Econ 401A: Microeconomic Theory

Final Exam

Answer four (4) questions only. If you are not sure of the meaning of a question please talk to a proctor.

You must write your answers in a blue book (or books). Please make sure that you indicate the questions to be graded on the cover of the bluebook as well as your name and ID. Please do not hand in the questions.

1. Cost function and monopoly

A monoply is a price taker in input markets. The input price vector is $r=(4,9)$ .The production function is $q=F(z)=z_1^{1/4}z_2^{1/4},\ z\geq 0$. The firm faces a demand price fucntion $p(q)=1200-12q$.

(a) Solve for the cost function of the firm.

(b) Depict the average and marginal cost curves in a neat figure.

(c) Solve for the profit maximizing output and output price.

(d) Suppose instead that no output can be produced without at least 12 units of input 1. The production function is $q=F(z)=(z_1-12)^{1/4}z_2^{1/4},\ z_1\geq 12,\ z_2\geq 0$. Solve for the cost function of the firm.

(e) Solve for the new profit maximizing output and output price**.**

(f) Suppose that $F(z)=(z_1-k)^{1/4}z_2^{1/4},\ z_1\geq k,\ z_2\geq 0$ where $k>12$. How would the profit-maximizing output be affected?

2. Walrasian equilibrium in a homothetic economy.

(a) What is a homothetic utility function?

(b) Let $x(p,I)$ be the utility maximizing choice of a consumer with income I . If utility is homothetic show that $x(p,I)=Ix(p,1)$.

Commodities 1 and 2 are both consumed and used as inputs. Commodity 3 is a consumption good. Commodity 4 is an input. The endowment vector in an economy is $\omega=(32,32,0,16)$. Commodity 3 is produced using the other three commodities as inputs. The production function is

$q_3=z_1^{1/4}z_2^{1/4}z_4^{1/2}$. The utility function is $U(x^h)=\ln x_1^h+\ln x_2^h+4\ln x_3^h$.

(c) Solve for the optimal inputs and hence the optimal consumption vector of the representative agent.

(d) If $p_1=1$, solve for the WE prices of commodities 2 and 3.

(e) What is the WE price of commodity 4?

3. Asset price and state claims prices

Consider a simple economy with two states and three assets. You may think of the assets returns as the number of coconuts on a plantation in each state. In state 2 the returns are lower because of a typhoon.

The asset a return is $z_a = (400,100)$. The asset b return is $z^b = (500,300)$. The asset c return is $z^c = (600,300)$. The probability of state 1 is $\frac{2}{3}$. The prices of assets b and c are

$$P_b = 2100 \text{ and } P_c = 2400 .$$

(a) Design a mutual fund containing shares in only assets b and c that pays off 10 in state 1 and zero in state 2.

(b) Design a second mutual fund containing shares in only assets b and c that pays off zero in state 1 and 10 in state 2.

(c) What is the market value of each mutual fund?

(d) Design a fund that pays off the 10 in each state (so it is a riskless asset.) What is its market value?

(e) What is the implied market value of (i) a unit claim to state 1 and (ii) a unit claim to state 2.

(f) What can you say about the market value of asset a ?

(g) Alex owns asset A and has no other asset holdings. His utility function is $v(x) = \ln(80 + x)$. If there were explicit markets in state claims, what optimization problem would he solve? (You do not have to actually solve the problem.)

(g) In the absense of explicit markets in state claims, could Alex purchase (or short sell) assets in the stock market and achieve the same outcome? Explain carefully.

4. Pareto Efficiency in an exchange economy

Alex and Bev have the same utility function

$U(x^h) = -e^{-x_1^h} - e^{-x_2^h}$. The aggregate endowmwent is $\omega = (100, 200)$.

(a) What must be true if the allocation $\{x^A, x^B\}$ is Pareto Efficent. Explain using an Edgeworth Box diagram.

(b) Show that if an allocation is PE then $x_2^A - x_1^A = k$ for some number k (to be determined).

(c) Suppose that the individual endowments are is $\omega^A = \omega^B = (50,100)$. Are these endowments Pareto effcient? Explain

(d) With these endowments, what is the WE price ratio? How much trade would there be?

(e) If the endowments were different, might the WE price ratio change? Explain carefully.

5. Strategic choice

$$p_1 = 38 - q_1 - \alpha q_2 \ , \ p_2 = 44 - \alpha q_1 - q_2 \ ,$$

$$C_1(q_1) = 6q_1 \ , \ C_2(q_2) = 6q_2$$

Suppose firms compete by setting quantities.

(a) What is a Nash equilibrium?

(b) What conditions must necessarily hold if $\overline{q} = (\overline{q}_1, \overline{q}_2)$ is a Nash equilibrium?

(c) If $\overline{q}$ is the unique NE whan the game is played once, is it still the unique NE whan the game is repeated 6 times? Explain carefully.

(d) If $\alpha = \frac{1}{2}$ depicted the best reponse lines in a neat figure with q_1 on the horizontal and q_2 on the vectical axis.

(e) Solve for the unique NE.

(f) Using the figure (or any other method) discuss the effect of a reduction in the parameter α .

6. Sealed bid auctions

Each buyer's value is between zero and 1 (million) and is a random draw from the distribution with c.d.f. $F(\theta) = \Pr\{v_i \leq \theta\} = \theta^2$.

(a) Explain carefully how to solve for the equilibrium bid function in a sealed high-bid auction with two buyers.

(b) If there are two buyers solve for the equilibrium bid function (It is linear.)

(c) If there are three buyer solve again for the equilibrium bid fucntion.

(d) The seller changes the rules of the auction. Every buyer must submit a **non-refundable** cash bid. As before the high bidder wins. Discuss how you might modify your answer to (a) to solve for the equilibrium bid function.