Econ 401A Microeconomic Theory

Homework 1

You are encouraged to work on homework in groups. But you must submit your own answer. To maximize your own learning you should not get too much help! If you work a lot with a particular group you should indicate the names of the group on the front page of your homework.

Exercise 1: Monopoly profit maximization

A firm selling a vector of commodities x has revenue is $R = f(x)$. To produce x the firm needs $b = g(x)$ units of a single input. The market price of the input is λ .

(a) Write down the profit maximization problem. Hence obtain the FOC for profit maximization assuming that the solution is $\bar{x} >> 0$.

Suppose that there are two commodities. The demand functions are

$p_1(x_1) = 180 - x_1$ and $p_2(x_2) = 320 - 4x_2$. The input requirement is $g(x) = 2x_1^{\ 2} + \frac{3}{2}x_2^{\ 2}$.

(b) Solve for the profit maximizing outputs if the input price is λ . Hence solve for the firm's demand for the input $b(\lambda)$, as a function of the input price. Explain why this is strictly positive for all $\lambda > 0$.

(c) If the input price is 4 show that that the demand is $\bar{b} = 584$.

(d) With $\lambda = 4$, draw the profit-maximizing lines in a neat figure with x_1 and x_2 on thee axes. (See lecture 1). Hence explain why the point of intersection, $\bar{x}$ is indeed the profit maximizing choice.

Exercise 2: Monopoly profit maximization with an initial stock of the input

The data is the same as in exercise 1 and $\lambda = 4$. The firm has $\bar{b} = 584$ units of the input in stock. Will it wish to purchases additional units at the price of 4? Will it wish to sell some of these units at this price?

Exercise 3: Constrained maximization

A firm selling a vector of commodities x has revenue $R = f(x)$. To produce x the firm needs $b = g(x)$ units of a single input. The firm has a fixed supply of $\bar{b}$ units. Its goal is to maximize its revenue utilizing this stock. Formally it seeks to solve the following problem

$$\underset{x \geq 0}{Max}\{ f(x) \mid g(x) \leq \bar{b} \} .$$

The functions $f(x)$ and $g(x)$ are the same as in exercises 1 and 2. Also $\bar{b} = 584$

(a) If the firm could purchases or sell additional units at a price of $\lambda = 4$ would it do so?

(b) Appealing to your answers to Exercises 1 and 2, what is the solution to the constrained maximization problem?

Exercise 4: Profit maximization

$p_1(q_1) = 56 - q_1$, $p_2(q_2) = 104 - 2q_2$, $C(q) = q_1^{\,2} + 4q_1q_2 + 4q_2^{\,2}$.

(a) Solve for the maximizing $q_1 = m_1(q_2)$ for any q_2 and for the maximizing $q_2 = m_2(q_1)$ for any q_1 .

(b) What is the intersection point $\bar{q}$?

(c) Depict the maximizing lines in a neat figure and hence explain whether or not the intersection point is the profit-maximizing output vector.

Exercise 5: Consumer choice with a budget constraint

$U(x) = \ln(10 + x_1) + 2\ln x_2 + 3\ln x_3$, $p_2 = p_3 = 1$.

(a) Solve for the utility maximizing consumption choice if $(p_1, I) = (1, 60)$

(b) Solve again if $(p_1, I) = (1, 28)$

(c) For what value of p_1 and I are all three commodities purchased?

HINT: Show that if $\bar{x} >> 0$, then the FOC imply that

$$\frac{1}{p_1(10 + x_1)} = \frac{2}{p_2x_2} = \frac{3}{p_3x_3}$$

Exercise 6: Numerical analysis of consumer choice

Use Solver to answer (a), (c) and (f).

$U(x) = 8x_1^{\,1/2} + 3x_2^{\,2/3}$ $p = (1,1)$.

(a) Solve numerically for the consumer's choice if $I = 128$.

(b) Plug this into the necessary conditions and so confirm that they are satisfied.

(c) Use Solver to show how the consumption ratio varies as income rises.

(d) Draw some level sets depicting your results.

(e) In economic terms, why is this the case?

(f) Is the result true for other prices?