COORDINATION FAILURES AND ASSET PRICES*

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MAY 2004

Abstract

We introduce endogenous price formation into the theoretical global games model of currency crises, using a noisy rational expectations equilibrium approach along the lines of Grossman and Stiglitz (1976, 1980) and Hellwig (1980). We explore the payoff and informational channels through which asset prices affect coordination outcomes, and examine the implications for equilibrium selection by global games methods. We show that, when we take into account the role of domestic interest rates in determining devaluation outcomes, as discussed by Obstfeld (1986, 1996), multiplicity of equilibria reemerges robustly, even when we allow for heterogeneous information as suggested by Morris and Shin (1998). On the other hand, when devaluations are triggered mainly by reserve losses (i.e. the size of a currency attack), we obtain uniqueness, but for reasons that differ from the global games analysis. The intuition behind these results relies on a comparison between the information and payoff effects of prices.

*This paper expands on an earlier paper written by the late Arijit Mukherji and Aleh Tsyvinski. We thank Marios Angeletos, Stephen Morris, Ivan Werning, and especially Andy Atkeson for useful comments and discussions.
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1 Introduction

Coordination games with multiple equilibria play a prominent role in understanding macroeconomic interactions such as currency crises (Obstfeld 1986, 1996). Recently, the theory of global coordination games (Carlsson and van Damme 1993, Morris and Shin 1998) has provided one way to resolve the multiplicity of equilibria by arguing that it is the unintended consequence of assuming that payoffs are common knowledge among all players. By perturbing the game so that players observe the underlying state with idiosyncratic noise, the global games literature shows that a unique equilibrium is sustainable.

This perturbation argument abstracts from the various channels through which coordination outcomes depend on asset markets and prices. Two general concerns arise. First, as suggested by Atkeson (2000), if prices aggregate private information, they may restore common knowledge and thereby overturn the argument for equilibrium uniqueness. Second, coordination failures often arise directly within asset markets, so that besides aggregating information, prices determine payoffs and may therefore play an important role in the argument underlying the coordination failure. For example, in the second generation currency crises models, the domestic interest rate not only determines how costly it is to attack a currency, but also influences the central bank’s decision to maintain or abandon a fixed exchange rate. Expectations of a devaluation may become self-fulfilling, either because the costs of high domestic interest rates exceed the value of maintaining a fixed exchange rate (Obstfeld 1996), or because the anticipation of inflationary policies after a devaluation leads to an interest rate increase which causes a collapse in the demand for the domestic currency, and an unsustainable loss of foreign reserves (Obstfeld 1986).

In this paper, we introduce endogenous price formation into the theoretical global games model of currency crises, using a noisy rational expectations equilibrium approach along the lines of Grossman and Stiglitz (1976, 1980) and Hellwig (1980). We explore the payoff and informational channels through which asset prices affect coordination outcomes, and examine the implications for equilibrium selection by global games methods. We show that, when we take into account the role of domestic interest rates in determining devaluation outcomes, as discussed by Obstfeld (1986, 1996), multiplicity of equilibria reemerges robustly, even when we allow for heterogeneous information as suggested by Morris and Shin (1998). On the other hand, when devaluations are

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1 Other relevant applications include bank runs (Diamond and Dybvig 1983), debt crises (Calvo 1988, Cole and Kehoe 2000) or asset price crashes (Barlevy and Veronesi 2003, Abreu and Brunnermeier 2003).
triggered mainly by reserve losses (i.e. the size of a currency attack), we obtain uniqueness, but for reasons that differ from the global games analysis. As we will discuss below, the intuition behind these results relies on a comparison between the information and payoff effects of prices.

The main framework of our analysis considers a stylized currency crises game with endogenous interest rate determination. However, as a useful benchmark, we first isolate the informational effects of asset prices by separating the coordination game from the asset market. For this, we consider a version of Morris and Shin’s currency crisis game, in which agents are allowed to trade derivative assets whose payoffs depend either on the same underlying fundamentals, or on the devaluation outcome itself.\footnote{Within the currency crises interpretation of our game, parallel assets with exogenous payoffs may be interpreted for example as an index of companies, whose valuation depends on the same economic fundamentals as the devaluation outcome. The second type of asset in our model may literally be interpreted as a currency option.} A similar environment is thoroughly studied by Angeletos and Werning (2004); our conclusions regarding parallel asset prices are similar to theirs. While information aggregation through parallel asset prices generally raises the possibility of multiple equilibria, the underlying economic mechanism behind this result depends on whether the asset payoff is conditioned on exogenous fundamentals, or on the endogenous coordination outcome. In the first case, better private information improves the informativeness of prices, and thereby improves the available public information, which leads to multiple equilibria in the coordination game. However, when asset payoffs are endogenously conditioned on the devaluation outcome, there may be multiple equilibrium price functions that clear the asset market: since a price increase indicates that successful coordination is more likely to occur, the asset demand may be locally increasing in the price, if this information effect more than outweighs the direct payoff effect. For certain realizations of the underlying fundamentals, there are multiple market-clearing prices, and hence multiple equilibrium asset price functions. Moreover, any equilibrium price function necessarily generates a ‘crash’, whereby the equilibrium price discontinuously changes with fundamentals, triggering a discrete change in the probability of a coordination failure.\footnote{The possibility of having multiple REE price functions within the context of coordination games is first discussed in Angeletos and Werning (2004).}

With these insights in place, we turn our attention to market-based coordination failures. We consider a stylized currency crises game, in which the endogenous domestic interest rate not only aggregates information, but also determines the opportunity cost of attacking, i.e. investing in foreign assets. When the central bank’s decision to abandon the fixed exchange rate is uniquely
determined by the loss of foreign reserves, this reverses the logic of the previous multiplicity result: the direct payoff effect associated with a change in the domestic interest rate more than offsets the information effect, which implies that there exists a unique equilibrium.

However, this uniqueness result is due to specific modeling assumptions that tie the devaluation outcome to the loss of foreign reserves (i.e. the size of the attack), which in equilibrium is independent of the interest rate. This is logically very different from the second generation currency crises models, in which the domestic interest rate plays a critical role in determining, whether devaluation occurs. To account for the direct and indirect role of interest rates in determining devaluation outcomes, we therefore extend our currency crises model by (i) reinterpreting the ‘fundamentals’ in part as the central bank’s willingness to bear the economic and political costs of high domestic interest rates, as suggested by Obstfeld (1996) and/or by (ii) assuming that the loss of foreign reserves is an increasing function of the domestic interest rate, as in Obstfeld (1986). Under any of these alternative assumptions, multiplicity of equilibria reemerges, provided that prices are sufficiently informative about fundamentals. Here the additional channels through which interest rates determine the devaluation outcome overcome the direct payoff role of interest rates as the opportunity cost of an attack, and thereby restore multiplicity of equilibria.

Our results lead to novel insights beyond examining the validity of global games equilibrium selection. First, information aggregation through asset prices tends to be destabilizing and induce multiple equilibria, especially when the asset payoffs are conditioned on the coordination outcome without directly affecting the primary market that gives rise to a coordination failure. One category of such ‘derivative’ assets are options. Our results thus present a potential argument why derivative markets may have a destabilizing effect on primary markets, since the derivative prices generate public information without the mitigating payoff effect of prices within the primary market.

Second, many implications of the second generation models, most notably the unpredictability of speculative attacks and the sudden jumps in domestic interest rates associated with currency crises, are robust to a formal model of information aggregation. Indeed, the logic behind the multiplicity result in Obstfeld (1986, 1996) is driven not so much by the assumption that fundamentals are common knowledge, but by the dual role that interest rates play in coordinating individual investment decisions, along with directly or indirectly determining the ultimate devaluation outcome. This, however, is more appropriately captured within a rational expectations equilibrium than by a coordination game, which focuses on reserve losses as the source of action complementarities and
abstracts from the role of prices, as in Morris and Shin (1998). Our results thus further highlight the difference between foreign reserve losses and interest rates as the driving forces behind a central bank’s decision to maintain or abandon a fixed exchange rate.

**Related Literature:** Following the original papers of Carlsson and van Damme (1993) and Morris and Shin (1998), several authors have studied the robustness of equilibrium selection to exogenous public information and the effect of public information on coordination outcomes (see for example, Morris and Shin 2003 and 2004, and Hellwig 2002). We build on their insights, but endogenize the information structure by considering the specific role of prices. Furthermore, in taking an agnostic view on equilibrium multiplicity vs. uniqueness and focusing on predictions that can be made across the equilibrium set, the paper follows similar methodological grounds as Angeletos, Hellwig and Pavan (2003, 2004).

The idea of using a noisy REE approach to model price formation in global coordination games appears in response to Atkeson’s (2000) critique in a brief note by Mukherji and Tsyvinski (2001), in Tarashev (2003) and, independently, in Angeletos and Werning (2004). Mukherji and Tsyvinski (2001) construct an example, in which traders interact not only in a coordination game, but also in a parallel CARA-normal asset market, and show that if the price is sufficiently noisy, a unique equilibrium is sustained; their paper did not have results on multiplicity of equilibria. Tarashev (2003) analyzes a version of Morris and Shin’s (1998) currency crises model with endogenous determination of interest rates, in which a devaluation is triggered mainly by reserve losses; his main result establishes the existence of a unique equilibrium. While our model of currency crises shares similarities with Tarashev’s, our focus is on the role of domestic interest rates for devaluation outcomes along the lines suggested by Obstfeld, showing that this robustly restores the argument for multiplicity.

The paper most closely related to ours is Angeletos and Werning (2004). They extensively study the effects of noisy public signals of aggregate activity and prices in parallel asset markets with a specific focus on information aggregation, showing that equilibrium multiplicity is restored by the endogenous public signal when private signals are sufficiently precise. They derive this conclusion both for assets exogenously conditioned on fundamentals, and for assets whose payoffs depend on the endogenous measure of agents participating in the attack. In the latter case, they are

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4 These papers study the informational effects of policy interventions (AHP 2003), as well as the consequences of dynamic information flows in global coordination games (AHP 2004).

5 Chari and Kehoe (2000) use a noisy REE approach to study price formation in herding models.
the first to point out that multiplicity may arise from the price function in a rational expectations equilibrium. While our asset market structure differs from theirs, our discussion of parallel asset markets in section 3 complements their results by considering assets which are conditioned on the ultimate success or failure of the attack, as is natural to assume for currency or debt crises. We also discuss the intuition underlying the multiplicity in noisy Rational Expectations Equilibria as a horse-race between the payoff and the information effect associated with the asset price, an insight that underlies our analysis of market-based coordination failures, which is the main focus of our analysis.

Finally, our analysis of multiplicity in rational expectations equilibria on the basis of information and payoff effects of prices is related to a recent paper by Barlevy and Veronesi (2003). They study asset price crashes in a model with informed and uninformed traders, in which multiple market-clearing prices and discontinuities in the equilibrium price function are the result of a locally increasing asset demand by uninformed traders. Our discussion is based on a similar insight, with the key difference being that the locally increasing asset demand schedule in our model follows from the underlying coordination game, not the interaction between differentially informed traders.

In section 2, we lay out our model of the coordination game and asset market. In section 3, we discuss information aggregation through prices in a parallel market with exogenous assets, and when asset payoffs are endogenous to the coordination outcome in an environment similar to the one studied more extensively by Angeletos and Werning (2004). In sections 4 and 5, we allow for coordination failures to take place directly within asset markets, and develop a stylized model of currency crises, that replicates the main arguments of Obstfeld (1986 and 1996) within a model with heterogeneously informed traders.

2 Environment

2.1 Coordination Game

Throughout this paper, we consider a game, in which a measure 1 continuum of risk-neutral agents interact both within a coordination game and an asset market. The coordination game is modelled as a standard regime change coordination game, examples of which include, for example, currency crises, bank runs, or debt crises. Agents simultaneously choose whether or not to attack. There is

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6 see Angeletos, Hellwig and Pavan (2003) for further discussion and various interpretations of this stage game.
an opportunity cost $c$ of attacking, and a benefit 1, if the attack is successful. Agents’ payoffs are represented by the following payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Not attack</td>
<td>$c$</td>
<td>$c$</td>
</tr>
</tbody>
</table>

The attack is successful, if and only if the measure of agents attacking, denoted by $A$, exceeds a fundamental threshold $\theta$. If $\theta$ is commonly known by all players, then there exist multiple equilibria, whenever $\theta \in [0, 1]$; in that case, it is optimal to attack, when all other agents attack so that coordination is successful, and optimal not to attack if none else attacks, so that the attack fails. If $\theta > 1$, no agent attacks in the unique equilibrium, while if $\theta < 0$, any attack will be successful, so that all players attack in the unique equilibrium.

We assume that $\theta$ is not commonly known. Initially, nature draws $\theta$ from a uniform distribution over the entire real line.\(^7\) Each agent then observes the realized state of fundamentals $\theta$ with idiosyncratic noise $\varepsilon_i$. Formally, the private signal $x_i$ is defined by $x_i = \theta + \varepsilon_i$, where idiosyncratic noise $\varepsilon_i$ is normally distributed with the mean of zero, and precision $\beta$, $\varepsilon_i \sim \mathcal{N}(0, \beta^{-1})$.

In addition to the private signals, agents observe an endogenous public signal $z$ about the state. Throughout, this public signal will be derived from the observation of an asset price, and will take the form $z = \theta + v$, where $v \sim \mathcal{N}(0, \alpha^{-1})$. In our analysis, we further restrict attention to equilibria in which strategies in the coordination game are monotonic, so that they are characterized by threshold rules $\{x^*(z), \theta^*(z)\}$, such that an agent attacks if and only if $x_i < x^*(z)$, and the attack is successful, if and only if $\theta < \theta^*(z)$.\(^8\) For further reference, it will be useful to review the existing results about equilibrium characterization and uniqueness in global games with exogenous information. With exogenous public and private signals, we find the following equilibrium characterization, the details of which are discussed at length by Morris and Shin (1998, 2003) or Hellwig (2002):

**Lemma 1** In the coordination game with exogenous public and private information about the state, the set of equilibrium thresholds $\theta^*(z)$, for a given realization $z$ of the public signal is implicitly

\(^7\)This improper prior assumption is not essential for our results.

\(^8\)While the restriction to monotonic equilibria is not without loss of generality regarding the characterization of equilibria, it does not affect the analysis of uniqueness vs. multiplicity: It follows from a standard iterated elimination argument, that whenever there is a unique monotone strategy equilibrium of the coordination game, then this is also the unique overall equilibrium of the game.
defined as the set of solutions to the following equation:

$$\Phi^{-1}(c) = \frac{\alpha}{\sqrt{\alpha + \beta}} (\theta^*(z) - z) - \frac{\sqrt{\beta}}{\sqrt{\alpha + \beta}} \Phi^{-1}(\theta^*(z))$$

(1)

The corresponding attack threshold $x^*(z)$ is defined by:

$$x^*(z) = \theta^*(z) + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\theta^*(z))$$

The equilibrium is unique, if and only if $\alpha/\sqrt{\beta} < \sqrt{2\pi}$.

>From the perspective of equilibrium selection, we are in particular interested in the case where $\alpha \to \infty$, and/or $\beta \to \infty$, i.e. the limiting cases, when both signals become infinitely precise. In particular, we have the following proposition:

**Proposition 1** (i) As $\alpha/\sqrt{\beta} \to 0$, the unique equilibrium threshold of the coordination game is independent of $z$ and is characterized by $\lim \theta^* = 1 - c$.

(ii) Whenever $z \in (0,1)$, the coordination game has multiple equilibria in the limit, as $\alpha/\sqrt{\beta} \to \infty$. In the limit, the set of equilibrium threshold solutions converges to $\lim \theta^*(z) = 0$, $\lim \theta^*(z) = 1$, and $\lim \theta^*(z) = z$.

Thus, as $\alpha/\sqrt{\beta} \to 0$, that is, as private signals become infinitely precise relative to the public information, there exists a unique equilibrium in the coordination game, with an equilibrium threshold $\lim \theta^* = 1 - c$ that is independent of the signal noise. On the other hand, as $\alpha/\sqrt{\beta} \to \infty$, there exists one equilibrium, in which there is a coordination success with probability 1 in the limit, and one equilibrium, in which there is a coordination failure with probability 1 (and one unstable equilibrium). 9

### 2.2 Asset market

Prior to the coordination game, agents interact in an asset market. Throughout the paper, we maintain the following general structure about the asset market: There is a single asset, which is traded at a market-determined price $p$. The asset pays 1 if $\theta \leq \tilde{\theta}$ and 0, otherwise, for some threshold $\tilde{\theta} \in \mathbb{R}$. Every agent can buy at most one unit of this asset. Throughout, we will assume that the asset supply is stochastic, so that the price is not fully revealing of $\theta$: specifically, suppose

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9For a proof of these two propositions, see, for example, Theorem 1 in Hellwig (2002).
that \( S = \Phi(s) \) where \( s \sim N(0, \delta^{-1}) \). The precision parameter \( \delta \) indicates how much noise there is in the trading process (or alternatively, to what extent the asset price is efficient at aggregating private information). In the limiting case where \( \delta \to \infty \), there is no noise in the asset price, so the price becomes fully revealing of the state.

Given the assumption of risk-neutrality, the agents’ asset trading is characterized by a threshold function \( \hat{x}(p) \), such that an agent is willing to buy the asset at a price \( p \), if and only if \( x_i \leq \hat{x}(p) \). \( \Phi(\sqrt{\beta}(\hat{x}(p) - \theta)) \) then denotes the total asset demand in state \( \theta \). The asset-market clearing condition then states that, in equilibrium,

\[
\Phi\left(\sqrt{\beta}(\hat{x}(p) - \theta)\right) = \Phi(s),
\]

or \( \hat{x}(p) = \theta + \frac{1}{\sqrt{\beta}}s \). \( z \equiv \hat{x}(p) \) is therefore a summary statistic of the public information conveyed by \( p \), and provides a normally distributed public signal about \( \theta \), \( z \sim N\left(\theta, (\beta \delta)^{-1}\right) \).

This framework allows us to explicitly study the different information and payoff effects of asset prices that we discussed in the introduction. We start with the assumption that the asset threshold \( \hat{\theta} \) is exogenously given. Although the asset market is separate from the coordination game, the asset payoffs are determined by the same underlying fundamental \( \theta \), and hence the asset price influences the coordination game through the information it conveys, even though it is not directly relevant for the coordination payoffs. We then consider a game, in which the asset payoff is endogenously determined by the outcome of the coordination game - that is, the asset payoff is 1, if coordination is successful, and 0 if it fails. If agents follow threshold strategies in the coordination game, this implies that the asset threshold \( \hat{\theta} \) is endogenously determined to be equal to the coordination threshold.

We then consider a game, in which the coordination problem arises within an asset market; a prime example of this would be the case of a currency run. Formally, we assume that agents decide whether or not to buy an asset, which pays off, if and only if coordination is successful; and a success occurs, if and only if a sufficiently large fraction of agents buy the asset. The opportunity cost of attacking is then given by the asset price \( p \), i.e. the model collapses to the previous one, with the additional requirement that \( c = p \). To conclude, we consider a scenario that we argue is a more accurate representation of the theoretical argument made within the second-generation currency crises models, in which \( p \) influences the coordination outcome directly. For this, we interpret our model directly as a model of currency crises, in which \( p \) represents the domestic interest rate differential of a currency under attack, \( \theta \) represents the economic and political costs that a country
is willing to absorb in defending its fixed exchange rate regime, and a devaluation occurs, if and only if $\theta \leq p$. We also consider variants of this last model, in which the stochastic asset supply depends positively on the price, and where the devaluation outcome depends directly on both $p$ and the size of the attack.

3 Prices and Coordinated Attacks

3.1 Exogenous Asset Market

We begin our analysis with the case, in which $\epsilon$ is exogenously given. An equilibrium of the coordination game with asset trading is then defined as follows:

Definition 1 An equilibrium is a collection of functions $\hat{x}(p)$, $p(\theta, s)$, $x^*(p)$ and $\theta^*(p)$ such that:

(i) $p = \Pr \{ \theta \leq \bar{\theta} | \hat{x}(p), p \}$;

(ii) $\Phi \left( \sqrt{\beta} (\bar{x}(p) - \theta) \right) = \Phi (s)$;

(iii) $c = \Pr \{ \theta \leq \theta^*(p) | x^*(p), p \}$; and

(iv) $\theta^*(p) = \Pr \{ x \leq x^*(p) | \theta^*(p) \}$.

Conditions (i) and (ii) describe the equilibrium in the asset market, while conditions (iii) and (iv) describe the equilibrium of the coordination game. The only link between the two is through the additional information provided by the asset price. We can hence analyze the two separately. The equilibrium for trading in the asset market is characterized by the following lemma:

Lemma 2 In the unique asset market equilibrium, $\hat{x}(p)$ and $p(\theta, s)$ are characterized as follows.

$$\hat{x}(p) = \bar{\theta} - \frac{1}{\sqrt{\beta (1 + \delta)}} \Phi^{-1} (p)$$

$$p(\theta, s) = \Phi \left( \sqrt{\beta (1 + \delta)} \left( \bar{\theta} - \theta - \frac{1}{\sqrt{\beta}} s \right) \right)$$

The information provided by $p$ is summarized by $\hat{x}(p) = \theta + \frac{1}{\sqrt{\beta}} s$, which conditional on $\theta$, is normally distributed with mean $\theta$ and precision $\beta \delta$.

Proof. It follows from market-clearing that $\hat{x}(p) = \theta + \frac{1}{\sqrt{\beta}} s$. Since agents can perfectly infer $\hat{x}(p)$ from $p$, $\hat{x}(p)$ also serves as a sufficient statistic for the public information about $\theta$ that is...
conveyed by \( p \) about \( \theta \), and this price signal leads to a normally distributed public signal with precision \( \beta \delta \). Therefore, \( \Pr\{\theta \leq \bar{\theta} | x_i, p\} \) is given by:

\[
\Pr\{\theta \leq \bar{\theta} | x_i, p\} = \Phi\left(\sqrt{\beta (1 + \delta)} \left(\theta - \frac{\beta x_i + \beta \delta \bar{x}(p)}{\beta (1 + \delta)}\right)\right)
\]

and we find as an indifference condition for \( \bar{x}(p) \):

\[
p = \Phi\left(\sqrt{\beta (1 + \delta)} \left(\bar{\theta} - \bar{x}(p)\right)\right)
\]

which can be solved for \( \bar{x}(p) \) to find the above formulation. Substituting back into \( \bar{x}(p) = \theta + \frac{1}{\sqrt{\beta}} s \)

and solving for \( p \) gives \( p(\theta, S) \).

We now characterize the equilibrium in the coordination game. Here, the public signal is summarized by \( z = \bar{x}(p) \) with precision \( \beta \delta \), while the private signals have precision \( \beta \). Applying Lemma 1, the following proposition characterize equilibria in the coordination game.

**Proposition 2** The threshold \( \theta^*(p) \) characterizing the coordination game equilibrium is implicitly defined by:

\[
\Phi^{-1}(c) = \frac{\sqrt{\beta} \delta}{\sqrt{1 + \delta}} \left(\theta^* - \bar{\theta} \right) - \frac{1}{\sqrt{1 + \delta}} \Phi^{-1}(\theta^*) + \frac{\delta}{1 + \delta} \Phi^{-1}(p)
\]

There exists a unique equilibrium, if and only if \( \sqrt{\beta} \delta < \sqrt{2\pi} \).

Proposition 2 states that the information aggregation provided by the asset price increases the amount of public information available, and thereby raises the potential for multiple equilibria. The information aggregation effect dominates, if \( \beta \) or \( \delta \) are sufficiently large, i.e. when private information is sufficiently precise, or when there is little noise in the supply. This result appears in similar form in Angeletos and Werning (2004), proposition 6.\(^\text{10}\)

Substituting the condition for \( p(\theta, s) \) into the equilibrium condition for \( \theta^* \), we implicitly characterize the equilibrium threshold and the equilibrium asset price as a function of \( \theta \) and \( s \):

\[
p(\theta, s) = \Phi\left(\sqrt{\beta (1 + \delta)} \left(\theta - \bar{\theta} - \frac{1}{\sqrt{\beta}} s\right)\right)
\]

\[
\Phi^{-1}(c) = \frac{\sqrt{\beta} \delta}{\sqrt{1 + \delta}} \left(\theta^* - \theta - \frac{1}{\sqrt{\beta}} s\right) - \frac{1}{\sqrt{1 + \delta}} \Phi^{-1}(\theta^*)
\]

\(^\text{10}\)We conjecture that it is not necessary that the fundamental for the asset market and the coordination game be perfectly correlated. As long as the correlation is positive and agents have a single signal about the fundamentals, the price provides public signal of the average of private signals. However, as is discussed in Hellwig (2002), this is sufficient to generate limit common knowledge within the information structure, and thereby multiplicity of equilibria.
Thus, in equilibrium, the information conveyed by the asset price is independent of $\tilde{\theta}$. The asset price is decreasing in $\theta$ and $s$. The multiplicity of equilibria is purely generated by the coordination game, that is, for any given exogenous asset, the information conveyed by the asset price leads to multiple equilibria in the coordination game. This is also seen from the determination of $p$ and $\theta^*$ as functions of $z \equiv \theta + \frac{1}{\sqrt{\beta}}s$: while $p$ is determinate, given $z$, $\theta^*$ is selected from the correspondence defined by (7). The latter may take on multiple values $\theta^*$ iff $\sqrt{\beta} \delta$ is sufficiently large. This finding is summarized graphically in figure 1: In the left panels, we plot the dependence of $\theta^*$ on $p$, which characterizes the coordination game equilibrium. In the middle panel, we plot $p$ as a function of $z$, which determines the outcome of the asset market in terms of the underlying fundamentals. In the right panel, we combine these two to plot $\theta^*$ as a function of $z$.

3.2 Endogenous asset markets

The previous section considered a simple asset market, in which the information conveyed by asset prices provided the only linkage between the asset market and the subsequent coordination game. In particular, the asset’s payoffs were unrelated to the eventual coordination outcome. In many applications, however, it may be more reasonable to assume that asset payoffs are directly conditioned on coordination outcomes. This assumption is natural for example for stocks and
bonds of companies whose balance sheet is exposed to foreign exchange rate risk; alternatively, one might think of currency options or other derivative assets whose payoff is directly conditioned on the occurrence of a successful attack.

Here, we augment the previous model to account for such payoff linkages from the coordination game to the asset market. Formally, we assume that asset payoffs are endogenous to the coordination outcome; that is, we consider a model, in which the asset pays 1, if and only if the attack is successful, and 0 otherwise. Assuming that the equilibrium of the coordination game follows a threshold rule, \( \tilde{\theta} \) is then endogenously determined as \( \tilde{\theta} = \theta^*(p) \).

The endogeneity of the asset threshold imposes an additional requirement that the equilibrium threshold function has to be consistent with the agents’ beliefs in the asset market. Formally, we use the previous solution, substituting \( \tilde{\theta} = \theta^*(p) \) in the characterization of proposition 2. This leads to the following lemma as an immediate consequence of previous results:

**Lemma 3** When the asset payoff is contingent on successful coordination, there exists a unique equilibrium in the coordination game, contingent on a price realization \( p \). The equilibrium threshold is implicitly defined by:

\[
\frac{\delta}{1 + \delta} \Phi^{-1}(p) - \Phi^{-1}(c) = \frac{1}{\sqrt{1 + \delta}} \Phi^{-1}(\theta^*). \tag{8}
\]

This lemma states that contingent on an asset price realization \( p \), there exists a unique threshold \( \theta^* \) that is consistent with being an equilibrium threshold in the coordination game, given the information conveyed by the asset price in equilibrium. Thus, the second-stage coordination game has a unique equilibrium. However, we show next that there may be multiple rational expectations equilibria in the first-stage asset market. For this, note that after substituting \( \tilde{\theta} = \theta^*(p) \) into \( p(\theta, s) \), we find

\[
p(\theta, s) = \Phi \left( \sqrt{\beta (1 + \delta)} \left( \theta^* - \theta - \frac{1}{\sqrt{\beta}} s \right) \right).
\]

There is a simple interpretation of this latter model as a run on a resource in fixed supply, such as gold: Suppose a monetary authority (MA) attempts to fix the price of its currency against gold. Traders decide whether or not to purchase one unit of gold (instead of alternative investments that yield return \( c \)). \( \theta \) represents the MA’s reserves of gold, so if more than \( \theta \) traders acquire gold, the MA is forced to abandon the fixed price and the price of gold increases by 1. If in addition, traders have access to an asset market in which they trade shares of gold-mining companies, whose fundamental value is dependent on the realized price of gold, then the asset price of these companies provides additional information about \( \theta \), but the value of the gold-mining stocks is also dependent on the threshold below which the fixed price has to be abandoned.
But \( \theta^* \) is itself given as a function of \( p \), and if we substitute in for \( \theta^* \), we find \( p \) as a function of \( z \equiv \theta + \frac{1}{\sqrt{\beta}}s \):

\[
\frac{1}{\sqrt{\beta(1+\delta)}} \Phi^{-1}(p) = \Phi\left(\frac{\delta}{\sqrt{1+\delta}} \Phi^{-1}(p) - \frac{1}{\sqrt{1+\delta}}\Phi^{-1}(c)\right) - z
\]

Both the RHS and the LHS of this equation are increasing in \( p \). The fact that the LHS is increasing in \( p \) reflects a price effect: as the price increases, the agents are less willing to buy (ceteris paribus). The fact that the RHS is increasing in \( p \) is due to an information effect: a higher price indicates that \( \theta^* \) will be higher, and hence the asset more likely to pay out, which makes it more attractive for agents to buy. Rearranging terms, one finds

\[
z = \Phi\left(\frac{\delta}{\sqrt{1+\delta}} \Phi^{-1}(p) - \frac{1}{\sqrt{1+\delta}}\Phi^{-1}(c)\right) - \frac{1}{\sqrt{\beta(1+\delta)}} \Phi^{-1}(p).
\] (9)

Whenever the RHS of this equation is monotonic in \( p \), the price effect strictly dominates the information effect, and there exists a unique equilibrium price function \( p(z) \). However, if the RHS is non-monotonic, then the information effect locally dominates. In that case, the price function \( p(z) \) has to be discontinuous in at least one point, and any price function selected from the correspondence implicitly defined by equation (9) is an equilibrium price function. When are there multiple such price functions? The RHS is non-monotonic in \( p \), whenever \( \sqrt{\beta\delta} < \sqrt{2\pi} \), i.e. under the same condition as in the previous case with exogenous assets. Moreover, when we substitute out \( p \) to express the resulting equilibrium threshold \( \theta^* \) as a function of \( z \), we find:

\[
\Phi^{-1}(c) = \frac{\sqrt{\beta\delta}}{\sqrt{1+\delta}}(\theta^* - z) - \frac{1}{\sqrt{1+\delta}}\Phi^{-1}(\theta^*)
\] (10)

This equilibrium threshold correspondence is equivalent to the one in the previous case with exogenous assets, and hence the set of equilibrium threshold functions \( \theta^* \) are identical in both cases. However, note that in this case, the multiplicity does not arise from the coordination game, but from the rational expectations equilibrium in the asset market. To summarize, we have shown the following proposition, the content of which is illustrated in figure 2:

**Proposition 3** When the asset payoff is contingent on successful coordination, the equilibrium asset price function and devaluation threshold are implicitly defined by:

\[
\Phi^{-1}(c) = \frac{\sqrt{\beta\delta}}{\sqrt{1+\delta}}(\theta^* - z) - \frac{1}{\sqrt{1+\delta}}\Phi^{-1}(\theta^*)
\]

\[
z = \Phi\left(\frac{\delta}{\sqrt{1+\delta}} \Phi^{-1}(p) - \frac{1}{\sqrt{1+\delta}}\Phi^{-1}(c)\right) - \frac{1}{\sqrt{\beta(1+\delta)}} \Phi^{-1}(p)
\]
There exist multiple equilibria, if and only if $\sqrt{3} \delta < \sqrt{2 \pi}$. Furthermore, the set of sustainable equilibrium threshold functions $\theta^* (z)$ is identical to the one in proposition 2.

The possibility of multiple rational expectations equilibria in the context of a coordination game is first derived by Angeletos and Werning (2004), proposition 7, which studies assets endogenously conditioned on the size of the attack.

The intuition behind the multiplicity of price functions comes from a ‘folding property’ in the asset demand that is discussed in depth by Barlevy and Veronesi (2003): The higher is $p$, the more costly the asset becomes, but a higher $p$ also indicates that a higher payoff is likely, and hence makes the asset more attractive to investors. For an intermediate range of $p$, this second information effect outweighs the direct payoff effect of a price increase, and leads to a locally increasing demand schedule, as a function of $z$. This implies that there are multiple market-clearing prices for certain realizations of $p$, the equilibrium price correspondence is multivalued, and any selection from this correspondence is sustainable as an equilibrium in the asset markets. As a particular property of any such equilibrium, the price function is necessarily discontinuous in $z$, which also translates into a discontinuity in the equilibrium threshold function $\theta^* (p)$. Thus the equilibrium in the coordination game and the asset market necessarily generates a crash, i.e. a discontinuity in the asset price function $p (z)$; moreover, this crash is also associated with a discontinuity in $\theta^* (z)$, and a discrete increase in the probability of successful coordination. Where this crash occurs is arbitrary. In
this case, the observation of prices \textit{endogenously} restores common knowledge about the success or failure of coordination: When $\delta$ is sufficiently large, within a certain range of fundamentals, there are multiple equilibria, of which one leads to a high asset price, a high $\theta^* (p)$, and a probability close to 1 that coordination is successful, and another one to a low asset price, a low $\theta^* (p)$, and a probability of success close to 0.

4 Coordination Failures in asset markets

In many applications, coordination problems occur directly within asset markets. In that case, asset prices not only aggregate information, but also determine payoffs within the coordination game. Consider for example a speculative attack against a currency: speculators have the choice between investing in domestic assets which earn some safe excess return over the foreign interest rate, or in foreign assets, with the potential of a large gain once a devaluation occurs. In this subsection, we extend the previous analysis to formally model coordination within an asset market: Specifically, we consider a market, in which agents decide whether or not to buy an asset, whose payoff is 1, if the fraction of agents who buy exceeds a threshold $\theta$, and 0 otherwise. This may be interpreted as a model of currency crises, in which a devaluation is triggered by reserve losses: Specifically, suppose that agents decide whether or not to purchase a domestic bond or a foreign bond. The domestic bond pays a safe excess return $p$, while the foreign bond pays a return of 1, if and only if a devaluation occurs, and 0 otherwise. A devaluation occurs, if and only if the fraction of traders purchasing the foreign bond, and hence the central bank’s loss of foreign reserves, exceeds $\theta$. With respect to the previous analysis, the key addition is that besides aggregating information, $p$ also determines the opportunity cost of “attacking”.

We define an equilibrium of this asset markets by a triplet of functions $\theta^*(p)$, $x^*(p)$ and $p(\theta, s)$ such that (i) $p = \Pr \{ \theta \leq \theta^*(p) | x^*(p), p \}$; (ii) $\theta^*(p) = \Pr \{ x \leq x^*(p) | \theta^*(p) \}$; and (iii) $\Phi \left( \sqrt{\frac{x^*(p) - \theta}{\delta}} \right) = \Phi (s)$. This replicates exactly the previous equilibrium definition in the game with the endogenous asset payoff, under the additional restriction that the return to the safe action $c$, is endogenously determined by $c = p$, in which case the threshold condition for buying the asset and the threshold condition for attacking are identical, and given by condition (i) above. The following lemma characterizes the equilibrium in the coordination game, given the information conveyed by $p$:
Lemma 4 Given $p$, there exists a unique $\theta^*$ given by:

$$
-\Phi^{-1}(p) = \sqrt{1 + \delta} \Phi^{-1}(\theta^*),
$$

(11)
such that $\theta^*$ is an equilibrium threshold of the coordination game, given the information provided by the price.

This result follows immediately from the previous lemma, after substituting in $c = p$. The additional payoff effect of a price change overturns the previous comparative static result: Whereas previously, the threshold $\theta^*$ was strictly increasing in $p$, as an increase in $p$ increased the conditional beliefs about the occurrence of a coordination success, in the present case, the opposite is the case: even though a higher $p$ indicates a higher conditional likelihood of a coordination success, this information effect is more than outweighed by the direct effect of a price increase that makes it more costly for agents to attack, and hence successful coordination less likely. We now characterize the unique rational expectations equilibrium in this asset market, which is illustrated in figure 3:

Proposition 4 In the unique rational expectations equilibrium in the asset market, $\theta^*(p)$ and $p(z)$ are implicitly defined by:

$$
\Phi^{-1}(p) = \sqrt{\beta (1 + \delta)} (\theta^* - z)
$$

(12)

$$
\frac{1}{\sqrt{\beta}} \Phi^{-1}(\theta^*) + \theta^* = z
$$

(13)

where $z \equiv \theta + \frac{1}{\sqrt{\beta}} s$.

Proof. >From the previous results, it follows that the equilibrium threshold $\theta^*$ satisfies

$$
\sqrt{1 + \delta} \Phi^{-1}(p) = \sqrt{\beta} \delta (\theta^* - z) - \Phi^{-1}(\theta^*),
$$

together with $-\Phi^{-1}(p) = \sqrt{1 + \delta} \Phi^{-1}(\theta^*)$. Substituting the second condition into the first to eliminate $p$, we find the condition for $\theta^*$, from which the first condition for $p$ follows immediately.

$p$ is decreasing in $z$, and there is a unique equilibrium solution $\theta^*(z)$ which is continuous and increasing in $z$.\textsuperscript{12} What is the intuition behind the uniqueness, and the fact that the coordination

\textsuperscript{12} Note that, even though $\theta^*$ is increasing in $z$, $\theta^* - z$ is decreasing in $z$. It follows that, as previously, a higher value of $p$ translates into a higher conditional probability of successful coordination.
threshold is increasing in the public signal $z$? As in the previous case, for a given $p$, there is a unique equilibrium function $\theta^*(p)$. Here, the additional payoff effect associated with a price change, which affects the cost of attacking in the coordination game, more than offsets the information effect (which, all else equal would lead to a higher threshold $\theta^*$). Hence, the asset demand function is no longer backwards bending, but instead is monotonically decreasing in $p$. It follows that there is also a unique equilibrium asset price function.

The equilibrium is unique, even if $\beta \to \infty$ or $\delta \to \infty$, i.e. as the price becomes a perfect aggregator of private information. Furthermore, $\theta \leq \theta^*$, if and only if $\theta \leq \Phi(s)$. In equilibrium the size of the attack is exogenously determined by $s$, and the price adjusts to clear the market, given the noisy asset supply. The occurrence of a successful attack is then uniquely determined by $\theta$ and $s$, and independent of the price realization.\textsuperscript{13} The probability, conditional on $\theta$, that an attack is successful, is then given by:

\[
\Pr(\theta \leq \theta^* | \theta) = \Pr(\theta \leq \Phi(s) | \theta) = 1 - \Pr(s \leq \Phi^{-1}(\theta) | \theta)
\]

This probability is decreasing in $\theta$, converges to $\frac{1}{2}$, as $\delta \to 0$, and as $\delta \to \infty$, the probability of a successful attack converges to 1, if $\theta < \frac{1}{2}$, and 0, if $\theta > \frac{1}{2}$. The logic behind this implication comes as a simple consequence of the model structure: The size of the attack here is given by the asset demand, which in equilibrium is equal to the total asset supply, and hence independent of the price realization; the price just adjusts to clear the asset market. In the case, where $\delta \to 0$,

\textsuperscript{13}The limit uniqueness result established by Tarashev (2002) is based on a similar logic, although his model allows for a limited impact of prices on the coordination outcome.
this implies that \( p \to 1 - \theta^* \); when \( \delta \to \infty \), however, \( p \to 1 \), if \( \theta < 1/2 \), and \( p \to 0 \), if \( \theta > 1/2 \). Thus, the present stark example resurrects a limit uniqueness result similar to what underlies much of the global games literature: As the noise trading vanishes, coordination will be successful with probability 1, whenever \( \theta < 1/2 \) while if \( \theta > 1/2 \), there will be a failure with probability 1.

5 Reconsidering the second generation models of currency crises

In the previous section, the asset supply was exogenously given, and did not respond to asset prices. Market-clearing then implied that the exogenous stochastic asset supply also determined the size of an attack in the coordination game, and hence the coordination outcome. This is logically quite different from the second-generation currency crises models developed by Obstfeld (1986 and 1996), where it is argued that a devaluation will occur once the domestic interest rate premium is sufficiently high, so that the resulting political or economic costs become unsustainable. High domestic interest rate differentials then become self-sustaining, as they are required to compensate investors for an expected loss in the case of a devaluation. In this section, we reconsider the arguments made in the second-generation models of currency crises, taking explicitly into account the role of prices within the context of a model with informational heterogeneity.

As before, we suppose that there is a continuum of risk-neutral investors and a domestic monetary authority. Agents each hold one unit of wealth, which they may invest either in a domestic bond, that pays a market-determined rate of return \( p \), or a foreign currency asset, which pays a premium normalized to 1, if the domestic fixed exchange rate is abandoned, and a return of 0 otherwise. The supply of the foreign currency asset is assumed to be perfectly elastic; the supply of the domestic asset on the other hand, is stochastic, and may depend on \( p \). Formally, we assume that the supply of domestic bonds is given by \( S = 1 - \Phi(s + \gamma \Phi^{-1}(p)) \), where, as before, \( s \sim \mathcal{N}(0, \delta^{-1}) \), and the coefficient \( \gamma \) denotes the degree to which domestic credit responds to the interest rate premium \( p \).

Investors are risk-neutral, and decide whether to invest their wealth into the domestic or the foreign asset. They form their beliefs about the likelihood of a devaluation on the basis of a normally distributed private signal \( x_i \) with precision \( \beta \), and they also observe the domestic interest rate \( p \). Assuming that investors follow threshold rules for their investment decisions, such that they invest in the foreign asset, iff \( x_i \leq x^*(p) \), and in the domestic asset otherwise, asset market-clearing implies that the investment in the foreign asset, or equivalently the central bank’s loss of foreign
reserves satisfies
\[
\Phi \left( \sqrt{\beta} \left( x^*(p) - \theta \right) \right) = \Phi \left( s + \gamma \Phi^{-1}(p) \right). \tag{14}
\]
Apart from the endogenous variation of the asset supply with the price, this asset market structure exactly replicates our previous analysis. To complete the description of the currency crisis game, we discuss under what conditions a devaluation takes place. For this, we first consider the case where a devaluation is triggered solely by high domestic interest rates, that is, whenever \( p \geq \theta \). The fundamental \( \theta \) then denotes the central bank's political or economic willingness to endure the costs of high domestic interest rates. This is meant to capture the essence of the analysis in Obstfeld (1996), where the multiplicity of equilibria is triggered by interest rates: In one equilibrium, investors expect a devaluation, which leads to a high domestic interest rate premium, whose political and economic costs are unsustainable. In an alternative equilibrium, investors do not expect a devaluation, and hence the resulting low interest rate becomes sustainable. To focus on the direct effect of high domestic interest rates, we set \( \gamma = 0 \) in this case.

We then consider a version of the model of Obstfeld (1986), in which a devaluation is triggered by a self-fulfilling collapse of money demand, in anticipation of an inflationary monetary policy following the devaluation. For this, we suppose that \( \gamma > 0 \), to capture the collapse in domestic money associated with high interest rates. As in the previous section, however, we assume that a devaluation occurs, if the total measure of investments in domestic bonds falls below \( 1 - \theta \) (or the loss of foreign reserves exceeds \( \theta \)). Finally, we consider a general version of the present model, in which a devaluation is triggered by a combination of high domestic interest rates and foreign reserve losses.

5.1 Reconsidering Obstfeld (1996)

Suppose that a devaluation occurs, if and only if \( p \geq \theta \); the devaluation threshold \( \theta^*(p) \) is then given by \( \theta^*(p) = p \). We suppose further for simplicity that \( \gamma = 0 \); it follows from market-clearing that the information conveyed by \( p \) is summarized by \( z \equiv x^*(p) \), which is a normally distributed public signal with precision \( \beta \delta \). \( x^*(p) \) satisfies:
\[
p = \Phi \left( \sqrt{\beta \left( 1 + \delta \right)} \left( \theta^*(p) - \frac{x^*(p) + \delta z}{1 + \delta} \right) \right). \tag{15}
\]
It follows that \( p = \Phi \left( \sqrt{\beta \left( 1 + \delta \right)} \left( \theta^*(p) - z \right) \right) \). From the above analysis, we immediately find the following proposition:
Proposition 5 If a devaluation is triggered iff \( p \geq \theta \), then in equilibrium \( \theta^*(p) = p \), and \( p \) as a function of the underlying shocks, is characterized by:

\[
p = \Phi \left( \sqrt{\beta (1 + \delta)} (p - z) \right)
\]

(16)

There exists a unique equilibrium, if and only if \( \sqrt{\beta (1 + \delta)} < \sqrt{2\pi} \).

If devaluation is solely triggered by unsustainably high domestic interest rates, then the argument for equilibrium multiplicity that arises in Obstfeld (1996) is maintained, provided that private information is sufficiently precise. This result does not even rely on information aggregation by prices: if \( \beta \) is sufficiently large, multiplicity arises for any value of \( \delta \), i.e. even if the domestic bond market is infinitely noisy, so that little or no public information is provided by prices. This stark result comes from a crucial difference between a model in which devaluation is triggered by a high interest rate, and a model in which devaluation is triggered by reserve losses: In the former, the coordination motive is channeled through prices, that is, conditional on observing \( p \), and forming beliefs about \( \theta \), no further assessment is necessary of the likely actions taken by other agents. On the other hand, in models, in which devaluation is triggered by reserve losses, even conditional on \( p \) and \( \theta \), agents wish to form beliefs about what others are likely to do in order to take decisions whether or not to attack the currency.
5.2 Reconsidering Obstfeld (1986)

In Obstfeld’s (1986) paper about self-fulfilling currency crises, it is argued that the anticipation of inflationary monetary policies after a devaluation leads to high domestic interest rates and a collapse of domestic money demand, which in turn leads to a loss of reserves and hence a devaluation, thus validating the initial concerns. In this subsection, we reconsider this argument within the context of our model. The idea that high interest rates lead to a collapse in domestic credit is captured by the assumption that $\gamma > 0$. As we did in the previous section of the paper, we assume that a devaluation occurs, if and only if the loss of reserves exceeds $\theta$, i.e. if and only if

$$\theta \leq \Phi\left(\sqrt{\beta} (x^*(p) - \theta)\right) = \Phi\left(s + \gamma \Phi^{-1}(p)\right).$$

The domestic interest rate thus affects the devaluation outcome indirectly through its impact on domestic credit, and hence the resulting outflow of foreign reserves. Using the market-clearing condition, the information conveyed by the asset price is summarized by:

$$z = x^*(p) - \frac{\gamma}{\sqrt{\beta}} \Phi^{-1}(p) = \theta + \frac{1}{\sqrt{\beta}} s. \quad (18)$$

As a public signal about $\theta$, $z$ is normally distributed with mean $\theta$ and precision $\beta \delta$. The threshold $x^*(p)$, such that agents buy the asset whenever their private signal $x_i$ falls below $x^*(p)$, is determined from

$$p = \Phi\left(\sqrt{\beta + \beta \delta \left(x^*(p) - \frac{\beta x^*(p) + \beta \delta z}{\beta + \beta \delta}\right)}\right). \quad (19)$$

Finally, the devaluation threshold $\theta^*(p)$ is given by $\theta^*(p) = \Phi\left(\sqrt{\beta} (x^*(p) - \theta^*(p))\right)$. These conditions lead to the following immediate equilibrium characterization:

**Lemma 5** (i) in equilibrium, $\theta^*(p)$ is given by

$$\Phi^{-1}(\theta^*) = \frac{1}{\sqrt{(1 + \delta)}} \left[\frac{\delta \gamma}{\sqrt{1 + \delta}} - 1\right] \Phi^{-1}(p), \quad (20)$$

(ii) Any equilibrium asset price function is implicitly characterized by

$$z = \Phi\left(\left(\frac{\delta \gamma}{1 + \delta} - \frac{1}{\sqrt{1 + \delta}}\right) \Phi^{-1}(p)\right) - \frac{1}{\sqrt{\beta (1 + \delta)}} \left[1 + \frac{\gamma}{\sqrt{1 + \delta}}\right] \Phi^{-1}(p). \quad (21)$$

Whether or not $\theta^*$ is increasing in $p$ depends on the sign of $\frac{\delta \gamma}{\sqrt{1 + \delta}} - 1$. The fact that the asset supply is increasing in the price amplifies the positive information effect: A higher price indicates
not only that a devaluation is more likely, but also that demand for domestic bonds is smaller, and hence the equilibrium loss of foreign reserves is larger - for both reasons, an increase in the price translates into a higher equilibrium threshold, which may now more than offset the adverse direct effect on the demand of domestic bonds that results from a price increase. Thus, when $\gamma$ is sufficiently large, indicating a large effect of price on supply, or when $\delta$ is large, indicating that information aggregation is very effective, the positive information effect more than outweighs the negative price effect, and hence implies that the equilibrium devaluation threshold is increasing in $p$.

Whether or not there are multiple equilibria then depends on the precision of the private signals, $\beta$. If $\frac{\delta \gamma}{1+\delta} < \frac{1}{\sqrt{(1+\delta)}}$ then the RHS of (21) is unambiguously decreasing in $p$, and there exists a unique equilibrium. On the other hand, if $\frac{\delta \gamma}{1+\delta} > \frac{1}{\sqrt{(1+\delta)}}$, the RHS may be non-monotonic in $p$, if the precision of private signals is sufficiently high - in that case, there are multiple equilibria. We summarize this result in the following proposition (see also figure 5):

Proposition 6 (i) if $\frac{\delta \gamma}{1+\delta} < 1$, there exists a unique equilibrium, in which $\theta^*$ is decreasing in $p$ and increasing in $z$, and $p$ is decreasing in $z$, and $\theta^* - z$ is decreasing in $z$.

(ii) if $\frac{\delta \gamma}{1+\delta} > 1$ and $\sqrt{\beta \frac{\delta \gamma}{1+\delta}} \sqrt{\frac{1+\delta}{1+\delta}} < \sqrt{2\pi}$, then there exists a unique equilibrium, in which $\theta^*$ is increasing in $p$, and $p$ is decreasing in $z$, and $\theta^* - z$ is decreasing in $z$.

(iii) if $\frac{\delta \gamma}{1+\delta} > 1$ and $\sqrt{\beta \frac{\delta \gamma}{1+\delta}} \sqrt{\frac{1+\delta}{1+\delta}} > \sqrt{2\pi}$, then there exist multiple equilibria. While $\theta^*$ is increasing in $p$, $p$ is necessarily discontinuous in $z$. Both the threshold and the price function exhibit a discontinuity with respect to $z$.

In case (i), the direct payoff effect of a price increase is sufficiently strong to overturn the information aggregation effect, the equilibrium threshold is strictly decreasing in $p$, and there exists a unique equilibrium as in the previous example, independently of the precision of private information. In case (ii), the information effect of the price dominates the direct payoff effect, and asset demand may be locally increasing in price; however private information is not sufficiently precise to generate enough information aggregation through the market price; consequently there remains a unique equilibrium, with properties similar to Morris and Shin. Finally, in case (iii) the information effect of a price increase dominates the price effect, so that the asset demand is locally increasing in the price; consequently, there exist multiple equilibrium price functions, all of which necessarily generate a downward discontinuous price function. In this case, the information aggregation effect dominates the direct price effect.
The multiplicity argument in Obstfeld (1986) is therefore robust to heterogeneity of information, provided that prices are sufficiently effective in generating public information, and/or the impact of a domestic interest rate increase on foreign reserves is sufficiently strong.

5.3 General Results

The previous analysis suggests that once the domestic interest rate $p$ has a sufficiently strong direct or indirect influence on the devaluation outcome, the information effect of price changes becomes sufficiently strong so as to generate multiplicity. On the other hand, when the devaluation outcome is independent of the price realization and asset supply shocks are sufficiently large, a unique outcome prevails. These results thus suggest that whether or not there exists a unique asset market equilibrium depends on the comparison between the information aggregation effects of prices and the direct payoff effects.

In this subsection, we corroborate this intuition by considering a more general version of our model, in which the devaluation outcome depends both on the interest rate and the overall loss of foreign reserves. Specifically, we assume that a devaluation occurs, if and only if $\theta \leq F(A, p)$, where $F$ is increasing in both the loss of foreign reserves $A$ and in the domestic interest rate $p$. To retain the tractability of our model, we assume that $F(\theta, p) = \Phi (a_0 \Phi^{-1}(A) + a_1 \Phi^{-1}(p))$. As before, we
assume that the domestic supply of credit may be decreasing in \( p \), and hence the equilibrium loss of reserves increasing in \( p \). This is captured by the assumption that the domestic demand for bonds is given by \( 1 - \Phi \left( s + \gamma \Phi^{-1} (p) \right) \), where \( \gamma \geq 0 \).\(^{14}\)

As before, an equilibrium of this asset market is defined by a set of functions \( x^* (p) \), \( \theta^* (p) \), and \( p (\theta, s) \), such that (i) agents purchase domestic bonds if and only if \( x_i > x^* (p) \), and foreign bonds otherwise, (ii) a devaluation occurs if and only if \( \theta \leq \theta^* (p) \), and (iii) the asset market clears. From the market-clearing condition, \( z = x^* (p) - \frac{\gamma}{\sqrt{\beta}} \Phi^{-1} (p) \) summarizes the information conveyed by \( p \), and \( z \sim \mathcal{N} \left( \theta, (\beta \delta)^{-1} \right) \). \( x^* (p) \) and \( \theta^* (p) \) satisfy

\[
\begin{align*}
\theta^* (p) &= \Phi \left( a_0 \Phi^{-1} (A) + a_1 \Phi^{-1} (p) \right) = \Phi \left( a_0 \sqrt{\beta} (x^* (p) - \theta^* (p)) + a_1 \Phi^{-1} (p) \right) \\
p &= \Phi \left( \sqrt{\beta} \left( 1 + \delta \right) \left( \theta^* (p) - \frac{\beta x^* (p) + \beta \delta z}{\beta (1 + \delta)} \right) \right).
\end{align*}
\]

Solving for \( \theta^* (p) \) and \( p \) as a function for \( z \), we find the following lemma:

**Lemma 6** In any equilibrium, \( \theta^* (p) \) is given by

\[
\Phi^{-1} (\theta^*) = \left[ a_1 + \frac{a_0}{1 + \delta} \left( \gamma \delta - \sqrt{1 + \delta} \right) \right] \Phi^{-1} (p) \tag{22}
\]

Any equilibrium price function \( p (z) \) is selected from

\[
z = \Phi \left( \left[ a_1 + \frac{a_0}{1 + \delta} \left( \gamma \delta - \sqrt{1 + \delta} \right) \right] \Phi^{-1} (p) \right) - \frac{\sqrt{1 + \delta} + \gamma}{\sqrt{\beta} (1 + \delta)} \Phi^{-1} (p) \tag{23}
\]

Any resulting equilibrium threshold function \( \theta^* (z) \) is selected from

\[
z = \theta^* - \frac{\sqrt{1 + \delta} + \gamma}{\sqrt{\beta} \left[ a_1 (1 + \delta) + a_0 \left( \gamma \delta - \sqrt{1 + \delta} \right) \right]} \Phi^{-1} (\theta^*) \tag{24}
\]

Consequently, whenever \( \theta^* (p) \) is decreasing in \( p \), there exists a unique equilibrium. This occurs, if and only if

\[
\frac{a_1}{a_0} \sqrt{1 + \delta} + \frac{\gamma \delta}{\sqrt{1 + \delta}} \leq 1, \tag{25}
\]

i.e. when \( \frac{a_1}{a_0} \), \( \gamma \) and \( \delta \) are all sufficiently small. When the above inequality is violated, there exists a unique equilibrium, if and only if \( \beta \) is not too large, or

\[
\frac{\sqrt{\beta}}{\sqrt{1 + \delta} + \gamma} \left[ a_1 (1 + \delta) + a_0 \left( \gamma \delta - \sqrt{1 + \delta} \right) \right] \leq \sqrt{2 \pi} \tag{26}
\]

\(^{14}\)Note that all our previous results are special cases of this general formulation.
If this inequality is violated, the information effect is sufficiently strong so that there are multiple equilibria. We summarize this result in the following proposition:

**Proposition 7** (i) If \( \frac{a_1}{a_0} \sqrt{1 + \delta} + \frac{\gamma \delta}{\sqrt{1 + \delta}} \leq 1 \), there exists a unique equilibrium, in which \( \theta^* \) is decreasing in \( p \) and increasing in \( z \). \( p(z) \) is strictly decreasing in \( z \).

(ii) If \( \frac{a_1}{a_0} \sqrt{1 + \delta} + \frac{\gamma \delta}{\sqrt{1 + \delta}} > 1 \), but \( \frac{\sqrt{\pi}}{\sqrt{1 + \delta}} \left[ a_1 (1 + \delta) + a_0 (\gamma \delta - \sqrt{1 + \delta}) \right] < \sqrt{2 \pi} \), there exists a unique equilibrium, in which \( \theta^* \) is decreasing in \( z \). \( p(z) \) is strictly decreasing in \( z \).

(iii) If \( \frac{a_1}{a_0} \sqrt{1 + \delta} + \frac{\gamma \delta}{\sqrt{1 + \delta}} > 1 \), and \( \frac{\sqrt{\pi}}{\sqrt{1 + \delta}} \left[ a_1 (1 + \delta) + a_0 (\gamma \delta - \sqrt{1 + \delta}) \right] > \sqrt{2 \pi} \), there are multiple equilibria. Any sustainable equilibrium price function is necessarily discontinuous.

In other words, uniqueness requires that the impact of the interest rate in determining the devaluation outcome is not too large. Furthermore, whenever either \( \frac{a_1}{a_0} > 0 \) or \( \gamma > 0 \), there exist multiple equilibria once \( \delta \) is sufficiently large. As long as the domestic interest rate has a sufficiently strong direct or indirect influence on the eventual devaluation outcome, there exist multiple equilibria, even when information aggregation through prices is noisy. We note that if \( \delta \) is large, then multiplicity arises for any value of \( \frac{a_1}{a_0} > 0 \) or \( \gamma > 0 \) - that is if prices are very effective at aggregating private information, but also serve to influence the eventual devaluation outcome, then the multiplicity of equilibria as in the second-generation currency crises models arises as a very robust feature of the equilibrium structure. When \( a_1 > a_0 \), then multiplicity arises for any \( \delta \), provided that \( \beta \) is sufficiently large, and the characterization is identical to the one in figure 4. On the other hand, if \( a_1 < a_0 \), there exists a unique equilibrium, if \( \delta \) is small, and the equilibrium characterization corresponds to figure 5.

### 6 Conclusion

In this paper, we have made a systematic attempt to embed price formation into a stylized global coordination game. We have taken a noisy Rational Expectations Equilibrium approach with heterogeneously informed traders, in which the market-clearing asset price serves to aggregate private information.

Our paper presents two main conclusions. First, whenever agents can trade in assets whose payoffs are determined by the same underlying fundamentals, or are directly determined by the coordination outcome, the public information endogenously generated by prices works against, and potentially overturns the equilibrium selection argument of the global games literature. When the
coordination failure arises within a market, the payoff effect associated with an asset price mitigates the information aggregation effect, however, the argument for multiplicity is generally robust, as long as asset prices have a direct or indirect impact on the ultimate coordination outcome.

Second, in many applications such as currency or debt crises, asset prices play an important role in determining the ultimate coordination outcome. This is not appropriately captured by a reduced form coordination game that abstracts from prices. When we fully take into account the role of prices in such coordination games, the original uniqueness argument may not be valid. Nevertheless, the global games methodology provides a useful framework for studying informational heterogeneity and information aggregation by prices in markets which give rise to coordination failures.

References


