Can Housing Collateral Explain Long-Run Swings in Asset Returns?

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September 22, 2006

Abstract

To explain the low-frequency variation in US equity and debt returns in the 20th century, we solve an equilibrium model in which households face housing collateral constraints. An increase in the ratio of housing to human wealth loosens these borrowing constraints, thus allowing for more risk sharing. The rate of return that households require for holding equity decreases as a result. Feeding the historical time series of US housing collateral into the model replicates four features of long-run asset returns. (1) It produces a fifteen percent equity premium during the 1930s and a slow decline of the equity premium from eleven percent in the 1960s to four percent in 2003. (2) It generates large unexpected capital gains for equity holders, especially in the 1990s. (3) The risk-free rate and the housing collateral ratio are strongly positively correlated at low frequencies. (4) The model mimics the slow decline in the volatility of stock returns and the riskless interest rate, but it generates too much overall volatility in the riskless interest rate.

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Introduction

Some of the most dramatic historical episodes in equity markets have coincided with changes in housing markets. For example, the equity premium was very high in the 1930s when the value of the housing stock was low relative to output, while the gradual decline in the equity premium in the post-war period coincided with a sustained increase in housing values. This time-variation in equity premia cannot be accounted for in standard asset pricing models (Breeden (1979) and Lucas (1978)). This paper explores whether changes in the value of the housing stock can account for the timing and magnitude of these changes in equity markets.

To explore this question, we model households who differ by their income histories. They share income risk by trading contingent claims, but they cannot borrow more than the value of their house. When housing collateral is scarce, this borrowing constraint limits risk sharing more; as a result, risk premia are higher. Thus, risk premia vary over time and with housing collateral. This modest friction is a realistic one for an advanced economy like the US.[1]

The contribution of this paper is to show that the long-run variation in the amount of housing collateral quantitatively accounts for a large part of the long-run variation in stock and bond returns in the US. The calibrated model replicates (1) the long-run decline in the equity premium in the post-war period, documented by Jagannathan, McGrattan and Scherbina (2000), (2) the associated unanticipated capital gains for stock holders -which were especially high in the 1990s-, documented by Fama and French (2002), (3) the long-run fluctuations in the risk-free return, section 4 documents a strong positive relationship between the housing wealth-to-income ratio and the risk-free rate in the data, and (4) the decline in the volatility of the equity premium and the risk-free rate, also documented in section 4. The model produces a high equity premium in the 1930s because that was a period of collateral scarcity. Post-war, the model-predicted equity premium falls as housing collateral services became more abundant. The increase in the mortgage to income ratio

[1] Our emphasis on housing, rather than financial assets, reflects three features of the US economy: the participation rate in housing markets is very high (2/3 of households own their home), the value of the residential real estate makes up over seventy-five percent of total assets for the median household (Survey of Consumer Finances, 2001), and housing is a prime source of collateral (75 percent of household borrowing in the data is collateralized by housing wealth, US Flow of Funds, 2003). To keep the model exposition simple, we abstract from financial assets or other kinds of capital (such as cars) that households may use to collateralize loans. However, in the calibration we explore the effects of using a broader measure of collateral.

from 12% to 100% relaxes borrowing constraints, enables more risk sharing, and produces a decline in the equity premium from 11% in the 1950s to 4% in 2002. For the same reason, the predicted volatility of stock returns and the riskless interest rate decreases substantially. Because housing collateral increases rapidly in the 1990s, risk premia drop unexpectedly and this generates large unanticipated capital gains for stock holders. Lastly, the model generates a positive correlation between the riskless interest rate and the amount of housing collateral because times with scarce collateral are times in which the price for insurance against binding collateral constraints is high; precautionary savings push down the interest rate. Overall, our model overpredicts the volatility of the risk-free rate.

The main challenge in the asset pricing literature since Breeden (1979) and Lucas (1978) has been to develop an equilibrium model that can generate returns on stocks and bonds with the right properties in the time-series and in the cross-section. Several classes of models have been shown to be consistent with unconditional asset pricing moments. More recently, the literature has shifted its attention towards the models' ability to generate time-varying risk premia. While empirical explanatory variables that predict returns abound, getting an equilibrium model to deliver sufficient time-variation has proven much more challenging (e.g. Lettau and Ludvigson (2003)). The extant models seem to deliver either limited time-variation in risk premia, or the time-variation rests on unobserved or hard-to-estimate driving forces.[2] Our model has the advantage that its driving force, housing collateral, is observable. This allows us to feed in the historically observed housing collateral series into the model and to ask whether the model's predicted returns are consistent with the historical ones. This historical accounting exercise, based solely on observables, makes the model's successes and failures more apparent, and ultimately lends credibility to the explanation. Moreover,

[2]While the habit literature successfully reproduces many of the empirically observed features of stock prices, such as time-varying risk premia, in a model with a single representative agent with habit-based preferences (e.g., Abel (1990), Constantinides (1990), Campbell and Cochrane (1999), and Menzly, Santos and Veronesi (2004)), this class of models is difficult to test directly because the aggregate habit is unobserved. Also, habit preferences aggregate have some unappealing public policy implications (Ljungqvist and Uhlig (2000)). The long-run risk literature introduces a small but persistent component in aggregate consumption and dividend growth (e.g., Bansal and Yaron (2004) and Hansen, Heaton and Li (2005)). This turns out to be very useful for matching smooth consumption data and volatile returns. However, it is hard to distinguish between i.i.d. consumption growth and a specification that includes a small, predictable component based on available quantity data (e.g., Colacito and Croce (2005)). Finally, both classes of models could only address the long-run decline in the US equity premium through a radical change in the time series process for aggregate consumption growth, which again may be hard to detect in the data.

our quantitative asset pricing results show that the housing collateral mechanism is able to induce the observed amount of time-variation in risk premia.

This paper fits into a literature that studies the impact of time variation in risk sharing on asset prices in a heterogeneous agent economy model (e.g., Telmer (1993), Constantinides and Duffie (1996), Storesletten, Telmer and Yaron (2006), and especially Alvarez and Jermann (2000, 2001)). Compared to the extant heterogenous agent models, the focus of our exercise is on generating more time-variation in risk premia. This comes at the cost of a volatile riskless interest rate. A version of the model with a higher degree of inter-temporal substitution mitigates this problem. It also connects closely to the work of Ortalo-Magné and Rady (2002, 2006), which studies the impact of borrowing constraints and income shock dispersion on equilibrium housing prices.

Competing explanations for the low frequency evolution of asset prices abound: changing demographics (Geanakoplos, Magill and Quinzii (2004)), taxation (McGrattan and Prescott (2005)), stock market participation (Calvet, Gonzalez-Eiras and Sadini (2003) and **?**), stock market regulation, globalization, consumption growth volatility (Lettau, Ludvigson and Wachter (2006)), and technology (Pastor and Veronesi (2005) and Jermann and Quadrini (2006)). Yet none of these competitors can simultaneously explain time-series and cross-sectional return variation. Lustig and Van Nieuwerburgh (2005) shows that housing collateral risk is priced in the cross-section of asset returns. Thus, housing collateral is an attractive mechanism because it offers a unified explanation for a broad set of long-run and short-run facts.

Section 1 models households who trade a complete menu of assets, as in Lucas (1978), but they face endogenous solvency constraints because they can repudiate their debts. When a household chooses to repudiate its debts, it loses all its housing wealth but its labor income is protected from creditors. The household is not excluded from trading. In Kehoe and Levine (1993), Krueger (2000), Kehoe and Perri (2002), and Krueger and Perri (2005), limited commitment is also the source of incomplete risk-sharing. But the outside option upon default is exclusion from all future risk sharing arrangements. Alvarez and Jermann (2000) show how to decentralize these Kehoe and Levine (1993) equilibria with sequential trade. Geanakoplos and Zame (2000) and Kubler and Schmedders (2003) consider a different environment in which individual assets collateralize individual promises in a standard incomplete markets economy. We model the outside option as bankruptcy with loss of all collateral assets; all promises are backed by all collateral assets. Section 2 explains the equilibrium dynamics of the two driving forces: the wealth distribution and the collateral ratio. These two forces interact to deliver high equity premia, volatile equity premia, and high Sharpe ratios when collateral is scarce. We calibrate the model in section 3.

We feed the model seven decades worth of US data on aggregate consumption growth and housing collateral ratio dynamics. Section 4 computes the model-implied equity premium and risk-free rate, and compares their low-frequency evolution to the one in the US data. Section 5 argues that the housing explanation is plausible because the model's return predictability, Sharpe ratio variability, and unconditional asset pricing moments mostly match the data. Especially the high volatility of the Sharpe ratio is a feature that most equilibrium models cannot account for. The appendix provides proofs of the propositions, details of the calibration and the computational algorithm, and it contains a detailed discussion of the unconditional asset pricing moments.

1 Model

1.1 Environment

Uncertainty The economy is populated by a continuum of infinitely lived households. The structure of uncertainty is twofold: $s = (y, z)$ is an event that consists of a household-specific component $y \in Y$ and an aggregate component $z \in Z$. These events take on values on a discrete grid $S = Y \times Z$. We use $s^t = (y^t, z^t)$ to denote the history of events. S^t denotes the set of possible histories up until time t. The state s follows a Markov process with transition probabilities π that obey:

$$\pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z) \; \forall z \in Z, y \in Y.$$

Because of the law of large numbers, $\pi_z(y)$ denotes both the fraction of households drawing y when the aggregate event is z and the probability that a given household is in state y when the aggregate state is z.[3]

Preferences We use $\{x\}$ to denote an infinite stream $\{x_t(s^t)\}_{t=0}^{\infty}$. There are two types of commodities in this economy: a consumption good c and housing services h. These commodities cannot be stored. The households rank consumption streams according to the criterion:

$$U(\{c\},\{h\}) = \sum_{s^t|s_0}\sum_{t=0}^{\infty} \delta^t \pi(s^t|s_0) u\left(c_t(s^t), h_t(s^t)\right), \tag{1}$$

where δ is the time discount factor. The households have power utility over a CES-composite consumption good:

$$u(c_t, h_t) = \frac{\left[c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}}}{1-\gamma}.$$

The parameter $\psi > 0$ converts the housing stock into a service flow, γ governs the degree of relative risk aversion, and ε is the intratemporal elasticity of substitution between non-durable consumption and housing services.[4]

Endowments The aggregate endowment of the non-durable consumption good is denoted $\{c^a\}$. The growth rate of the aggregate endowment depends only on the current aggregate state: $c^a_{t+1}(z^{t+1}) = \lambda(z_{t+1})c^a_t(z^t)$. Each household is endowed with a labor income stream $\{\eta\}$. The labor income share $\hat{\eta}(y_t, z_t) = \eta(y_t, z^t)/c^a(z^t)$, only depends on the current state of nature. Since the aggregate endowment is the sum of the individual endowments,

$$\sum_{y' \in Y} \pi_z(y')\hat{\eta}_t(y', z) = 1, \ \forall z, t \geq 0.$$

The aggregate endowment of housing services is denoted $\{h^a\}$ and $\rho(z^t)$ denotes the relative price of a unit of housing services. The calibration specifies a process for the ratio of non-housing expenditures and housing services expenditures $\{r\}$, $r(z^t) = \frac{c^a(z^t)}{\rho(z^t)h^a(z^t)}$, rather than for $\{h^a\}$ directly.

[3]The usual caveat applies when applying the law or large numbers. We implicitly assume the technical conditions outlined by Uhlig (1996) are satisfied.

[4]The preferences belong to the class of homothetic power utility functions of Eichenbaum and Hansen (1990). Special cases are separability ($\varepsilon = \gamma^{-1}$) and Cobb-Douglas preferences ($\varepsilon = 1$).

Trading To keep the notation simple, there is no net positive wealth other than housing and no trade in stocks in the model. Since a complete menu of assets is traded, explicitly allowing trade in stocks would obviously not change anything, other than through its effect on the supply of collateral. In our calibration, we do include other sources of collateralizeable financial wealth. We also choose the simplest model of housing markets, without any frictions, to examine the collateral mechanism.

Each household is assigned a label (ℓ, s_0), where ℓ denotes the time-zero collateral wealth of this household. The cross-sectional distribution of initial non-labor wealth and income states (ℓ, s_0) is denoted $\mathcal{L}_0$. So, ℓ denotes the value of the initial claim to housing wealth as well as any financial wealth that is in zero net aggregate supply.

We let $\{c(\ell, s_0)\}$ denote the stream of consumption and we let $\{h(\ell, s_0)\}$ denote the stream of housing services of a household of type (ℓ, s_0). The financial markets are complete: households trade a complete set of contingent claims a in forward markets, where $-a_t(\ell, s^t, s')$ is a promise made by agent (ℓ, s_0) to deliver one unit of the consumption good if event s' is realized in the next period. These claims are in zero net supply, and trade at prices $q_t(s^t, s')$.[5] All prices are quoted in units of the non-durable consumption good. There are frictionless rental markets and markets for home ownership; ownership and housing consumption are separated. The rental price is $\rho_t(z^t)$; $p_t^h(z^t)$ denotes the (asset) price of the housing stock. Because of the law of large numbers, these prices only depend on aggregate histories.

At the start of each period, the household purchases non-housing consumption in the spot market $c_t(\ell, s^t)$, housing services in the rental market $h_t^r(\ell, s^t)$, contingent claims in the financial market and ownership shares in the housing stock $h_{t+1}^o(\ell, s^t)$ subject to a wealth constraint:

$$c_t(\ell, s^t) + \rho_t(z^t)h_t^r(\ell, s^t) + \sum_{s'} q_t(s^t, s')a_t(\ell, s^t, s') + p_t^h(z^t)h_{t+1}^o(\ell, s^t) \leq W_t(\ell, s^t). \qquad (2)$$

Next period wealth is labor income, plus assets, plus the cum-dividend value of owned

[5]This setup is equivalent to having financial intermediaries trade in state contingent claims and provide insurance to the households (Atkeson and Lucas (1993)).

housing:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(z^t, z') + \rho_{t+1}(z^t, z')\right]. \qquad (3)$$

Collateral Constraints Households can default on their debts. When the household defaults, it keeps its labor income in all future periods. The household is not excluded from trading, even in the same period. However, all collateral wealth is taken away. As a result, the markets impose a solvency constraint that keeps the households from defaulting: all of a household's state-contingent promises must be backed by the cum-dividend value of its housing owned at the end of period t, h^o_{t+1}. In each node s^t, households face a separate *collateral constraint* for each future event s':

$$-a_t(\ell, s^t, s') \leq h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(z^{t+1}) + \rho_{t+1}(z^{t+1})\right], \text{ for all } s^t, s'. \qquad (4)$$

As in Alvarez and Jermann (2000), these constraints are not too tight: they allow for the maximal degree of risk sharing, given that agents cannot be excluded from trading, while preventing default.

1.2 Equilibrium Asset Prices

Competitive Equilibrium. *Given a distribution over initial non-labor wealth and initial states $\mathcal{L}_0$, a competitive equilibrium is a feasible allocation $\{c(\ell, s^t), h^r(\ell, s^t), a(\ell, s^t), h^o(\ell, s^t)\}$ and prices $\{q, p^h, \rho\}$ such that (i) for given prices and initial wealth, the allocation solves each household's maximization problem (1) s.t. (2), (3) and (4), and (ii) the markets for the consumption good, the housing services, the contingent claims and housing ownership shares clear.*

To rule out arbitrage opportunities, payoffs in each state of the world are priced by the unconstrained agents at every date and state (Alvarez and Jermann (2000)). These unconstrained households have the highest intertemporal marginal rate of substitution (IMRS):

$$m_{t+1} = max_{i\in[0,1]} \left\{ \delta \frac{u_c(c^i_{t+1}, h^i_{t+1})}{u_c(c^i_t, h^i_t)} \right\} = max_{i\in[0,1]} \left\{ \delta \left(\frac{c^i_{t+1}}{c^i_t} \right)^{-\gamma} \left(\frac{1 + r^{-1}_{t+1}}{1 + r^{-1}_t} \right)^{\frac{1-\varepsilon\gamma}{\varepsilon-1}} \right\}. \quad (5)$$

The second equality follows directly from the form of the utility function, the definition of the expenditure ratio $r = \frac{c^a}{\rho h^a}$, and market clearing in the housing market.[6] No arbitrage implies that the return on any security j, R^j_{t+1}, satisfies the standard Euler equation $E_t[m_{t+1} R^j_{t+1}] = 1$.

2 Inside the Model

This section explains in detail how movements in the housing collateral ratio induce variation in conditional asset pricing moments. To characterize the equilibrium consumption dynamics and link these to the state prices of consumption, we use stochastic consumption weights. This section can be skipped by readers mainly interested in the asset pricing results.

2.1 Equilibrium Consumption Dynamics and the Collateral Ratio

Following Lustig (2003), we use a simple risk-sharing rule to characterize the equilibrium consumption choices for each household in an equivalent time zero trading environment.[7]

Let $\xi_t(\ell, s^t) = \frac{\chi}{\zeta_t(\ell, s^t)}$ be the cumulative Lagrange multiplier on the collateral constraint (4) at time t for household (ℓ, s_0), where $\zeta_t(\ell, s^t) = 1 - \sum_{s^\tau \preceq s^t} \gamma_\tau(\ell, s^\tau)$ adds all the multipliers γ along the path leading to that node; χ is the inverse of the multiplier on the time zero budget constraint. When the constraint does not bind, its Lagrange multiplier is zero, and the household's cumulative multiplier remains unchanged. But when the constraint binds, the multiplier increases to a cutoff level $\underline{\xi}_t(y_t, z^t)$. This cutoff is the consumption share at which the collateral constraint holds with equality. Note that the cutoff only depends on the

[6]The equilibrium rental price is $\rho_t = u_h(c^i_t, h^i_t)/u_c(c^i_t, h^i_t) = \psi(h^i_t/c^i_t)^{-\frac{1}{\varepsilon}}$, $\forall i$. Since there is one economy-wide rental market, the rental price only depends on aggregate quantities: $\rho_t(z^t) = \psi(h^a_t(z^t)/c^a_t(z^t))^{-\frac{1}{\varepsilon}}$. Consequently, all households equate their non-housing to housing consumption ratios $r(z^t)$.

[7]A separate appendix derives the equivalence of equilibria in these two trading environments if interest rates are high enough in the sense of Alvarez and Jermann (2000). It also shows the necessary and sufficient first order conditions for the household.

current event y_t, not on the entire history. Summarizing the dynamics:

$$\xi_t(\ell, y^t, z^t) = \begin{cases} \xi_{t-1}(\ell, s^{t-1}) & \text{if } \xi_{t-1}(\ell, s^{t-1}) > \underline{\xi}_t(y_t, z^t) \\ \underline{\xi}_t(y_t, z^t) & \text{if } \xi_{t-1}(\ell, s^{t-1}) \leq \underline{\xi}_t(y_t, z^t) \end{cases} \tag{6}$$

The aggregate weight process $\xi_t^a(z^t)$ summarizes to what extent collateral constraints bind *on average, i.e. across households*:

$$\xi_t^a(z^t) = \sum_{y^t} \int \xi_t^{\frac{1}{\gamma}}(\ell, y^t, z^t) d\mathcal{L}_0 \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)}.$$

Household (ℓ, s^t)'s consumption shares $\frac{c_t(\ell,s^t)}{c_t^a}$ and $\frac{h_t(\ell,s^t)}{h_t^a}$ are completely determined by the ratio $\frac{\xi_t(\ell,s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)}$. This ratio captures the household's position in the wealth distribution.

$$c_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} c_t^a(z^t) \text{ and } h_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} h_t^a(z^t). \tag{7}$$

We verify in a separately available appendix that this rule satisfies the first order condition for non-housing and housing consumption and the market clearing conditions. Combining (7) and (6), when a household switches to a state with a binding constraint, its consumption share increases. Everywhere else, its consumption share is drifting downwards at the rate $\frac{\xi_{t+1}^a}{\xi_t^a}$. Shocks to $\xi_t^a(z^t)$ reflect aggregate shocks to the wealth distribution. Because they follow from an inability to insure against human wealth shocks, these can be interpreted as liquidity shocks.

Combining the risk-sharing rule (7) for unconstrained households ($\xi_{t+1}(\ell, s^{t+1}) = \xi_t(\ell, s^t)$) and (5), we obtain a new expression for the SDF:

$$m_{t+1} = \delta \left(\frac{c_{t+1}^a}{c_t^a}\right)^{-\gamma} \left(\frac{1 + r_{t+1}^{-1}}{1 + r_t^{-1}}\right)^{\frac{1-\varepsilon\gamma}{\varepsilon-1}} \left(\frac{\xi_{t+1}^a}{\xi_t^a}\right)^{\gamma}. \tag{8}$$

For future reference, we denote the equilibrium price of a claim to some payoff stream $\{d\}$

in units of z^t consumption as

$$\Pi_{z^t}\left[\{d\}\right] = \sum_{k=1}^{\infty} \left(\prod_{j=1}^{k} M_{t,t+j}(z^{t+j})\pi(z^{t+j}|z_t) \right) d_{t+k},$$

where $M_{t,t+j} = m_{t+1}m_{t+2}\dots m_{t+j}$ is the j-period ahead pricing kernel.

A Benchmark Economy The perfect insurance environment provides a useful benchmark for understanding asset prices. Because households are never constrained, the individual multiplier stays constant at its initial value: $\xi_t(\ell, s^t) = \xi_0(\ell, s_0)$. The aggregate weight process reflects the initial wealth distribution and is constant: $\xi_t^a(z^t) = \xi_0^a(z_0) = \int \xi_0(\ell, s_0)^{\frac{1}{\gamma}} d\Phi_0(z_0)$.[8] Consumption shares are constant, consumption levels only move with aggregate consumption and there is full insurance. All agents equate their IMRS and the SDF is the standard Breeden-Lucas pricing kernel, adjusted for a composition factor that arises from the non-separability between non-housing and housing consumption. It contains only the first two factors in (8). As it turns out, the composition effect alone is unable to generate enough variation in conditional asset pricing moments, and it generates a low unconditional equity premium.

A Multiplicative Discount Factor Adjustment The bulk of the action comes from the collateral effect instead. It does not hinge on the non-separability of preferences, but relies on imperfect consumption insurance among heterogenous households induced by occasionally binding collateral constraints. The third term of the SDF in (8) reflects this departure from perfect insurance and it measures the risk of binding solvency constraints. It is the growth rate of the aggregate consumption weight process ξ_{t+1}^a, raised to the power of risk aversion γ. When many households are severely constrained in state z^{t+1} ($\xi^a(z^{t+1}) >> \xi^a(z^t)$), the price of consumption in that state is much higher, and the unconstrained households experience high marginal utility growth: When lots of households run into binding constraints and experience consumption share increases, the unconstrained households have to experience large decreases in their consumption shares. The increase in the price of consumption

[8] Φ_0 is the initial distribution of multipliers ξ_0, a monotone transformation of the initial wealth distribution $\mathcal{L}_0$.

induces them to accept low consumption growth rates. When nobody is constrained, the aggregate consumption weight process stays constant, $\xi^a(z^{t+1}) = \xi^a(z^t)$, and the representative agent SDF re-emerges. The risk of binding solvency constraints endogenously creates *heteroscedasticity* in the SDF.

Housing Collateral The novel feature of this model is that the tightness of the constraints, and therefore the size of the multiplicative adjustment to the SDF, depends on the ratio of aggregate housing wealth to aggregate total wealth, $my\,(z^t)$:

$$my(z^t) = \frac{\Pi_{z^t}\left[\{h^a \rho\}\right]}{\Pi_{z^t}\left[\{c^a + h^a \rho\}\right]} = \frac{\Pi_{z^t}\left[\{c^a r^{-1}\}\right]}{\Pi_{z^t}\left[\{c^a\left(1+r^{-1}\right)\}\right]}. \tag{9}$$

The numerator measures the value of collateralizable wealth; it equals the price of a claim to the aggregate housing dividend. The denominator is the sum of collateralizable housing wealth and non-collateralizable human wealth. If the expenditure ratio r is constant, the collateral ratio equals $\frac{1}{1+r}$. If r varies over time in a persistent manner, then the housing collateral ratio my inherits this persistent variation. This variation in the collateral ratio affects $\frac{\xi^a_{t+1}}{\xi^a_t}$ and therefore the amount of risk sharing that can be sustained and equilibrium asset prices. We formalize this in the following two propositions, proven in appendix A.

If the total housing claim is sufficiently valuable, then perfect risk sharing can be sustained.

Proposition 2. *Let $\Pi^*[\{\cdot\}]$ denote the price of that claim under perfect risk-sharing and let $r = r_{max}$ denote the maximum expenditure ratio. Perfect risk sharing can be sustained if*

$$\Pi^*_z\left[\left\{c^a(z)\left(1+r^{-1}_{max}\right)\right\}\right] \geq \Pi^*_{z,y}\left[\{\eta(y,z)\}\right] \text{ for all } (y,z) \in Y \times Z$$

This condition guarantees that each household can consume the average endowment without violating its collateral constraint. The following proposition states that an economy with more housing collateral (lower r) has lower cutoffs $\underline{\xi}$, thereby allowing for more consumption smoothing. Such an increase in the supply of collateral brings the cutoff consumption share closer to its lower bound of zero. In the limit perfect risk-sharing obtains. Conversely, a decrease in the supply of collateral (higher r) brings the cutoff rules closer to their upper bound, the labor income shares $\hat{\eta}$. In the limit, as the collateral disappears, the economy reverts to autarky (no risk sharing).

Proposition 3. *Assume utility is separable and consider two economies, denoted by superscripts 1 and 2. If* $r_\tau^1(z^\tau) < r_\tau^2(z^\tau)$, $\forall z^\tau \geq z^t$ *then the cutoffs satisfy* $\underline{\xi}^1(y_t, z^t) \leq \underline{\xi}^2(y_t, z^t)$. *As* $r_\tau(z^\tau) \to \infty$ *for all* $z^\tau \geq z^t$, $\underline{\xi}(y_t, z^t) \to \hat{\eta}(y_t, z^t)$. *Conversely, as* $r_\tau(z^\tau) \to 0$ *for all* $z^\tau \geq z^t$, $\underline{\xi}(y_t, z^t) \to 0$.

Differences in the r process affect the equilibrium aggregate multiplier process ξ^a. An economy with a uniformly lower housing collateral (higher r) process has higher liquidity shocks and lower average interest rates (or equivalently higher average state prices):

Corollary 1. *Assume utility is separable and consider two economies, 1 and 2. Fix the distribution of initial multipliers across economies:* $\Phi_0^1(z_0) = \Phi_0^2(z_0)$. *If* $r_t^1(z^t) < r_t^2(z^t)$, $\forall z^t$ *then* $\left\{\xi_t^{a,1}(z^t)\right\} \leq \left\{\xi_t^{a,2}(z^t)\right\}$, *and the state prices are higher on average in the second economy.*

The last proposition and corollary compare two economies with different collateral processes $\{r\}$ to illustrate the mechanism that underlies time-variation in the market price of risk.

Computation These aggregate weight shocks play a key role in the numerical computation of equilibria. To solve the model numerically, we rely on an approximation of the growth rate of the *aggregate weight shock* $g_t(z^t) \equiv \frac{\xi_t^a(z^t)}{\xi_{t-1}^a(z^{t-1})}$ using a truncated history of aggregate shocks. This is discussed in detail in appendix B.

2.2 Two Driving Forces

To build intuition for the asset pricing results, we first explain the two main driving forces of the model: shocks to the wealth distribution, operating at business cycle frequencies, and variation in the housing collateral ratio, operating at low frequencies. Both of these forces affect the SDF m_{t+1} in (8) through its third term g_{t+1}^γ, which is a function of the aggregate weight shock $g_{t+1} = \frac{\xi_{t+1}^a}{\xi_t^a}$.

Shocks to the Wealth Distribution We build in a higher cross-sectional income dispersion in a low aggregate consumption growth state, a mechanism pioneered in Mankiw

(1986) and Constantinides and Duffie (1996) (see calibration below). Because risk sharing is imperfect, this income dispersion effect results in more wealth and consumption dispersion. First, the household cutoff levels are higher in low aggregate consumption growth states, $\underline{\xi}(y_t, z^{t-1}, re) > \underline{\xi}(y_t, z^{t-1}, ex)$, and this makes the consumption increase for households that switch to a state with a binding constraint larger. Second, low aggregate consumption growth states are short-lived in our model and agents are more constrained in these states as a result, because of their desire to smooth out its effect on their consumption. As the combined result of these two forces, the size of the aggregate weight shock increases more in low aggregate consumption growth states ($g_{t+1}(z^t, re) > g_{t+1}(z^t, ex)$). However, after a low aggregate consumption growth shock accompanied by a large aggregate weight shock g_{t+1}, the left tail of the wealth distribution is cleansed, and subsequent aggregate weight shocks are much smaller. This *cleansing* mechanism lowers the conditional market price of risk $\sigma_t[m_{t+1}/E_t[m_{t+1}]$ and increases the interest rate after a bad shock. These wealth distribution dynamics operate at business cycle frequencies and are also present in Lustig (2003). They are a first source of heteroscedasticity in the SDF, and will allow the model to match year-to-year variation in stock returns.

Housing Collateral Mechanism There is another source of heteroscedasticity: low frequency changes in the housing collateral ratio. This paper's novel feature are movements in the housing collateral ratio that come from exogenous movement in the non-housing expenditure ratio r together with endogenous movements in the SDF (equation 9). It is these low frequency movements in the housing collateral ratio that allow the model the match asset prices at low frequencies.

Figure 1 illustrates the collateral mechanism for a typical two hundred period simulation of the benchmark model. The calibration is in section 3 below. Panel 1 plots the housing collateral ratio my (bold, right axis) together with the expenditure ratio r (single line, left axis). It shows that the housing collateral ratio increases when households spend a larger share of income on housing. The persistence of my comes from this relationship. Panel 2 plots the cross-sectional consumption growth dispersion (single line, left axis) against the housing collateral ratio my (bold line, right axis). It summarizes the risk sharing dynamics in the model. When collateral is scarce, more households run into binding collateral constraints. To prevent default, the consumption share of the constrained households increases. At the same time, the unconstrained households' consumption share decreases precipitously (see equation 7). As a result, the cross-sectional standard deviation of consumption growth increases, evidence of less risk-sharing. For example, in a period of collateral abundance (period 126), $\sigma_t[\Delta \log c_{t+1}]$ is 8.1%, whereas in a period of collateral scarcity (period 174), it is only 0.9%.[9] The aggregate weight shock g_{t+1}, plotted in panel 3, measures the economy-wide extent to which the solvency constraints bind. It also governs the new component to the SDF g_{t+1}^{γ}. The panel illustrates that when collateral is scarce, constraints bind more frequently and more severely and this is reflected in a large aggregate weight shock. For example, in period 126 the liquidity shock is 1.07, whereas in period 174 it is only 1.01. The SDF is higher and more volatile in such periods of collateral scarcity, and quite different from the representative agent SDF. The next panels illustrate how this impacts asset prices.

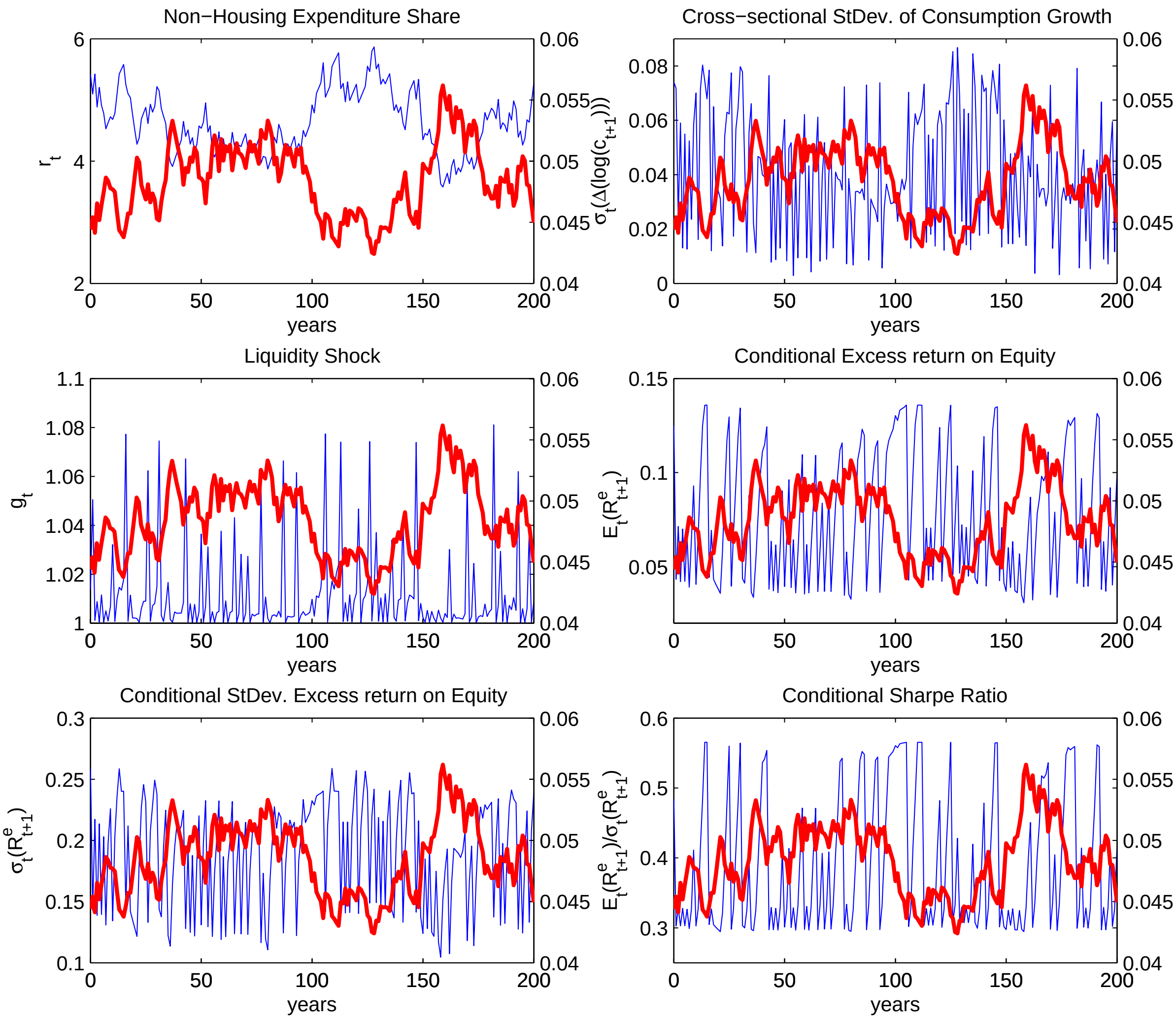
Equity premium The fourth panel of figure 1 shows that the conditional expected excess return on a (non-levered) claim to aggregate consumption (dotted line, left axis) is higher in periods of collateral scarcity (full line right axis). The conditional equity premium is 13.5% when my is low (period 126), but only 4.5% when the housing collateral ratio is high (period 174). The fifth panel shows that the conditional volatility of the excess return on the consumption claim (left axis) is 10.3% when collateral is abundant (period 174) and more than doubles to 24% when collateral is scarce (period 126). The net result of the collateral mechanism is a conditional Sharpe ratio ($E_t[R^{c,e}_{t+1}]/\sigma_t[R^{c,e}_{t+1}]$) that is higher in times of collateral scarcity (sixth panel). It is .34 in period 174 and almost .57 in period 126.

Figure 2 summarizes the conditional asset pricing moments somewhat differently. It plots the averages of these conditional asset pricing moments against the value of the collateral ratio. The entire time-series of conditional asset pricing moments is computed, then averaged over histories of the aggregate state $(z_{t-1}, \cdots, z_{t-k})$, sorted according to whether the last aggregate shock realization z_t was high (dashed line) or low (full line), and then sorted

[9]As an aside, even though the consumption shares change in important ways when collateral constraints bind, the *unconditional* volatility of consumption growth for an individual household is moderate. In our benchmark model it is less than 10% of the unconditional volatility of individual income growth. There is still a considerable amount of risk-sharing.

Figure 1: Risk Sharing, Conditional Asset Pricing Moments and Collateral Ratio

The graphs display a two hundred period model simulation under the benchmark parametrization (see Table 1). The shocks are the same in each panel. The first panel plots the non-housing expenditure ratio r. The second panel plots the cross-sectional standard deviation of consumption growth across households ($\sigma_t[\Delta \log c_{t+1}]$). The third panel is the aggregate weight shock g_{t+1}. The fourth panel plots the equity premium predicted by the model, i.e. the expected excess return on a non-levered claim to aggregate consumption $E_t\left[R^{c,e}_{t+1}\right]$. The fifth panel is the conditional standard deviation of this excess return $\sigma_t\left[R^{c,e}_{t+1}\right]$. The sixth panel is the conditional Sharpe ratio $E_t\left[R^{c,e}_{t+1}\right]/\sigma_t\left[R^{c,e}_{t+1}\right]$. Each of these series are measured against the left axis and plotted in a single blue line. The housing collateral ratio my is measured against the right axis and plotted in a bold red line.



according to my. Concentrating on the dashed lines, the equity premium is 9% higher when collateral is scarce ($my = .04$) than when it is abundant ($my = .10$) in the first panel. The other two panels in the top row match our earlier findings of higher conditional volatility and

Sharpe ratios when collateral is scarce.[10] The bottom row shows that the conditional market price of risk $\sigma_t[m_{t+1}]/E_t[m_{t+1}]$, an upper bound on the Sharpe ratio, is higher when collateral is scarce (panel 4). The price-dividend ratio in panel 5 is also higher when collateral is scarce because the demand for insurance against binding solvency constraints drives up the price of stocks. It also drives up the price of bonds. So, the model simultaneously generates a high equity premium and a high price-dividend ratio because the risk-free rate is low when collateral is scarce (panel 6).

These are the dynamics of asset prices that underly our main results in section 4. We first turn to the calibration of the model.

3 Calibration

There are two driving forces in the model: the income process and the non-housing expenditure ratio.

Income Process The first driving force in the model is the Markov process for the non-durable endowment process. It has an aggregate and an idiosyncratic component. The aggregate endowment growth process is taken from Mehra and Prescott (1985) and replicates the moments of aggregate consumption growth in the 1871-1979 data. Aggregate endowment growth, λ, follows an autoregressive process:

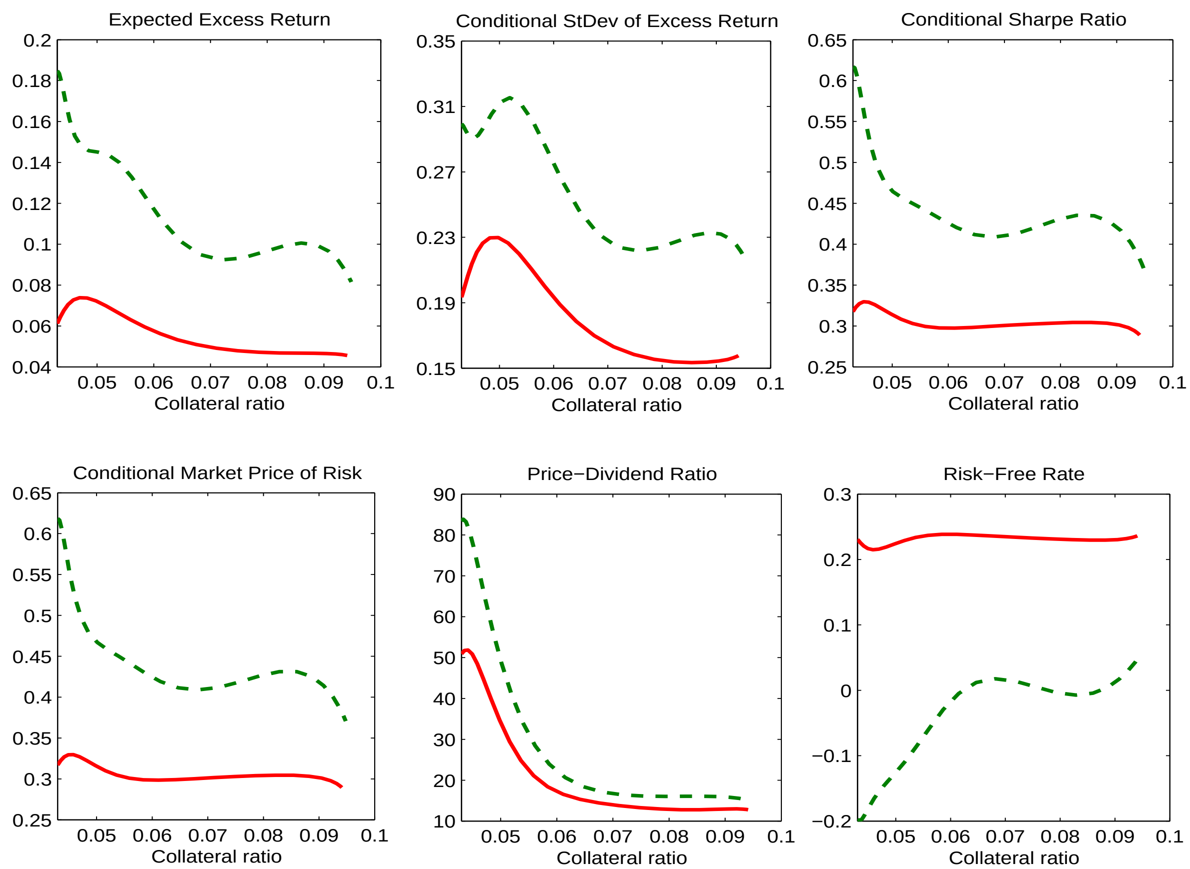
$$\lambda_t(z_t) = \rho_\lambda \lambda_{t-1}(z_{t-1}) + \varepsilon_t,$$

with $\rho_\lambda = -.14, E(\lambda) = .0183$ and $\sigma(\lambda) = .0357$. We discretize the AR(1) process with two aggregate growth states $z = (ex, re) = [1.04, .96]$ (for expansion and recession) and an aggregate state transition matrix $[.83, .17; .69, .31]$. The implied ratio of the probability of a high aggregate endowment growth state to the probability of a low aggregate endowment growth state is 2.65. The unconditional probability of a low endowment growth state is 27.4%. This matches the observed frequency of recessions.

[10] The non-monotonicity for low collateral ratios comes from the Chebychev approximation used to compute policy functions.

Figure 2: Summary Conditional Asset Pricing Moments.

This graph reports average asset pricing moments from a long model simulation under the benchmark parametrization. All series are averaged over histories $(z_{t-1}, \cdots, z_{t-k})$, sorted into low z_t ($\lambda(z_t) = 0.96$, full line) and high z_t observations ($\lambda(z_t) = 1.04$, dashed line) and plotted against the housing collateral ratio (horizontal axes). The first row of the figure plots the expected excess return on a claim to aggregate consumption (panel 1), its conditional standard deviation (panel 2) and its Sharpe ratio (panel 3). The second row plots the conditional market price of risk (panel 4), the conditional price-dividend ratio (panel 5), and the risk-free rate (panel 6).



An important stylized fact about idiosyncratic labor income volatility in the US is that it increases in recessions (Storesletten, Telmer and Yaron (2004)). Our calibrated labor income process is designed to capture this feature. Following Alvarez and Jermann (2001), log labor income shares follow an AR(1) process with autocorrelation of .92, and a conditional variance of .181 in low and .0467 in high aggregate endowment growth states. Discretization into a four-state Markov chain results in individual income states $(\eta^1(hi, ex), \eta^1(lo, ex)) =$ $[.6578, .3422]$ in the high and $(\eta^1(hi, re), \eta^1(lo, re)) = [.7952, .2048]$ in the low aggregate endowment growth state.[11] We refer to the counter-cyclical labor income share dispersion as the Mankiw (1986) effect.

Expenditure Ratio The second driving force in the model is the process for the ratio of non-housing to housing expenditures $\{r\}$. Calibrating the expenditure ratio is equivalent to calibrating the evolution of the aggregate housing stock $\{h\}$ and imposing the intra-temporal optimality condition. Following Piazzesi, Schneider and Tuzel (2006), we specify an autoregressive process which also depends on aggregate endowment growth λ:

$$\log r_{t+1} = \bar{r} + \rho_r \log r_t + b_r \lambda_{t+1} + \sigma_r \nu_{t+1}, \tag{10}$$

where ν_{t+1} is an i.i.d. standard normal process with mean zero, orthogonal to λ_{t+1}. In our benchmark calibration we set $\rho_r = .96$, $b_r = .93$ and $\sigma_r = .03$. The parameter values come from estimating equation (10) on US data.[12] We discretize the process for $\log(r)$ as a five-state Markov process. A second calibration switches off the effect of consumption growth by setting $b_r = 0$. Both calibrations fix $\sigma_r = .03$. We choose the constant $\bar{r}$ to match the average housing expenditure share of 19% in the data (NIPA, 1929 to 2004).

Average Housing Collateral Ratio A key quantitative question is whether collateral is sufficiently scarce for our borrowing constraints to have a large effect. Because this question is an important one, we consider two measures to calibrate the average ratio of collateral wealth to total wealth. The first measure focuses on housing collateral, the second measure includes non-housing sources of collateral.

We measure factor payments to housing wealth as total US rental income and factor payments to human wealth as labor income (compensation of employees). NIPA data show that rental income was 3.4% of rental income plus labor income in 1946-2002 and 4.3% in 1929-2002. Because the factor payments ratio maps directly into the housing collateral ratio,

[11] The one difference with the Storesletten et al. (2004) calibration is that recessions are shorter in our calibration. In their paper the economy is in the low aggregate endowment growth state half of the time. That implies that the unconditional variance of our labor income process is lower.

[12] Table 1 in a separate appendix shows regression estimates for ρ_r and b_r.

the data suggest a housing collateral ratio less than 5%.[13]

Even though non-housing wealth is less collateralizable, our second estimate is a broad collateral measure. It includes financial wealth, the market value of the non-farm non-financial corporate sector in the US. We add interest payments and dividend payments to the income stream from collateralizable wealth and we add proprietary income to the income stream from non-collateralizable wealth. The factor payment ratio increases to 8.6% in the post-war sample and 9.4% in the full sample (row 2), suggesting a housing collateral ratio less than 10%.

An alternative approach is to compare the collateralizable wealth to income ratio in model and data. Assuming that the expected return on total collateralizable assets is 9% and the expected dividend growth rate is 3%, then a collateral ratio of 5% implies a collateral wealth-to-income ratio of 85% according to Gordon's growth formula: $.85 = .05/(.09 - .03)$. Likewise, the implied wealth-to-income ratio is 150% when the collateral ratio is 10%. In US data, the 1929-2004 average ratio of mortgages to income is 55%. If we include financial wealth, that ratio increases to 155%. This approach also points towards a housing collateral ratio of 5% and a broad collateral ratio of 10%.[14] Our benchmark calibration ($my = 0.05$) produces a collateralizable wealth-to-income ratio of 96%. Section C in the appendix explains that a higher my implies a lower wealth/income ratio in the model, through its effect on the risk-free rate.

We take the model with a 5% collateral ratio as our benchmark and consider the economy with a 10% collateral ratio as an alternative. To simultaneously match the average expenditure share of housing services ($\bar{r}$) of 19% and the average ratio of housing wealth to total wealth (my) of 5% or 10%, we scale up the aggregate non-housing endowment.

Preference Parameters In the benchmark calibration, we use additive utility with discount rate $\delta = .95$, coefficient of relative risk aversion $\gamma = 8$, and intratemporal elasticity of substitution between non-housing and housing consumption $\varepsilon = .05$. We fix the relative

[13]If r is constant, the housing collateral ratio or the ratio of housing wealth to total wealth is $\frac{1/r}{1+1/r} = 1/(1+r)$. This is a very good approximation for the average collateral ratio in the model with stochastic r.

[14]The Gordon growth model is an approximation. Appendix C provides a detailed analysis of this asset value approach to calibrating the collateral share.

weight on housing in the utility function $\psi = 1$ throughout.[15] Because our goal is to explain conditional moments of the market return, we choose the parameter γ to match the unconditional market risk premium. We also compute the model for $\gamma \in \{2, 5, 10\}$ and $\varepsilon \in \{.15, .75\}$. A choice for the parameter ε implies a choice for the volatility of rental prices:

$$\sigma(\Delta \log \rho_{t+1}) = \left| \frac{1}{\varepsilon - 1} \right| \sigma(\Delta \log r_{t+1}). \tag{11}$$

In NIPA data (1930-2004), the left hand side of (11) is .046 and the right-hand side is .041. The implied ε is .098. By choosing a low ε, we impose that rental prices are consistent with the expenditure ratio. A choice for ε closer to one helps to generate a higher average equity premium and lower risk-free rate, at the cost of excessive rental price volatility.

Stock Market Return We define the stock market return as the return on a *leveraged* claim to the aggregate consumption process $\{c_t^a\}$ and denote it by R^l. In the data, dividends are more volatile than aggregate consumption. We choose leverage parameter $\kappa = 3$, where $\sigma(\Delta \log d_{t+1}) = \kappa \sigma(\Delta \log c_{t+1}^a)$.[16] We also price a non-levered claim on the aggregate consumption stream, denoted R^c. The excess returns, in excess of a risk-free rate, are denoted $R^{l,e}$ and $R^{c,e}$. Table 1 summarizes the benchmark parametrization and the other values we consider in the sensitivity analysis.

Computation Our computational strategy is to keep track of cross-sectional distributions over wealth and endowments that change over time. Appendix B provides the algorithm.

4 Model Meets Twentieth Century Data

We explore the model's long-run predictions, using the last seven decades in the US as a testing ground. The value of housing wealth to income shifts dramatically over this period.

[15]The Arrow-Pratt measure of relative risk aversion $\frac{-cu_{cc}}{u_c} = (\frac{r_t}{1+r_t})\gamma + (\frac{1}{1+r_t})\varepsilon^{-1}$ is a linear combination of γ and ε with weights depending on the non-durable expenditure ratio r_t. In the simulations $r = 4.26$ on average, so that the weight on γ is .81 on average. Because r_t is not very volatile, neither is the degree of risk aversion.

[16]For the period 1930-2004, the volatility of annual nominal dividend growth is 14.8%, whereas the volatility of annual nominal consumption growth (non-durables and services excluding housing services) is 5.6%, a ratio of 2.6.

Table 1: Parameter Calibration

Parameter	Benchmark	Sensitivity Analysis
γ	8	[2,5,10]
ε	.05	[.15,.75]
ψ	1	·
$\bar{r}$	4.26	·
ρ_r	.96	·
b_r	.93	0
σ_r	.03	·
$E[my]$	.05	.10
λ	[1.04,.96]	·
η	[.6578,.7952,.3422,.2048]	[.6935,.6935,.3065,.3065]
κ	1	3

At the onset of the Great Depression, the mortgage-to-income ratio increases from 25 percent to 50 percent because house prices do not decline as quickly as national income. The ratio subsequently decreases to a minimum of 12 percent by the end of WW-II. After that, the ratio increases almost without interruption to a value of 100 percent today. We focus on three key features of the data: (i) the decline in the volatility of returns and the risk-free rate, (ii) the low-frequency variation in the average risk-free rate, and (iii) the long-run decline in the equity premium since WW-II. Taking as given the observed evolution of the housing collateral ratio, the model replicates all three features.

We use two distinct measures of the housing collateral stock: the value of outstanding home mortgages (MO) and the market value of residential real estate wealth (RW). The data are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds (Federal Board of Governors) for 1945-2001. We use both the value of mortgages and the total value of residential wealth to allow for changes in the extent to which housing can be used as a collateral asset. National income is labor income plus net transfer income from the Historical Statistics of the US for 1926-1930 and from the National Income and Product Accounts for 1930-2001.[17]

[17] The data appendix in Lustig and Van Nieuwerburgh (2005) provides detailed sources.

We feed the observed aggregate consumption growth shocks and the observed housing collateral ratio between 1929 and 2003 into the model. To match the frequency of recessions in the data, we define a recession as a year in which aggregate consumption growth drops one standard deviation below its sample mean. We use both measures of the collateral ratio: mortgages to national income, $\frac{MO_t}{Y_t}$, and residential wealth to national income $\frac{RW_t}{Y_t}$. We equate the percentage deviations of $\{\frac{MO_t}{Y_t}\}$ and $\{\frac{RW_t}{Y_t}\}$ from their sample average in the data to the percentage deviations of my in the model by feeding in the right r_t process.

Declining Risk Premium and Risk-free Rate Volatility The first panel of Table 2 documents a stunning long-run decrease in the volatilities of excess stock market returns and risk-free rates. The standard deviation of excess stock returns declines from 30% in the 1930s to 10% in the 1990s, while the standard deviation of the risk-free rate declines from around 7% to 2%. While inflation was more volatile in the early decades, the volatility of the risk-free rate cannot be accounted for by inflation surprises alone. The second column ($\sigma(r^f_{ex-post})$) reports annual risk-free rates computed from annualizing the difference between the monthly three-month T-bill rate minus the inflation rate in the same month. The third column subtracts the previous month's inflation rate instead ($\sigma(r^f_{ex-ante})$). The small difference between the two suggests this volatility is not exclusively due to inflation surprises. Most asset pricing models target a stable risk-free rate, but the stability of the risk-free rate is a recent phenomenon. Our model can account for this radical decline in volatility.

The model matches the volatility decline in returns. In the benchmark calibration (panel 2 of Figure 2), the standard deviation of the return on an un-levered claim to aggregate consumption declines from 36% percent in the 1930s to 12% in the 1990s when we use the mortgage-based collateral measure (column 1); it declines from 23% to 12% for the residential wealth-based measure (column 3). The model also delivers a steep decline in risk-free rate volatility: from 21% to 11% in column 2 and from 16% to 11% in column 4. While this decline is consistent with the data, the model induces too much volatility in the risk-free rate. A modified version of our model with Epstein and Zin (1991)-type preferences mitigates this problem.[18] An increase in the intertemporal elasticity of substitution to 0.2 from .125,

[18]The details of the model with Epstein-Zin preferences are available in a separate appendix.

while keeping the risk aversion coefficient constant at its benchmark value of 8, allows us to roughly match the volatility. Panel 3 shows that the risk-free volatility now declines from 10% to 3%, in line with the data. At the same time, this model preserves the steep decline in the stock return volatility: from 29% to 4%.

Level of the Risk-free Rate The risk-free rate is low when housing collateral is scarce, both in the model and in the data, because the demand for insurance pushes up the price of future consumption. To focus on the long-run dynamics we compute the 9-year moving average of the one-year risk-free rate in the data and in the model. The top row of Figure 3 plots the data, the bottom row plots the model-generated data; the left panel uses the mortgage-based measure and the right panel uses the residential wealth-based measure.

The data reveal a strong positive correlation between the long-run risk-free rate and the housing collateral measure: 0.75 in the left panel and 0.83 in the right panel. The initial increase in housing collateral in the late 1920s coincides with an increase in the risk-free rate. At the start of the 1930s, the risk-free rate declines precipitously and this decline coincides with a decline in the housing collateral ratio. During WW-II, the federal government did keep real interest rates artificially low. In the post-war period, the two series continue to co-move until the mid-1990s.

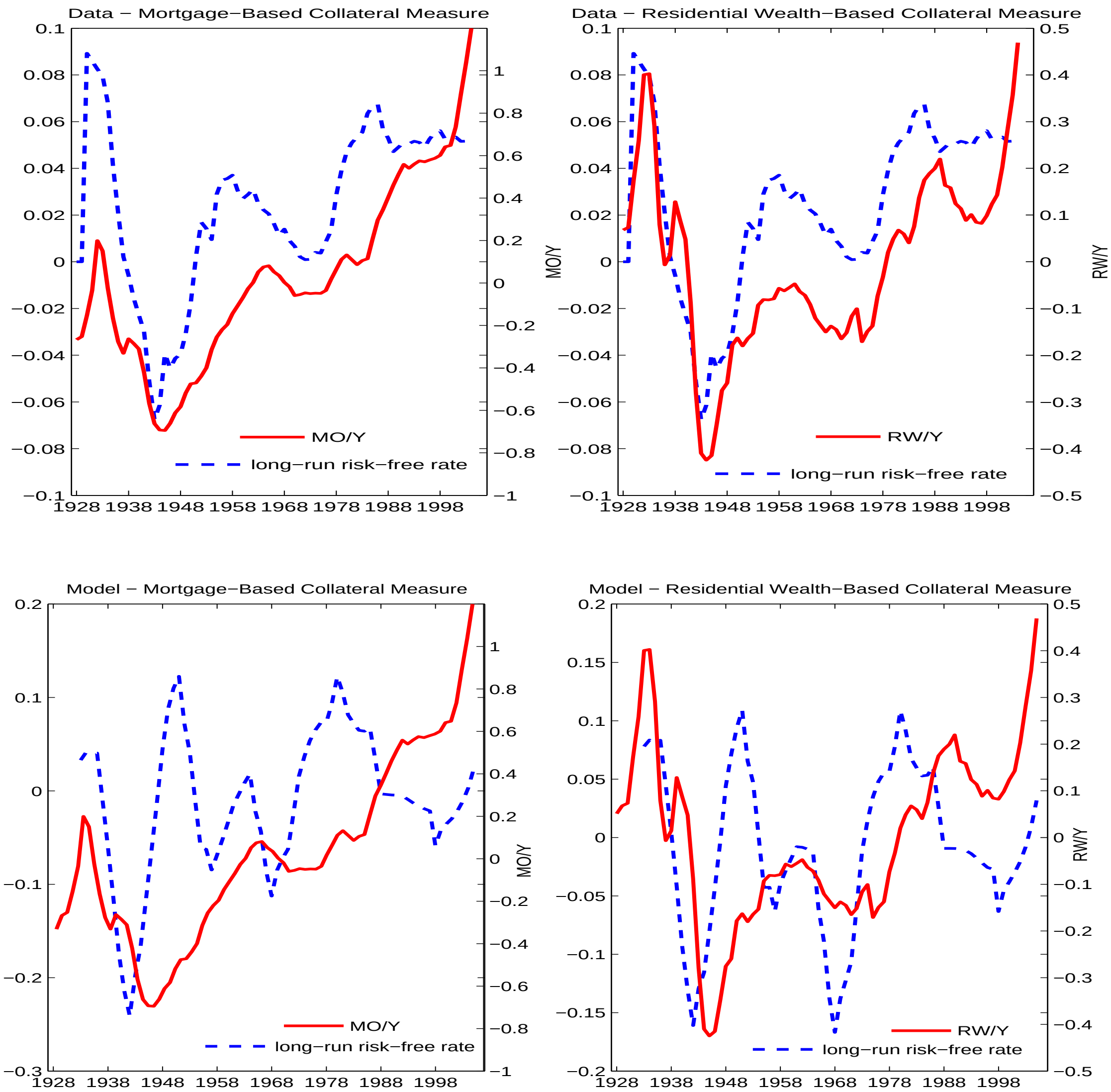
The model produces a similar low-frequency pattern for the risk-free rate. The bottom row of Figure 3 shows that the model predicts the decline in the risk-free rate in the Great Depression, the increase in the late 1940s, the decline in the 1960s, the rise in the 1970s, and the decline in the 1980s and early 1990s. Since the mid-1990s, the model predicts an increase in the risk-free rate because housing collateral has become more abundant. One notable divergence is that the increase in housing collateral in the last 10 years did not lead to a commensurate increase in the interest rate.[19]

Equity Premium Finally, our model generates a long-run decline in the equity premium as well as large unexpected return in the 1990s. Many authors have argued that the equity premium has declined substantially over the last four decades. Jagannathan et al. (2000)

[19] We conjecture that this may be due to the unprecedented outflow of tradeable wealth from the US in the last decade. This is a topic for future research; the current model abstracts from this.

Figure 3: Time-Variation in Risk-free rate: Data and Model.

The two plots in the first row of the figure plot the 9-year moving average of the annual T-bill rate and the collateral ratio in the data (dashed line). In the left panel the housing collateral ratio is measured as the percentage deviation of the mortgage-based collateral measure from its long-run trend (MO/Y). The right panel uses the residential wealth-based collateral measure (RW/Y). The annual risk-free rate is computed from monthly deflated T-Bill returns: $r^f_{t,t+1} = 12 \times ((1 + i_{t,t+1})^{(1/12)} - (1 + \pi_{t,t+1}))$, where i is the annual nominal holding period return on 3-month T-bills from CRSP and π is the monthly inflation computed from the BLS consumer price index. The panels in the second row plot the same statistics for the benchmark model. I.e., we feed in the observed aggregate consumption growth and housing collateral data and compute the model-implied 9-year moving average of the annual risk-free rate.



use Gordon's growth formula to back out the equity premium and conclude it has declined from 8% in the 1940s to 1% in the 1990s. Fama and French (2002) argue that, because of a decrease in the equity premium, capital gains were much higher than expected, especially in the 1990s. Because housing collateral became more abundant since the 1940s, our model delivers this slow decline in the equity premium. In the benchmark economy, the average equity premium ($E\left[E_t[R^e_{t+1}]\right]$) declines from 10.6% in the 1940s and a high of 11.2% in the 1960s to 5.5% in the 2000s (first column of Table 3). The model also generates the large unexpected capital gains of the 1990s. The second column reports the sample average of the realized excess return in each decade, $E[R^e_{t+1}]$. The realized return in the 1990s is 15.4%, much higher than the equity premium of 7.5%.

The decade-by-decade averages somewhat understate the extent of the decline in the equity premium. The left panel on the top row of Figure 4 contrasts the low and the high frequency variation by plotting the model-predicted annual equity premium (dashed line) alongside the 9-year moving-average (solid line). The vertical bars denote recession years. The equity premium is always higher at the onset of a recession. The equity premium peaks at 15% in the early 1940s, while it reaches a low of 3.5% in 2002. At the same time, the conditional volatility of excess stock returns declines from a high of 25% in the early 1930 to a low of 15% in 2002 (left panel on the middle row). Over the same period, the conditional Sharpe ratio declines from .70 to .35 (left panel bottom row).

The predicted variation in conditional excess return moments looks similar to the data. The right panels of figure 4 plot the empirical counterpart to the equity premium, the conditional volatility and the conditional Sharpe ratio of excess returns. To construct these measures, we project realized excess stock return and its realized volatility (constructed from daily data) on the housing collateral measure and the real risk-free rate. Because the housing collateral ratio is slow-moving, we can interpret the projected series as capturing a long-run equity premium and long-run conditional volatility. The conditional Sharpe ratio is the ratio. The equity premium also peaks in the early 1940s around 15% and declines to 5% at the end of the sample (top right panel). The conditional volatility also goes from 25% to 15% (middle panel), and the Sharpe ratio falls by more than half (bottom panel).

5 Time-Varying Asset Returns

To conclude, we show that the model captures two more important features of conditional asset pricing moments: (i) the same return predictability as in the data, and, (ii) highly volatile Sharpe ratios. At the end of this section, we summarize our findings for unconditional asset price moments.

Long Horizon Predictability If expected returns are vary over time with the housing collateral ratio, then we should find that the housing collateral ratio predicts returns. An important question is whether the model can quantitatively replicate the predictability coefficients found in the data. Panel 1 of table 4 shows results from predictability regressions of long-horizon excess returns on the lagged housing collateral scarcity measure in the data.[20] Results are reported for horizons up to 8 years and for two samples, and are taken from Lustig and Van Nieuwerburgh (2005). The main findings are that excess returns are higher when collateral is scarce ($b_1 > 0$). The effect becomes larger and statistically more significant with the horizon and the R^2 increases. Panel 2 shows that the model replicates the pattern of predictability coefficients surprisingly well. It reports regression results *inside* the model of excess returns on our measure of housing collateral ratio scarcity. When housing collateral is scarce (my is low), the excess return is high. The magnitude of the slope coefficients is close to the one we find in the data. Moreover, the R^2 of the predictability regression increase with the predictability horizon, just as in the data. We find this negative relationship between my_t and the excess return for a non-levered claim, as well as for a levered claim to aggregate consumption ($\kappa = 3$).

Sharpe Ratio in the Data Does the model generate enough volatility in the Sharpe ratio and does the Sharpe ratio co-move correctly with the housing collateral ratio? To evaluate our model against the data, we estimate the Sharpe ratio on annual data from

[20]Collateral scarcity is measured as $\widetilde{my}_t = \frac{\max(my_t)-my_t}{\max(my_t)-\min(my_t)}$, where $\max(my_t)$ and $\min(my_t)$ are the sample maximum and minimum of $\{my_t\}$. This ratio is always between 0 and 1. The measure is based on outstanding residential mortgages. Collateral is scarcer when my_t is lower. The housing collateral ratio my_t is estimated as the residual from a cointegration relationship between MO and Y, and is therefore a stationary variable. Details are provided in Lustig and Van Nieuwerburgh (2005) and the data are downloadable from the authors' web sites.

1927-1992 and compare it to the variation in the Sharpe ratio generated by the model. The conditional mean return is the projection of the excess return on the housing collateral ratio, the dividend yield and the ratio of aggregate labor income to consumption, all of which have been shown to forecast annual returns.[21] Likewise, the conditional volatility is the projection of the standard deviation of intra-year monthly returns on the same predictors. We form the Sharpe ratio as the ratio of the predicted excess returns and predicted volatility. Table 5 shows the estimation results for 1 year returns (column 1), but also for 5 year and 10-year cumulative excess returns (columns 2 and 3). The last three rows of the table indicate the unconditional mean and standard deviation of the Sharpe ratio as well as its correlation with the housing collateral ratio. In the estimation, the correlation between the Sharpe ratio and the measure of collateral scarcity $\widetilde{my}$ is positive in the data and equal to .25, .32, and .50 for 1, 5 and 10 year cumulative excess returns. The volatility of the Sharpe ratio on 1, 5 and 10 year cumulative excess returns is .10, .18, and .20. Lettau and Ludvigson (2003) even report a volatility of .45 for quarterly returns between 1952 and 2000. Similar to the data, our model generates volatile Sharpe ratios, and Sharpe ratios that co-move correctly with the housing collateral ratio. The correlation between the Sharpe ratio and $\widetilde{my}$ is also positive: .50, .59 and .39 for 1, 5 and 10 year cumulative excess returns on a non-levered consumption claim. The unconditional Sharpe ratio volatility is .40, .42, .40. Other models have a hard time generating this much volatility. For example, the unconditional standard deviation of the Sharpe ratio is .09 for the Campbell and Cochrane (1999) model and the consumption volatility model of Lettau and Ludvigson (2003). The volatility of the Sharpe ratio in the representative agent model is even smaller.

Unconditional Asset Pricing Moments Finally, Appendix D discusses the model's unconditional asset pricing moments reported in Table 6. The model matches the mean equity premium and its volatility, the mean Sharpe ratio, and the mean risk-free rate for the benchmark parametrization. A representative agent economy is unable to deliver these results, even if preferences are non-separable between housing and non-housing consumption. Our model has one major drawback: it generates too much volatility in the risk-free rate.

[21]See Lustig and Van Nieuwerburgh (2005), Lettau and Ludvigson (2001), and Santos and Veronesi (2006) respectively.

We show that a modest increase in the elasticity of intertemporal substitution goes a long way towards mitigating this problem. Furthermore, the lower volatility of the risk-free rate in the post-war era masks an important, and often overlooked stylized fact: The volatility of the risk-free rate has changed substantially from decade to decade in the US. We argued in section 4 that this is not simply a measurement issue due to changes in inflation volatility, and we showed that our model is consistent with this long-run decline in risk-free rate volatility.

6 Conclusion

Our paper shows how endogenous, state-contingent borrowing constraints interact with the housing market to deliver plausible asset pricing predictions. Equilibrium changes in the value of the housing stock change the degree to which risk sharing takes place. The housing collateral mechanism, in combination with wealth distribution shocks, endogenously generates time-varying volatility in the Sharpe ratio on equity. When confronted with the actual aggregate consumption growth and housing collateral ratio series, the model delivers the same low frequency changes in the level and volatility of the equity premium and the risk-free rate than those we document in the data. It generates the same predictability patterns and volatile Sharpe ratios as in the data.

This paper is part of a broader research agenda. In Lustig and Van Nieuwerburgh (2006a), we show that the same model is also able to deliver a quantitatively meaningful return spread on book-to-market sorted portfolios and link this feature to the term structure of equity risk premia. In Lustig and Van Nieuwerburgh (2006b), we test the model's risk sharing implications using quantity data only. We show that regional consumption growth in the US is more cross-correlated when the housing collateral supply increases. Conversely, less risk sharing takes place when collateral is scarce. This finding offers direct support for the collateral mechanism.

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A Technical Appendix

This section contains the proofs of the propositions in the main text. For more details on the model (definition of the cumulative multipliers, derivation and optimality of the risk sharing rule and the optimality of the law of motion for the cumulative multipliers), we refer the reader to section 2 of the separate appendix to this paper, available on our web sites.

Condition 1 Section 2 in the separate appendix explains the equivalence between the static and sequential budget constraints and solvency constraints. This equivalence holds only if interest rates are high enough (see Alvarez and Jermann (2000)). We impose the following condition. Let η_{max} denote the highest possible labor endowment realization in each future, aggregate node z^t.

Condition 1. *Interest rates are said to be high enough if*

$$\Pi_{z_0,y_0}[\{\eta_{max}\}] < \infty,$$

This is the equivalent of the condition in Alvarez and Jermann (2000) that interest rates be high enough, translated to an economy with a continuum of consumers. In an economy with a finite number of agents, it is sufficient to require the time zero value of a claim to the aggregate endowment to be finite, but here it is not sufficient for the value of a claim to the average endowment to be finite. □

Proof of Proposition 2 Denote the price of a claim under perfect risk-sharing by $\Pi^*[\{\cdot\}]$. Perfect risk sharing can be sustained if and only if

$$\Pi_z^*\left[\left\{c^a\left(1+\frac{1}{r}\right)\right\}\right] \geq \Pi_{z,y}^*\left[\{\eta(y,z)\}\right]$$

for all (y,z,r) with nonzero measure; Φ is the joint measure defined on $\mathcal{P}(Y)\times\mathcal{P}(Z)\times\mathcal{B}(R)$. If this condition is satisfied, each household can get a constant and equal share of the aggregate non-durable and housing endowment at all future nodes. That immediately implies that perfect risk-sharing is feasible. If there is a value r_{max} such that any $r' > r_{max}$ is measure zero, then perfect risk sharing can be sustained if

$$\Pi_z^*\left[\left\{c^a(z)\left(1+r_{max}^{-1}\right)\right\}\right] \geq \Pi_{z,y}^*\left[\{\eta(y,z)\}\right] \text{ for all } (y,z)\in Y\times Z$$

This condition is sufficient, but not necessary. □

Proof of Proposition 3 Assume utility is separable. Let $C(\ell,y_t,z^t)$ denote the cost of claim to consumption in state (y_t,z^t) for a household who enters the period with weight ξ . The cutoff rule $\underline{\xi}(y_t,z^t)$ is determined such that the solvency constraint binds exactly: $\Pi_{y,z^t}\left[\{\eta\}\right] = C(\underline{\xi},y_t,z^t)$, where $C(\xi, y_t, z^t)$ is defined recursively as:

$$C(\xi, y_t, z^t) = \frac{\underline{\xi}(y_t, z^t)}{\xi_t^a(z^t)}\left(1 + \frac{1}{r_t}\right) + \delta \sum_{z_{t+1}} \pi(z_{t+1}|z_t) \sum_{y'} \frac{\pi(y_{t+1}, z_{t+1}|y_t, z_t)}{\pi(z_{t+1}|z_t)} m_{t+1}(z^{t+1}) C(\xi', y_{t+1}, z^{t+1}),$$

and ξ' is determined by the cutoff rule (6). Note that the stochastic discount factor $m_{t+1}(z^{t+1})$ does not depend on $r_t(z^t)$ because we assumed that utility is separable. This also implies that the cost of a claim to labor income $\Pi_{y,z^t}[\{\eta\}]$ does not depend on r.

We prove the result for a finite horizon version of this economy. We first assume some arbitrary state prices $\{p_t(s^t|s_0)\}$ for both of these economies. $\{m_t(z^t)\}$ denotes the SDF process implied by these state prices. Finally, we use T^i to denote the operator that maps the aggregate weight functions $\{\xi_t^a(z^t)\}$ we start with into a new aggregate function $\{\xi_t^{\prime,a}(z^t)\}$.

In the last period T, the cutoff rule is determined such that:

$$\eta(y_{T-1}, z^{T-1}) = \frac{\underline{\xi}(y_{T-1}, z^{T-1})}{\xi_T^a(z^{T-1})}\left(1 + \frac{1}{r_{T-1}(z^{T-1})}\right) + \delta \sum_{z_{t+1}} \pi(z_T|z_{T-1})$$
$$\sum_{y'} \frac{\pi(y_T, z_T|y_{T-1}, z_{T-1})}{\pi(z_T|z_{T-1})} m_T(z^T|z_{T-1}) \left(\frac{\xi'^{1/\gamma}}{\xi_T^a(z^T)}\left(1 + \frac{1}{r_T(z^T)}\right) - \eta(y_T, z^T\right),$$

where $\frac{\xi'^{1/\gamma}}{\xi_T^a(z^T)}\left(1 + \frac{1}{r_T}\right) \geq \eta\left(y_T, z^T\right)$. Given $r_T^1 < r_T^2$ and $r_{T-1}^1(z^{T-1}) < r_{T-1}^2(z^{T-1})$, this implies that $\ell^{1,c}(y_{T-1}, z^{T-1}) < \ell^{2,c}(y_{T-1}, z^{T-1})$ for all (y_{T-1}, z^{T-1}). By backward induction we get that, for a given sequence of $\left\{\xi_t^a(z^t)\right\}$, $\ell^{1,c}(y_t, z^t) < \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t) in the finite horizon economy. This in turn implies that $T^1(\left\{\xi_t^a(z^t)\right\}) \leq T^2(\left\{\xi_t^a(z^t)\right\})$ for all z^t, with strict inequality if at least one of the constraints binds. This follows directly from the definition of

$$\begin{aligned}
\xi_t^a(z^t) &= \sum_{y^t} \int \xi_t(\ell, y^t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \\
&= \sum_{y^t} \int_{\underline{\xi}(y_t, z^t)} \xi_{t-1}(\ell, y^t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \\
&\quad + \sum_{y^t} \int^{\underline{\xi}(y_t, z^t)} \underline{\xi}(y_t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \qquad (12)
\end{aligned}$$

$\xi_t^a(z^t)$ is non-decreasing in $\underline{\xi}(y_t, z^t)$. The proof extends to the infinite horizon economy if the transition matrix has no absorbing states. The reason is that $lim_{T\to\infty} E_t\left[\beta^{T-t} m_T(z^T|z_t)\pi_{z^T, y_T}\right]$ does not depend on the current state (y_t, z_t). Now, this also implies a new state price function, for each s^t:

$$p_t'(s^t|s_0) = p_t^a(s^t|s_0) \frac{\xi_t^a(z^t)^{\gamma}}{\xi_0^a(z^0)},$$

where $p_t^a(s^t|s_0)$ is the representative agent state price. So if we start with the equilibrium state prices for the second economy $\{p_t^2(s^t|s_0)\}$, the implied aggregate weights for the first economy will be smaller:

$$T^1\left(\{\xi_t^{2,a}(z^t)^\star\}\right) < \{\xi_t^{2,a}(z^t)^\star\} = T^2\left(\{\xi_t^{2,a}(z^t)^\star\}\right),$$

where the last equality follows because we started with the equilibrium prices for the second economy, and similarly,

$$T^1(\{\xi_t^{1,a}(z^t)^\star\}) = \{\xi_t^{1,a}(z^t)^\star\} < T^2(\{\xi_t^{1,a}(z^t)^\star\}),$$

if we start with the equilibrium prices in the first economy. Now, it can be shown that $T^i(\{\xi_t'(z^t)\}) \leq T^i(\{\xi_t(z^t)\})$ if $\{\xi_t'(z^t)\} > \{\xi_t(z^t)\}$ (Lustig (2003). Finally, using the previous results:

$$T^1\left(\{\xi_t^{2,a}(z^t)^\star\}\right) < \{\xi_t^{2,a}(z^t)^\star\} \textit{ and } T^1(\{\xi_t^{1,a}(z^t)^\star\}) = \{\xi_t^{1,a}(z^t)^\star\},$$

we obtain that $\{\xi_t^{1,a}\left(z^t\right)^\star\} < \{\xi_t^{2,a}\left(z^t\right)^\star\}$. □

Proof of Corollary 1 Follows from the definition of the cutoff level in the previous proof. For a given sequence of $\left\{\xi_t^a(z^t)\right\}$, it is obvious that $\ell^{1,c}(y_t, z^t) < \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t). This in turn implies that $\left\{\xi_t^{a,1}(z^t)\right\} \leq \left\{\xi_t^{a,2}(z^t)\right\}$. This follows directly from the definition of the aggregate weight shock (12). As a result, $\xi_t^a(z^t)$ is non-decreasing in $\underline{\xi}(y_t, z^t)$. This implies the state prices at time 0 for consumption to be delivered in s^t are higher, and this is true for all nodes s^t.

Interest rates between time zero and time t are given by $R_{0,t}^f = E_0[M_{0,t}]^{-1}$, where the pricing kernel between time 0 and time t is $M_{0,t} = m_0 \cdot m_1 \cdots m_t$. A lower aggregate weight shock $\frac{\xi_t^a}{\xi_0^a}$ at time t *in all nodes* s^t implies a lower pricing kernel on average and higher interest rates on average. □

B Computing Stationary Equilibria

In this appendix we show how to compute stationary equilibria. As we noted in section 2, the aggregate weight shock depends on the entire history of aggregate shocks z^∞. To avoid the curse of dimensionality, we follow Lustig (2003) and truncate aggregate histories. Households only keep track of the last k lags of the aggregate state, $z_t^k = (z_t, z_{t-1}, \cdots, z_{t-k})$, and the current expenditure ratio $r_t(z^t)$. The current expenditure ratio r_t contains additional information not present in the truncated history z_t^k, namely r_{t-k}. We use $\mathcal{R}$ to denote the ergodic set for the process r. For a household starting the period with weight $\xi \in \Xi$, the policy function $l(y', z'; \xi, r, z^k) : \Xi \times \mathcal{R} \times Z^k \rightarrow \mathbb{R}$ produces the new individual weight in state (y', z'). There is one policy function $l(\cdot)$ for each pair $(y', z') \in Y \times Z$. The policy function $g^*(z'; r, z^k) : \mathcal{R} \times Z^k \rightarrow \mathbb{R}$ forecasts the aggregate weight shock when moving to state z' after history (z^k, r).

Competitive Equilibrium. *A stationary stochastic equilibrium is a time invariant distribution* $\Phi^*_{(r,z^k)}(\xi, y)$ *over individual weights, individual endowments, current expenditure ratio, and truncated aggregate histories, and updating rules* $l(\cdot)$ *and* $g^*(\cdot)$. *For each* $\left(z^{k\prime}, z^k\right)$ *with* $z^{k\prime} = \left(z', z^k\right)$, *the distribution satisfies:*

$$\Phi^*_{(r,z^{k\prime})} = \sum_{z^k} \pi(z^{k\prime}|z^k) \int Q\left(\xi, y, r, z^k\right) \Phi^*_{(r,z^k)}(d\xi \times dy)$$

where $Q\left(\xi, y, r, z^k\right)$ *is the transition function induced by the policy functions. The forecast of the aggregate weight shock is given by:*

$$g^*(z'; r, z^k) = \sum_{y' \in Y} \int l(y', z'; \xi, r, z^k)^{\frac{1}{\gamma}} \Phi^*_{r,z^k}(d\xi \times dy) \frac{\pi(y', z'|y, z)}{\pi(z'|z)}, \ \forall z' \in Z.$$

Intertemporal prices are pinned down by the stochastic discount factor in equation (8), using the forecasted shock $g^*(\cdot)$ as an approximation to the actual $g(\cdot)$. For any given realization $\{z\}$, the actual aggregate weight shock $g(\cdot)$ differs from the forecast $g^*(\cdot)$ because the distribution over individual weights and endowments $\Phi^*(\cdot)$ differs from the actual distribution $\Phi(\cdot)$, which depends on z^∞. The definition of a stationary equilibrium implies that, *on average, across aggregate histories*, $\Phi^*(\cdot) = \Phi(\cdot)$ and markets clear: The difference between actual consumption and consumption based on a truncated history is zero on average, but not state-by-state. In each state z', the approximation error equals the percentage difference between the actual aggregate weight shock and aggregate weight shock based on a truncated history:

$$c^a(z'; r, z^k) - c^a(z'; z^\infty) = \frac{g^*(z'; r, z^k) - g(z'; r, z^\infty)}{g^*(z'; r, z^k)} \tag{13}$$

This is the difference between consumption and the endowment. As the truncation parameter k increases, the approximation error decreases because market clearing holds on average in long histories. We use $k = 5$ lags in all our computations. The percentage allocation errors in (13) provide a clear measure of the closeness to the actual equilibrium. For our benchmark calibration, the average error in a simulation of 10,000 periods is only 0.0011 with standard deviation .0035. The largest error in absolute value is 0.0282.

We compute the approximating equilibrium as follows. The aggregate weight shock process is initialized at the full insurance value ($g^* = 1$) and the corresponding stochastic discount factor is computed. The cutoff rule for the individual weight shocks ensure that the solvency constraints hold with equality. Then we generate a panel of data by simulating the model: $\{z_t\}_{t=1}^T$ for $T = 10,000$ and $\{y_t\}_{t=1}^T$ for a cross-section of $5,000$ households. For each truncated history, we store $\left\{\frac{l(y',z';\xi,r,z^k)^{\frac{1}{\gamma}}}{g^*(z';r,z^k)}\right\}$ as the household's identifying label ξ (the re-scaling keeps the state variables stationary), and compute the sample mean of the aggregate weight shock $\{g_t^*(z', r, z^k)\}_{t=1}^T$ and the resulting stochastic discount factor $\{m_t^*(z', r, z^k)\}_{t=1}^T$. A new cut-off rule is computed with these new forecasts. These two steps are iterated on until convergence.

C Asset Value Approach to Calibrating Housing Collateral Ratio

In the main text we used factor payments on collateralizable and non collateralizable wealth to calibrate the housing collateral ratio. Those data were taken from Table 1.12. National Income by Type of Income (NIPA), except for the net interest series which comes from Table 1.13 (net interest paid by domestic corporations, line 8). Here we describe a second approach, based on measuring collateral wealth directly. More precisely, we compare the ratio of *collateral wealth to total income* in the model and in the data.

We start by measuring the collateral wealth-to-income ratio in the data. Housing collateral wealth is measured as the market value of outstanding mortgages. The residential mortgage series is from the Flow of Funds Tables and is available for the post-war period. Over that period, the average ratio of residential mortgages to labor income plus rental income is 0.55. Financial wealth is measured as the market value of non-farm non-financial corporations in the US This series is constructed based on Flow of Funds data; Lustig and Van Nieuwerburgh (2006c) provides the details. Our broad measure of total collateral wealth to total income is constructed as the ratio of residential mortgages plus financial wealth to labor income plus interest income plus dividend income plus proprietor's income. That ratio is 1.55 in post-war data.

We compare these numbers to the housing collateral wealth-to-income ratio in the model. More precisely, we fix an average housing collateral ratio my, simulate the model for a long period, and compute the housing wealth to total income ratio. When $my = 0.05$, the collateral wealth to income ratio is 0.90, in between the narrow and the broad empirical measure.

What is the effect of a higher housing collateral ratio in the model? When my is higher than 0.05, the collateral wealth-to-income ratio is actually *lower* than 0.90. To understand this, consider the Gordon growth formula:

$$\frac{collateral\,wealth}{total\,income} = \frac{\frac{collateral\,income}{total\,income}}{R-g},$$

where R is the expected rate of return on total wealth and g is the growth rate of total income. The numerator is effectively the housing collateral ratio, for example 0.075. In the denominator, the aggregate endowment growth rate g is the same across calibrations. Not so for the discount rate R. This discount rate is the sum of the risk premium on a claim to aggregate consumption (the equity premium) and the risk-free rate. In an economy with more collateral, the equity premium goes down, but the interest rate goes up. This interest rate effect dominates the risk premium effect, so that the denominator is increasing in my. It does not help to increase my to generate a large collateral wealth to total income ratio. We find a higher wealth to income ratio of 0.96 for $my = 0.035$ than the 0.90 for $my = 0.05$ and the 0.84 for $my = 0.075$.

The main point is that, with a five percent collateral ratio, our benchmark model allows for a lot of collateral: 90% of the value of national income on average.

D Unconditional Asset Pricing Moments

The model succeeds in matching most unconditional asset pricing moments, when we set γ equal to eight, except for the volatility of the risk-free rate.

Risk Premium Table 6 compares the unconditional first and second moments of asset returns in US data (panel 1), in the collateral model (panels 2 and 4), and in the representative agent model (panel 3). The benchmark calibration in panel 2 generates a 8.6% risk premium on an un-levered equity claim, with a volatility of 21.7%. These numbers line up with the 7.9% excess return in the data and its 20.7% volatility. The Sharpe ratio is 0.397, close to the 0.384 Sharpe ratio observed in 1927-2004. Because consumption growth is less volatile in the data than dividend growth, we also compute a levered claim to aggregate consumption in the model ($\kappa = 3$). The model with lower risk aversion ($\gamma = 5$) now also generates a sizeable and volatile (levered) risk premium: 3.9% expected excess return with 16.7% standard deviation. We contrast this with a representative agent economy. The equity premium on an un-levered (levered) consumption claim is less than one-third (one-half) as big as in the collateral model, even though preferences are non-separable between non-housing and housing consumption. Finally, doubling the collateral ratio to 10% brings this economy closer to the representative agent economy because the solvency constraints are looser. The expected excess return on a levered consumption claim is still high (5.7%) and volatile (21%) for ($\gamma = 8, \varepsilon = .05$).

Risk-free Rate The model with $\gamma = 8$ ($\gamma = 5$) generates an average risk free rate of 2.6% (7.7%), close to the 1.9% in the 1871-1979 data. The risk of binding collateral constraints increases the expected SDF more when risk aversion γ is high, and pushes down the risk-free rate. There are two reasons for this fall in the risk-free rate: Households cannot borrow as much, and they accumulate more precautionary savings. When households are more risk averse, the precautionary motive is stronger. They bid up the price of risk-free assets which provide insurance against the risk of binding constraints. In contrast, the risk-free rate *increases* with γ in the representative agent economy (panel 3). A more risk-averse representative agent is less willing to substitute intertemporally and wants to borrow more against growing labor income; this drives up the risk-free rate. The level of the risk-free rate is much too high: 15.8% in the benchmark calibration.

The biggest shortcoming of the collateral model is the high unconditional volatility of the risk-free rate. It is 7% in the economy with $\gamma = 5$, 15.6% in the economy with $\gamma = 8$, but only 4.2% in the 1927-2002 data and 5.2% in the 1871-1979 data. When the average collateral ratio is 10% instead, the risk-free rate is higher but less volatile (panel 4). Two forces drive this volatility: variation in the expected fraction of households facing binding constraints due to shocks to the wealth distribution (at higher frequencies), and shocks to the risk-sharing technology due to changes in value of housing collateral (at lower frequencies). Both modulate the demand for insurance. While our benchmark calibration generates more risk-free rate volatility that other heterogenous agent models (e.g., 5% in Alvarez and Jermann (2001) and **?**), these models typically don't generate time-variation in equity premia because their market prices of risk are constant. As we emphasized in section 4, risk-free rates are far from constant. Our model generates a large decline in the risk-free rate volatility between the 1930s and the 1990s, a pattern consistent with the data.

Sensitivity: Recursive Utility One way to mitigate the risk-free rate volatility is to use recursive preferences to de-couple risk aversion from the willingness to substitute consumption over time. Modestly raising the intertemporal elasticity of substitution from 0.0125 to 0.20, while keeping $\gamma = 8$, reduces the volatility of the risk-free rate from 15.6% to 6.9% (panel 5). The equity premium is still 4.9% with a volatility of 19.5%.The separate appendix reports more detailed results.

Sensitivity: Composition Effect A higher intratemporal elasticity of substitution (ε) increases the equity premium and the Sharpe ratio and lowers the risk-free rate (panel 6). This effect also shows up in the representative agent economy; Piazzesi et al. (2006) refer to it as a *negative composition* effect. Assets that pay off in low non-housing expenditure share growth states are risky as such states occur in recessions. An increase in ε from .05 to .75 increases the equity premium by 4% and decreases the risk-free rate by 4% in both our model and the representative agent economy. Detailed results for the representative agent economy are available upon request. However, these empirically plausible return moments come at the expense of an implausibly high rental price growth volatility. For $\varepsilon = .75$, the rental price growth volatility $\sigma(\Delta \log \rho)$ is 19% per annum (see equation 11), whereas observed rental price growth volatility is below 5%.Driving ε even closer to 1 leads to exponentially increasing rental price growth volatility. For $\varepsilon > 1$ the representative agent model generates a negative equity premium. We choose $\varepsilon = .05$ for our benchmark calibration because the rental price growth volatility then matches the data. A higher ε also increases risk-free rate volatility.

Sensitivity: Risk Aversion, Expenditure Share, and Income Heteroscedasticity Varying the coefficient of relative risk aversion γ from 2 to 10 (panel 7) increases the equity premium on a levered (non-levered) consumption claim from 0.8% to 15.7% (.3% to 13.3%). The Sharpe ratio increases from .07 to 0.55. The risk-free rate falls from 8.6% to -2.1%. When the expenditure share does not depend on consumption growth ($b_r = 0$ in the specification of r), the housing collateral ratio becomes less volatile. The equity risk premium is now 6.9% with volatility 23.7%. The risk-free rate is 1.8%, and its volatility is 3% lower than in the benchmark model (panel 8). Finally, we shut down the Mankiw (1986) mechanism by having income shocks with the same dispersion in booms as in recessions (panel 9). The housing collateral mechanism alone generates a sizeable 5.7% equity premium. The risk-free rate is 5% higher, but 5% less volatile.

Table 2: Decade-by-Decade Volatility of Asset Returns.

Panel 1 reports the volatility of excess stock returns and the risk-free rate in the data. R^e is the excess return on the CRSP value-weighted stock index. The nominal risk-free rate $i_{t,t+1}$ is the annual return on the CRSP 3-month T-bill rate. Inflation $\pi_{t,t+1}$ is computed from the BLS consumer price index series. Column 1 reports the annualized standard deviation of excess stock returns, computed as the sample standard deviation of monthly excess returns, where $R^e_{t+1} = 12 \times ((1 + R_{t,t+1}) - (1 + i_{t,t+1})^{1/12})$. The second column reports the annualized standard deviation of the ex-post risk-free rate $r^f_{t,ex-post} = 12 \times ((1 + i_{t,t+1})^{(1/12)} - (1 + \pi_{t,t+1}))$. To minimize the effect of inflation surprises, column 3 reports the annualized standard deviation of an ex-ante risk-free rate which subtracts out the previous month's inflation rate instead: $r^f_{t,ex-ante} = 12 \times ((1 + i_{t,t+1})^{(1/12)} - (1 + \pi_{t-1,t}))$. To compute decade-averages, we only use the last month in each year. Panel 2 reports the same statistics for the model under the benchmark parametrization. The model-simulated data were generated by feeding in observed aggregate consumption growth $\{z_t\}_{t=1929}^{T=2000}$ and the observed collateral ratio, measured either based on mortgages $\{\frac{MO_t}{Y_t}\}_{t=1929}^{T=2000}$ or on residential wealth $\{\frac{RW_t}{Y_t}\}_{t=1929}^{T=2000}$. If aggregate consumption growth at t is one standard deviation below the mean, z_t is the low consumption growth state, else z_t is classified as the high consumption growth state. This procedure matches the unconditional probability of a low aggregate consumption growth in the model to that in the data. In the model, R^e is the excess return on an un-levered claim to aggregate consumption. Panel 3 reports the same statistics as panel 2, but for a model with Epstein and Zin (1991) preferences. The intertemporal elasticity of substitution is set to 0.2 and the coefficient of relative risk aversion is held at its benchmark value of 8.

Panel 1: Data			
	$\sigma(R^e_t)$	$\sigma(r^f_{t,ex\text{-}post})$	$\sigma(r^f_{t,ex\text{-}ante})$
1931-1940	0.31	0.06	0.07
1941-1950	0.13	0.08	0.10
1951-1960	0.10	0.03	0.04
1961-1970	0.12	0.02	0.02
1971-1980	0.15	0.02	0.03
1981-1990	0.15	0.03	0.02
1991-2000	0.11	0.01	0.02

Panel 2: Benchmark Model				
	MO/Y		*RW/Y*	
	$\sigma(R^e_t)$	$\sigma(r^f_t)$	$\sigma(R^e_t)$	$\sigma(r^f_t)$
1931-1940	0.36	0.21	0.23	0.16
1941-1950	0.33	0.26	0.43	0.22
1951-1960	0.18	0.19	0.17	0.17
1961-1970	0.10	0.12	0.08	0.15
1971-1980	0.16	0.15	0.16	0.16
1981-1990	0.14	0.14	0.16	0.14
1991-2000	0.12	0.11	0.12	0.11

Panel 3: Model with Epstein-Zin Preferences				
	MO/Y		*RW/Y*	
	$\sigma(R^e_t)$	$\sigma(r^f_t)$	$\sigma(R^e_t)$	$\sigma(r^f_t)$
1931-1940	0.29	0.10	0.25	0.07
1941-1950	0.17	0.10	0.27	0.08
1951-1960	0.09	0.08	0.09	0.07
1961-1970	0.07	0.04	0.04	0.05
1971-1980	0.08	0.06	0.08	0.06
1981-1990	0.05	0.05	0.09	0.05
1991-2000	0.04	0.03	0.06	0.04

Table 3: Decade-by-Decade Equity Premium.

This table reports the benchmark model's predictions for the equity premium and the realized excess returns on stocks. $R^e_{t+1} = R^{c,e}_{t+1}$ is the excess return on an un-levered claim to aggregate consumption. We report the sample average of the conditional expected excess return $\widehat{E}(E_t[R^e_{t+1}])$ and the sample average of the realized excess return $\widehat{E}(R^e_{t+1})$ for each decade. The model-simulated data were generated by feeding in observed aggregate consumption growth $\{z_t\}_{t=1929}^{T=2000}$ and the observed collateral ratio, measured either based on mortgages $\{\frac{MO_t}{Y_t}\}_{t=1929}^{T=2000}$ (column 2 and 3) or on residential wealth $\{\frac{RW_t}{Y_t}\}_{t=1929}^{T=2000}$ (column 4 and 5).

	MO/Y		RW/Y	
	$\widehat{E}(E_t[R^e_{t+1}])$	$\widehat{E}(R^e_{t+1})$	$\widehat{E}(E_t[R^e_{t+1}])$	$\widehat{E}(R^e_{t+1})$
1931-1940	0.097	0.175	0.080	0.093
1941-1950	0.097	0.055	0.092	0.093
1951-1960	0.106	0.086	0.101	0.112
1961-1970	0.112	0.204	0.128	0.265
1971-1980	0.071	−0.015	0.077	−0.019
1981-1990	0.072	0.072	0.080	0.079
1991-2000	0.075	0.154	0.094	0.147
2001-2004	0.055	0.039	0.057	0.013

Figure 4: Time-Variation in Equity Premium: Model and US Data

This panel plots the conditional expected excess stock market return, its conditional standard deviation and the conditional Sharpe ratio in the model (left column) and in the data (right column). The model-simulated data were generated by feeding in observed aggregate consumption growth $\{z_t\}_{t=1929}^{T=2000}$ and the observed collateral ratio, measured based on mortgages $\{\frac{MO_t}{Y_t}\}_{t=1929}^{T=2000}$ into the benchmark model. The top panel plots the expected excess return on a claim to aggregate consumption. The middle panel plots the conditional standard deviation of the excess return on a claim to aggregate consumption. The bottom panel plots the conditional Sharpe ratio, the ratio of the expected excess return over its standard deviation. In the right column all conditional asset pricing moments are constructed from the data in the following way. The expected excess return, plotted in the top right panel, is computed by projecting the annual CRSP value-weighted stock return in excess of the annual return on the Fama 3-month T-bill return on the mortgage-based housing collateral ratio and the Fama 3-month T-bill return (less inflation over the previous year). The middle panel plots the conditional standard deviation of the excess return. It is computed from daily data as $\sum_{k \in year}(R_k^{CRSP} - (1+i)^{1/360})^2$. We use daily S&P500 data from Global Financial Data before 1960, and daily CRSP data afterwards. The expected standard deviation is computed by projecting this standard deviation on two lags, the mortgage-based housing collateral ratio and the Fama 3-month T-bill return (less inflation over the previous year). The bottom right panel plots the conditional Sharpe ratio, the ratio of the expected excess return over its standard deviation.

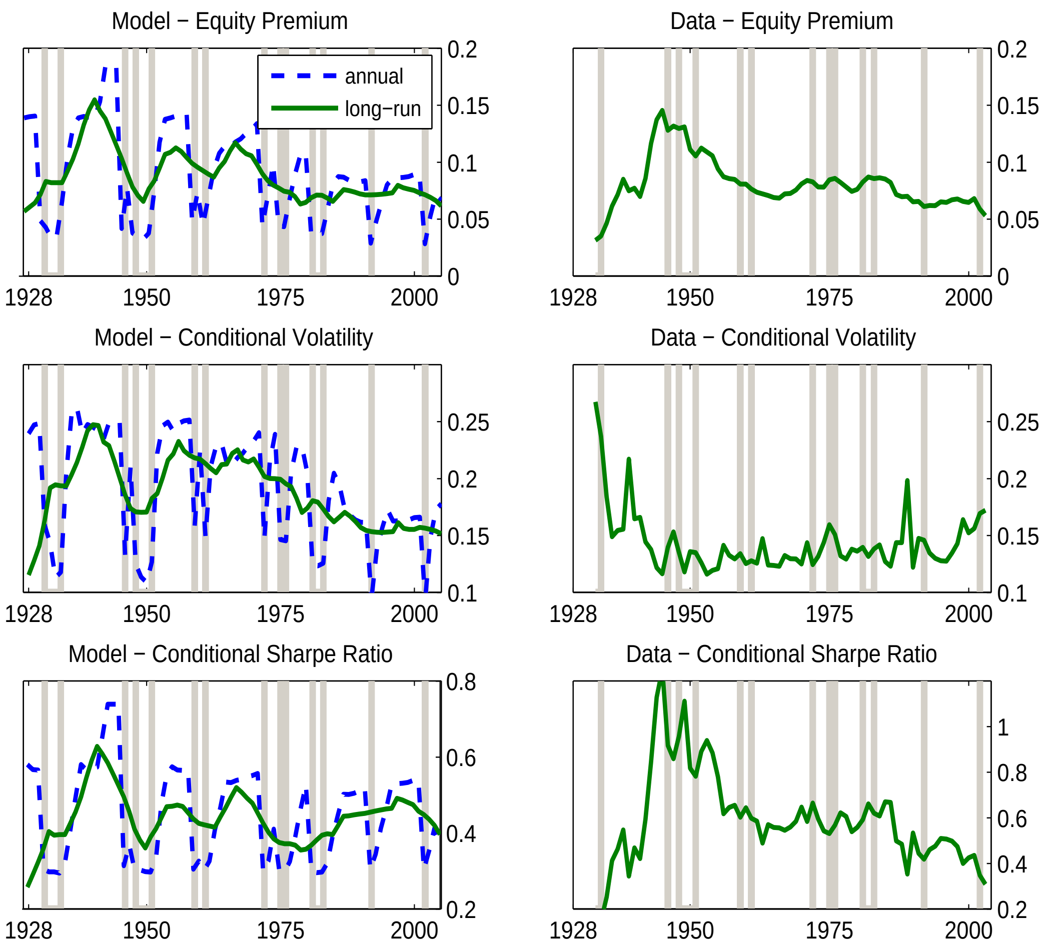

Table 4: Predictability of K-Year Excess Returns: Data and Model.

Results of regressing log K-horizon excess returns on the housing collateral ratio. The intercept is b_0, the slope coefficient is b_1. The first panel reports the results in the data. The t-stats in brackets are computed using the Newey West covariance matrix with K lags. The returns are cum-dividend returns on the value-weighted CRSP index. The collateral scarcity measure $\widetilde{my}_t$ is based on the market value of outstanding mortgages. The long sample contains annual data from 1930-2003. The post-war sample is from 1945-2003. The second panel reports the same regressions inside the model. The regressions were obtained by simulating the model for 10,000 periods under the benchmark parametrization with leverage parameter $\kappa = 1$ (left columns) and $\kappa = 3$ (right columns).

	b_0	b_1	R^2	b_0	b_1	R^2
			Panel 1: Data			
Horizon	**Entire Sample**			**Post-War Sample**		
1	0.01	0.15	0.02	0.02	0.15	0.04
	[0.24]	[1.25]		[0.63]	[1.73]	
2	0.07	0.25	0.03	0.06	0.31	0.07
	[0.66]	[1.11]		[0.83]	[1.75]	
3	0.16	0.31	0.03	0.12	0.46	0.09
	[1.02]	[0.97]		[1.04]	[1.80]	
4	0.29	0.32	0.02	0.22	0.56	0.09
	[1.50]	[0.80]		[1.42]	[1.82]	
5	0.38	0.47	0.03	0.34	0.66	0.08
	[1.83]	[1.16]		[1.67]	[1.77]	
6	0.41	0.79	0.07	0.43	0.89	0.10
	[1.70]	[1.73]		[1.58]	[1.86]	
7	0.41	1.20	0.12	0.49	1.26	0.14
	[1.32]	[2.11]		[1.30]	[2.02]	
8	0.37	1.80	0.18	0.50	1.79	0.19
	[0.91]	[2.71]		[1.01]	[2.26]	
			Panel 2: Model			
Horizon	**Leverage =1**			**Leverage=3**		
1	−0.00	0.13	0.01	0.03	0.12	0.00
2	−0.07	0.32	0.02	0.01	0.29	0.01
3	−0.16	0.54	0.03	−0.04	0.47	0.01
4	−0.29	0.81	0.04	−0.12	0.70	0.02
5	−0.45	1.11	0.06	−0.23	0.96	0.03
6	−0.62	1.42	0.07	−0.36	1.24	0.04
7	−0.81	1.77	0.09	−0.50	1.54	0.05
8	−1.01	2.12	0.11	−0.65	1.86	0.05

Table 5: Long-Term Sharpe Ratios in Data.

the table reports coefficient estimates for $R^e_{t+1} = b_0 + b_1 R_t + b_2 dp_t + b_3 lc_t + b_4 \widetilde{my}_t + \varepsilon_{t+1}$ and $Vol_{t+1} = a_0 + a_1 dp_t + a_2 lc_t + a_3 \widetilde{my}_t + a_4 Vol_t + a_5 Vol_{t-1}$. The variables dp, lc and $\widetilde{my}$ are the dividend yield, the labor income-consumption ratio, and the housing collateral scarcity measure based on the value of mortgages (see Lustig and Van Nieuwerburgh (2005) for a detailed description of the data). In particular $\widetilde{my}_t = \max(my_t) - my_t/(\max(my_t) - \min(my_t))$, where $\max(my_t)$ and $\min(my_t)$ are the sample minimum and maximum of the housing collateral ratio my. R^e denotes the value weighted market return in excess of a 1 month T-bill return. Vol_t is the standard deviation of the 12 monthly returns in year t. $R1, R5, R10$ denote the 1-year, 5-year and 10-year ahead cumulative excess returns. The estimation is by GMM with the OLS normal conditions as moment conditions. Standard errors are Newey-West with lag length 3. The estimation period is 1927-1992, the longest common sample. The predicted Sharpe ratio is formed as the ratio of the predicted mean excess return to the predicted standard deviation. The last three rows indicate the sample mean of the predicted Sharpe ratio, its sample standard deviation, and the sample correlation between the Sharpe ratio and the housing collateral scarcity measure $\widetilde{my}$.

Regressors	R1	Vol1	R5	Vol5	R10	Vol10
constant	-.24	.09	.71	-.004	.76	.05
(s.e.)	(.34)	(.05)	(0.28)	(.04)	(.42)	(.04)
lag ret	.04		.74		.76	
(s.e.)	(.13)		(.13)		(.08)	
dp	1.07	.33	-1.20	.40	.26	.12
(s.e.)	(2.00)	(.27)	(3.16)	(.22)	(3.00)	(.21)
lc	.22	-.07	-.76	.03	-.92	-.03
(s.e.)	(.32)	(.05)	(.33)	(.03)	(.45)	(.04)
$\widetilde{my}$	.02	-.01	.48	-.04	.70	.01
(s.e.)	(0.20)	(.02)	(.20)	(.02)	(.23)	(.02)
lag vol		.51		.96		.80
(s.e.)		(.20)		(.12)		(.12)
2 lag vol		-.18		-.19		.03
(s.e.)		(.17)		(.11)		(.10)
E[Sharpe]	.40		1.02		1.12	
σ[Sharpe]	.10		.18		.20	
$\rho[Sharpe, \widetilde{my}]$	.26		.32		.50	

Table 6: Unconditional Asset Pricing Moments for Collateral Model.

Averages from a simulation of the model for 5,000 agents and 10,000 periods. In the first column, $R^{l,e}$ denotes the excess return on a levered claim to aggregate consumption growth, with leverage parameter $\kappa = 3$. $R^{c,e}$ denotes the excess return on a non-levered claim to aggregate consumption growth. The third column reports the unconditional mean of the risk-free rate. Columns four to six report unconditional standard deviations of levered and non-levered consumption claims and risk-free rate. The last two columns report Sharpe ratios on levered and non-levered consumption claims. Panel 1 reports historical averages for the annual S&P500 return and for the annual real return on a 3-month Treasury bill for the samples 1927-2004 and 1871-1979 (data from Global Financial data). Panels 2-4 are for the benchmark parametrization. Panel 2 reports the results for the economy with a 5 percent average collateral ratio. Panel 3 reports results for the representative agent economy. Panel 4 reports the moments for the collateral economy with 10 percent collateral on average. Panels 5-9 report results from various sensitivity exercises. Panel 5 reports the unconditional asset pricing moments for the model with recursive preferences. Risk aversion is $\gamma = 8$ and the intertemporal elasticity of substitution is 0.2. Panel 6 reports results for different parameters of intratemporal elasticity of substitution between non-housing and housing services consumption ε. All other parameters are held constant at their benchmark level and there is 5 % collateral. Panel 7 varies the coefficient of relative risk aversion γ. Panel 8 is the benchmark calibration but with an expenditure share process is an AR(1): $\log r_t = \bar{r} + \rho_r \log r_{t-1} + \sigma_r \nu_t$, i.e. $b_r = 0$. Panel 9 is the benchmark model with 5% collateral, but without conditional heteroscedasticity in the income share process: $\eta = [.6935, .6935, .3065, .3065]$ instead of [.6578,.7952,.3422,.2048].

	$E(R^{l,e})$	$E(R^{c,e})$	$E(r^f)$	$\sigma(R^{l,e})$	$\sigma(R^{c,e})$	$\sigma(r^f)$	$\frac{E(R^{l,e})}{\sigma(R^{l,e})}$	$\frac{E(R^{c,e})}{\sigma(R^{c,e})}$
Sample				**Panel 1: Data**				
1927-2004	0.079		0.010	0.207		0.042	0.384	
1871-1979	0.058		0.019	0.200		0.052	0.289	
(γ, ε)				**Baseline Analysis**				
				Panel 2: Benchmark 5 percent Collateral Model				
(5, .05)	0.039	0.026	0.077	0.167	0.106	0.070	0.230	0.246
(8, .05)	0.103	0.086	0.026	0.266	0.217	0.156	0.388	0.397
				Panel 3: Representative Agent Model				
(5, .05)	0.024	0.010	0.129	0.127	0.057	0.031	0.186	0.177
(8, .05)	0.045	0.023	0.158	0.146	0.074	0.055	0.309	0.310
				Panel 4: 10 percent Collateral Model				
(5, .05)	0.031	0.018	0.111	0.149	0.084	0.047	0.205	0.210
(8, .05)	0.079	0.057	0.084	0.210	0.152	0.114	0.376	0.379
(γ, ε)				**Sensitivity Analysis**				
				Panel 5: Sensitivity - Recursive Preferences				
(8, .05)	0.049	0.033	0.048	0.195	0.144	0.069	0.252	0.229
				Panel 6: Sensitivity - Varying Intratemporal Elasticity of Substitution				
(8, .15)	0.108	0.086	0.026	0.277	0.226	0.157	0.392	0.381
(8, .75)	0.142	0.110	−0.018	0.241	0.191	0.157	0.590	0.577
				Panel 7: Sensitivity - Varying Coefficient of Relative Risk Aversion				
(2, .05)	0.008	0.003	0.086	0.108	0.041	0.011	0.075	0.070
(10, .05)	0.157	0.133	−0.021	0.289	0.243	0.205	0.545	0.547
				Panel 8: Sensitivity - AR(1) process for Log Non-Housing Expenditure Ratio r				
(8, .05)	0.069	0.051	0.018	0.237	0.192	0.129	0.292	0.264
				Panel 9: Sensitivity - No Income Heteroscedasticity				
(5, .05)	0.036	0.021	0.088	0.161	0.099	0.061	0.222	0.210
(8, .05)	0.057	0.040	0.078	0.196	0.131	0.102	0.291	0.305