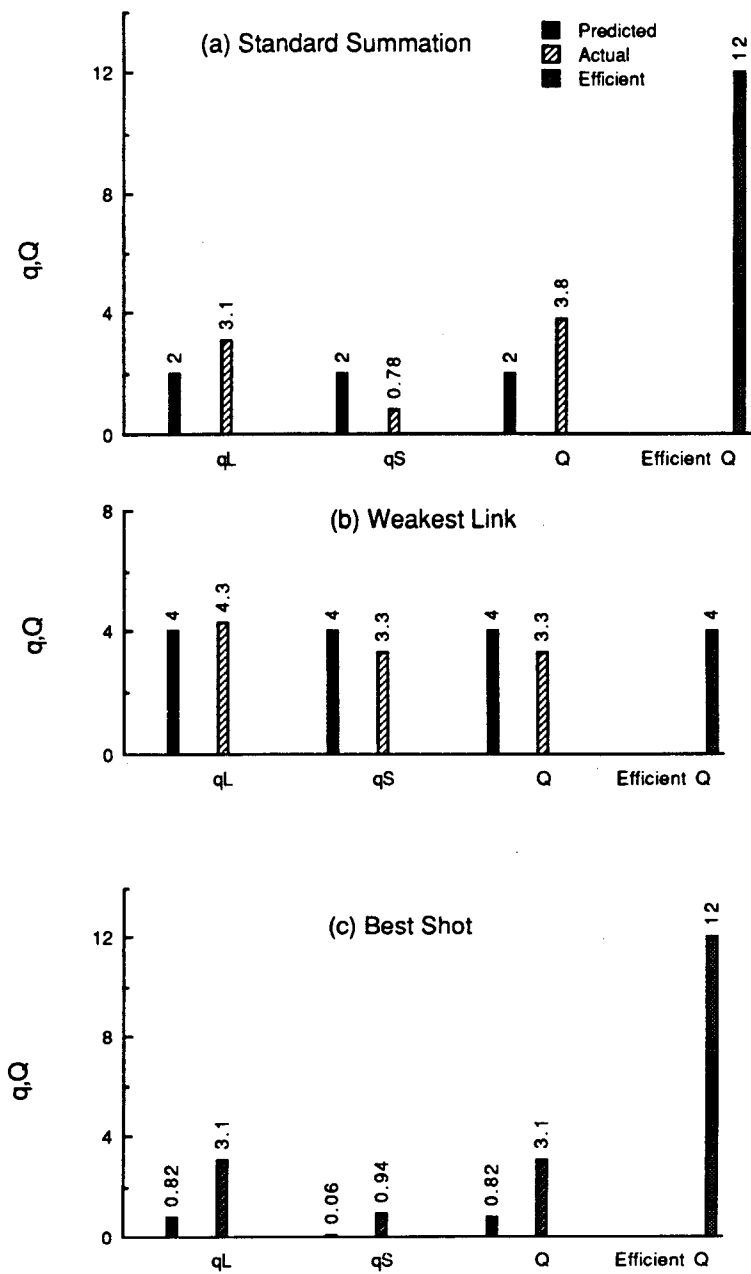


FIG. 3.—Outcomes of sealed-bid experiments

be defined as a strategy pair such that neither player would find it advantageous to revise his choice given the other's selected strategy. But it turns out that the Nash equilibrium is not unique in any of the cases considered; hence a supplementary principle or principles had to be appealed to. We called on two such principles. The first is symmetry. Given the completely parallel situations of the two players in the sealed-bid experiments, we selected as our predicted solution only among those Nash equilibria such that the members of each pair make equal contributions to the public good. (In the Best Shot setting, however, we must also examine asymmetric solutions, for reasons to be made clear below.) We also had to employ one other supplementary principle, Pareto dominance, to be discussed shortly when the Weakest Link case is taken up below.

Under Standard Summation, the Nash equilibria constitute an infinite class of outcomes, to wit, the continuum of paired nonnegative public-good provisions that sum to four. Among the possibilities are $(q_1, q_2) = (0, 4), (3, 1), (2.5, 1.5)$, and so on. If, for example, the parties had chosen the respective provisions $(q_1, q_2) = (3, 1)$, neither would be able to profit by a unilateral revision of his choice. The sole symmetrical member of this class of solutions is obviously $(2, 2)$. Hence our predicted provisions of the public good are two for each player. In summarizing the actual data here, table 7 reports separately the average of the larger provisions in each pair, denoted q_L , and of the smaller, denoted q_S . In this symbolism, our prediction under Standard Summation is $q_L = q_S = 2$, so that the social aggregate is $Q = 4$. Notice that while this predicted aggregate is the same as that for Standard Summation under the previous sequential protocol, the predicted distribution within pairs has changed drastically from $(4, 0)$ to $(2, 2)$.

Under Weakest Link, the Nash equilibria once again comprise a continuum of outcomes: to wit, all the pairs of the form (x, x) such that $0 \leq x \leq 4$. Possible instances include $(0, 0), (1.5, 1.5), (3, 3)$, and $(4, 4)$. Reference to figure 1 will indicate that, for example, if the parties had each chosen to provide three units, then neither member of the pair would want to unilaterally revise his choice. Here, since *all* the Nash equilibria are symmetrical, we must call on our second supplementary principle: Pareto dominance. The justification is that since all the Nash equilibria pay off equally to the parties, the most attractive and only reasonable Nash equilibrium should be a "meeting of the minds" such that the mutual profit is as favorable as possible. With this supplementary principle in addition to symmetry, the predicted provisions are $q_L = q_S = 4$. Under the Weakest Link social composition function, the aggregate quantity of the public good would then be $Q = 4$.

Finally, under Best Shot, matters are somewhat complicated. There are only two deterministic Nash equilibria, in each of which one player provides four units of the public good and the other none. Both of these solutions evidently fail to satisfy the symmetry principle. There is, however, a symmetric mixed-strategy Nash equilibrium. Although actually finding it posed quite a daunting problem for our experimental subjects, that was nevertheless the “prediction” we adopted. By varying the contribution level one cent at a time, subjects could effectively provide units of the public good in increments of $q = .012195$. A continuous approximation of the equilibrium mixed strategy involves providing zero units of the public good with probability .8, the remaining probability being distributed over the interval from $q = 0$ to $q = 4.1$.⁹

As summarized in table 7 and figure 3, the results in the sealed-bid experiments fall considerably short of the excellent matches between predicted and actual results achieved under the sequential protocol.

⁹ The exact Nash equilibrium mixed strategy in the sealed-bid Best Shot case is a discrete probability distribution for q rising in steps of .012195 (which is the fraction of a unit purchased by a one-cent incremental contribution) over the range $q = 0$ to $q = 4$. Following the suggestion of an anonymous referee, to achieve a computationally feasible solution we calculated a continuous approximation of the equilibrium discrete strategy using the function $V(q) = 1.025q - .025q^2$ in place of the step function for total redemption value shown in the last column of table 3. If we let $F(q)$ be the cumulative density function and $f(q)$ the probability density function of the mixed strategy employed by the opponent, a player’s expected “profit” from providing any amount q is

$$\pi(q) = [F(q)(1.025q - .025q^2 - .82q)] + \int_q^\infty (1.025x - .025x^2 - .82q)f(x)dx.$$

The first term on the right-hand side here shows the expected profit when the amount q provided by the player is larger than the opponent’s level of provision, while the second term shows the profit when it is the opponent who makes the bigger contribution. For $F(q)$ to be a symmetrical mixed-strategy Nash equilibrium, the profitability must be the same for all levels of q employed with positive probability density. That is, $\pi'(q) = 0$. Differentiating the term in brackets in the expression above for π , we obtain

$$F(q)(1.025 - .05q - .82) + (1.025q - .025q^2 - .82q)f(q).$$

The derivative of the integral is

$$-(1.025q - .025q^2 - .82q)f(q) + \int_q^\infty \frac{d(1.025x - .025x^2 - .82q)}{dq} f(x)dx.$$

Canceling and making obvious substitutions leads to the simple result

$$\pi'(q) = F(q)(1.025 - .05q) - .82 = 0.$$

Thus the equilibrium mixed strategy is given by

$$F(q) = \frac{.82}{1.025 - .05q}.$$

Notice that there is a discrete probability mass at $q = 0$, to wit, $F(0) = .8$ precisely. The probability density is positive over the range from $q = 0$ to $q = 4.1$.

The one exception is the Weakest Link case, in which the observed results here do track the theoretical prediction reasonably well, though still not nearly as closely as under the sequential protocol.

For Standard Summation, the predicted equal distribution of the public-good provision, $q_L = q_S = 2$, was not borne out. It looks as if the partners were groping in the dark, trying out all kinds of possibilities, as evidenced by the huge spread between the average of the larger provisions ($q_L = 3.063$) and of the smaller ($q_S = 0.782$). Surprisingly, the average social aggregate $Q = 3.845$ was quite close to the theoretical prediction $Q = 4$. But this average is misleading since it hides the serious undershooting and overshooting that occurred in many cases and caused a loss of profit to the players. We discuss this further in the next subsection.

Finally, for Best Shot the shoe is somewhat on the other foot. Here the larger versus smaller *relative* provisions are heavily disproportionate as predicted. But, in aggregate, far more units are being provided than predicted. In consequence, however, the parties are getting substantially closer to the efficient solution; in fact, they are generating an average 26.1 percent of the efficient number of units rather than the mere 11.7 percent that the theory indicated.

Provision versus Profit

Up to now our evaluation of the experimental results has run entirely in terms of individual and social provisions of the public good: the efficient, predicted, and experimentally observed magnitudes q_i and Q . For some purposes, particularly with regard to degree of efficiency achieved, it is more accurate to think in terms of individual and group "profit," a term used here in place of what the textbooks would call "consumer surplus."

In comparing the results in terms of public-good provisions versus profits, the following are some points of interest.

1. Quite commonly the partner contributing the *smaller* provision reaps the *larger* profit, a result stemming from the nature of public goods and the benefit of free-riding.
2. In terms of efficiency achieved, the results tend to "look better" when scaled in terms of aggregate profit Π rather than in terms of aggregate social provision Q . The reason is that an efficiency failure essentially always takes the form of a shortfall in the social provision of the public good; given the fact of diminishing returns, the shortfall involves units of lower marginal benefit than the units actually provided. This argument also indicates why efficiency is more correctly measured in terms of aggregate profit rather than number of units of the public good provided.

3. One notable exception to the preceding generalization is the result for Weakest Link under the sealed-bid protocol. Here the observed individual and aggregate provisions q_S , q_L , and Q all are not too far from the 100 percent efficiency predictions, but the π_L and Π profit observations are way off the mark. In fact, π_L is so heavily negative as to just tilt Π into the negative region. But these anomalies are somewhat "accidental." It so happened that in Weakest Link the theoretical aggregate profits are very small in magnitude compared with the other two social composition functions: \$0.84 versus \$7.56. Since each single unit provided costs \$0.82, any substantial error made by any individual—particularly an overshooting—even on a single trial was liable to seriously affect the overall average. What occurred here, specifically, is that in the very first period when the subjects were still operating entirely in the dark, one player in each of two experimental pairs overshot by enough to generate a relatively huge negative profit. These two instances were numerically heavy enough to dominate the average calculated over 50 trials since in all the other cases the observed profits were (as predicted) quite close to zero in numerical magnitude.

4. In one case (Standard Summation under the sealed-bid protocol), the aggregate Q observed squares nicely with prediction whereas the aggregate Π observed does not. The reason is, as indicated earlier, that the average aggregate Q represented a canceling-out of some instances of serious undershooting and overshooting. Hence in this case the efficiency achieved as measured by profit looks worse (and actually is worse) than the efficiency indicated by the Q measure.

V. Summary

The experiments reported on here were designed to test whether voluntary private provision of public goods met theoretical expectations under the assumption of rational, self-interested behavior. We go beyond the previous experimental literature in examining individual and group choices under alternative social composition functions, making explicit use of alternative decision protocols.

Three social composition functions were studied: Standard Summation, Weakest Link, and Best Shot. Theoretical considerations indicated that the traditional result as to "underprovision" of public goods under Standard Summation would be substantially mitigated under Weakest Link but aggravated under Best Shot.

Under our sequential protocol, we conducted a trio of experiments, one for each of the social composition functions. In each group (pair) of subjects the second mover had enough information to make an

explicit optimizing choice, but the first mover had to act in ignorance of his or her partner's likely later behavior. The actual results averaged over periods and replications squared remarkably with those predicted.

Under our sealed-bid protocol, both group members were in the dark as to partners' likely behavior. To overcome nonuniqueness of the Nash equilibrium here, two supplementary principles were appealed to: (i) only symmetrical solutions were considered, and (ii) among symmetrical solutions the Pareto-dominant one was chosen. Since the informational problem was notably more difficult under sealed-bid, and just what would constitute rational behavior subject to some question, we anticipated a poorer fit between observed and predicted results. While this indeed occurred, in some respects the subjects did manage to go a surprising distance toward theoretical anticipations.

For the sequential experiments, our results may be regarded as strongly confirming a compound hypothesis that the subjects (*a*) acted in a rational, self-interested way, (*b*) believed that their partners would behave similarly, and (*c*) correctly conjectured that their payoffs were symmetrical with those of their partners. For the experiments conducted under the sealed-bid protocol, this compound hypothesis was less adequately confirmed. Further study will be necessary to specify which portions failed and to what degree.

Appendix

Instructions for Participants

You are about to participate in a decision process in which one of numerous competing alternatives will be chosen. This is part of a study intended to provide insight into certain features of decision processes. The instructions are simple. If you follow them carefully and make good decisions, you might earn a considerable amount of money. You will be paid in cash.

This decision process will proceed as a series of ten periods. In *each* period the level of a *project* will be determined and financed. The "level" can be at zero "units" or more. Attached to the instructions you will find a sheet called the *Redemption Value Sheet*. It describes the *value to you* of decisions made in each period. *You are not to reveal this information to anyone*. It is your own private information.

During each period a level of the project will be determined. For the first unit provided during a period you will receive the amount listed in row 1 of the Redemption Value Sheet. If a second unit is also provided during the period, you will receive the additional amount listed in row 2 of the Redemption Value Sheet. If a third unit is provided, you will receive, in addition to the two previous amounts, the amount listed in row 3, etc. As you can see, your individual total payment in each period is computed as a sum of the redemption values of specific units. These totals of redemption values are tabulated for your convenience on the right-hand side of the page.

The earnings per period, which are yours to keep, are the *differences* be-

tween the total of redemption values of units of the project provided and *your individual expenditures* on the project. Suppose, for example, your Redemption Value Sheet was as below and two units were provided.

REDEMPTION VALUE SHEET (Example)

Provided Level (Units)	Redemption Value of Specific Units	Total Redemption Value of All Units
1	600	600
2	500	1,100
3	400	1,500

Your redemption value for the two units would be 1,100 and your earnings would be computed by subtracting your individual expenditures from this amount. If 2.5 units were provided, the redemption value would be determined by the redemption values of the first and second unit plus half of the third unit, that is, $600 + 500 + (0.5)400 = 1,300$.

The process by which the level of the project is decided will proceed as follows. Each unit of the project costs \$0.82. At the beginning of each period you are to write on the Expenditure Form the amount you will spend individually. This number should also be recorded on row 2 of your Individual Record of Earnings. These individual Expenditure Forms will be collected. The number of units of the project is then determined by applying *one* of the following three Rules:

- Rule I:* The number of units provided is the *total* of the individual expenditures divided by the cost per unit.
- Rule II:* The number of units provided is the *smallest* of the individual expenditures divided by the cost per unit.
- Rule III:* The number of units provided is the *largest* of the individual expenditures divided by the cost per unit.

You will be told at the beginning of each period which of these Rules applies to you in that period. After the level of the project has been determined it will be announced. Your individual expenditures will not be made public. Note that your individual expenditures are binding on you, irrespective of the Rule used to determine the level of the project.

When the level of the project is announced, you should enter the Total Redemption Value of all units obtained from the Redemption Value Sheet on row 1 of your Individual Record of Earnings. You should then subtract row 2 from row 1 on this record to determine your earnings for this period. Row 4 provides a place for you to record the number of units of the project provided in each period.

During this process you are not to speak to anyone or otherwise attempt to communicate. There may be several groups making decisions at once. You will be told which group you are participating with in each period and how many members are in your group. Your Individual Record of Earnings identifies your Individual Number, and has a row for you to note your Group Number in each period. The group you are assigned to in making the decision in each period will be dissolved immediately thereafter, and your new group assignment will be different each period. Furthermore, in no event will you ever be told who else is in the group with you.

Are there any questions?

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