Due next Thursday before class.

Based on questions today I have made some minor changes. These are marked in red. Do not submit your spread-sheet. I have added a question that can be quickly answered using Solver. Just submit the results of the numerical maximization.

Note that question 3 is very similar to the question you considered in the TA session. I have tried to organize the question so that the method of approach is clear.

1. Profit maximization

A firm has a cost function $C(q) = 40q_1 + 60q_2 + (2q_1 + 5q_2)^2 - q_2^2$.

The firm is a price take in output markets. The price vector is $p = (168, 372)$.

(a) Solve for the unique critical point $\bar{q} >> 0$ of the profit function.

(b) Explain why this is not the profit-maximizing output vector.

(c) Is $\bar{q}$ a local maximum?

(d) What output vector does maximize the profit of the firm.

(e) Use Solver in Excel to solve the maximization problem numerically. Then choose some number b between 2 and 10 (not necessarily an integer and solve numerically if the cost function is

$$C(q) = 40q_1 + 60q_2 + (2q_1 + 5q_2)^2 + bq_2^2$$

How does your numerical answer change when b rises?

Note: You are not expected to do part (e) analytically. Simply submit two values of b and the associated outputs.

2. Output maximization

A firm has production function $q = F(z) = (\frac{1}{z_1} + \frac{1}{z_2})^{-1}$ **.**

(a) Show that if inputs are scaled up by θ , then output is also scaled up by θ , i.e. $F(\theta z) = \theta F(z)$.

(b) The input price vector is $p >> 0$. The manager is asked to maximize output given a budget of B dollars. Explain why the solution to this maximization problem is also the solution to the following simpler maximization problem

$$\underset{z\geq 0}{Max}\{-\frac{1}{z_1}-\frac{1}{z_2}\mid B-p\cdot z\geq 0\}$$

(c) Use the Lagrange method for the simpler problem and show that $p_1 z_1 = \frac{p_1^{1/2}}{\lambda^{1/2}}$. (By an identical argument it follows that $p_2 z_2 = \frac{p_2^{1/2}}{\lambda^{1/2}}$.)

(d) Solve for $\lambda^{1/2}$ and hence show that $\frac{1}{z_1} = \frac{p_1^{1/2}(p_1^{1/2}+p_2^{1/2})}{B}$

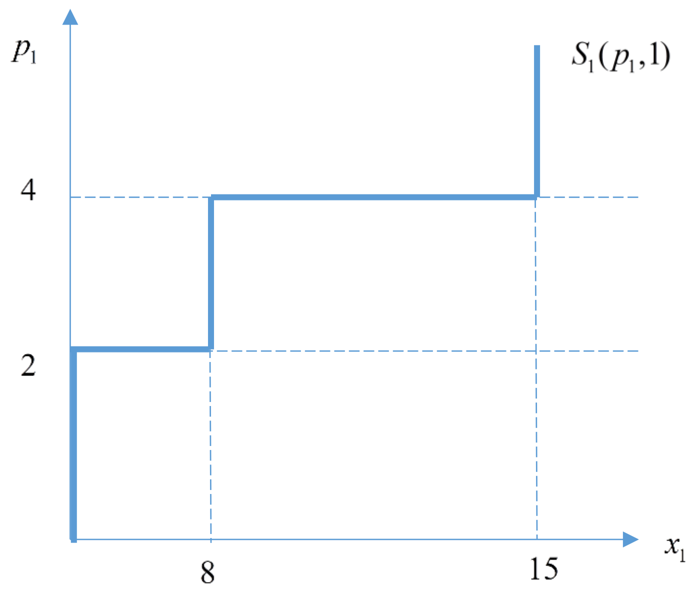
(e) Solve also for $\frac{1}{z_2}$. Hence obtain an expression for maximized output

3. Walrasian equilibrium

Alex has an endowment $\omega^A = (8,8)$ and Bev has an endowment of $\omega^B = (7,12)$. Alex likes each unit of commodity 1 twice as much as each unit of commodity 2. Bev likes each unit of commodity 1 four times as much as each unit of commodity 2.

(a) Explain why the optimal choice for each consumer is not affected if the price vector changes from $p = (p_1, p_2)$ to $\theta p = (\theta p_1, \theta p_2)$ for any $\theta > 0$. Thus we can always "normalize" by choosing $\theta = \frac{1}{p_j}$ so that the price of commodity j is equal to 1.

(b) Suppose that $p_2 = 1$. Explain why the market supply of commodity 1, $S_1(p_1, 1)$ is as depicted below.



(c) For what price of commodity 1 will (i) neither (ii) one (iii) two consumers have a strictly positive demand for commodity 1.

(d) Fully determine the market demand, $D_1(p_1,1)$ for every price p_1 and depict it in a separate figure. Explain any horizontal segments.

(e) What is the equilibrium price of commodity 1.

(f) Let $x_j(p) = x_j^A(p) + x_j^B(p)$ be total demand for commodity j given price vector p and let $\omega = \omega^A + \omega^B$ be the total endowment vector. Write down the budget constraints and hence show that

$$p_1(x_1(p) - \omega_1) + p_2(x_2(p) - \omega_2) = 0$$

Use this result to show that if supply equals demand in the market for commodity 1, then supply must also equal demand in the market for commodity 2.

4. Equilibrium trades in an exchange economy

Alex has a utility function $U^A(x^a) = x_1^a x_2^a$ and Bev has a utility function $U^B(x^b) = -\frac{1}{x_1^b} - \frac{1}{x_2^b}$.

(a) Explain why the slope of a level set at x is $-\frac{\partial U^h}{\partial x_1}(x) / \frac{\partial U^h}{\partial x_2}(x)$. Why is $\frac{\partial U^h}{\partial x_1}(x) / \frac{\partial U^h}{\partial x_2}(x)$ called the consumer's marginal rate of substitution?

(b) Suppose both consumers have the same consumption bundle $\overline{x}$.

Show that $MRS^a(\overline{x}) = MRS^b(\overline{x})$ if $\overline{x}_2 = \overline{x}_1$. i.e. the consumption bundle is on the 45° line.

Who has the higher MRS at if (i) $\overline{x}_2 > \overline{x}_1$ (ii) $\overline{x}_2 < \overline{x}_1$?

You might try drawing the level sets of Alex and Bev through $\overline{x} = (2,1)$ in a single diagram.

(c) Suppose $\omega^a = \omega^b = (\alpha, \alpha)$ so that the endowment **is on** the 45° line.

Explain why demand cannot be equal to supply if $p_1 \neq p_2$. What if $p_1 = p_2$?

(d) Suppose $\omega^a = \omega^b$ and the endowment is above the 45° line (more endowment of commodity 2.)

In the Walrasian Equilibrium, which consumer will sell commodity 1?

(e) Suppose $\omega_2^a / \omega_1^a > \omega_2^b / \omega_1^b$. In the Walrasian Equilibrium, is it clear which consumer will sell commodity 1?

Remark: In parts (d) and (e) you are not expected to solve for the WE price vector.