John Riley Third Draft 11 October 2016

Econ 401A: Microeconomic Theory[1]

Homework set 2

Due date: Tuesday October 18. I will have extra office hours on Monday October 17 (1:00 PM to 3:00PM) in case you need some tips on how to proceed.

1. Constant returns to scale production function

The production set of a firm is

$$S = \{(z_1, z_2, q) \mid 16 z_1^3 z_2 - q^4 \geq 0\}.$$

where $z = (z_1, z_2)$ is the input vector and q is the output of the firm.

(a) Show that if $(z_1, z_2, q) \in S$ then $(\theta z_1, \theta z_2, \theta q) \in S$ for all $\theta > 0$.

(b) If the input prices are $r = (r_1, r_2)$ and the manager has a budget of B solve for the maximum output as a function of the budget and the input prices.

(c) Hence solve for the minimum cost of producing q units.

(d) What is the average and marginal cost of the firm?

(e) What must the output price $p = p(r_1, r_2)$ be in an economy in which this firm is producing?

2. Three commodity economy

Commodities 1 and 2 are used as input in the production of commodity 3. The aggregate production function is

$$q = 2 z_1^{3/4} z_2^{1/4} .$$

The endowment is $\omega = (\frac{64}{3}, 32, 0)$. Consumer $h,\ h = 1,...,H$ has a Cobb-Douglas utility function

$$U(x^h) = (x_1^h)(x_2^h)(x_3^h)^4 .$$

(a) Solve for the input vector z^* that maximizes the utility of the representative consumer.

HINT: Transform the utility function into something easier.

[1] My thanks to Allen for pointing out an error in question 3. I hope he will let me know if I am still wrong.

(b) Hence solve for the consumption of the representative consumer $x^* = (x_1^*, x_2^*, x_3^*)$

(c) Use your answer to question 1 to explain why, if firms are price-takers, the equilibrium profit in this three commodity economy must be zero.

(d) Let the price vector be $p = (p_1, p_2, p_3)$. Suppose that the price of commodity 3 is 1. Consider the maximization problem of the firm. Use the FOC to determine the input prices (p_1, p_2) if (z_1^*, z_2^*, q^*) is profit-maximizing.

(e) At these prices will the representative consumer choose the consumption vector x^*? Explain briefly.

3. Elasticity of substitution in a two commodity economy

A consumer has a utility function $U(x_1, x_2)$. Let $M(p, \bar{U})$ be the smallest income for which the consumer's utility is at least $\bar{U}$.

(a) Assuming that the solution, $x^c(p, \bar{U}) >> 0$, explain why $MRS(x^c) = \dfrac{p_1}{p_2}$.

(b) Show that if $U(x_1, x_2) = a_1 x_1^{1/2} + a_2 x_2^{1/2}$, then

$$\frac{x_2^c}{x_1^c} = \left(\frac{a_2}{a_1}\right)^2 \left(\frac{p_1}{p_2}\right)^2 .$$

(c) Hence show that $\sigma = \mathcal{E}(\frac{x_2}{x_1}, p_1) = 2$

(d) Solve for $\dfrac{x_2^c}{x_1^c}$ if (i) $U(x_1, x_2) = a_1 x_1^{1/3} + a_2 x_2^{1/3}$ (ii) . $U(x_1, x_2) = (a_1 x_1^{-1} + a_2 x_2^{-1})^{-1}$. In each case solve for $\sigma = \mathcal{E}(\frac{x_2}{x_1}, p_1)$

(e) In each of the cases in (d) depict the level set $U(x) = U(1,1)$.