Asset Price Support Policy During Crises: How Aggressive Should it Be?

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Abstract

Should a Central Bank (CB) aim at smoothing out all asset price volatility in crisis times? What trade-offs does it face? To address this question we consider an economy where leverage is endogenously determined by the CB asset price support policy during crises. By keeping the price of distressed assets above a critical level, the CB can induce a high-leverage equilibrium with high output but with infrequent financial crises. But how agressively should the Central Bank support the price of distressed assets? The optimal CB policy depends on whether the interventions necessary to support the high-leverage equilibrium are costly or not. If the CB does not require any net wealth to credibly promise the minimal intervention that will keep asset prices above the critical level, it is optimal to commit all its wealth to intervention—this seems to be the case in the US in the aftermath of the 2008 crisis. In contrast, if interventions are costly, there is a trade-off between enjoying higher leverage and output now, but withstanding a lower number of crises before falling into a low-leverage low-output trap, and a more prudent policy that can keep the economy longer in the high-leverage equilibrium. We solve the Central Bank problem in closed-form by converting it into a linear HBJ equation. Key determinants of the shape of policy functions is the number of future crises the Central Bank can withstand and the net expected social value of leverage. We find that more prudent policies tend to be optimal when leverage is more socially valuable or the Central Bank has more wealth.

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PRELIMINARY DRAFT

1 Introduction

During systemic crises, Central Banks or Government-backed bailout agencies intervene by buying distressed assets from financial intermediaries. They do so in order to avoid the bankruptcy of financial intermediaries, mitigate losses of their creditors, and limit the size of the output contraction. Large equity injections into banks have similar effects since they shift non-performing assets from private to public sector balance sheets. Recent examples of such interventions include the US troubled asset relief program (authorized for about 5% of GDP) and the Irish Bank Bailout (40.7% of GDP). These policies push explicitly or implicitly the prices of distressed assets well above the prices that would have resulted from unavoidable large-scale fire-sales. These interventions carry the risk of large fiscal losses if the value of the purchased assets does not recover enough.

How generous should Central Banks' asset price support policies be? How large should the associated transfers be? As Hall and Reiss (2015) have argued, Central Banks committed to control inflation do face resource constraints. The scale and aggressiveness of their asset price support policies thus depends on their financial position and on the fiscal backstop they can expect from the Treasury. In this paper, we characterize a key intertemporal trade-off faced by Central Banks that undertake such support policies. The generosity of Central Bank policy is a key driver of leverage and output pre-crisis but also of the potential fiscal loss experienced during a crisis, and so by promising to maxing up its support during the *next crisis*, the Central Bank can be forced to reduce its ability to intervene in *subsequent future crises.* A central bank might therefore optimally choose a less generous asset price support policy in order to save ammunitions for future interventions.

We consider an infinite horizon economy in which financial intermediaries face leverage constraints due to moral hazard concerns. In this set-up, the Central Bank can choose to intervene during crises by buying distressed assets from financial intermediaries. In such an environment, risk-taking, leverage, investment and output, are endogenously determined by the Central Bank's asset price support policy, which is fully anticipated. An aggressive enough Central Bank's policy will support an equilibrium with high leverage and high output until the next crisis occurs. Otherwise, there will be low leverage, no risk-taking, and stable but low output. The more the Central Bank attempts to limit the distressed asset price collapse during a crisis, the greater are leverage and output before the crisis, but also, possibly, the larger are the ultimate net fiscal costs of intervention.

An important factor that determines the shape of the optimal policy is

whether Central Bank's interventions in the high-leverage equilibrium are necessarily costly or not. In the case of *costless interventions*, where the Central Bank does not require any available fiscal space to credibly commit to future interventions, it is optimal for the Central Bank to intervene to the full extent of its available resources. The cost incurred during crises is always outweighed by the benefits of higher leverage in normal times. In other words, the central bank crisis policy pays for itself. The large-scale financial sector intervention undertaken by the Federal Reserve System and the US Treasury in the aftermath of the 2008 appear ex-post to fall in the category of costless interventions. According to US Treasury (2013), The Troubled-Asset Relief Program (TARP) Bank Investments and Credit Market Programs, the AIG rescue package, the Fed Emergency Credit & Liquidity Programs, have all turned a positive net profits after five years.[1] Figure 2 in the appendix plots disbursements and cash backs for all TARP programs. Less than five years after the launch of the TARP programs, the initial disbursement associated with asset purchases appear are compensated by the proceeds of assets sale.

Things are a lot different when Central Bank's interventions necessarily carry a positive net fiscal cost. In this case, the Central Bank faces an intertemporal trade-off. By increasing the resources committed to limit asset price collapse, the Central Bank can boost risk-taking and leverage but face the risk of running out of fiscal space after the next crisis. In such case, the central bank will not be able to promise even the smallest intervention necessary to sustain the high-leverage equilibrium, and the economy will fall into a low leverage–low output trap. The trade-off is then between enjoying higher leverage now, but withstanding a smaller number of crises, and choosing a more prudent policy with lower leverage which allows to withstand a larger number of crises. The record of fiscal costs of crises provided by Laeven and Valencia (2012) indicates that most central bank intervention are indeed *costly*.

We solve the Central Bank optimal policy in closed-form by converting it into the solution of a linear stochastic continuous time Hamilton-Jacobi-Bellman equation. The central bank optimal policy can be simply characterized by the number of future crises the Central Bank can commit to respond to. This in turn implicitly determines how far the economy is from falling into a low output trap. We find that the optimal solution depends on three key factors: (i) how large are the net social gains of being in a high-leverage

[1] Note that is is not the case for TARP Auto Industry Investments and the TARP Housing Program which by 2013 exhibit significant fiscal losses.

economy as compared to a low-leverage economy; (ii) how fast can the Central Bank restore its balance sheet in the aftermath of a crisis, and in doing so maintain its ability to intervene in the next crisis; (iii) how high is the stochastic discount factor, i.e., how worth is the value of a dollar of intervention in this crisis vs. the value of a dollar used to intervene in the next crisis. The combination of these factors determines the optimal degree of *prudence* of a central bank, that is how many crises away it chooses to remain until its capacity to intervene and to sustain high leverage is depleted.

According to our findings, Central Bank's prudence should be greater the larger the net social benefits of leverage, the faster the Central Bank can restore its balance sheet, and the larger the stochastic discount factor. Intuitively, with a low discount factor, the Central Bank is impatient and prefers higher leverage now even if that implies falling into the low leverage–low output trap sooner. On the contrary, with high benefits of leverage, the Central Bank has a lot to lose when falling into a low leverage–low output trap, which should favor prudence. Finally, the faster the Central Bank can restore its balance-sheet after a crisis, the more resources it will have to re-leverage the economy post-crisis: this increases the benefits of staying in a high-leverage equilibrium for a longer time and favors prudence.

The large scale of the distressed assets purchase programs observed during the 2008 crisis reflects the high degree of leverage of modern finance, and in particular the growth of the shadow banking sector in which the holdings of risky and potentially illiquid assets are backed by short-term uninsured liabilities. According to IMF (2014), the size of the shadow banking sector has broadly increased, between 2002 and 2008, from 110 to 160 percent of GDP in the US, from 120 to 180 percent oh GDP in the UK, from 100 to 150 percent of GDP in the Euro area, and from 10 to 40 percent of GDP in Emerging Market Countries. Figure 3 in the appendix compares the total liabilities of banks and shadow banks in the US between 1950 and 2008. The shadow banking sector, that was quasi non-existant until the 70s, reached the size of banking sector in 1992, became close to be twice bigger by 2007, before shrinking dramatically in 2008.

A large share of the shadow banking sector consisted of conduits set up by traditional banks to limit their capital requirements (regulatory arbitrage). When the funding liquidity for shadow banks dried out, their assets got either liquidated or repatriated to the balance sheets of their mother banks, thus threatening their solvency.

During crises, and under a sense of emergency if not panic, policy makers tend to scale up their asset purchase programs as much as possible with the feeling that the bigger the better. While the moral hazard consequences of

such behavior have been well studied, very little is known about its effects on the *future ability* of a central bank to intervene during future crises. This paper fills such a gap by offering a normative perspective on Central Bank's crisis policy. In doing so this paper naturally complements the burgeoning literature on the intertemporal resources constraints faced by Central Banks (Hall and Reis, 2015, Del Negro and Sims, 2015).

The rest of the paper is organized as follows. Section 2 presents a static model of leverage and risk-taking by financial intermediaries. Section 2 embeds it into a macroeconomic model and studies how the Central Bank policy affects leverage and risk-taking. Section 3 characterizes the Central Bank optimal policy.

2 Model

We consider a minimal real model that allows us to characterize analytically the dilemma faced by a Central Bank (CB). Investment is determined by the financing firms obtain from financial intermediaries, who in turn need to fund themselves by issuing debt. Intermediaries are subject to moral hazard and so in equilibrium they may face borrowing constraints imposed by investors. Final output is subject to exogenous aggregate shocks and so firms may have little revenue to repay financial intermediaries with. Intermediaries must decide whether to absorb this insolvency risk—by issuing non-contingent debt—or to transfer this risk to investors by issuing contingent debt.

In equilibrium, leverage and expected output are larger if intermediaries issue contingent debt as this endogenously relaxes borrowing constraints. We first show that the choice of a safe vs. risky financing structure depends on the price of distressed loans (q) during crises. A risky debt structure—in which financial intermediaries issue non-contigent debt—can only be an equilibrium if the distressed assets price is high enough. The intervention of the CB sets a floor on the price of distressed assets ex-post by promising to buy any amount at that price. Then, it also determines ex-ante the debt structure and leverage. If the floor on distressed assets prices is high enough, the intermediaries will find it optimal to issue non-contingent debt. The larger the floor, the less the moral hazard problem, the more investors are willing to lend, and so the higher leverage and expected output.

The CB however has a budget constraint and so it may face a trade-off which is the core of the paper. If the CB's policy is to intervene such that asset prices don't fall below a floor q, then the CB is implicitly choosing that it will only be able to intervene $N(q)$ times, in a probalistic sense.

If q is below a threshold $\underline{q}$, then financial intermediaries choose a financial structure that never leads to default, but leverage and average output are too low. This low-leverage equilibrium is an absorbing state.

We start by presenting a simple static framework of financial intermediation. This framework will then be embedded in a macroeconomic model in the next section. In this section, the price of distressed assets q_t is taken as given. In Section 3, it will be endogenized as the result of Central Bank intervention.

2.1 A simple microeconomic model of leverage and risk-taking

We consider a model of financial intermediation with three types of agents: final investors, financial intermediaries, and firms. All agents are risk-neutral with a common discount factor β.

Firms

Firms launch investment projects. By investing k_t in a project in period t, a firm can produce $A_{t+1}k_t$ in period $t+1$. The return on investment A_{t+1} is stochastic and common across firms. There are two states of nature: good (corresponding to normal times) with probability $1-u$, and bad (corresponding to crisis times) with probability u. The return is equal to $\bar{A}$ in normal times and to $\underline{A}$ in crisis times, with $\bar{A} > \beta^{-1} > \underline{A}$. The state of nature is revealed at the beginning of each period. Firms finance their investment by selling equity to financial intermediaries.

Financial intermediaries

Financial intermediaries are the only agents able to fund firms. A financial intermediary starts with an exogenous endowment w_t used as internal funds. Then, she funds investment k_t with her internal funds w_t and by issuing liabilities l_t to final investors:

$$k_t = w_t + l_t.$$

Denoting m_t the loan-to-value ratio (l_t/k_t), the *leverage ratio*, that is, the ratio of assets to internal funds, is given

$$k_t/w_t = 1/(1 - m_t)$$

There are two types of liabilities: contingent and non-contingent. For expositional clarity, suppose that each financial intermediary only issues

one type of liability. Contingent liabilities are promises to pay an amount $A_{t+1}P_{t+1}$. The intermediary commits in period t on the level of P_{t+1}, but the full repayment is contingent on the realized return A_{t+1}. When issuing contingent liabilities, financial intermediaries pass on the risk of investment projects to final investors. Therefore, they are always solvent.

A financial intermediary can also issue non-contingent liabilities: promises to pay a given amount P_{t+1} in all states of nature. In that case, the intermediary absorbs part of the risk with her internal funds. If the total return $A_{t+1}k_t$ of the funded investment projects falls short of the promised repayment P_{t+1} to final investors, the intermediary is insolvent and is forced to default. Her assets are liquidated and sold at a distressed price q_{t+1}, which is known at time t. Final investors are paid $q_{t+1}k_t$ and the financial intermediary gets nothing. The distressed price q will be endogeneized in Section 3.

In addition, financial intermediaries are subject to moral hazard. When funding investment projects and issuing liabilities in period t, a financial intermediary can decide to set up a diversion scheme. Then, if she is solvent in period $t+1$, she manages to divert all her assets and does not pay anything to final investors. Insolvent financial intermediaries, which are subject to liquidation, are not able to carry out their diversion scheme. Setting up a diversion scheme entails a utility cost h per unit of investment. This cost is paid in period $t+1$, regardless of whether the intermediary is solvent or not, i.e. regardless of the success or failure of the scheme.

Financial intermediaries choose how much to invest (k_t), what type of liability to issue, and whether to set up a diversion scheme to maximize discounted expected profits $\beta\,\mathrm{E}_t\,\pi_{t+1}$.

Final investors

Final investors have deep pockets and offer a perfectly elastic supply of funds at the expected rate of return β^{-1}.

Parametric restrictions

We make the following assumptions.

Assumption 1. *Investment projects of firms have a strictly positive net present value (NPV):*

$$(1-u)\bar{A} + u\underline{A} > \beta^{-1}.$$

Assumption 2. *The diversion cost satisfies*

$$(1-u)\underline{A} < h < \min(\beta^{-1}, (1-u)\bar{A}).$$

Assumption 3. *The price of assets during a crisis, q_t, does not exceed the following threshold:*

$$q_t < \frac{1-\beta h}{\beta u}. \tag{1}$$

From Assumption 2, the numerator $1-\beta h$ on the right-hand side of (1) is strictly positive. The upper bound on the asset price q_t set by Assumption 3 insures that leverage always remains finite.

2.2 Leverage and asset issuance by financial intermediaries

We now solve the financial intermediary optimization problem. The financial intermediary chooses the size of its balance-sheet k_t, the amount and type of liability issued, and whether or not to set up a diversion scheme to optimize expected profits next period.

There are three different cases: (i) a financial intermediary issuing contingent liabilities and not setting a diversion scheme, (ii) a financial intermediary issuing uncontingent liabilities and not setting a diversion scheme, and (iii) a financial intermediary issuing uncontingent liabilities and setting a diversion scheme. The case of contingent liabilities with a diversion scheme can be discarded: as the financial intermediary would always succeed in diverting the assets, final investors would get nothing. Therefore, no final investor would agree to finance such an intermediary.

Contingent Liabilities Consider first the case of a financial intermediary issuing contingent liabilities and not setting a diversion scheme, for which we use the superscript c. The corresponding loan-to-value ratio is m_t^c. Profits received in period $t+1$ are equal to $\pi_{t+1}^c = A_{t+1}k_t^c - A_{t+1}P_{t+1}^c$. Since final investors require a gross rate of return β^{-1}, the promised repayment must satisfy the following breakeven condition:

$$\mathrm{E}[A]P_{t+1} = \beta^{-1}m_t^c k_t^c \tag{2}$$

No diversion by the financial intermediary implies that expected profits net of debt repayment must exceed the expected return of investment minus the diversion cost.

$$\mathrm{E}[A](k_t^c - P_{t+1}^c) \geq \mathrm{E}[A]k_t^c - hk_t^c. \tag{3}$$

This condition simplifies to $\mathrm{E}[A]P^c_{t+1} \leq hk^c_t$. The intermediary cannot promise an expected repayment larger than the diversion cost. Together with the breakeven condition, this sets an upper bound on the loan-to-value ratio: $m^c_t \leq \beta h$. This upper bound is strictly lower than 1 from the second inequality of Assumption 2. Given Assumption 1 of a strictly positive NPV of investment projects, financial intermediaries choose the maximum loan-to-value ratio

$$m^c_t = \beta h.$$

The corresponding leverage ratio (the ratio of assets to internal funds) is $1/(1-m^c)$, which gives

$$k^c_t = \frac{1}{1-m^c_t} w_t = \frac{1}{1-\beta h} w_t. \tag{4}$$

The discounted profit of the financial intermediary is then

$$\beta \, \mathrm{E}_t \, \pi^c_{t+1} = \beta \frac{\mathrm{E}[A]-h}{1-\beta h} w_t. \tag{5}$$

Note that the NPV of the financial intermediary is equal to

$$-w_t + \beta \, \mathrm{E}_t \, \pi^c_{t+1} = \frac{\beta \, \mathrm{E}[A]-1}{1-\beta h} \, w_t. \tag{6}$$

The fraction is the leveraged excess return on investment projects, and is strictly positive by Assumption 1. Taking a leveraged position in investment projects with contingent liabilities always dominates lending funds at the riskless rate of return β^{-1} (or consuming them).

Non-Contigent Liabilities Consider now the case of a financial intermediary issuing non-contingent liabilities and not setting a diversion scheme, for which we use the superscript u. The corresponding loan-to-value ratio is m^u_t. Profit in period $t+1$ is then $\pi^u_{t+1} = \max(0, A_{t+1}k^u_t - P^u_{t+1})$. Suppose the financial intermediary is solvent in the good state of nature but insolvent in crisis times. The breakeven condition for the final investor is now

$$\big((1-u)P^u_{t+1} + uq_{t+1}k^u_t\big) = \beta^{-1} m^u_t k^u_t. \tag{7}$$

Final investors only receive the promised payment P^u_{t+1} in the good state of nature, when the intermediary is solvent. Otherwise, they get the value

of assets liquidated at the distressed assets price q_{t+1}. The no-diversion condition for the financial intermediary is now:

$$(1-u)(\bar{A}k_t - P^u_{t+1}) \geq (1-u)\bar{A}k^u_t - hk^u_t. \tag{8}$$

On the left hand-side, the intermediary only makes profits in the good state of nature, with probability $1-u$. On the right hand-side, the diversion scheme is decided in advance, and paid for whatever the outcome, but it is only successful when the intermediary is solvent. The condition simplifies to $(1-u)P^u_{t+1} \leq hk^u_t$. As before, the intermediary cannot promise an expected repayment larger than the diversion cost, but the repayment only takes place in the good state of nature. Given the breakeven condition, this leads to a larger upper bound on the loan-to-value ratio: $m^u_t \leq \beta(h + uq_{t+1})$. Intuitively, since assets cannot be diverted during a liquidation process, the same diversion cost can be used to back up a larger promised repayment in the good state of nature.

If q is not too large as assumed in (1), this maximum loan-to-value ratio is strictly lower than 1. If this investment strategy is profitable, financial intermediaries choose the highest possible loan-to-value ratio and level of investment:

$$m^u_t = \beta(h + uq_{t+1}), \tag{9}$$

$$k^u_t = \frac{1}{1-m^u_t}w_t = \frac{1}{1-\beta(h+uq_{t+1})}w_t. \tag{10}$$

The discounted expected profit is

$$\beta\,\mathrm{E}_t\,\pi^u_{t+1} = \beta\frac{(1-u)\bar{A}-h}{1-\beta(h+uq_{t+1})}w_t, \tag{11}$$

and the corresponding NPV is equal to

$$-w_t + \beta\,\mathrm{E}_t\,\pi^u_{t+1} = \frac{\beta[(1-u)\bar{A}+uq_{t+1}]-1}{1-\beta(h+uq_{t+1})}\,w_t. \tag{12}$$

The loan-to-value ratio m^u, leverage $1/(1-m^u)$, investment k^u and the NPV are all increasing in the price of distressed assets q. The upper limit to q of Inequality (1) implies that leverage and the demand for funds to final investors remain finite. This guarantees that (deep pocket) final investors are able to fund financial intermediaries at the expected rate of return β^{-1}.

The promised rate of return is equal to

$$\frac{P^u_{t+1}}{m^u_t k^u_t} = \frac{\beta^{-1}}{1-u+u\frac{(1-u)q_{t+1}}{h}}. \tag{13}$$

Since the promised repayment is equal to $hk^u_{t+1}/(1-u)$, Assumption 2 makes sure that the intermediary is indeed solvent (insolvent) in the good (bad) state of nature.

The Choice of the Debt Structure How does expected profit compare between issuing contingent liabilities, Equation (5), and issuing non-contingent liabilities, Equation (11)? On the one hand, the numerator of expected profit is lower with uncontingent liabilities, as the financial intermediary only makes profits in the good state of nature. On the other hand, leverage is higher (the denominator is lower) with non-contingent liabilities. Since leverage increases with the price of distressed assets q, it is optimal for the financial intermediary to issue uncontingent liabilities when this price is large enough, that is, if $q \geq \underline{q}$ with

$$\underline{q} = \underline{A}\frac{\beta^{-1} - h}{\mathrm{E}[A] - h}. \tag{14}$$

Otherwise, when $q < \underline{q}$, financial intermediaries prefer to issue contingent liabilities with a lower leverage.

Finally, consider the case of an intermediary with non-contingent liabilities and a diversion scheme, for which we use the superscript d. The corresponding loan-to-value ratio is m^d. This case is only possible if the intermediary is solvent in the good state of nature and insolvent in the bad state of nature. If she were always solvent, diversion would always be successful and final investors would get nothing. If she were always insolvent, she would get nothing herself. The breakeven constraint of final investors is now:

$$\beta u q_{t+1} k^d_t = m^d_t k^d_t, \tag{15}$$

which sets the loan-to-value ratio to $m^d_t = \beta u q_{t+1}$. Diversion is optimal if

$$(1-u)(\bar{A}k^d_t - P^d_{t+1}) \leq (1-u)\bar{A}k^d_t - hk^d_t, \tag{16}$$

which implies a large enough promised repayment to $P^d_{t+1} \geq hk^d_{t+1}/(1-u)$. Expected profits of the financial intermediary become

$$\beta\,\mathrm{E}_t\,\pi^d_{t+1} = \beta\frac{(1-u)\bar{A} - h}{1 - \beta u q_{t+1}} w_t. \tag{17}$$

Comparing Equations (11) and (17), it is clear that expected profits without diversion are strictly larger than with diversion. This comes from the fact that the loan-to-value ratio is lower with diversion.

The following Proposition summarizes this discussion.

Proposition 1 (Distressed Price and the Optimal Debt Structure)**.** *Financial intermediaries never set up a diversion scheme.*

With a low distressed price $q_{t+1} < \underline{q}$, they choose a safe financial structure: they issue contingent liabilities, are always solvent, have a low loan-to-value ratio $m^c = \beta h$, a corresponding low leverage $1/(1-m^c)$, and their expected profits are equal to $\beta \, \mathrm{E}_t \, \pi^c_{t+1} = \beta \frac{\mathrm{E}[A]-h}{1-m^c} w_t$.

With a high distressed price $q_{t+1} > \underline{q}$, financial intermediaries choose a risky financial structure: they issue uncontingent liabilities, are insolvent in the bad state of nature, have a high loan-to-value ratio $m^u(q_{t+1}) = \beta(h + uq_{t+1})$, a corresponding high leverage $1/(1-m^u(q_{t+1}))$, and their expected profits are equal to $\beta \, \mathrm{E}_t \, \pi^u_{t+1} = \beta \frac{(1-u)\bar{A}-h}{1-m^u(q_{t+1})} w_t$.

When the distressed price is $q_{t+1} = \underline{q}$, financial intermediaries are indifferent between the safe and the risky financial structure.

Proposition 1 relates the financing structure to the distressed price q_t. In the next section, the Central Bank intervention during crises will make the distressed price, and therefore the financing structure, leverage and output, endogenous.

3 Central Bank interventions and aggregate leverage

This Section embeds the simple microeconomic framework of Section 2.1 into a macroeconomic model and studies how Central Bank interventions affect the portfolio choice of financial intermediaries. By intervening on the market for disressed assets during financial crises, the Central Bank can support the asset price q_t. As the previous section showed, this price plays a key role in the choice of financial structure by financial intermediaries. Ex ante, the expectation of future interventions by the Central Bank during financial crises translates into a choice of risky financial intermediation, with high leverage and large uncontingent liabilities.

3.1 The setup

Technology

There is a single all-purpose good produced by firms as described in Section 2.1.

Households

There is a representative household made of a measure-one continuum of members who share the same consumption C_t^H, as in Gertler and Karadi (2011). Each member receives an exogenous endowment w_t in period t. For the sake of simplicity, we assume a constant endowment and drop the time subscript. At the beginning of period t, each member has a probability $\eta < 1$ of becoming a financial intermediary until the beginning of the following period. Then, she returns to the household with her profits π_{t+1}. Therefore, there is a measure-η continuum of financial intermediaries. The rest of the household members are final investors.

The representative household is risk-neutral with discount factor β. Utility is given by $U_t = \sum_{s\geq 0} \mathrm{E}_t\, \beta^s C_{t+s}^H$. The budget constraint of the representative household in period t is:

$$(1-\eta)w + \Pi_t + (1+\rho_t)D_{t-1} + (1+r_{t-1})S_{t-1} + r_{t-1}B_{t-1} = C_t^H + D_t + S_t + T_t. \tag{18}$$

The household receives the endowments of final investors $(1-\eta)w$, the aggregate profits of exiting financial intermediaries Π_t, the (risky) realized return ρ_t on their loans to financial intermediaries D_{t-1}, and the (safe) return r_{t-1} on safe asset holdings S_{t-1}. In addition, the household also receives interests r_{t-1} on the net assets held by the Central Bank, B_{t-1}, as will be explained below. The household uses these funds to consume, lend to financial intermediaries D_t, buy safe assets S_t, and pay taxes T_t. Note that S_t can be negative as we do not impose any credit constraints on the representative household. Taxes T_t are directly transferred to the Central Bank.

Financial intermediaries borrow and invest in firm equity as modeled in Section 2.1. Denote x_t the fraction of financial intermediaries choosing a risky financial structure in period t. We can aggregate across financial intermediaries to get the following expressions:

$$\begin{aligned} L_t &= (1-x_t)\eta m^c k_t^c + x_t \eta m^u(q_{t+1}) k_t^u \\ &= (1-x_t)\frac{1}{1-m^c}\eta w + x_t \frac{1}{1-m^u(q_{t+1})}\eta w, \end{aligned} \tag{19}$$

$$\begin{aligned} Y_t &= (1-x_{t-1})\eta A_t k_{t-1}^c + x_{t-1}\eta A_t k_{t-1}^u \\ &= (1-x_{t-1})\frac{1}{1-m^c} A_t \eta w + x_{t-1}\frac{1}{1-m^u(q_t)} A_t \eta w, \end{aligned} \tag{20}$$

$$\begin{aligned} \beta\, \mathrm{E}\, \Pi_{t+1} &= (1-x_t)\eta\beta\, \mathrm{E}\, \pi_{t+1}^c + x_t \eta\beta\, \mathrm{E}\, \pi_{t+1}^u \\ &= (1-x_t)\beta\frac{\mathrm{E}[A]-h}{1-m^c} w + x_t \beta \frac{(1-u)\bar{A}-h}{1-m^u(q_{t+1})} w. \end{aligned} \tag{21}$$

The variable L_t represents the aggregate liabilities of financial intermediaries, an indicator of the size of the financial sector; Y_t is aggregate output; $\beta\Pi_{t+1}$ is aggregate discounted expected profit. Note that aggregate output depends on both the realization of productivity A, an exogenous shock, and the leverage ratio $1/(1-m)$, which is endogenously chosen by financial intermediaries.

The Central Bank and the market for distressed assets

The Central Bank has the possibility to intervene during crisis by buying distressed assets. This allows the Central Bank to set a floor on asset prices and mitigate losses of final investors.

Distressed Asset Purchases More precisely, there is a market for distressed assets which determines the asset price q_t. This market opens at the beginning of the period, after the state of nature is revealed. Financial intermediaries who turn out to be insolvent are forced into liquidation and sell their assets on the distressed market. Both the Central Bank and private final investors can intervene on this market. At the end of the period, buyers of distressed assets are able to recover some fraction of their value. We assume that the Central Bank is able to recover a fraction v of the value of the assets, while final investors can only recoup a fraction $v^* < v$.

When the Central Bank does not intervene, the distressed price is set by a zero profit condition for final investors: $v^*\underline{A} - q_t = 0$. Denoting q^* the *fire-sale price* absent Central Bank interventions, we have:

$$q^* = v^*\underline{A}. \tag{22}$$

By intervening on the market for distressed assets, the Central Bank can price out fire-sale investors by setting a price in the range $[q^*, (1-\beta h)/(\beta u))$. The upper bound restates Inequality (1). It is a natural bound for Central Bank interventions since infinite leverage would imply an infinite cost of intervention (see below), which would violate any intertemporal budget constraint. More precisely, we assume an intervention policy of the following form.

Definition 1 (Central Bank policy)**.** *At the beginning of each period t, the Central Bank announces its intervention policy $q_{t+1} \in [q^*, (1-\beta h)/(\beta u))$ in the next period. Then, in period $t+1$, the Central Bank buys distressed assets to target the price q_{t+1} on the distressed assets market if and only if more than half of the financial intermediaries are insolvent.*

The threshold of 50% of defaults is arbitrary but captures the fact that Central Bank interventions take place during systemic events.

Direct Fiscal Cost of Interventions When the Central Bank intervenes, it pays q and recoups $v\underline{A}$ per unit of asset. For a high enough price q, the Central Bank then incurs losses $q - v\underline{A} > 0$ per unit of asset. Crisis interventions are then potentially costly, with a total cost $x_{t-1}C(q_t)$ where $C(q_t)$ is given by:

$$C(q_t) = \eta k_{t-1}^u (q_t - v\underline{A}) = \eta w \frac{q_t - v\underline{A}}{1 - m^u(q_t)}. \tag{23}$$

In practice, Central Banks are likely to benefit from a fiscal backstop in such cases. However, the ability of the Government to provide this fiscal backstop may not be infinite. We assume that the amount of fiscal resources available for crisis interventions is limited but increases over time as the Government gradually accumulates fiscal surpluses and builds a war-chest for future crises.

The Central Bank budget constraint More specifically, denote B_t the amount of fiscal resources available to the Central Bank at the end of period t, with

$$B_t \geq 0. \tag{24}$$

For expositional convenience, we will refer to B_t as the net assets of the Central Bank, even though in practice these resources might be in the balance sheet of the Government. The budget constraint of the Central Bank is given by

$$B_t = (1 + r_{t-1})B_{t-1} - r_{t-1}B_{t-1} + T_t - \delta_t x_{t-1} C(q_t) \tag{25}$$

where $\delta_t = 1$ in the bad state of nature and 0 in the good state of nature. The Central Bank receives interest payments on its assets, which it rebates to the household, and a transfer T_t from the household. In crisis time, the Central Bank also pays the cost of intervening on the distressed assets market. This cost can be both negative or positive.

The sequence of transfers T_t are supposed to be set in such a way that the Central Bank always satisfies the usual transversality condition:

$$\lim_{T \to \infty} \beta^T B_T = 0. \tag{26}$$

For the sake of simplicity, we consider the following sequence:

$$T_t = \begin{cases} T & \text{if } B_t < \bar{B}, \\ 0 & \text{if } B_t \geq \bar{B}, \end{cases} \tag{27}$$

for some arbitrary upper bound on net assets $\bar{B}$. In the following, we consider the case when $B_t \ll \bar{B}$, i.e. we take the limit $\bar{B} \to +\infty$. In other words, the level of transfers received by the Central Bank, i.e., the speed at which it can restore its balance sheet, is exogenously set at T.

Asset market clearing

To close the model, borrowing by financial intermediaries has to be equal to lending by the representative household:

$$L_t = D_t, \tag{28}$$

and safe assets are in zero net supply:

$$S_t + B_t = 0. \tag{29}$$

Risk-neutrality implies that the household provide an infinitely elastic supply of funds at an expected gross rate of return β^{-1}, provided the endowment w is large enough, which we assume throughout. This sets the safe interest rate and the expected risky rate of return to

$$r_t = \mathrm{E}_t\, \rho_{t+1} = \beta^{-1} - 1. \tag{30}$$

The realized return ρ_t on loans to financial intermediaries depends on the state of nature and the structure of their balance-sheet as describe in Section 2.1.

Parametric restrictions

In addition to Assumptions 1 and 2, we make two other parametric assumptions.

Assumption 4. *The share of recovery by private agents is low:* $v^* < \dfrac{\beta^{-1} - h}{\mathrm{E}\, A - h}$.

Assumption 5. *The share of recovery by the Central Bank satisfies:* $\dfrac{\beta^{-1} - (1-u)\bar{A}}{u\underline{A}} < v < \dfrac{\beta^{-1} - h}{u\underline{A}}$.

3.2 Discussion of the Set-Up

Central Bank interventions In this paper, we assume that policy interventions take the form of asset purchases by the Central Bank. These assets are initially held by financial institutions. During a crisis, they are liquidated by financial institutions to meet their liabilities. Asset purchase sustain asset prices and supports the balance-sheet of financial intermediaries. Such interventions have been observed in recent years in Ireland (2008), United States (2008), United Kindgdom (1998), Switerland (2008) etc. The dataset of Laeven and Valencia (2012) reports such intervention in more than half of the systemic banking crises. While we model the policy intervention as the purchase of distressed assets by the Central Bank, the model could be easily reformulated to encompass other forms of banks' rescue policies such as direct equity injection by the Treasury, or government guarantees on banks' liabilities. Such policies are *de facto* equivalent to the purchase of distressed assets at a price higher than the firesale price.

Distressed asset prices In the model, distressed assets during crises are either bought by the central banks or sold to private investors through a firesale. We assume that the price targeted by the Central Bank is above the firesale price ($q^* < q$). Otherwise the Central Bank intervention would have no effects.

We model the low price in the firesale market as reflecting low asset valuation by private investors (i.e. a low share of recovery v^*) but it would be equivalent to have this low price determined by *cash-in-the-market pricing* as in Allen and Gale (1998). The Assumption 4 of a low asset valuation v^* implies that absent a Central Bank intervention, the liquidation price of assets is too low to sustain a high leverage equilibrium. This assumption is needed to create moral hazard: expectations of policy intervention during crises is what induces financial intermediaries to take on risk that will make crises possible.

We also assume that $v^* < v$, which implies that even when the Central Bank makes no losses, it is able to sustain asset prices at a higher level than the firesale price. This allows us to consider the case of asset purchases by the Central Bank where it ultimately ends up making profits. We refer to these as "costless interventions" and study them in Section 4.2. If v^* were larger than v, the Central Bank would necessarily have to make losses in order to support asset prices. The case of "costly interventions" is treated in Section 4.3. Overall, the assumption $v^* < v$ allows for more generality.

Appendix A.1 presents a model to microfound it as the result of easier liquidity provision by the Central Bank.[2]

Finally, the upper bound on v in Assumption 5 is a sufficient and necessary condition for the intervention $C(q)$ to be strictly increasing in q: larger interventions are more costly. The lower bound means that investment projects net of recovery costs by the Central Bank have a strictly positive net present value, that is, $\beta\big((1-u)\bar{A} + uv\underline{A}\big) - 1 > 0$. While this assumption seems to favor the largest possible intervention by the Central Bank, we will see that this is not the case once fiscal space is taken into account.

The Central Bank resource constraint The modeling of the Central Bank resource constraint is building up on Hall and Reis (2015). The main result of Hall and Reis (2015) is that a Central Bank, with a dual mandate to control inflation and to intervene by buying distressed assets in times of crisis, would need to receive periodic backstop payments from the US Treasury order to stabilize its level of interest-bearing reserves. Our model includes such a backstop policy. The initial position of the Central Bank, the backstop policy and past interventions all contribute to determine the resources available for future interventions.

3.3 Equilibrium

Given initial conditions x_{-1}, B_{-1}, and a (contingent) sequence of Central Bank policy $\{B_t, q_t\}_{t\geq 0}$ satisfying the constraint on fiscal resources (24) and the budget constraints (25), an equilibrium is a (stochastic) sequence $\{x_t\}_{t\geq 0}$ such that financial intermediaries behave optimally as described in Proposition 1.

We can now fully characterize the equilibria of the model. Under Assumption 4, the fire-sale price set by private investors $q^* = v^*\underline{A}$ is too low to sustain a risky financial sector and induce financial intermediaries to issue uncontingent liabilities. Therefore, they will only choose a risky financial structure if they expect a Central Bank intervention. The within-period equilibrium is described by the following proposition.

Proposition 2. *When the Central Bank announces a distressed assets price $q_{t+1} \in [q^*, \underline{q})$, there is a unique low-leverage within-period equilibrium with*

[2] Intuitively, intervening on the fire-sale market requires final investors to keep liquid assets in their portfolio, which is costly, whereas the Central Bank can create liquidity at will. This translates into lower net profits for final investors, compared to the Central Bank.

$x_t = 0$, that is where all financial intermediaries choose a safe financial structure with contingent liabilities and a low loan-to-value ratio m^c, and without systemic defaults in period $t+1$.

When the Central Bank announces a distressed assets price $q_{t+1} \in (\underline{q}, (1-\beta h)/(\beta u))$, there are two within-period equilibria: (i) a low-leverage equilibrium with $x_t = 0$ identical to the previous one, and (ii) a high-leverage within-period equilibrium with $x_t = 1$ where all financial intermediaries choose a risky financial structure with uncontingent liabilities and a high loan-to-value ratio $m^u(q_{t+1})$, and where systemic defaults take place in period $t+1$ in the bad state of nature.

When the Central Bank announces a distressed assets price $q_{t+1} = \underline{q}$, there is a continuum of high-leverage within-period equilibria where a fraction $x \in \{0\} \cup (0.5, 1]$ of financial intermediaries choose a risky financial structure with uncontingent liabilities and a high loan-to-value ratio $m^u(\underline{q})$, and with systemic defaults in period $t+1$ in the bad state of nature. The remaining financial intermediaries choose a safe financial structure with a loan-to-value ratio m^c.

As the Proposition shows, financial intermediaries face a coordination problem. A promise by the Central Bank to sustain asset prices at a high enough level $q \geq \underline{q}$ acts as a put option which relaxes the moral hazard problem and allows financial intermediaries to leverage up and take up more risk. However, this put is only valued when enough financial intermediaries default, i.e. during a systemic event. This requires that enough financial intermediaries engage in risk taking ex ante.

In the following, we assume a simple coordination rule with a financially safe absorbing state.

Assumption 6 (Equilibrium selection)**.** *In period t, if $q_{t+1} \geq \underline{q}$, all financial intermediaries choose a risky financial structure with loan-to-value ratio $m^u(q_{t+1})$ if and only if they had already done so in the previous period. Otherwise, they all choose a safe financial structure with loan-to-value ratio m^c.*

This selection rule implies that the low-leverage equilibrium is an absorbing state. Denote ζ_t a dummy variable equal to 1 when the economy is in a high-leverage equilibrium and to 0 when it is in the absorbing low-leverage equilibrium. Assumption 6 implies that the fraction of financial intermediaries choosing a risky financial structure is given by $x_t = \zeta_t$.

Suppose $\zeta_t = 1$ initially. The Central Bank has two possibilities. If it announces $q \geq \underline{q}$ for next period, the economy stays in a high-leverage

equilibrium. If it announces $q < \underline{q}$, the economy suddenly deleverages and switches to a low-leverage equilibrium with safe financial intermediation where it stays forever. The state variable ζ_t then evolves according to

$$\zeta_t = \mathbb{1}_{q_{t+1} \geq \underline{q}} \, \zeta_{t-1} \tag{31}$$

where $\mathbb{1}$ is the indicator function, for some initial value $\zeta_0 = 1$.

The rationale for this assumption of an absorbing state is that switching from a high-leverage to a low-leverage equilibrium implies a complete change in the structure of financial intermediation. This makes it unlikely that the economy can quickly reswitch to a high-leverage equilibrium. In general, we could have a markov process for equilibrium selection, whereby the economy would re-switch to the high-leverage equilibrium with some probability $p < 1$. Our assumption of an absorbing state consists in taking the limit $p \to 0$. In practice, we expect this to be a good approximation since the switch to a low-leverage equilibrium might go together with substantial institutional change. A large crisis that depletes central bank's resources is likely to be followed by a backlash from society against risk-taking in the financial sector, and the extent of financial losses it implies for the intervening Central Bank. Historically, this has sometimes been reflected in the enactment of laws and regulations that make it more costly to take on credit risk, or impose outright bans are on certain activities of financial intermediaries. In some other cases, it could lead to a change in the charter of the Central Bank that either curb its possibility of intervention in distressed asset markets or reduce the fiscal backstop.

As long as the economy is in the high-leverage equilibrium with risky financial intermediation, policy interventions during systemic events are potentially costly. The cost associated to stabilizing the price of distressed assets at a level q_t is equal to $C(q_t)$ and given by Equation (23). Under Assumption 5, it is strictly increasing in q_t and becomes positive for a high enough price q_t. However, intervention can be socially desirable since the social return of an investment project, net of recovery cost by the Central Bank, is strictly positive as implied by the first inequality of Assumption 5. The optimal policy then depends on the ability by the Central Bank to bear the possible losses associated to its interventions. This is the topic of the next section.

4 Optimal Central Bank policy

This section characterizes the optimal policy of the Central Bank when its ability to bear losses is constrained.

4.1 The policy problem

The optimal policy consists in choosing a (contingent) sequence of distressed assets prices $\{q_{t+s}\}_{s\geq 0} \in [q^*, (1-\beta h)/(\beta u))$ to maximize household utility U_t under the budget constraint (25), the constraint on fiscal resources (24), and the equilibrium selection rule given in Assumption 6.

Using the budget constraints of the household and the Central Bank (18) and (25), the market-clearing conditions (28) and (29), and the equilibrium selection rule, we get

$$C_t^H = (1+\rho_t)L_{t-1} - L_t + (1-\eta)w + \Pi_t - \delta_t \zeta_{t-1} C(q_t)$$

where L_t is the aggregate liabilities of financial intermediaries defined in Equation (19). We can discard the first term $(1+\rho_t)L_{t-1}$ which depends on past choices and the realization of the shock. Then, given (30), maximizing U_t amounts to

$$\max \sum_{s=1}^{\infty} \mathrm{E}_t\, \beta^s [(1-\eta)w + \Pi_{t+s} - \delta_{t+s}\zeta_{t+s-1} C(q_{t+s})].$$

Denote $V(B_t, \zeta_{t-1})$ the value function associated to this policy problem. It is useful to write $V(B_t, \zeta_{t-1}) = J(B_t, \zeta_{t-1}) + W$ where W is the value of the absorbing state where all financial intermediaries choose a safe financial structure with contingent liabilities. We have

$$\begin{aligned} W &= \sum_{s=1}^{\infty} \mathrm{E}\, \beta^s [(1-\eta)w + \eta \pi_t^c] \\ &= \frac{\beta}{1-\beta} \big[(1-\eta)w + \eta\, \mathrm{E}\, \pi^c\big]. \end{aligned}$$

Then, $J(B_t, \zeta_{t-1})$ is given by

$$J(B_t, \zeta_{t-1}) = \max_{q \in [q^*, (1-\beta h)/(\beta u))} \zeta_t \beta \big[\eta \, \mathrm{E}\big(\pi^u(q) - \pi^c\big) - uC(q) + \mathrm{E}_t \, J(B_{t+1}, \zeta_t)\big] \tag{32}$$

$$\text{s.t.} \quad B_{t+1} \geq 0 \tag{33}$$

$$B_{t+1} = \begin{cases} B_t + T - \delta_{t+1} C(q) & \text{if } B_{t+1} < \bar{B}, \\ B_t - \delta_{t+1} C(q) & \text{if } B_{t+1} \geq \bar{B}, \end{cases} \tag{34}$$

$$\zeta_t = \mathbb{1}_{q \geq \underline{q}} \zeta_{t-1}. \tag{35}$$

Equation (33) just restates Equation (24) and reflects the fact the fiscal resources are limited. Equation (34) comes from the Central Bank budget constraint (25), the definition of taxes (27), and the fact that $x_t = 1$ in the high-leverage equilibrium. Finally, Equation (35) comes from (31) and describes the equilibrium selection rule.

The period-utility that enters the value function J has two terms. The first term is the increase in profits of financial intermediaries brought by moving to a risky financial structure with high leverage. As explained is Section 1, this term is positive when $q \geq \underline{q}$. The second term is the expected cost of policy intervention bore by the rest of the household.

The choice of the policy instrument q directly affects the value function through these two terms. It also indirectly affects the value function through its impact on the intervention cost $C(q)$, which depletes available resources in case of crisis. By sustaining asset prices at a higher level q during a crisis, the Central Bank also increases the cost of intervention $C(q)$, which reduces next period resources B_{t+1}. This affects utility through the continuation value $\mathrm{E}_t \, J(B_{t+1}, \zeta_t)$. Finally, should the Central Bank not intervene (either because it chooses not to, or because its resources are too low), it would make the economy switch to the low-leverage equilibrium forever. Formally, we would then have $\zeta_{t+s} = 0$ for all $s \geq 0$, and J would fall to 0.

A change of variable Because period-utility and the cost of intervention are non-linear in the policy instrument q, the optimization problem is hard to solve. However, it can be simplified by expressing it in terms of the Central Bank's expected intervention cost $uC(q)$ rather than the asset price q. The change of variable $q \to uC(q)$ maps $[q^*, (1-\beta h)/(\beta u))$ into $[uC^*, +\infty)$, where $C^* = C(q^*)$. Denote $\underline{C} = C(\underline{q})$ is the smallest possible cost of intervention associated to risky financial intermediation: Assumption 4 is equivalent to $C^* < \underline{C}$. It turns out that the excess expected profit

from high-leverage financial intermediation is linear in the expected intervention cost. Indeed, we have: $\eta\,\mathrm{E}\big(\pi^u(q) - \pi^c\big) = (1+\kappa)[uC(q) - u\underline{C}]$, where

$$\kappa = \frac{\beta[(1-u)\bar{A} + vu\underline{A}] - 1}{1 - \beta(h + uv\underline{A})} \tag{36}$$

This transforms the optimization problem into a simpler linear problem.

More precisely, period-utility becomes an affine function of the expected intervention cost uC:

$$\kappa uC - (1+\kappa)u\underline{C}.$$

Consider first the linear term, κuC. By committing one additional unit to the expected cost of intervention, the Central Bank increases period-utility by κ. This coefficient κ is the marginal leveraged excess social return of risky financial intermediation associated to 1 unit of expected intervention cost. This can be seen more clearly by looking at the definition in Equation (36). The expression for κ has the same form as the NPV of safe projects (6) and risky projects (12). Its numerator is the NPV of 1 unit of investment, taking into account the recovery cost by the Central Bank, that is, the excess social return. This numerator is multiplied by a leverage ratio equal to $1/(1 - m^u(v\underline{A}))$, where distressed assets are valued at their social return $q = v\underline{A}$. Overall, κ is equal to the NPV of one unit of risky financial intermediation, as given in Equation (12), when $q = v\underline{A}$. The second term of the period utility, the intercept $(1+\kappa)u\underline{C}$, has a higher coefficient ($1+\kappa$ instead of κ), which comes from the fact the financial intermediaries do not take into account the intervention cost in their choice of financial structure. When $C = \underline{C}$, their profits are equal to what they would be in the safe absorbing state, by definition of $\underline{q}$ and $\underline{C}$. But the Central Bank still has to pay the expected cost $u\underline{C}$, which shows up in period-utility.

Continuous time To solve this policy problem, we move to the limit of continuous time. Let dN be a Poisson process with arrival rate u and size 1, corresponding to the occurrence of the bad state of nature. The notation t refers to time t before the poissonian jump, while t^+ refers to time t after the poissonian jump. The constraint on available fiscal resources (33) becomes

$$B_t - \mathbb{1}_{C \geq \underline{C}} C \geq 0. \tag{37}$$

This follows from the fact the B_t must be positive at every instant, including immediately after a poissonian jump. The budget constraint (34) becomes

$$dB_t = \begin{cases} Tdt - \mathbb{1}_{C \geq \underline{C}} C dN & \text{if } B_t < \bar{B}, \\ -\mathbb{1}_{C \geq \underline{C}} C dN & \text{if } B_t \geq \bar{B}, \end{cases} \tag{38}$$

Finally, the equilibrium selection rule (35) becomes

$$d\zeta_t = -(1 - \mathbb{1}_{C \geq \underline{C}})\zeta_t. \tag{39}$$

In the continuous time limit, the Bellman equation (32) can be rewritten as a Hamilton-Jacobi-Bellman (HJB) equation:

$$rJ(B_t, \zeta_t) = \max_{C \in [C^*, +\infty)} \zeta_t \big[u\big(\kappa C - (1+\kappa)\underline{C}\big) + TJ_1'(B_t, \zeta_t) + u\big(J(B_t - C, \zeta_{t^+}) - J(B_t, \zeta_t)\big)\big] \tag{40}$$

where maximization is subject to (37), (38), and 39. The right-hand side of the HJB has three terms. The first term is the period-utility associated to the intervention cost C. The second term is the gain from increasing assets B_t through accumulating taxes in the absence of crisis. The last term is the expected change in value when a crisis takes place. The maximization over the intervention cost C is subject to two constraints. First, one must have $C \geq C^*$, corresponding to $q \geq q^*$ where q^* is the price of distressed assets that would obtain without Central Bank intervention. The Central Bank cannot target a price lower than the market price. Second, from (37), one has $C \leq B_t$: the intervention cost cannot exceed available resources.

The solution of the HBJ equation is subject to a boundary condition at the upper bound $B_t = \bar{B}$, when taxes fall to zero as described by (27):

$$rJ(\bar{B}, \zeta_t) = \max_{C \in [C^*, +\infty)} \zeta_t \big[u\big(\kappa C - (1+\kappa)\underline{C}\big) + u\big(J(\bar{B} - C, \zeta_{t^+}) - J(\bar{B}, \zeta_t)\big)\big]. \tag{41}$$

Reduced form HBJ equation Note that in the safe absorbing state we have $J(B_t, 0) = 0$. Therefore, the HBJ equation only needs to be solved when $\zeta_t = 1$. Using this, we can further simplify the HBJ equation to:[3]

$$(r+u)J(B_t, \zeta_t) = \max_{C \in [C^*, +\infty)} \zeta_t \big[u\big(\kappa C - (1+\kappa)\underline{C}\big) + TJ_1'(B_t, \zeta_t) + uJ(B_t - C, \zeta_{t^+})\big]$$

It will be useful to work with a slightly modified value function $\tilde{J}(B_t, \zeta_t) = (u + r)J(B_t, \zeta_t)$. We get:

$$\tilde{J}(B_t, \zeta_t) = \max_{C \in [C^*, +\infty)} \zeta_t \big[u\big(\kappa C - (1+\kappa)\underline{C}\big) + \tilde{T}\tilde{J}_1'(B_t, \zeta_t) + \gamma \tilde{J}(B_t - C, \zeta_{t^+})\big] \tag{42}$$

[3] This equation is the same as (40) when $\zeta_t = 1$. When $\zeta_t = 0$, both equations simplify to $J(B_t, 0) = 0$.

with boundary condition

$$\tilde{J}(\bar{B}_t, \zeta_t) = \max_{C \in [C^*, +\infty)} \zeta_t \big[u\big(\kappa C - (1+\kappa)\underline{C}\big) + \gamma \tilde{J}(\bar{B}_t - C, \zeta_{t^+}) \big] \tag{43}$$

where $\tilde{T} = T/(u+r)$ is the net present value of expected taxes received until the next crisis, and $\gamma = u/(u+r)$ is the expected discount factor at the time of the next crisis. Intuitively, this modified value function changes the time unit to the expected time interval between two crises, taking discounting into account.

An important factor that determines the shape of the optimal policy is whether Central Bank interventions in the high-leverage equilibrium are necessarily costly or not, that is, whether $\underline{C} > 0$. We briefly cover the case where $\underline{C} \leq 0$, which we dub the case of "costless interventions" before handling in more detail the case of "costly interventions."

4.2 Costless interventions

In this case the Central Bank does not require any net wealth (or fiscal space) to credibly promise the minimal intervention that will keep asset prices above the critical level. We will show that it is then optimal for the Central Bank to always commit all its wealth to intervention. The cost incurred is always outweighed by the benefits of higher leverage.

More precisely, the lowest value of the distressed assets price q_t consistent with a high-leverage equilibrium is $\underline{q}$ given by Equation (14). The corresponding intervention cost in case of crisis is $\underline{C}$. From (23), this cost is negative when $\underline{q} \leq v\underline{A}$, that is when $v \geq (\beta^{-1} - h)/(\mathrm{E}\, A - h)$. In that case, the Central Bank is able to recoup a high enough fraction v of distressed assets to actually make profits by intervening during a crisis.

When $\underline{C} \leq 0$, we also have $C^* < \underline{C} \leq 0$, and the range of admissible intervention costs is $[C^*, B_t]$, which includes 0. The period-utility $\kappa u C - (1+\kappa) u \underline{C}$ is positive for at least $C = 0$ and for larger intervention costs. Thus, it is both possible and optimal to always stay in the high-leverage equilibrium with $\zeta_t = 1$.

What is the optimal cost of intervention C? Given the linearity of the value function, the optimal cost of intervention is the highest possible one. The only limit comes from the constraint ((37)) on available resources B_t. Therefore, the Central Bank chooses the highest possible intervention cost $C = B_t$.

Substituting $C = B_t$ in the HBJ equation, we can solve for the value

function $\tilde{J}(B,1)$. We find

$$\tilde{J}(B,1) = u\kappa B + u\kappa\tilde{T} - (1+\kappa)u\underline{C} + \gamma\tilde{J}(0,1) \\ + e^{-\frac{\bar{B}-B}{\tilde{T}}}\left[\tilde{J}(\bar{B},1) - \left(u\kappa\bar{B} + u\kappa\tilde{T} - (1+\kappa)u\underline{C} + \gamma\tilde{J}(0,1)\right)\right].$$

Substituting for the boundary condition (43) and for $\tilde{J}(0,1)$, and taking the limit $B \ll \bar{B}$, we get

$$\tilde{J}(B,1) = u\kappa B + \frac{u\kappa\tilde{T} - (1+\kappa)u\underline{C}}{1-\gamma}. \tag{44}$$

The interpretation is straightforward. When the Central Bank sets $C = B$, it allows the economy to be in the high-leverage equilibrium and gets the instantaneous utility $u\kappa B - (1+\kappa)u\underline{C}$ (with respect to the low-leverage equilibrium). Then, as time goes by, it accumulates taxes that can be used to increase C. This goes on until the next crisis. The net present value of future taxes up to the next crisis is $\tilde{T}$, which gives an additional value $u\kappa\tilde{T}$. When the next crisis takes place, assets B_t are fully depleted and the Central Bank can only commit to an intervention cost $C = 0$, before starting accumulating taxes again, until the following crisis. The value corresponding to the time interval between the next crisis and the following one is then $\gamma[u\kappa\tilde{T} - (1+\kappa)u\underline{C}]$, since γ is the expected discount factor at the time of the next crisis. The total value is then

$$\tilde{J}(B,1) = u\kappa B + [u\kappa\tilde{T} - (1+\kappa)u\underline{C}](1+\gamma+\gamma^2+\dots) = u\kappa B + \frac{u\kappa\tilde{T} - (1+\kappa)u\underline{C}}{1-\gamma}.$$

This case of costless interventions has a number of interesting features. First, this is a case where crisis policy can be a free lunch. Then, the Central Bank should not hesitate to intervene during crisis. As discussed in the introduction, the large scale distressed asset purchases undertaken by the Federal Reserve System and US Treasury, in the aftermath of the 2008, can be classified in the category of costless intervention because they all end up turning net profits after five years. However we shall recall that this lucky outcome was not necessarily the most likely ex-ante and so the Fed and the US Treasury were ready to incur losses to stabilize the financial sector.

In fact, according to our result, the Central Bank should not only intervene, but commit all its available (fiscal) resources to intervention in case of crisis, thereby committing to support asset prices at the highest possible level. Crisis intervention has then no net social cost. Admittedly, this

result derives from our modeling choices and the linear structure of the optimization problem. In doing so, we have abstracted from potential costs of high leverage, such as non-performing loans or failures in the allocation of resources, which would naturally limit the optimal size of financial intermediation. However, despite a framework rigged in favor of large interventions, we will see in Section 4.3 that this result does not generalize to the case of costly interventions.

Second, when the optimal policy is implemented, the economy features an endogenous financial cycle, with phases of gradual leveraging interrupted by crises associated with sudden deleveraging. This financial cycle is driven by the evolution of fiscal space. Between crises, available fiscal resources increase and the Central Bank is expected to be able to support asset prices q at an ever higher level in case of crisis. This endogenously builds up leverage. The financial system, whose size is given by $L = \frac{1}{1-m^u(q)}\eta w$, gradually expands. When the bad shock hits, the Central Bank intervenes and uses up its resources. Immediately after the crisis, with no fiscal space, it can only commit to support prices at the lower level $\underline{q}$: the economy then suddenly deleverages, before gradually leveraging up again. As for output, it is given by $Y = \frac{1}{1-m^u(q)}A\eta w$, and gradually expands between crises, thanks to increasing leverage. During a crisis, output falls both because of the bad shock $A = \underline{A}$, and because of lower leverage. By contrast, in a low-leverage equilibrium, leverage, and the size of the financial system would be constant, and output would follow the dynamics of the productivity shock A.

Third, this implies an inverse relationship between the frequency of crises and their severity (as measured by the cost of intervention). Consider a large sample of economies. Lucky economies would experience fewer crises. But they would have a lot of time to leverage up between crises, and the fewer crises would be associated with more costly interventions. On the contrary, unlucky economies would experience more crises, but because their fiscal space would stay limited, these crises would only have low fiscal costs. This result is consistent with the experience of Eastern Europe in the 1990s and 2000s. Those countries who had longer and steeper lending booms were also those who experienced a deeper crisis in 2008 (Ranciere, Tornell, Vamvakidis, 2010). It is also consistent with the view that the great moderation has planted the seed for a larger crisis.

4.3 Costly interventions

We now move to the case where even the smallest intervention is costly: $\underline{C} > 0$. This case differs from the previous one in an important way. In

the previous case, the Central Bank was able to keep the economy in the high-leverage equilibrium even without resources, that is, when $B = 0$. This is no longer true. The Central Bank now needs at least $B \geq \underline{C}$ to induce financial intermediaries to choose the high-leverage equilibrium. When the resources available to the Central Bank are not sufficient, the economy falls in the absorbing safe equilibrium.

This creates an intertemporal trade-off. If the Central Bank targets a very high leverage today, yielding high output and high instantaneous utility, it can deplete enough resources in case of crisis to fall into the absorbing safe equilibrium. The trade-off is then between leveraging a lot now but withstanding a smaller number of crises, and choosing a more prudent policy with a lower leverage which allows to withstand a larger number of crises.

Before proceeding, let us simplify notation by normalizing variables with the minimum expected cost of intervention $u\underline{C}$. Let $b_t = B_t/(u\underline{C})$, $c = C/(u\underline{C})$, $c^* = C^*/(u\underline{C})$, $\tau = \tilde{T}/(u\underline{C})$ and $j(b_t, \zeta_t) = \tilde{J}(B_t, \zeta_t)/(u\underline{C})$. The HBJ equation (42) can be restated

$$j(b_t, \zeta_t) = \max_{c \in [c^*, +\infty)} \zeta_t \big[\kappa c - (1+\kappa) + \tau j_1'(b_t, \zeta_t) + \gamma j(b_t - c, \zeta_t)\big]. \tag{45}$$

Accordingly, $d\zeta_t = -(1 - \mathbb{1}_{c \geq 1})\zeta_t$. To induce financial intermediaries to stay in a high-leverage equilibrium, the Central Bank has to choose $c \geq 1$.

With this formulation, the structure of the model is fully captured by a vector of three deep parameters (κ, τ, γ). As explained before, κ is the marginal leveraged excess social return of risky financial intermediation. The reduced variable τ is the net present value of taxes received until the next crisis, in units of the minimum expected cost of intervention, and γ is the expected discount factor at the time of the next crisis.

The HBJ equation (45) has the form of an integro-differential equation. While such equations are notoriously difficult to solve analytically, we can actually get closed-form solutions for some parameter values by solving it recursively over subsequent intervals $b \in [0, 1)$, $[1, 2)$, $[2, 3)$, ..., each interval being at least one crisis further away from the absorbing low-leverage equilibrium than the previous one. As explained above, the trade-off is between enjoying higher leverage now or withstanding a larger number of crises before falling in the low-leverage absorbing state. Then, what matters for the shape of policy functions is the distance to this absorbing state.

Distance to absorbing state Consider first the case of low fiscal resources with $b \in [0, 1)$. The constraint of limited fiscal resources (37) implies $c \leq b < 1$. Then, the Central Bank is unable to commit to a policy

$c \geq 1$ that would sustain a high-leverage equilibrium. Therefore, when $b \in [0, 1)$, the economy is in the absorbing state of low-leverage equilibrium and $j(b, \zeta) = j(b, 0) = 0$. This is a direct consequence of the fact that even the smallest possible intervention induces a positive fiscal cost and therefore requires resources.

Suppose now that $b \in [1, 2)$. The Central Bank has the choice between committing to interventions of size $c \in [1, b]$ or switching to the low-leverage equilibrium. In the former case, a crisis would bring the economy back to the previous interval $b \in [0, 1]$, that is, to the absorbing low-leverage equilibrium. In other words, the economy is one crisis away from the absorbing equilibrium. Suppose the Central Bank indeed chooses to intervene, what should be the size of intervention? In the HBJ equation (45), the Central Bank maximizes $\kappa c + \gamma j(b - c) = \kappa c$ since $b - c$ falls in the interval $[0, 1]$ where $j = 0$. We get a bang-bang solution: the Central Bank should either choose the largest possible intervention $c = b$ or let the economy fall in the absorbing low-leverage equilibrium.

Suppose now that we are further away from the absorbing state: $b \in [2, 3)$. The Central Bank can set a large intervention cost $c \in (b-1, b]$. Then, we have $b - c < 1$. This is the same case as before: a crisis would bring the economy to the absorbing equilibrium, and interventions should be set at the largest level with $c = b$. However, the Central Bank can also choose a lower intervention cost $c \in [1, b - 1]$: a crisis would then bring the economy back to the previous case $b \in [1, 2)$. In other words, the Central Bank can now choose to stay 2 crises away from the absorbing state. Given the linearity of the optimization problem, we also expect a bang-bang solution of the form $c = b - 1$.[4]

Based on this discussion, we look at bang-bang policies of the following type. Given some integer $n \geq 1$, consider the policy defined by $c = b-(n-1)$ when $b \in [n, +\infty)$, by $c = b - (n - 1) + p$ when $b \in [n - p, n - p + 1)$ for $p = 1 \ldots (n - 1)$, and by any $c < 1$ when $b \in [0, 1)$ (letting the economy fall into the absorbing state). We call this policy the n-crisis policy. For example, the 1-crisis policy sets $c = b$ when $b \in [1, +\infty)$. It promises the largest possible intervention and lets the economy fall in the absorbing state after the next crisis. The 2-crisis policy sets $c = b - 1$ when $b \in [2, +\infty)$ and $c = b$ when $b \in [1, 2)$. On the former interval, it promises a smaller intervention that brings its net assets to $b = 1$ in case of crisis (instead of $b = 0$ in the 1-crisis policy), so that it is still able to keep the economy in the

[4] Once we have computed the value function, we can indeed check that the policy $c = b - 1$ dominates the other corner solution $c = 1$.

high-leverage equilibrium after the next crisis: this policy can withstand two crises in a row. Similarly, the n-crisis policy consists in promising sufficiently low interventions to keep the economy (at most) n crises away from the absorbing state. These policies are illustrated by Figure 1. In addition, the policy of moving immediately the economy to the low-leverage equilibrium (by setting any $c < 1$) is always available.

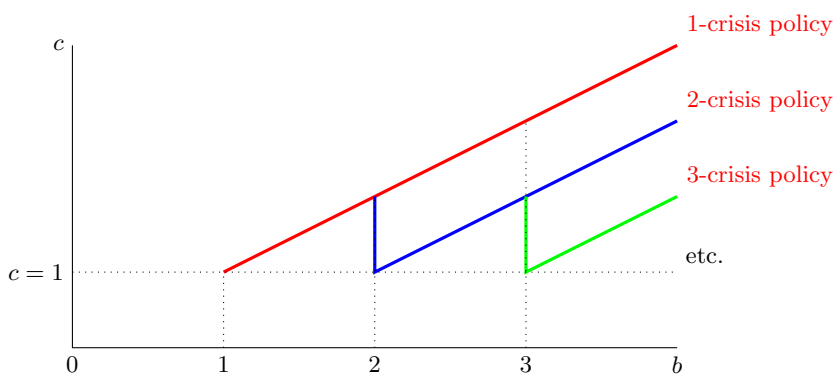


Figure 1: Policies

The trade-off the Central Bank is facing is then between the instantaneous payoff of leveraging the financial system and the resilience of the economy to crises. By committing to larger interventions, the Central Bank is able to leverage the economy more, increasing production and current welfare, but at the cost of withstanding a lower number of crises.

Solving for the value function We can now solve for the value function. Consider first the 1-crisis policy $c = b$ for $b \in [1, +\infty)$. The associated value function is given by $j(b) = \kappa b - (1 + \kappa) + \tau j'(b) + \gamma j(0)$ (where the dependence in ζ has been dropped for expositional convenience), together with two boundary conditions: (i) $j(b) = 0$ for $b \in [0, 1)$ and (ii) $j(\bar{b}) = \kappa\bar{b} - (1+\kappa) + \gamma j(0)$. We find $j(b) = \kappa(b+\tau) - (1+\kappa) - \kappa\tau e^{-(\bar{b}-b)/\tau}$. Taking the limit $b \ll \bar{b}$, we get

$$j(b) = \begin{cases} 0 & \text{if } 0 \leq b < 1 \\ \kappa(b+\tau) - (1+\kappa) & \text{if } b \geq 1. \end{cases} \tag{46}$$

When $b \leq 1$, the Central Banks gets the instantaneous utility $\kappa b - (1 + \kappa)$ associated to $c = b$. In addition, as time goes by, it accumulates taxes that are used to increase c, until the next crisis, which gives the term $\kappa\tau$.

After the next crisis, the economy falls into the absorbing state with $j = 0$. It is instructive to compare this value function with the case of costless interventions (44), where the policy function takes the same form but the economy never falls into the absorbing state. The value functions obtained only differ by the multiplier term $1/(1-\gamma)$, which represents the leveraged value of taxes received after the next crisis. This multiplier is absent from Equation (46): indeed, taxes cannot be used to leverage the economy after the next crisis since the economy falls into the absorbing state.

The 1-crisis policy dominates the low-leverage absorbing state when $\kappa(b+\tau) \geq (1+\kappa)$. There are two cases. If $\tau \leq \kappa^{-1}$, the 1-crisis policy only dominates the absorbing state when $b - 1 \geq (\kappa^{-1} - \tau)$. For a higher NPV of taxes $\tau > \kappa^{-1}$, the 1-crisis policy always dominates the low-leverage absorbing state when $b \geq 1$.

Consider now the trade-off between the 1- and the 2-crisis policy. When $b \geq 2$, the Central Bank could choose $c = b - 1$ instead of $c = b$. From the HBJ equation (45), this would yield a value $\kappa(b-1)-(1+\kappa)+\tau j'(b)+\gamma j(1)$ instead of $j(b) = \kappa b-(1+\kappa)+\tau j'(b)+\gamma j(0)$. The 2-crisis policy is preferred when $\kappa < \gamma[j(1)-j(0)]$. In that case, by foregoing the instantaneous return κ, the Central Bank gets the higher value $j(1)$ after the next crisis. Using the expression of the value function (46), this condition becomes

$$\kappa < \gamma(\kappa\tau - 1)$$

which has a very clear interpretation. By choosing the 2-crisis policy, the Central Bank decreases the promised intervention by 1 unit, and looses κ on the left-hand side. On the right-hand side, it gets the benefit of sustaining the high-leverage equilibrium after the next crisis and until the following one. First, it can still use the 1 unit of fiscal resources that was saved before the crisis to get the instantaneous utility $\kappa - (1+\kappa)$. Then, it also gets the new fiscal resources up to the following crisis, which gives an additional term $\kappa\tau$. The total benefits is then equal to $\kappa\tau - 1$ and has to be discounted by γ. Rearranging terms, we get a very simple condition in terms of the underlying deep parameters (κ, τ, γ):

$$\tau > \kappa^{-1} + \gamma^{-1}. \tag{47}$$

This condition states that a country should implement a prudent crisis policy and try to stay at least two crises away from the exhaustion of fiscal resources and complete deleveraging if it has: (i) a large fiscal capacity τ, (ii) a large leveraged excess rate of return κ, and (iii) a large discount factor γ. Going back to its definition (36), κ is large when the economy is productive and when the recovery share v of the Central Bank is large. The

discount factor γ is large when the representative household is patient and when the probability of a bad state of nature is large.

To go further, we can solve for the value function associated to the 2-crisis policy. On $b \in [1, 2)$, it is given by the same differential equation as the 1-crisis policy. It can be expressed as a function of the boundary $j(2)$: $j(b) = \kappa(b + \tau) - (1 + \kappa) + e^{-(2-b)/\tau}[j(2) - (\kappa\tau + \kappa - 1)]$. Then, on $b \geq 2$, the value function is given by: $j(b) = \kappa(b - 1) - (1 + \kappa) + \tau j'(b) + \gamma j(1)$, together with the boundary condition $j(\bar{b}) = \kappa(\bar{b} - 1) - (1 + \kappa) + \gamma j(1)$. The value function can be solved by imposing continuity of j at $b = 2$. We get:

$$j(b) = \begin{cases} 0 & \text{if } 0 \leq b < 1 \\ \kappa(b + \tau) - (1 + \tau) + e^{-(2-b)/\tau} \frac{\gamma(\kappa\tau - 1) - \kappa}{1 - \gamma e^{-1/\tau}} & \text{if } 1 \leq b < 2 \\ \kappa(b + \tau) - (1 + \kappa) + \frac{\gamma(\kappa\tau - 1) - \kappa}{1 - \gamma e^{-1/\tau}} & \text{if } b \geq 2. \end{cases} \tag{48}$$

It is easy to check that this 2-crisis value function is strictly larger than the 1-crisis value function (46) when (47) is satisfied.

Compare the two value functions when $b \geq 2$. With the 2-crisis policy, the value is reduced by κ and increased by $\gamma(\kappa\tau - 1)$ as explained before. But this is not all. If the economy is lucky after the next crisis, it can accumulate enough taxes to get back to $b = 2$. This event is discounted by $e^{-1/\tau}$. Then, the economy reswitches back to the policy $c = b - 1$, which gives an additional term $-\kappa + \gamma(\kappa\tau - 1)$, and so on, yielding a multiplier $1/(1 - \gamma e^{-1/\tau})$. The total net benefit from adopting the 2-crisis policy is indeed:

$$\begin{aligned} &-\kappa + \gamma\big[\kappa\tau - 1 + e^{-1/\tau}[-\kappa + \gamma(\kappa\tau - 1) + e^{-1/\tau} \dots]\big] \\ &\qquad = -\kappa + \frac{\gamma}{1 - \gamma e^{-1/\tau}}\big[\kappa\tau - 1 - \kappa e^{-1/\tau}\big] = \frac{\gamma(\kappa\tau - 1) - \kappa}{1 - \gamma e^{-1/\tau}}. \end{aligned}$$

This term also appears in the value function on the interval $b \in [1, 2)$, discounted by $e^{-(2-b)/\tau}$. It corresponds to the value of reaching $b = 2$ and switching to a policy $c = b - 1$.

What about the 3-crisis policy? With the same reasoning as above, we can show that the 2-crisis policy is strictly dominated by the 3-crisis policy when

$$\tau > \kappa^{-1} + \gamma^{-1} + (\gamma^{-1} - 1)\frac{\gamma^{-1} - e^{-1/\tau}}{1 - e^{-1/\tau}}.$$

This happens for even larger τ, κ, and γ than in condition (47).

The following proposition summarizes these results.

Proposition 3. *On the set made of n-crisis policies and the policy of switching to the low-leverage equilibrium:*

- *If $\tau \leq \kappa^{-1}$, the optimal policy is the low-leverage equilibrium when $b - 1 < \kappa^{-1} - \tau$ and $c = b$ when $b - 1 \geq \kappa^{-1} - \tau$.*
- *If $\kappa^{-1} < \tau \leq \kappa^{-1} + \gamma^{-1}$, the optimal policy is the 1-crisis policy.*
- *If $\kappa^{-1} + \gamma^{-1} < \tau \leq \kappa^{-1} + \gamma^{-1} + (\gamma^{-1} - 1)\frac{\gamma^{-1} - e^{-1/\tau}}{1 - e^{-1/\tau}}$, the optimal policy is the 2-crisis policy.*
- *If $\tau > \kappa^{-1} + \gamma^{-1} + (\gamma^{-1} - 1)\frac{\gamma^{-1} - e^{-1/\tau}}{1 - e^{-1/\tau}}$, the 2-crisis policy is dominated by the 3-crisis policy.*

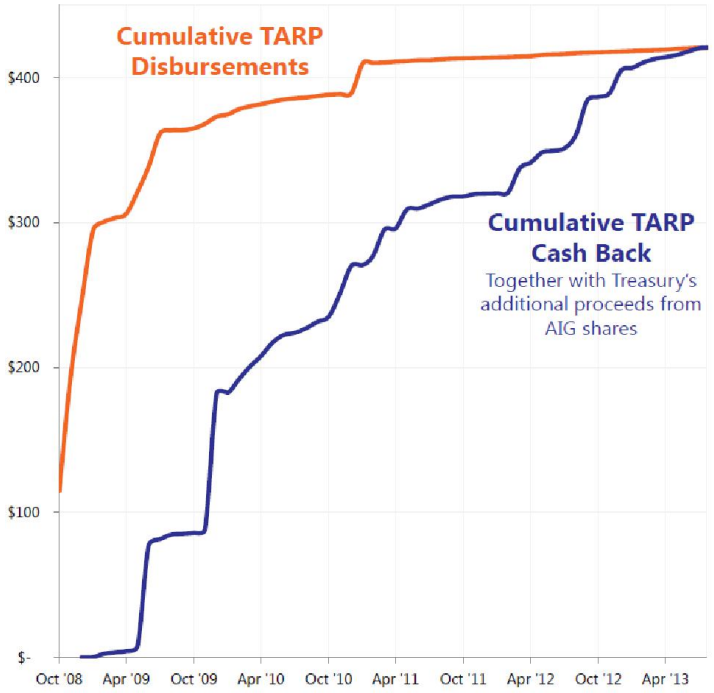
5 Conclusion

[TO BE ADDED]

A Appendix

A.1 The market for distressed assets

[to be added]



Note: As of September 6, 2013, TARP disbursements were $420.9 billion and TARP collections together with the proceeds from Treasury's additional AIG shares were $421.8 billion. TARP

Figure 2: TARP Disbursement and TARP Cash Back

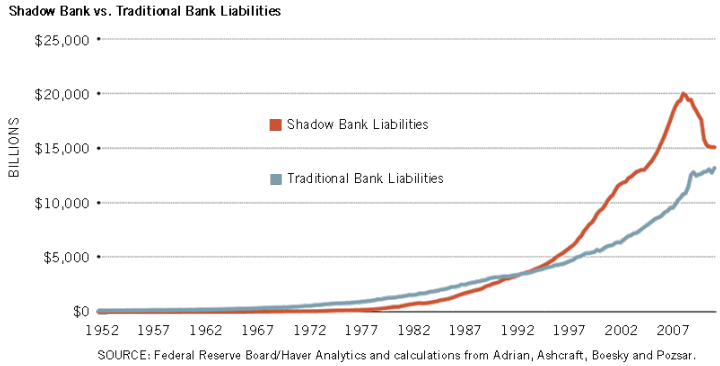


Figure 3: Shadow Bank vs. Traditional Bank Liabilities