

Modeling Aggregate Behavior and
Fluctuations in Economics: Stochastic
Views of Interacting Agents

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Preface

This book is a sequel to Aoki (1996) in a loose sense that it is motivated by a similar set of considerations as its predecessor, and shares some of the same objectives. It records my efforts, since the publication of my last book in 1996, at evaluating and reformulating macroeconomic models that are employed by the mainstream economic profession. A stochastic point of views is taken to construct models for finite numbers of interacting agents in this book. In other words, the book emphasizes models that focus on economic phenomena that involve stochastic laws, or stochastic regularities that govern economic phenomena.

To make this book more readily accessible to traditionally trained economists and graduate students in economics, this book is more narrowly focussed than my previous one, and attempts to establish better links with some well-known models in the macroeconomic literature than my previous one. This book is motivated by my strong desire to persuade some traditionally trained economists to phrase their questions in stochastic ways, and apply some of the methods in this book in their works.

Mainstream economists and graduate students of economics may wonder why use stochastic models or what additional or new insights do they gain, or if stochastic laws in economics are so useful, why have they not heard of it before.

A short answer is that models with finite numbers of agents in appropriate stochastic context reveal interesting economic phenomena that are invisible in deterministic models with infinite numbers of (representative) agents. Traditional models wash out some important information about economies, but one would not know them. This finitary and stochastic approach provides more information about economy than deterministic economic laws permit.

There are many areas of economics to which my approach applies. In speaking of inflation and unemployment, Tobin, in his presidential address at the American Economic Association meeting in 1971, comes close to describing stochastic laws, and aggregate dynamics and fluctuations (in terms of Fokker-Planck equations, say) of my ways of modelings, when he says, "... stochastic macro-equilibrium, stochastic, because random intersectoral shocks keep individual labor markets in diverse states of disequilibrium; macro-equilibrium, because the perpetual flux of particular markets produces fairly definite aggregate outcomes of unemployment and wages ...".

Another major class of examples is building business cycle models. All kinds of theories are found in the literature, and new theories keep crop-

ping up. The real business cycle (RBC) theory by Kydland and Prescott (1982) may arguably be the most influential current theory of the mainstream economists. As typified by the RBC, a natural research strategy to study business cycles is to explain fluctuations as a direct outcome of the behavior of *individual agents*. The more strongly one wishes to interpret aggregate fluctuations as something 'rational' or 'optimal', the more likely one is led to this essentially microeconomic approach. The mission of this approach is to explain fluctuations as responses of individual agents to changes in their economic environments. The consumer's intertemporal substitution, for example, is a device to achieve this goal. This has been the standard approach in the mainstream economics in the last twenty years or so.

Surely, we would like to know the distributions of durations of "good" times and "bad" times. When models admit multiple equilibria, which equilibrium if any, the model will settle in? How long will the system expected to stay in one basin of attraction before it moves to another? And so on. This book presents a different approach to fluctuations. This alternative approach is based on the fact that economy consists of a large number of agents or sectors. (Population of a large industrialized economy, for example, consists of the order of 10^8 agents.) Even if agents intertemporally maximize their respective objective functions, their environments or constraints all differ, and are always subject to idiosyncratic shocks. Our alternative approach emphasizes that an outcome of interactions of a large number of agents facing such incessant idiosyncratic shocks cannot be described by a response of the representative agent, and calls for a model of stochastic processes. In a seminal work, Slutsky (1937) proposed a stochastic approach. We follow his lead in this book to build a stochastic model of fluctuations and growth.

Although studies of macroeconomy with many heterogeneous agents are not new, dynamic behavior of economies in disequilibrium is not satisfactorily analyzed. The traditional Walrasian economy is the egregious example. It focuses on price adjustment with the help of the non-existent auctioneer.

In a nutshell, this book formulates and analyzes a large but a finite number of interacting economic agents as continuous time Markov chains with discrete state spaces. Dynamics are described in terms of the backward Chapman-Kolmogorov equations, also known as the master equations. We are interested in such questions as the existence of stationary probability distributions for some variables, of critical points of aggregate dynamics, and fluctuations about locally stable equilibria, distributions of relative sizes of the basins of attractions and associated probabilities, and how they relate to the lengths of business cycles, and so forth. The agents are assumed to be exchangeable rather than representative, and have either a finite or

countably infinite choices or decisions to choose from, or they are of a finite or a countably infinite types or categories.

Unlike jump Markov processes treated in standard textbooks on probability or stochastic processes, transition rates of the processes in this book are endogenously determined via the value maximizations by the agents in the model. Using this framework, we take fresh looks at some well-known search models, such as the Diamond model, disequilibrium quantity adjustment models, as well as models for diffusions of innovations and endogenous growth. Formulations of a few large clusters of agents in markets, and the implications on volatility of returns on financial asset market, which may develop from interaction of many agents, are also examined using some random combinatorial analysis. Such investigations lead to results not usually discussed in the traditional macroeconomic literatures, such as existence of power-laws for some variables of interest, discoveries that some common laws apply to some seemingly unrelated areas, and so on.

This book is aimed at advanced graduate students and practicing professionals in economics, as well as in some related areas, such as recently formed area of econo-physics. Some of the topics have been discussed by the author at graduate courses at UCLA, and Keio University, Tokyo, and at several conferences, workshops, and seminars. The author wishes to express his appreciations to Profs. R. Craine, K. Kawamata, A. Kirman, M. Marchesi, T. Lux, W. Semmler, H. Yoshikawa, and J.-B. Zimmermann for opportunities for presenting talks, and to Profs. Y. Shirai, D. Costantini, U. Garibaldi, and D. Sornette for their useful comments on some parts of the topics in the book. I am particularly indebted to Professors Yoshikawa, Costantini, and Garibaldi for their helps and guidance in overcoming my ignorance and misunderstandings. Simulations reported in this book were programmed by a former and current graduate student at UCLA, J. Nagamine, and R. Singh. I thank them for their help.

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