Soft Landings*

Martin Schneider
UCLA

Aaron Tornell
UCLA and NBER

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Abstract

A country experiencing a lending boom goes through a period of unusually fast growth in credit. This paper proposes a theory of lending booms that incorporates two distortions which are prevalent in emerging markets: the imperfect enforceability of contracts and government bailout guarantees. The first of distortion implies that there may be an underinvestment problem and that shocks are propagated through their effect on borrower wealth. The second distortion amplifies shocks since it encourages excessive risk taking and overinvestment. Although they appear to affect borrowing in opposite ways, the tow distortions do not neutralize each other. They combine to rationalize the gradual buildup of lending booms, the excess volatility of credit and asset prices and the slow recovery if a lending boom ends in a financial crisis. The interaction also introduces a nonlinearity in the response to shocks. This explains why most lending booms do not require a large negative shock to end, but rather come to a 'soft landing'. In addition to accounting for the main characteristics of a typical lending boom episode the model also surprising policy implications.

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1. Introduction

A country experiencing a lending boom goes through a period of unusually fast growth in credit. Lending booms occur frequently in emerging markets.[1] They are often accompanied by asset price inflation and strong investment growth, especially in traditionally risky sectors.[2] In addition, many recent financial crises were preceded by lending booms.[3] A large literature now argues that lending booms are the result of mistaken government policy: the existence of bailout guarantees creates a moral hazard problem that entails overborrowing, excessive investment and risk taking. The boom inevitably ends when it is realized that futher guarantees are not credible: a crisis ensues.[4]

While there is considerable evidence that bailout guarantees are present, the standard account of lending booms suffers from two drawbacks. First, the typical lending boom does not end in a crisis. Around 85% of booms lead to a *soft landing,* with credit and asset prices gradually reverting to trend.[5] Second, the formal underpinnings of the standard story derive from the deposit insurance literature, which was developed to study optimal financing and risk taking by competitive banks in developed countries.[6] It is thus maintained that the extent of external financing does not affect the cost of capital. This assumption makes gambling with borrowed money particularly attractive in the existing models. However, it is implausible in the light of recent evidence on ownership structure in emerging economies. Controlling shareholders typically hold large stakes, even in large firms. This not only suggests that external finance is costly, but also that incentives for inefficient risk taking may not exist.

This paper develops a theory of lending booms in economies where production is controlled by wealthy entrepreneurs. We show that, in such economies, soft landings are a natural outcome. In our model, entrepreneurs hold large stakes in their firms, because contracts cannot be enforced perfectly. Bailout guarantees are also present. When deciding on investment and financing, entrepreneurs thus trade off gains from the subsidy implicit in the guarantee against losses to their own capital. At very low levels of wealth, there is inefficient underinvestment. As entrepreneurs become richer, the moral hazard problem becomes more severe, and the boom 'overheats': inefficient overinvestment and risk taking occur. However, as net worth rises even further relative to existing investment opportunities, inefficient and highly risky projects

[1] Aaron: here cite Pierre, and what else ?

[2] Aaron: here we need micro evidence, Pomerleano etc.

[3] Aaron: cite your own empirical stuff here, and Kaminsky etc.

[4] Aaron: papers are mckinnon pill, corsetti, krugman, dooely and as many imf guys as possible...; also can put burnside et al but need to put extra snetence

[5] Aaron: cite Pierre and papers you have with Frank

[6] cite Kareken wallace, kahn marcus, maybe some newer stuff; also freixas rochest as an overview,

are foregone: a soft landing occurs.

As one building block of our model, we provide an explicit microeconomic framework to clarify why financing constraints can bind in an economy with bailout guarantees. This is not a foregone conclusion: if a bailout always occurs in case of default, why should lenders care whether borrowers can commit to repay ? This argument overlooks the fact that bailout guarantees typically insure lenders only against systemic risk. A bailout will not occur if just an isolated firm defaults, especially not a small one. Instead, bailouts happen only when there is a critical mass of defaults. Collateral then still matters for credit, because lenders have to guard against idiosyncratic default risk.

In a world with bailout guarantees, entrepreneurs and lenders try to collude to exploit the bailout guarantee. There are two implications for policy. First, while better enforceability of contracts may avoid inefficient underinvestment early on during a lending boom, it also fosters more inefficient overinvestment as the lending boom overheats. The reason is that a better contracting technology provides entrepreneurs and lenders with a more effective tool to exploit the guarantee. This contradicts conventional wisdom that better contract enforcement should improve the allocation of resources. It follows that institutional changes that improve contract enforcement may not be desirable unless at the same time a regulatory framework is put in place that contains excessive risk taking. Second, with systemic guarantees, there can be a *self-fulfilling* correction to an overheated lending boom: if entrepreneurs suddenly believe that their peers will no longer take excessive risk, they are better off reverting to prudent investment policies themselves. The resulting drop in output is not a crisis that entails underinvestment, but a correction that restores efficiency.

We extend our basic model to include land which serves both as a factor of production and as collateral. We show that the model explains various stylized facts about the behavior of asset prices during lending booms. During a boom, the prices of productive assets are often inflated in a way that is hard to reconcile with historical fundamentals. In our model, this arises because they also capitalize future subsidies implicit in bailout guarantees. The effect is likely to be reinforced if the country recently experienced an improvement in contract enforecement. Second, returns in the beginning of a boom are volatile and negatively skewed. The latter feature is arises naturally from the asymmetric adjustment costs implied by financing constraints. Finally, asset prices typically peak well before the lending boom ends: they anticipate the soft landing. This is also in line with the data.[7]

AARON: how about the land price - soft landing mechanism here ??

Our setup is related to existing 'financial accelerator' models of a small open economy. These models are also driven by financing constraints and feature entrepreneurial net worth as the key state variable. However, the presence of bailout guarantees overturns several results typically associated with financing accelerator models. First, our model gives rise to *both* over- and underinvestment, whereas typically financing constraints

[7] Aarron: herre need a quote; perrhaps also on the otherr facts in this paragrraph

induce underinvestment. Better contract enforcement or infusions of net worth thus not necessarily improve efficiency of investment. Second, because of the soft landing effect, a positive shock to net worth may *decrease* investment in our model. This means that the link between cash flow and investment is nonlinear, positive for firms with low net worth, but negative when net worth is higher. Simple linear regression analysis of the relationship between cash flow and investment may thus not be able to uncover the importance of financing constraints.

The paper proceeds as follows. The remainder of this introduction summarizes stylized facts on lending booms and reviews related literature. Section 2 presents the simplest version of the model. Here we abstract from fixed assets and posit an exogenous bailout policy to focus on the dynamics. This minimal setup shows why soft landings occur. Section 3 describes a more elaborate framework which justifies the bailout policy and introduces land. Section 4 collects results for this model. Section 5 concludes. All proofs are collected in the appendix.

1.1. Empirical Evidence on Lending Booms

The existing evidence on lending booms can be read as answering three questions. First, what are the salient features of macroeconomic aggregates and relative prices during an episode? During a typical boom, investment rises along with credit.[8] In addition, asset prices, in particular that of real estate, rise and the real exchange rate appreciates.[9] On average, lending booms do not end in a financial crisis, but rather with a 'soft landing'. Asset prices tend to revert before the lending boom ends.[10] Although abrupt collapses of booms are not the norm, it is true that almost all banking and currency crises in emerging markets have been preceded by lending booms.[11] Those lending booms that have ended in a crisis have typically been followed by a credit crunch. That is, in the aftermath of crises new lending falls sharply and recuperates only gradually. [12]

A second empirical question is whether the quality and composition of investment is different during

[8] Pomerleano (1998) considers data from 734 South East Asian corporations from 1992 to 1996. For the case of Thailand, the average investment rate during this period was 29% (3% in the US). Furthermore, 78% of this investment was financed with debt (8% in the US). Claessens, et.al. (1998), using a database of 5550 firms in nine Asian countries, find that during the early 1990s investment and leverage were very high and increasing, while profitability was declining. In Thailand during 1988-95 the investment rate increased from 10% to 14.5%, the debt-to-equity ratio increased from 1.6 to 2.2. The corresponding figures for the US are 3.8 to 3.7 and 0.8 to 1.1.

[9] Gourinchas, et. al (1999) document this fact for the real exchange rate. Guerra (1998) and Hernandez and Landerretche (1998) document the appreciation of real estate and stock prices.

[10] Gourinchas et. al. (1999) find that in a sample of 91 countries over the past 35 years, the probability that a lending boom will end in a currency crisis is less than 20%. Furthermore, the build-up and ending phases of an average boom are similar in magnitude and duration.

[11] See Corsetti, Pesenti and Roubini (1998), Kaminsky and Reinhart (1999) and Tornell (1999).

[12] See for example Krueger and Tornell (1999) or Sachs, Tornell and Velasco (1995).

lending booms, when compared to normal times. Naturally, an answer to this question is not as easily quantifiable, and the existing evidence is largely anecdotal. Nevertheless, there does seem to be a tendency for the quality of investment to deteriorate during lending booms.[13] Moreover, firms and banks have been noted to shift to activities that have traditionally been considered to be more risky, such as investment in real estate.[14] Not all firms experience booms and busts in the same way. Small, bank-dependent firms and firms in the nontradable sector have grown more strongly during the boom, but have been slower to recover after the crisis than large exporting firms with access to direct finance.[15]

Third, there is some direct evidence that the two distortions we focus on are present especially in emerging markets. While such guarantees exist in many countries, in emerging markets they are tend not to be accompanied by a strong regulatory framework.[16] In addition, in many emerging markets contracts are not as easily enforceable as in developed countries.[17]

1.2. Related Literature

In this section we will discuss in more detail how our model relates to previous theoretical work. In stressing the role of borrowers' wealth as a key state variable, our model belongs to the class of 'financial accelerator' models. Following Bernanke and Gertler (1989) a number of authors have explored propagation mechanisms derived from the dynamics of lender-borrower relationships. Whenever the relationship between borrowers and lenders is subject to one of the imperfections familiar from standard static models of the debt constrained entrepreneurial firm, the marginal cost of external finance depends on borrowers' wealth.[18] Independent exogenous shocks, for example to borrower wealth, may then have persistent effects to the extent that they affect borrowers' wealth and hence the cost of external finance in the following period. Kiyotaki and Moore (1997) have enriched the basic mechanism by pointing out the role of durable assets, which are desired by borrowers as both factors of production and collateral[19]. The feedback effects between borrower wealth and asset prices provide an important amplification device for the basic financial accelerator effect. Small open economy models with a financial accelerator include Aghion, Bacchetta and Banerjee (1998) and Antinolfi and Huybens (1998).

[13] Pomerleano (1998) finds that the return on assets in his sample of Thai firms fell from 9% in 1992 to 5% in 1996 (9% and 13% in the US). Claessens, et.al. (1998) document that the real return on assets fell from 11% to 8%.

[14] See, for example, Bank for International Settlements (1999).

[15] See Krueger and Tornell (1999).

[16] See Bank for International Settlements (1999).

[17] See, for example, Johnson et al. (1999).

[18] Several ways of modeling imperfections have been proposed. For example, moral hazard with costly state verification (introduced by Townsend (1979)), is employed e.g. in Bernanke and Gertler (1989,1998) and Carlstrom and Fuerst (1996); ex ante moral hazard, studied by Holmstrom and Tirole (1995), is used by Aghion and Bolton (1995); Kiyotaki and Moore (1997) rely on a version of the Hart and Moore (1994,1997) incomplete contracting theory of debt finance.

[19] While in the KM model the durable good is land, other papers (e.g. Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1998)) have considered a setup in which the price of capital is allowed to fluctuate due to adjustment costs.

The main difference between our model and existing financial accelerator models is that in our lending booms entrepreneurs take on highly risky, negative net present value projects. It thus accounts for the excess risk taking and low investment quality of observed in many lending booms. Financial accelerator models explain neither this fact, nor soft landings because lending would stop if negative NPV projects were undertaken in equilibrium. Note also that comparative statics differ. For instance, a worse contractual enforceability is usually 'bad' in financial accelerator models because it prevents borrowers from undertaking good projects. In contrast, in our model it prevents wasteful investment and makes the emergence of lending booms more difficult.

Ljungqvist (1995), Krugman (1998a,b) and Corsetti et al. (1999) have both studied the role of the moral hazard problem arising from government guarantees in the context of a growth model. They show that overinvestment and mispricing of capital can occur if borrowers' liabilities are guaranteed by the government. Ljungqvist (1995) also compares stationary equilibria of economies with and without guarantees and finds that the former are subject to larger swings in asset prices. In contrast to our model, borrowers in these papers are competitive firms who do not have wealth of their own (there is free entry into operating the technology). The gradual development of lending booms is then generated by invoking adjustment cost of investment, rather than a gradual increase in entrepreneurial wealth. Thus, there is no gradual interplay between lending and asset prices in the build-up phase of a lending boom. More importantly, these models generate neither soft landings, nor balance sheet effects, such as those that were reported in the aftermath of crises.

Another related paper is Allen and Gale (1998). This paper explores the effects of risk shifting behavior by borrowers who issue standard debt to lenders. As in the literature on firm capital structure, the convex payoff profile of the borrower's residual income turns risk neutral borrowers into risk lovers. Allen and Gale exploit the effect of this on asset prices: since borrowers are the only agents who have access to the asset market, assets are priced according to their (distorted) marginal utility. This is how a bubble can develop. We invoke the same principle to get mispricing in our model, except that the distortion in entrepreneurs' 'effective' preferences arises from government guarantees. A second key difference between our analysis and that of Allen and Gale is that they are not concerned with the gradual development of lending booms. In their model, the supply of credit granted by lenders is exogenous, and there is no feedback between asset prices and lending.

2. The Soft Landing Mechanism

2.1. Setup

We consider a small open economy, populated by overlapping generations of risk neutral entrepreneurs. Every entrepreneur in generation t owns a risky production technology, which turns k_t units of the single numeraire good invested in period t into

$$y_{t+1} = z_{t+1} f(k_t)$$

units of the good in period $t+1$. The productivity shock z_{t+1} is i.i.d.: it equals one with probability α, and zero otherwise. An entrepreneur begins the period with internal funds w_t. He can raise additional funds b_t by issuing debt with a promised interest rate ρ_t to risk neutral foreign investors. Foreigners have 'deep pockets': they are willing to lend any amount, provided that they expect to earn at least the riskless rate world interest rate, fixed at r. Entrepreneurs also have access to alternative investment opportunities that earn the riskless rate, which we refer to as 'riskless savings' s_t. The budget constraint is thus

$$s_t + k_t = w_t + b_t.$$

Distortions

There are two distortions in this economy. First, entrepreneurs cannot commit to repay debt: they may default strategically. If they do so, lenders can try to seize their assets. However, lenders can only recover multiples ψ_k and ψ_s of fixed capital and riskless savings, respectively, with $\psi_k, \psi_s \leq 1 + r$. Default is thus attractive to entrepreneurs *unless* the promised debt repayment is lower than the 'collateral value' of assets:

$$(1 + \rho_t) b_t \leq \psi_s s_t + \psi_k z_{\bar{t}+1} k_t. \tag{2.1}$$

The second distortion is the existence of an agency which guarantees entrepreneurs' debt. In particular, we assume that, if entrepreneurs default in the 'bad state' $z_t = 0$, foreign lenders are bailed out by this agency, so that they recover principal plus riskless rate, $(1 + r) b_t$. There is no bailout in the good state. This bailout scheme will be derived endogenously in the next section as the outcome of a game between entrepreneurs and lenders.

2.2. Optimal Investment and Financing

The goal of an entrepreneur in period t is to maximize expected profits, subject to the constraint that lenders must break even. It is optimal to not default in the good state, and to offer the riskless rate $\rho_t = r$.[20] This implies a *collateral constraint*

$$(1 - \beta\psi_k) k_t + (1 - \beta\psi_s) s_t \leq w_t, \tag{2.2}$$

where $\beta = \frac{1}{1+r}$. Internal funds thus bound the amount of investment, a familiar property of financially constrained firms. Moreover, in contrast to most existing setups, our model features several investment opportunities that differ in the extent to which returns can be pledged to lenders. The entrepreneur must

[20] Indeed, from (3.2), it is not credible to promise more than $(1 + r)b_t$ in the bad state. Therefore, the lender can only break even if at least $(1 + r) b_t$ is repaid in the good state. Of course, it is never optimal to repay more than that.

thus allocate scarce internal funds among these opportunities. The 'price' he assigns to an opportunity depends negatively on the fraction of proceeds that can be pledged, ψ_s or ψ_k.

In this section, we focus on the case where riskless savings can be fully pledged to lenders, $\psi_s = 1 + r$. Expected profits

$$\begin{aligned} \Pi(k_t, w_t) &= (1+r) w_t + [\alpha f(k_t) - (1+r) k_t] \\ &\quad + (1-\alpha)(1+r) \max\{k_t - w_t, 0\} \end{aligned}$$

must be maximized subject to the budget and collateral constraints. Here the second term represents profits from running the firm, while the third term captures the subsidy due to the bailout guarantee. This subsidy can only be claimed by picking a *risky plan*, which entails default in the bad state $(k_t > w_t)$. In contrast, a safe plan $(k_t \leq w_t)$ does not involve any borrowing: the firm is entirely financed internally.

Optimal Safe and Risky Plans

Suppose the entrepreneur opts for a safe plan. He will then try to operate the technology at the first best level k_t^*, which equates the marginal product of capital to the interest rate:

$$\alpha f'(k^*) = 1 + r$$

If internal funds are insufficient to finance k^*, the marginal product is higher than $1 + r$ and all internal funds should be invested. The optimal safe investment is thus $k^s(w_t) := \min\{w_t, k^*\}$. In contrast, under a risky plan, the subsidy provided through the bailout is increasing in the amount borrowed. This artificially lowers the marginal cost of capital. Under the best risky plan, the entrepreneur would like to invest up to the level of capital $\hat{k} > k^*$ that would be optimal if the technology were riskless:

$$f'\left(\hat{k}\right) = 1 + r$$

This amount can again only be financed if sufficient internal funds are available, since the collateral constraint limits firm leverage. The optimal risky investment is

$$k^r(w_t) = \begin{cases} \frac{w_t}{1-\beta\psi_k} & \text{if } w_t < (1-\beta\psi_k)\hat{k} \\ \hat{k} & \text{if } (1-\beta\psi_k)\hat{k} < w_t \leq \hat{k} \\ w_t & \text{if } w_t > \hat{k} \end{cases}$$

Here the first subcase reflects the collateral constraint, and the last subcase follows from our definition of a risky plan.

Trading off Subsidy versus Efficiency

Profits under the best risky and safe plan, $\Pi(k^r(w), w)$ and $\Pi(k^s(w), w)$, respectively, are compared in Figure 1. The figure shows three investment regions, the existence of which is proven formally in the appendix. First, for low internal funds, risky plans are always optimal. They permit the entrepreneur to leverage the firm, and if the marginal product of capital is high, this is clearly desirable. In fact, as long as $w < k^*$, the collateral constraint keeps the firm in an *underinvestment* region, relative to the first best. Second, risky plans remain optimal even as $w > k^*$, giving rise to a region of inefficient *overinvestment* and risk taking. As investment is increased beyond k^*, the marginal decrease in expected profits due to inefficient investment is initially second order. It is outweighed by the first order increase in profits due to the guarantee. The latter increase occurs as long as firm leverage is sufficiently high.[21] Finally, there is an *efficient investment* region. If internal funds are high enough, the expected loss of the entrepreneur's own stake in the firm cannot be compensated by the subsidy. The entrepreneur is better off to self-finance the firm, and run it at the first best scale.

These results of our model are reminiscent of the literature on deposit insurance and capital requirements. In both cases, high leverage (a low capital ratio) goes along with high risk taking. However, a key difference is in what is exogenous to the firm in a given period. The deposit insurance literature typically considers banks of a fixed scale, but with variable capital chosen by shareholders with 'deep pockets'. The models predict changes in leverage and risk as a result of changes in regulation. For example, if capital requirements are relaxed, shareholders prefer higher leverage and riskier loan portfolios. This setup is motivated by large US banks. In contrast, our entrepreneurial firms have variable scale, but their capital is predetermined by the wealth of the entrepreneur. It thus predicts changes in leverage and risk as a result of changes in past profits. This gives rise to our dynamic analysis, to be considered next.

2.3. Dynamics: Lending Booms with a Soft Landing

We assume that entrepreneurs bequeathe their firms to their heirs. More precisely, if the good state $z_{t+1} = 1$ is realized, they consume a fraction c of profits Π_t, and pass on the rest, together with the technology. If the bad state is realized, we assume that the young generation receives a small aid payment ε. The wealth of a representative dynasty is thus evolves over time according to

$$w_{t+1} = (1 - c)\, z_{t+1}\Pi_{t+1} + (1 - z_{t+1})\, \varepsilon.$$

Figure 2 sketches the dynamics of the model. There are two transition functions: one for the good and one for the bad state. This allows to trace out paths of wealth and investment, given a realization of the shocks.

[21] For example, for $w < (1 - \beta\psi_k)\,\hat{k}$, the leverage ratio under a risky plan is constant.

For a concrete example, suppose the economy is initially at point A in Figure 2. Entrepreneurs are relatively poor and a lack of collateral prevents them from investing at an efficient scale. If good shocks occur, internal funds are gradually built and firms grow. However, during this phase output is volatile and growth rates are negatively skewed: downturns are sharp and short ‘crises’ that have persistent output effects, since lending booms are long and gradual. If a boom is long enough, it will eventually *overheat*: entrepreneurs begin to finance projects that have negative net present value. In this phase, vulnerability to crises remains a problem: in fact, output falls even more if a crisis occurs. In addition, any crisis will feature the failure of a fair number of ‘white elephant’ project that everybody knew up front to be unreasonable.

How does a lending boom end ? If it is not punctuated by a crisis, entrepreneurial wealth will eventually reach the efficient region. Investment in the risky technology drops down to the first best level: there is a *soft landing*. The negative net present value projects that correspond to investment levels between k^* and $\hat{k}$ are only worthwhile if they can be financed with other people’s money, and subsidized by the bailout agency. Once entrepreneurs have become sufficiently rich, this is no longer possible. Inevitably, the failure of ‘white elephant’ projects would eat up too much entrepreneurial capital. Therefore, entrepreneurs reduce the scale of operations; they prefer to invest in riskless projects. If good shocks to continue to occur, the ‘lucky path’ leads the economy to the steady state at point B, where investment and savings remain constant. Of course, if bad shock occurs, entrepreneurs bounce back to lower wealth levels. However, the effect of a bad shock on output is milder and less persistent in the efficient region.

Since entrepreneurial net worth is a key state variable in our model, it is interesting to examine unanticipated changes in it. This impulse response differs across the three investment regions. First, there is *no* effect on output in the efficient region, unless the shock manages to move the economy out of this region. In contrast, in the underinvestment and most of the overheating region, an increase in net worth increases output. However, if a shock pushes the economy from the overheating to the efficient region, an increase in net worth may actually decrease investment and output. It follows that the response to shocks in our model is highly nonlinear. In particular, in an economy with bailout guarantees, simple test based on regressions of investment on net worth measures may well understate the importance of financing constraints. This will be true especially for firms thatare relatively large.

It is interesting to compare the dynamics of our model with alternative setups that rely on only one of the two distortions. First, suppose there are no bailout guarantees. Entrepreneurs do no contribute any wealth of their own and they can borrow any amount they want at the riskless rate. This situation corresponds to the case in which $\psi_s = 0$ and $\psi = \beta^{-1}$. It follows that at time 0 capital will jump to the Pangloss level $\widetilde{k}$. The model then has no internal propagation mechanism: gradual development of lending booms and persistent effects of crises must follow from the persistence of shocks, unless some other adjustment cost is imposed. Moreover, a soft landing cannot occur: all lending booms must end in a crisis.

Second, suppose there are no bailout guarantees. The model then reduces to a fairly standard 'financial accelerator model' along the lines of Bernanke and Gertler (1989). Without guarantees, interest rates reflect the riskiness of the projects. Thus, entrepreneurs never invest beyond the efficient level k^*; there is no overheating. The financial accelerator model also features internal propagation through net worth and a nonlinear response to shocks. However, higher cash flow can never lead to *less* investment, as is the case in a soft landing. This additional source of nonlinearity is unique to our model. Another difference is in the relationship between returns and constraints. In the financial accelerator model, a constrained firm always has a high marginal product of capital. In contrast, in our model, constraints bind at both high and low MPK firms: the unconstrained firms in the efficient region have intermediate returns.

3. A Model with Land

In this section we introduce a second productive factor in order to characterize. We consider the same setup as in the previous Section excep that now there are two goods: an internationally tradable good, which is the numeraire, and land, which is inelastically supplied. Both, the competitive and the crony sectors produce the numeraire good. We proceed in two steps. First, we characterize the interaction between lending and asset prices by considering a finite terminal time T. Then, we study the mechanism that generates the soft landing by endogenizing T.

Technology

As before entrepreneur i has two investment opportunities: a safe storage technology that transforms one unit of the numeraire into $1+r$ units the next period, and a risky technology that combines k units of the numeraire and land l to produce the numeraire good according to

$$f(k_t^i, l_t^i, \theta_{t+1}) = \theta_{t+1} g(l_t^i) k_t^i, \qquad g' > 0, g'' < 0$$

Linearity in k implies that the leverage ratio will be constant in equilibrium, which greatly simplifies the analysis.[22] However, it rules out switches.

The productivity parameter θ_t may take two values: θ or 0. It follows a Markov process with transition matrix $\begin{bmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{bmatrix}$. In other words, the economy can experience a string of good luck, but once this

[22] To draw the connection to the previous section think of the previous section's production function as representing a set of perfectly correlated indivisible investment projects each with 'capacity' dk_t.

string ends, it never starts again. In section 5 we will drop this assumption and consider a standard non-degenerate transition matrix in which the elements $(0,1)$ are replaced by $(\alpha, 1-\alpha)$. As we will see, the results stated in Propositions 1 and 2 will remain unchanged.

In order to have overinvestment in equilibrium it is necessary that the risky technology have a greater return than the safe return in good times, but a lower expected return. This is captured by the following assumption:

Assumption 1 *The return on risky capital, $\theta g(l)$, satisfies $\theta g(0) = 1 + r$ as well as*

$$\alpha\theta g(l) < 1 + r < \theta g(l) < \infty \qquad \text{for all } l > 0 \tag{3.1}$$

Markets, Borrowing and Bailouts

The supply of land is fixed at one, and there exists a competitive market for land. We will denote by $p_t(\theta_t)$ period t's price of land if the state is θ_t. As in the previous section, lenders will only lend if there is collateral, which they can grab in case of default.

Assumption 2 *An entrepreneur can pledge as collateral his land, a share $\psi < 1 + r$ of his risky capital, and a share $\psi_s < 1 + r$ of his wealth invested in the safe technology.*

Collateral can be costlessly appropriated by lenders in case of default. It follows that at time $t + 1$ the value of the collateral will be $p_{t+1}(\theta_{t+1})l_t^i + \psi k_t^i + \psi_s s_t$.[23] As before, a bailout occurs if and only if more than 50% of entrepreneurs default in a given period.

The Credit Market Game

The timing of events for a given generation of entrepreneurs and lenders is as follows. Entrepreneurs announce plans for risky investment (k_t^i) and land purchases (l_t^i) as well as how much debt they want to issue (b_t^i) and the interest rate they promise (ρ_t^i). Lenders then decide whether to accept or reject these offers. During the next period the shock is realized and borrowers make their default decision. A bailout occurs if more than half of the entrepreneurs defaults[24].

[23] To motivate the term $\psi < 1 + r$ let $\psi_s = 0$ and suppose that in case of bankruptcy the entrepreneur can divert a share δ of his cash flow, which is $\theta_{t+1}g(l_t^i)k_t^i$. Thus, the lender will be able to appropriate the entrepreneur's land and the remaining cash flow $[1-\delta]\theta_{t+1}g(l_t^i)k_t^i$. It then follows that the "pledgeability coefficient" corresponds to the lower bound of this expression: $\psi = [1-\delta]\theta g(0)$. Condition (3.1) then implies that $0 \leq \psi \leq 1 + r$.

[24] In several countries credit flows occur in two stages. In the first stage domestic banks borrow from foreign creditors. These loans are typically not collateralized, but enjoy government bailout guarantees. In a second stage domestic banks, in turn, make collateralized loans to domestic firms. In the model we merge both stages.

We will relegate the details of this credit market game to the appendix and will merely state two important results. First, there always exists a safe equilibrium in which nobody invests in the risky technology and there is no lending. Since in this case there is no bailout in the bad state, it does not pay for an individual entrepreneur to deviate and invest in the risky technology, because the latter has negative NPV (Assumption 1).

Second, in a symmetric equilibrium with positive investment in the risky technology (this will be referred to as a 'risky equilibrium'), there cannot be default in the good state. If everyone were to always default, a bailout would always occur. But then an entrepreneur could deviate by issuing even more debt and investing it, say, in land.

Consider then the case in which all entrepreneurs repay their debt in the good state. From the nature of the commitment problem, sufficient collateral is required to prevent an individual borrower from defaulting in the good state. At the same time, lenders will break even if they are promised the riskless interest rate. This is because in the bad state everyone will default and there will be a bailout. Since lenders are competitive, the riskless rate is precisely what borrowers will offer. The bottom line is that lenders are willing to charge the riskless interest rate, but lend to an entrepreneur only up to the present value of his collateral in the good state

$$b_t^i \leq \bar{b}_t^i \equiv \beta \left[p_{t+1}(\theta) l_t^i + \psi k_t^i + \psi_s s_t \right], \qquad \beta \equiv \tfrac{1}{1+r} \tag{3.2}$$

Suppose an entrepreneur were to deviate by borrowing more than $\bar{b}_t^i$. He would default in the good state at time $t+1$. Lenders will agree to such a plan only if the payment they receive in the bad state (through the bailout) is large to enough to compensate them for the loss in the good state. But, by assumption, the payment per dollar invested from a bailout is only the riskless rate. Consequently, the expected payoff to lenders under the proposed deviation would be strictly less than the riskless rate, and lenders will not agree to the proposed plan. It follows that equilibrium plans are those that yield the entrepreneur maximum payoff among all plans that satisfy (3.2).

During each period, given his inherited wealth w_t^i, a young entrepreneur solves the following problem.

Problem I *Taking as given current and expected land prices, choose how much to borrow* ($b_t^i \geq 0$)*, land-holdings* ($l_t^i \geq 0$)*, capital* ($k_t^i \geq 0$)*, and how much to store* ($s_t^i \geq 0$) *in order to maximize the expected present value of next period's profits*

$$\begin{aligned} E_t \pi_{t+1}^i \;\; = \;\; & \alpha\beta \left[\theta g(l_t^i) k_t^i + p_{t+1}(\theta) l_t^i + \tfrac{1}{\beta} \left[s_t^i - b_t^i \right] \right] \\ & + [1-\alpha]\beta \max \left\{ (1+r)(s_t^i - b_t^i), \;\; (1+r-\psi_s)\, s_t^i \right\} \end{aligned} \tag{3.3}$$

subject to borrowing constraint (3.2), and to budget constraint

$$s_t^i + k_t^i + p_t l_t^i \leq w_t^i + b_t^i, \tag{3.4}$$

Since each entrepreneur consumes a share c of his profits, wealth evolves according to

$$w_t^i(\theta_t) = \begin{cases} [1-c]\pi_t^i(\theta_t) & \text{if } t \geq 1 \\ e & \text{if } t = 0 \end{cases} \tag{3.5}$$

We will only consider symmetric equilibria. Thus, in what follows we will omit the superscript i and consider a representative entrepreneur. A symmetric rational expectations equilibrium (REE) consists of sequences of entrepreneur's policies $\{b_t^*, l_t^*, k_t^*, s_t^*\}_{t=0}^T$, of wealth $\{w_t^*\}_{t=0}^T$, and of land prices $\{p_t^*\}_{t=0}^T$, such that, every period the land market clears ($l_t^* = 1$), and either of the following statements is true: (1) $s_t^* = w_t^*$ and $b_t^* = l_t^* = k_t^* = 0$; or (2) the quadruple $(b_t^*, l_t^*, k_t^*, s_t^*)$ solves Problem I taking as given wealth w_t^*, current prices p_t^* and future prices $\left(p_{t+1}^*(0),\ p_{t+1}^*(\theta)\right)$.

The two cases considered in the preceding definition correspond to the two types of credit market equilibria: a safe equilibrium in which no one invests in the risky technology and there is no lending, and a risky equilibrium. This implies that there are multiple REE. In particular, one might construct REE in which a lending boom breaks down because some 'sunspot' causes agents to revert to the safe equilibrium. Since the focus in this paper is on the sustainability of lending booms, for the most part we will consider equilibria where the lending boom continues as long as possible.

3.1. The Lucky Path and Temporary Equilibria

Note that if there were no bailouts, lenders would be willing to lend only at interest rates no lower than $\frac{1+r}{\alpha} - 1 > r$. Since entrepreneurs must contribute some equity to the risky project, they would not be willing to borrow because their expected rate of return would be lower than the riskless rate. As a result, in a no-bailout regime there would be no investment in the risky technology and the price of land would be zero. Using this benchmark we will say that there is a lending boom, an investment boom, and asset price inflation if b_t, k_t, and p_t follow increasing paths, respectively.

Lending booms may occur in a world with bailouts because entrepreneurs may find it profitable to undertake negative NPV projects. This is because there is an implicit bailout subsidy that arises from the fact that lenders will be willing to charge the riskless interest rate. The size of the subsidy depends on how much entrepreneurs can borrow. With the presence of a commitment problem, the subsidy may be too small to make negative NPV investment profitable. Consequently, the existence of bailout guarantees does not ensure the existence of lending booms and asset price inflation. Our objective is to determine the conditions under which they will occur.

Since the value of today's collateral depends on tomorrow's possible land prices, we need to solve for the entire sequence of future prices to obtain today's equilibrium allocation. This computation is greatly simplified because $\theta_t = 0$ implies $p_{t+j} = 0$ for all $j \geq 0$. Recall that if the productivity parameter $\theta_t = 0$, then $\theta_{t+j} = 0$ for all $j > 0$. This implies that there will be no more investment after time t, and thus no reason to borrow. Since land is only used as either collateral or as an input in risky production, it becomes worthless once $\theta_t = 0$. In section 5 we will show that the main results hold if we drop the assumption that 0 is an absorbing state.

Since $\theta_t = 0$ implies $p_{t+j} = 0$, we can characterize all risky REE by considering only *"lucky paths"* along which θ_t is always equal to θ. We will construct REE by first deriving risky temporary equilibria (TE) for a given level of wealth w and for given future prices $\{\hat{p}(\theta) = \hat{p},\ \hat{p}(0) = 0\}$. A symmetric risky TE is a collection (k, l, s, b, p) such that (k, l, s, b) solve Problem I taking land prices as given and the land market clears: $l = 1$.[25] The next Proposition states conditions under which risky equilibria can exist. Let

$$\gamma(l) \equiv \frac{\theta g'(l) l}{\theta g(l) - \psi} \tag{3.6}$$

Proposition 1. For every wealth level w and expected land price $\hat{p}$, there is a threshold for the pledgeability parameter $\underline{\psi} \in (0, \beta^{-1})$ such that if $\psi \geq \underline{\psi}$, there exists a unique symmetric risky TE. In this equilibrium: $s^* = 0$,

$$k^* = \frac{w}{[1 - \beta\psi][1 + \gamma(1)]} \tag{3.7}$$

$$b^* = \beta[\hat{p} + \psi k^*] \tag{3.8}$$

$$p^* = \beta\hat{p} + \tfrac{\gamma(1)}{1+\gamma(1)} w \tag{3.9}$$

The threshold $\underline{\psi}$ is uniquely determined by $\frac{1}{\beta} = R(\underline{\psi}) :\equiv \frac{\alpha[\theta g(1) - \underline{\psi}]}{[1 - \beta\underline{\psi}][1+\gamma(1)]}$.

Observe that the threshold $\underline{\psi}$ is strictly positive. If only land could be pledged as collateral ($\psi = 0$), as e.g. in Kiyotaki and Moore (1997), then risky equilibria would not exist.

The proof of Proposition 1 is in the appendix. Here we simply present the intuition. Consider an entrepreneur who contemplates investing in the negative NPV technology. This can only be profitable if he defaults in the bad state and thus claims the subsidy provided by the government through the implicit bailout guarantee. Indeed, the guarantee must be large enough to outweigh the expected loss the entrepreneur makes by putting his own wealth at risk. Let $w^p := w - (1 - \beta\psi_s)s$ denote the amount of wealth which lenders can appropriate in case of default. We can think of pledgeable wealth w^p as the 'equity' an entrepreneur puts up for a firm operating the risky technology. Given that the entrepreneur follows a risky policy that leads her to default in the bad state (i.e., $(1 + r)(s_t - b_t) > (1 + r - \psi_s)\, s_t$), budget constraint (3.4) implies that the objective function can be rewritten as

$$E\pi = w + \alpha[\beta\theta g(l)k - [p - \beta\widehat{p}]l - k] - [1 - \alpha]w^p \tag{3.10}$$

[25] To simplify notation, when we will omit the dependence of a variable on θ_t, we will refer to the value of the variable along the lucky path.

The second term is the expected excess return from operating the risky technology, while the third term is the expected loss from default. Since the government subsidy is increasing in the amount borrowed, entrepreneurs would like to leverage up their firms as much as possible[26]. Equivalently, they want to make the excess return in (3.10) large relative to pledgeable wealth. However, the degree of leverage is limited by the borrowing constraint

$$ul + (1-\beta\psi)k \leq w^p \tag{3.11}$$

Since (3.11) must hold with equality, we can rewrite (3.10) as

$$E\pi = w + [\alpha\beta[\theta g(l) - \psi]\mathcal{K} - 1]\, w^p, \qquad \mathcal{K} \equiv \tfrac{k}{w^p} \tag{3.12}$$

Where $\mathcal{K}$ is the capital-equity ratio. Equation (3.12) makes clear that investing in the risky technology is profitable only if $\mathcal{K}$ is large enough. In particular, if the firm were entirely equity financed, then $\mathcal{K} \leq 1$, and the excess return on equity would be negative.

To determine how high of a $\mathcal{K}$ entrepreneurs can attain in equilibrium, note from (3.11) that unless debt can be fully secured by invested capital ($\psi = \beta^{-1}$), every dollar invested in k requires *extra* equity of $1 - \psi\beta$. Similarly, every unit of land purchased requires extra equity of $p - \beta\widehat{p}$. At an optimum the ratio of marginal products of k and l must equal the ratio of their opportunity costs (here, their per unit equity requirements)[27]

$$\frac{\alpha[\theta g'(l^*)k^*]}{\alpha[\theta g(l^*) - \psi]} = \frac{p^* - \beta\widehat{p}}{1 - \beta\psi} \tag{3.13}$$

Since in a risky equilibrium $l^* = 1$, price equation (3.9) in Proposition 1 follows from (3.13). Similarly, equilibrium capital (3.7) follows from (3.11) and (3.13). Thus, the equilibrium capital-equity ratio is $\mathcal{K}^*(\psi) = \frac{1}{[1-\beta\psi][1+\gamma(1)]}$.

Thus, $\mathcal{K}^*$ and the risky return depend on the importance of land in production $\gamma(1)$, and on the pledgeability parameter ψ. To understand the role of $\gamma(1)$ note that in equilibrium $\gamma(1) = \frac{p^*-\beta\widehat{p}}{(1-\beta\psi)k^*}$ (by (3.6) and (3.13)). That is, $\gamma(1)$ equals the ratio of land's equity requirements to capital's equity requirements. The more important land is in production, the higher its equilibrium opportunity cost, and the more equity (own wealth) is needed to support it. A greater $\gamma(1)$ reduces $\mathcal{K}$ and the risky return for the following reason. Since the supply of land is fixed, in equilibrium the marginal return to land derives from capital gains. However,

[26] To illustrate the role of the bailout guarantee we can write $E\pi = w + \{\alpha\beta\theta g(l)k - k - (p - \alpha\beta\hat{p})l - (1-\alpha)\beta\psi_s s\} + (1-\alpha)b$, where the term in braces is expected profit in the absence of a bailout guarantee and the last term is the bailout subsidy.

[27] The probability of the lucky state α vanishes from the "pricing condition" (3.13). Since lenders are bailed out in the bad state, they simply set the borrowing constraint to ensure that they are repaid in the lucky state (i.e., they set $\bar{b} = \beta[\hat{p} + \psi k + \psi_s s]$ instead of $\bar{b} = \alpha\beta[\hat{p} + \psi k + \psi_s s]$). This implies that the opportunity cost of land is $[p - \beta\hat{p}]$ instead of the "true cost" $[p - \alpha\beta\hat{p}]$. Similarly, the opportunity cost of capital $1 - \beta\psi$ differs from the true cost $1 - \alpha\beta\psi$. In other words, the bailout guarantee induces borrowers and lenders to evaluate future payoffs under the best possible scenario, rather than using expected values. This is the "mispricing" of debt in a world of deposit insurance which has been used by Krugman (1998) and Mckinnon and Pill (1997).

since the equilibrium price of land cannot grow faster than the safe return, it follows that greater investment in land reduces excess risky returns in equilibrium[28].

An increase in ψ has two opposing effects on the risky return. First, it allows a greater share of risky capital to be pledged as collateral, increasing the leverage of the firm. Second, it increases $\gamma(1)$. We show in the appendix that the first effect dominates (Lemma 7.6). Thus, there exists a threshold $\underline{\psi}$ such that the risky return is no lower than the safe return if and only if $\psi \geq \underline{\psi}$, as stated in Proposition 1.

To sum up, a risky TE exists only if (i) leverage is high enough, and (ii) land does not eat up too much equity. These two conditions hold if the degree of pledgeability of capital is high enough.

3.2. Rational Expectations Risky Equilibrium

Along a lucky path the conditions that characterize a temporary risky equilibrium must hold for every $t \leq T$. Furthermore, since the entrepreneur repays to the lender $b_t = \beta[p_t l + \psi k_{t-1}]$, profit equation (3.12) and price equation (3.9) imply that along the lucky path:

$$\begin{aligned} w_t &= [1-c][\theta g(1) - \psi]k_{t-1} && (3.14)\\ p_t &= \beta p_{t+1} + \frac{\gamma(1)}{1+\gamma(1)} w_t && (3.15) \end{aligned}$$

In addition, terminal condition $p_T(\theta_T) = 0$ must be satisfied. This results from a zero demand for land during the last period (T). In what follows we will construct capital, price and wealth sequences that satisfy (3.7), (3.14) and (3.15). It follows from (3.7) and (3.14) that along the lucky path wealth and the capital stock satisfy the following recursions

$$w_t = \zeta w_{t-1}, \qquad k_t = \zeta k_{t-1}, \qquad \text{where } \zeta \equiv \frac{[\theta g(1) - \psi][1-c]}{[1+\gamma(1)][1-\beta\psi]} \tag{3.16}$$

Substituting (3.16) in price equation (3.15) we have that

$$p_t(p_T) = \beta^{T-t} p_T + w_t \frac{\gamma(1)}{1+\gamma(1)} \sum_{i=0}^{T-t-1} \beta^i \zeta^i \tag{3.17}$$

Finally, to obtain the risky REE recall that the first generation of entrepreneurs are born with an endowment e. Thus, (3.7) implies that $k_0^*(\theta) = \frac{e}{[1+\gamma(l)][1-\beta\psi]}$. For future reference we will summarize our results in the following Proposition.

Proposition 2. *A symmetric risky REE exists if and only if the pledgeability coefficient $\psi \geq \underline{\psi}$. Along the lucky path safe storage is zero for any $t < T$.*

[28] Note that if $\gamma(1)$ is sufficiently large, the expected return on risky investment falls below the safe return and, the risky temporary equilibrium ceases to exist. This occurs even if the technology has positive NPV (i.e., $\alpha = 1$).

- *Wealth and risky capital evolve according to*

$$w_t^* = e\zeta^t \qquad 0 \le t \le T \tag{3.18}$$

$$k_t^* = \frac{1}{[1+\gamma(1)][1-\beta\psi]} w_t^* \qquad 0 \le t < T \tag{3.19}$$

- *If* $\zeta\beta \neq 1$, *prices and borrowing are given by*

$$p_t^* = \frac{1-(\beta\zeta)^{T-t}}{1-\beta\zeta} \frac{\gamma(1)}{1+\gamma(1)} w_t^* \qquad 0 \le t \le T \tag{3.20}$$

$$b_t^* = \frac{\beta}{1+\gamma(1)} \left[\frac{1-(\beta\zeta)^{T-t+1}}{1-\beta\zeta} \zeta\gamma(1) + \frac{\psi}{1-\beta\psi} \right] w_t^* \qquad 0 \le t < T \tag{3.21}$$

3.3. Equilibrium Dynamics

We can use Proposition 2 to replicate stylized facts 2 and 4. Is there an equilibrium path where investment in negative expected NPV projects grows gradually? What is the nature of the feedback between asset prices, investment and lending? As we mentioned earlier, by setting terminal time $T < \infty$ we are imposing the existence of a soft landing. In the next section we will endogenize T.

The equilibrium equation for capital (3.19) makes it clear that the mere existence of bailout guarantees and a commitment problem in financial markets does not generate investment booms. Even if the necessary conditions (stated in Proposition 1) for investment in negative NPV projects are satisfied, investment will be decreasing, unless $\zeta \equiv \frac{[\theta g(1)-\psi][1-c]}{[1+\gamma(1)][1-\beta\psi]} > 1$. This condition holds if the savings rate $(1-c)$ and the pledgeability of future cash flows (ψ) are high, or if the technological parameter $\gamma(1)$ is small. The first two conditions ensure that an increasing amount of resources is invested in risky projects. A low $\gamma(1)$ ensures that these resources do not end up invested in land. Recall that $\gamma(1)$ determines the equilibrium ratio of land equity requirements to those of capital.

In order to verify that there may be investment booms we need to check whether $\zeta > 1$ is consistent with the restriction on parameters necessary for the existence of a risky REE $(\psi \ge \underline{\psi})$. Since we might rewrite this condition as $R(\psi) \ge 1/\beta$ (see Proposition 1) and since $\zeta = R(\psi)[1-c]/\alpha$, it follows that $\psi \ge \underline{\psi}$ is equivalent to $\zeta \ge \frac{1-c}{\alpha\beta}$. Therefore, $\zeta > 1$ is always consistent with $(\psi \ge \underline{\psi})$. Hence, we have that an investment boom can always take place.

To analyze the feedback between investment and asset prices we use price equation (3.20). Note that the term $\frac{1-(\zeta\beta)^{T-t}}{1-\zeta\beta}$ in (3.20) is decreasing in time for any $\zeta\beta \neq 1$, while w_t might go either up or down. This has two implications. If risky investment is non-increasing $(\zeta \le 1)$, prices must fall. However, increasing risky investment $(\zeta > 1)$ does not necessarily lead to increasing prices. Furthermore, even if there were inflation, prices can go up only on a certain time interval, then they must decline. This path is consistent with the stylized fact that in the initial phase of a LB episode asset prices increase, but fall gradually after the LB has peaked. The different paths that investment and prices can follow are illustrated in Figure 1.

The intuition is as follows. The return to holding land has two components: capital gains and dividends derived from the fact that land is an input in production. In the case that investment is declining, the marginal product of land, and dividends, must fall over time. Since the terminal price of land is zero, capital gains cannot be increasing if dividends are not. Therefore, if investment is declining, so is the price of land. In the case that investment is increasing p_t cannot increase forever if the horizon is finite. Agents know that at time T the price of land will be zero. To prevent arbitrage opportunities the price cannot experience an anticipated jump at T. Thus, it must start to decline at some earlier time. Therefore, prices can go up only on a certain time interval, and then they must decline.

Finally, since risky lending is determined by collateral values, the lending path will reflect the paths of investment and prices. Equation (3.21) reveals that risky lending will either be increasing or follow an inverted U-shaped path whenever there is an investment boom (i.e., $\zeta > 1$). However, if risky investment is declining (i.e., $\zeta \leq 1$), so is risky lending. For future reference we will summarize these results in the following Corollary.

Corollary 3.1 (Characterization of a Lending Boom).

- *If entrepreneurs need to contribute wealth of their own to risky projects, bailout guarantees need not lead to investment booms. These occur only if the pledgeability coefficient ψ, and the savings rate $1 - c$ are high.*
- *Asset price inflation arises only if there is an investment boom. However, if the horizon is finite, prices will start declining before the investment boom ends.*
- *In the presence of an investment boom, risky lending might follow an increasing or a inverted U-shaped path.*

It is illustrative to compare the price dynamics with traditional bubble models. If one were to consider the price in the absence of bailouts as the "fundamental", then any deviation from this fundamental solution could be interpreted as a bubble. We should warn the reader that this interpretation is not without controversy as one could also argue that given the existence of bailouts, the fundamental price is given by (3.20). Given this caveat, and assuming for the moment that $p_t > 0$ represents a bubble, Proposition 2 shows that in our model there can be a bubble in the presence of a finite horizon with certain terminal time, and that this bubble need not be increasing. Indeed with a finite horizon it must eventually start to decrease so that the asset's price converges to its fundamental value at terminal time T. Thus, in our model asset price bubbles crash only if the lucky path ends unexpectedly before time T. This stands in contrast to traditional bubble models, where rational bubbles must increase at the rate of interest, until they unexpectedly crash.

3.4. Self-Fulfilling Crises

There are two types of crises in this economy: fundamental and self-fulfilling. The first occur when $\theta_t = 0$. An unexpected self-fulfilling crisis occurs if each agent believes that a majority of entrepreneurs will not invest in the risky technology. If this were the case at some time t, everyone would expect that a bailout would not be granted at $t+1$ if $\theta_{t+1} = 0$. As a result lenders would be willing to lend only at interest rates greater than the riskless rate (i.e., $\rho \geq \frac{1+r}{\alpha} - 1 > r$). This hike in interest rates would make the risky projects' returns on equity negative. Therefore, at t all entrepreneurs would store all their wealth in the safe technology. Since the demand for land would fall to zero, its price would unexpectedly jump to zero.

4. Switching Equilibria

In this section we will rationalize the third stylized fact that, on average, LBs do not end abruptly, but with soft landings. In the previous section soft landings were imposed *exogenously* by assuming that there is a date T by which the LB must come to an end. For instance, T can represent the date at which agents anticipated an event, such as a regulatory reform, after which they will not be able to access risky projects. In this section we will explore under which circumstances a LB might end *endogenously*, even if no reform is on the horizon ($T = \infty$). The point we will make is that once entrepreneurs become rich enough, they stop investing in negative NPV projects (i.e., risky investment drops to zero). The key ingredient for this to occur is the existence of an upper bound on the capital that can be invested in the risky project $\bar{k}$.

$$k_t^i \leq \overline{k} \tag{4.1}$$

As we will show in the next section, this upper bound can be considered as reflecting the existence of decreasing returns to capital in the risky technology.

Recall that equity or *pledgeable wealth* is the wealth that creditors can appropriate in case of default. That is, $w^p = w - (1 - \beta\psi_s)s$. In Section 2 we showed that entrepreneurs adopt risky projects if they can leverage their firms sufficiently, and do not need spend too much on land, so that they can attain a high capital-equity ratio ($\mathcal{K} = k/w^p$). In the model of Section 2 a switch cannot occur because along any risky REE $\mathcal{K}$ is constant. As we will see, in the presence of a capacity constraint $\bar{k}$ it is possible for equilibrium $\mathcal{K}$ to fall as wealth increases. Thus, a switch might occur when wealth reaches a certain threshold.

We will consider two scenarios depending on whether safe storage s is fully pledgeable ($\psi_s = \beta^{-1}$) or not ($\psi_s < \beta^{-1}$). If $\psi_s = \beta^{-1}$and wealth is increasing along the lucky path, then $\mathcal{K}$ must fall once $\bar{k}$ is reached. Therefore, there is a time at which *all* wealth is switched to the safe storage technology. This argument is similar to that emphasized in the deposit insurance literature, according to which only undercapitalized banks are subject to the 'moral hazard problem due to deposit insurance'. Banks with sufficiently high capital net of contingent liabilities will behave prudently (Mishkin (1998)).

Under the second scenario $\psi_s < \beta^{-1}$ the effective capital argument does not apply because entrepreneurs can keep their safe storage off limits from lenders. Thus, in principle, equity (i.e., pledgeable wealth (w^p)) could be kept constant while wealth (w) was increasing. Surprisingly, under some conditions this argument does not hold. This is because once $\bar{k}$ is reached entrepreneurs might be tempted to buy more land. Since land is in fixed supply its opportunity cost will increase. We will show that under some circumstances the increase in land's opportunity cost is so large that it drives the equilibrium $\mathcal{K}$ below the profitability level. Therefore, all resources are shifted to the safe storage technology. Note that in both scenarios there is a soft landing. That is, in anticipation of the switch, at some point in time the price of land peaks and starts to gradually fall, smoothly reaching zero at the time of the switch.

The only difference relative to section 2 is the addition of capacity constraint (4.1). We focus again on risky equilibria in which there is a bailout if the lucky path ends, but not otherwise. Entrepreneurs solve Problem I with the additional constraint (4.1). As in Section 2 we will construct REE by splicing together temporary equilibria (TE). Our objective is to show that under the conditions of Section 2, there may be *no* risky TE for large wealth levels. Note that, as in Section 2, TE in which everyone uses only the riskless technology exist for all wealth levels. The results that follow are that under certain conditions, in *every* REE, all wealth must eventually be shifted away from the risky technology.

4.1. Returns from storage are fully pledgeable

Since *pledgeable wealth* equals $w^p = w - (1 - \beta\psi_s)s$, when $\psi_s = \beta^{-1}$ all wealth equals equity in the risky firms that entrepreneurs run (i.e., $w^p = w$). Now, for a risky TE to exist, the capital to equity ratio $\mathcal{K} = k/w^p$ must be larger than a certain threshold $\widetilde{\mathcal{K}}$. Since $\mathcal{K}$ inevitably falls with wealth as soon as $\bar{k}$ is reached, $\mathcal{K}$ must drop below the threshold $\widetilde{\mathcal{K}}$ for sufficiently large wealth levels. Formally, this leads us to:

Lemma 4.1 (TE: fully pledgeable storage). *If $\psi_s = \beta^{-1}$ and $\psi \geq \underline{\psi}$, then there exists a critical value of wealth $\widetilde{w}$ (defined in (7.17)) such that a symmetric risky TE exists if and only if $w \leq \widetilde{w}$.*

Note that the only condition on ψ required in this Lemma is one already invoked in section 2 ($\psi \geq \underline{\psi}$), which ensures that a risky TE exists for *some* level of wealth. To establish that in *any* symmetric REE there must be a switch, we need only make sure that wealth grows even after investment capacity is reached. As Proposition 3 will show, this is the case if the savings rate is not smaller than the discount factor.

4.2. Returns from safe storage are *not* fully pledgeable

The existence of switching in the case $\psi_s = \beta^{-1}$ relies on the fact that all the bank accounts of entrepreneurs can be confiscated in case of default. This discourages rich entrepreneurs from gambling. What happens if entrepreneurs' accounts are off limits ($\psi_s = 0$)? At first glance, it appears that there should be no switching.

After all, why shouldn't the entrepreneur simply store most of his wealth abroad, and put up just enough collateral $w^p = ul + (1 - \beta\psi)\overline{k}$ to invest up to capacity, thereby running a highly leveraged business at home? It turns out that this strategy may not be part of an equilibrium.

The reason is that once entrepreneurs have more wealth than is required to reach capacity, they will be tempted to use this extra wealth to buy more land. Of course, since the supply of land is fixed, the end result is simply a bidding up of the opportunity cost of land ($u_t = p_t - \beta p_{t+1}(\theta)$). This, in turn, increases the required collateral, causing $\bar{\mathcal{K}} = \overline{k}/w^p$ to fall. It may therefore be that the equilibrium $\mathcal{K}$ is high enough while capacity has not been reached, but once $k = \bar{k}$ equilibrium $\mathcal{K}$ drops enough to destroy the risky equilibrium.

In contrast to the previous case, a switch is not inevitable. In the previous case, since $\psi_s = \beta^{-1}$ and pledgeable wealth is equal to w, there is always a critical $\widetilde{w}$ above which there is a switch. Here, the drop in the capital-to-equity ratio is caused by the rise in the equilibrium opportunity cost of land u. Since u is bounded above by the marginal product of land, $\overline{u} = \alpha\beta g'(1)\overline{k}$, LBs can go on forever if $\mathcal{K}$ is sufficiently high at $u = \overline{u}$. Only if the capital-to-equity ratio is sufficiently small to begin with (e.g. if ψ is not too large), can it be driven below the threshold $\widetilde{\mathcal{K}}$. The next Lemma more precisely captures this.

Lemma 4.2 (TE: not fully pledgeable storage). *Suppose that $\psi_s < \beta^{-1}$.*

- *There exists a threshold $\overline{\psi} \in [\underline{\psi}, \beta^{-1})$ such that, for any $\psi \in (\underline{\psi}, \overline{\psi})$, there is a critical value of wealth $\widetilde{w}$ (given by (7.16)) with the property that a risky symmetric TE exists if and only if $w \leq \widetilde{w}$.*
- *If land is useless in production, there exists no critical wealth level $\widetilde{w}$. That is, $\overline{\psi} \longrightarrow \underline{\psi}$ as $g'(1) \longrightarrow 0$.*

The threshold $\overline{\psi}$ must be strictly less than β^{-1}, reflecting the fact that ψ cannot be too large. In addition the effect behind this switching result is unique to an economy with land. In the limit, as land becomes useless ($g'(1) = 0$), LBs can persist forever unless (as in the previous Lemma) storage is fully pledgeable.

As in the previous case, if wealth continues to grow after capacity is reached, a switch must eventually occur. The following Proposition states the conditions under which a switch must occur in any symmetric risky REE.

Proposition 3. *Suppose that there is a capacity constraint $k \leq \overline{k}$, and the savings rate is no lower than the discount factor ($1 - c \geq \beta$).*

- *Then in every symmetric risky REE there exists a finite time $\bar{\tau}(\psi_s)$ at which wealth reaches the switching level $\widetilde{w}(\psi_s)$, defined by (7.16)-(7.17), if:*
  1. *Returns from storage are fully pledgeable ($\psi_s = \beta^{-1}$), or*
  2. *Returns from storage are not fully pledgeable ($\psi_s < \beta^{-1}$) and $\psi \in (\underline{\psi}, \overline{\psi})$.*

- *Along the lucky path, there is a switch when* $w = \widetilde{w}(\psi_s)$. *Risky investment is given by*

$$k_t^* = \begin{cases} \frac{\zeta^t e}{[1+\gamma(1)][1-\beta\psi]} & \text{if } t < \bar{\tau}'(\psi_s) \\ \bar{k} & \text{if } \bar{\tau}'(\psi_s) \leq t < \bar{\tau}(\psi_s) \\ 0 & \text{if } t \geq \bar{\tau}(\psi_s) \end{cases}$$

where $\bar{\tau}(\psi_s) \leq \bar{\tau}'(\psi_s) + 1$ *if* $\psi_s < \beta^{-1}$.

- *If* $\psi_s < \beta^{-1}$, *the path of prices is as in Proposition 1 replacing* T *by* $\bar{\tau}(\psi_s)$.

Note that risky investment follows the same path as that described by Proposition 2 up to the point where k_τ reaches $\bar{k}$. At that time there might be a switch ($k = 0$ and $s = w$), or the economy might continue for a while investing in the risky and safe technologies for a while ($k = \bar{k}$ and $s > 0$). However, there must eventually be a switch. Note that if $\psi_s < \beta^{-1}$, the switch must occur either when capacity is reached or during the following period.

5. Sequences of Booms and Busts

In section 2 we assumed that after a bust occurs for the first time the opportunity to undertake white elephant investment projects ceases to be available. In some situations this is certainly appropriate. For instance, one might expect that after the first bust a regulatory reform would take place which would prevent banks from undertaking high risk low return strategies. However, history has produced several examples in which financial crises and bailouts were followed by more lending booms. To capture this pattern we drop the assumption that the productivity process θ_t remains at zero forever after it has hit zero once. Instead we assume that the process $\{\theta_t\}$ is independent and identically distributed according to (**??**), as in Section 4.

In order to get the economy off the ground after a bad shock we assume that young entrepreneurs receive a (small) endowment whenever $\theta_t = 0$. We assume that this endowment equals a very small share ε of last period entrepreneur's wealth (i.e., $\varepsilon\zeta w_{t-1}$).[29] In all other respects the model is as in section 2. In particular, a bailout takes place if and only if a majority of entrepreneurs cannot repay their debts.

Under the process (**??**) the price of land will not drop to zero when $\theta_t = 0$, as happened in section 2. As a result, the analysis becomes more complicated. In this section we will derive a stationary risky REE under this milder assumption. The relevant state variable is entrepreneurial wealth. As before, A symmetric stationary lending RE equilibrium consists of entrepreneurs' policy functions (b^*, l^*, k^*, s^*), as well as a land price function $p^*(w)$, such that, for each state w, the land market clears: $l^*(w) = 1$, and the quadruple (b^*, l^*, k^*, s^*) solves Problem I taking as given wealth w, prices $p^*(w)$ and price expectations

[29] Note that if we transfered the same endowment (say e) whenever a bad shock were to hit, then in some cases entrepreneurs' wealth would be greater in the bad state than in the good state (i.e., $\zeta w < e$).

$(p^*([1-c]\pi(w;\theta)), p^*([1-c]\pi(w;0)))$. Note that the conditional profit function $\pi(w;\tilde{\theta})$ is the *equilibrium* profit of an entrepreneur conditional on the future productivity shock. Note also that in every state w, expected prices are the prices that would arise in the two possible successor states. We prove in the appendix the following Proposition.

Proposition 5.1. *If the pledgeability parameter $\psi \geq \underline{\psi}$, the equilibrium growth rate $\zeta < 1/\beta$, and the horizon is infinite, then there exits a stationary symmetric risky REE defined on the state space given by $\tilde{W} = \{w \in \Re_+\}$. In every state $w \in \tilde{W}$:*

- *Entrepreneurs pay the riskless interest rate, and default if and only if a bad shock occurs.*
  - *There is no storage. Risky capital, borrowing and the price of land are given by:*

$$k^*(w) = \frac{w}{[1+\gamma(1)][1-\beta\psi]} \tag{5.1}$$

$$b^*(w) = \beta[p^*(w) + \psi k^*(w)] \tag{5.2}$$

$$p^*(w) = \frac{1}{1-\beta\zeta}\frac{\gamma(1)}{1+\gamma(1)}w \tag{5.3}$$

Note that the equilibrium paths of capital, borrowing and prices are made equal to those in Proposition 2 by setting $T = \infty$. Recall that Proposition 2 was derived under the assumption that the price in the bad state was always equal to zero. Thus, any entrepreneur who borrowed was automatically unable to repay in the bad state because all his wealth was wiped out. We showed that in those circumstances, the optimal policy for an entrepreneur was to borrow up to the value of his collateral. Given this, we derived k^*, l^*, and s^*.

In this section land's price in the bad state is positive ($p^*(e) > 0$). As before, an entrepreneur can follow two types of borrowing policies: a 'risky' one and a 'safe' one. Under a risky policy he will not be able to repay his debt in the bad state. In contrast, under a safe policy he borrows less than what he will be able to repay in the bad state. Proposition 5.1 is proven by showing that borrowing up to the borrowing constraint is always preferred to any safe borrowing policy. This implies that entrepreneurs solve Problem I of section 2. Therefore, the equilibria are made the same as those characterized in Propositions 1 and 2 by setting $T = \infty$.

Following a safe borrowing policy might be more profitable than setting $b = \bar{b}$ because land's price in the bad state is positive. The argument that shows that an entrepreneur will not find it profitable to deviate and choose a safe policy is as follows. Suppose an entrepreneur deviates by choosing a safe borrowing policy $b^i < \beta l^i p^*(\varepsilon w)$. Since equilibrium prices are increasing in wealth, this deviations entails setting $b^i < \bar{b}^i$. Thus, the deviant faces an opportunity cost of capital of 1 instead of $1 - \beta\psi$. This deviation implies that he will set $k^i = 0$ because the expected marginal product of capital $\alpha\theta g'(l^i)$ is lower than the safe return $1/\beta$. As a result, the return to investing in land will be composed only of capital gains. However, if all other entrepreneurs use land in production, the price along the lucky path grows at a lower rate than the safe rate of return. Thus, the expected gain on land holdings is lower than that on storage. Therefore, the best strategy for the deviant is to store all wealth. Lastly, if $\psi \geq \underline{\psi}$, storing all wealth does not generate more expected profits than in equilibrium (for the same reasons as in section 2). Therefore, such a unilateral deviation is not profitable.

Along the equilibrium path wealth evolves according to

$$w_{t+1} = \begin{cases} \zeta w_t & \text{with probability } \alpha \\ \varepsilon \zeta w_t & \text{with probability } 1-\alpha \end{cases} \tag{5.4}$$

Suppose we start the economy off at $w = e$. It then embarks on a path in which wealth grows (or shrinks!) at rate ζ. It stays on this path as long as good shocks ($\theta_t = \theta$) prevail. The first bad shock returns wealth to $\varepsilon \zeta w_t$. The slope of these paths depends (as does the slope of the lucky path in section 2) on whether ζ is greater or smaller than one. On one hand, if $\zeta < 1$, wealth and prices shrink as long as good shocks prevail. If we set the initial endowment e low, we may think of this case as an equilibrium in which the price is approximately zero throughout.[30] In other words, if the consumption rate and the expenditure share on land $\gamma(1)$ are high (so that $\zeta < 1$), then lending booms practically do not develop. On the other hand, if $\zeta > 1$, the situation changes dramatically. In this case, an economy starting with wealth e will experience boom paths of the type characterized in Proposition 2, with prices, lending and investment increasing hand in hand. While subsequent booms vary in length, with probability one they all end in a crash which leads to another bailout.

6. Conclusion

We have presented a model that combines two distortions prevalent in numerous economies: government bailout guarantees and the imperfect enforceability of contracts. This has allowed us to rationalize several stylized facts associated with lending booms. Also, the model allows us to address issues that have been at the forefront of the policy debate in recent years. We can address issues such as why it is that financial reforms have often lead to crises, what the correct policy towards capital inflows should be and whether lending booms are necessarily harmful.

We will expand on the last issue by making reference to the privatization of the Mexican banks, although the same type of debate has been going in several emerging economies. Mexican officials were well aware of the LB that was going on during the years following the privatization. However, it was argued that after decades of statism, the financial reforms were essential to building up an entrepreneurial class, accelerating financial deepening and promoting long-run growth. The idea underlying this policy stance runs as follows.

[30] Recall that the role of the endowment e is only to provide a very small seed for the economy to take off if $\theta_t = \mathbf{0}$.

The economy is composed of an entrepreneurial sector and a competitive sector, and the size of the former has positive externalities on the latter. For instance, a larger entrepreneurial sector increases the growth rate of the competitive sector. However, the entrepreneurial sector can only grow gradually (e.g., its capital stock is limited by entrepreneurial wealth). Thus, policies that accelerate the growth of the entrepreneurial sector might be beneficial to the economy. Clearly, these policies might generate LBs, in which there will be asset price inflation and some inefficient investment (or looting so to speak). This, in turn, will make the economy vulnerable to crises during the transition period. However, in this case the benefits might outweigh the costs. Thus, Mexican officials might very well say there was nothing wrong ex-ante with their reform policies.

Our model can readily be extended to rationalize this widespread policy stance. In our model when entrepreneurial wealth reaches a certain threshold, there is a switch in which all wealth is shifted away from inefficient projects. Thus, there is a transitional period during which there is asset price inflation and overinvestment. This LB path might end with a soft landing, or with a crash. By modeling the competitive sector and introducing a policymaker, one could then investigate the class of objective functions that might render an LB path optimal ex-ante.

Government bailout guarantees and agency problems in financial markets are pervasive in numerous countries. Understanding the macroeconomic implications of these conditions is essential to understanding the economic experiences of these countries, and to designing economic policies.

7. Appendix

7.1. The Credit Market Game

Here we will analyze the credit market game described in section 2. A typical entrepreneur in cohort t begins with inherited wealth w_t. He observes the price of land (p_t), and holds price expectations $(p_{t+1}(\theta), p_{t+1}(0))$ for the two states of nature that are possible in period $t+1$. At time t entrepreneur i makes a take-it-or-leave-it offer $(\varkappa_t^i, b_t^i)$ to lenders, where $\varkappa_t^i$ is the return he offers and b_t^i is the amount he is borrowing. Simultaneously he announces his plan (k_t^i, l_t^i, s_t^i), which is required to satisfy budget constraint (3.4). Lenders then decide whether to accept or reject the contract offered. In period $t+1$, the productivity shock hits and entrepreneurs simultaneously decide whether to default or not. The payoff to an entrepreneur who defaults is

$$\pi_{t+1}^i(def;\theta_{t+1}) = \theta_{t+1} g(l_t^i) k_t^i + (1 + r - \psi_s) s_t^i - \frac{\theta_{t+1}}{\theta} \psi k_t^i \tag{7.1}$$

While if he repays the promised amount $\varkappa^i b^i$, his payoff is

$$\pi_{t+1}^i(no - def;\theta_{t+1}) = \theta_{t+1} g(l_t^i) k_t^i + p_{t+1}(\theta_{t+1}) l_t^i + (1 + r) s_t^i - \varkappa_t^i b_t^i \tag{7.2}$$

The lenders who have lent to entrepreneur i get the promised amount $\varkappa_t^i b_t^i$ if either i repays, or i defaults and a bailout occurs (i.e. more than half of entrepreneurs default). If i defaults but lenders are not bailed out, their payoff is

$$\pi_{t+1}^L(def,\ no-bailout;\theta_{t+1}) = \min\left[\varkappa_t^i b_t^i,\ p_{t+1}(\theta_{t+1})l_t^i + \frac{\theta_{t+1}}{\theta}\psi_t^i k_t^i\right] \tag{7.3}$$

We will consider symmetric subgame perfect equilibria of this credit market game. We will assume $p_{t+1}(\theta_{t+1}) < \beta^{-1}p_t$. Otherwise the demand for land would be infinite.

Lemma 7.1. *1. There is a symmetric SPE of the credit market game in which every entrepreneur stores all wealth.*

2. In every symmetric SPE that allows for positive investment, there cannot be default in the good state.

Proof. In the last stage, entrepreneurs do not default if $\pi_{t+1}^i(def;\theta_{t+1}) \leq \pi_{t+1}^i(no-def;\theta_{t+1})$, or equivalently

$$\varkappa_t^i b_t^i \leq p_{t+1}(\theta_{t+1})l_t^i + \psi_s s_t^i + \frac{\theta_{t+1}}{\theta}\psi k_t^i \tag{7.4}$$

In particular, if they default in the good state, they will also default in the bad state. Hence, there can never be an equilibrium in which there is a bailout in the good state, but not in the bad state.

Lenders' strategies can be characterized by their reservation interest rate. They require the riskless rate if either (i) there is a bailout in both states, (ii) there is a bailout in the bad state and (7.4) holds for $\theta_{t+1} = \theta$, or (iii) there is no bailout but (7.4) holds for $\theta_{t+1} = 0$. They accept the risky rate $\frac{1-r}{\alpha} - 1$ if either (i) there is no bailout and (7.4) holds for $\theta_{t+1} = \theta$ or (ii) there is a bailout in the bad state, but (7.4) does not hold for $\theta_{t+1} = \theta$.

Suppose everyone stores all wealth. Then there is no bailout in either state. A deviation can only be profitable if it involves investment in capital. But lenders will require the risky rate $\frac{1-r}{\alpha} - 1$ which is strictly higher than the net return on capital for all l. This demonstrates part 1.

Suppose now that there was an equilibrium where a bailout takes place in the good state. As noted above, there must then be a bailout in both states. But then an entrepreneur could deviate by borrowing more and buying more land. Lenders would still accept the new offers since the reservation rate is independent of the borrower's plan. This would increase profits in the good state since g is increasing in land, while leaving the profit in the bad state unchanged. This demonstrates part 2.■

7.2. Proofs of Sections 2 and 3

In this subsection we provide the solution to Problem I, which is not a standard convex problem. Note first that there are two types of policies that solve Problem I: a risky policy, that leads to default in the bad state; and a safe policy under which a default never occurs. Note also that in any equilibrium $p > \alpha\beta\widehat{p}(\theta)$. Otherwise the demand for land would be infinite. It then follows that the best safe policy is given by $k = l = 0$ and $s = w$. This yields an expected payoff of w. To find the best risky policy, we have to maximize (3.10) subject to borrowing constraint (3.11) and $k \leq \bar{k}$. Finally, we will have to check whether the best risky policy yields an expected payoff higher than w. Let $I = ul + (1 - \beta\psi)k$ denote the total collateral required for running the risky technology. We can search for the best risky policy in two stages: first the entrepreneur chooses an amount of collateral I and storage s and then he allocates collateral optimally between capital k and land l. The 'second stage' problem is

$$\begin{aligned} \rho(I) = \quad & \max_{k,l} \alpha\beta[\theta g(l) - \psi]k \\ & \text{subject to} \quad ul + [1 - \beta\psi]k = I, \quad k \leq \bar{k} \end{aligned} \tag{7.5}$$

Here ρ represents expected revenue from the risky technology. Substituting $I = ul + [1 - \beta\psi]k$ into (3.10) and (3.11) yields the 'first stage' problem

$$\begin{aligned} & \max_{I,s} \ \pi(I) := \rho(I) + \alpha[w - I] + [1 - \alpha][1 - \beta\psi_s]s \\ & \text{subject to} \quad I + [d1 - \beta\psi_s]s \leq w, \qquad I \geq 0, \quad s \geq 0 \end{aligned} \tag{7.6}$$

It is clear that if $\psi_s < \beta^{-1}$, the first constraint must bind. Meanwhile, if $\psi_s = \beta^{-1}$, storage drops out from the first stage problem. Thus, the first stage problem becomes

$$\max_{I,s} \ \pi(I) := \rho(I) + \eta[w - I] \quad \text{subject to} \quad 0 \leq I \leq w \tag{7.7}$$

where $\eta = 1$ if $\psi_s < \beta^{-1}$ and $\eta = \alpha$ if $\psi_s = \beta^{-1}$.

Lemma 7.2 (Solution to the second stage problem). *Problem (7.5) has a maximizing solution $\{l^*(I), k^*(I)\}$ if and only if the opportunity cost of land, $u = p - \beta\widehat{p}$, is non-negative. Furthermore,*

1. *There exists a unique level of risky investment, $\overline{I}$, such that $k^*(I) = \bar{k}$ if and only if $I \geq \overline{I}$.*

2. *Profit function $\rho(I)$ is increasing and* $\rho''(I) \begin{cases} > 0 & \text{if } I < \overline{I} \\ < 0 & \text{if } I \geq \overline{I} \end{cases}$

Proof. The Lagrangian associated with problem (7.5) is

$$\mathcal{L} = \alpha\beta[\theta g\,(l) - \psi]k + \lambda\,[I - ul - [1 - \beta\psi]k] + v[\bar{k} - k] \tag{7.8}$$

The necessary conditions for a maximum are $b^i = \bar{b}^i$,

$$0 = \alpha\beta\theta g'(l)k - \lambda u, \quad \lambda \geq 0 \tag{7.9}$$

$$0 = \alpha\beta[\theta g(l) - \psi] - \lambda[1 - \psi\beta] - v \tag{7.10}$$

$$0 = [\bar{k} - k]\nu, \text{ with } k \leq \bar{k}, \quad \nu \geq 0 \tag{7.11}$$

Since $g'(l) > 0$, (7.9) holds only if $u \geq 0$. Since $g''(l) < 0$, $\psi \leq \beta$ and $u \geq 0$, the bordered Hessian is unambiguously positive: $B \equiv \alpha\beta\theta[1-\psi\beta][2ug'(l) - g''(l)k[1-\psi\beta]] > 0$. Thus, the above conditions are also sufficient for a maximum. First, we characterize the optimum in the case $k < \bar{k}$. Since $v = 0$ and $B > 0$, k and l are uniquely determined by

$$u\left[\theta g(l) - \psi\right] \;=\; \theta g'(l)k[1-\psi\beta] \tag{7.12}$$

$$I \;=\; ul + [1-\psi\beta]k, \quad \text{and} \tag{7.13}$$

$$\frac{dl}{dI} = \frac{[1-\psi\beta]g'(l)}{B[\alpha\beta\theta]^{-1}} > 0, \qquad \frac{dk}{dI} = \frac{ug'(l) - k[1-\psi\beta]g''(l)}{B[\alpha\beta\theta]^{-1}} > 0 \tag{7.14}$$

Since $\frac{dk}{dI} > 0$, there exists a unique level of I, call it $\overline{I}$, such that $k(I) \geq \bar{k}$ if and only if $I \geq \overline{I}$. This proves part 1 of Lemma 7.2. To derive part 2 note that since λ is the Lagrange multiplier associated with the budget constraint in (7.5), $\rho'(I) = \lambda = \frac{\alpha\beta[\theta g(l)-\psi]}{1-\psi\beta}$. Thus, if $I < \overline{I}$, $\rho''(I) = \frac{\alpha\beta\theta g(l)'}{1-\psi\beta}\frac{dl}{dI} > 0$ (by (7.14)). Second, we consider the case $k = \bar{k}$. In this case λ and l are determined by (7.9) and (7.13) substituting $\bar{k}$ for k. Condition (7.10) can then be expressed as:

$$\nu = \left\{I - [1-\psi\beta][1+\gamma(l)]\bar{k}\right\}\frac{\alpha\beta[\theta g(l) - \psi]}{u}, \qquad \gamma(l) \equiv \frac{\theta g'(l) l}{\theta g(l) - \psi} \tag{7.15}$$

It follows that (7.11) is satisfied if and only if $I \geq \overline{I} \equiv [1-\psi\beta][1+\gamma(l)]\bar{k}$. Condition (7.13) implies that $\frac{dl}{dI} = \frac{1}{u} \geq 0$. Since $\rho'(I) = \lambda = \frac{\alpha\beta\theta g'(l)\bar{k}}{u}$, we have that $\rho''(I) = \frac{\alpha\beta\theta g''(l)\bar{k}}{u}\frac{dl}{dI} < 0$ in the case $I \geq \overline{I}$.■

Note that it is not necessary to provide a complete solution to Problem I for any set of prices. It suffices to consider the case in which the land market clears ($l = 1$). Problem I has three types of risky solutions (i.e., with $I > 0$). There could be either an extremal optimum ($I = w$) with $k < \overline{k}$, an extremal optimum ($I = w$) with $k = \overline{k}$ or an interior optimum ($\rho'(I) = \eta$). We consider each case in turn. We will refer repeatedly to the following three functions

$$\begin{aligned}\widehat{w}(\psi) &\equiv \alpha\beta[\theta g(1) - \psi]\bar{k} \\ \bar{w}(\psi, \eta) &\equiv [1-\beta\psi]\left[1 + \frac{\gamma(1)\alpha\beta}{\eta}\frac{\theta g(1) - \psi}{1-\beta\psi}\right]\bar{k} \\ \underline{w}(\psi) &\equiv [1-\beta\psi][1+\gamma(1)]\bar{k}\end{aligned} \tag{7.16}$$

Lemma 7.3 (TE: $k < \overline{k}$ and $s = 0$). *If*

$$w < \underline{w}(\psi) < \widehat{w}(\psi),$$

the unique symmetric risky TE is given by $k^* = \frac{w}{[1+\gamma(1)][1-\beta\psi]} < \bar{k}$, $l^* = 1$, $b^* = \bar{b}$, $s^* = 0$ *and* $u^* = p - \beta\widehat{p} = \frac{\gamma(1)}{1+\gamma(1)}w$.

Proof. By substituting $(k^*, 1)$ into the first order conditions (7.9)-(7.11), one can verify that $(k^*, 1)$ is a solution to (7.5) given prices u^* and $I = w$. This yields expected revenue $\rho(w) = \frac{\alpha\beta(\theta g(1)-\psi)}{[1+\gamma(1)][1-\beta\psi]}w$. The shadow price of collateral (i.e. the multiplier on the constraint in (7.5)) at $I = w$ is $\lambda(u^*, w) = \frac{\alpha\beta(\theta g(1)-\psi)}{[1-\beta\psi]}$. The condition $\underline{w}(\psi) < \widehat{w}(\psi)$ implies that $\lambda(u^*, w) > 1$. It follows that $\pi'(w) = \rho'(w) - \eta = \lambda(u^*, w) - \eta > 0$, so that π has a local maximum at $I = w$. In addition, since $k^* < \overline{k}$ is optimal, we must have $w < \overline{I}$ at the given prices. Since $\rho(I)$ (and hence π) is convex for all $I < w < \overline{I}$ (by Lemma 7.2), there cannot exist another local maximum. Finally, the condition $\underline{w}(\psi) < \widehat{w}(\psi)$ is equivalent to $\pi(w) = \rho(w) > w$. Thus, the best risky solution to Problem I is indeed better than the best riskless one.■

We now ask what happens if capacity is reached. First, there is a wealth range where entrepreneurs invest up to capacity, but do not store. Then there is a wealth range where they are indifferent between storage and investment. The following two Lemmas consider these cases.

Lemma 7.4 (TE: $k = \overline{k}$ and $s = 0$). *If*

$$\underline{w}(\psi) \leq w < \min\{\widehat{w}(\psi), \bar{w}(\psi, \eta)\},$$

the unique symmetric risky TE is given by $k^* = \bar{k}$, $l^* = 1$, $b^* = \bar{b}$, $s^* = 0$ *and* $u^* = p - \beta\widehat{p} = w - [1-\beta\psi]\bar{k} > 0$.

Proof. The proof parallels that of the previous Lemma. Again, it can be verified from the first order conditions that $(\bar{k}, 1)$ is optimal in the second stage problem for $I = w$. Expected revenue is now $\rho(w) = \alpha\beta(\theta g(1) - \psi)\overline{k} = \widehat{w}(\psi)$ and the shadow price of collateral is $\lambda(u^*, w) = \alpha\beta g'(1)\bar{k}/u^*$. The condition $w < \bar{w}(\psi, \eta)$ implies that $\lambda(u^*, w) > 1$, so that $\pi'(w) > 0$ and π has a local maximum at $I = w$. Suppose there was another (interior) local maximum, say I^o, Then $\pi'(I^o) = \rho'(I^o) - \eta = 0$ and $\pi''(I^o) = \rho''(I^o) < 0$ so we must have $\overline{I} < I^o$. But $\rho''(I^o) < 0$ for all $I > \overline{I}$, contradicting $\pi'(w) > 0$. Finally, the condition $w < \widehat{w}(\psi)$ is equivalent to $\pi(w) = \rho(w) > w$, so that the best risky solution to Problem I is better than the best riskless one.■

Lemma 7.5 (TE: $k = \overline{k}$ and $s > 0$). *If*

$$\bar{w}(\psi, \eta) \leq \ w \ < \frac{\widehat{w}(\psi) - \eta\bar{w}(\psi, \eta)}{(1-\eta)w},$$

then the unique symmetric risky TE is given by $k^* = \bar{k}$, $b^* = \bar{b}$, $s^* = w-\ \bar{w}(\psi, \eta)$ *and* $u^* = p^* - \beta\widehat{p} = \eta^{-1}\alpha\beta g'(1)\bar{k}$

Proof. Regardless of the value of η, it can be verified from the first order conditions that $(\bar{k}, 1)$ is optimal in the second stage problem for $I = u^* + (1-\beta\psi)\overline{k}$. Expected revenue is $\rho(u^* + (1-\beta\psi)\overline{k}) = \alpha\beta(\theta g(1) - \psi)\overline{k} = \widehat{w}(\psi)$. Consider the first stage problem. In the case $\eta = 1$ the shadow price of collateral is $\lambda(u^*, w) = \alpha\beta g'(1)\bar{k}/u^* = 1$. This together with $k^* = \overline{k}$ implies that π has a local maximum at $I = u^* + (1-\beta\psi)\overline{k}$. Since π is convex for low I and concave for high I, there can be no other local maximum, and $\pi(u^*+(1-\beta\psi)\overline{k}) > \pi(w)$. Finally $\pi(u^*+(1-\beta\psi)\overline{k}) = \widehat{w}(\psi)+\alpha(w-(u^*+(1-\beta\psi)\overline{k})) = \widehat{w}(\psi)+w-\bar{w}(\psi, \eta)$.

Thus, the condition $\widehat{w}(\psi) > \bar{w}(\psi,\eta)$ ensures that the best risky solution to Problem I is better than the best riskless one.

Now consider the case $\eta = \alpha$. The shadow price of collateral is $\lambda(u^*, w) = \alpha\beta g'(1)\bar{k}/u^* = \alpha$. As in the case $\eta = 1$, π has a unique local maximum at $I = u^* + (1-\beta\psi)\bar{k}$. But now $\pi(u^* + (1-\beta\psi)\bar{k}) = \widehat{w}(\psi) + \alpha(w - (u^* + (1-\beta\psi)\bar{k})) = \widehat{w}(\psi) + \alpha(w - \bar{w}(\psi,\eta))$. Thus, the condition stated in Lemma 7.5 ensures that the best risky solution to Problem I is better than the best riskless one.■

If (w,ψ) are such that none of the cases considered in Lemmas 7.2-7.5 apply, then the only TE is the safe one where everybody stores all wealth. This characterizes the symmetric TE for all levels of wealth. In order to prove that the conditions stated in Lemmas 7.2-7.5 are not vacuous, we need to show that in fact $\underline{w}(\psi) < \widehat{w}(\psi)$, so that a risky TE exists for low wealth levels. The next lemma shows that this is the case.

Lemma 7.6. *There exists a unique $\underline{\psi} \in (0, \beta^{-1})$ such that $\underline{w}(\psi) < \widehat{w}(\psi)$ if and only if $\psi > \underline{\psi}$. The threshold $\underline{\psi}$ is independent of $\bar{k}$.*

Proof. Define the functions $\widehat{J}(\psi) = \alpha\frac{\theta g(1)-\psi}{\beta^{-1}-\psi}$ and $\underline{J}(\psi) = \frac{\theta(g(1)+g'(1))-\psi}{\theta g(1)-\psi}$. We have $\underline{w}(\psi) < \widehat{w}(\psi)$ if and only if $\underline{J}(\psi) < \widehat{J}(\psi)$. Now $\underline{J}(0) > \widehat{J}(0)$ by Assumption 1. Moreover $\widehat{J}(\psi) \to \infty$ as $\psi \to \beta^{-1}$, whereas $\underline{J}(\beta^{-1}) < \infty$. By continuity, it follows that there is at least one intersection, say $\underline{\psi}$. Finally, $\underline{J}'(\psi) = \frac{\theta g'(1)}{(\theta g(1)-\underline{\psi})^2} = \alpha\frac{\theta g'(1)}{(\theta(g(1)+g'(1))-\underline{\psi})(\beta^{-1}-\psi)} < \alpha\frac{\theta g(1)-\beta^{-1}}{(\beta^{-1}-\underline{\psi})^2} = \widehat{J}'(\underline{\psi})$ where the last inequality follows from the fact g is concave so that $\theta g(1) - \beta^{-1} = \theta(g(1)-g(0)) > \theta g'(1)$. Hence $\widehat{J}(\underline{\psi})$ must cut $\underline{J}(\psi)$ from below, so that the intersection is unique.■

Proof of Proposition 1.

The setup of section 2 has $\bar{k} = \infty$. Therefore $w < \underline{w}(\psi)$ for every w. Let $\underline{\psi}$ be defined by Lemma 7.6. Then for all $\psi > \underline{\psi}$, Lemma 7.3 applies and delivers the result.■

Proof of Lemma 4.1.

Since $\psi_s = \beta^{-1}$, we have that $\eta = \alpha$. The condition $\psi > \underline{\psi}$ implies that there is a risky TE for every $w \le \underline{w}(\psi)$. Now if $\underline{w}(\psi) < w \le \bar{w}(\psi,\eta)$, Lemma 7.4 implies that there exists a risky TE for $w \le \widehat{w}(\psi)$, while if $w > \bar{w}(\psi,\eta)$, then Lemma 7.5 implies that there is a risky TE for $w \le \frac{\widehat{w}(\psi)-\alpha\bar{w}(\psi,\eta)}{1-\alpha}$. Combining these facts, the critical value is

$$\widetilde{w} = \max\left\{\widehat{w}(\psi), \frac{\widehat{w}(\psi)-\alpha\bar{w}(\psi,\eta)}{1-\alpha}\right\} \tag{7.17}$$

Proof of Lemma 4.2.

If $\eta = 1$, Lemmas 7.2-7.5 imply that there is a critical value $\widetilde{w} = \widehat{w}(\psi)$ if and only if

$$\underline{w}(\psi) \le \widehat{w}(\psi) < \bar{w}(\psi,1)$$

It follows from the definitions that $\underline{w}(\psi) \leq \widehat{w}(\psi)$ implies $\underline{w}(\psi) < \bar{w}(\psi, 1)$ for all ψ. Define the function $\overline{J}(\psi) = \frac{\theta g(1)-\psi}{\theta(g(1)-g'(1))-\psi}$. Then $\overline{w}(\psi, 1) > \widehat{w}(\psi)$ if and only if $\overline{J}(\psi) < \widehat{J}(\psi)$ where $\widehat{J}(\psi) = \alpha \frac{\theta g(1)-\psi}{\beta^{-1}-\psi}$. Clearly $\overline{J}(0) > \widehat{J}(0)$ and $\widehat{J}(\beta^{-1}) = \infty > \overline{J}(\beta^{-1})$ so that there is an intersection, say $\overline{\psi}$. Finally $\widehat{J}'(\overline{\psi}) = \alpha \frac{\theta g(1)-\beta^{-1}}{(\beta^{-1}-\overline{\psi})^2} = \frac{\theta g(1)-\beta^{-1}}{(\theta(g(1)-g'(1))-\overline{\psi})^2} > \frac{\theta g'(1)}{(\theta(g(1)-g'(1))-\overline{\psi})^2} = \overline{J}'(\underline{\psi})$ which implies that the intersection is unique.■

Proof of Proposition 3.

We consider first the case $\psi_s = \beta^{-1}$. Since $\psi \geq \underline{\psi}$ by assumption, Lemma 4.1 implies that there is a critical wealth $\widetilde{w}$ defined by (7.17). Note that $\widehat{w}(\psi) < \overline{w}(\psi, \alpha)$ if and only if $\widetilde{w} = \widehat{w}(\psi)$. Lemma 7.3, implies that for any wealth level $w_t < \underline{w}(\psi)$, in any risky symmetric REE $k_t^* = \frac{w_t}{[1-\beta\psi][1+\gamma(1)]}$ and $s_t^* = 0$. Wealth then grows at the rate $\zeta - 1 > 0$ in this region. Denote by τ the first time that wealth is no smaller than $\underline{w}(\psi)$

$$\tau = \inf \{t \geq 0 \mid w_t \geq \underline{w}(\psi)\}$$

First, if $w_\tau \geq \widetilde{w}$, there is a switch at τ. Second, if $w_\tau < \widetilde{w}$ and $\widehat{w}(\psi) < \overline{w}(\psi, \alpha)$ (i.e. $\widetilde{w} = \widehat{w}(\psi)$), then Lemma 7.4 implies that at τ entrepreneurs invest $\overline{k}$ but do not store. Thus, $w_{\tau+1} = \frac{1-c}{\alpha\beta}\widehat{w}(\psi)$. Since we have assumed that $1 - c > \beta$ there must be a switch at $\tau + 1$. Third, we consider the case $w_\tau < \widetilde{w}$ and $\widehat{w}(\psi) > \overline{w}(\psi, \alpha)$, so that $\widetilde{w} = \frac{\widehat{w}(\psi)-\alpha\overline{w}(\psi,\alpha)}{1-\alpha}$. Note that in this case w must grow beyond $\overline{w}(\psi, \dot{\alpha)}$. Indeed, if $w_\tau < \overline{w}(\psi, \dot{\alpha)}$, then Lemma 7.4 implies that $w_{\tau+1} = \frac{1-c}{\alpha\beta}\widehat{w}(\psi) > \overline{w}(\psi, \alpha)$. Let τ' be such that $w_{\tau'} > \overline{w}(\psi, \dot{\alpha)}$ (τ' could be τ or $\tau + 1$). Lemma 7.5 then implies that wealth satisfies $w_{t+1} = \frac{1-c}{\alpha\beta}\widehat{w}(\psi) + (1 - c)\beta^{-1}(w_t - \bar{w}(\psi, \alpha)) = \frac{1-c}{\beta}w_t + (\alpha^{-1}\widehat{w}(\psi) - \bar{w}(\psi, \alpha))$ for all $t \geq \tau'$. Since $1 - c > \alpha\beta$ and in this case $\widehat{w}(\psi) > \overline{w}(\psi, \alpha)$, the wealth path must be strictly increasing. Hence, there is a finite time at which wealth becomes greater that $\widetilde{w}$, and a switch must occur.

Next we consider the case $\psi_s < \beta^{-1}$. Since $\psi \in (\underline{\psi}, \overline{\psi})$ by assumption, Lemma 4.2 implies that there is a critical wealth level $\widetilde{w} = \widehat{w}(\psi)$. As in the case $\psi_s = \beta^{-1}$, there is a smallest time τ such that $w_\tau \geq \underline{w}(\psi)$. If $w_\tau \geq \widehat{w}(\psi)$, there is a switch at τ. If instead $w_\tau < \widehat{w}(\psi)$, then Lemma 7.4 implies that $w_{\tau+1} = \frac{1-c}{\alpha\beta}\widehat{w}(\psi) > \widehat{w}(\psi)$, so that there must be a switch at $\tau + 1$. ■

7.3. Proofs of Section 4

We will refer repeatedly to the efficient (k^*) and Pangloss ($\widetilde{k}$) investment levels. They are defined by $\alpha\beta\theta f'(k^*) = 1$ and $\beta\theta f'(\widetilde{k}) = 1$, respectively. To prove Proposition 4 we will characterize the value functions associated with the safe and risky investment policies (**??**) and (**??**). Replacing them in profit function (**??**), we get

$$\pi^s(w) = \begin{cases} \alpha\beta\theta f(w) & \text{if } w \leq k^* \\ \alpha\beta\theta f(k^*) + w - k^* & \text{if } w > k^* \end{cases} \tag{7.18}$$

$$\pi^r(w) = \begin{cases} \alpha\beta\theta f(\frac{w}{1-\beta\psi}) - \alpha\left[\frac{w}{1-\beta\psi} - w\right] & \text{if } w \leq \widetilde{k}[1-\beta\psi] \\ \alpha\beta\theta f(\widetilde{k}) - \alpha\left[w - \widetilde{k}\right] & \text{if } \widetilde{k} > w > \widetilde{k}[1-\beta\psi] \\ \alpha\beta\theta f(w) & \text{if } w \geq \widetilde{k} \end{cases} \tag{7.19}$$

The next two Lemmas characterize the value functions associated with the safe and risky policies that an entrepreneur might follow.

Lemma 7.7. *If $\alpha\beta\theta f'(0) > 1$, then $\pi^r(w) > \pi^s(w)$ for any w on $(0, k^*]$.*

Proof. Define the function $\Pi(w) = \pi^r(w) - \pi^s(w)$. Equations (7.18) and (7.19) imply that $\Pi(w)$ is continuous. There are two cases. First, in the case $k^* \leq \widetilde{k}[1-\beta\psi]$

$$\Pi(w) = \alpha\beta\theta\left[f\left(k^r\right) - f(w)\right] - \alpha\left[k^r - w\right], \qquad w \in (0, k^*] \tag{7.20}$$

where $k^r = \frac{w}{1-\beta\psi}$. The mean value theorem implies that there exists a constant $a \in (w, k^r)$ such that $\beta\theta f'(a) = \beta\theta\frac{f(k^r)-f(w)}{k^r-w}$. Concavity of f implies that $\beta\theta f'(a) > \beta\theta f'(k^r) \geq \beta\theta f'(\frac{k^*}{1-\beta\psi}) \geq \beta\theta f'(\widetilde{k}) = 1$. Since $\beta\theta f'(a) > 1$, it follows that (7.20) is positive for any w on $(0, k^*]$.

Consider now the case $k^* > \widetilde{k}[1-\beta\psi]$. For $w \leq \widetilde{k}[1-\beta\psi]$ the argument is the same as the previous one. For $w > \widetilde{k}[1-\beta\psi]$ replace k^r by $\widetilde{k}$ in (7.20), and note that there is a constant $b \in (w, \widetilde{k})$ such that $\beta\theta f'(b) = \beta\theta\frac{f(\widetilde{k})-f(w)}{\widetilde{k}-w}$. Moreover, $\beta\theta f'(b) > 1$ because $b < \widetilde{k}$.□

Lemma 7.8. *There is a unique wealth level $(\widetilde{w}(\psi))$ such that $\pi^r(w) < \pi^s(w)$ if and only if $w > \widetilde{w}(\psi)$. Furthermore, $\widetilde{w}(\psi) > \widetilde{k}(1-\beta\psi)$ if and only if $\psi > \widetilde{\psi}$, where*

$$\widetilde{\psi} \equiv \frac{\alpha\beta\theta[f(k^*)-f(\widetilde{k})]-[k^*-\widetilde{k}]}{[1-\alpha]\beta\widetilde{k}} < \frac{1}{\beta} \tag{7.21}$$

Proof. We consider first the case $k^* \leq \widetilde{k}(1-\beta\psi) \equiv \bar{w}$. Equations (7.18) and (7.19) imply that $\Pi(w)$ has the following three properties. First, for $w \geq k^*$, $\Pi(w)$ is concave. That is, $\Pi'(w)$ is declining. Second, $\Pi'(w)$ is negative for any $w \geq \bar{w}$. Third, $\Pi(\bar{w}) > 0$ if and only if $\psi > \widetilde{\psi}$. It follows that there is a unique $\widetilde{w}$ such that $\Pi(\widetilde{w}) = 0$. Furthermore, for $\psi > (<)\widetilde{\psi}$, we must have $\widetilde{w}(\psi) > (<)\widetilde{k}(1-\beta\psi)$.

We consider now the case $\bar{w} < k^*$. Lemma 7.7 implies that $\Pi(\bar{w}) > 0$. Since $\Pi'(w) < 0$ for $w \geq \bar{w}$, there is a unique $\widetilde{w}$ such that $\Pi(\widetilde{w}) = 0$. Furthermore, $\widetilde{w} > k^* > \bar{w}$.

Finally, we show that $\widetilde{\psi} < \beta^{-1}$. The mean value theorem implies that there is a constant $a \in (k^*, \widetilde{k})$ such that $\widetilde{\psi} \equiv \frac{1-\alpha\beta\theta f'(a)}{[1-\alpha]\beta\widetilde{k}/[\widetilde{k}-k^*]}$. Since, $\beta\theta f'(a) > \beta\theta f'(\widetilde{k}) = 1$, we have that $\widetilde{\psi} < \frac{[1-\alpha][\widetilde{k}-k^*]}{[1-\alpha]\beta\widetilde{k}} < \frac{1}{\beta}$. The second inequality follows from $\widetilde{k} > k^*$.□

Proof of Proposition 4

Lemma 7.7 implies that the wealth threshold $\widetilde{w}(\psi)$ at which it becomes profitable to switch from the risky to the safe investment policy is greater than k^*. Lemma 7.8 states that the threshold $\widetilde{w}(\psi)$ is unique, and establishes that $\widetilde{k}$ is reachable if and only if $\psi \geq \widetilde{\psi}$.□

7.4. Proof of Proposition 5

Consider the entrepreneur's portfolio problem given that a bailout will occur in the bad state, but not in the good state. If borrowing constraint (3.2) is binding, a borrower owes at the beginning of each period

$$\bar{b}(w)/\beta = lp(\zeta w) + \psi k(w) > lp(\varepsilon\zeta w) \tag{7.22}$$

Since $\varepsilon < 1$ and price function (5.3) is increasing in wealth, this amount is always greater than the maximum that an entrepreneur is able to repay if the bad shock hits (which is just the value of his land in state $\varepsilon\zeta w$, i.e. $lp(\varepsilon\zeta w)$).

The entrepreneur can follow two types of borrowing policies: risky and safe. 'Risky borrowing policies' satisfy

$$b/\beta > lp^*(\varepsilon\zeta w) \tag{7.23}$$

In contrast, 'safe borrowing policies' do *not* satisfy (7.23). If an entrepreneur follows a risky policy, he will default in the bad state because the value of his land does not cover his debt obligations. It follows that in this case the entrepreneur solves Problem I in section 2. Thus, it is straightforward to show that he will borrow the maximum amount possible $b = \bar{b}$ and the temporary equilibrium will be the same as the one of Proposition 1 for the case $\psi \geq \underline{\psi}$.

Suppose that an entrepreneur deviates and adopts a 'safe borrowing policy' that does *not* satisfy (7.23). Under such a policy the entrepreneur will be able to repay his debt in the bad state. Thus, his expected profits are (note that $\psi_s = 0$)

$$E\pi = \beta\alpha[\theta k g(l) + p^*(\zeta w)l - \frac{b}{\beta}] + \beta[1-\alpha][p^*(\varepsilon\zeta w)l - \frac{b}{\beta}] + s$$

Note that (7.22) and $b/\beta \leq p(\varepsilon\zeta w)l$ imply that $b < \bar{b}(w)$. Since $b < \bar{b}(w)$, the opportunity cost of capital is 1 instead of $1 - \beta\psi$. Since the marginal product of risky capital has negative expected net present value (i.e., $\alpha\theta g(l) < \beta$), it is optimal for the deviant to set $k^* = 0$. It follows that the expected return on land is composed only of capital gains. Thus, land purchases are positive only if

$$\alpha p^*(\zeta w)l + [1-\alpha]p^*(\varepsilon\zeta w)l \geq \tfrac{p^*(w)l}{\beta}$$

The left hand side is the expected return on land, while the right hand side is the safe return. Since $\varepsilon < 1$, this inequality can never be satisfied. To see this note that $p^*(x) = \frac{\gamma(1)x}{[1-\beta\zeta][1+\gamma(1)]}$. Thus, $\alpha p^*(\zeta w) + [1-\alpha]p^*(\varepsilon\zeta w)$ $\leq \frac{\gamma(1)\zeta w}{[1-\beta\zeta][1+\gamma(1)]}$. This expression is smaller than $\frac{p^*(w)}{\beta} = \frac{\gamma(1)w}{\beta[1-\beta\zeta][1+\gamma(1)]}$ because $\gamma(1) > 0$ and $\frac{\beta\zeta}{1-\beta\zeta} < \frac{1}{1-\beta\zeta}$ for any $\beta\zeta \neq 1$. It follows that if $b/\beta \leq p^*(\varepsilon\zeta w)l$, it is optimal to store all wealth. Using the same arguments as in section 2 we can show that since $\psi \geq \underline{\psi}$, expected profits with $s = w$ are not greater than with $s = 0$. Therefore, deviating by adopting a safe borrowing policy does not yield higher expected profits than the equilibrium policy. ■

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