## Homework 4

### Due in class Tuesday November 29

### 1. WE in an economy with constant rturns to scale and identical homothetic preferences

Commodity a and b are produced using inputs $z_1$ and $z_2$. The production fucntions are

$$q_a = (z_1^a)^{1/3}(z_2^a)^{2/3} \text{ and } q_b = (z_1^b)^{2/3}(z_2^b)^{1/3}.$$

The aggregate endowment of the two inputs is $\omega = (\omega_1, \omega_2) = (1,64)$ . The WE price of commodity $b$ is $p_b = 1$ .

All consumers have the same utility function

$$U(q) = \ln q_a + \ln q_b .$$

(a) Show that to maximize utility $z^a = (\frac{1}{3}, \frac{128}{3})$ and so $z^b = (\frac{2}{3}, \frac{64}{3})$.

(b) Hence show that the WE input price ratio is $\frac{r_1}{r_2} = 64$ .

(c) Solve for the maximizing outputs $(\overline{q}_a, \overline{q}_b)$ and show that their ratio is $4$.

(d) Solve for the WE output price of commodity 1.

(e) Solve for the WE input prices.

Suppose that commodity a is produced in the north of the country and those who live in the north own the inputs used in its production. Commodity b is produced in the south of the country and those who live in the south own the inputs used in the south.

(f) Show that the total value of the regional endowments is equal.

(g) What is the value of the commodities shipped between the two regions?

BONUS QUESTIONS (worth 1 point each only)

(h) Suppose that inputs in both region are tripled so that $\omega^N = (1,128)$ and $\omega^S = (2,64)$. Is it true that the WE prices will be unaffected and that production and consumption will simply triple?

(i) Would the prices remain WE prices if the two regions split into independent countries but imposed no constraints on trade?

**2. Pricing game**

If a firm cannot quickly change its capacity, then the quantity setting model examined in class makes sense. But what if capacity can be easily changed. Then a firm can lower a price and still guarantee delivery, or raise a price and sell off unused capacity. Here is the data for the model examined in class.

$C_f(q_f) = c_f q_f$ where $c_1 = 4$ and $c_2 = 7$ .

Demands are as follows:

$q_1(p) = 60 - 20p_1 + 10p_2$ , $q_2(p) = 150 + 10p_1 - 20p_2$ .

Then the profit of firm $f$ is

$$\pi_f(p_f, q_f(p)) = p_f q_f(p) - C_f(q_f(p)) = (p_f - c_f) q_f(p)$$

(a) For each firm solve for the best response function $p_1 = b_1(q_2)$ and $p_2 = b_2(p_1)$ .

(b) Depict the response functions in a neat figure. What are the equilibrium prices in this game?

(c) If firm 2 sets a price of 7 (so that profit is zero), what is firm 1's best response? Plot this in the figure.

(d) Starting from this price pair, examine the adjustment process proposed by Cournot.

(e) Compare the equilibrium prices with those in the quantity setting game.

**3. Sealed high-bid auction with more than 2 buyers**

Suppose that there are three buyers and that buyer 2 and buyer 3 bid according to the linear strategy $B_i(\theta_i) = k\theta_i$ . Values are uniformly distributed on $\theta = [0,10]$ so that buyer $i$'s value is below $\theta_i$ with probability $F(\theta_i) = \frac{1}{10}\theta_i$

(a) Explain why buyer 1's best reponse will never exceed $10k$ .

(b) Obtain an expression for buyer 1's payoff for all bids.

(c) Show that his best reponse is $B_1(\theta_1) = Min\{\frac{2}{3}\theta_1, 10k\}$ .

(d) Hence solve for the equilibrium bidding strategies .

(e) Solve also for the equilibrium bidding strategies if there are 4 buyers.

**4. Sad loser auction**

There are two bidders. A single item is to be sold to the high bidder. Each bidder puts some money in an envelope. When all the envelopes have been submitted, they are opened and the person submitting the envelope with the most cash is declared the winner of the item. None of the cash subnitted is returned to any of the bidders.

(a) If the values are uniformly distributed on $\Theta = [0,100]$ so that $\Pr\{v_i \le \theta\} = \dfrac{\theta}{100}$ , solve for the equilibrium bid function.

Claim: The symmetric equilibrium bid function is a quadratic function i.e. $B(\theta_i) = k\theta_i^2$ . Therefore assume that bidder 2 is bidding according to this strategy and solve for the strategy that that maximizes the expected payoff to bidder 1. Use this to solve for the value of $k$ that leas to a symmetric equilibrium.

(b) With three bidders the equilibrium bid function is a cubic function, i.e. for some $\hat{k}$

$$B(\theta_i) = \hat{k}\theta_i^3 .$$

Is this statement true or false?