Homework 4 due Thursday December 7

Answer questions 1-3 and either 4A or 4B.

1. **Cost functions and output prices in a price taking economy**

$$q_A = (\tfrac{3}{2} z_1^A)^{2/3}(3z_2^A)^{1/3} = \frac{3}{2^{2/3}}(z_1^A)^{2/3}(z_2^A)^{1/3} \text{ ,}$$

$$q_B = (3z_1^B)^{1/3}(\tfrac{3}{2} z_2^B)^{2/3} = \frac{3}{2^{2/3}}(z_1^A)^{1/3}(z_2^A)^{2/3}$$

(a) Solve for the cost functions if the input price vector is $r = (r_1, r_2)$.

(b) If commodities A and B are both produced, show that in a price taking economy

$$\frac{p_A}{p_B} = (\frac{r_1}{r_2})^{1/3} .$$

Hence if the output price ratio $\frac{p_A}{p_B}$ rises, then $\frac{r_1}{r_2}$ rises.

(c) Suppose this is a two commodity economy. For what aggregate endowments of inputs, ω , is the production of commodity 1 more input 1 intensive?

(d) If the price of commodity B is 1 and the price of commodity A rises, show that one input price must fall and the other must rise.

(e) What is the elasticity of the input price ratio with respect to the change in the price of commodity A?

Remark: Some of the results in question 1 will be helpful in answering question 2.

2. **Two-input, two-output CRS economy**

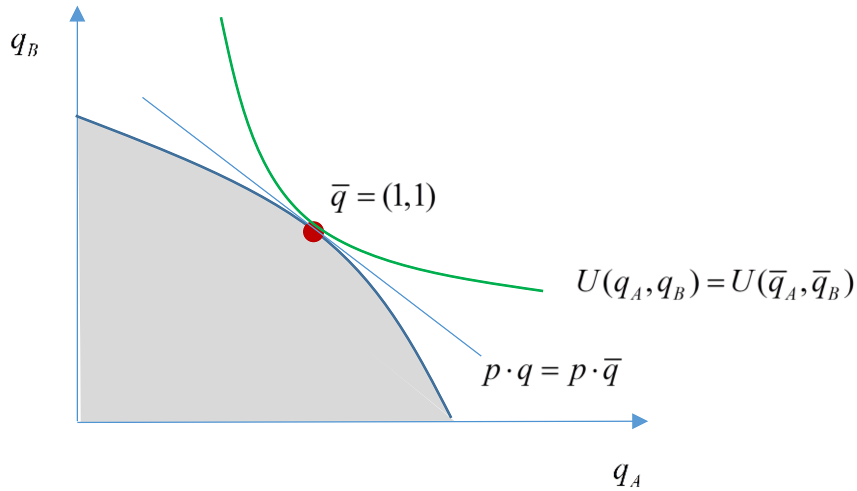
Consider an economy with the production functions in question 1. The aggregate supply of each input is 1. All consumers have the same logarithmic preferences $U^h = \ln q_A^h + \ln q_B^h$. Therefore we can consider a single representative consumer.

(a) Solve for the optimal allocation of inputs $\bar{z}^A = (\bar{z}_1^A, \bar{z}_2^A)$ and $\bar{z}^B = (\bar{z}_1^B, \bar{z}_2^B)$. Hence show that the optimal outputs are $\bar{q} = (1,1)$.

(b) Throughout we will normalize and assume that $p_B = 1$. What is the equilibrium price of commodity A. What is the equilibrium input price ratio? What is the equilibrium MRTS in bothe indistries?

(c) What are the equilibrium input prices?

Remark: The equilibrium is depicted below. The shaded region is the set of feasible ouput vectors. At the optimum, the steepness of the boundary is equal to the steepness of the level set and hence the steepness of the revenue line $p \cdot q = p \cdot \overline{q}$.



(d) Initially transportation costs make international trade unprofitable. Suppose that transportation costs suddenly drop to zero and that the world prices of the two outputs are both 1. Explain why this would have no effect on this economy.

(e) Suppose that preferences change so that we can no longer use the representative agent. Would there still be on effect on equilibrium input prices? Might there be trade?

Another economy has the same aggregate inputs and the same technology. However consumers have a stronger preference for commodity B. Preferences can be represented by the utility fucntion

$$U = \ln q_A + 2 \ln q_B \ .$$

(f) Use the figure above to compare the equilibrium price of commodity 1 in the two economies if there is no international trade. Given this conclusion, compare the input price ratio and input prices with those in the economy of parts (a)-(c).

(g) Finally suppose that there are no barriers to trade. The world prices of commodities A and B are both 1. Explain why both countries will produce $\overline{q} = (1,1)$. How much will this second country export?

3. Price v. quantity competition

$$q_1 = 12 - p_1 + \tfrac{1}{2} p_2 \ , \ q_2 = 12 - p_2 + \tfrac{1}{2} p_1 \ ,$$

$$C_1(q_1) = 6q_1 \ , \ C_2(q_2) = 6q_2$$

Suppose firms compete by setting prices rather then quantities.

(a) Solve for the symmetric Nash Equilibrium prices and hence show that the equilibrium output of each firm is 6.

(b) Show that the demand functions can be inverted into the following two demand price fucntions

$p_1 = 24 - \frac{4}{3}q_1 - \frac{2}{3}q_2$ and $p_2 = 24 - \frac{4}{3}q_2 - \frac{2}{3}q_1$.

(c) Suppose firms compete by setting quantities. Solve for the symmetric NE quantities and hence show that the equilibrium output of each firm is lower and hence equilibrium prices are higher than with price competition.

4A. Symmetric equilibrium in a sealed high-bid auction with two identical items for sale

There are three buyers. There are two identical items for sale. Each buyer wishes to purchase one unit only. Each buyer's value is between zero and 1 (million). Each buyer's value is a random draw from a uniform distribution, i.e.,

$$F(\theta) = \Pr\{v_i \le \theta\} = \theta \ .$$

The buyers submitting the two highest bids each win an item and pay their own bid.

If all bid according to the same bid function $B(\theta)$, it is the buyers with the two highest values that win.

There are three ways in which buyer 1 can win:

(i) Buyer 1 has the highest value
(ii) Buyer 2 has a higher value and buyer 3 has a lower value
(iii) Buyer 3 has a higher value and buyer 2 has alower value

(a) Compute these probabilities and hence the win probbaility $w(\theta)$.

(b) Another way to compute the win probability is to compute that probaility that buyer 1 loses and then subtract this from 1. Compute this probaility in order to check your answer to (a).

(c) Expalin carefully why the equilibrium payoff $U(v_i) = \int_0^{v_i} w(\theta)d\theta$.

(d) Use this result to solve for the equilibrium bid function.

Suppose the seller sets a reserve price of $v_0 = \frac{1}{2}$.

(e) Explain why the equilbrium expected payoff of a buyer with value v_i is

$$U(v_i) = \int_{\frac{1}{2}}^{v_i} w(\theta)d\theta \ .$$

(f) Hence solve for the new equilibrium bid function.

4B. Entry in a symmetric quantity setting model

If there are n firms in the industry the market clearing price is

$p = 30 - q_1 - q_2 - ... - q_n$. The cost of production of firm i is $C_i(q_i) = 60 + q_i^2$.

(a) Depict the average and marginal cost curve of a firm.

(b) Suppose first that there are two firms in the industry. Solve for the Nash Equilibrium outputs and profits.

(c) Suppose next that firm 3 enters. Solve for the new NE outputs and profits.

(d) If the fixed cost of firm 3 were 64 rather than 60 would your answer change?

(e) Is there a Nash Equilibrium with 4 firms? Explain.