Fighting Currency Depreciation: Intervention or Higher Interest Rates?[1]

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Abstract

Central banks typically respond to pressures on their currencies by a combination of foreign exchange market intervention and interest rate changes. We build a simple model of a small open economy in order to (a) understand this observed policy response, and (b) investigate the optimal mix of these two policy instruments. The model has two crucial features. First, the presence of nominal wage rigidities provides an incentive for the policymaker to prevent nominal exchange rate fluctuations. Second, the dependence of some firms on bank finance implies that higher domestic interest rates extract an output cost. The output effect of interest rate changes implies that, in general, policymakers would choose some combination of interest rate policy and direct market intervention to insulate the economy. Moreover, in the presence of costly intervention, the optimal policy response also involves allowing some currency fluctuation. We analyze how the optimal policy mix depends on key parameters.

1 Introduction

Central banks typically respond to weakening currency values by using a combination of higher domestic interest rates and foreign exchange market intervention. Moreover, recent evidence seems to suggest that most developing countries are loathe to let their currencies float freely. This paper is an attempt at providing a rationalization for these two features of observed policymaker behavior.

In response to a depreciating currency, the first line of defense for central banks is to raise some short-term interest rate under their control. The idea is that, by making domestic assets more attractive, higher interest rates should strengthen the currency. This was, for instance, the standard IMF prescription for monetary policy in the recent programs in East Asia. In effect, as discussed in Ghosh and Phillips (1999), the essential task of monetary policy in the IMF programs in Indonesia, Korea, and Thailand was to prevent a further slide in the currency and thus cut short a vicious circle of depreciation and inflation. Some critics – most notably Jeff Sachs and Joe Stiglitz – have strongly disagreed with this policy. In fact, Sachs and Radelet (1998) even question whether high interest rates will indeed lead to a stronger currency to begin with. Even if higher interest rates were to help in stabilizing the domestic currency, the resulting output costs could be substantial.

At the same time, central banks do not limit themselves to raising interest rates to defend the domestic currency in times of turbulence. Typically they also intervene in foreign exchange markets by selling international reserves. In practice, therefore, a high interest rate defense of the currency is often part of a broader monetary policy framework which also includes some amount of intervention. The IMF program in Indonesia, for example, explicitly allowed for significant intervention by setting a substantially lower target for international reserves relative to the program's central scenario. In the cases of Thailand and Korea, intervention was also part of the picture but was limited by a lower stock of international reserves and an informal understanding that, by and large, higher interest rates would be the main means of achieving currency stability. Clearly, there was thus a policy decision to be made on the extent to which higher interest rates and/or intervention would be used to prevent further currency depreciation.

More generally, the choice of how much to intervene and/or raise interest rates in response to a negative shock that tends to weaken the domestic currency is related to the optimal choice of exchange rate regimes. An important literature in the 1980's emphasized the fact that the choice was not limited to fixed versus fully flexible exchange rates but rather entailed a choice of the optimal degree of foreign exchange market intervention (with fixed and flexible rates just being the extreme cases).[1] More recently, Calvo and Reinhart (2000) and Sturzenegger and Levy (2000) have reignited this debate at an empirical level by arguing that, in practice, most countries with *de jure* floating rates in fact intervene quite heavily in foreign exchange markets. The former have coined the term "fear of floating" to describe the stylized fact that, based on revealed preferences, policymakers seem to dislike severe exchange rate fluctuations and, as a result, intervene and/or use high interest rates to stabilize the domestic currency in response to negative shocks.

To illustrate empirically some of the issues involved, Table 1 presents volatility indicators for Indonesia, Korea, Thailand, Mexico, Peru, and, for comparison purposes, Argentina and Japan. Argentina provides a nice benchmark having had a currency board during all the period covered in Table 1. Since the exchange rate is fixed, changes in the monthly nominal exchange

[1] See the classic contribution by Aizenman and Frenkel (1985) and the references therein. Aizenman and Frenkel (1985) study the joint optimal determination of wage indexation and foreign exchange market intervention in the context of a Fischer-Gray type of model.

rate fall within a plus/minus 2.5 percent band 100 percent of the time. Although the amount of intervention in a fixed exchange rate regime will depend on the underlying shocks, one would expect intervention to be heavy. Indeed, only in 36.7 percent of the times does the monthly variation in international reserves fall within a similar band. Finally, in only 31.6 percent of the times, the monthly variation in nominal interest has fallen in a plus/minus 50 basis point band. At the other extreme, Japan provides the benchmark for a (relatively) clean floater.

Focusing first on the Asian countries for the post-crisis period, Table 1 indicates that the volatility of exchange rates has been high. There is, however, evidence of heavy intervention. Intervention appears to have been the largest in Korea, followed by Indonesia, and Thailand. In fact, for all three countries, foreign exchange rate variability is either close to or below that of Argentina's. This clearly suggests that, even when the IMF programs may not have contemplated it, intervention was heavy. It would also appear that there was intense use of changes in short-term interest rates to stabilize the currency, particularly in Indonesia and Thailand.[2] In sum, Table 1 is clearly consistent with the notion that all three countries relied heavily on both intervention and higher interest rates to stabilize the currency.

The cases of Mexico and Peru provide additional evidence. After the December 1994 crisis, Mexico is known to have made heavy use of interest rate policy to stabilize its currency (particularly in the aftermath of the Russian crisis in August 1998). This is amply borne by the fact that, as indicated in Table 1, there is only a probability of 9.4 percent that interest rate changes will fall in the band, a "degree of interest rate activism" only matched by Indonesia and Thailand. In addition, Mexico seems to intervene

[2] Needless to say, using variability of short-term market interest rates as an indicator of interest rate activism on the part of the monetary authority is, to say the least, fraught with difficulties and should be seen as suggestive at best.

heavily (in fact, to the same degree as Korea and Indonesia). Again, it is certainly remarkable that variations in international reserves in Mexico are larger than in Argentina, an emerging market with a fixed exchange rate and subject to large shocks. Peru, on the other hand, is known for being a "phony" floater (i.e., intervening heavily), which is consistent with the figures shown in Table 1 (see Moron and Castro (2000) for a detailed analysis). In particular, Peru's central bank has sold large amount of foreign exchange in turbulent moments (for instance, in January 1999, when Brazil abandoned the peg).

For all its practical relevance, there is little theoretical work on the optimality of using *both* higher interest rates and foreign exchange market intervention in response to negative shocks. Having such a model would enable us to ask questions such as: (i) what is the relative effectiveness of higher interest rates and intervention in strengthening the domestic currency? (ii) if both policies help in strengthening the currency, what is the optimal policy mix? and (iii) what are the main factors that determine the optimal policy mix?

The main purpose of this paper is thus to develop a simple model that can provide a conceptual apparatus to answer such important policy questions. In our model, the "fear of floating" stems from the fact that exchange rate variability leads to output costs. In the presence of nominal wage rigidities, changes in the exchange rate lead to changes in the actual real wage, which in turn leads to "voluntary unemployment" (to use Barro and Grossman's (1971) terminology) if the real wage falls below its equilibrium value or "involuntary unemployment" if the real wage rises above its equilibrium level. (Notice that exchange rate variability is costly regardless of whether the exchange rate depreciates or appreciates.) We model the active interest rate defense of the currency along the lines of Calvo and Vegh (1995) by assuming that it basically entails paying interest on some interest-bearing liquid asset.[3] As in Lahiri and Vegh (2000b), we incorporate into the model an output cost of raising interest rates. Hence, in our model, higher interest rates raise the demand for domestic liquid assets but at the cost of a fall in output.

In the context of such a model, consider a negative shock to real money demand. Under perfectly flexible exchange rates and wages, real money balances would fall through an increase in the exchange rate (with no changes in international reserves or interest rates). Suppose now that nominal wages are sticky. In that case, the exchange rate depreciation is costly because it leads to a lower real wage and lower employment (since actual labor is given by the short end of the market, in this case labor supply). Given this adjustment cost, there may be room for policies that prevent the exchange rate from moving. In fact, we show that if intervention is costless (more on this below), it is optimal to keep the exchange rate completely stable. This is achieved by a combination of higher interest rates (which raise real money demand relative to the benchmark case) and a sale of international reserves (to accommodate the fall in real money demand). Our model thus endogenously generates an optimal policy mix of higher interest rates and foreign exchange market intervention. It thus provides a simple and yet complete conceptual framework for the "fear of floating" phenomenon.

A drawback of the model so far is that policymakers find it optimal to leave the exchange rate completely unchanged in response to a monetary shock. To capture the possibility of letting some part of the adjustment take place through a depreciation of the currency, we assume that foreign exchange market intervention is costly (i.e., it entails some resource cost). While not

[3] This paper is therefore related to an incipient theoretical literature that analyzes the active use of interest rates to defend an exchange rate peg (see Drazen (1999a,b), Flood and Garber (2000) and Lahiri and Vegh (2000a,b)) or strenghthen the currency under a pure float (Lahiri and Vegh (2000c)).

explicitly modeled, this cost could capture the fact that policymakers value some positive level of international reserves for, say, precautionary motives. Alternatively, one could think of this as being a direct institutional restriction imposed on the central bank by multilateral agencies. We show that when intervention is costly, it is optimal, in response to a negative money demand shock, for policymakers to let the exchange rate adjust (dirty floating), raise interest rates, and also engage in some foreign exchange market intervention.

The next question is: how does the optimal policy mix vary with changes in key parameters? We show that the bigger the money demand shock, the less aggressive is interest rate policy and the more policymakers let exchange rate movements and interventions facilitate the adjustment. Also, the more dependent is the economy on bank credit (which makes the output cost of higher interest rates more severe), the more policymakers will let the exchange rate adjust. Lastly, the faster the speed of adjustment of nominal wages the lower are both the optimal exchange rate depreciation and the level of intervention.

The paper proceeds as follows. Section 2 develops the model. Section 3 analyzes the effect of a negative money demand shock. Section 4 studies the optimal policy response to this shock and shows that it entails some combination of higher interest rates and foreign exchange market intervention. Section 5 shows that, if intervention is costly, then the optimal response also involves letting the domestic currency depreciate in response to the negative money demand shock (in combination with higher interest rates and intervention). It also provides a numerical solution of the model to show how the optimal policy mix varies with changes in key parameters. Section 6 concludes.

2 The Model

We consider an infinitely-lived, representative household model of a small open economy. The economy consumes and produces two goods – y and x – both of which are freely traded. The economy takes the world prices of the two goods as given and the law of one price is assumed to hold for both goods. The foreign currency price of good y is assumed to be constant and, for convenience, normalized to unity. The world relative price of good x in terms of good y is p which is also assumed to be constant over time. The economy has access to perfectly competitive world capital markets where it can borrow and lend freely in terms of good y at the constant world interest rate r.

2.1 Households

The representative household derives utility from consuming the two goods and disutility from supplying labor. The lifetime welfare of the household is given by

$$W \equiv \int_0^\infty \frac{1}{1-1/\sigma}\{[c_t - \zeta(l_t^s)^\nu]^{1-1/\sigma} - 1\}e^{-\beta t}dt, \quad \sigma > 0, \quad \zeta > 0, \quad \nu > 1, \tag{1}$$

where

$$c = (c^y)^\rho (c^x)^{1-\rho} \tag{2}$$

is a consumption composite index. We use l_t^s to denote labor supply. Throughout the paper we maintain a notational distinction between labor supply and labor demand since in the presence of nominal wage rigidities, labor supply will not necessarily equal labor demand at all times. In order to rule out secular dynamics of consumption we make the standard assumption that $\beta = r$, i.e., the rate of time preference equals the world real rate of interest.[4]

The household's flow budget constraint in terms of good y (or foreign currency) is given by

$$\dot{a}_t = ra_t + w_t l_t^s + \Omega_t^y + \Omega_t^x + \Omega_t^b + \tau_t - c_t^y - pc_t^x - I_t^d h_t - \alpha v(\hat{h}_t) \qquad (3)$$

where w denotes the wage rate in foreign currency (henceforth referred to as the real wage), h are demand deposits in terms of the foreign currency and $I^d \equiv i - i^d$ is the deposit spread with i^d being the interest paid on deposits. Ω^y and Ω^x are dividends received from firms in sectors y and x respectively while Ω^b are dividends from commercial banks. Also, τ denotes the lump sum transfers from the government to households while $a = b + h$ gives net household assets in terms of foreign currency (or equivalently, in terms of good y).

$\hat{h} = \frac{H}{P}$ denotes real demand deposits held by the household where H denotes nominal demand deposits while P gives the domestic currency price index of the composite consumption good c. $\alpha v(\hat{h})$ denotes the transactions costs that are incurred by the household where $\alpha > 0$ is a positive constant. As is standard, we assume that the function $v(.)$ is strictly convex so that $v' < 0$ and $v'' > 0$. Thus, the household can reduce its transactions costs by holding more demand deposits in terms of the composite consumption good. Lastly, we will assume that $v(\hat{h})$ is a social cost, so that it will enter in the country's resource constraint.

Given (2) it is easy to establish that the domestic currency price index is given by

$$P = \frac{p^{1-\rho}}{\rho^\rho (1-\rho)^{1-\rho}} E \equiv \frac{E}{B} \qquad (4)$$

[4] We adopt these preferences for analytical convenience. In particular, they imply that the labor supply decision becomes independent of wealth effects which simplifies the analysis significantly. Moreover, Correia et al (1995) have shown that these preferences provide a better description of current account dynamics of small opoen economies relative to those under standard CES preferences.

where E denotes the nominal exchange rate (domestic currency price of the foreign currency) while $B = \frac{\rho^{\rho}(1-\rho)^{1-\rho}}{p^{1-\rho}}$ is a positive constant.[5] Since $h = H/E$, equation (4) implies that $\hat{h} = Bh$. Hence, transactions costs are given by $\alpha v(\hat{h}) = \alpha v(Bh)$. Since the relative price p is constant over time, it is also easy to see from (4) that the rate of inflation in this economy must equal the rate of currency depreciation at all points in time. Hence, we must have $\frac{\dot{P}}{P} = \frac{\dot{E}}{E} = \varepsilon$. Perfect capital mobility then implies that the nominal interest rate must be given by $i = r + \varepsilon$.

Integrating the household's flow constraint subject to the transversality condition on a gives

$$a_0 + \int_0^{\infty} (w_t l_t^s + \Omega_t^y + \Omega_t^x + \Omega_t^b + \tau_t) e^{-rt} dt = \int_0^{\infty} \left[c_t^y + p c_t^x + I_t^d h_t + \alpha v(\hat{h}_t) \right] e^{-rt} dt. \tag{5}$$

The household chooses time paths for c^y, c^x, l^s and h to maximize (1) subject to (5) taking as given the paths for w, r, p, I^d, Ω^f and Ω^b. The first-order conditions for utility maximization are:

$$\rho c_t [c_t - \zeta (l_t^s)^{\nu}]^{-1/\sigma} = \lambda c_t^y, \tag{6}$$

$$(1-\rho) c_t [c_t - \zeta (l_t^s)^{\nu}]^{-1/\sigma} = p \lambda c_t^x, \tag{7}$$

$$\nu \zeta (l_t^s)^{\nu - 1} = B w_t, \tag{8}$$

$$-\alpha B v'(B h_t) = I^d. \tag{9}$$

The first order conditions (6) – (9) can be used to derive the following relationships:

$$\frac{1-\rho}{\rho} \frac{c_t^y}{c_t^x} = p. \tag{10}$$

[5] P is the consumption-based price index which is defined as the minimum expenditure required to purchase one unit of the composite consumption index $(c^y)^{\rho}(c^s)^{1-\rho}$.

$$l_t^s = \left(\frac{Bw_t}{\nu\zeta}\right)^{\frac{1}{\nu-1}} \tag{11}$$

$$Bh_t = \hat{h}_t = L\left(\frac{I_t^d}{\alpha B}\right), \qquad L' < 0. \tag{12}$$

Equation (10) says that marginal rate of consumption substitution between the two goods must equal their relative price. Equation (11) shows that labor supply by households must be rising in the wage rate. Lastly, equation (12) says that real money demand in terms of good y must be falling in the cost of holding deposits I^d. Note that for a given I^d, a higher α implies that h must go up. Hence, the parameter α can be thought of as a shock to money demand.

2.2 Firms

Since there are two distinct sectors in this economy, there are two types of firms – those that produce good y and those that produce good x. Both sectors are assumed to be perfectly competitive.

2.2.1 Sector y firms

The industry producing good y is characterized by perfectly competitive firms which hire labor to produce the good using the technology

$$y_t = \left(l_t^d\right)^\eta \tag{13}$$

where l^d denotes labor demand. Firms may hold foreign bonds, b^y. Thus, the flow constraint faced by the firm is

$$\dot{b}_t^y = rb_t^y + \left(l_t^d\right)^\eta - w_t l_t^d - \Omega_t^y. \tag{14}$$

Integrating forward equation (14), imposing the standard transversality condition, and using equation (13) yields

$$\int_{t=0}^{\infty} e^{-rt}\Omega_t^y dt = b_0^y + \int_{t=0}^{\infty} \left[(l_t^d)^\eta - w_t l_t^d\right] e^{-rt} dt. \tag{15}$$

The firm chooses a path of l^d to maximize the present discounted value of dividends, which is given by the right hand side of equation (15) taking as given the paths for w_t, I_t^l, r and the initial stock of financial assets b_0^f. The first order condition for this problem is given by

$$\eta (l_t^d)^{\eta - 1} = w_t. \tag{16}$$

Equation (16) yields the firm's demand for labor:

$$l_t^d = \left(\frac{w_t}{\eta} \right)^{\frac{1}{\eta - 1}}, \tag{17}$$

which shows that labor demand by firms is decreasing in the real wage (note that $0 < \eta \leq 1$).

One should note that in the case of a linear production function or $\eta = 1$, the first order condition for profit maximization reduces to

$$w_t = 1.$$

Hence, the labor demand schedule in this case is given by

$$l_t^d \Big|_{\eta=1} = \begin{cases} 0 & \text{for } w_t > 1 \\ \infty & \text{for } w_t < 1 \end{cases} \tag{18}$$

In other words, labor demand is zero for any real wage above 1 and "infinite" for any real wage below 1. So actual employment for $w > 1$ is zero and for $w < 1$ it is given by labor supply.

2.2.2 Sector x firms

Sector x is also characterized by perfectly competitive firms which produce good x. Firms in this sector use an imported input q to produce good x. The technology for producing good x is given by

$$x_t = q_t^\theta \qquad \theta \in (0, 1), \tag{19}$$

where q denotes the imported input. The world relative price of q in terms of good y is p^q which is assumed to be constant. To economize on notation and with no loss of generality we assume $p^q = 1$. Sector-x firms are however dependent on bank loans for their working capital needs. In particular, we assume that firms face a credit-in-advance constraint to pay for imported input:

$$n_t \geq \psi q_t \qquad \psi > 0, \tag{20}$$

where n denotes loans from commercial banks. This constraint introduces a demand for bank loans and, hence, a credit channel into the model. Our maintained assumption throughout the paper is that equation (20) holds as an equality throughout. As is well known, this constraint will hold as an equality along all paths where the cost of loans, I^l, is positive. We will focus on precisely such paths.

These firms may hold foreign bonds, b^x. Hence, the real financial wealth of the representative firm at time t is given by $a_t^x = b_t^x - n_t$. Using i^l to denote the lending rate charged by banks and letting $I^l \equiv i^l - i$ denote the lending spread, we can write the flow constraint faced by the firm as

$$\dot{a}_t^x = r a_t^x + p x_t - q_t - I_t^l n_t - \Omega_t^x. \tag{21}$$

Integrating forward equation (21), imposing the standard transversality condition, and using equations (19) and (20) yields

$$\int_{t=0}^{\infty} e^{-rt} \Omega_t^x dt = a_0^x + \int_{t=0}^{\infty} \left[p q_t^{\theta} - q_t \left(1 + \psi I_t^l \right) \right] e^{-rt} dt. \tag{22}$$

Note that the proportion of imports which have to be paid for through loans-in-advance introduces an extra cost of inputs to the firm, ψI^l, which is over and above the world price of the input.

The firm chooses a path of q to maximize the present discounted value of dividends, which is given by the right hand side of equation (22) taking as given the paths for I_t^l, r and the initial stock of financial assets a_0^x. The representative firm's first order condition for profit maximization is

$$p\theta q_t^{\theta-1} = 1 + \psi I_t^l. \tag{23}$$

Equation (23) implies that the demand for bank loans is given by

$$q_t = \left(\frac{p\theta}{1+\psi I_t^l}\right)^{\frac{1}{1-\theta}}. \tag{24}$$

Hence, the firm's demand for the imported input is decreasing in the lending spread. This is the credit channel in the model. Lastly, the loan demand by sector-x firms can be determined from equation (24) as

$$n_t = \psi\left(\frac{p\theta}{1+\psi I_t^l}\right)^{\frac{1}{1-\theta}}. \tag{25}$$

2.3 Banks

The economy is assumed to have a perfectly competitive banking sector. We formalize the banking sector along the lines of Lahiri and Végh (2000b). The representative bank accepts deposits from consumers, lends to sector-x firms (n), and holds domestic government bonds (z).[6] The bank charges an interest rate of i^l to firms and earns i^g on the government bonds. It also holds required cash reserves, m (high powered money). The bank pays depositors an interest rate of i^d. Thus, the balance sheet identity of the bank implies that $m_t + n_t + z_t = h_t$.[7]

[6] Commercial bank lending to governments is particular common in developing countries. Government debt is held not only as compulsory (and remunerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin America countries during the 1980's that Rodriguez (1991) aptly refers to such governments as "borrowers of first resort". For evidence, also see Druck and Garibaldi (2000).

[7] Similar results would go through if we allowed banks to hold foreign bonds in world capital markets as long as banks face a cost of managing domestic assets (along the lines

Letting $I^g = i^g - i$ denote the real return from lending to the government, the flow constraint of the representative bank is

$$\Omega_t^b = I_t^l n_t + I_t^d h_t + I_t^g z_t - i_t m_t. \tag{26}$$

Note that since required reserves are non-interest bearing, the opportunity cost of holding required reserves for banks is the foregone nominal interest rate i. Lastly, we assume that the central bank imposes a reserve-requirement ratio $\delta > 0$. Since required reserves do not earn interest, at an optimum the bank will not hold any excess reserves. Hence, we must have

$$m_t = \delta h_t. \tag{27}$$

Thus, the representative commercial bank's balance sheet identity can be written as

$$(1-\delta)h_t = n_t + z_t. \tag{28}$$

The bank maximizes profits by choosing sequences of n_t, z_t, h_t and m_t subject to equations (27) and (28) taking as given the paths of I^l, I^d, I_t^g, δ and i. The first order conditions for the banks' optimization problem is (assuming an interior solution)

$$(1-\delta)\, I_t^l + I_t^d = \delta i_t, \tag{29}$$

$$(1-\delta)\, I_t^g + I_t^d = \delta i_t. \tag{30}$$

Conditions (29) and (30) simply say that, at an optimum, the representative bank equates the marginal cost of deposits (RHS) to the marginal revenue

of Edwards and Végh (1997), Burnside, Eichenbaum, and Rebelo (1999), or Agenor and Aizenman (1999). Put differently – and as is well-known – some friction needs to exist at the banking level for banks to play a non-trivial role in the credit-transmission mechanism. We chose the specifaction with no foreign borrowing because it is analytically simpler. Moreover, it is not a bad description of commercial banking in most developing countries.

from an extra unit of deposits (LHS). Note that the marginal revenue from an additional unit of deposits has two components. The first, given by I_t^d, is due to the fact that borrowing from consumers is cheaper for banks (whenever $I_t^d > 0$) than borrowing in the open market. The second, given by either $(1-\delta)\, I_t^l$ or $(1-\delta)\, I_t^g$, captures the fact that banks can lend a fraction $1-\delta$ of each additional unit of deposits to either firms or to the government.

Equations (29) and (30) imply that we must have

$$I_t^g = I_t^l. \tag{31}$$

This also implies that $i^l = i^g$, i.e., the lending rate to firms must equal the interest rate on government bonds. Intuitively, loans and government bonds are perfect substitutes in the bank's asset portfolio. Since the bank can get i^g by lending to the government, it must receive at least as much from firms in order to extend loans to them.

From equation (30), it is also easy to see that the deposit spread, I^d, is given by

$$I_t^d = i_t - (1-\delta) i_t^g. \tag{32}$$

Since $I^d = i - i^d$, it follows immediately that we must have $i_t^d = (1-\delta) i_t^g$ for all t. Thus, *ceteris paribus*, a rise in the domestic interest rate i^g must result in a higher deposit rate for consumers and, hence, an increase in demand deposits. Since i^g is controlled by policymakers, the preceding shows that interest rate policy in this model effectively amounts to the government being able to pay interest on money.

Lastly, we will restrict attention to parameter ranges for which I^d and I^l are non-negative. Thus, we will confine attention to environments where $i^d \leq i \leq i^g$. This restriction is needed to ensure a determinate demand for both loans and demand deposits. Note that this amounts to restricting the relevant interest rates to be in the range $0 \leq i^g - i \leq \delta i^g$.

2.4 Government

The government in this economy issues high powered money $m\,(=\delta h)$ and domestic bonds z, makes lump-sum transfers τ to the public, and sets the reserve requirement ratio δ on deposits. Domestic bonds are interest bearing and pay i^g per unit. The central bank also holds foreign exchange reserves R which bear the world rate of interest r. The consolidated government's flow budget constraint is thus given by

$$\dot{R}_t = rR_t + \dot{m}_t + \dot{z}_t + \varepsilon_t m_t + (\varepsilon_t - i_t^g)z_t - \tau_t. \tag{33}$$

The central bank balance sheet identity (in terms of foreign currency) is given by

$$R_t + \frac{D_t}{E_t} = m_t \tag{34}$$

where $D = D^g - Z$ is net nominal domestic credit. Note that D^g denotes nominal gross domestic credit while Z is the nominal value of government bonds. We assume that the monetary authority sets the rate of growth of net nominal domestic credit to zero. Hence,

$$\frac{\dot{D}_t}{D_t} \equiv \mu_t = 0 \qquad \text{for all } t,\ D_0 \text{ given} \tag{35}$$

Substituting the central bank balance sheet in equation (33) and using the domestic credit rule given by (35) gives

$$\tau_t = rR_t + \dot{z}_t + \varepsilon_t(m_t - d_t) + (\varepsilon_t - i_t^g)z_t \tag{36}$$

In effect, the lump sum transfers to the representative consumer consists of the seignorage revenue from issuing cash (net of real net domestic credit, d) and bonds *net* of the interest payments on the domestic bond. Note that the inflation tax is given by $(\varepsilon_t - i_t^g)z_t$ in the case of domestic bonds.

We shall assume throughout that i^g is an actively chosen policy instrument. For analytical convenience, we will think of I^g as the policy instrument (recall that, by definition, $I^g = i^g - i$). Given i, the central bank can always set an i^g to implement the desired value of I^g. We shall also assume that the government sets the rate of money growth μ to zero (see equation (35)) and lets fiscal transfers τ adjust endogenously so that equation (36) is satisfied.[8]

It is useful at this stage to restate the two key effects of interest rate policy in the model. First, since government bonds and bank credit to firms are perfect substitutes in the banks' portfolio, a higher interest rate on government bonds leads to an increase in the lending rate. This reduces bank credit and causes an output contraction (see equations (24) and (31)). This effect will be referred to as the *output effect* of interest rate policy. Second, the higher interest rate on government bonds induces banks to also pay a higher rate on bank deposits (recall that $i^d = (1-\delta)i^g$) and thus, increase the demand for bank deposits. We will refer to this as the *money demand effect*.

2.5 Resource constraint

By combining the flow constraints for the consumer, the firm, the bank and the government (equations (3), (14), (21), (26) and (33) we get the economy's flow resource constraint:

$$\dot{k}_t = rk_t + y_t + px_t - c_t^y - pc_t^x - q_t - \alpha v(Bh_t). \tag{37}$$

where $k = R + b + b^y + b^x$. Note that the right hand side of equation (37) is simply the current account for this economy. Integrating forward subject to the No-Ponzi game condition $\lim\limits_{t\to\infty} k_t e^{-rt} = 0$ gives

$$k_0 + \int_0^\infty [y_t + px_t - c_t^y - pc_t^x - q_t - \alpha v(Bh_t)]e^{-rt}dt = 0. \tag{38}$$

[8]In this class of models if the policymaker were free to choose both I^g and μ then the optimum in a flexible wage world would involve setting $i = i^g = 0$ at all times (see Lahiri and Végh (2000c)).

2.6 Flexible wages equilibrium

We start by describing the perfect foresight equilibrium path (PFEP) for this economy under flexible wages and floating exchange rates. As is standard for the case of floating exchange rates, we will assume that the central bank chooses to keep the time path of international reserves piecewise constant. In other words, the central bank keeps $\frac{\dot{R}}{R} = 0$. However, we will allow the central bank to undertake *discrete* interventions in the foreign exchange market. Notice that under floating exchange rates, μ and i^d are policy instruments (under a peg, the policy instruments would be ε and i^d). Our maintained assumption is that equation (35) holds at all times so that nominal net domestic credit is constant while $i^d = (1-\delta)i^g$ is also kept constant over time.

The labor market clearing condition dictates that labor demand must equal labor supply, i.e., $l_t^s = l_t^d$. Imposing this condition on equations (11), (16) and (25) yields the equilibrium labor and real wage for this economy (equilibrium values are denoted with a bar):

$$\bar{l} \equiv \left(\frac{B\eta}{\zeta\nu}\right)^{\frac{1}{\nu-\eta}}, \tag{39}$$

$$\bar{w} = \eta\left(\frac{B\eta}{\zeta\nu}\right)^{\frac{\eta-1}{\nu-\eta}} \tag{40}$$

In other words, along a PFEP, both employment and the real wage are constant.

Next, notice that the evolution of real high powered money in the economy is given by $\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \varepsilon$ where M denotes nominal money and where we have used the fact that $m = \frac{M}{E}$. Time differentiating the central bank balance sheet identity and equating the result with the expression for $\frac{\dot{m}}{m}$ derived above gives

$$\dot{m}_t = \left[r - i^d + \alpha B v'(Bh_t)\right] d_t. \tag{41}$$

Note that in deriving (41) we have used the household's first order condition for demand deposits, equation (9), to substitute out for ε. Equation (41) shows that the steady state level of demand deposits, $\bar{h}$, is implicitly defined by the condition

$$-\alpha B v'(B\bar{h}) = r - i^d \tag{42}$$

It follows from the convexity of the $v(.)$ function that the steady state level of h is increasing in both i^d and α.

It is easy to check that (41) is an unstable differential equation around the steady state. Hence, m $(= \delta h)$ must jump to its steady state level, $\bar{m}$, instantaneously at time $t = 0$ itself and remain constant thereafter. Furthermore, the rate of currency depreciation must equal the rate of nominal domestic credit growth at all times. Since $\mu = 0$ under our experiment, we must have $\varepsilon_t = 0$ for all t. Thus, the nominal exchange rate is constant over time.

Using the central bank balance sheet, the equilibrium level of the nominal exchange rate is easily determined to be

$$\bar{E} = \frac{D_0}{\bar{m} - R_0}, \tag{43}$$

where $\bar{m} = \delta\bar{h}$ and D_0 is given. Equation (43) shows that policymakers have two avenues of influencing the exchange rate. First, for a given R_0, they can use interest rate policy to affect i^d. This will influence $\bar{m}$ directly and, hence, change E. Second, for a given $\bar{m}$, they can intervene in the foreign exchange market and alter the level of R_0 and, hence, E. The determination of the optimal mix of these two policies is an issue that we will come back to later.

In order to determine steady state consumption, notice that equation (10) implies that the ratio $\frac{c^x}{c^y}$ is a constant. Hence, $\frac{c}{c^y}$ must also be constant. This fact when combined with the first order condition for consumption along with the already established fact that the equilibrium level of employment $\bar{l}$ is constant implies that c^y, c^x, and c must all be constant. The country resource constraint then implies that the constant levels of consumption of the two goods are:

$$\bar{c}^y = \rho\left[rk_0 + \bar{l}^{\eta} + p\left(\frac{p\theta}{1+\psi I^l}\right)^{\frac{\theta}{1-\theta}} - \left(\frac{p\theta}{1+\psi I^l}\right)^{\frac{1}{1-\theta}} - \alpha v(B\bar{h})\right], \quad (44)$$

$$p\bar{c}^x = (1-\rho)\left[rk_0 + \bar{l}^{\eta} + p\left(\frac{p\theta}{1+\psi I^l}\right)^{\frac{\theta}{1-\theta}} - \left(\frac{p\theta}{1+\psi I^l}\right)^{\frac{1}{1-\theta}} - \alpha v(B\bar{h})\right] \quad (45)$$

2.6.1 Money demand shocks under flexible wages

As a benchmark case, consider an unanticipated and permanent fall in α (i.e., a negative money demand shock) under floating rates and flexible wages. Since real money demand decreases, the nominal exchange rate depreciates instantaneously to accommodate the lower real money demand. Furthermore, the nominal wage rises by the same proportion as the exchange rate. Thus, with an unchanged interest rate policy the real side of the economy remains completely insulated. Consumption of the two goods may rise or fall depending on the elasticity of the transactions cost function with respect to h. Note that under a peg (i.e., full intervention), the economy would adjust instantaneously through the private sector reducing its holdings of nominal money balances and the central bank decreasing its international reserves by the corresponding amount.

3 Nominal Wage Rigidities

We now depart from the flexible wages paradigm by introducing a nominal wage rigidity into the model. However, we will retain the assumption of flexible exchange rates. We assume that nominal wages cannot jump at any point in time. Hence, the labor market clearing condition $l^d = l^s = \bar{l}$ does not necessarily hold at all points in time. In particular, it is assumed that nominal wages W adjust according to the following dynamic equation:

$$\dot{W}_t = \gamma(\bar{w} - \frac{W_t}{E_t}), \qquad W_0 \text{ given} \tag{46}$$

where $\gamma \in (0, \infty)$ captures the speed of adjustment towards the equilibrium real wage $\bar{w}$. Recall that $\bar{w}$ is given by equation (40). The implication of introducing sticky nominal wages (as we will see below) is that an exchange rate depreciation will now lead to a fall in the real wage and cause a temporary labor market disequilibrium and concomitant output losses in sector-y.

Using the previously shown result that along any PFEP with flexible exchange rates, E must jump to its steady state value $\bar{E}$ at time $t = 0$ itself, one can solve equation (46) to get

$$w_t = \bar{w} + e^{-\frac{\gamma}{\bar{E}}t}\left(w_0 - \bar{w}\right) \tag{47}$$

where $w_t = W_t/\bar{E}$ and $w_0 = W_0/\bar{E}$. As is easily checked, $\lim_{t\to\infty} w_t = \bar{w}$. Moreover, $\dot{w}_t \gtreqless 0$ for $w_t \lesseqgtr \bar{w}$. Lastly, the equilibrium nominal wage is given by $\bar{W} = \bar{w}\bar{E}$.

As in standard disequilibrium models, we will assume that actual employment is given by the short end of the market. In other words, when the real wage is below (above) its equilibrium value, actual labor is determined by labor supply (demand). Notice that this disequilibrium model implies that only one of the two labor optimality conditions will hold. If the real wage is below its equilibrium value, the household's labor condition will hold but the firm's will not. Conversely, if the real wage is above its equilibrium value, the firm's first order condition will hold but the household's will not.

There are two potential cases of disequilibrium. For $w_0 < w_t < \bar{w}$ we have $l_t^a = l_t^s = \left(\frac{Bw_t}{\nu\zeta}\right)^{\frac{1}{\nu-1}}$. Substituting in for w_t from equation (47) and simplifying the result yields the path for actual employment:

$$l_t^a = \bar{l}\left[1+\left(\frac{w_0}{\bar{w}}-1\right)e^{-\frac{\gamma}{E}t}\right]^{\frac{1}{\nu-1}} \tag{48}$$

with $l_0^a = \left(\frac{Bw_0}{\nu\zeta}\right)^{\frac{1}{\nu-1}}$. Analogously, for the case $w_0 > w_t > \bar{w}$ we have $l_t^a = l_t^d = \left(\frac{w_t}{\eta}\right)^{\frac{1}{\eta-1}}$. The path for actual employment is now given by

$$l_t^a = \bar{l}\left[1+\left(\frac{w_0}{\bar{w}}-1\right)e^{-\frac{\gamma}{E}t}\right]^{-\frac{1}{1-\eta}} \tag{49}$$

with $l_0^a = \left(\frac{w_0}{\eta}\right)^{\frac{1}{\eta-1}}$. Substituting these relations into equation (13) yields the time path of output of good y for each case.

It useful to note that in both cases $l^a < \bar{l}$ throughout the transition. Intuitively, any deviation of the real wage from its equilibrium value implies that the short end of the labor market determines actual employment. In the case of an unanticipated increase in the real wage, labor demand falls while labor supply goes up (relative to the equilibrium). Since labor demand is the short side of the market, actual employment equals labor demand. Hence, output of sector-y falls. Conversely, when the real wage is below the equilibrium, labor supply is smaller while labor demand is greater relative to the equilibrium. In this event, actual employment is supply determined. Hence, employment falls and output of sector-y declines.[9]

This result is extremely important in order to understand the real effects of exchange rate fluctuations within this model. It implies that exchange rate appreciations and depreciations are *both* contractionary. This result stands in stark contrast to the standard Mundell-Fleming model with rigid prices wherein depreciations are expansionary while appreciations are contractionary. The difference arises because the standard models in the

[9] This case is exactly what Barro and Grossman (1991) called "voluntary unemployment" in their analysis of disequilibrium models.

Mundell-Fleming tradition postulate output to be demand determined with demand being a function of the real exchange rate. As this model shows, introduction of an explicit supply-side alters the implications quite dramatically.

The consumption dynamics along the adjustment path can be determined directly from the employment dynamics. Noting that λ is constant along a PFEP and c^x/c^y and c/c^y are both constants at all times, one can differentiate the first order condition (6) with respect to time to get:

$$\dot{c}_t = \zeta\nu(l_t^a)^{\nu-1}\dot{l}_t^a > 0, \tag{50}$$

which says that consumption rises along with employment during the transition. There is a unique time path of consumption that satisfies (50)and the intertemporal resource constraint. Given the paths for c and l^a, the values of c_0 and l_0^a would then determine the value of the multiplier through the first order condition (6). Clearly, welfare should be lower than it would be under flexible wages (and floating rates) because either firms in sector-y are forced to accept a path for labor that does not satisfy their first-order condition (16) or the first order condition for households, equation (8) is violated.

3.1 Money demand shocks under rigid nominal wages

Starting from the steady state, consider a negative shock to money demand, i.e., a fall in α. Suppose the central bank allows the exchange rate to float freely. On impact, the negative money demand shock induces an immediate exchange rate depreciation. Nominal wage rigidities imply that the market real wage, W/E, will fall while the equilibrium real wage, $\bar{w}$, remains unchanged. Hence, the labor market goes into a situation of excess demand. As shown above, employment and output will fall and then slowly return to the steady state.

We conclude this section by noting that this model reproduces the standard Mundell-Fleming results regarding the optimal exchange rate regime under fixed and flexible rates. Under fixed exchange rates, sticky wages do not make any difference to the adjustment path of the economy. The economy would adjust instantaneously under both flexible and sticky wages as the central bank would buy and sell reserves to keep E unchanged. Thus, relative to flexible exchange rates, fixed exchange rates are better for insulating the real side of the economy from money market shocks.

To think about real shocks in this model, consider a shock to p – the relative price of good x. This shock changes the equilibrium real wage and, hence, requires a change in the market real wage. Under flexible rates, this would happen instantaneously through a change in the nominal exchange rate. Under fixed exchange rates and rigid nominal wages, the economy cannot adjust instantaneously because neither the nominal wage nor the nominal exchange rate can jump. The economy returns to the long run equilibrium only through a slow adjustment of the nominal wage which is accompanied by an output contraction in sector-y (unless, of course, there is policy change in the exchange rate). Thus, for the purposes of insulating the real side of the economy from real shocks, flexible exchange rates are better than fixed exchange rates.

4 Optimal Stabilization Policy

Having described the adjustment of the economy to money demand shocks under flexible and fixed exchange rates, we now turn to the issue of the optimal policy response to such a shock. In our environment, the policymaker has two free instruments available – interest rate policy and a discrete one-shot intervention in the foreign exchange market. In particular, they can control the domestic interest rate I^g and they can choose the degree of intervention to jointly determine reserves and the nominal exchange rate. To see this, rewrite the central bank balance as

$$E = \frac{D}{m - R}$$

Since domestic credit D is given, any choice of m combined with a choice of R implies a unique choice of E. Note that m depends on I^d since $m = \delta h$ which, in turn, depends on I^g (see equation 32). Hence, from the policymaker's perspective, choosing I^g is equivalent to choosing m. Put differently, any change in m can be accommodated by the policymaker through some combination of changes in R and E. If R is left unchanged then the entire change in real money balances must occur through a change in E (the case of fully flexible exchange rates). Alternatively, if R is changed by the full amount of the change in m then E remains unchanged (fixed exchange rates). Any intermediate choice of R is a case of a dirty float.

Starting from a steady state with $E = \bar{E}$, $m = \bar{m}$ and $R = \bar{R}$, we consider a negative money demand shock at time $t = 0$, i.e., α falls. The policymaker's goal is to choose I^g and R_0 to maximize welfare of the representative agent. Solving the optimal policy problem gets immensely simplified due to the following proposition:

Proposition 1 *Given any choice of m (or, equivalently, I^g) by the policymaker in response to an α shock, it can never be optimal for the central bank to choose an R_0 such that $E_0 \neq \bar{E}$.*

Proof. Recall that any $E_0 \neq \bar{E}$ implies that the market real wage $W_0/E_0 \neq \bar{w} = W_0/\bar{E}$ where $\bar{w}$ is the equilibrium real wage. Hence, output and employment must fall on impact and then rise gradually back towards the long run steady state. The central bank can always choose R_0 such that $\bar{m} - \bar{R} = m_0 - R_0$. Such a choice of R_0 would imply that $E_0 = \bar{E}$ which would leave the labor market completely unaffected and hence, output of sector-y unchanged. Moreover, output of sector-x is independent of the size of the intervention. Since intervention is costless from the perspective of the country as a whole, country wealth is unaffected by the size of intervention (a larger R just corresponds to a lower private foreign bond holdings b leaving k unchanged). Hence, this particular choice of R_0 dominates any other choice of initial reserves. ■

Since the only source of dynamics in the model is the labor market friction, the preceding implies that along any optimal policy plan the economy has to remain stationary throughout. In other words, the policymaker will respond to a money market shock by always keeping the nominal exchange rate unchanged so as to insulate the economy from any labor market frictions. The problem thus reduces to one of choosing optimal real money balances in a stationary economy through an appropriate choice of I^g. Once m is chosen, the optimal intervention involves choosing an R_0 such that $E_0 = \bar{E}$.

The stationarity of the economy implies that lifetime welfare of the representative household is given by

$$W = \frac{1}{r\left(1-\frac{1}{\sigma}\right)}\left[\left\{c-\zeta\bar{l}^{\nu}\right\}^{1-\frac{1}{\sigma}}-1\right],$$

where we have used the fact that the policymaker will ensure that the labor market will always be in equilibrium. Hence, $l^a = \bar{l}$. For a stationary economy the country resource constraint given by equation (38) implies that

$$c^y + pc^x = rk_0 + \bar{l}^{\eta} + pq^{\theta} - q - \alpha v(\hat{h}).$$

Moreover, the first order conditions for consumption imply that $c^y + pc^x = c^y/\rho$ and $c/B = c^y/\rho$ where $B = \frac{\rho^{\rho}(1-\rho)^{1-\rho}}{p^{1-\rho}}$. Hence, the country budget constraint reduces to

$$c/B \equiv \hat{c} = rk_0 + \bar{l}^{\eta} + pq^{\theta} - q - \alpha v(\hat{h}).$$

Since W is monotonically rising in c for a given $\bar{l}$, the policymaker's problem reduces to choosing I^g to maximize c/B $(=\hat{c})$ subject to equations (12), (24) and (30) for a given α, k_0 and $\bar{l}$. Differentiating $\hat{c}$ with respect to I^g gives

$$\frac{d\hat{c}}{dI^g} \equiv \Gamma = -\frac{\psi(p\theta)^{\frac{1}{1-\theta}}\psi I^g}{(1-\theta)}\left(\frac{1}{1+\psi I^g}\right)^{1+\frac{1}{1-\theta}} - \frac{(1-\delta)}{B}\frac{I^d}{\alpha B}L'\left(\frac{I^d}{\alpha B}\right), \quad (51)$$

where we have used equation (31) to get $I^l = I^g$. In the following we shall use $\tilde{I}^g$ to denote the optimal value of I^g. $\tilde{I}^g$ is defined by the relation $\Gamma|_{\tilde{I}^g} = 0$. It is easy to check from equation (51) that $\Gamma|_{I^g=0} < 0$ and $\Gamma|_{I^g=\frac{\delta}{1-\delta}r} > 0$. Hence, $\tilde{I}^g \in \left(0, \frac{\delta}{1-\delta}r\right)$. In other words, the optimal domestic interest rate lies strictly in the interior of the permissible range. Note that $I^g = \frac{\delta}{1-\delta}r$ corresponds to $I^d = 0$ which is equivalent to implementing the Friedman rule within this model.

Equation (51) clearly shows the two key margins over which the policy-maker chooses the optimal I^g. First, a higher I^g implies that I^l goes up. Hence, the cost of funds for sector-x firms goes up which implies that output (net of the import bill), and hence, consumption falls. This effect is captured by the first term on the right hand side of (51). However, a higher I^g also implies a higher deposit rate for depositors and hence, a lower opportunity cost of holding deposits I^d. This causes money demand to go up which, in turn, reduces transactions costs and thereby increases consumption. This is the positive money demand effect of higher domestic interest rates which is captured by the second term on the right hand side of (51). Note that the Friedman rule – $I^d = 0$ – emerges as the optimum when $\psi = 0$. When $\psi = 0$, higher lending rates do not have any output effect since firms do not rely on bank credit at all. Hence, it is optimal to raise the domestic interest rate all the way to $I^g = \frac{\delta}{1-\delta}r$ which implies that $I^d = 0$ and, hence, the lowest possible transactions costs.

For $\tilde{I}^g$ to be an optimum we also need to ensure that the second order condition for a maximum is satisfied.

Lemma 1 *Define* $\hat{I}^d = \frac{I^d}{\alpha B}$, *and* $\xi^h \equiv -\frac{\hat{I}^d}{\hat{h}}L'(\hat{I}^d)$. *The conditions* $\psi I^g < 1-\theta$ *and* $\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} > 0$ *are jointly sufficient for the second order condition for the* *government's welfare maximization problem to be satisfied. Moreover, these conditions imply that $\hat{c}$ is globally concave in I^g and, hence, the optimal solution, $\tilde{I}^g$, is unique.*

We omit a detailed statement of the proof since it follows simply from differentiating Γ with respect I^g. Note that in deriving the above we have used equation (12) which says that $\hat{h} = L(\hat{I}^d)$. Hence the second term on the right hand side of (51) can be rewritten as $\frac{-(1-\delta)}{B}\hat{h}\xi^h$. It is easy to see from the definition that ξ^h is the elasticity of money demand ($\hat{h}$) with respect to its opportunity cost $\hat{I}^d$. However, the condition $\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} > 0$ always holds for the quadratic transactions costs case. Moreover, in the case of a Cagan money demand function this condition will hold as long as we restrict attention to ranges where $\xi^h < 1$. In the following we shall confine attention to parameter ranges for which the conditions identified in Lemma 1 are satisfied.[10]

The optimal domestic interest rate is a function of, amongst other parameters, the money demand shock α. Of key interest to us is the behavior of the optimal domestic interest rate with respect to α. In particular,

$$\frac{\partial \tilde{I}^g}{\partial \alpha} = -\frac{\partial \Gamma / \partial \alpha}{\partial \Gamma / \partial I^g}.$$

Under the conditions of Lemma 1, the sign of this derivative is negative, i.e., $\frac{\partial \tilde{I}^g}{\partial \alpha} < 0$. This follows from the fact that $\partial \Gamma / \partial I^g < 0$ from the second order condition for welfare maximization while $\partial \Gamma / \partial \alpha = \frac{(1-\delta)}{B} \frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} \frac{\partial \hat{I}^d}{\partial \alpha} < 0$.

[10] A necessary condition for $\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} > 0$ is that $\frac{\partial \xi^h}{\partial \hat{I}^d} > 0$. In words, the interest elasticity of money demand must be increasing in the effective interest rate. This is a characterisitic of both the quadratic transactions costs case and the Cagan money demand case. Obviously, it is violated for the case of a constant elasticity of money demand. Since Cagan-type money demands seem to provide the best econometric fit for developing countries (see, for instance, Easterly, Mauro, and Schmidtt-Hebbel (1995)), we see the case of non-constant semi-elasticities as the most relevant from a practical point of view.

Proposition 2 *The optimal domestic interest rate policy, $\tilde{I}^g$, is a decreasing function of the money demand parameter α. Hence, along any optimal policy plan, a negative (positive) money demand shock (fall in α) must induce a rise (fall) in $\tilde{I}^g$.*

The proposition says that a social welfare maximizing policymaker must react to a negative money demand shock by raising domestic interest rates. Since a lower level of domestic money holdings implies a higher transactions cost and, hence, lower consumption, at the margin it is optimal for the policymaker to raise I^g in order to economize on transactions costs even though it reduces output.

The implication of the above for the policymaker's optimal intervention behavior depends on the movement in real money balances h in response to an unexpected fall in α along an optimal policy path. Since $h = B^{-1}\hat{h}$, we can deduce the effect on h by examining the behavior of $\hat{h}$.

Lemma 2 *Along an optimal path for I^g, $\frac{d\hat{I}^d}{d\alpha} < 0$ and hence, $\frac{d\hat{h}}{d\alpha} > 0$.*

Proof. See Appendix.

In other words, even though the policymaker reacts to a fall in α by raising I^g and, hence, reducing I^d, the policy response is never strong enough to completely offset the fall in α. Hence, along the optimal policy path, the opportunity cost of holding real money balances rises in response to a negative money demand shock which implies that $\hat{h}$ and h must fall.

The preceding analysis is sufficient for us to tie down the behavior of all the endogenous variables in the model in response to a money demand shock. We summarize them in the following proposition:

Proposition 3 *Along an optimal policy path, an unexpected fall (rise) in α causes real money balances to fall (rise). The central bank responds to the* *shock by raising (lowering) the domestic interest rate I^g, as well as intervening in the foreign exchange market by selling (buying) international reserves in order to keep the nominal exchange rate unchanged at the pre-shock level. Output of sector-x falls while sector-y remains unaffected.*

The model thus generates the stylized fact that was outlined in the introduction. In particular, in response to a monetary shock which puts pressure on the exchange rate, optimal behavior by the central bank entails a mix of higher interest rates and foreign exchange market intervention in order to insulate the exchange rate completely. We should note that the existence of nominal wage rigidities is key for generating this insulation. Absent a nominal wage rigidity, exchange rate fluctuations are costless. In that event, the central bank has no incentive to intervene which implies that a free float is optimal.[11]

5 Costly Interventions and Dirty Floats

A drawback of the model, as developed thus far, is that along the optimal path the nominal exchange rate remains constant. The model predicts that policymakers use a combination of interest rate policy and interventions to completely insulate the nominal exchange rate from monetary shocks. This prediction is at odds with the evidence cited in Table 1 which suggests that central banks often resort to dirty floating.

The model generates the complete insulation prediction because interventions are costless for the central bank while exchange rate fluctuations have

[11] Within the context of the model, without nominal wage rigidities the policymaker is essentially indifferent between intervening and not intervening since allocations are independent of the level of the exchange rate. However, even an an infinitesimal cost of intervening would imply that the optimal policy would be a free float and no foreign exchange market interventions at all.

output costs. In order to generate some exchange rate fluctuation along an optimal policy path we now add a social cost of altering the stock of international reserves. For simplicity, this cost is assumed to be symmetric, i.e., it applies to both increasing or decreasing the stock of reserves. In particular, we assume that the cost of effecting a discrete change in reserves at time 0 is given by:

$$g = \phi \left(\bar{R} - R_0\right)^2. \tag{52}$$

Clearly, if $\phi = 0$, the model reduces to the one analyzed earlier where the optimal response is to fully insulate the exchange rate from all money shocks. Under this respecification, the resource constraint for the economy now becomes:

$$k_0 - g + \int_0^\infty \left[y_t + px_t - c_t^y - pc_t^x - q_t - \alpha v(Bh_t)\right] e^{-rt} dt = 0. \tag{53}$$

Defining $\hat{E} \equiv \frac{E_0 - \bar{E}}{E_0}$, one can use the central bank balance sheet to compute the change in E time 0 as a function of the change in high powered money m $(= \delta h)$ and reserves R. In particular,

$$\hat{E} = \frac{dm - dR}{D_0/\bar{E}}, \tag{54}$$

where $dm \equiv \bar{m} - m_0$ and $dR \equiv \bar{R} - R_0$. Note that as before, we use bars over variables to denote their initial steady state levels before the negative money demand shock. Equation (54) shows that if the fall in real balances exceed the sales of international reserves by the central bank then the exchange rate must depreciate. Combining (52) and (54), we get:

$$g(\hat{E}) \equiv \phi \left(\bar{m} - m_0 - \frac{D_0}{\bar{E}}\hat{E}\right)^2, \qquad \hat{E} \leq \hat{E}^{flex}. \tag{55}$$

where $\hat{E}^{flex} \equiv \frac{\bar{m}-m_0}{D_0/\bar{E}}$ is the proportional change in the exchange rate in the pure floating case. Notice that $g(\hat{E}^{flex}) = 0$, $g(\hat{E}) \geq 0$, $g(0) = \phi\left(\bar{m} - m_0\right)^2 > 0$, and $g'(\hat{E}) = -2\phi\left(\bar{m} - m_0 - \frac{D_0}{\bar{E}}\hat{E}\right)\frac{D_0}{\bar{E}} \lesseqgtr 0$ as $R_0 \lesseqgtr \bar{R}$.

The last relation says that the intervention costs are falling (rising) in the size of the currency depreciation when the central bank is forced to sell (buy) reserves, i.e., $R_0 < (>)\bar{R}$. Intuitively, for say $R_0 < \bar{R}$ a bigger currency depreciation makes the required reserve sales smaller and thereby lowers the intervention cost. Analogously, for $R_0 > \bar{R}$, a bigger *appreciation* makes the intervention cost smaller.

The policy choices at time 0 are E_0 and I_0^g. Since $\bar{E}$, D_0, and $\bar{m}$ are given exogenously, a given choice of E_0 along with the value of h_0 that is implied by the chosen I_0^g allows us to uniquely determines R_0 from equation (54). Moreover, all private sector behavior can be expressed as functions solely of E_0 and I_0^g. Before stating the government's optimization problem, it is useful to note that the first order conditions for household optimization imply that $c^y + pc^x = c/B$ where $c = (c^y)^\rho (c^x)^{1-\rho}$. Further, $\rho c/c^y = B$ where $B = \frac{\rho^\rho(1-\rho)^{1-\rho}}{p^{1-\rho}}$ is a positive constant. Hence, the government's problem can be formalized as:

$$\underset{\{E_0, I_0^g\}}{\text{Max}} \quad W(E_0, I_0^g) = (1 - 1/\sigma)^{-1} \int_0^\infty \left\{\left[c_t - \zeta((l_t^a)^\nu\right]^{1-1/\sigma} - 1\right\} e^{-rt} dt$$

subject to equations (6), (11), (12), (24), (31), (32), (47), (48), (53) and (54). Note that under a negative shock to α we must have $l^a = l^s$.

Conceptually, the recursivity of the problem implies that it can be solved in two stages. In the first stage one can compute the optimal level of intervention given any choice of I^g. This would yield an optimal intervention schedule as a function of I^g. In the second stage one can then choose I^g to maximize welfare subject to the optimal intervention schedule derived from the first stage. However, the inherent dynamics of the model due to slow labor adjustment makes it difficult to solve the model analytically. Instead, we numerically simulate the model to study the behavior of the optimal policy functions and their implications for the endogenous variables of interest to us, namely the nominal exchange rate and output.

For the simulations we assume that the transactions cost function is given by

$$v(\hat{h}) = \kappa - \hat{h}\left(F - G\log\hat{h}\right), \qquad F > 0, G > 0$$

where κ is a positive constant large enough to ensure that transactions costs are positive at all times. The key feature of this specification is that it generates the Cagan money demand function. In particular, the money demand equation given by (12) now reduces to

$$\hat{h} = e^{\frac{F-G}{G}} e^{-\frac{I^d}{\alpha BG}}. \tag{56}$$

In the baseline simulation of the model we set $r = 0.04$ and initial value of the money demand parameter to $\alpha = 1.$ and initial central bank reserves $\bar{R}$ both to zero. Note that $\bar{R} = 0$ implies that $\bar{E} = 1$ where $\bar{E}$ is the initial steady state nominal exchange rate.[12] We use α' to denote the post-shock value of the money demand parameter α. Hence, the lower is α' the larger is the negative money demand shock. As anticipated, the optimal response to the fall in α (the money demand shock) is for the policymaker to follow a partial insulation strategy using a combination of exchange rate depreciation, higher interest rates and sales of foreign reserves. The optimal policy now allows the real side of the economy to respond to the shock.

We then turn to the more interesting exercise of determining the response of the optimal policy mix to changes in key parameters. Figures 1-4 report

[12] The assumed values for the preference parameters are $\{\zeta, \sigma, \nu, \rho\} = \{0.9, .5, 1.12, 0.5\}$. The sector-$x$ production function parameters are $\{\theta, \psi\} = \{0.6, 1\}$ while the sector–y production function parameter is $\eta = 0.7$. The relative price of good-x is $p = 0.25$. The transactions costs parameters are $\{\kappa, F, G\} = \{1, 0.9, 1\}$. Lastly, we set $k_0 = 0$ and $\delta = 0.4$. We should note that we are only interested in the qualitative nature of the solutions. There is no attempt at replicating a particular economy.

simulations of these comparative static effects. Figure 1 depicts the effect of varying α' on the *optimal* post-shock solutions for four key variables – the level of intervention (Panel A), real balances (Panel B), the nominal exchange rate (Panel C), and the present discounted value of the implied output loss relative to the steady state level (Panel D). Figure 2 shows the corresponding plots when $\alpha' = 0.05$ but the credit-in-advance parameter ψ is varied. Figure 3 shows the comparative static effects of varying the semi-elasticity of money demand G while Figure 4 shows the effects of varying γ which is the speed of adjustment of nominal wages.

As can be seen from Figure 1, a bigger money demand shock (a smaller α') induces the policymaker to allow a bigger exchange rate depreciation and intervene more heavily. Moreover, it is optimal for the policymaker to now allow for a greater output contraction. Figure 2 shows that the more dependent is the economy on bank credit (a higher ψ) the more the exchange rate should be allowed to depreciate. In effect, the policymaker should allow most of the adjustment to occur through exchange rate movements and interventions rather than through interest rate policy. Figure 3 shows that the lower is the semi-elasticity of money demand (a higher G) the less useful is interest rate policy (Panel B). Hence, more of the adjustment should occur through echange rate depreciations and reserve sales. Lastly, Figure 4 says that the faster the speed of adjustment of nominal wages (a higher γ) the lower is the optimal level of intervention as well as the exchange rate depreciation. Further, the higher is γ the more one should raise the domestic interest rate, i.e., the greater should real balances be (Panel B). All these results accord with standard economic intuition.

6 Conclusions

The sequence of currency crisis episodes in the 1990's has brought to the fore the issue of the optimal design of policies to counteract currency pressures. The starting point for the work in this paper has been two distinct but related strands of the literature. First, central banks typically react to currency pressures by both raising domestic interest rates and by intervening in foreign exchange markets. Second, recent empirical work has unearthed fairly compelling evidence that even countries with *de jure* floating exchange rates behave like *de facto* fixed exchange rate economies – catchily referred to by Calvo and Reinhart (2000) as the "fear of floating". These two strands are naturally linked by the question of the optimal exchange rate regime with a pure float and a completely fixed peg just being two extremes amongst the possible regimes.

This paper has been an attempt at providing a rationalization for both the observed policy responses mentioned above. We have presented a model which endogenously generates a rational "fear of floating". In particular, the existence of a nominal wage rigidity implies that all nominal exchange rate fluctuations are contractionary (as opposed to only appreciations being contractionary as in the standard Mundell-Fleming model). This provides an incentive for the policymaker to prevent exchange rate fluctuations. We have shown that, in general, policymakers can accomodate monetary shocks through any combination of: (a) nominal exchange rate fluctuations which affect the level of domestic assets of the central bank; (b) interest rate policy to affect central bank liabilities; and (c) foreign exchange market intervention which alters the level of central bank foreign assets directly.

The paper has shown that the optimal policy mix depends on a number of key parameters. First, if interventions are costless then it is optimal for policymakers not to allow the exchange rate to fluctuate at all. This can be done through an appropriate mix of interest rate policy and intervention. Second, when interventions are costly, it is optimal to allow some exchange rate fluctuations. The precise policy mix depends on, amongst other factors, the size of the output cost of interest rate policy, the size of the money demand shock, the elasticity of money demand, and the speed of adjustment of nominal wages.

We view this work as the first step in developing full-fledged macroeconomic models which can then be calibrated to specific economies in order to derive sharp prescriptions regarding the design of optimal policy responses to shocks – both real and monetary. Developing richer models along these lines would be extremely useful for both understanding the actual responses observed in the data as well as for devising implementable and usable policy rules for central bankers.

Appendix This appendix establishes Lemma 2. Noting that $\hat{h} = L(\hat{I}^d)$, it follows that $\frac{d\hat{h}}{d\alpha} = \left(\frac{d\hat{I}^d}{d\alpha}\right) L'$ where $L' < 0$. Now $\frac{d\hat{I}^d}{d\alpha} = \frac{\partial \hat{I}^d}{\partial I^d}\frac{\partial I^d}{\partial I^g}\frac{\partial \tilde{I}^g}{\partial \alpha} - \frac{\hat{I}^d}{\alpha}$. By definition $\hat{I}^d = \frac{I^d}{\alpha B}$, while $I^d = \delta r - (1-\delta)I^g$ from equation (32). Hence,

$$\frac{d\hat{I}^d}{d\alpha} = -\left(\frac{1-\delta}{\alpha B}\right)\frac{\partial \tilde{I}^g}{\partial \alpha} - \frac{\hat{I}^d}{\alpha}$$

where $\frac{\partial \tilde{I}^g}{\partial \alpha} = \frac{-\partial \Gamma/\partial \alpha}{\partial \Gamma/\partial \tilde{I}^g}$. Differentiating equation (51) with respect to α and I^g and substituting in the resulting expressions gives (after some algebraic manipulation):

$$\frac{d\hat{I}^d}{d\alpha} = -\frac{\hat{I}^d}{\alpha}\left[1 - \frac{\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d}}{\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} + (1-\theta-\psi I^g)q\alpha\left(\frac{B\psi}{(1-\delta)(1-\theta)(1+\psi I^g)}\right)^2}\right] < 0 \quad (57)$$

Recall that under the conditions of Lemma 1 $\frac{\partial(\hat{h}\xi^h)}{\partial \hat{I}^d} > 0$ and $1-\theta > \psi I^g$ which establishes the fact that $\frac{d\hat{I}^d}{d\alpha} < 0$. Noting that $L' < 0$ completes the proof that along the optimal policy path we must have $\frac{d\hat{h}}{d\alpha} > 0$.

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Table 1: Volatility in exchange rates, international reserves, and nominal interest rates

Country	Period	Probability that the monthly change in ...		
		nominal exchange rate falls in +/- 2.5 % band	international reserves falls in +/- 2.5 % band	nominal interest rates falls in +/- 50 basis points
Indonesia	1997/7 - 4/1999	14.3	29.9	0
Korea	1997/11 - 4/1999	17.7	5.6	19.9
Thailand	1997/7 - 4/1999	38.1	40.9	9.1
Mexico	1994/12 - 1999/4	63.5	28.3	9.4
Peru	1990/8 -1999/4	71.4	48.1	32.3
Argentina	3/1991 - 4/1999	100.0	36.7	31.6
Japan	2/1973 - 4/1999	61.2	74.3	86.4

Source: Calvo and Reinhart (2000)

Figure 1: Variations in the money demand shock

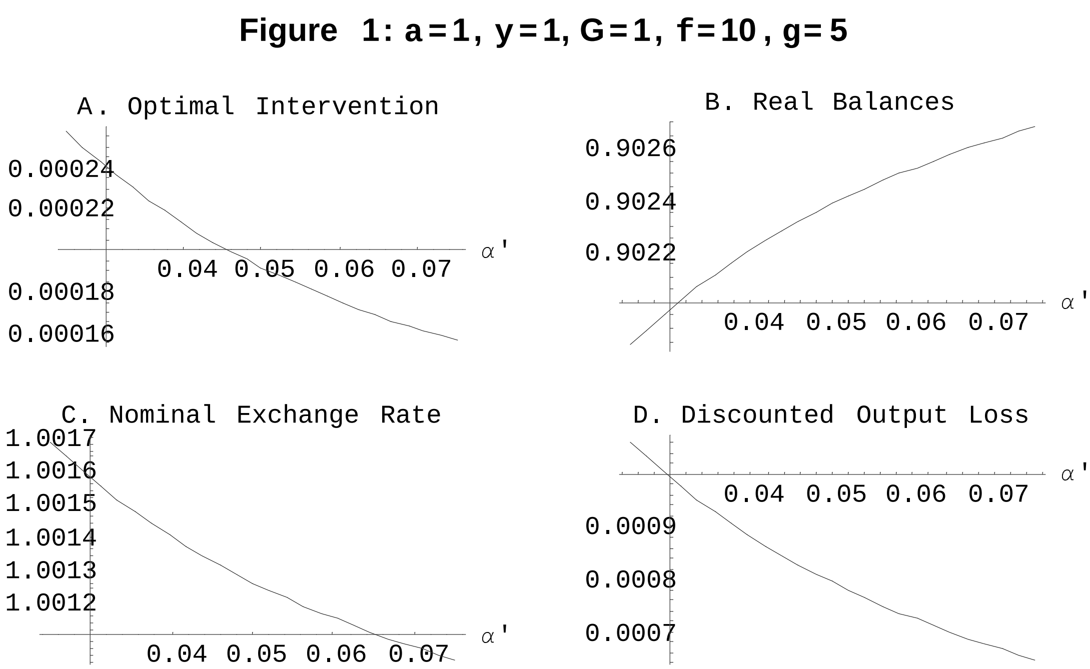

Figure 2: Dependence on bank credit

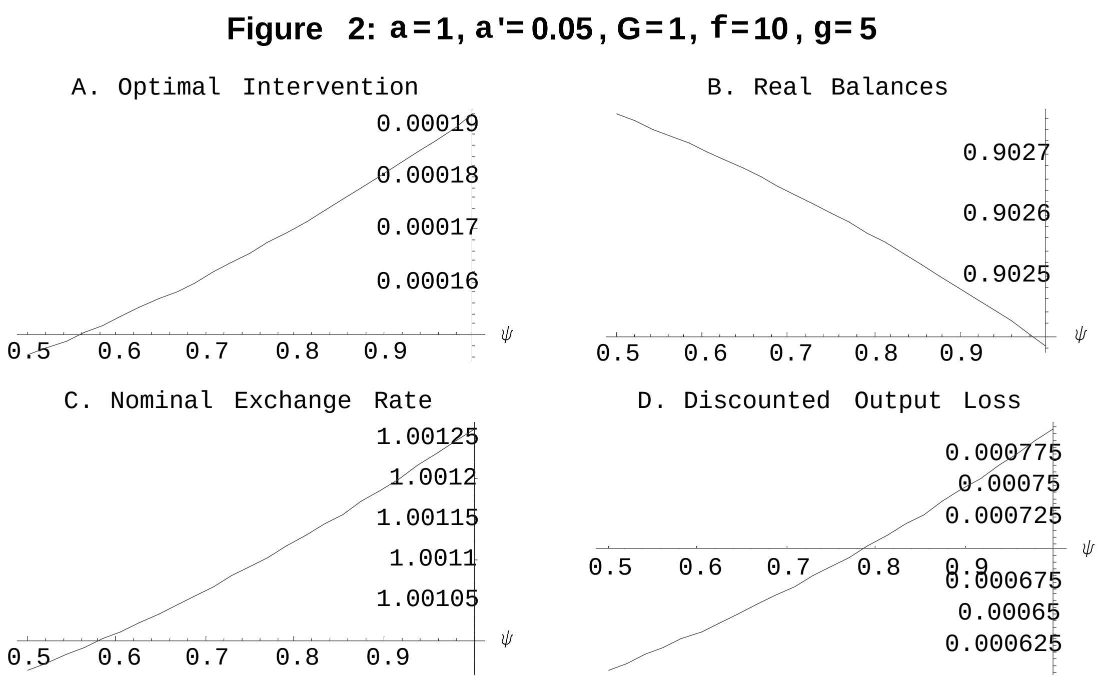

Figure 3: The semi-elasticity of money demand

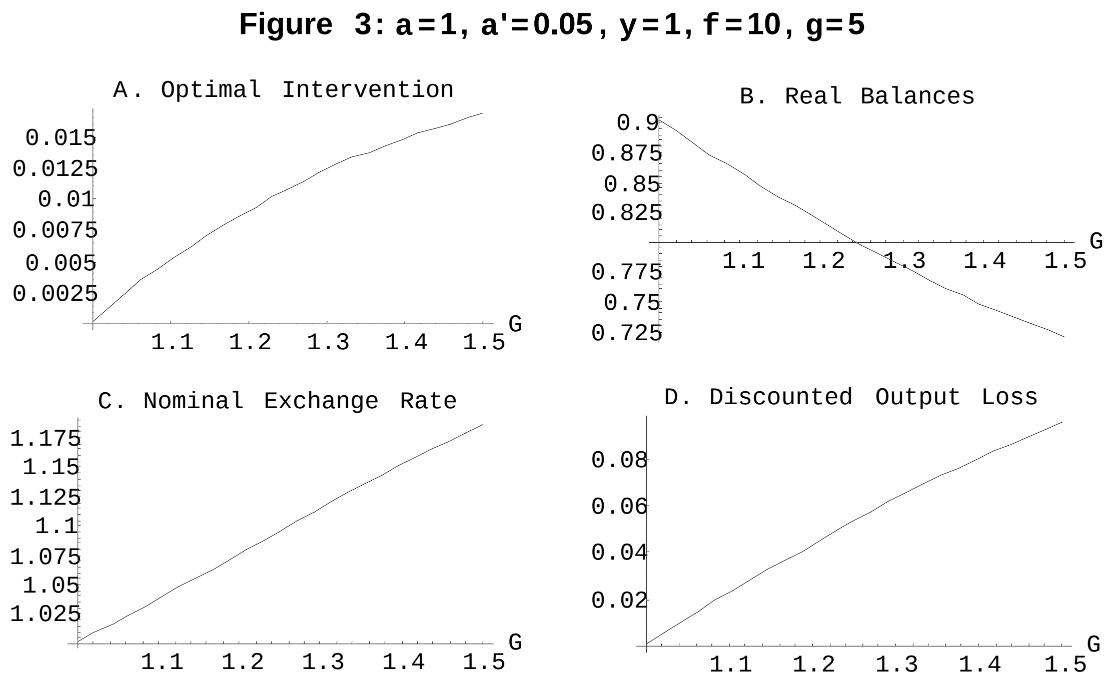

Figure 4: The speed of wage adjustment