Econ 401A: Economic Theory Mid-term

Starting time: 9:30 End time 10:50. Attempt three (3) questions only. Each will be graded out of 10. There are lots of points for partial answers.

1. Labor supply

A worker has utility function $U(x_1, x_2) = x_1(x_2 + a)$ where x_1 is consumption of leisure hours and x_2 is consumption of the produced commodity. The hourly wage rate is w. The price of commodity 2 is p_2.

The worker has 24 available hours per day and can choose how many hours, z, in which to work. This determines her salary wz and hence her expenditure on commodity 2. Her endowment of commodity 2 is zero.

(a) Solve for the utility maximizing labor supply.

(b) Analyze income and substitution effects for this consumer as the wage rises. Are they reinforcing or opposing effects on her labor supply?

(c) What condition does the parameter a have to satisfy, in order for this worker to choose not to work?

Points (a)=6, (b)=3, (c)=1

2. Walrasian Equilibrium (WE) in an exchange economy

Consumer $h,\ h = 1,...,H$ has utility function $U(x_1^h, x_2^h) = \alpha_1 (x_1^h)^{1/2} + \alpha_2 (x_2^h)^{1/2}$. The aggregate endowment is $\omega = (100, 400)$.

(a) Explain why a WE price vector $p = (p_1, p_2)$ is also a WE price vector for the representative consumer economy.

(b) Use the representative consumer to solve for the WE price ratio.

(c) Reinterpret the model as two consumers in a two state economy. For a Pareto Efficient allocation, explain why the final consumption ratio for each consumer is equal to the aggregate endowment ratio.

(d) For a Walrasian Equilibrium allocation, is the final consumption ratio for each consumer equal to the aggregate endowment ratio?

Points (a)+(b)=7, (c)+(d)=3

3. Multi-product monopoly

The demand price function in market 1 is $p_1(q_1) = 60 - q_1$. In market 2 it is $p_2(q_2) = 90 - 2q_2$. The cost of production is

$$C(q) = K + (q_1 + q_2)^2 + 2q_2^{\,2} = K + q_1^{\,2} + 2q_1q_2 + 3q_2^{\,2}\ .$$

(a) Prove that the sum of concave functions is concave. (It is enough to provide a proof for two concave functions.)

(b) Is total profit $\pi(q_1, q_2)$ a concave problem if the constant $K = 0$? Explain.

(c) Solve for the profit-maximizing outputs if $K = 0$.

(d) Solve for the profit-maximizing outputs for all K .

Points (a)=2, (b)=2, (c)=5, (d)=1

4. **Walrasian Equilibrium in a three commodity model with production**

Commodity 1 is both consumed and used as an input in the production of commodity 3. Commodity 2 is poisonous so is not consumed. It is used only as an input in the production of commodity 3.

The production function of the representative firm is $q_3 = 8z_1^{\,1/2}z_2^{\,1/2}$.

The aggregate endowment is $\omega = (16, 16, 0)$.

Each consumer has the same utility function $U(x^h) = \ln x_1^h + 2\ln x_3^h,\ h = 1, ..., H$.

(a) Solve for the optimal z_1 for the representative consumer.

(b) Illustrate in a figure with x_1 (and z_1) on the horizontal axis and x_3 on the vertical axis.

(c) Let $p = (p_1, p_2, p_3)$ be a price vector that supports the production decision of the firm. If $p_2 = 1$ what are the other prices?

(d) What is the firm's profit?

(e) Are the three prices WE prices? Explain.

Points (a)=4, (b)=2, (c)=2, (d)=1, (e)=1