Homework 2+

1. Pricing attributes

Consider a simple exchange economy with three commodities. A unit of each commodity contains attributes valued by consumers (protein, carbohydrates, etc. There are four attributes. The aggregate endowment in this economy is $\omega=(16,8,32)$. The levels of each attribute are listed in the matrix $[a_{ij}]$ below.

a_{ij}	Commodity j		
Attribute i	Commodity 1	Commodity 2	Commodity 3
Attribute 1	1	2	1
Attribute 2	3	2	0
Attribute 3	1	2	0
Attribute 4	2	4	1

(a) What is the total endowment $\bar{z}_i$ of each attribute?

Let z_i be consumption of attribute i . Then if a consumer purchases commodity vector x her consumption of attribute i is

$$z_i(x)=a_{i1}x_1+a_{i2}x_2+a_{i3}x_3 \; .$$

Then her consumption of attribute 1 is $z_1(x)=1x_1+2x_2+1x_3$

Consumers purchase commodities because of their attributes. Each consumer has the same utility function $U(z)=4\ln z_1+2\ln z_2+4\ln z_3+6\ln z_4$.

(b) Explain the two different ways of solving for the equilibrium market prices.

(c) Consider virtual markets for the four attributes. Solve for the equilibrium shadow prices of the four attributes. Choose the smallest possible integers.

(d) Use these shadow prices to solve for the commodity prices.

(e) Convert the problem into a standard problem with utility function $u(x)$ and show that the prices in part (d) are indeed Walrasian Equilibrium commodity prices.

2. Walrasian Equilibrium

There are three consumers, each with a utility function of the form $U^h(x^h)=u(n^h+x^h)$ where $u(\cdot)$ is homothetic. Consumer h has an endowment of ω^h . One way to interpret this model is that consumer

h has a tradable endowment of ω^h and also a non-tradable endowment of n^h . Under this interpretation, consumer h has a total consumption of $t^h = n^h + x^h$.

(a) Explain why the budget constraint for total consumption t^h can be written as follows:

$$p \cdot t^h \leq p \cdot (\omega^h + n^h) \ .$$

Hence the choice of consumer h is $\bar{t}^h$ that solves

$$\underset{t^h \geq n^h}{Max}\{u(t^h) \mid p \cdot (\omega^h + n^h) - p \cdot t^h \geq 0\}, \ h = 1,...,H \qquad (1.1)$$

Note that the lower bound for total consumption is now n^h as these are non-tradable. Except for this change, the economy looks very like the representative consumer economy.

This suggests a possible short-cut. Consider the relaxed problem (1.2) below and solve for the WE price vector.

$$\underset{t^h \geq 0}{Max}\{u(t^h) \mid p \cdot (\omega^h + n^h) - p \cdot t^h \geq 0\}, \ h = 1,...,H \ . \qquad (1.2)$$

In the relaxed model the WE prices can be obtained by considering a representative consumer.

If the solution satisfies the constraints $\bar{t}^h \geq n^h$, then these are WE prices for the economy with non-tradable commodities.

(b) Suppose that $U^1(x^1) = v(1 + x_1^1) + v(x_2^1)$, $U^2(x^2) = v(3 + x_1^2) + v(x_2^2)$, $U^3(x^3) = v(x_1^3) + v(x_2^3)$

where $v(t^h)$ is a strictly concave function.

Endowments $\omega^1 = (4,3), \ \omega^2 = (8,13), \ \omega^3 = (0,0)$.

Solve for the WE prices for the relaxed economy.

(c) Solve for the WE consumption vectors.

HINT: Each consumer purchases a fraction of the aggregate consumption bundle.

Confirm that the prices are WE prices for the model with non-tradable as well as tradable endowments.

(d) Suppose instead that

$$\omega^1 = (4,2), \ \omega^2 = (6,6), \ \omega^3 = (2,8) .$$

Repeat the analysis. Hence explain whether the approach again yields the WWE prices for the economy with non-tradable as well as tradable endowments.

(e) Suppose next that $v(t) = t^{1/2}$, the non-tradable endowments are unchanged and the tradable endowments are

$$\omega^1 = (3,40), \ \omega^2 = (9,24), \ \omega^3 = (0,0) .$$

Repeat parts (b) and (c).

3. Firm production functions and industry production functions

There are four firms in an industry. The four production functions are as follows:

$$q_1 = z_1^{1/2} \text{ , } q_2 = 2z_2^{1/2} \text{ , } q_3 = 2z_3^{1/2} \text{ , } q_4 = 4z_4^{1/2}$$

(a) If the total supply of inputs to the industry is $\overline{z}$, solve for the input vector $(\overline{z}_1, \overline{z}_2, \overline{z}_3, \overline{z}_4)$ that maximizes industry output.

HINT: From the Necessary conditions solve for each firms input as a function of λ . Then substitute back into the constraint $\sum_1^4 z_j = \hat{z}$.

(b) Show that the industry production function is $q = 5z^{1/2}$.

4. Walrasian Equilibrium with production (more detailed description)

Consider a two commodity economy. Commodity 1 is both consumed and used as an input to produce commodity 2. The endowment of commodity 1 is ω_1 and z_1 units are used in the production of commodity 2. The remainder of the endowment is consumed.

There are four firms with the production functions in question 3. It follows that the industry production function is $q = 5z_1^{1/2}$ The aggregate endowment vector is $\omega = (\omega_1, \omega_2) = (84, 0)$. Every consumer has the same utility function

$$U(x^h) = 4\ln x_1^h + 6\ln x_2^h,\ \ h = 1,...,H.$$

(c) Appeal to (b) to obtain an expression for utility as a function only of the total input z_1 used by the four firms. Then solve for the optimal input of commodity 1.

(d) What is the optimal consumption vector $\overline{x} = (\overline{x}_1, \overline{x}_2)$?

(e) At what price vector would the representative agent not wish to trade away from $\overline{x}$?

(f) What is the profit-maximizing input and maximized profit at these prices?