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Econ 401a Final Examination

(minor revisions after exam)

Time allowed: three (3) hours. Attempt four (4) questions only. Indicates on the outside of your blue book the four answers to be graded. There is a ten minute reading period. GOOD LUCK!

1. Choice over time

Consider a two period model in which x_1 is first period consumption and x_2 is second period consumption. The interest rate on lending and borrowing is r .

(a) If a consumer has an income stream of $y=(y_1,y_2)$ show that she faces a single life-time budget constraint. (2 pts)

(b) Depict this constraint and the consumer's choice if she is a saver. What are the income and substitution effects of an increase in the interest rate. Are they opposing or reinforcing (i) for x_1 (ii) for x_2 ? (You should assume that both commodities are "normal goods".) (3 pts)

(c) Would the answers be any different if the consumer were a borrower? (1 pt)

(d) Suppose that the utility function is $U(x_1,x_2)=x_1^{1/2}+x_2^{1/2}$. Solve for the equilibrium consumption bundles as a function of the interest rate and the present value of the income stream $W=y_1+\frac{y_2}{1+r}$.

(3 pts)

(e) Generalize the model and solve for the equilibrium consumption bundle if there are three periods and $U(x_1,x_2)=x_1^{1/2}+x_2^{1/2}+x_3^{1/2}$. Again solve as a function of the interest rate and the present value of the three period income stream $W=y_1+\frac{y_2}{1+r}+\frac{y_3}{(1+r)^2}$. (1 pt)

2. Walrasian Equilibrium with production

There are three commodities. Commodities 1 and 2 are used both as inputs in the production of commodity 3 and consumed. The aggregate endowment of the three commodities is $\omega=(80,4,0)$. All consumers have the same utility function

$$U(x^h)=3\ln x_1^h+2\ln x_2^h+\ln x_3^h .$$

The production function for commodity 3 is $q_3=24z_1^{1/3}z_2^{2/3}$.

(a) Explain carefully why the WE of this economy can be solved by considering the demand of a "representative consumer" (2 pts)

(b) Solve for the utility maximizing inputs and consumption levels. (4 pts)

(c) If $p_3 = 1$, what prices of the inputs are consistent with profit-maximization? (3 pts)

(d) For these three prices, are the necessary conditions for utility maximization satisfied? Prove your claim. (1 pt)

3. State claims equilibrium prices and asset prices

There are three (weather) states and two plantations that produce coconuts. The vector of "returns" for asset a is $z^a = (40,10,3)$ and for asset plantation b is $z^b = (24,15,6)$ so that the aggregate endowment of coconuts (in thousands) in each state is $\omega = (64,25,9)$. Each consumer has the same utility function $u(x) = x^{1/2}$. The probability vector is $\pi = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. There are three markets, one for each state.

(a) Show that the expected utility function $U(x,\pi) = U(x_1, x_2, x_3, \pi_1, \pi_2, \pi_3)$ is a homothetic function. (2 pts)

(b) Solve for the Walrasian Equilibrium state claims price if $p_1 = 1$. (5 pts)

(c) What is the market value of each asset? (1 pt)

(d) If a consumer's initial holdings of asserts are 1% of the value of the entire market, could the consumer achieve his optimum by holding 1% of each asset? Explain. (1 pt)

(e) Discuss whether trading only the two assets in a stock market would yield the same outcome as trading in the state claims markets. (1 pt)

4. Production and cost

A firm has a fixed cost of 64 and a production function $q = F(z) = z_1^{1/4} z_2^{1/2}$. The input price vector is $r = (4,8)$.

(a) If the manager has a budget of B solve for the maximum output. (4 pts)

(b) Explain why you can use your answer to solve for the cost function of the firm. (1 pt)

(c) Show that it has the form $C(q) = 64 + Aq^{4/3}$ and solve for the parameter A. (2 pts)

(d) Depict both MC and AC in a neat figure. (2 pts)

(e) If there is free entry into the industry, what will be the equilibrium price? (1 pt)

5. Quantity competition

Two firms produce products that are imperfect substitutes. Each for has a marginal cost of 10. The demand price functions are as follows:

$$p_1 = 70 - 2q_1 - q_2 \text{ and } p_2 = 70 - q_1 - 2q_2 .$$

(a) If firms choose their output levels, show that firm 2's best response to firm 1 is

$$q_2^b(q_1) = 15 - \tfrac{1}{4}q_1 \quad \text{(1 pt)}$$

(b) Solve for the Nash equilibrium quantities and hence prices of this game. (3 pts)

(c) Next suppose that there are three firms each with MC=10 and

$$p_1 = 70 - 2q_1 - (q_2 + q_3)\ ,\ \ p_2 = 70 - 2q_2 - (q_1 + q_3)\ ,\ p_3 = 70 - 2q_3 - (q_1 + q_2)$$

Solve for the equilibrium quantities (and hence prices). (2 pts)

(d) Returning to the two firm case, suppose that firm 1 chooses its output first. Having observed this output, firm 2 chooses its best response, $q_2^b(q_1)$ (See part (a)). Write down an expression for firm 1's profit

$$U_1(q_1, q_2^b(q_1)). \quad \text{(1 pt)}$$

(e) Solve for firm 1's new profit-maximizing output. (2 pts)

(f) Provide the intuition as to why this output is greater than the output in (b). (1 pt)

6. All pay auction

There are two buyers. Values are uniformly distributed on the interval $[1,2]$. Each buyer submits a bid $b \geq 0$. The high bidder wins and both must pay their bid.

(a) Explain carefully why the equilibrium marginal payoff is

$$U'(\theta) = \theta - 1.$$

(You may wish to use a diagrammatic explanation. (2 pts)

(b) Solve for the equilibrium payoff and hence for the equilibrium bid function. (4 pts)

(c) Suppose next that the seller chooses the minimum bid $\underline{b}$ so that a buyer with value $\frac{3}{2}$ is indifferent between bidding and staying out. What is this minimum bid? (2 pts)

(d) Solve for the new equilibrium bid function. (2 pts)

7. Auction with multiple units for sale

There are three identical units for sale. There are 4 buyers. Each wishes to obtain one unit. Buyer i's value is a random draw from a uniform distribution with values in $\Theta = [0,1]$. The item is to be sold by sealed bid auction. The three high bidders will each win one item. If there is more than one low bidder, then the loser with be chosen randomly from the low bidders.

(a) Suppose that the three winners must pay the highest losing bid. Is it an equilibrium strategy for the buyers to bid their true values? (2 pts)

(b) Is this a dominant strategy equilibrium? (1 pt)

(c) Suppose instead that each winner must pay his or her own bid. Explain why the equilibrium probability of losing is $(1-\theta)^3$. (1 pt)

(d) Solve for the equilibrium expected payoff $U(\theta)$. (3 pts)

(e) Hence solve for the equilibrium bid function $B(\theta)$. (2 pts)

(f) Comment on whether expected revenue would be lower in the auction in which each pays only the losing bid. (1 pt)