Concentrated Ownership and Bailout Guarantees*

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Abstract

This paper studies the effect of bailout guarantees in an economy where ownership of firms is concentrated. In contrast to standard models of deposit insurance, bailout guarantees need not generate excessive risk taking, but may instead alleviate underinvestment. While the economy can experience wasteful lending booms, such booms often end in a self-correcting soft landing, as in the data. However, an economy that operates efficiently can also relapse into episodes of inefficient over- or underinvestment. Financial development has unintended consequences as it provides markets with tools to better exploit the bailout guarantee.

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1. Introduction

A country experiencing a lending boom goes through a period of unusually fast growth in credit. Lending booms occur frequently in emerging markets and are often accompanied by asset price inflation and strong investment growth.[1] In addition, many recent financial crises were preceded by lending booms. It is often argued that lending booms are the result of mistaken government policy: the existence of bailout guarantees creates a moral hazard problem that entails overborrowing, excessive investment and risk taking. A boom then ends once it is realized that further guarantees are not credible and a crisis ensues.

The standard moral hazard account of lending booms suffers from two drawbacks. First, the typical boom does not end in a crisis. Instead, many lending booms end in a *soft landing,* with credit and asset prices gradually reverting to trend (Gourinchas et al. 2001). Second, the formal underpinnings of the standard story derive from the deposit insurance literature, which was developed to study optimal financing and risk taking by banks in developed countries. In particular, it assumes frictionless capital markets: leverage does not affect the cost of capital. This assumption makes gambling with borrowed money particularly attractive. However, it is implausible in the light of recent evidence on ownership structure.[2] In emerging economies, controlling shareholders often hold large stakes, even in large firms. This suggests that external finance is costly, and therefore that incentives for inefficient risk taking may be much weaker.

This paper develops a theory of lending booms in economies where production is controlled by wealthy entrepreneurs. We show that, in such economies, lending booms fuelled by guarantees can occur, but tend to naturally end in a soft landing. In our model, entrepreneurs hold large stakes in their firms, because contracts cannot be enforced perfectly. Bailout guarantees encourage overinvestment and risk taking, as in existing models. However, the new theme is that entrepreneurs trade off any gains from exploiting a guarantee (through risk-taking) against losses to their own capital. This tradeoff changes over time with the level of entrepreneurial net worth, which gives rise to rich dynamics for investment and asset prices.

In equilibrium, our economy moves into and out of three distinct phases. At low levels of entrepreneurial net worth, the high cost of external finance hampers investment. In this phase of inefficient *under*investment, bailout guarantees actually foster growth since they provide a substitute for scarce capital. The moral hazard problem emerges only at intermediate levels of net worth, where it leads to inefficient *over*investment. A third phase occurs when net worth is high enough relative to existing investment opportunities. Once entrepreneurs have a sufficient amount of capital at stake, they forego inefficient and highly risky projects and investment is efficient.

One reason why our economy undergoes these different phases is the presence of exogenous shocks. For example, a sequence of bad terms-of-trade shocks might lower profits and hence entrepreneurial net worth

[1] A set of references on these stylized facts is provided below.

[2] Bekaert and Harvey (2003) survey studies that provide evidence on capital market imperfections in emerging economies.

and move the economy into the underinvestment region. Alternatively, a string of good productivity shocks might cause a boom to overheat as it moves the economy into the moral hazard region. However, we show that transitions between phases also occur endogenously. The nonlinear dependence of investment on entrepreneurial net worth implies that the economy can exhibit endogenous cycles; in particular, it can exhibit a relapse into an inefficient region even without negative shocks. The interaction of bailout guarantees and credit market frictions is thus by itself a source of volatility.

We also show that the model helps understand the behavior of asset prices during lending booms. The prices of productive assets often rise in booms to levels that are hard to reconcile with historical fundamentals. In our model, this happens because asset prices also capitalize future subsidies implicit in bailout guarantees. The effect is reinforced if the country recently experienced an improvement in contract enforcement which makes it easier to exploit guarantees. In addition, returns in the beginning of a boom tend to be volatile and negatively skewed. In our model, this feature arises naturally from the asymmetric adjustment costs implied by financing constraints. Finally, asset prices in our model typically peak well before the lending boom ends: they anticipate the soft landing.

As one building block of our model, we provide an explicit microeconomic framework to clarify why financing constraints can bind in an economy with bailout guarantees. This is not a foregone conclusion: if a bailout always occurs in case of default, why should lenders care whether borrowers can commit to repay? This argument overlooks the fact that bailout guarantees typically insure lenders only against systemic risk. A bailout will not occur if just an isolated firm defaults, especially not a small one. Instead, bailouts happen only when there is a critical mass of defaults. Collateral then still matters for credit, because lenders have to guard against idiosyncratic default risk.

A key feature of our model is that entrepreneurs and lenders implicitly collude to exploit the bailout guarantee. This has immediate implications for policy. While better enforceability of contracts may avoid inefficient underinvestment early on during a lending boom, it also fosters more inefficient overinvestment as the lending boom overheats. The reason is that a better contracting technology provides entrepreneurs and lenders with a more effective tool to exploit the guarantee. This contradicts conventional wisdom that better contract enforcement should improve the allocation of resources. It follows that institutional changes that improve contract enforcement may not be desirable unless at the same time a regulatory framework is put in place that contains excessive risk taking.

The literature on moral hazard due to bailout guarantees is large. Roubini and Setser (2004) provide an overview of recent work that applies this concept to emerging economies.[3] In terms of formal dynamic analysis, Krugman (1998), Corsetti et al. (1999), Ljungqvist (2002) and Eichenbaum et al. (2004) have studied moral hazard in the context of a neoclassical growth model with frictionless credit markets. Borrowers are

[3] See Freixas and Rochet (1998) for an exposition of the theory of deposit insurance.

then competitive firms and shareholder wealth does not matter for investment. The key mechanism generating soft landings in our model is thus absent. More generally, our new results derive from the interaction of bailout guarantees with lack of contract enforceability that we view as an important institutional feature of emerging economies.

Our model is also related to financial accelerator models of the business cycle. Following Bernanke and Gertler (1989), a number of authors have explored the macroeconomic implications of credit market frictions.[4] However, we show that the presence of bailout guarantees overturns several results typically associated with financing accelerator models. First, our model gives rise to *both* over- and underinvestment, whereas typically financing constraints induce only underinvestment. As a result, better contract enforcement or infusions of net worth do not necessarily improve efficiency of investment in our model. Second, because of the soft landing effect, a positive shock to net worth may *decrease* investment in our model. This means that the link between cash flow and investment is nonlinear: it is positive for firms with low net worth, but negative when net worth is higher. Simple linear regression analysis of the relationship between cash flow and investment may thus not be able to uncover the importance of financing constraints.

The paper proceeds as follows. The remainder of this introduction briefly points to evidence on lending booms and the distortions we focus on. Section 2 presents our model of a entrepreneurial firms in a small open economy. Section 3 derives optimal leverage policy when bailout guarantees are present. Section 4 derives proporties of the business cycle and asset prices. Some proofs are collected in an appendix.

Empirical Evidence on Lending Booms and Distortions

The existing evidence on lending booms can be read as answering two questions. The first question is *what are the salient features of macroeconomic aggregates and relative prices during an episode.* During a typical boom, investment and asset prices rises along with credit. For example, Pomerleano (1998) considers data from 734 South East Asian corporations from 1992 to 1996. For the case of Thailand, the average investment rate during this period was 29% (3% in the US). Furthermore, 78% of this investment was financed with debt (8% in the US). Claessens et.al. (1999), using a database of 5550 firms in nine Asian countries, find that during the early 1990s investment and leverage were very high and increasing. In Thailand during 1988-95 the investment rate increased from 10% to 14.5%, the debt-to-equity ratio increased from 1.6 to 2.2. The corresponding figures for the US are 3.8 to 3.7 and 0.8 to 1.1. Guerra (1998) and Hernandez and Landerretche (1998) document the appreciation of real estate and stock prices.

On average, lending booms do not end in a financial crisis, but rather in a "soft landing". Asset prices

[4] These papers embed frictions that are familiar from standard static models of the debt constrained entrepreneurial firm into a macro framework. For example, moral hazard with costly state verification (Townsend (1979), is employed in Bernanke and Gertler (1989,1998) and Carlstrom and Fuerst (1996); ex ante moral hazard, studied by Holmstrom and Tirole (1995), is used by Aghion and Bolton (1995); Kiyotaki and Moore (1997) rely on a version of the Hart and Moore (1994,1997) incomplete contracting theory of debt finance.

tend to revert before the lending boom ends. For example, Gourinchas et. al. (2001) find that in a sample of 91 countries over the past 35 years, the probability that a lending boom will end in a currency crisis is less than 20%. Furthermore, the build-up and ending phases of an average boom are similar in magnitude and duration. Although abrupt collapses of booms are not the norm, it is true that almost all banking and currency crises in emerging markets have been preceded by lending booms (see, for example, Corsetti et al. 1998, Kaminsky and Reinhart 1999 and Tornell 1999). Moreover, those lending booms that have ended in a crisis have typically been followed by a credit crunch. That is, in the aftermath of crises new lending falls sharply and recuperates only gradually (Krueger and Tornell 1999; Sachs et al. 1995).

A second empirical question is *whether the quality and composition of investment is different during lending booms, when compared to normal times.* Naturally, an answer to this question is not as easily quantifiable, and the existing evidence is largely anecdotal. Nevertheless, there does seem to be a tendency for the quality of investment to deteriorate during lending booms. Pomerleano (1998) finds that the return on assets in his sample of Thai firms fell from 9% in 1992 to 5% in 1996 (9% and 13% in the US). Claessens, et.al. (1999) document that the real return on assets fell from 11% to 8%. Moreover, firms and banks have been noted to shift to activities that have traditionally been considered to be more risky, such as investment in real estate (BIS, 1999). Not all firms experience booms and busts in the same way. Small, bank-dependent firms and firms in the nontradable sector have grown more strongly during the boom, but have been slower to recover after the crisis than large exporting firms with access to direct finance (Krueger and Tornell 1999).

There is also some direct evidence that the two distortions we focus on are present especially in emerging markets. On the one hand, bailout guarantees are present in many countries, and they tend not to be accompanied by a strong regulatory framework (Roubini and Setser, 2004). On the other hand, claims on firm insiders are not as easily enforceable as in developed countries, as external corporate governance tends to be weak (Johnson et al. 2000, Klapper and Love 2002). The role of concentrated ownership in emerging markets as a response to this has been emphasized by Himmelberg et al. (2002). The results of Harvey et al. (2004) suggest that debt is used in part to alleviate the agency problems between controlling and minority shareholders.

2. The Model

We consider a small open economy, populated by overlapping generations of risk neutral entrepreneurs. The riskless world interest rate is fixed at r. Every entrepreneur in generation t owns a risky production technology, which turns k_t units of the single numeraire good invested in period t into

$$y_{t+1} = z_{t+1} f(k_t)$$

units of the good in period $t+1$. The productivity shock z_{t+1} is i.i.d. over time and equals one with probability α, and zero otherwise. In addition, it is perfectly correlated across entrepreneurs. The production function f is continuous, increasing and concave, with $f(0) = 0$ and $\lim_{k \to 0} f'(k) = \infty$.

An entrepreneur of generation t begins period t with internal funds w_t. He can raise additional funds b_t by issuing one period bonds with a promised interest rate ρ_t to risk neutral foreign investors. Foreigners have 'deep pockets': they are willing to lend any amount, provided that they expect to earn at least the world interest rate. Entrepreneurs also have access to alternative investment opportunities that earn the riskless rate, which we refer to as riskless savings s_t. The budget constraint is thus

$$s_t + k_t = w_t + b_t. \tag{2.1}$$

Distortions

The economy is subject to two distortions. First, entrepreneurs cannot commit to repay debt. In particular, entrepreneurs of generation t may default strategically at date $t+1$. Once they do so, lenders seize entrepreneurs' assets. However, the physical assets cannot be fully recovered: if k_t was invested in capital, lenders obtain only $\psi z_{t+1} k_t$, where $\psi \leq 1 + r$. The payoff to an entrepreneur who defaults is thus

$$\Pi^d_{t+1}(z_{t+1}) = z_{t+1}(f(k_t) - \psi k_t). \tag{2.2}$$

The second distortion is the existence of bailout guarantees. We assume that an aid agency steps in whenever more than half of all entrepreneurs default. The agency then takes over recovery of the delinquent loans, and it pays lenders $(1+r)b_t$ for every b_t dollars lent. We do not address where the agency obtains funds for a bailout, but treat the bailout as a windfall to the domestic entrepreneurial sector. One can imagine either foreign aid or the domestic government levying taxes on the (unmodelled) household sector, for example.

In the absence of strategic default, profits realized by generation t in $t+1$ are

$$\Pi_{t+1}(z_{t+1}) = z_{t+1} f(k_t) + (1+r) s_t - (1+\rho_t) b_t. \tag{2.3}$$

Every "old" entrepreneur in $t+1$ consumes $c\Pi_{t+1}$. He then passes on the firm to his heir, a member of generation $t+1$. This "young" entrepreneur thus starts operations with internal funds $w_{t+1} = (1-c)\Pi_{t+1}$. In contrast, if the firm is in default in $t+1$, the young entrepreneur starts over with $w_{t+1} = \varepsilon$.[5] Throughout, we take ε to be a number close to zero.

Equilibrium

[5] An alternative and perhaps more natural assumption would be that *every* young entrepreneur receives an endowment ε, together with a share of profits $(1-\delta)\Pi_{t+1}$. This would not significantly change the nature of the dynamics, but would make the algebra significantly less transparent. We adopt the present assumption for simplicity.

The timing of events for a given generation of entrepreneurs and lenders is as follows. At date t, every entrepreneur announces plans for risky investment k_t^i and savings s_t^i and debt b_t^i that satisfy 2.1, as well as a promised interest rate ρ_t^i. Lenders then decide whether to accept or reject these offers. At date $t+1$, the shock z_{t+1} is realized and entrepreneurs decide whether or not to default. A bailout occurs if more than half of the entrepreneurs default.

Since the bailout depends on the action of all entrepreneurs, individual entrepreneurs' payoffs are interdependent. Formally, the above description of actions and payoffs defines a "credit market game". Since all entrepreneurs are identical and the shocks are perfectly correlated, it is natural to focus on symmetric equilibria of this game. We thus define an equilibrium of the model as a stochastic process $(k_t, s_t, b_t, \rho_t, w_t)$, such that (i) given w_t, (k_t, s_t, b_t, ρ_t) is an offer by entrepreneurs that is accepted in a symmetric subgame perfect equilibrium of the credit market game and (ii) $w_t = \varepsilon$, if the credit market equilibrium played by generation $t-1$ calls for default, and $w_t = (1-c)\,\Pi_t$ otherwise, where profit Π_t is defined by 2.3.

3. Credit Market Equilibrium

We characterize equilibria of the model in two steps. In this section, we discuss the interaction of a given generation t of entrepreneurs with its lenders – the equilibrium of the date t credit market game. This interaction determines what happens in the credit market at date t, and also whether a bailout occurs at date $t+1$. The second step of the analysis will consider dynamics, where a sequence of credit market games is connected by the passing down of firms and internal funds.

3.1. Frictionless benchmark

As a benchmark, it is helpful to review what happens when firms have no capital at stake and face no commitment problem, that is, if $w_t = 0$ and $\psi = 1+r$. This special case replicates familiar results from competitive models in the literature. Suppose first that no bailout is expected. It is then optimal to invest in physical capital to the point k^* where the expected marginal product of capital is equal to the expected rate of return that lenders must earn:

$$\alpha f'(k^*) = 1 + r.$$

Since investment must be financed by borrowing, the firm will be forced into default in the bad state $(z_{t+1} = 0)$. The firm's debt is thus risky. In fact, it does not pay for entrepreneurs to save at all $(s_t = 0)$, so that lenders do not receive any payoff in the bad state. To nevertheless ensure that lenders finance the investment, entrepreneurs must pay an interest rate $\rho_t = \frac{1+r}{\alpha} - 1 > r$.

Now suppose instead that a bailout occurs in the bad state. Lenders know that the aid agency will pay them $(1+r)$ per dollar lent in that state. As a result, entrepreneurs can borrow at the rate $\rho_t = r$ that does not provide compensation for default risk. While it is still not optimal to save, investment is now optimally driven up to the point k^{**} where the marginal product of capital *conditional on the good state* equals the riskless rate:

$$f'(k^{**}) = 1 + r.$$

This is because the firm itself pays the interest rate only in the good state – the aid agency picks up the tab in the bad state. We have $k^{**} > k^*$, so that bailout guarantees increase investment. However, the extra investment $k^{**} - k^*$ is channelled to "white elephant" projects that have ex ante *negative* net present value.

In the two special cases presented so far, our model replicates the investment levels and interest rates that occur in a standard model with competitive firms that operate the same technology, but have access to perfect capital markets. However, we emphasize that our model differs from the deposit insurance literature in what is exogenous to the firm in a given period. The deposit insurance literature considers banks of fixed scale, but with variable capital chosen by shareholders with "deep pockets". The models then predict changes in leverage and risk as a result of changes in regulation. For example, if capital requirements are relaxed, shareholders prefer higher leverage and riskier loan portfolios. This setup is motivated by large US banks. In contrast, in our world of concentrated ownership, firms have variable scale, but their capital is predetermined by the wealth of the entrepreneur. Our model thus predicts changes in leverage and risk as a result of changes in past profits. This will also be important below in our dynamic analysis.

3.2. Internal funds and strategic default risk

The key question is how the above results change if entrepreneurs (i) have limited access to external funds ($\psi < 1 + r$), but (ii) do have access to internal funds ($w_t > 0$). We focus on symmetric equilibria of the credit market game, where all entrepreneurs choose the same strategies. Two types of equilibria of the credit market game are of interest. In a *risky* equilibrium, investment is financed at least in part by borrowing and firms default in the bad state. In contrast, in a *safe* equilibrium, investment is fully financed internally and there is no default. The following proposition provides necessary and sufficient conditions for both types of equilibria to exist.

Proposition 3.1. *(Credit Market Equilibria)*

1. There is a threshold level of internal funds $\tilde{w} \in \left(k^*, \tilde{k}\right)$ *such that a risky equilibrium exists if and only if* $w \leq \tilde{w}$. *In this equilibrium, entrepreneurs do not save and pay the rate* $\rho_t = r$ *on their debt. Investment is given by*

$$k = \min\left\{\frac{w}{1 - \beta\psi}, \tilde{k}\right\},$$

where $\beta = \frac{1}{1+r}$. *All entrepreneurs default if* $z_{t+1} = 0$, *and a bailout occurs in that state.*

2. A safe equilibrium exists if and only if $w \geq k^*$. *In this equilibrium, entrepreneurs invest* $k = k^*$ *and save* $s = w - k^*$. *They do not borrow.*

A proof of the proposition is contained in Subsection 3.3 below. The main points are represented graphically in Figure 3.1. The figure shows optimal investment in both safe (dashed line) and risky (solid line) equilibria as a function of internal funds. It also distinguishes four equilibrium regions. First, for low internal funds (the area shaded in light gray that satisfies $w < k^* (1 - \beta\psi)$), there is a unique risky equilibrium in which all firms invest less than the first best level k^*. This *underinvestment* region arises because a binding collateral constraint prevents firms from borrowing as much as they would need to invest up to the efficient level.

Underinvestment

In the presence of bailout guarantees, it is not obvious that the collateral constraint should bind at low levels of internal funds. For example, if a simple deposit insurance scheme was in place, an entrepreneur could simply default for sure in all states of the world – lenders would be willing to provide external funds as long as the aid agency pays them back. The reason the collateral constraint binds in the present model is that the guarantees are systemic. By construction, they insure lenders against widespread default by many entrepreneurs, but not against strategic default by an individual entrepreneur. In a risky equilibrium, a bailout occurs in the bad state, where profits are actually low. However, to credibly commit not to default in the good state, entrepreneurs must finance a certain of fraction of investment internally, that is, some of their own wealth must be invested in collateral.

While bailout guarantees do not neutralize the collateral constraint, they do relax it. To illustrate this fact, Figure 3.1 plots equilibrium investment in a model without bailout guarantees, but with a commitment problem – the dotted line that joins the safe equilibrium investment function at $w = k^*$. This "no bailout" version of the model shares a familiar property with many models of financing constrained firms in the literature: since entrepreneurs must finance a fraction of investment internally, investment is constrained by the level of internal funds as long as the latter is small. The presence of bailout gaurantees now leads to higher investment throughout the underinvestment region. The reason is that the subsidy provided through the expected bailout payment effectively works like an increase in internal funds.

Overinvestment

A second region is characterized by internal fund levels $w \in (k^* (1 - \beta\psi), k^*)$. There is still a unique, risky, equilibrium. However, investment in this equilibrium now exceeds the efficient level. This reflects the bailout guarantee: as in the competitive benchmark, an artificially low cost of capital encourages *overinvestment*. However, investment intially still depends on internal funds. Only at $w = k^{**} (1 - \beta\psi)$ – the kink in the investment function – does investment reach the level k^{**} that would occur in a frictionless model with bailout guarantees. This regions marks a second key difference between our model and the "no bailout" model of financing constrained firms. The latter cannot give rise to overinvestment – instead, once internal funds are sufficiently high (point $w = k^* (1 - \beta\alpha\psi)$ in the figure), investment is constant at the efficient level.

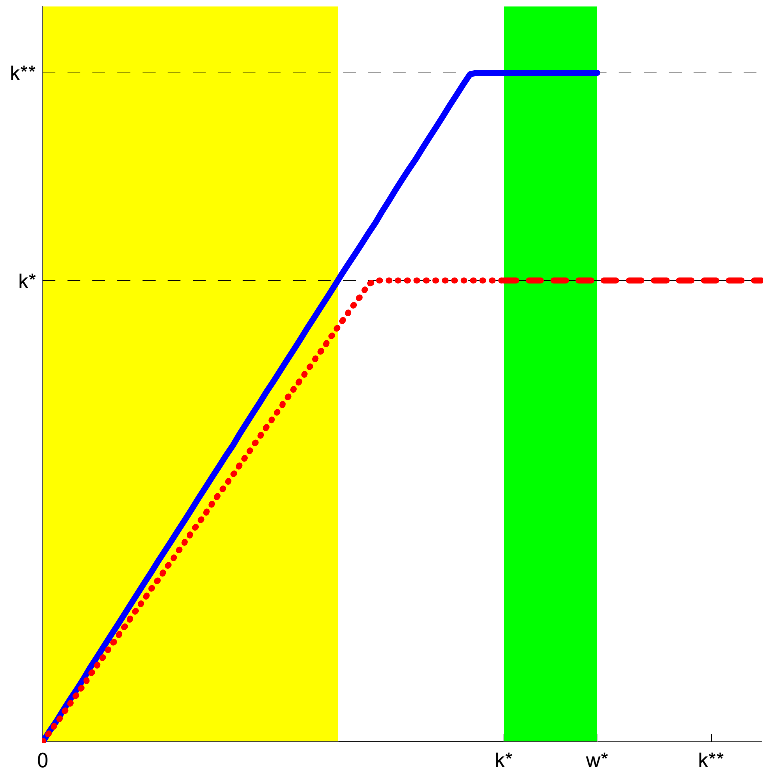

Figure 3.1: **Investment and equilibrium regions.**

Investment in risky (solid line) and safe equilibrium (dashed line) are shwon as a function of internal funds. For comparison, the dotted line shows nvestment when there is no bailout.

Coordinated Risk-Taking and Multiple Equilibria

The third region comprises internal fund levels $w \in [k^*, w^*]$ and is characterized by *multiple credit market equilibria.* In this region, shaded in dark gray in the figure, the coordination problem posed by systemic bailout guarantees takes center stage. If every entrepreneur believes that a bailout will occur in the bad state, then it is optimal for everyone to undertake a risky plan that leads to default in that state. This is because anticipation of a bailout lowers the cost of capital for entrepreneurs that actually plan on defaulting. But if all entrepreneurs undertake risky plans, they actually fulfill the bailout expectations though their actions. In contrast, if no entrepreneur expects a bailout, the cost of capital is not distorted. This leads entrepreneurs to finance investment internally, which in turn fulfills expectations that no bailout will occur.

The proposition shows that a safe equilibrium is *not* possible at levels of internal funds below k^*. This is because a safe equilibrium requires that any type of default – strategic or non-strategic – be ruled out. The absence of a bailout eliminates strategic default: lenders would not fund plans that entail strategic default. However, it does not eliminate non-strategic default that occurs simply because of bad productivity shocks. If investment cannot be fully financed internally (that is, $w < k^*$), a high expected marginal product of capital makes it optimal for entrepreneurs to borrow and increase investment beyond w even if there is no anticipation of a bailout. But then there would be widespread (nonstrategic) default in the bad state, which would trigger a bailout. It follows that no anticipation of a bailout is thus not compatible with rational expectations at low levels of internal funds.

Full Internal Finance

The fourth and final region consist of internal funds levels $w > w^*$. In this region, there is a unique safe equilibrium where all investment is financed internally. The previous paragraph explains why a safe equilibrium can occur when internal funds are high enough. The important feature of the fourth region is that a risky equilibrium cannot occur. The reason is concentrated ownership. For rich entrepreneurs, the expected benefit from the bailout guarantee does not outweigh the loss of own capital in the event of a default. Unfair bets only pay off when they can be financed with money borrowed at subsidized rates. Rich entrepreneurs will therefore forego overinvestment and excessive risk taking even when a bailout is expected. Instead, they prefer to invest at the efficient level k^*, which they can finance internally. But this implies that no bailout can be expected in equilibrium, since no defaults will take place.

3.3. Proof of Proposition 3.1

This section provides a proof of Proposition 3.1 that focuses on the economic intuition, with technical details relegated to the appendix. The reader mostly interested in macroeconomics may wish to skip this subsection proceed directly to the dynmaic analysis in Section 4. In what follows, we refer to plans that involve no savings and lead to default in the bad state, but not in the good state, as *risky plans.* We also call plans that involve no borrowing *safe* plans. The appendix shows that only these two types of plans can ever occur in equilibrium. To prove part 1 of the proposition, we now establish that, if a bailout is expected in the bad state only, then it is optimal for a firm to offer a risky plan if and only if $w < w^*$. If all firms adopt a risky plan, a bailout indeed occurs in the bad state, and a risky equilibrium is shown to exist.

The Role of Distortions: Subsidy and Collateral Constraint

Suppose a bailout is expected in the bad state only. The guarantee implies that lenders are happy to accept the riskless rate $\rho_t = r$ on risky plans. Using this interest rate, the budget constraint and the definition of profits, entrepreneurs' expected payoff can be written as

$$V(k_t, w_t) = (1+r) w_t + [\alpha f(k_t) - (1+r) k_t] + (1-\alpha)(1+r) \max\{k_t - w_t, 0\} \tag{3.1}$$

Here the second term represents profits from physical investment, while the third term captures the subsidy due to the bailout guarantee. The latter can be claimed by picking a risky plan (which implies $k_t \geq w_t$), but is foregone when picking a safe plan $(k_t \leq w_t)$.

The objective (3.1) is maximized subject to the constraint that there be no default in the good state,

$$(1+\rho_t) b_t \leq \psi k_t + (1+r) s_t. \tag{3.2}$$

This condition is a *collateral constrant* – the entrepreneur can pledge all his savings, but only a limited amount of the funds invested in physical capital. Using again the budget constraint to substitute, we obtain the equivalent formulation

$$(1-\beta\psi) k_t \leq w_t.$$

Since capital cannot be fully pledged, at least a share $1-\beta\psi$ of any investment in capital must be financed internally.

Optimal Safe and Risky Plans

It is now straightforward to calculate the best risky and the best safe plan. If the entrepreneur opts for a safe plan, he will try to operate the technology at the first best level k^*. In case internal funds are insufficient to finance k^*, the marginal product is higher than $1+r$ and all internal funds should be invested. The optimal safe investment is thus $k^s(w_t) := \min\{w_t, k^*\}$. In contrast, under a risky plan, the subsidy provided through the bailout is increasing in the amount borrowed. As under the competitive benchmark, this artificially lowers the marginal cost of capital and the entrepreneur would like to invest up to $\hat{k}$. However, this desired amount can again only be financed if sufficient internal funds are available, since the collateral constraint limits firm leverage. As long as $w_t \leq k^{**}$, the optimal risky investment is thus

$$k^r(w_t) = \begin{cases} \frac{w_t}{1-\beta\psi_k} & \text{if } w_t < (1-\beta\psi_k)\hat{k} \\ \hat{k} & \text{if } (1-\beta\psi_k)\hat{k} < w_t \leq \hat{k} \end{cases}$$

For $w_t > k^{**}$, there does not exist an optimal risky plan – the safe plan that sets $k_t = w_t$ dominates all risky plans, and the entrepreneur would like to be as close as possible to that safe plan.

Trading off Subsidy versus Efficiency

It remains to compare profits under the best risky and safe plan, $V(k^r(w), w)$ and $V(k^s(w), w)$, respectively, to determine the overall optimal plan. This is done in Figure 3.2, which reveals three different investment regions. First, for low internal funds, risky plans are always optimal. This is because leverage increases profits that can be made from physical investment. To illustrate, consider the slope of both profit functions at $w = 0$. Since we have assumed $f'(0) > 1 + r$, we have

$$\frac{\partial}{\partial w} V\left(k^r(w), w\right)\bigg|_{w=0} = \frac{\alpha f'(0)}{1-\beta\psi} - \frac{\psi}{1-\beta\psi} > \alpha f'(0) = \frac{\partial}{\partial w} V\left(k^s(w), w\right)\bigg|_{w=0}.$$

Under a risky plan with maximum leverage, a dollar of internal funds permits $1 - \beta\psi$ dollars of investment. Even though a share ψ of this investment (in future value terms) must be be pledged to lenders, the high marginal product makes the risky plan overall more profitable than a safe plan that permits only one dollar of investment per dollar of internal funds.

As long as $w \leq k^*(1 - \beta\psi)$, firms *underinvest* relative to the first best since they have too little collateral. However, as internal funds increase beyond $k^*(1 - \beta\psi)$, investment is not fixed at k^*. Risky plans remain optimal at least as long as $w \leq k^*$ and firms therefore *overinvest* in inefficient, negative NPV projects in that region. Intuitively, as investment is increased beyond k^*, the marginal decrease in expected profits due to inefficient investment is initially smaller than the increase in expected profits due to the guarantee. The latter increase occurs as long as firm leverage is sufficiently high.[6]

Finally, there is an *efficient investment* region. If internal funds are high enough, the expected loss of the entrepreneur's own stake in the firm cannot be compensated by the subsidy from the guarantee. The entrepreneur is better off financing the firm internally, investing at the first best scale and saving any surplus internal funds. We have already argued above that, for $w > k^{**}$, any risky plan is dominated by the safe plan that sets $k = w$. Since the latter plan is worse than the best safe plan that sets $k = k^*$, the safe profit function is higher than the risky profit function for $w = k^{**}$. By continuity of both profit functions, there must be a unique cutoff w^*, such that the best risky plan is optimal if and only if $w \leq w^*$.[7]

Safe Equilibria

We now assume that there is no bailout, and show that the optimal plan is safe if and only if $w \geq k^*$. Since there is indeed no bailout if everyone adopts a safe plan, this establishes existence of a safe equilibrium (part 2 of the proposition). In the absence of bailouts, lenders will only fund a risky plan if they are paid the interest $\rho_t = \frac{1+r}{\alpha} - 1$ that compensates them for default in the bad state. As a result, the entrepreneur's expected profit no longer includes the expected benefit from the guarantee and reduces to

$$V(k_t, w_t) = (1+r)\, w_t + \left[\alpha f(k_t) - (1+r)\, k_t\right]. \tag{3.3}$$

In addition, the collateral constraint becomes more stringent: substituting into (3.2) using the new interest rate delivers

$$(1 - \alpha\beta\psi)\, k_t \leq w_t.$$

[6] For example, for $w < (1 - \beta\psi_k)\, \hat{k}$, the leverage ratio under a risky plan is constant.

[7] It is also easy to see why the cutoff $\tilde{w}$ must lie above k^*. Inspection of the profit function shows that, for $w = k^*$, any risky plan with k larger than, but arbitrarily close to, $w = k^*$ is a feasible risky plan and hence by construction worse than the best risky plan. By continuity, the best safe plan must also be worse than the best risky plan.

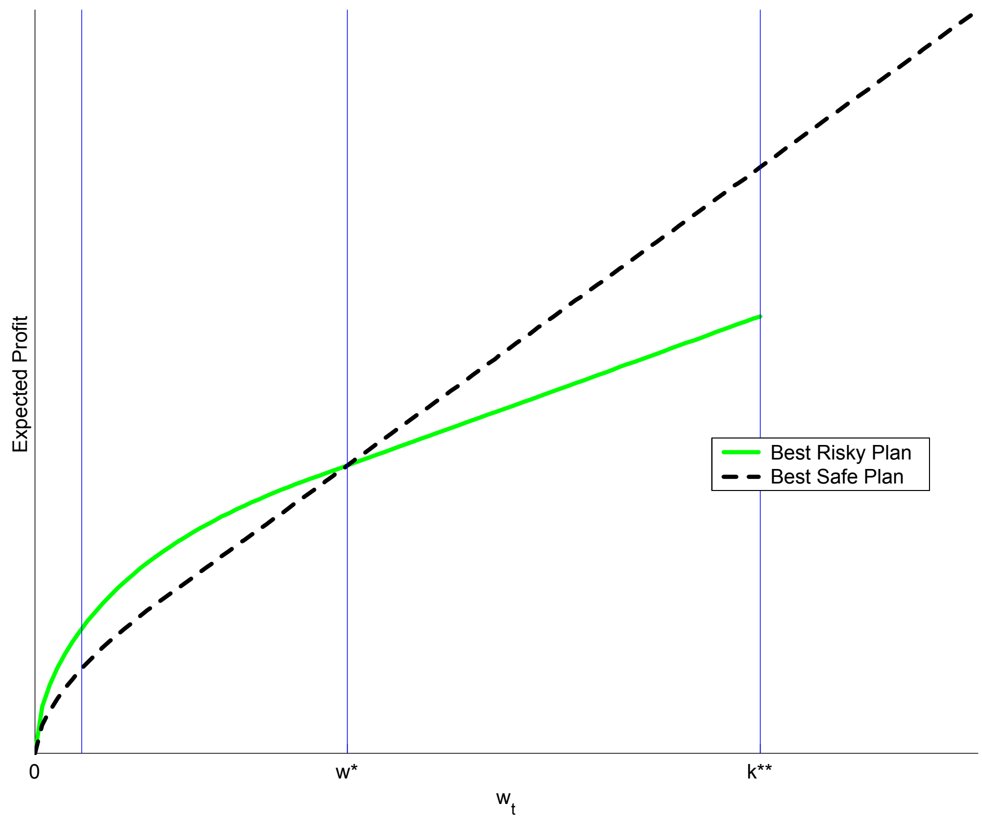

Figure 3.2: **Expected profit as a function of internal funds, under the best risky and safe plan.**

Since capital is lost in the bad state, the entrepreneur can effectively only pledge a share $\alpha\beta\psi$ and must come up with $(1-\alpha)\,\beta\psi$ of additional internal funds. This is less than in the bailout case, where the aid agency effectively insured the pledged capital in the bad state.

This formulation leads directly to the optimal plan: investment is

$$\min\left\{\frac{w_t}{1-\alpha\beta\psi}, k^*\right\}.$$

Clearly, the optimal plan is safe if and only if $w_t \geq k^*$. At any lower level of internal funds, investment is partly financed by borrowing, so that the firm must default in the bad state.

4. Dynamics

To characterize equilibria, it is convenient to introduce some additional notation. Every equilibrium of the model implies a sequence of equilibria of the credit market game. Let $\eta_t \in \{r, s\}$ indicate whether a risky or a safe equilibrium is played in period t. In those periods in which w_t lies in the multiple equilibria region $[k^*, w^*]$, we need a selection rule. We assume that there is a stochastic process x_t, valued in $\{r, s\}$ that determines which credit market equilibrium is played. This process represents entrepreneurs' *mutual trust* in each others' willingness to take risk and hence trigger a bailout in the bad state. The sequence η can then be expressed as a function of internal funds and the mutual trust variable x:

$$\eta_t = \eta(w_t, x_t) = \begin{cases} r & \text{if } w_t < k^* \\ s & \text{if } w_t > w^* \\ x_t & \text{if } w_t \in [k^*, w^*] \end{cases}$$

Using the results from Proposition 3.1, an equilibrium process of internal funds $\{w_t\}$ is a solution to the stochastic difference equation

$$w_{t+1} = \begin{cases} \begin{array}{c} (1-c)\, z_{t+1} \left(f\left(\min\left\{ \frac{w_t}{1-\beta\psi}, k^{**} \right\} \right) + (1+r)\left(w_t - \min\left\{ \frac{w_t}{1-\beta\psi}, k^{**} \right\} \right) \right) \\ + (1-z_{t+1})\, \varepsilon \end{array} & \text{if } \eta_t(w_t, x_t) = r \\ (1-c)\left(z_{t+1} f(k^*) + (1+r)(w_t - k^*) \right) & \text{if } \eta_t(w_t, x_t) = s \end{cases} \tag{4.1}$$

The current level of internal funds – and possibly mutual trust x – determine investment and borrowing in t. The productivity shock z_{t+1} then determines internal funds in $t+1$. If a safe equilibrium was played, the economy has a "cushion" $w_t - k^*$ to fall back on – a bad productivity shock will not lead to defaults. In contrast, a risky equilibrium entails widespread default when $z_{t+1} = 0$ and the economy will restart at $w_t = \varepsilon$.

Consider now an economy that starts at the state $w_t = \varepsilon$. As long as productivity is high ($z_t = 1$) and trust is strong ($x_t = r$), the economy will follow a path of high investment and high entrepreneurial profits. We refer to this path as the *lucky path*. An economy on the lucky path can experience four types of notable events. On the one hand, either one of the exogenous shocks may throw the economy off the lucky path. We call a period of unusually low productivity ($z_t = 0$) a *crisis*. In a crisis, both ex ante good and ex ante excessive (negative NPV) projects fail and output is zero. In contrast, a *correction* is a drop in investment, brought about by a breakdown in mutual trust ($x_t = s$). A correction does not affect productivity and current output, although it does affect output with a lag through its effect on investment.

On the other hand, we emphasize two events that occur *along* the lucky path and are not triggered by shocks. A *soft landing* is a drop in investment that occurs as the economy transits from the regions of risky or multiple credit market equilibria to the region of safe credit market equilibria. A soft landing is similar to a correction in that it does not affect current output and productivity. However, it is different because it does not rely on an exogenous breakdown of trust. If internal funds grow sufficiently, the economy is forced into a soft landing even when trust is always strong. Finally, a *relapse* is an increase in investment that occurs as the economy transits from the region of safe credit market equilibria to a region of risky or multiple credit market equilibria.

To clarify the endogenous propagation mechanisms of the model, in particular the concept of a soft landing, we now focus on "maximum risk" equilibria. These equilibria are special in two ways. First, they are driven only by fundamental shocks – fluctuations in mutual trust will be reintroduced below. Second, maximum risk equilibria give maximal force to the traditional effect that bailout guarantees lead to excessive risk taking and overinvestment: we assume that a risky credit market equilibrium is played whenever it exists. In particular, we assume that a risky credit market equilibrium is always played in the multiple equilibria region ($x_t = r$ for all t). The following proposition shows that, nevertheless, lending booms frequently end in a soft landing, at least when borrowing constraints are not too tight.

Proposition 4.1. *(Maximum Risk Equilibria).*

Suppose that risky equilibria are played whenever they exist ($x_t \equiv r$). *Assume further that* $\varepsilon < (1 - \beta\psi)\, k^*$ *and* $c > 1 - \beta$.

1. *There is a unique stationary equilibrium.*

2. *There are thresholds* c_1 *and* c_2, *with* $1 - beta < c_1 < c_2 < 1$, *that define three cases for the lucky path:*

a. *(stable soft landing) for* $1 - \beta < c < c_1$, w_t *converges along the lucky path to a constant* $w_\infty \geq \tilde{w}$. *It enters the safe region once and never exits again.*

b. *(endogenous cycles) for* $c_1 \leq c \leq c_2$, *after starting out in the risky region, the lucky path eventually displays oscillatory behavior, with* w_t *moving back and forth between the risky and safe regions.*

c. *for* $c > c_2$, w_t *converges along the lucky path to a constant* $w_\infty < \tilde{w}$. *It never reaches the safe region.*

A complete proof of the proposition is contained in the Appendix. The next two subsections present simple, graphical arguments for cases (a) and (b). In either case, the proposition requires two assumptions on parameters. First, the assumption $\varepsilon < (1 - \beta\psi)\, k^*$ says that the labor endowment of an individual young entrepreneur, earned outside of the firm in a state of bankruptcy, is small relative to the scale of the firm in good times. In light of Proposition 3.1, this implies that, at $w = \varepsilon$, investment is less than the efficient level k^*. In other words, a crisis that leads to widespread defaults depletes internal funds sufficiently to move the economy to the underinvestment region. Second, assuming that $c > 1 - \beta$ ensures that entrepreneurs consume profits fast enough to not make their savings grow without bound in the safe region. This appears reasonable given that there is no productivity growth in the model, and we would expect any model that determines c endogenously to have this property.

4.1. Stable Soft Landings

The case of stable soft landings – case (a) of Proposition 4.1 – is illustrated in Figures 4.1 and 4.2. This case is relevant when the dividend rate is not too high. It describes economies where any lending boom that is not punctured by a crisis ends in a soft landing. Moreover, every soft landing ushers in a phase of stability that can only be ended by a crisis, that is, a bad productivity shock.

Mechanics

Figure 4.1 depicts the transition functions for internal funds (4.1) under two realization of the productivity shock. Parameters are chosen so that they fall into case (a) of Proposition 4.1: in particular, the dividend payout rate c is not too high. The top, solid, line in the figure is the transition function if the economy remains on the lucky path ($z_{t+1} = 1$). There is always a discontinuity at $w_t = w^*$, the boundary between the multiple equilibria region, where a risky equilibrium is played, and the safe region. Since $w^* > k^*$, the risky equilibrium played at $w = w^*$ entails overinvestment, that is $k > k^*$. As w becomes larger than w^*, we move to the safe region where investment drops to k^*. This implies lower output realized in the good state, which accounts for the discontinuity. The bottom, dashed, line is the transition function when the economy is hit by a crisis ($z_{t+1} = 0$). As long as risky equilibria are played ($w \leq w^*$), this is simply equal to ε. There is again a discontinuity at w^*, since for $w > w^*$ the economy develops a cushion of internal funds that are not wiped out by a crisis.

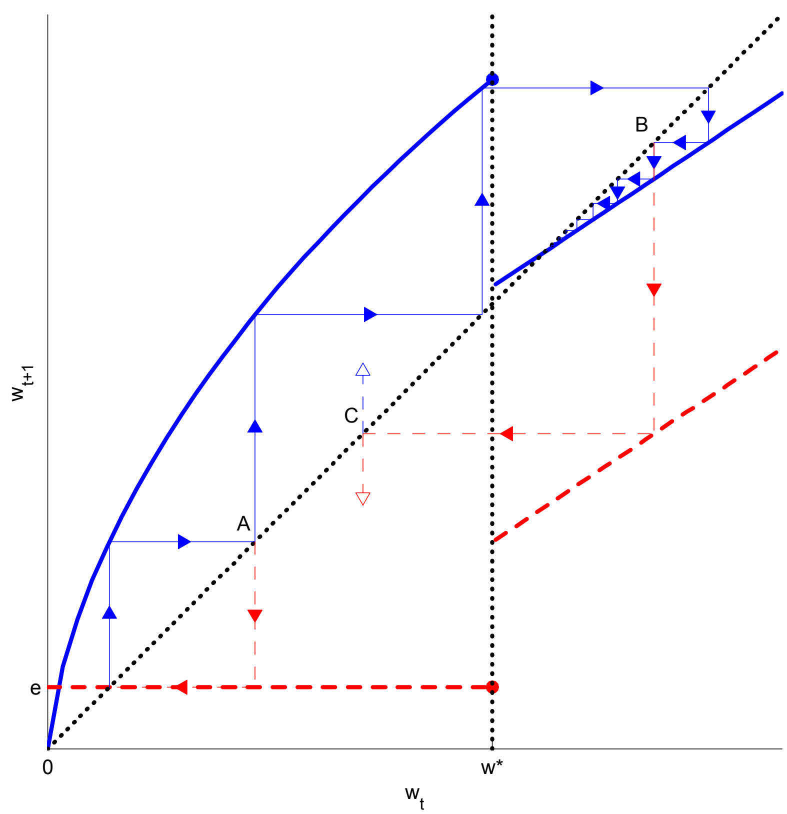

Figure 4.1: **Equilibrium in case (a) – stable soft landing.** Transition functions for internal funds are shown for $z_{t+1} = 1$ (solid) and $z_{t+1} = 0$ (dashed). The thin solid line with arrows indicates motion along the lucky path towards w_∞.

The slope of the lucky ($z_{t+1} = 1$) transition function for $w > w^*$ is $(1-c)(1+r)$, which is less than one given that $c > 1 - \beta$. The conditions for case (a) ensure c is large enough to make the lower branch of the lucky transition function actually intersect the 45 degree line. As long as the economy remains on the lucky path, it must therefore converge to the intersection point. Of course, at any point along the lucky path, a crisis can move the economy along the unlucky ($z_{t+1} = 0$) transition function, and hence off the lucky path.

The drop in output associated with a crisis always leads to a sharp drop in internal funds. If the crisis occurs during the lending boom, for example at point A, internal funds drop back to $w = \varepsilon$. In contrast, if the economy was previously in the safe region, such as at point B, firms have a cushion of internal funds that tempers the fall. Nevertheless, it is apparent from the figures that a finite *sequence* of crises is always sufficient to return the economy to $w = \varepsilon$ (in the figure, suppose that a second crisis occurs immediately after the first crisis has moved the economy to point C). This property essentially guarantees existence of a unique invariant distribution, as shown in the Appendix.

Intuition

Figure 4.2 shows the evolution of internal funds and investment along the lucky path as a function of time, when the economy starts at $w_1 = \varepsilon$. Initially, a cash-strapped entrepreneurial sector is forced to invest below the efficient level k^*. As profits increase along the lucky path, both internal funds and investment then rise at an increasing rate. After four periods, the entrepreneurial sector has become rich enough that it can afford the first-best investment level. However, this does not mark the end of the lending boom: encouraged by bailout guarentees, entrepreneurs now leverage up their firms to invest in negative NPV projects. Investment continues to rise at an increasing rate, and it includes a large fraction of “white elephant” projects.

At any point in time, a negative productivity shock – a crisis – can end the lending boom. This returns the economy to the state $w = \varepsilon$. The shape of the lucky path implies that the *conditional volatility* of major economic aggregates also increases during a lending boom. For example, the one-period-ahead conditional variance of investment is given by

$$var_t(y_{t+1}) = \alpha(1-\alpha)(f(k_t) - \varepsilon)^2 .$$

In contrast, the benchmark economy without financing contraints would have constant conditional variance $\alpha(1-\alpha)(f(k^*) - \varepsilon)^2$. Financing constraints thus introduce conditional heteroskedasticy. On the one hand, volatility is lower in the early stages of a boom. On the other hand, conditional volatility rises above the benchmark level as the boom overheats and investment rises beyond k^*. Entrepreneurs thus not only reduce the expected return on investment by funding white elephants, but also introduce excessive risk into the economy.

The distinctive feature of the model is that every lending booms comes to an end even if there is no bad productivity shock. As entrepreneurs become rich enough, highly risky overinvestment strategies become

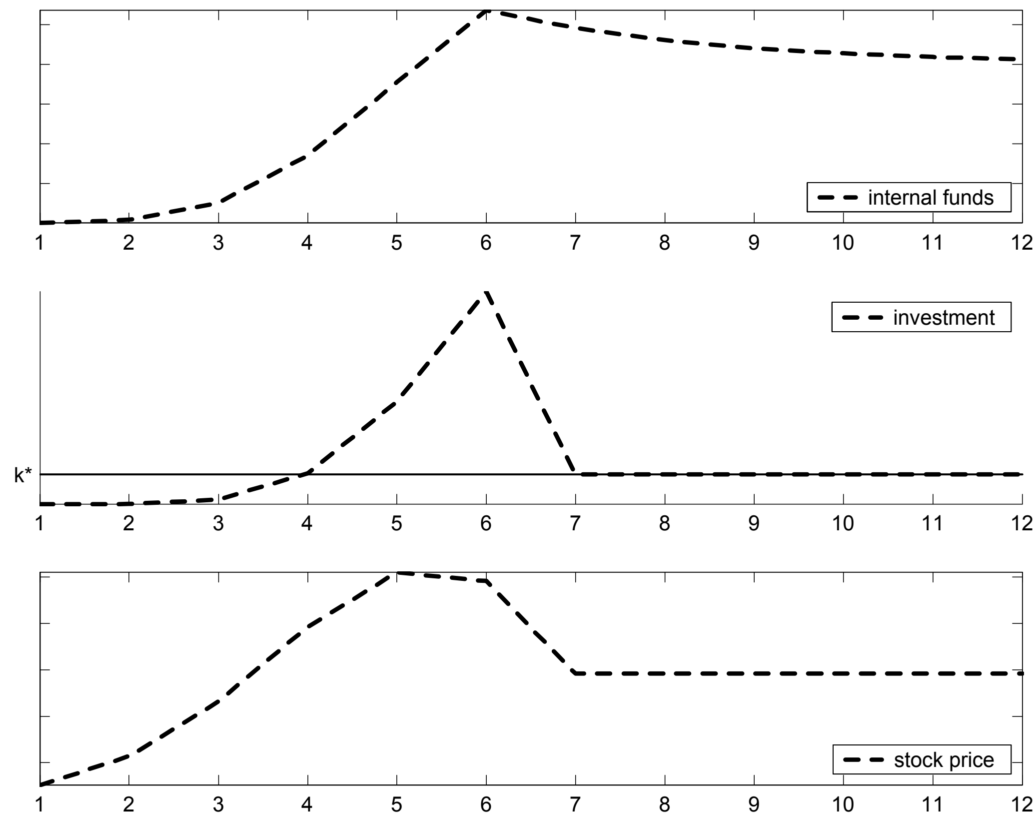

Figure 4.2: **The lucky path in case (a) – stable soft landings.**

Panels show internal funds, investment and stock price as a function of calendare time, assuming that $w_1 = \varepsilon$ and that no bad productivity shocks occur.

unprofitable, as the losses entrepreneurs would sustain on their own capital outweight any benefits from the bailout guarantee. Beginning in period six, entrepreneurs thus reduce investment to the efficient level k^* – the boom ends in a soft landing. In our example, investment then remains constant at the first-best level, as entrepreneurs draw down internal funds to the long run level w_∞. At the same time, the conditional volatility of output remains at the constant value it would attain in the frictionless benchmark economy.

Although a soft landing generates a drop in investment, and – with a one period lag – also in output, it does not resemble a recession. Instead, the drop in investment occurs because the economy does not keep up the wasteful investment and excessive conditional volatility characteristic of an overheating lending boom. The economy returns to normal behavior and behaves like a frictionless economy, at least as long as there is no bad productivity shock. In sharp contrast, a crisis *during the lending boom* does resemble a recession: it induces a persistent underutilization of resources ($k < k^*$). By depleting internal funds, it is followed by a "hangover" that recedes only slowly as internal funds grow back to boom levels.

While a soft landing typically begins a stable phase of the business cyle, bad productivity shocks can still occur. However, the effect of such shocks is quite different in this phase, compared to when the shocks hit during a lending boom. Indeed, suppose internal funds have moved close to w_∞, to a point such as point B in Figure 4.1. A bad productivity shock decreases output and internal funds, moving the economy down to point C. But at this point, the economy is back in the overinvestment region: investment will be higher than at point B! The entrepeneurial sector responds to the crisis, and resulting loss in output and internal funds by investing its remaining funds, but at the same time leveraging the firms in anticipation of a bailout guarantee. A crisis in the safe region therefore is not followed by a persistent slump: instead, it triggers another overheated lending boom.

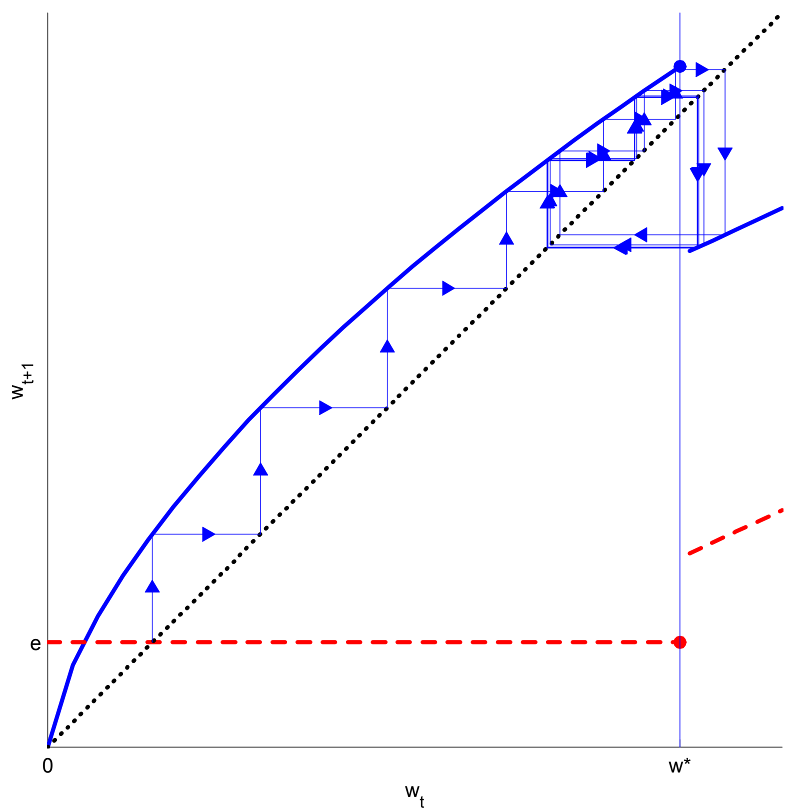

Figure 4.3: **Equilibrium in case (b) – endogenous cycles.**

Transition functions for internal funds are shown for $z_{t+1} = 1$ (solid) and $z_{t+1} = 0$ (dashed). The thin solid line with arrows indicates motion along the lucky path, which eventually osillates.

4.2. Endogenous Cycles

The previous subsection has shown that the stable phase of the cycle entered through a soft landing cannot last forever – bad productivity shocks can move the economy back to a situation of high leverage and lending booms. In this subsection, we show that if the dividend rate is high, then the force that pulls the economy back into the high leverage regime is even stronger – a *relapse* to a high leverage regime necessarily occurs even if there is no bad productivity shock. This is case (b) of Proposition 4.1, illustrated in Figure 4.3. Mechanically, for high enough c the 45 degree line passes through the discontinuity in the lucky transition function. As a result, the lucky path cannot converge, but must oscillate, bouncing back and forth between the risky and safe region.

The key to endogenous cycles is again the nonlinear relationship between internal funds and investment. As in the previous subsection, gambling and overinvestment are profitable as long as internal funds are not too high. Once too much of entrepreneurs' own money is at stake, investment reverts to the first best – as in case (a) a soft landing occurs. What is different in case (b) is that the high dividend payout rate depletes internal funds sufficiently quickly to make gambling profitable again. This cannot happen in the stable case (a) unless a crisis occurs first. Figure **??** shows the behavior of internal funds and investment in the absence of bad productivity shocks. Gradual lending booms inevitably end as investment is briefly reduced to the efficient level, before the next boom starts.

Figure 4.3 also clarifies that there can be no soft landing if the payout rate c is too high. If c is close to one, almost all profits would be paid out along the lucky path. The lucky transition function then cuts the 45 degree line at some $w < w^*$ and the lucky path would converges to that value.

4.3. Stock Price Behavior

In recent boom-bust episodes, stock prices have often peaked well before a crisis. At the same time, crises have proved remarkably hard to predict using financial indicators. How can these seemingly contradictory facts be reconciled? We now show that the model suggests a natural explanation. To define stock prices, we need to describe the financial policy of entrepreneurs in more detail. The analysis of section 3 assumes that there is only inside equity, and that all external financing is obtain in the debt market. Here we define dividends and price a claim on a dividend stream, which we call outside equity. The interpretation is that there is a – negligible – set of minority shareholders, who buy and sell outside equity in the stock market, while the majority ownership remains with the entrepreneur. Since investors are risk neutral, the dividend stream will be worth exactly its expected present value.

We assume that the entrepreneur runs a company that produces output, but that his other wealth is not held within the company, and is therefore not used to pay dividends in times of crisis. Outside equity is a claim to the profits delivered by the physical investment projects:

$$d_{t+1} = z_{t+1} \left(f \left(k_t \right) - \left(1 + r \right) k_t \right).$$

If there is a bad productivity shock, no dividend is paid. In addition, in any crisis the company is reorganized and outside equity becomes worthless. Of course, the entrepreneur, who is the controlling shareholder of

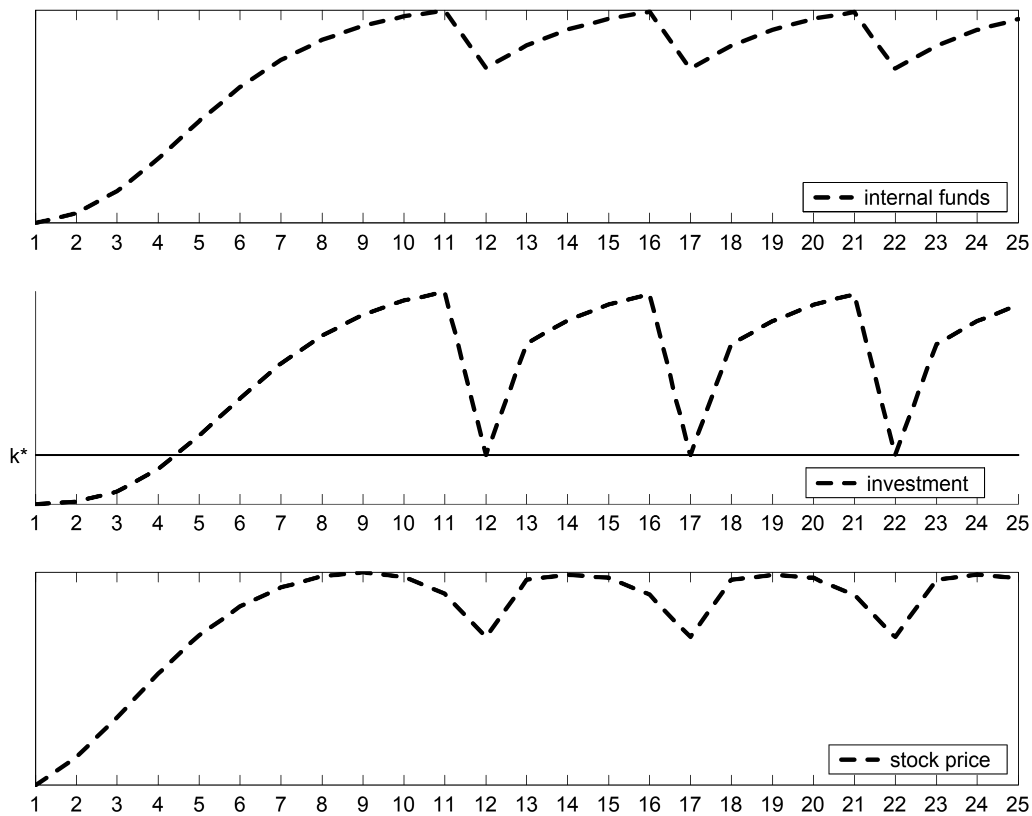

Figure 4.4: **The lucky path in case (b) – endogenous cycles.**

Panels show internal funds, investment and stock price as a function of calendare time, assuming that $w_1 = \varepsilon$ and that no bad productivity shocks occur.

the firm, may recapitalize the company in a crisis – this is what happens in the safe region, where the entrepreneur actually has sufficient funds to avert default. Nevertheless, outside equityholders receive zero.[8]

This setup leads to a simple recursion for the stock price along the lucky path. Let P_0 denote the stock price in the state $w = \varepsilon$. Then P_n, the stock price in the nth period that the economy remains on the lucky path can be found from

$$P_n = \beta\alpha\left(f\left(k_n\right) - (1+r)\,k_n + P_{n+1}\right),$$

where k_n is investment along the lucky path. This path for the stock price is plotted in the third panels of Figures 4.2 and 4.4.

The two plots shed light on the whole evolution of stock prices. In the risky region, the stock price moves

[8] An alternative scenario would be to have the entrepreneur's wealth as a part of the firm's asset. One could then define dividends as proportional to total entrepreneurial profits. Given the limited power of minority shareholders in many middle income countries, it seems more natural to assume that minority shreholders lose out in a crisis.

along the lucky path until a crisis occurs, at which point it reverts to P_0. Throughout the safe region, the stock price is constant at

$$\frac{\beta\alpha}{1-\alpha\beta}\left(f\left(k^*\right)-\left(1+r\right)k^*\right).$$

Any transition from the safe to the risky region caused by a crisis or a reversal implies a restart of the recursion. We do not show numerical results, but it is clear from the recursiom formula that the qualitative behavior of any price sequence that follows a crisis or relapse must qualitatively very similar to the stock market boom that starts in the state $w = \varepsilon$.

The key feature of the stock price is that it peaks before a soft landing occurs. This is because asset prices are forward looking – they factor in the lower dividends the company expects to pay after a soft landing. In the safe region, no bailout is expected – and the controlling shareholders – the entrepreneurs – have to recapitalize the firm in case of default. The resulting cutback of investment lowers dividends in the good state, while dividends are always zero in the bad state. Therefore, expected dividends fall.

The hump shape of the lucky stock price path helps understand why prices can peak before a crisis. Any country that experiences a crisis while being far enough along the lucky path will exhibit such a pattern. However, this does not mean that a peak of the stock market helps predict a crisis. Indeed, in our model the incidence of a crisis is an iid event, so that knowledge of the stock price does not help forecast it. The peak only signals that a soft landing is now closer – it tells investors nothing about future productivity shocks.

4.4. Fluctuations in Trust

So far, we have ignored fluctuations in trust by assuming that $x_t \equiv r$, that is, a risky equilibrium is played whenever it exists. In this subsection we relax this assumption. The following proposition characterizes equilibria in this case.

Proposition 4.2. *(Fluctuations in Trust)*

Suppose that $\varepsilon < (1-\beta\psi)\,k^*$ and $c > 1-\beta$.

1. For any stationary Markov trust process $\{x_t\}$, independent of $\{z_t\}$, there is a unique stationary equilibrium.

2. (Minimum Risk Equilibria) Suppose x_t is held constant at $x_t = s$. There are thresholds c_3 and c_4, with $c_1 < c_3 < c_4 < 1$, that define three cases for the lucky path

(a) if $c \leq c_3$, internal funds converge along the lucky path to a constant w_∞.

(b) if $c \in (c_3, c_4]$, internal funds oscillate between the risky and safe region.

(c) if $c > c_4 > c_2$, the lucky path converges to a constant in the risky region.

3. In any state (w, x) where there is low trust $(x = s)$, the 1-period-ahead conditional variances of internal funds, investment and output are smaller than in the corresponding state of high trust $(x = r)$.

Part 1 of the Prosposition says that adding exogenous shocks to trust does not change the basic nature of the dynamics. An equilibrium is described by a stationary Markov equilibrium; in this respect the model resembles standard business cycle models. The difference to Proposition 4.1 is that there are now two exogenous forcing processes – productivity shocks $\{z_t\}$ and trust, which does not affect payoffs directly. However, it can matter for real activity, because it acts as a coordination device for entrepreneurs. Formally, the equilibrium may thus be viewed as a stochastic sunspot equilibrium.

The new transition functions for internal funds are illustrated in Figure 4.5. This figure resembles Figure 4.1. The top solid line is still the unique lucky transition function for $w_t < k^*$. In addition, in the multiple equilibria region $w_t \in [k^*, w^*]$, the solid black line is the lucky transition function that is relevant when trust is high, $x_t = r$. The cases of Proposition 4.1 thus remain relevant for understanding the dynamics when $x_t = r$ – here we have chosen a consumption rate c that lies below c_2. What is new in the figure is that there is now a separate lucky transition function that becomes relevant when $x_t = s$. This lucky transition function is the dotted black line defined over the range $[k^*, w^*]$. Similarly, there is an unlucky transition function for $x_t = s$, drawn as a dotted gray line. In any state within the multiple equilibria region, the level of trust determines which pair of transition functions is relevant. The shock z_{t+1} then selects the lucky or unlucky transition from the relevant pair.

To understand the dynamics in general it is helpful to focus first on *minimum risk equilibria*. These occur when $x_t \equiv s$: a safe credit market equilibrium is played whenever it exists. Minimum risk equilibria are thus the polar opposite of the maximum risk equilibria of Proposition 4.1. They minimize the risk taking potential of bailout guarantees because entrepreneurs do not trust each other enough to coordinate risk taking. Part 2 of Proposition 4.2 characterizes the luckypath in a minimum risk equilibrium. The argument parallels that for maximum risk equilibria. Indeed, the transition functions for a minimum risk equilibrium have discontinuities at $w = k^*$, just like the transition function for a maximum risk equilibrium had discontinutities at $w = w^*$.

In the figure, the lucky transition function for the case $x_t = s$ is above the 45 degree line at $w = k^*$. This is implies that we are in case (a) of Proposition 4.2. The lucky path in a minimum risk equilibrium, drawn as a thin solid line with arrows, converges to a constant. Along the lucky path investment first increases, possibly beyond the efficient level. However, it quickly reaches the safe region of efficient investment. The entrepreneurial sector then proceeds to accumulate a cushion of internal funds. In the present example, this cushion eventually becomes large enough that a single crisis cannot displace the economy out of the safe region! In other words, the economy can withstand a bad productivity shock, keeping investment at the efficient level in its aftermath. Of course, a sufficiently long sequence of bad productivity is always sufficient to move the economy back to the risky region.

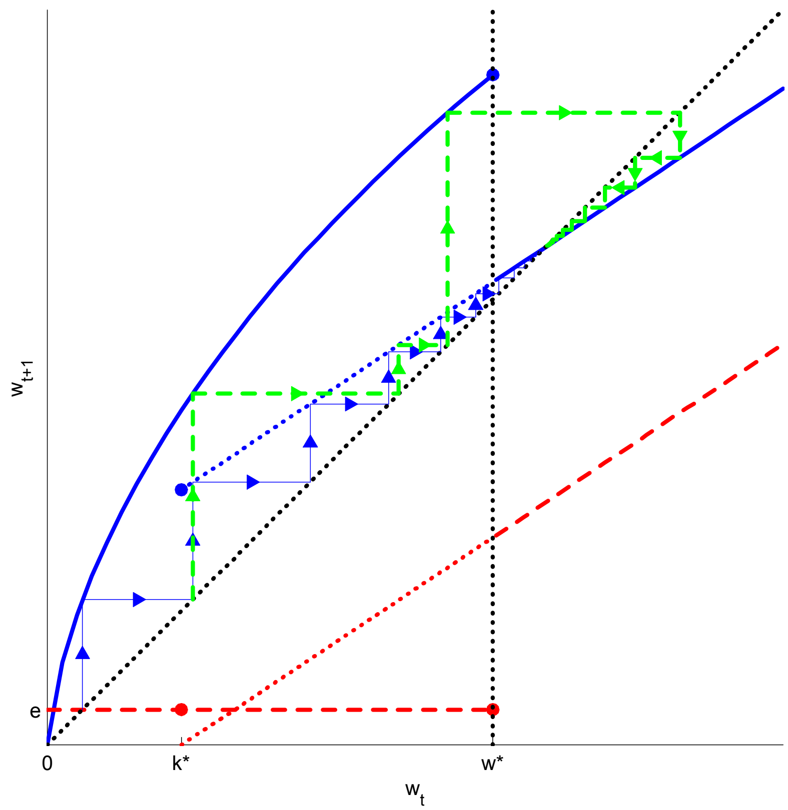

Figure 4.5: **Equilibrium in case (a) – stable soft landing**.

Transition functions for internal funds are shown for $x_t = r$ and $z_{t+1} = 1$ (solid), $x_t = r$ and $z_{t+1} = 0$ (dashed), $x_t = s, z_{t+1} = 1$ (dotted, dark) as well as $x_t = s, z_{t+1} = 0$ (dotted light). The thin solid line with arrows indicates motion along the lucky path in a minimum risk equilibrium towards w_∞. The dashed light line with arrows describes a path along which trust fluctuates.

The gray dashed line illustrates some of the additional dynamics that are possible when trust fluctuates. The two paths coincide up to the point A. However, at this point the dashed path represents what happens with high trust: investment is much higher than in the minimum risk equilibrium (the solid path). If entrepreneurs' gamble is successful and the economy remains on the lucky path, growth is much higher than in the minimum risk equilibrium as the economy moves up to point B. We assume that at this point, the dashed path encounters a breakdown of trust, a *correction*. As a result, there can be at most a small increase in internal funds along the lucky path. However, if the breakdown in trust is temporary, investment and trust are again high. Starting from point C, a lending boom develops that leads to a large amount of wasteful investment before ending in a soft landing.

The bottom line is that fluctuations in trust lead to much more erratic lending booms. This general theme is also present in the other cases (not shown). A breakdown in trust can, at any point in time, reduce investment to the efficient level. At the same it will slow down or even diminish the growth of internal funds that fuels the boom. Of course, the flip side of low growth of the lucky path is that a cushion of internal funds is built in low trust periods. The figure clarifies part 3 of the proposition: the *conditional* volatility of real variables is reduced in low trust periods.

5. Appendix

Proof of Proposition 3.1.

We have defined a safe plan as a plan with $b_t = 0$, and a risky plan as a plan with $s_t = 0, b_t > 0$ and default in the bad, but not in the good state. We first show that entrepreneurs never pick any other plans, whether or not a bailout is expected in the bad state. Given this fact, it is sufficient to show that (i) when a bailout is expected, a risky plan is optimal if and only if $w \leq w^*$, and (ii) when no bailout is expected, a safe plan is optimal if and only if $w \geq k^*$. Claim (ii) is established in the text. An analytical proof of claim (i) that does not rely on the graphical argument in the text, is offered below.

Step 1: Only safe or risky plans can ever be optimal.

First, it never makes sense to offer a plan that leads to default for sure. Whether or not there is a bailout in the bad state, lenders will accept such a plan only if they obtain at least $1 + r$ per dollar lent in the good state. However, they can only seize this much if all the borrowed funds are invested in riskless savings – after all, they can only recover $\psi < 1 + r$ per dollar invested in capital. But if entrepreneurs invest all borrowed funds in savings, they make zero profits on these funds and might as well not borrow.

Second, inspection of the conditions for default show that there does not exist a plan under which default is optimal in the good state, but not in the bad state.

We are thus left with plans that either (i) never default, or (ii) default in the bad state, but not in the good state. We show that for both types, without loss of generality, borrowing and riskless saving can be taken to be mutually exclusive activities.

Indeed, any savings are seized by lenders in case of default. This means that the expected return on a dollar saved under a type (ii) plan is $\alpha (1 + r)$. At the same time, the expected cost of a dollar borrowed is $\alpha (1 + r)$ when a bailout is expected (and $1 + r$ must only be paid in the good state), and it is $1 + r$ when no bailout is expected. Therefore, saving a borrowed dollar never leads to positive profits under a type (ii) plan. Since type (i) plan triggers default in the good state, it must necessarily involve some borrowing. The previous argument implies that wlog it can be taken to be a risky plan (with $s_t = 0$).

Under a type (i) plan, the expected return on a dollar saved or invested cannot be less than $1 + r$. At the same time, the expected cost of a dollar borrowed is $1+r$ whether or not a bailout is expected (the bailout does not affect a safe plan that does not lead to default). Again, saving a borrowed dollar cannot lead to positive profits. A type (ii) plan that involves borrowing must necessarily involve some riskless savings, since otherwise could not be paid in the bad state. The previous argument thus implies that wlog borrowing can be taken to be zero, so that the type (i) plan can be taken to be a safe plan.

Step 2: When a bailout is expected, a risky plan is optimal if and only if $w \leq w^*$, for some $w \in (k^*, k^{**})$.

The profit functions for the best safe and risky plan, derived in the text, are

$$V(k^s(w), w) = \begin{cases} \alpha f(w) & \text{if } \ w \leq k^* \\ \alpha f(k^*) + (1+r)(w-k^*) & \text{if } \ w > k^* \end{cases} \tag{5.1}$$

$$V(k^r(w), w) = \begin{cases} \alpha f(\frac{w}{1-\beta\psi}) - \alpha \frac{\psi w}{1-\beta\psi} & \text{if } \ w \leq \widetilde{k}[1-\beta\psi] \\ \alpha f(\widetilde{k}) + \alpha(1+r)\left[w-\widetilde{k}\right] & \text{if } \ \widetilde{k} > w > \widetilde{k}[1-\beta\psi] \end{cases} \tag{5.2}$$

The following lemma characterize these functions and thereby complete the proof.

Lemma 5.1. *If $f'(0) > 1+r$, then $\pi^r(w) > \pi^s(w)$ for any w on $(0, k^*]$.*

Proof. Define the function $\Pi(w) = \pi^r(w) - \pi^s(w)$ on $[0, k^*]$. Equations (5.1) and (5.2) imply that $\Pi(w)$ is continuous. There are two cases. First, in the case $k^* \leq \widetilde{k}[1-\beta\psi]$

$$\Pi(w) = \alpha\left[f(k^r) - f(w)\right] - \alpha(1+r)\left[k^r - w\right], \qquad w \in (0, k^*] \tag{5.3}$$

where $k^r = \frac{w}{1-\beta\psi}$. The mean value theorem implies that there exists a constant $a \in (w, k^r)$ such that $\beta f'(a) = \beta\frac{f(k^r)-f(w)}{k^r - w}$. Concavity of f implies that $\beta f'(a) > \beta f'(k^r) \geq \beta f'(\frac{k^*}{1-\beta\psi}) \geq \beta f'(\widetilde{k}) = 1$. Since $\beta f'(a) > 1$, it follows that (5.3) is positive for any w on $(0, k^*]$.

Consider now the case $k^* > \widetilde{k}[1-\beta\psi]$. For $w \leq \widetilde{k}[1-\beta\psi]$ the argument is the same as the previous one. For $w > \widetilde{k}[1-\beta\psi]$ replace k^r by $\widetilde{k}$ in (5.3), and note that there is a constant $b \in (w, \widetilde{k})$ such that $\beta f'(b) = \beta\frac{f(\widetilde{k})-f(w)}{\widetilde{k}-w}$. Moreover, $\beta f'(b) > 1$ because $b < \widetilde{k}$.□

Lemma 5.2. *There is a unique wealth level $\widetilde{w}(\psi)$ such that $\pi^r(w) < \pi^s(w)$ if and only if $w > \widetilde{w}(\psi)$. Furthermore, $\widetilde{w}(\psi) > \widetilde{k}(1-\beta\psi)$ if and only if $\psi > \widetilde{\psi}$, where*

$$\widetilde{\psi} \equiv \frac{\alpha\beta[f(k^*)-f(\widetilde{k})]-[k^*-\widetilde{k}]}{[1-\alpha]\beta\widetilde{k}} < \frac{1}{\beta} \tag{5.4}$$

Proof. We consider first the case $k^* \leq \widetilde{k}(1-\beta\psi) \equiv \bar{w}$. Equations (5.1) and (5.2) imply that $\Pi(w)$ has the following three properties. First, for $w \geq k^*$, $\Pi(w)$ is concave. That is, $\Pi'(w)$ is declining. Second, $\Pi'(w)$ is negative for any $w \geq \bar{w}$. Third, $\Pi(\bar{w}) > 0$ if and only if $\psi > \widetilde{\psi}$. It follows that there is a unique $\widetilde{w}$ such that $\Pi(\widetilde{w}) = 0$. Furthermore, for $\psi > (<)\widetilde{\psi}$, we must have $\widetilde{w}(\psi) > (<)\widetilde{k}(1-\beta\psi)$.

We consider now the case $\bar{w} < k^*$. Lemma 5.1 implies that $\Pi(\bar{w}) > 0$. Since $\Pi'(w) < 0$ for $w \geq \bar{w}$, there is a unique $\widetilde{w}$ such that $\Pi(\widetilde{w}) = 0$. Furthermore, $\widetilde{w} > k^* > \bar{w}$.

Finally, we show that $\widetilde{\psi} < \beta^{-1}$. The mean value theorem implies that there is a constant $a \in (k^*, \widetilde{k})$ such that $\widetilde{\psi} \equiv \frac{1-\alpha\beta f'(a)}{[1-\alpha]\beta\widetilde{k}/[\widetilde{k}-k^*]}$. Since, $\beta f'(a) > \beta f'(\widetilde{k}) = 1$, we have that $\widetilde{\psi} < \frac{[1-\alpha][\widetilde{k}-k^*]}{[1-\alpha]\beta\widetilde{k}} < \frac{1}{\beta}$. The second inequality follows from $\widetilde{k} > k^*$.■

Proof of Proposition 4.1.

It is useful to first prove the properties of the lucky path in part 2.

Part 2. Define the thresholds c_1 and c_2 as follows:

$$(1-c_1)\left(f(k^*) + (1+r)(w^* - k^*)\right) = w^*, \tag{5.5}$$

$$(1-c_2)\left(f(\min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}) + (1+r)\left(w^* - \min\left\{\frac{w_t}{1-\beta\psi}, k^{**}\right\}\right)\right) = w^*. \tag{5.6}$$

In terms of Figure **??**, c_1 is the unique value of c for which the lower branch of the lucky transition function intersects the 45 degree line exactly at $w = w^*$. Also, c_2 is the unique value of c for which the upper branch intersects the 45 degree line at w^*.

We have $c_1 > 1 - \beta$. Indeed, suppose that $c = 1 - \beta$. By (4.1), the lower branch of the lucky transition function is always above the 45 degree line at $w = w^*$:

$$\begin{aligned}
& (1-c)\left(f(k^*) + (1+r)(w^* - k^*)\right) \\
= \; & w^* + \beta f(k^*) - k^* \\
> \; & w^* + \beta\left(\alpha f(k^*) - (1+r) k^*\right) \\
> \; & w^*,
\end{aligned}$$

where the last inequality follows from the fact that f is continuous and concave, $\alpha f'(0) > 1 + r$ and $\alpha f'(k^*) = (1+r)$. Now varying c does not affect the optimal plans or what equilibrium is played − all it does is scale the lucky transition function. Therefore, there must exist c_1 with $\beta > 1 - c_1 > 0$ such that (5.5) holds.

We also have $c_2 > c_1$. Indeed, the best risky plan for $w = w^*$ derived in the proof of Proposition 3.1 maximizes $f(k) + (1+r)(w^* - k)$ subject to the collateral contraint $(1-\beta\psi)k \leq w^*$. But $k = k^*$ was in the contraint set of this problem since Proposition 3.1 implies that $w^* > k^*$. Therefore,

$$\left(f(\min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}) + (1+r)\left(w^* - \min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}\right)\right) > \left(f(k^*) + (1+r)(w^* - k^*)\right).$$

As a result, there must exist c_2 with $1 - c_1 > 1 - c_2 > 0$ such that (5.6) holds. Graphically, the upper branch of the lucky transition function is always strictly above the lower branch at $w = w^*$.

Consider $c \in (1-\beta, c_2)$. Since the upper branch is strictly above the 45 degree line for all $w \leq w^*$, the lucky path must grow beyond w^* – there must be a soft landing. Cases a and b distinguish different behaviors inside the safe region ($w > w^*$).

First. consider $c \in (1-\beta, c_1)$. In this case, the lower branch of the lucky transition function is above the 45 degree line. The slope of the lower branch is $(1-c)(1+r) < 1$, so that the lower branch must cut the 45 degree line from above at some value $w_\infty > w^*$. Therefore, within the safe region, the lucky path converges to w_∞. For $c = c_1$, we have $w_\infty = w^*$, that is the lucky path converges to w^*. This establishes case a.

Second, consider $c \in (c_1.c_2)$. In this case, the lower branch is below the 45 degree line at $w = w^*$, whereas the upper branch remains above it – the lucky transition function does not intersect the 45 degree line at all. In addition, for $w > w^*$, the fact that the slope of the lower branch is less than one implies that w_t decreases monotonically. Since the lower branch is strictly below w^* at $w = w^*$, it follows that the lucky path must again transit to $w < w^*$. In this region, it will again monotonically increase until it must transit to $w > w^*$ and so on. This establishes case b.

Finally, consider $c \geq c_2$. The lucky transition function cuts the 45 degree from above at some w_∞ with $0 < w_\infty < w^*$. Therefore, the lucky path increases monotonically and converges to w_∞, and therefore it never reaches the safe region.

$(1-c)\left(f(\min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}) + (1+r)\left(w^* - \min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}\right)\right)$

Part 1. Step 1. There is a number $\bar{w}$, such that the process $\{w_t\}$ lives on the interval $[\varepsilon, \bar{w}]$.

The upper bound $\bar{w}$ is defined as follows. Let w_∞ denote the largest value where the lucky transition function crosses the 45 degree line and define

$$w^{hi} := (1-c)\left(f(\min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}) + (1+r)\left(w^* - \min\left\{\frac{w^*}{1-\beta\psi}, k^{**}\right\}\right)\right),$$

the value of the lucky transition function at $w = w^*$. Now define $\bar{w} = \max\left\{w^{hi}, w_\infty\right\}$.

We now show that the upper bound $\bar{w}$ is respected by any sample path that starts in $[\varepsilon, \bar{w}]$ and for which the shock realization is $z_t = 1$ throughout. We verify this separately for the parametrizations of cases a-c.

In case c, $w_\infty \in (0, w^*)$. We must have $w \leq w_\infty$, because all paths starting at $w < w_\infty$ converge monotonically to w_∞ from below.

In case b, $w_\infty = 0$. However, as shown in the proof of part 1, any path must be decreasing in the safe region. Therefore, $w \leq w^{hi}$, since w^{hi} is the highest point at which a path can possibly enter the safe region.

In case a, there can be two subcases. If $w_\infty \geq w^{hi}$, then all paths converge monotonically to w_∞ from below. In contrast, if $w_\infty < w^{hi}$ then there exist paths that enter the safe region at $w > w_\infty$. However, the slope of the lower branch of the lucky path all paths must be monotonically decreasing as they converge to w_∞. Therefore we have $w \leq w^{hi}$ as in case b.

It remains to show that sample paths that experience crises ($z_t = 0$) at least once also respect the upper bound. But holdings fixed w_t, a crisis outcome of w_{t+1} is always strictly below the outcome if there is no crisis. Therefore all sample paths respect the upper bound.

Finally, it is clear that $w_t \geq \varepsilon$: since $\varepsilon < (1 - \beta\psi) k^*$, the lucky path is monotonically increasing at $w = \varepsilon$, while the outcome for w in a crisis is bounded below by ε.

Step 2. There is an integer n and a number $\gamma > 0$ such that, for every point $v \in [\varepsilon, \bar{w}]$,

$$\Pr(w_{t+N} = \varepsilon | w_t = v) > \gamma.$$

To construct γ and n, consider first $v \in (w^*, \bar{w}]$. We know that the "unlucky" transition function (for $z_{t+1} = 0$) has a slope less than one and that its value at $w = w^*$ is

$$(1 - c)(1 + r)(w^* - k^*) < w^* - k^* < w^*,$$

so that it lies below the 45 degree line for all $w > \varepsilon$. This implies that for every $v \in (w^*, \bar{w}]$, a *finite* sequence of crises (realizations $z_t = 0$) – occurring one after the other – implies that w_t returns to the region where $w < w^*$. But for $w \leq w^*$, a crisis wipes out all internal funds in that region so that $\Pr(w_{t+1} = \varepsilon | w_t = v) = 1 - \alpha$. Therefore no more than a finite number of realizations $z_t = 0$ leads from any $v \in (w^*, \bar{w}]$ to $w = \varepsilon$. In particular, the most steps are required for $v = \bar{w}$. We thus let N be the smallest integer such that $\Pr(w_{t+N} = \varepsilon | w_t = \bar{w}) = (1 - \alpha)^N$ and pick $\gamma = (1 - \alpha)^N / 2$.

For $v < \bar{w}$, the smallest integer n such that $\Pr(w_{t+n} = \varepsilon | w_t = \bar{w}) = (1 - \alpha)^n$ is less or equal to N. (In particular, for $v < w^*$, $n = 1$). Moreover, $w_t = \varepsilon$ and $z_{t+1} = 0$ implies $w_{t+1} = \varepsilon$, since the endowment is always below w^*. Therefore, $\Pr(w_{t+N} = \varepsilon | w_t = \bar{w})$ is at least $(1 - \alpha)^N$ and thus by construtio larger then γ.

Step 3. There is a unique invariant measure associated with the Markov operator defined by (4.1).

We verify the conditions of Theorem 11.10 in Stokey and Lucas (1989).

First, Step 2 implies that the Markov operator satisfies Doeblin's condition: there is a finite measure ϕ on the state space $[\varepsilon, \bar{w}]$, an integer $N \geq 1$, and a number $\gamma > 0$, such that, for every Borel subset of $[\varepsilon, \bar{w}]$, if $\phi(B) \leq \gamma$ then $\Pr\{w_{t+N} \in B | w_t = v\} \leq 1 - \gamma$ for all $v \in [\varepsilon, \bar{w}]$. Indeed, let ϕ be a Dirac measure on the point ε, that is $\phi(\{\varepsilon\}) = 1$, $\phi(B) = 1$ for all B such that $\varepsilon \in B$, and $\phi(B) = 0$ for all B such that $\varepsilon \notin B$.

Second, our construction of ϕ also implies that for any Borel set B of positive ϕ-measure (in our case, any B containing ε), then for each $v \in [\varepsilon, \bar{w}]$, there is an integer $n \geq 1$ such that $\Pr(w_{t+n} = \varepsilon | w_t = v) > 0$. Indeed, Step 2 has shown that the probability of reaching ε itself from any starting point v is positive.■

Proof of Proposition 4.2.

Part 1. Step 1 in the proof of Proposition 4.1 goes through if the interval for internal funds w_t is expanded to $(0, \bar{w}]$. Indeed, for given w_t, the level w_{t+1} that obtains for $z_{t+1} = 1$ is bounded above by the value that obtains if $x_t = r$, the case covered in the proof of Proposition 4.1. This implies that the upper bound $\bar{w}$ need not change. The lower bound now becomes zero, since internal funds may drop below ε if a crisis occurs in a safe equilibrium (cf. Figure 4.5). However, internal funds never reach zero, since $w_{t+1} = \varepsilon$ whenever a crisis reduces profits to zero.

Step 2 in the proof of Proposition goes through with minor modification. Indeed, one can follow the above proof to construct an integer n and a number $\gamma > 0$ such that, for every point $(v, x) \in [\varepsilon, \bar{w}] \times \{r, s\}$, $\Pr(w_{t+N} = \varepsilon, x_{t+1} = r | w_t = v, x_t = x) > \gamma$. The key to that step is that internal funds return to the value ε in a finite number of steps. This can be engineered as before by selecting a sufficiently long sequence of crises. Since x and z are indepdendent, an accompanying sequence of x values can be picked without upsetting this key property.

Given the result of Steps 1 and 2, Step 3 can be applied to the Markov operator (4.1) as before.

Part 2. We define thresholds c_3 and c_4 by

$$\begin{aligned}(1-c_3)\, f(k^*) &= k^* \\ (1-c_4)\left(f(\min\left\{\frac{k^*}{1-\beta\psi}, k^{**}\right\}) + (1+r)\left(k^* - \min\left\{\frac{w_t}{1-\beta\psi}, k^{**}\right\}\right)\right) &= k^*.\end{aligned}$$

We have $c_3 > 1 - \beta$. Indeed, suppose that $c = 1 - \beta$. Since k^* is defined by $\alpha f'(k^*) = 1 + r$, strict concavity of f implies

$$(1-c)\, f(k^*) > (1-c)\, f'(k^*)\, k^* = (1-c)\, \frac{1+r}{\alpha} k^* > (1-c)(1+r)\, k^* = k^*.$$

Therefore, the lucky transition function in a safe equilibrium at $w = k^*$ which equals $f(k^*)$ is above the 45 degree line if $c = 1 - \beta$.

The equations defining c_3 and c_4 are the same as those defining c_1 and c_2 above, except that w^* has been replaced by k^*. Similar arguments as above imply that $c_4 > c_3$. In addition, $w^* > k^*$ implies that $c_4 > c_2$ and $c_3 > c_1$.

Suppose now that $x_t = s$ always. The lucky transition function now has a discontinuity at $w = k^*$; it follows the lucky safe transition function for all $w \geq k^*$. The rest of the proof parallels the arguments in the proof of Proposition 4.1

In case (a), the lower branch of the lucky transition function is above the 45 degree line. It follows that the lucky path converges.

In case (b), the 45 degree line passes through the discontinuity in the lucky transition function. It follows that the lucky path must oscillate. Finally, in case (c), the lucky transition function intersects the 45 degree line at a point below k^*. The lucky path converges montonically to that point.■

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