Optimal Transport Networks in Spatial Equilibrium

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Introduction

- Central questions in international trade and economic geography involve counterfactuals with respect to trade costs

- Transport infrastructure is one important determinant of trade costs (e.g., Limao and Venables, 2001)
  - How should infrastructure investments be allocated across regions?
  - What are the aggregate gains?
  - How important are the inefficiencies in infrastructure investments?

- Answering these questions requires identifying the best set of infrastructure investments in a network
This Paper

- Develop a framework to study optimal transport networks in general equilibrium

- Solve a global optimization over the space of networks
  - given any primitive fundamentals
  - in a neoclassical trade framework (with labor mobility)

- Apply to actual road networks in 25 European countries
  - how large are the gains from expansion and the losses from misallocation of current networks?
  - how to these effects vary across countries?
  - what are the regional effects?
Key Features

- **Neoclassical Trade Model on a Graph**
  - Infrastructure impacts shipping cost in each link

- **Sub-Problems:**
  - how to ship goods through the network? ("Optimal Flows")
  - how to build infrastructure? ("Optimal Network")

- **Optimal flows = well known problem from Optimal Transport literature**
  - Numerically very tractable
  - Especially using dual approach (convex optimization in the space of prices)

- **Full problem (Flows+Network+GE) inherits numerical tractability**
  - Map infrastructure investments in each link to equilibrium prices
    - Sidestep direct search in space of networks
  - For sufficiency, add congestion in transport → global optimum
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Literature Background


- First-order welfare impact of changes in infrastructure: Allen and Arkolakis (2016)

- Optimization or search over networks: Felbermayr and Tarasov (2015), Alder (2016)


Here

- Global optimization over networks, in neoclassical environment, given any primitive fundamentals
- Related to optimal flow problem on a network (e.g., Chapter 8 of Galichon, 2016)

★ OT literature does not typically embed optimal-transport in GE or optimize over network
Model
Preferences and Technologies

- $\mathcal{J} = \{1, \ldots, J\}$ locations
  - $N$ traded goods aggregated into $c_j$
  - 1 non-traded in fixed supply (can make it variable)
  - $L_j$ workers located in $j$ (fixed or mobile)

- Homothetic and concave utility in $j$,
  $$U(c_j, h_j)$$

  where
  $$c_j L_j = C_j^T \left( C_j^1, \ldots, C_j^N \right)$$

  - $C_j^T(\cdot)$ homogeneous of degree 1 and concave

- Output of sector $n$ in location $j$ is:
  $$Y_j^n = F_j^n \left( L_j^n, V_j^n, X_j^n \right)$$

  - $F_j^n(\cdot)$ is either neoclassical or a constant
  - $V_j^n, X_j^n = \text{other primary factors and intermediate inputs}$

- Special cases
  - Ricardian model, Armington Specific-factors, Heckscher-Ohlin, Endowment economy, Rosen-Roback...
Underlying Graph

- The locations are arranged on an *undirected* graph
  - $\mathcal{J} = \{1, \ldots, J\}$ nodes
  - $\mathcal{E}$ edges

- Each location $j$ has a set $\mathcal{N}(j)$ of “neighbors” (directly connected)
  - Shipments flow through neighbors
  - “Neighbors” may be geographically distant
    - Fully connected case $\mathcal{N}(j) = \mathcal{J}$ is nested

- Example: square network, $\#\mathcal{N}(j) = 8$
Transport Technology

- Per-unit cost of shipping $Q^n_{jk}$ units of a commodity $n$ from $j$ to $k \in \mathcal{N}(j)$:
  - $\tau_{jk} \left( Q^n_{jk}, I_{jk} \right)$ nominated in units of good itself (iceberg)
  - (alternatively, cross-good congestion: $\tau_{jk} \left( \sum_n Q^n_{jk}, I_{jk} \right)$ nominated in the bundle of traded goods)

- Decreasing returns to transport:
  \[
  \frac{\partial \tau_{jk}}{\partial Q^n_{jk}} \geq 0
  \]
  - “Congestion” in short, may account for travel times, road damage, fixed factors in transport technologies...

- Returns to infrastructure:
  \[
  \frac{\partial \tau_{jk}}{\partial I_{jk}} < 0
  \]
  - On roads: number of lanes, whether road is paved, signals,...
  - The transport network is defined by $\{I_{jk}\}_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$
Decentralized Allocation Given the Network

- **Free entry of atomistic traders into shipping each** \( n \) **from** \( o \) **to** \( d \) **for all** \((o, d) \in J^2\)
  - Problem of traders shipping from \( o \) to \( d \)

\[
\min_{r=(j_0, \ldots, j_\rho) \in R_{od}} \left( p_{o}^n T_{r,0}^n + \sum_{k=0}^{\rho-1} p_{jk+1}^n t_{jk,i_{k+1}}^n T_{r,k+1}^n \right)
\]

- \( T_{r,k}^n \) **is the accumulated iceberg cost from** \( k \) **to** \( d \) **along path** \( r \) **(product of** \( 1 + \tau_{jk}^n \) **)**
- \( R_{od} \) **are all possible routes from** \( o \) **to** \( d \)
- Corresponds to minimum-cost route problem from gravity literature in the absence of taxes

- **Conservation of flows constraint**

\[
C_j^n + \sum_{n'} X_{j_{n'}}^n + \sum_{k \in \mathcal{N}(j)} \left( 1 + \tau_{jk}^n \right) Q_{jk}^n \leq Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n
\]

- **Exports**
- **Imports**

- Remaining features correspond to standard decentralized competitive equilibrium (given \( l_{jk} \))
Decentralized Allocation Given the Network

- Free entry of atomistic traders into shipping each $n$ from $o$ to $d$ for all $(o, d) \in J^2$

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  - Exports
  - Imports

- Remaining features correspond to standard decentralized competitive equilibrium (given $l_{jk}$)
Planner’s Problem Given Network (Labor Mobility)

**Definition**

The planner’s problem given the infrastructure network is

\[ W_0 \left( \{ l_{jk} \} \right) = \max_{c_j, L_j, V^n_j, X^n_j, L_j} \max u \]

subject to (i) availability of traded and non-traded goods,

\[ c_j L_j \leq C_j^T (C_j) \text{ and } h_j L_j \leq H_j \text{ for all } j; \]

(ii) the balanced-flows constraint,

\[ C^n_j + \sum_{n'} X^n_{jn'} + \sum_{k \in N(j)} \left( Q^n_{jk} + \tau^n_{jk} (Q^n_{jk}, l_{jk}) Q^n_{jk} \right) = F^n_j \left( L^n_j, V^n_j, X^n_j \right) + \sum_{i \in N(j)} Q^n_{ij} \text{ for all } j, n; \]

(iii) free labor mobility,

\[ L_j u \leq L_j U (c_j, h_j) \text{ for all } j; \]

(iv) local and aggregate labor-market clearing; and

(v) factor market clearing and non-negativity constraints.

**Proposition**

(Decentralization) Given the network, the welfare theorems hold under Pigovian taxes:

\[ 1 - t^n_{jk} = \frac{1 + \tau^n_{jk}}{1 + \tau^n_{jk} + \frac{\partial \tau^n_{jk}}{\partial Q^n_{jk}} Q^n_{jk}}. \]
Planner’s Problem Given Network (Labor Mobility)

**Definition**

The planner’s problem given the infrastructure network is

\[ W_0 (\{ I_{jk} \}) = \max_{c_j, L_j, V^n_j, X^n_j, L_{j}, Q^n_{jk}} \max u \]

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Features of Optimal Flows Problem

- Multiplier $P^n_j$ of conservation of flows constraint $=$ price of good $n$ in $j$ in the decentralization

- No-arbitrage conditions

  $$\frac{P^n_k}{P^n_j} \leq 1 + \tau^n_{jk} + \frac{\partial \tau^n_{jk}}{\partial Q^n_{jk}} Q^n_{jk}, \text{ if } Q^n_{jk} > 0$$

- Dual solution coincides with primal

  ▶ Dual = convex optimization with linear constraints in smaller space (just prices)
  ▶ Efficient algorithms are guaranteed to converge to global optimum (Bertsekas, 1998)
Network Building Technology

- Building infrastructure $l_{jk}$ takes up $\delta_{jk}^l l_{jk}$ units of a scarce resource ("asphalt")
  - Building cost $\delta_{jk}^l$ may vary across links
    - e.g. due to ruggedness, distance...
  - Asphalt in fixed aggregate supply $K$

- Bounds:
  $$0 \leq l_{jk} \leq \bar{l}_{jk} \leq \tilde{l}_{jk} \leq \infty$$
  - E.g. due to existing infrastructure or space restrictions

- Alternatively, can also impose endogenous supply of resources in infrastructure:
  $$\delta_{jk}^l l_{jk} = F^l \left( L^l_j + L^l_k, H^l_j + H^l_k \right)$$
Optimization over Transport Network

Definition

The full planner’s problem with labor mobility is

\[ W = \max \{ W_0 \, (\{ l_{jk} \}) \} \]

subject to:

(a) the network building constraint, \( \sum_j \sum_{k \in \mathcal{N}(j)} \delta_{jk} l_{jk} = K \); and

(b) the bounds \( \underline{l}_{jk} \leq l_{jk} \leq \bar{l}_{jk} \)

At the global optimum, the optimal network satisfies

\[ \mu \delta_{jk} \geq \sum_n P_j^n Q_{jk}^n \left( -\frac{\partial \tau_{jk}^n}{\partial l_{jk}} \right), \quad \text{if} \ l_{jk} > \underline{l}_{jk} \]

Reduces numerical search to space of prices → Full problem inherits tractability of optimal flows

Proposition

If the function \( Q_{\tau_{jk}} (Q, I) \) is convex in \( Q \) and \( I \), the full planner’s problem with mobile labor (immobile labor) is a quasiconvex (convex) optimization problem.

Ensures that our solution is indeed a global optimum for the transport network
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Example: Log-Linear Transport Technology

- Log-linear transport technology:
  \[ \tau_{jk}(Q, I) = \delta_{jk}^\tau \frac{Q^\beta}{I^\gamma} \]
  
  - Global convexity if \( \beta > \gamma \)

- Optimal network
  \[ I_{jk}^* \propto \left[ \frac{1}{\delta_{jk}^l \left( \delta_{jk}^\tau \right)^{\frac{1}{\beta}}} \left( \sum_{n: P^n_k > P^n_j} P^n_j \left( \frac{P^n_k}{P^n_j} - 1 \right)^{\frac{1+\beta}{\beta}} \right) \right]^{\frac{\beta}{\beta - \gamma}} \]
  
  where \( P^n_j \) are the equilibrium prices in GE
Additional Properties

1. Tree property in non-convex cases
2. Inefficiencies and externalities in the market allocation
Example: One Good on a Regular Geometry

One Traded Good, Endowment Economy, Output 10x Larger at Center, Uniform Fixed Population

(a) Population

(b) Productivity
Example: One Good on a Regular Geometry

Optimal Network, $K = 1$
Example: One Good on a Regular Geometry

Optimal Network, $K = 100$
Role of Building Costs

20 randomly placed cities

Building Cost: $\delta_{jk}^l = \delta_0 \text{Distance}_{jk}^{\delta_1}$
Role of Building Costs

Adding a mountain, a river, bridges, and water transport

Building Cost:  \( \delta_{jk}^I = \delta_0 \text{Distance}_{jk}^{\delta_1} \left( 1 + |\Delta \text{Elevation}_{jk}| \right)^{\delta_2} \delta_3^{\text{CrossingRiver}_{jk}}\delta_4^{\text{AlongRiver}_{jk}} \)
Application
Application

Questions

- What are the aggregate gains from optimal expansion of current networks?
- What would be the regional effects?
- How important are the inefficiencies in infrastructure investments?

Study these questions across European countries
Underlying Graph and Observed Infrastructure

- In 25 European countries we observe, at high spatial resolution
  - Road networks with features of each segment: number of lanes and national/local road (EuroGeographics)
  - Value Added (G-Econ 4.0)
  - Population (GPW)

- Construct the graph \((\mathcal{J}, \mathcal{E})\) and observed road network for each country
  - \(\mathcal{J}\): population centroids of 0.5 degree (\(\sim\)50 km) square cells
  - \(\mathcal{E}\): all links among contiguous cells (8 neighbors per node)
  - \(I_{jk}^{obs}\): observed infrastructure between all connected \(jk \in \mathcal{E}\)
Example: Spain

(a) Underlying Graph

(b) Actual Road Network

(c) Measured Infrastructure $I_{jk}^{obs}$
Calibration

- Production technologies: $Y^n_j = z^n_j L^n_j$

- Transport technologies: $\tau^n_{jk} = \delta^n_{jk} \left( \frac{Q^n_{jk}}{I^n_{jk}} \right)^\beta$
  - Geographic frictions $\delta^n_{jk} = \delta^n_0 \text{dist}_{jk}$ to match intra-regional share of intra-national trade in Spain
  - Congestion $\beta = 1.24$ to match elasticity of travel times to road use (Wang et al., 2011)
  - Returns to infrastructure: $\gamma \in \{0.5\beta, \beta, 1.5\beta\}$

- Preferences: $U(c, h) = c^\alpha h^{1-\alpha}$
  - $N = 10$ tradeable sectors with CES demand ($\sigma = 5$)

- Fundamentals: $\{z_j, H_j\}$ such that $\{GDP_{j}^{obs}, L_{j}^{obs}\}$ is the model’s outcome given $\{I_{jk}^{obs}\}$
  - Model fit
  - Trade-distance elasticity (not targeted) close to 1

- Building costs $\delta^I_{jk}$: 2 approaches
  - Assume that observed road networks are optimal → Back out $\delta^I_{jk}^{FOC}$ from FOC’s using $I_{jk}^{obs}$
  - Use estimates from Collier et al. (2016) → Set $\delta^I_{jk}^{GEO}$ as function of distance and ruggedness
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- **Production technologies:** $Y_j^n = z_j^n L_j^n$

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  - Trade-distance elasticity (not targeted) close to 1

- **Building costs $\delta_{jk}^I$**: 2 approaches
  - Assume that observed road networks are optimal $\rightarrow$ Back out $\delta_{jk}^{I, FOC}$ from FOC’s using $I_{jk}^{obs}$
  - Use estimates from Collier et al. (2016) $\rightarrow$ Set $\delta_{jk}^{I, GEO}$ as function of distance and ruggedness
Optimal Expansion and Reallocation

- **Optimal expansion**
  - Increase $K$ by 50% relative to calibration in every country
  - Build on top of existing network ($I_{jk} = I_{jk}^{obs}$)
  - Using both $\delta_{jk}^{I,FOC}$ and $\delta_{jk}^{I,GEO}$

- **Optimal reallocation**
  - $K$ is equal to calibrated model
  - Build anywhere ($I_{jk} = 0$)
  - Using $\delta_{jk}^{I,GEO}$
### Average Aggregate Effects

Each element of the table shows the average welfare gain in the corresponding counterfactual across the 25 countries in our data.

<table>
<thead>
<tr>
<th>Returns to Scale:</th>
<th>$\gamma = 0.5\beta$</th>
<th>$\gamma = \beta$</th>
<th>$\gamma = 1.5\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Labor:</strong></td>
<td>Fixed</td>
<td>Mobile</td>
</tr>
<tr>
<td></td>
<td><strong>Optimal Reallocation</strong></td>
<td>$\delta = \delta_l,GEO$</td>
<td>3.2%</td>
</tr>
<tr>
<td></td>
<td><strong>Optimal Expansion</strong></td>
<td>$\delta = \delta_l,GEO$</td>
<td>3.9%</td>
</tr>
<tr>
<td></td>
<td>$\delta = \delta_l,FOC$</td>
<td></td>
<td>1.2%</td>
</tr>
</tbody>
</table>
Cross-Country Effects

Optimal Reallocation, $\delta^{GEO}$

Linear regression slope (robust SE): Mobile Labor: -2.523 (1.269); Fixed Labor: -2.035 (.712)
Regional Effects (Spain)

Optimal Reallocation
Regional Effects (Spain)

Optimal Expansion
Regional Effects (Spain)
Optimal Expansion, Non-Convex
Where is infrastructure placed?

**Dependent variable:** Infrastructure growth in the counterfactual

<table>
<thead>
<tr>
<th></th>
<th>Reallocation</th>
<th>Expansion ($\delta = \delta^{I,GEO}$)</th>
<th>Expansion ($\delta = \delta^{I,FOC}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>0.308***</td>
<td>0.104***</td>
<td>0.004</td>
</tr>
<tr>
<td>Income per Capita</td>
<td>0.127</td>
<td>0.007</td>
<td>-0.020</td>
</tr>
<tr>
<td>Consumption per Capita</td>
<td>0.290**</td>
<td>0.179***</td>
<td>0.130</td>
</tr>
<tr>
<td>Infrastructure</td>
<td>-0.362***</td>
<td>-0.195***</td>
<td>-0.067**</td>
</tr>
<tr>
<td>Differentiated Producer</td>
<td>0.271***</td>
<td>0.133***</td>
<td>-0.099***</td>
</tr>
</tbody>
</table>

$R^2$  

0.38  

0.32  

0.38  

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.
Which regions grow?

Dependent variable: Employment growth in the counterfactual

<table>
<thead>
<tr>
<th></th>
<th>Reallocation</th>
<th>Expansion ($\delta = \delta^{l,GEO}$)</th>
<th>Expansion ($\delta = \delta^{l,FOC}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.002**</td>
</tr>
<tr>
<td>Income per Capita</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.030**</td>
</tr>
<tr>
<td><strong>Consumption per Capita</strong></td>
<td><strong>-0.147</strong>*</td>
<td><strong>-0.139</strong>*</td>
<td><strong>-0.179</strong>*</td>
</tr>
<tr>
<td>Infrastructure</td>
<td>0.002</td>
<td>0.005***</td>
<td>0.000</td>
</tr>
<tr>
<td>Infrastructure Growth</td>
<td>0.013*</td>
<td>0.032**</td>
<td>0.003**</td>
</tr>
<tr>
<td>Differentiated Producer</td>
<td>0.013**</td>
<td>0.023***</td>
<td>0.031***</td>
</tr>
</tbody>
</table>

$R^2$ 0.57 0.67 0.90

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.
Conclusion

- We develop and implement a framework to study optimal transport networks
  1. Neoclassical model (with labor mobility) on a graph
  2. Optimal Transport with congestion
  3. Optimal Network

- Application to road networks in Europe
  - Larger gains from optimal expansion and losses from misallocation in poorer economies
  - Optimal expansion of current road networks reduces regional inequalities

- Other potential applications
  - Political economy / competing planners
  - Model-based instruments for empirical work on impact of infrastructure
  - Optimal investments in developing countries
  - Optimal transport of workers
  - Absent forces: agglomeration, dynamics
Planner’s Problem (Immobile Labor)

**Definition**

The planner’s problem without labor mobility given the infrastructure network is

\[
W_0 (\{ l_{jk} \}) = \max_{c_j, L_j, V^n_j, X^n_j} \max_{Q^n_{jk}} \sum_j \omega_j L_j U (c_j, h_j)
\]

subject to (i) availability of traded and non-traded goods,

\[
c_j L_j \leq C_j^T (C_j) \text{ and } h_j L_j \leq H_j \text{ for all } j;
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(ii) the balanced-flows constraint,

\[
C_j^n + \sum_{n'} X^n_{jn'} + \sum_{k \in \mathcal{N}(j)} \left( Q^n_{jk} + \tau^n_{jk} (Q^n_{jk}, l_{jk}) Q^n_{jk} \right) = F_j^n (L^n_j, V^n_j, X^n_j) + \sum_{i \in \mathcal{N}(j)} Q^n_{ij} \text{ for all } j, n;
\]

(iii) local labor-market clearing; and

(iv) factor market clearing and non-negativity constraints.
The “price field” decentralizes the optimal flows

- Flows follow the gradient of prices
- Consumption locations appear as local peaks of the field
Fictitious Planner Problem with Externalities

- We may assume that the market allocation is inefficient, e.g. due to
  - No corrective taxes: $t^n_{jk} = 0$
  - Externalities from population: $F^n_j (\cdot; L_j)$

**Definition**

The fictitious-planner’s problem with externalities given the infrastructure network is

$$W_0 \left( \{ l_{jk} \}; \overline{L}, \overline{Q} \right) = \max_{c_j, l_j, v^n_j, x^n_j, L_j} \max_u \left( \sum_{n} Q^n_{jk} \right)$$

subject to conditions (i), (iii), (iv), (v) from before, and

$$C_j^n + \sum_{n'} X_{j}^{nn'} + \sum_{k \in \mathcal{N}(j)} \left( Q^n_{jk} + \tau^n_{jk} \left( Q^n_{jk}, l_{jk} \right) Q^n_{jk} \right) = F^n_j \left( L_j, v^n_j, x^n_j; \overline{L}_j \right) + \sum_{i \in \mathcal{N}(j)} Q^n_{ij} \text{ for all } j, n.$$

- This problem satisfies the same convexity+duality properties of the standard planner’s given the network.
- Letting $L_0 \left( \overline{L}, \overline{Q} \right)$ and $Q_0 \left( \overline{L}, \overline{Q} \right)$ be the solution to the fictitious-planner problem with externalities:

**Proposition**

(Decentralization with Externalities) $\left( \overline{L}, \overline{Q} \right)$ corresponds to an inefficient market allocation if and only if $\overline{L} = L_0 \left( \overline{L}, \overline{Q} \right)$ and $\overline{Q} = Q_0 \left( \overline{L}, \overline{Q} \right)$. 
Fictitious Planner Problem with Externalities

- We may assume that the market allocation is inefficient, e.g. due to
  - No corrective taxes: \( t^n_{jk} = 0 \)
  - Externalities from population: \( F^n_j (\cdot; L_j) \)

**Definition**

The fictitious-planner’s problem with externalities given the infrastructure network is

\[
W_0 \left( \{ I_{jk} \}; \overline{L}, \overline{Q} \right) = \max_{C_j, L_j, V^n_j, X^n_j, Q^n_{jk}} \max_u \left( C_j, L_j, V^n_j, X^n_j, Q^n_{jk} \right)
\]

subject to conditions (i), (iii), (iv), (v) from before, and

\[
C^n_j + \sum_{n'} X^n_{jn'} + \sum_{k \in N(j)} \left( Q^n_{jk} + \tau^n_{jk} \left( \overline{Q^n_{jk}}, I_{jk} \right) Q^n_{jk} \right) = F^n_j \left( L^n_j, V^n_j, X^n_j, \overline{L}_j \right) + \sum_{i \in N(j)} Q^n_{ij} \text{ for all } j, n.
\]

- This problem satisfies the same convexity+duality properties of the standard planner’s given the network.
- Letting \( L_0 \left( \overline{L}, \overline{Q} \right) \) and \( Q_0 \left( \overline{L}, \overline{Q} \right) \) be the solution to the fictitious-planner problem with externalities:

**Proposition**

(Decentralization with Externalities) \( \left( \overline{L}, \overline{Q} \right) \) corresponds to an inefficient market allocation if and only if

\[
\overline{L} = L_0 \left( \overline{L}, \overline{Q} \right) \text{ and } \overline{Q} = Q_0 \left( \overline{L}, \overline{Q} \right).
\]
Nonconvex Case: Economies of Scale in Transport

- What if $\gamma > \beta$, i.e., returns to transport technology are increasing?
  - KKT no longer sufficient: local vs. global optimality
- Computations show the network becomes extremely sparse
  - The planner prefers to concentrate flows on few large “highways” (branched transport)

**Proposition**

(Netowrk Shape in Non-Convex Cases) *In the absence of a pre-existing network (i.e., $I^0_{jk} = 0$), if the transport technology is satisfies $\gamma > \beta$ and there is a unique commodity produced in a single location, the optimal transport network is a tree.*

- Intuition: cycles are sub-optimal because it pays off to remove an edge and concentrate flows elsewhere

![Diagram](attachment:image.png)

(a) tree (no loops)  
(b) non-tree
Tree Property with Economies of Scale: Intuition

- Two types of elementary cycles can occur:

  ![Case (i) and Case (ii) diagrams](image)

  - **Argument:**
    - Cycles of type (i) waste resources and can be ruled out
    - With cycles of type (ii), better off to redirect flows to one branch:

  ![Initial and After redirection diagrams](image)
Computation via Duality

- FOCs: nonlinear system of many variables
  
  **Primal**
  \[
  \sup_{C,L,Q} \inf_P \mathcal{L}(C,L,Q;P)
  \]
  
  ▶ Even if convex, slow to converge (high dimension in \(C, L, Q\))

- **Dual**
  \[
  \inf_P \mathcal{L}(C(P), L(P), Q(P); P)
  \]
  
  ▶ Use FOCs and substitute for \(C, Q, \ldots\), as function of \(P\), then minimize over Lagrange multipliers
  
  ▶ Convex minimization problem in fewer variables with just non-negativity constraints (just \(P\))
Congestion Parameter $\beta$

- A well documented empirical relationship in transportation science is the speed-density relationship.
- Using data from various segments of US roads, Wang et al. (2011) estimate the logistic relationship:

$$speed = v_b + \frac{v_f - v_b}{1 + \exp \left( \frac{density - k_t}{\theta_1} \right)^{\theta_2}}$$

- Assuming that transport costs are proportional to travel times (inverse of speed) and density is proportional to flows $Q$, we can identify $\beta$ in

$$\log \text{ (travel time per km)} = const + \beta \times \log \text{ (density)}$$

and find $\beta = 1.2446$

*Figure: Pooled estimates of travel times against density*
Discretization of the Observed Network

Road network from EuroGeographics data provides the following characteristics:

- Number of lanes, national/primary/secondary road, paved/unpaved, median, etc.

For each pair of locations (j,k), we identify the least cost route on the observed network using relative user costs from Combes and Lafourcade (2005).

In the discretized network, we construct an infrastructure index

\[ I_{jk} = \text{lanes}_{jk} \times \chi_{nat}^{1-nat_{jk}} \]

where

- \( \text{lanes}_{jk} \) is the average observed number of lanes on the least cost route from j to k
- \( nat_{jk} \in [0, 1] \) is the fraction of that route spent on national roads
- \( \chi_{nat} \) is the building and maintenance cost per km. of national roads relative to other types (Doll et al., 2008)
### Average Infrastructure Across Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Code</th>
<th>Length (Km.)</th>
<th>Number of Segments</th>
<th>Average Lanes per Km.</th>
<th>Number of Cells</th>
<th>Length (Km.)</th>
<th>Average Infrastructure Index</th>
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</tbody>
</table>

- **Infrastructure index:**
  - Across countries: correlated with income per capita
  - Within countries: 0.7 correlation with travel times (GoogleMaps) across all pairs

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Average Infrastructure and Income Per Capita

Linear regression slope (robust SE): 10200.4 (3603.5)
Average Infrastructure and Speed

Regression slope (robust SE): 5.445 (.188). Pools all links. Includes country fixed effects.
The table reports the average and standard deviation of the trade-distance and elasticity and intra-regional trade share across the 25 countries in our data.
Model Fit

Linear regression slope (robust SE): Mobile Labor: 1.025 (.006)

Linear regression slope (robust SE): Mobile Labor: 1.029 (.007); Fixed Labor: .992 (.01)

Return
Cross-Country Effects
Optimal Expansion, $\delta^{GEO}$

Linear regression slope (robust SE): Mobile Labor: -2.427 (1.209); Fixed Labor: -1.928 (.584)
Cross-Country Effects

Optimal Expansion, $\delta^{FOC}$

Linear regression slope (robust SE): Mobile Labor: -.715 (.337); Fixed Labor: -.549 (.191)