Final exam

Time allowed: Three (3) hours. There is also a 10 minute reading period. If you use a pencil make sure that your answers are clearly readable. There are six (6) questions. Your grade will be based on your answers to four (4) questions. Please help the grader by writing down the numbers of the four answers to be graded on the front cover of your blue book.

If you finish four questions and want to compete for a small prize, you may answer extra questions. On the cover you must clearly indicate which answers are extra answers. These will be considered by me only after the grades have been assigned.

1. **Production and cost**

A firm has a production function

$$q = F(z) = (48z_1)^{1/2} + (48z_2)^{1/2} + (48z_3)^{1/2} = 48^{1/2}(z_1^{1/2} + z_2^{1/2} + z_3^{1/2}) \ .$$

The price of each input is 4.

(a) If the manager of a firm is given a budget of B and told to maximize output what would his input choice be and what would be the maximized output?

(b) Provide a definition of the cost function of a firm with a production function, $q = F(z)$.

(c) What is the cost function of this firm?

(d) Suppose that the firm is a monopoly. The demand price function is $p = 2 - \frac{1}{36}q$. Solve for the profit-maximizing output $q(p)$.

Suppose instead that the production function is

$$q = F(z)^2 = [(48z_1)^{1/2} + (48z_2)^{1/2} + (48z_3)^{1/2}]^2 .$$

(e) What is the new cost function?

(f) Suppose that there are many competing firms with the same production function. What will be the equilibrium price?

2. **Pareto Efficient allocations**

$U^a(x^a) = x_1^a + 50\ln x_2^a$, $U^b(x^b) = x_1^b + 150\ln x_2^b$. The aggregate endowment is $\omega = (100, 200)$.

Let $\bar{x}^1 >> 0$ and $\bar{x}^2 >> 0$ be a Pareto Efficient allocation.

(a) Use an Edgeworth Box diagram to explain what must be true for an allocation inside the box to be a PE allocation.

(b) Solve for these PE allocations and depict them in the Edgeworth Box.

(c) What is the supporting price ratio for these allocations?

(d) Are there other PE allocations? Explain.

(e) Suppose that consumer 1's endowment is $\omega^a = (20,80)$. What is the WE price ratio?

(f) Suppose that consumer 1's endowment is $\omega^a = (70,170)$. Is the price ratio in (e) again the WE price ratio?

3. The first welfare theorem for an exchange economy

Consumer $h,\ h=1,...,H$ has a strictly increasing utility function. $U^h(x^h)$. Consumer h has an endowment vector ω^h. Let p be the WE price vector. Let $\{\bar{x}^h\}_{h=1}^H$ be a WE allocation.

(a) What does it mean for the allocation $\{\bar{x}^h\}_{h=1}^H$ to be a WE allocation?

(b) What does it mean to say that the allocation $\{x^h\}_{h=1}^H$ Pareto dominates $\{\bar{x}^h\}_{h=1}^H$?

(c) What is a Pareto Efficient allocation?

(d) Suppose that $U^h(x^h) > U^h(\bar{x}^h)$. Explain clearly why $p\cdot x^h > p\cdot \bar{x}^h = p\cdot \omega^h$.

(e) Suppose that $U^h(x^h) \geq U^h(\bar{x}^h)$. Explain clearly why $p\cdot x^h \geq p\cdot \bar{x}^h = p\cdot \omega^h$.

(f) Define $\bar{x} = \sum_{h=1}^{H} \bar{x}^h$, $x = \sum_{h=1}^{H} x^h$ and $\omega = \sum_{h=1}^{H} \omega^h$. Explain why $p\cdot x > p\cdot \omega$.

(g) Hence show that there exists no feasible allocation that Pareto dominates the WE allocation.

4. State claims prices and financial markets

There are two assets. Asset 1 has a state dependent dividend vector $z_1 = (40,30,10,90)$. Asset 2 has a state contingent dividend vector $z_2 = (80,10,50,30)$. The states are equally likely. Each consumer has the same VNM utility function $v(x) = \ln x$.

(a) Show that the expected utility function of each consumer is homothetic.

(b) Suppose that there are insurance (i.e. state claims) markets. Solve for the WE state claims prices if $p_1 = 1$.

(c) What is the market value of each asset? What would be the value of a financially engineered asset with dividend vector $z_{SP} = (12, 4, 6, 12)$?

(d) What is a final consumption $\bar{x}^h$ of a consumer whose asset shares have a value equal to a fraction α^h of the total wealth in the economy?

(e) Suppose that there are no state claims markets but consumers can trade shares of assets 1 and 2 and the financially engineered asset. Discuss how a consumer h can achieve the consumption vector $\bar{x}^h$ by trading these assets.

(f) Suppose that there is no financially engineered asset. Could consumer h still achieve the consumption vector $\bar{x}^h$ by trading his initial shareholdings of assets 1 and 2.

5. Strategic quantity choices

There are two firms. They produce products that are imperfect substitutes. The demand price functions are $p_1 = 100 - 2q_1 - q_2$ and $p_2 = 55 - q_1 - 2q_2$. The unit cost of production for each firm is $c = 10$. Each firm chooses its output.

(a) What must be true for $\bar{q} = (\bar{q}_1, \bar{q}_2)$ to be Nash Equilibrium strategies?

(b) Solve for the Nah Equilibrium strategies of this game.

(c) If they were to collude, what would be the joint profit-maximizing outputs?

(d) Comment on the feasibility of collusion in this game.

Suppose instead there are three firms. The demand price functions are $p_1 = 100 - 2q_1 - q_2 - q_3$, $p_2 = 55 - q_1 - 2q_2 - q_3$ and $p_3 = 55 - q_1 - q_2 - 2q_3$. The unit cost of production for each firm is $c = 10$. Each firm chooses its output.

(e) Write down the necessary conditions for $\bar{q} = (\bar{q}_1, \bar{q}_2, \bar{q}_3)$ to be a Nash Equilibrium.

(f) Is there a Nash equilibrium of the form $\bar{q} = (\bar{q}_1, \bar{q}_2, \bar{q}_3) = (4x, x, x)$? If not, why not. If so what are the NE strategies?

6. Equilibrium bidding in a sealed high-bid auction

There are 2 buyers. Each buyer's value is continuously distributed on an interval $[0,1]$ in millions. The c.d.f. is

$$\Pr\{\tilde{v}_i \leq v_i\} = F(v_i)$$

Let $B(v)$ be the symmetric equilibrium bid function in a sealed high-bid auction for a buyer with value v and let $w(v)$ be the equilibrium win probability.

(a) Explain why the equilibrium payoff is $U(v) = w(v)(v - B(v))$.

(b) Explain why a buyer's equilibrium win probability is $w(v) = F(v)$.

(c) Explain carefully why a buyer's equilibrium marginal payoff is $U'(v) = w(v)$.

(d) Hence solve for the equilibrium payoff if $F(v) = 2v - v^2$.

What is the maximum payoff $U(1)$ and hence the maximum bid $B(1)$?

(e) Solve also for the equilibrium bid function $B(v)$.

(f) Explain how to solve for the new equilibrium bid function if the seller sets a reserve price $r = \frac{1}{2}$.